INSTITUTO DE FÍSICA DE PARTÍCULAS Y DEL COSMOS
IPARCOS
UNIVERSIDAD COMPLUTENSE DE MADRID

Ignazio Scimemi

q_T Drell-Yan spectrum in TMD formalism

LHC EW precision sub-group meeting (pT W/Z benchmarking)

11/09/2019

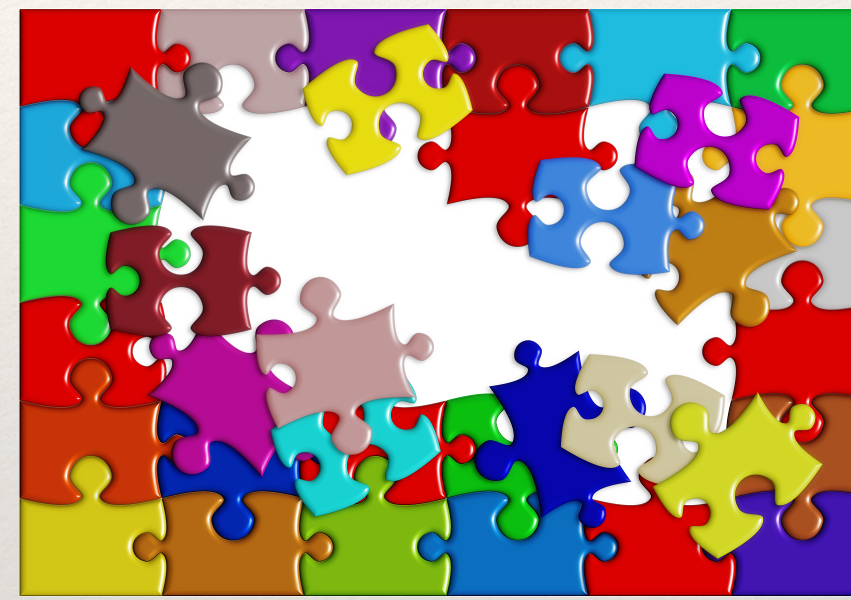
Artemide code and fits with A. Vladimirov

STRONG2020

Factorization

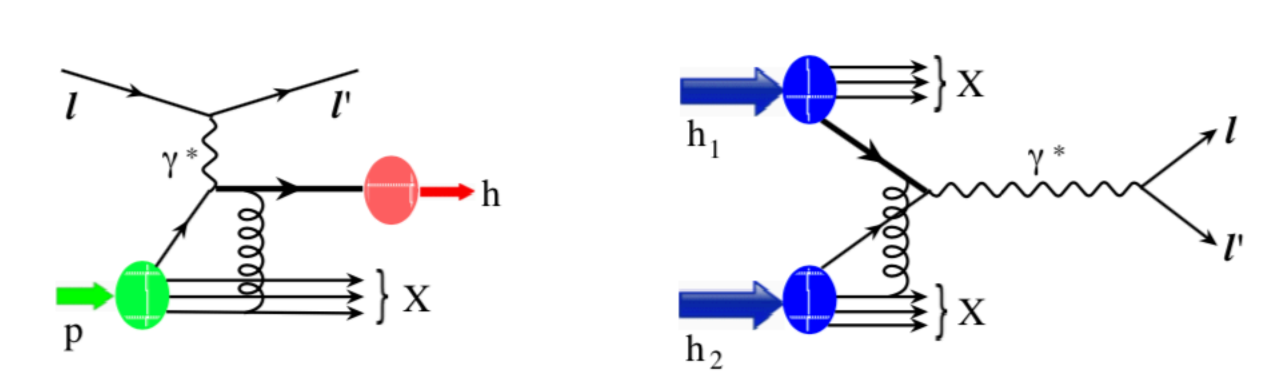
Outline

Implementation/coding



Outline

Extraction/statistics/LHC impact



Drell-Yan classical knowhow

Factorization!

$$\frac{d\sigma}{dQ^2} = \sum_{i,j=q,\bar{q},g} \int_0^1 dx_1\, dx_2\, \mathcal{H}_{ij}(x_1, x_2, Q^2, \mu^2)\, f_{i/P}(x_1, \mu^2)\, f_{j/\bar{P}}(x_2, \mu^2)$$

PDF: Parton Distribution Functions

PDF describe the probability of finding a parton "i" in the hadron "P":

We can define the probability of one parton in one hadron **independently** of the probability of the other parton.

The quark states are totally disentangled!

PDF: Parton Distribution Functions

All non-perturbative QCD information is encoded in PDFs

Is this true for all processes that we know?

No,...

Do exist similar processes with different functions which encode a non-perturbative information different from PDFs?

Trying to extend our knowledge naively

Suppose to measure more differential cross sections, can we write (Collins, Soper, Sterman '80)

$$\frac{d\sigma}{dQ^2 dq_T dy} = \sum_q \sigma_q^\gamma H(Q^2,\mu^2) \int \frac{d^2\mathbf{b}}{4\pi} e^{-i\mathbf{q_T}\cdot\mathbf{b}} \Phi_{q/A}(x_A,\mathbf{b},\mu)\Phi_{q/B}(x_B,\mathbf{b},\mu)?$$

Unfortunately we cannot be so naive

$$\Phi_{q\leftarrow q}(x,\mathbf{b},\mu) = \delta(1-x) + a_s 2C_F \mathbf{B}^\varepsilon \Gamma(-\varepsilon)\left(p_{qq}(x) - \varepsilon(1-x) - 2\delta(1-x)\lambda_\delta\right) + \ldots$$

$p_{qq} =$ splitting function

$\mathbf{B} = \mathbf{b}^2/4$

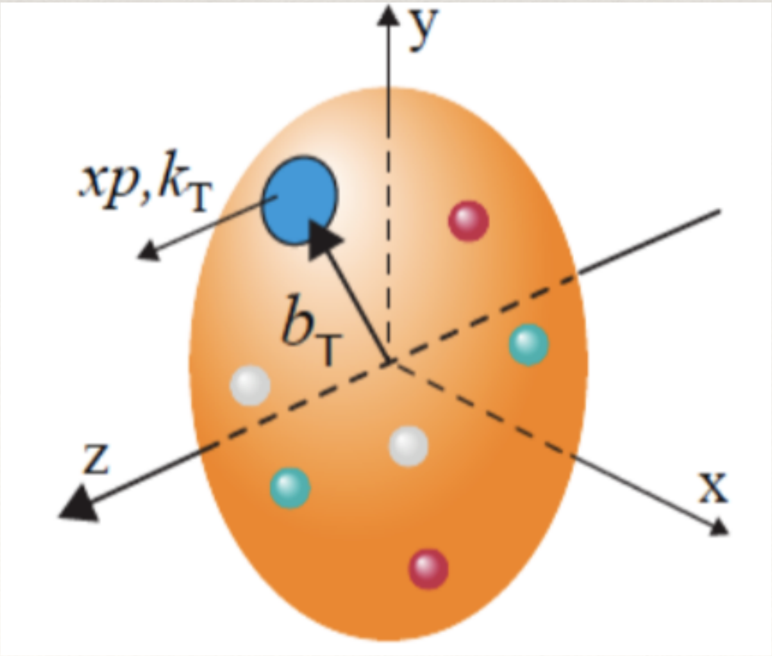
$\lambda_\delta = \ln \delta^+/p^+$

One does not achieve a full separation of UV and IR singularities!

*In standard QCD calculations this new **rapidity divergent** part cancels in the whole cross section computation: it seems that at small transverse momentum the initial states are **entangled** …*

Factorization theorem for TMDs

(J.C. Collins 2011, M.G. Echevarria, A. Idilbi, I. Scimemi, 2012, Chiu et al. 2012, T. Becher, M. Neubert 2010, J. Gaunt 2014, V. Vladimirov 2017, ..)

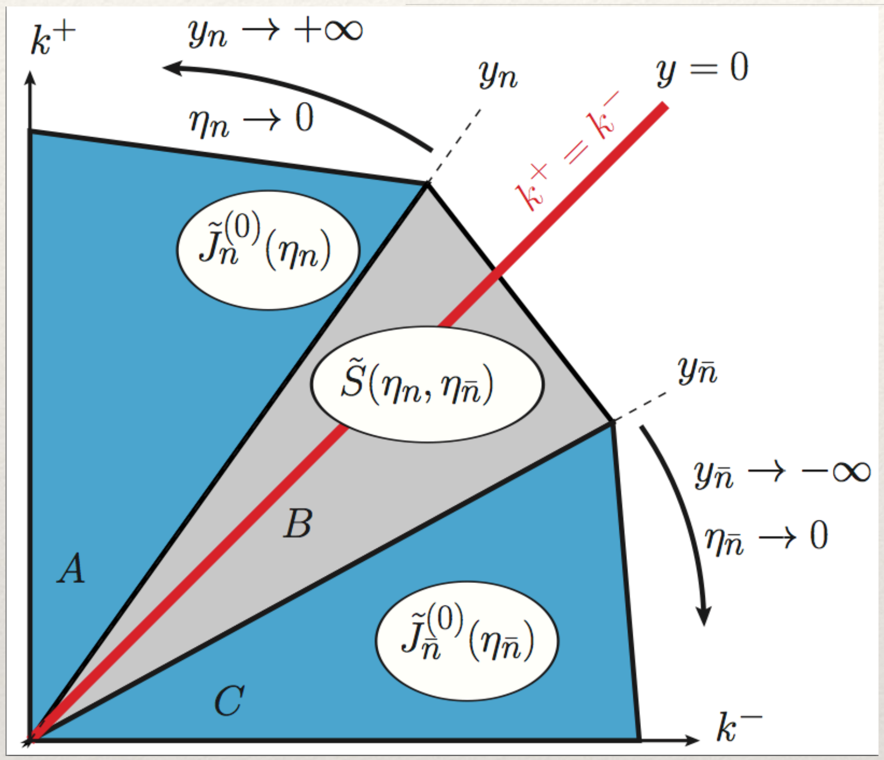


Factorization theorem basics

TMDPDF-1 TMDPDF-2

- PDF, Soft Factor have "rapidity divergences"
- Soft Factor mixes 2 TMD-PDF through rapidity divergent pieces ..

Factorization theorem basics: siDIS case



$$d\sigma \sim \int d^4x e^{iqx} \sum_X \langle h_1 | J^\mu(x) | X, h_2 \rangle \langle X, h_2 | J^\nu(0) | h_1 \rangle$$

$$d\sigma \sim \int d^2 b_T e^{-iq_T b_T} H(Q^2) \Phi_{h_1}(z_1, b_T) S(b_T) \Delta_{h_2}(z_2, b_T) + Y$$

TMDPDF — Soft Factor — TMDFF

- PDF, Soft Factor, FF have "rapidity divergences"
- Soft Factor mixes PDF and FF through rapidity divergent pieces ..
- We have the **splitting of rapidity singularities in the Soft Factor**:

$$S(b_T) = \sqrt{S(b_T, \zeta)} \sqrt{S(b_T, \zeta^{-1})} \quad \Rightarrow \quad d\sigma \sim H(Q) \int d^2 b_T e^{-iq_T b_T} F(x; b_T) D(z; b_T)$$

Each TMD is now free of rapidity singularity: nice renormalizable non-perturbative object

TMD factorization in nutshell

.. for DY and heavy boson production we have (Collins 2011, Echevarria, Idilbi, Scimemi (EIS) 2012)

$$\frac{d\sigma}{dQ^2 dq_T dy} = \sum_q \sigma_q^\gamma H(Q^2,\mu^2) \int \frac{d^2\mathbf{b}}{4\pi} e^{-i\mathbf{q_T}\cdot\mathbf{b}} \Phi_{q/A}(x_A,\mathbf{b},\zeta_A,\mu)\Phi_{q/B}(x_B,\mathbf{b},\zeta_B,\mu)$$

$$\sqrt{\zeta_A \zeta_B} = Q^2$$

…and similar formulas are valid for SIDIS (EIC) and hadron production in e+e- colliders

The renormalization of rapidity divergences is responsible for a new rapidity evolution scale

We have new non-perturbative effects which cannot be included in PDFs.
We have a new renormalization scale for each TMD

TMD factorization in nutshell

.. for DY and heavy boson production we have

(Collins 2011, Echevarria, Idilbi, Scimemi (EIS) 2012,Vladimirov 2017)

$$\frac{d\sigma}{dQ^2 dq_T dy} = \sum_q \sigma_q^\gamma H(Q^2,\mu^2) \int \frac{d^2\mathbf{b}}{4\pi} e^{-i\mathbf{q_T}\cdot\mathbf{b}} \Phi_{q/A}(x_A,\mathbf{b},\zeta_A,\mu)\Phi_{q/B}(x_B,\mathbf{b},\zeta_B,\mu)$$

$$\sqrt{\zeta_A \zeta_B} = Q^2$$

The renormalization of the rapidity divergences is responsible for the a new resummation scale: **2-D evolution**

THE CASE OF UNPOLARIZED TMDS:
THE PERTURBATIVE CALCULABLE PART OF UNPOLARIZED TMDS IS KNOWN AT NNLO!

WHICH SCALE PRESCRIPTION ALLOWS AN OPTIMAL EXTRACTION OF TMD'S?

WHAT IS THE RANGE OF VALIDITY OF THE TMD FACTORIZATION THEOREM?

DO LHC DATA HAVE AN IMPACT ON TMD EXTRACTION?

Cross section and TMD structure

Lepton tensor cuts

E.w. factors

Hard Coefficient

$$\frac{d\sigma}{dQ^2 dy d(q_T^2)} = \frac{4\pi}{3N_c}\frac{\mathcal{P}}{sQ^2}\sum_{GG'} z_{ll'}^{GG'}(q)\sum_{ff'} z_{ff'}^{GG'}|C_V(q,\mu)|^2\int\frac{d^2\vec{b}}{4\pi}e^{i(\vec{b}\vec{q})}$$

$$\times F_{f\leftarrow h_1}(x_1,\vec{b};\mu,\zeta)F_{f'\leftarrow h_2}(x_2,\vec{b};\mu,\zeta) + Y$$

Only small qT data

Evolution factor

$$F_{f\leftarrow h}(x,\mathbf{b};\mu_f,\zeta_f) = R^f[\mathbf{b};(\mu_f,\zeta_f)\leftarrow(\mu_i,\zeta_i)]F_{f\leftarrow h}(x,\mathbf{b};\mu_i,\zeta_i)$$

$$R^f[\mathbf{b};(\mu_f,\zeta_f)\leftarrow(\mu_i,\zeta_i)] = \exp\left[\int_P\left(\gamma_F^f(\mu,\zeta)\frac{d\mu}{\mu} - \mathcal{D}^f(\mu,\mathbf{b})\frac{d\zeta}{\zeta}\right)\right]$$

ζ-prescription

2-D TMD evolution

COUPLED EVOLUTION OF TMD ...

TMD (standard) anomalous dimension

$$\mu^2 \frac{d}{d\mu^2} F_{f\leftarrow h}(x,b;\mu,\zeta) = \frac{\gamma_F(\mu,\zeta)}{2} F_{f\leftarrow h}(x,b;\mu,\zeta)$$

$$\zeta \frac{d}{d\zeta} F_{f\leftarrow h}(x,b;\mu,\zeta) = -\mathcal{D}(\mu,b) F_{f\leftarrow h}(x,b;\mu,\zeta)$$

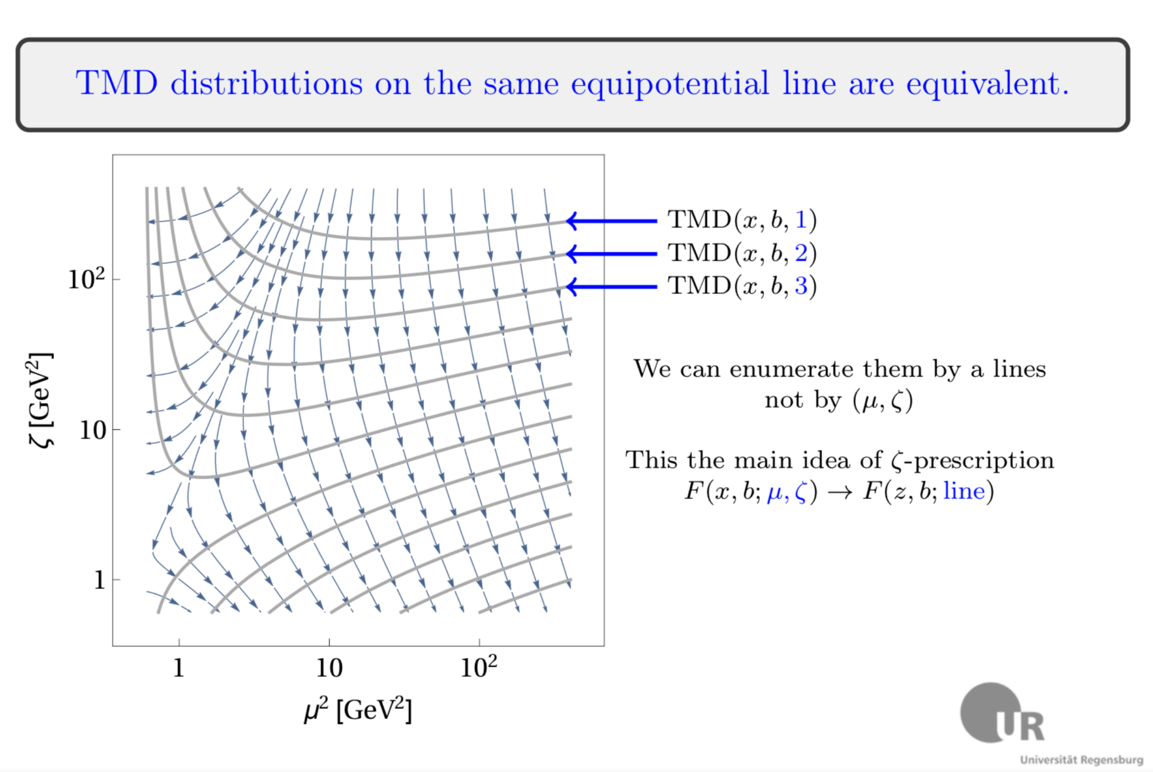
TMD rapidity anomalous dimension

Collinear overlap

$$\zeta \frac{d}{d\zeta} \gamma_F(\mu,\zeta) = -\Gamma(\mu)$$

$$\mu \frac{d}{d\mu} \mathcal{D}(\mu,b) = \Gamma(\mu)$$

Double-scale evolution, perturbative series truncation, zeta-prescription

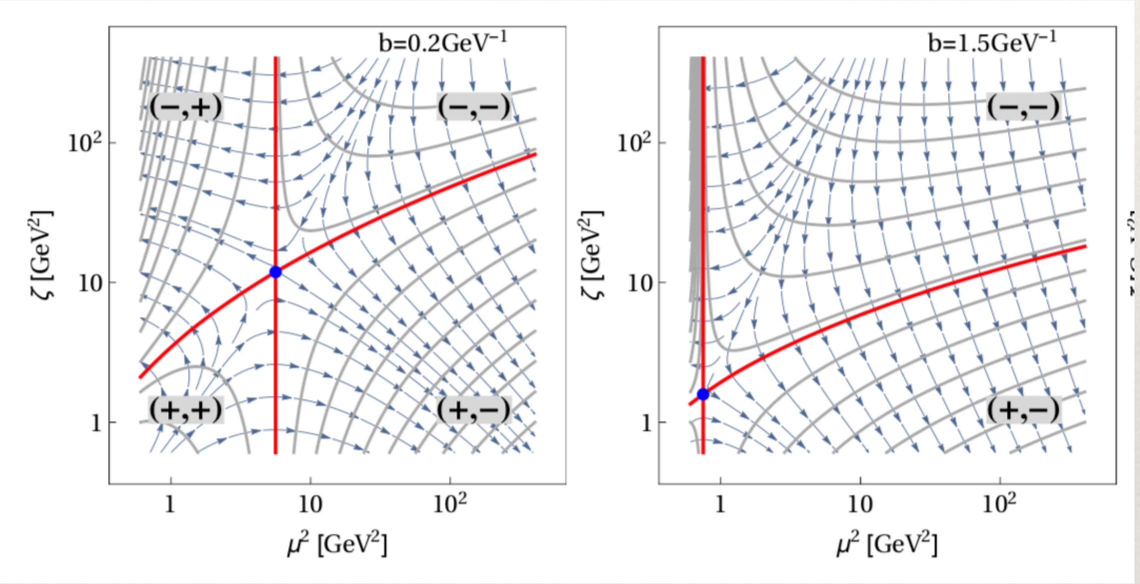


The truncation of the perturbative series spoils the path-indepence. This is recovered using the zeta-prescription.

Optimal TMD definition

Null evolution curves have a unique saddle point!!

$$\mathcal{D}(\mu_{\text{saddle}}, b) = 0$$
$$\gamma_F(\mu_{\text{saddle}}, \zeta_{\text{saddle}}) = 0$$



The saddle point moves with *b*.

- *For every value of b one can find a saddle point. We prevent the reaching of Landau pole.*
- *The D-function should include also a non-perturbative part.*
- *It is possible to provide a zeta-prescription including a perturbative and non-perturbative part. (I. Scimemi, A. Vladimirov, 1803.11089, A. Vladimirov 1907.10356)*

TMD evolution kernel: perturbative and non-perturbative parts

$$F_{f\leftarrow h}(x,\boldsymbol{b};\mu,\zeta) = R^f[\boldsymbol{b};(\mu,\zeta)\to(\mu_0,\zeta_{\mu_0}(\boldsymbol{b}))]F_{f\leftarrow h}(x,\boldsymbol{b})$$

in optimal prescription:

- **F(x,b) is scaleless**
- **No dependence of mu_0 (saddle point)**

in optimal prescription:

- **Easy algebraic form of the evolution kernel**

$$R[\boldsymbol{b};(\mu,\zeta)\to(\mu,\zeta_\mu(\boldsymbol{b}))] = R[\boldsymbol{b};(\mu,\zeta)] = \left(\frac{\zeta}{\zeta_\mu(\boldsymbol{b})}\right)^{-\mathcal{D}(\mu,\boldsymbol{b})}$$

$$\mathcal{D}^f(\mu, b^*(\mathbf{b})) = \mathcal{D}^f_{res}(\mu, b^*(\mathbf{b})) + \mathcal{D}^f_{NP}(\mathbf{b})$$ Resummed and non-perturbative part

$$b^*(\mathbf{b}) = \sqrt{\frac{B^2_{NP}\mathbf{b}^2}{B^2_{NP}+\mathbf{b}^2}}$$

Avoid Landau pole

$$\mathcal{D}^f_{NP}(\mathbf{b}) = c_0|\mathbf{b}|b^*(\mathbf{b})$$

$$f = q,\ g$$

Non-perturbative models are chosen such that the perturbative part were valid on the largest interval

Remember: Evolution is Universal

TMD: perturbative and non-perturbative parts

Asymptotic limit of TMD for large transverse momentum

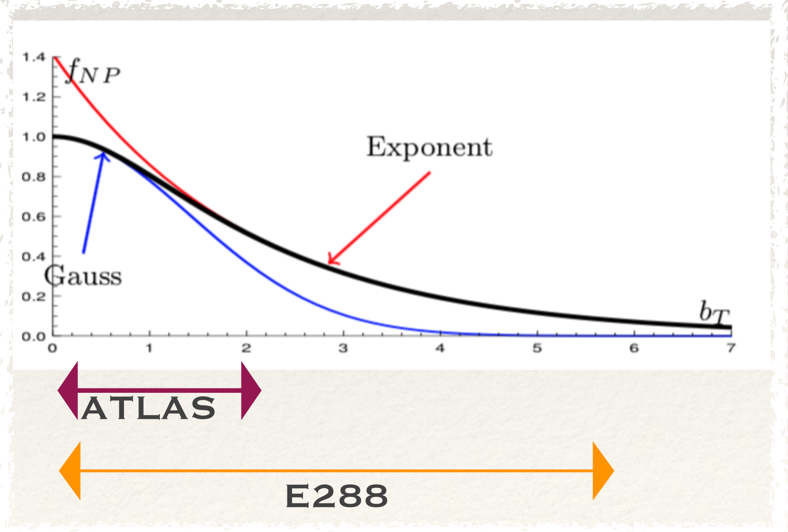
$$\mu = \frac{2e^{-\gamma_E}}{|\boldsymbol{b}|} + 2\,\mathrm{GeV}$$

$$F_{f\to h}(x,\boldsymbol{b}) = \sum_{f'} \int_x^1 \frac{dy}{y} C_{f\leftarrow f'}\left(\frac{x}{y}, \ln\left(\boldsymbol{b}^2\mu^2\right)\right) f_{f'\leftarrow h}(y,\mu) f_{NP}(x,\boldsymbol{b})$$

Perturbative Wilson coefficient matching

PDF

Non-perturbative non-asymptotic part



$$f_{NP}(x,\boldsymbol{b}) = \exp\left(-\frac{(\lambda_1(1-x) + \lambda_2 x + \lambda_3 x(1-x))\boldsymbol{b}^2}{\sqrt{1 + (\lambda_4 + \lambda_5 x^{\lambda_6})\boldsymbol{b}^2}}\right)$$

	H	$C_{f\leftarrow f'}$	$\mathcal{D}^f_{res}$	ζ_μ	PDF	$\Gamma_{\text{cusp}},\ \gamma_F$
Pert. order	α_s^2	α_s^2	α_s^2	α_s^2	NNLO	α_s^3

Summary of theory input

CROSS-SECTION

$$\frac{d\sigma}{dQ^2 dy dq_T^2} = \sigma_0 \sum_{f_1,f_2} \int \frac{d^2\boldsymbol{b}}{4\pi} e^{i(\boldsymbol{b}\cdot\boldsymbol{q}_T)} H_{f_1 f_2}(Q,Q)\{R[\boldsymbol{b};(Q,Q^2)]\}^2 F_{f_1\leftarrow h_1}(x_1,\boldsymbol{b}) F_{f_2\leftarrow h_2}(x_2,\boldsymbol{b}).$$

TMD

$$F_{f\rightarrow h}(x,\boldsymbol{b}) = \sum_{f'} \int_x^1 \frac{dy}{y} C_{f\leftarrow f'}\left(\frac{x}{y}, \ln\left(\boldsymbol{b}^2\mu^2\right)\right) f_{f'\leftarrow h}(y,\mu) f_{NP}(x,\boldsymbol{b})$$

EVOLUTION FACTOR

$$R^f[\boldsymbol{b};(\mu_1,\zeta_1)\rightarrow(\mu_2,\zeta_2)] = \exp\left[\int_P \left(\frac{\gamma_F^f(\mu,\zeta)}{2}\frac{d\mu^2}{\mu^2} - \mathcal{D}^f(\mu,\boldsymbol{b})\frac{d\zeta}{\zeta}\right)\right]$$

ZETA-PRESCRIPTION

$$R^f[\boldsymbol{b};(\mu,\zeta)\rightarrow(\mu,\zeta_\mu(\boldsymbol{b}))] = R^f[\boldsymbol{b};(\mu,\zeta)] = \left(\frac{\zeta}{\zeta_\mu(\boldsymbol{b})}\right)^{-\mathcal{D}^f(\mu,\boldsymbol{b})}$$

WE HAVE TWO UNCORRELATED SOURCES OF NP PHYSICS

$$\mathcal{D}^f(\mu, b^*(\mathbf{b})) = \mathcal{D}^f_{res}(\mu, b^*(\mathbf{b})) + \mathcal{D}^f_{NP}(\mathbf{b})$$

$$f_{NP}$$

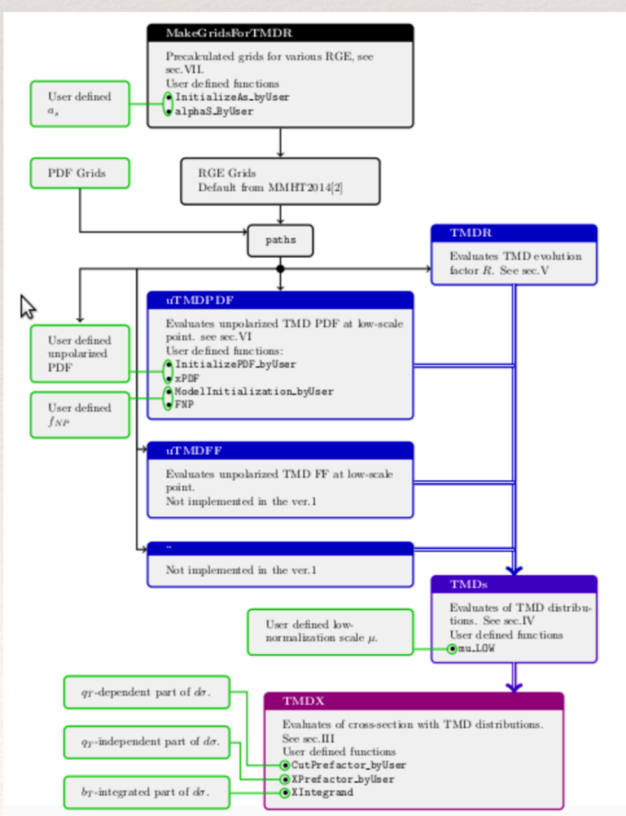
arTeMiDe

- FORTRAN 90 code
- Module structure
- Convolutions, evolution (LO,NLO,NNLO)
- Fourier to q_T-space, integrations over phase space
- Scale-variation (ζ-prescription)
- User defined PDFs, scales, f_{NP}
- Efficient code ($\sim 10^9$ TMDs $\sim$ 6 min at NNLO)

Currently ver ~~1.1~~ 2.0

Available at: https://teorica.fis.ucm.es/artemide

Future plans: add modules for fragmentations, and polarized TMDs



arTeMiDe

- Python interface, Manual.
- All processes at NNLO. Theory article published.
- PDFs from LHAPDF.
- Many processes already included, not only DY.
- artemide repository,
  https://teorica.fis.ucm.es/artemide/
  https://github.com/vladimirovalexey/artemide-public.
- Latest version 2.0.

Data and limits of TMD analysis

The limits of the TMD analysis are defined by the limit of factorization and are independent of the non-perturbative parametrization of TMDs or perturbative order

$$\delta_t = q_t/M \lesssim 0.2$$

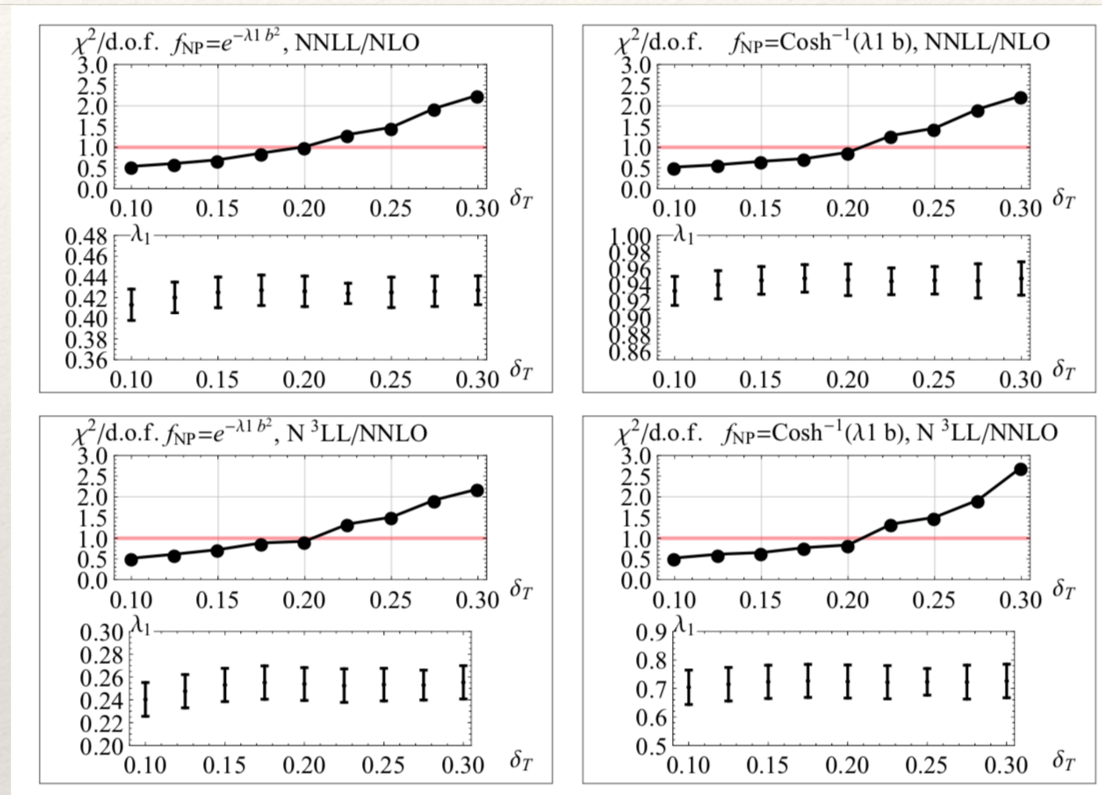


Table from arXiv:1706:01473

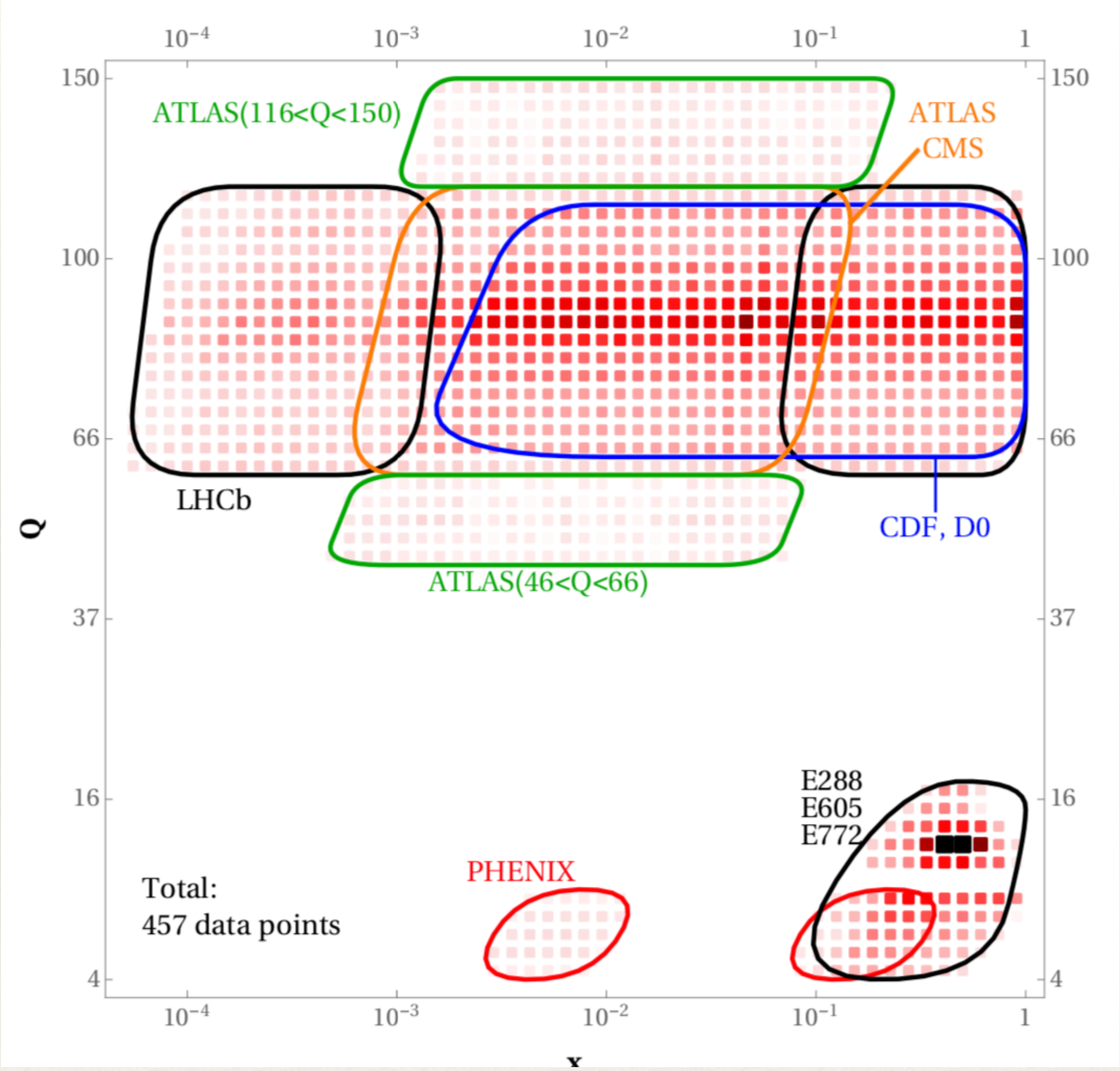
ATLAS experiment has an extraordinary precision:

$$\delta < 0.1, \qquad \text{or} \qquad \left(\delta < 0.25 \quad \text{and} \quad \delta^2 < \sigma\right)$$

qT spectrum DATA SETS for TMD

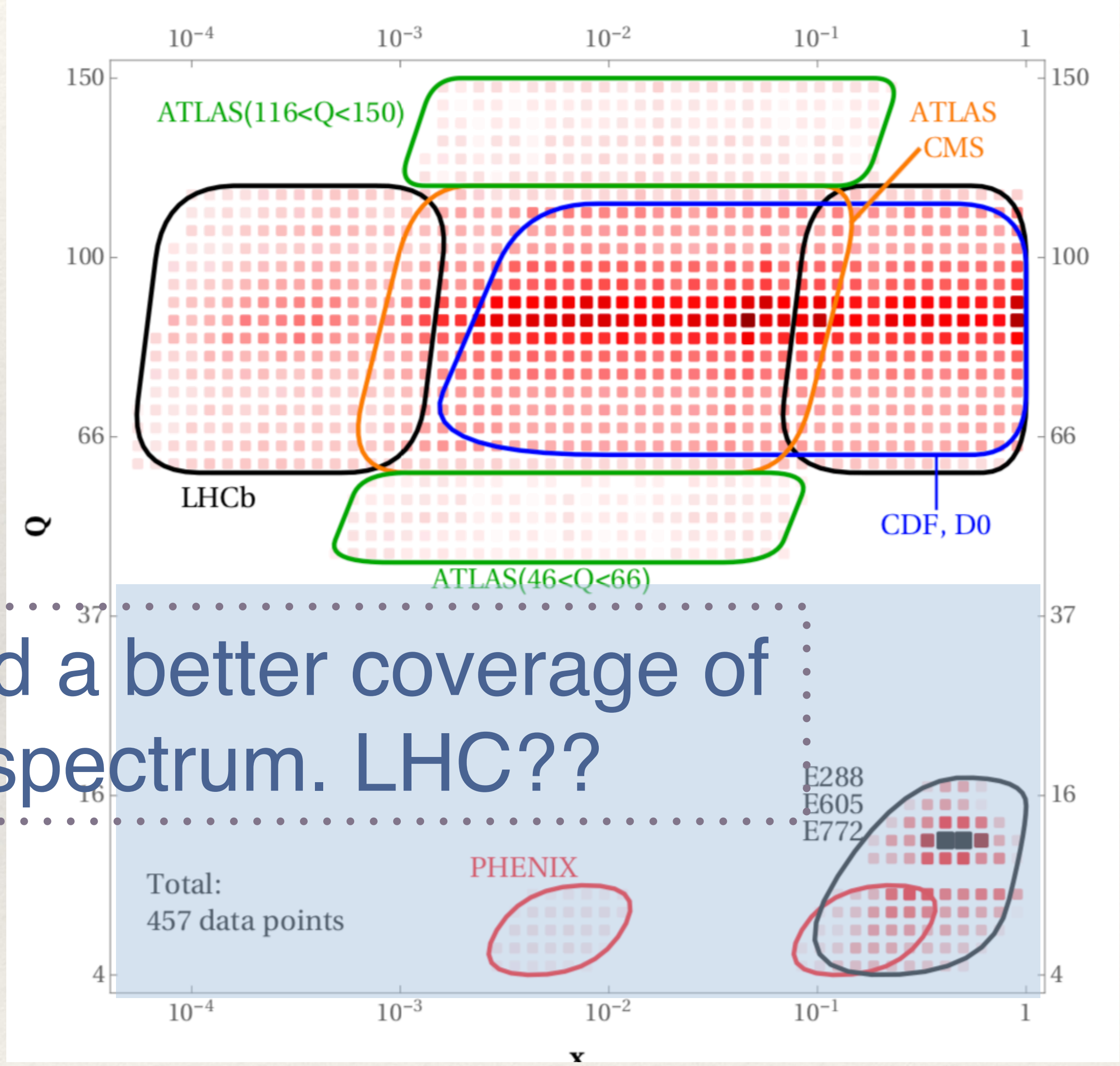
High-energy data

Low-energy data



Largest DY set ever
457 POINTS:
low energy 263
high energy 194

DATA SETS for qT spectrum



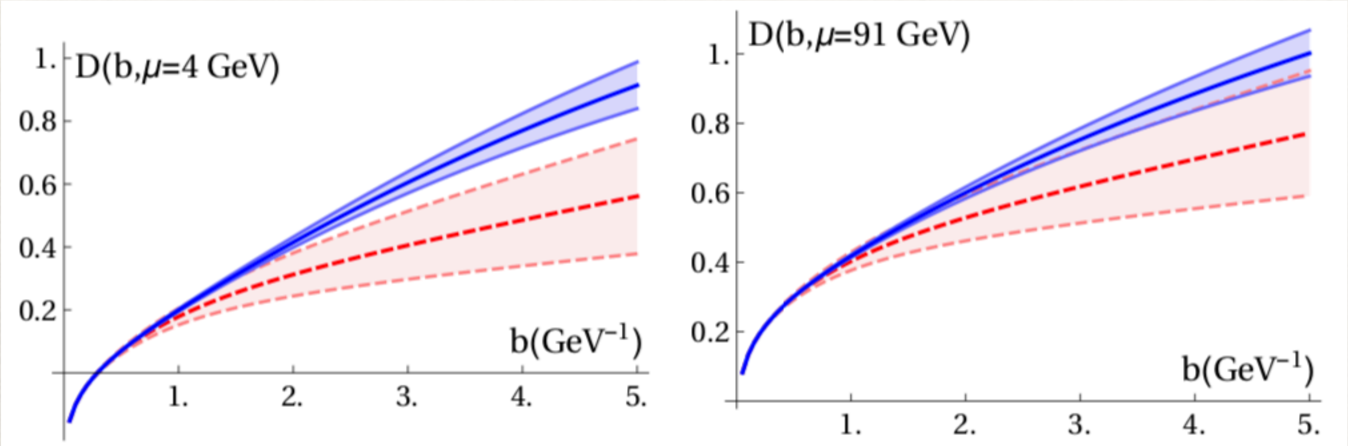
Largest DY set ever
457 POINTS:
low energy 263
high energy 194

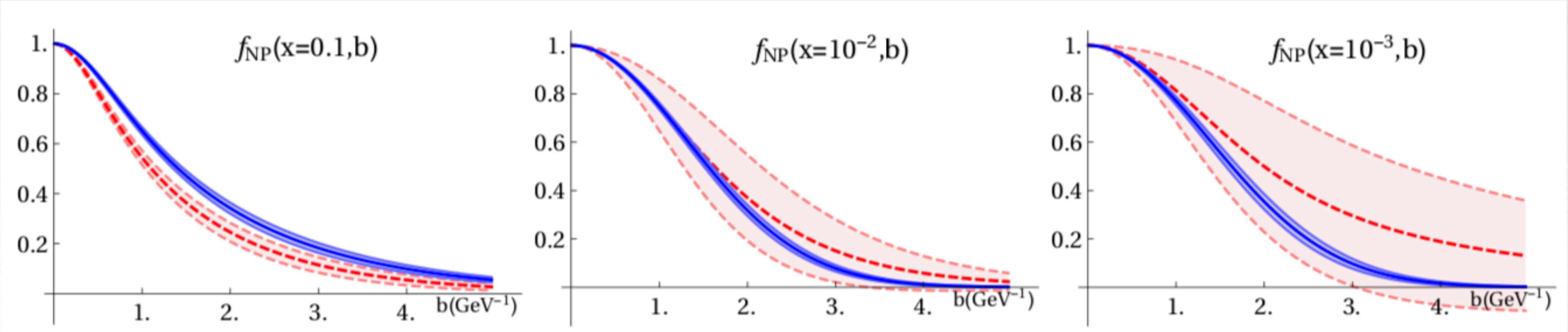
TMD knowledge need a better coverage of this part of the spectrum. LHC??

Requests: smallest possible binning(<1 GeV?), qT/Q<0.2.

TMD extraction V. Bertone, I.S., A. Vladimirov JHEP 1906 (2019) 028

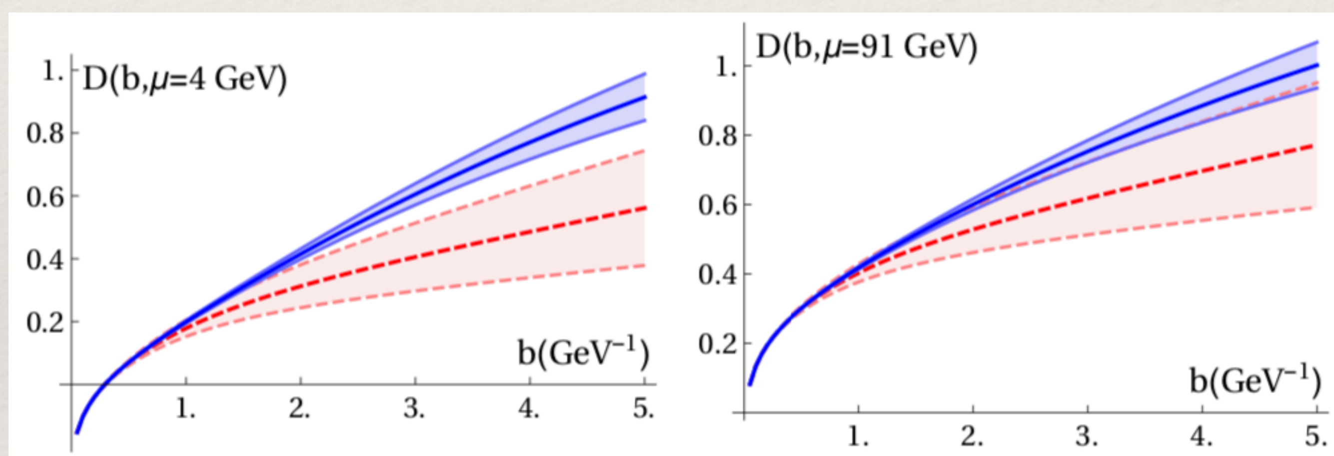
B_{NP}	c_0	λ_1	λ_2	λ_3	λ_4	λ_5
Full data set						
3.31 ± 0.28	0.024 ± 0.006	0.258 ± 0.022	8.18 ± 1.00	-4.76 ± 1.38	$300. \pm 89.$	2.44 ± 0.12
2.5(fixed)	0.037 ± 0.007	0.248 ± 0.025	8.15 ± 1.40	-4.96 ± 1.60	$275. \pm 53.$	2.52 ± 0.13
Excluding LHC-data						
1.21 ± 0.50	0.057 ± 0.038	0.21 ± 0.17	12.1 ± 4.4	-3.51 ± 5.40	$316. \pm 196.$	2.11 ± 0.28
2.5(fixed)	0.014 ± 0.012	0.14 ± 0.08	11.2 ± 3.8	-2.48 ± 3.96	$413. \pm 277.$	2.07 ± 0.21

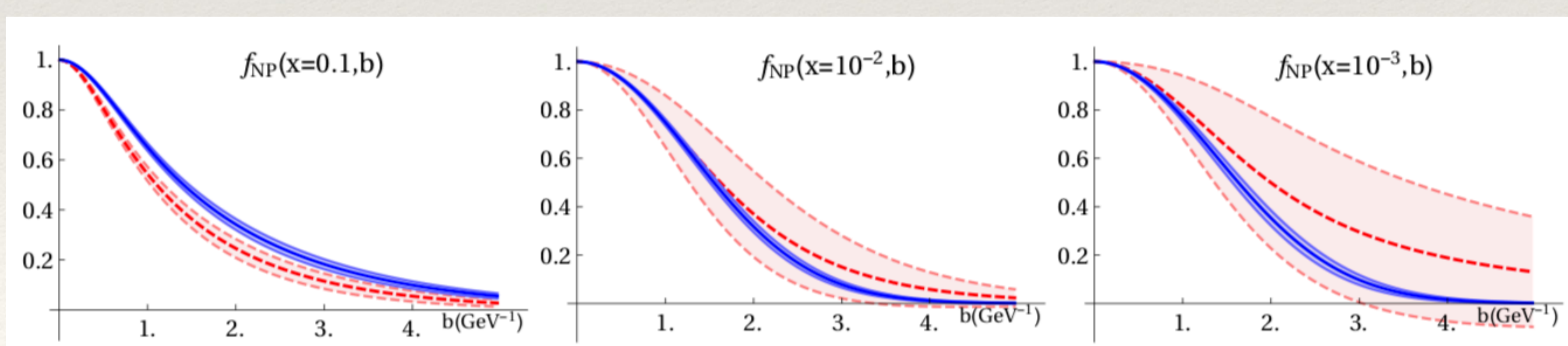




TMD extraction V. Bertone, I.S., A. Vladimirov JHEP 1906 (2019) 028

**Because the cross section
is dominated by
the QCD perturbative part, the LHC DY-spectrum strongly
constrains the TMD shape**





Numerical results.

Data set	N_{pt}	χ^2_D/N_{pt}	$\chi^2_\lambda/N_{\text{pt}}$	χ^2/N_{pt}	$\langle d/\sigma \rangle$
E288 (200)	43	0.79	0.06	0.86	41.15%
E288 (300)	53	0.89	0.04	0.93	35.72%
E288 (400)	76	0.78	0.01	0.80	26.52%
E605	53	0.49	0.05	0.54	24.74%
E772	35	1.65	0.05	1.70	13.24%
PHENIX	3	0.28	0.02	0.30	4.08%
Low energy data	**263**	**0.86**	**0.04**	**0.90**	
CDF (run1)	33	0.54	0.14	0.68	8.42%
CDF (run2)	39	1.37	0.01	1.37	2.90%
D0 (run1)	16	0.76	0.00	0.76	0.12%
D0 (run2) •	8	1.51	0.00	1.51	0.00%
D0 $(\text{run2})_\mu$ •	3	0.33	0.36	0.68	0.33%
Tevatron	99	0.97	0.06	1.03	
ATLAS (7 TeV) $\|y\| < 1$ •	5	2.16	0.00	2.17	−0.05%
ATLAS (7 TeV) $1 < \|y\| < 2$ •	5	5.13	0.00	5.14	−0.07%
ATLAS (7 TeV) $2 < \|y\| < 2.4$ •	5	1.08	0.00	1.08	−0.02%
ATLAS (8 TeV) $\|y\| < 0.4$	5	1.86	0.33	2.19	3.68%
ATLAS (8 TeV) $0.4 < \|y\| < 0.8$	5	2.41	0.68	3.09	3.66%
ATLAS (8 TeV) $0.8 < \|y\| < 1.2$	5	1.02	0.54	1.56	3.77%
ATLAS (8 TeV) $1.2 < \|y\| < 1.6$	5	1.24	0.49	1.73	4.29%
ATLAS (8 TeV) $1.6 < \|y\| < 2.0$	5	0.42	0.59	1.01	4.93%
ATLAS (8 TeV) $2.0 < \|y\| < 2.4$	5	1.55	1.21	2.76	5.56%
ATLAS (8 TeV) 46 - 66 GeV	3	0.43	0.07	0.49	1.45%
ATLAS (8 TeV) 116 - 150 GeV	7	0.74	0.13	0.87	1.96%
ATLAS total	55	1.65	0.37	2.02	
CMS (7 TeV) •	8	1.26	0.00	1.26	0.00%
CMS (8 TeV) •	8	0.85	0.00	0.85	0.00%
CMS total	16	1.06	0.00	1.06	
LHCb (7 TeV)	8	2.05	0.90	2.95	5.69%
LHCb (8 TeV)	7	3.85	1.69	5.54	5.65%
LHCb (13 TeV)	9	0.60	0.29	0.89	6.34%
LHCb total	24	2.03	0.90	2.93	
High energy data	**194**	**1.30**	**0.25**	**1.55**	
Global	**457**	**1.05**	**0.12**	**1.17**	

Are TMD relevant for qT spectrum at LHC? (F. Hautmann, I.S., A. Vladimirov, in preparation)

Common assumptions check

$$\mathcal{D}^f(\mu, b^*(\mathbf{b})) = \mathcal{D}^f_{res}(\mu, b^*(\mathbf{b})) + \mathcal{D}^f_{NP}(\mathbf{b}) \qquad b^*(\mathbf{b}) = \sqrt{\frac{B^2_{NP}\mathbf{b}^2}{B^2_{NP} + \mathbf{b}^2}}$$

$$\mathcal{D}^f_{NP}(\mathbf{b}) = c_0|\mathbf{b}|b^*(\mathbf{b})$$

$$\mathcal{D}^f_{NP}(\mathbf{b}) = c_0\mathbf{b}^2$$

Data	χ^2/N_{pt}	$B_{\rm NP}$	c_0	λ_1	λ_2	λ_3	λ_4	λ_5
D_{NP}-fit $f_{NP}=1$								
LHC only	2.38	1.0	0.164	-	-		-	-
Full set	4.67	5.5	0.105	-	-	-	-	-
$D_{NP}=c_0 b^2$-fit $f_{NP}=1$								
LHC only	3.5	5.5	0.12	-	-	-	-	-
Full set	5.05	5.4	0.103	-	-	-	-	-

Using just simple models for evolution gives poor description of data

Data	χ^2/N_{pt}	$B_{\rm NP}$	c_0	λ_1	λ_2	λ_3	λ_4	λ_5
D_{NP}-fix $f_{NP}=e^{-\lambda_1 b^2}$-fit								
LHC set	1.79	4.5	0.	0.362	-	-	-	-
Full set	4.54	4.5	0.	0.326	-	-	-	-

A too simple model for evolution and TMD gives poor description of LE data

Data	χ^2/N_{pt}	$B_{\rm NP}$	c_0	λ_1	λ_2	λ_3	λ_4	λ_5
D_{NP}-fix f_{NP}-full fit								
LHC set	1.697	4.5	0.	0.334	0.9743	191.3	3.07	0.803
Full set	1.18	4.5	0.	0.313	7.995	230.1	2.314	-5.212

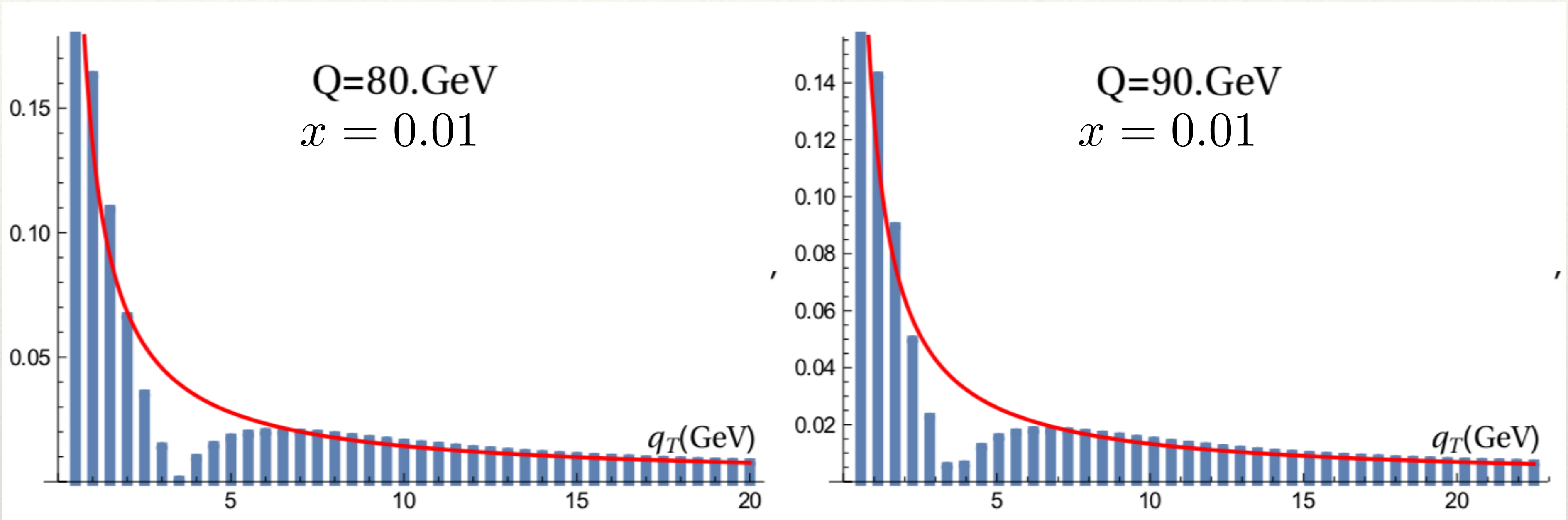
A saturation model for evolution + TMD Is also possible

IMPACT OF TMDS ON LHC DATA

Comparing
cross sections w./w.o.
f_{NP}

$f_{NP} \neq 1$ $\quad$ $f_{NP} = 1$

$$\frac{d\sigma_{\text{Tot}} - d\sigma_{\text{w/o TMD-NP}}}{d\sigma_{\text{Tot}}}$$

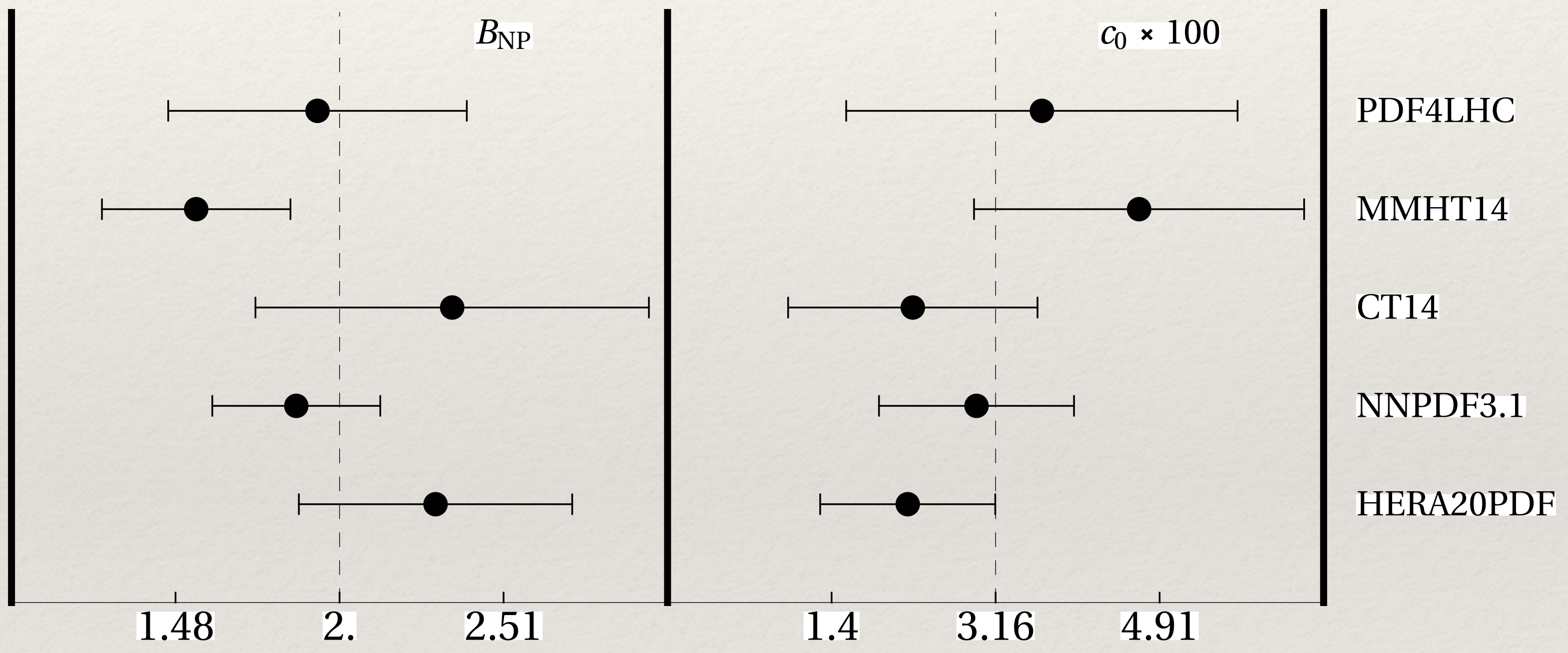


Red line: $0.001 + \frac{0.134}{q_T}$ $\qquad$ Red line: $0.001 + \frac{0.127}{q_T}$

TMD vs PDF sets

PRELIMINARY

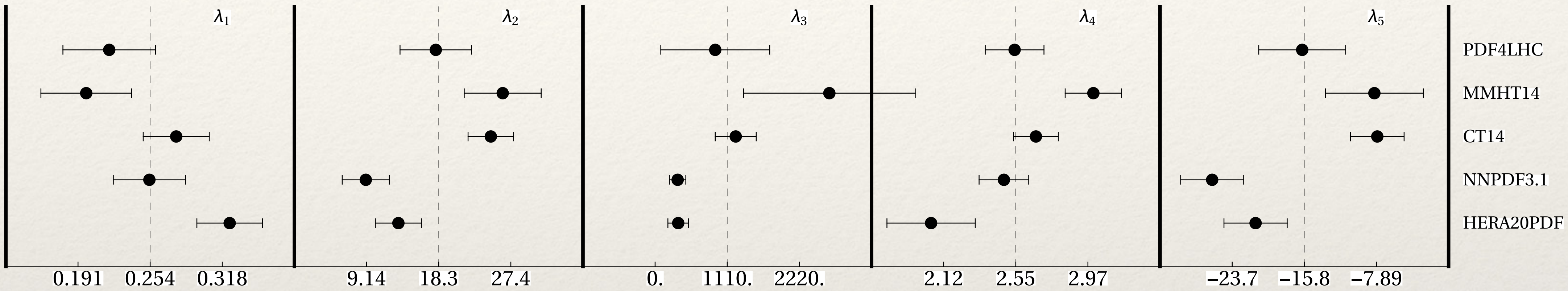
EVOLUTION



TMD vs PDF sets

PRELIMINARY

FNP



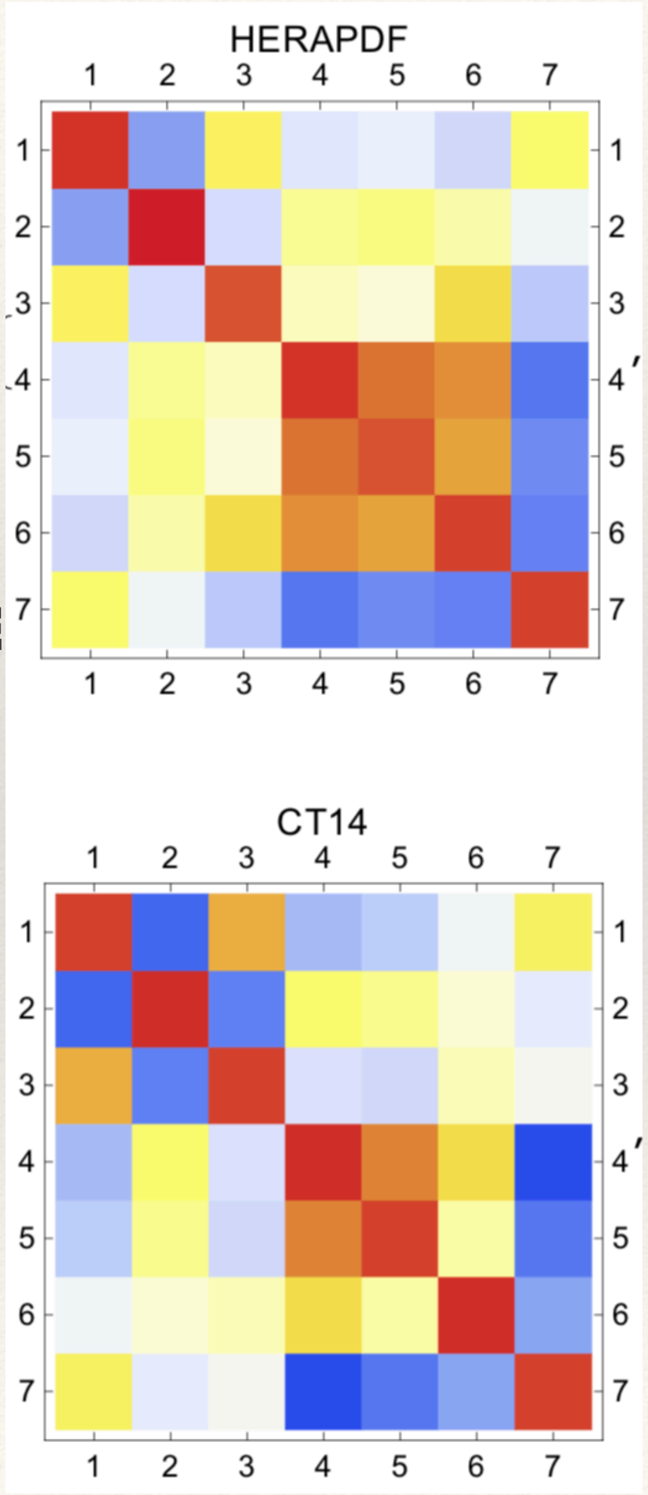
The spread in the constants is related to PDF sets

PDF set	χ^2/N_{pt}
HERAPDF	0.95
NNPDF3.1	1.17
MMHT14	1.36
PDF4LHC	1.52
CT14	1.63

TMD vs PDF sets

PRELIMINARY

IN PRINCIPLE THE CONSTANTS OF THE EVOLUTION KERNEL ARE UNCORRELATED TO THE REST.
THIS IS REALIZED BY THE SET OF PDF
WHICH PROVIDE THE BETTER FIT



$1{=}B_{NP},\ 2{=}c_0,$
$3,4,5,6,7{=}\lambda_{1,2,3,4,5}$

Summary

- In q_T spectrum, for $q_T/Q \lesssim 0.2$ the TMD are a relevant source of NP physics
- One can study TMD with 2 GeV$< Q <$ 150 GeV, and we miss measurements away from Z-boson peak. Many regions are un-explored, a small q_T binning is desirable/necessary.
- We have tried to quantify the amount of TMD contribution for each q_T bin at LHC.
- There is an interesting interplay between PDF sets and quality of TMD fits.
- The W-spectrum should be studied in a similar way.

Back up

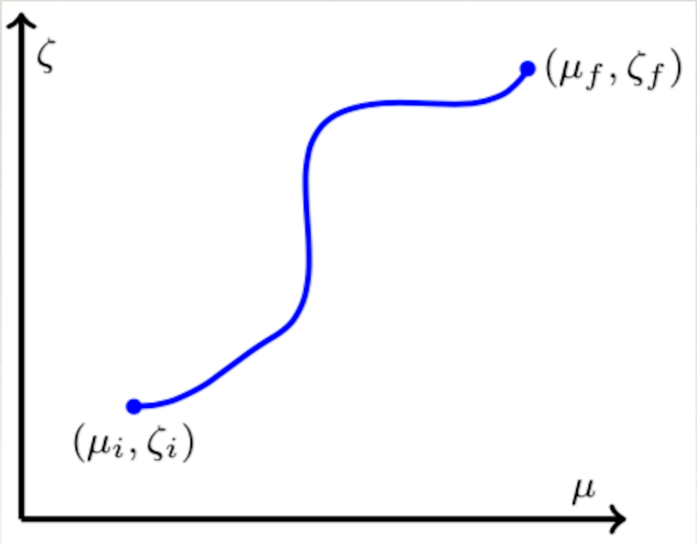
Ambiguity in the TMD evolution

Coupled evolution of TMD and truncation of the perturbative series

Integrability Condition…

$$\zeta \frac{d}{d\zeta} \gamma_F(\mu, \zeta) = -\mu \frac{d}{d\mu} \mathcal{D}(\mu, b)$$

…ensures the path independence of the evolution factor…



$$R[b; (\mu_f, \zeta_f) \to (\mu_i, \zeta_i)] = \exp\left[\int_P \left(\gamma_F(\mu, \zeta)\frac{d\mu}{\mu} - \mathcal{D}(\mu, b)\frac{d\zeta}{\zeta}\right)\right]$$

2D Evolution field: Notation and ideal case

The evolution scales are treated equally

$$\vec{\nu} = \left(\ln \frac{\mu^2}{1\ \mathrm{GeV}^2}, \ln \frac{\zeta}{1\ \mathrm{GeV}^2}\right)$$

Differentiation

$$\vec{\nabla} = \frac{d}{d\vec{\nu}} = (\mu^2 \frac{d}{d\mu^2}, \zeta \frac{d}{d\zeta}), \qquad \mathbf{curl} = (-\zeta \frac{d}{d\zeta}, \mu^2 \frac{d}{d\mu^2})$$

Evolution field

$$\mathbf{E}(\vec{\nu}, b) = (\frac{\gamma_F(\vec{\nu})}{2}, -\mathcal{D}(\vec{\nu}, b))$$

TMD Evolution

$$\vec{\nabla} F(x, b; \vec{\nu}) = \mathbf{E}(\vec{\nu}, b) F(x, b; \vec{\nu})$$

Integrability Condition and Scalar Potential

$$\vec{\nabla} \times \mathbf{E} = 0 \Rightarrow \mathbf{E}(\vec{\nu}, b) = \vec{\nabla} U(\vec{\nu}, b)$$

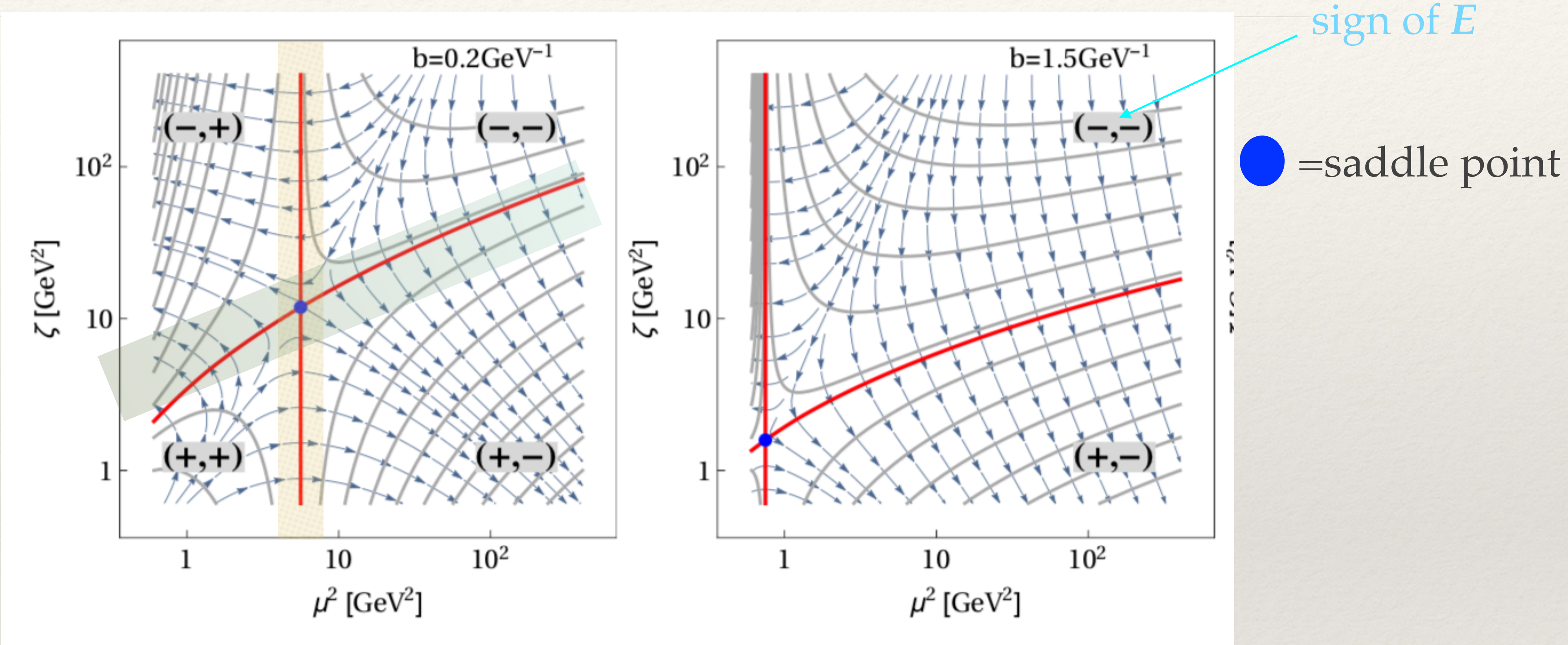
Evolution kernel

$$\ln R[b; \vec{\nu}_f \to \vec{\nu}_i] = U(\vec{\nu}_f, b) - U(\vec{\nu}_i, b)$$

with

$$U(\vec{\nu}, b) = \int^{\nu_1} \frac{\Gamma(s)s - \gamma_V(s)}{2} ds - \mathcal{D}(\vec{\nu}, b)\nu_2 + \mathrm{const}(b)$$

2D Evolution field: Notation and ideal case



Singularities: Landau pole (on the left, not shown) and saddle point $\mathbf{E}(\vec{\nu}_{\text{saddle}}, b) = \vec{0}$

Equipotential/null-evolution curves: $\vec{\omega}(t, \vec{\nu}_B, b) = (t, \omega(t, \vec{\nu}_B, b)) \quad \to \frac{d\vec{\omega}}{dt} \cdot \vec{\nabla} U(\vec{\omega}, b) = 0$

Special null-evolution curves: $\mu = \mu_{\text{saddle}}$ and $\vec{\nu}_B = \vec{\nu}_{\text{saddle}}$

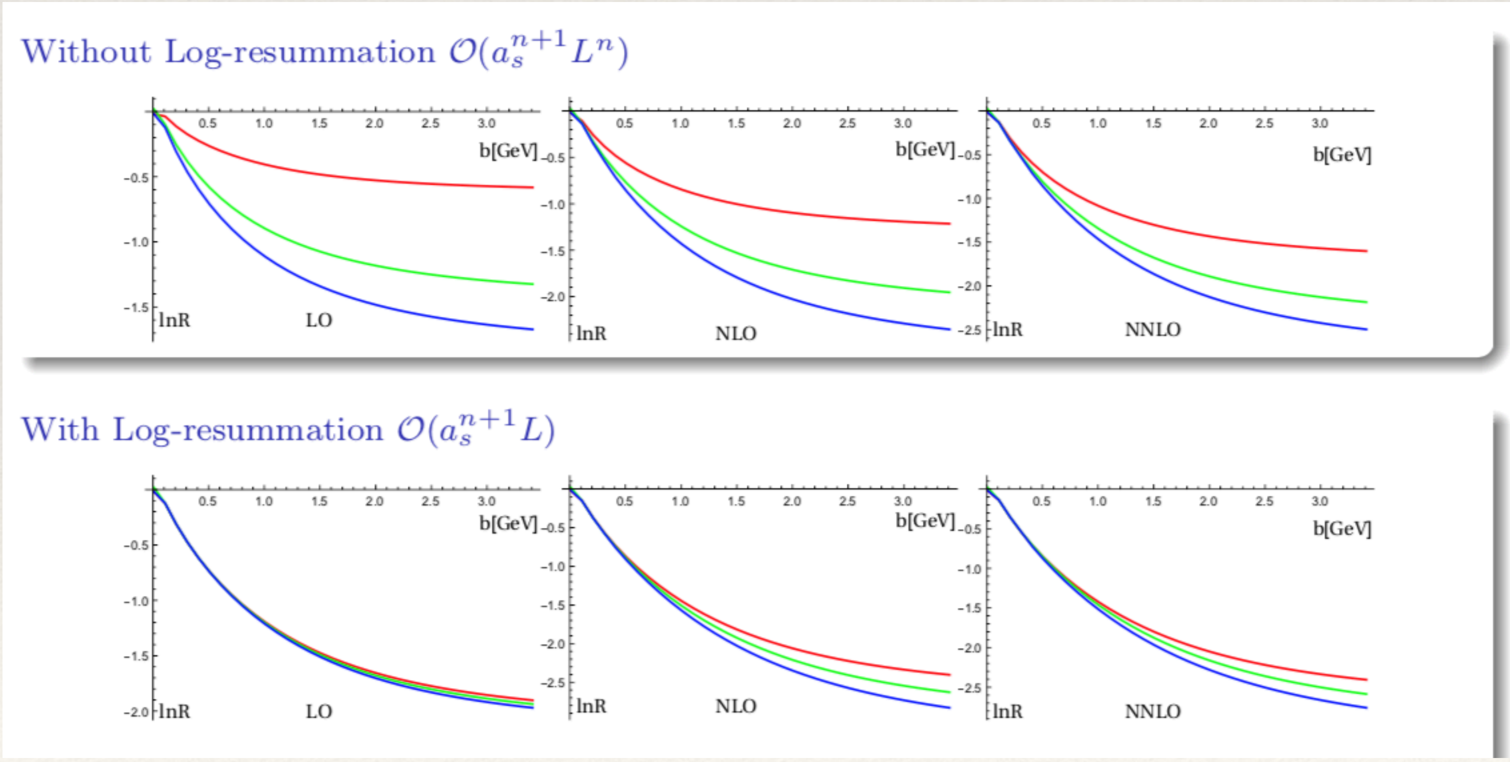
Series truncation in TMD evolution

Coupled evolution of TMD and truncation of the perturbative series

In practice due to the truncation of the perturbative series:
Transitivity and reversibility of evolution is lost

$$\zeta \frac{d}{d\zeta}\gamma_F(\mu,\zeta) \neq -\mu \frac{d}{d\mu}\mathcal{D}(\mu,b)$$

For Q=Mz the solution path dependence enormous



Truncation of the perturbative series

The truncation introduces a path difference

$$\delta\Gamma(\mu,b)=\Gamma(\mu)-\mu\frac{d\mathcal{D}(\mu,b)}{d\mu},$$

$$\delta\Gamma^{(N)}=2\sum_{n=1}^{N}\sum_{k=0}^{n}n\bar{\beta}_{n-1}(a_s)a_s^{n-1}d^{(n,k)}\mathbf{L}_\mu^k$$

$$\text{with } \bar{\beta}_n(a_s)=\beta(a_s)-\sum_{k=0}^{n-1}\beta_k a_s^{k+2}$$

$$\mathbf{L}_\mu=\ln\left(\frac{X^2b^2}{4e^{-2\gamma_E}}\right)$$

$\delta\Gamma^{(N)}\sim\mathcal{O}(a_s^{N+1}\mathbf{L}_\mu^N)$ with perturbative D

$\delta\Gamma^{(N)}\sim\mathcal{O}(a_s^{N+1}\mathbf{L}_\mu)$ with resummed D

$$\ln\frac{R[b;\{\mu_1,\zeta_1\}\xrightarrow{P_1}\{\mu_2,\zeta_2\}]}{R[b;\{\mu_1,\zeta_1\}\xrightarrow{P_2}\{\mu_2,\zeta_2\}]}=\frac{1}{2}\int_{\Omega(P_1\cup P_2)}d^2\nu\,\delta\Gamma(\vec{\nu},b)=\int_{\mu_2}^{\mu_1}\frac{d\mu}{\mu}\delta\Gamma(\mu,b)\ln\left(\frac{\zeta_1(\mu)}{\zeta_2(\mu)}\right)$$

The path dependence is enhanced by the difference in rapidity scale

At large value of impact parameter the breaking of integrability condition becomes crucial

Recovering path independence

Helmholtz decomposition of evolution fields

$$\mathbf{E}(\vec{\nu}, b) = \tilde{\mathbf{E}}(\vec{\nu}, b) + \boldsymbol{\Theta}(\vec{\nu}, b)$$

Basic properties of evolution fields

$$\text{curl}\tilde{\mathbf{E}} = 0, \qquad \vec{\nabla} \cdot \vec{\Theta} = 0, \qquad \tilde{\mathbf{E}} \cdot \boldsymbol{\Theta} = 0.$$

Scalar potentials

$$\tilde{\mathbf{E}}(\vec{\nu}, b) = \vec{\nabla}\tilde{U}(\vec{\nu}, b) \qquad \boldsymbol{\Theta}(\vec{\nu}, b) = \mathbf{curl}\, V(\vec{\nu}, b)$$

Ideally one could repair the truncation using decomposition of the evolution field

$$\text{curl}\mathbf{E} = \text{curl}\boldsymbol{\Theta} = \frac{\delta\Gamma(\vec{\nu}, b)}{2} \neq 0$$

THE INTEGRABILITY CONDITION IS RE-ESTABLISHED DEFINING THE EVOLUTION KERNEL AS

$$\ln R[b; \vec{\nu}_f \to \vec{\nu}_i] = \tilde{U}(\vec{\nu}_f, b) - \tilde{U}(\vec{\nu}_i, b)$$

$$\nabla^2 \tilde{U}(\vec{\nu}, b) = \frac{1}{2} \frac{d\gamma_F(\vec{\nu})}{d\nu_1}$$

However in order to fix completely the evolution potential one needs boundary condition for the evolution field:
at the moment no theoretically solid non-perturbative input is known

Recovering path independence

We modify anomalous dimensions such that integrability restored

$$\mu \frac{d\mathcal{D}(\mu,b)}{d\mu} = -\zeta \frac{d\gamma_F(\mu,\zeta)}{d\zeta}$$

It can be done from both sides of the equation.

Improved $\mathcal{D}$

Facilitate

$$\mu \frac{d\mathcal{D}}{d\mu} = \Gamma.$$

by

$$\mathcal{D}(\mu,b) = \int_{\mu_0}^{\mu} \frac{d\mu}{\mu}\Gamma(\mu) + \mathcal{D}(\mu_0,b)$$

- In the spirit of [Collins' text book].
- Already used in many studies
- However, it is not the best way

Improved γ

We set

$$\zeta \frac{d\gamma_F}{d\zeta} \equiv -\mu \frac{d\mathcal{D}}{d\mu} = \delta\Gamma - \Gamma$$

Or

$$\gamma_F(\mu,\zeta) \to \gamma_M(\mu,\zeta,b)$$

$$\gamma_M = (\Gamma - \delta\Gamma) \ln\left(\frac{\mu^2}{\zeta}\right) - \gamma_V$$

- Completely self consistent
- Very natural

Universität Regensburg

Improved γ scenario

$$\gamma_M(\mu,\zeta,b) = (\Gamma(\mu) - \delta\Gamma(\mu,b))\mathbf{l}_\zeta - \gamma_V(\mu)$$

$$\tilde{U}(\vec{\nu},b) = \int^{\nu_1} \frac{(\Gamma(s) - \delta\Gamma(s,b))s - \gamma_V(s)}{2} ds - \mathcal{D}(\vec{\nu},b)\nu_2 + const(b)$$

$$\ln R[b;(\mu_f,\zeta_f) \to (\mu_i,\zeta_i)] = -\int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu}(2\mathcal{D}(\mu,b) + \gamma_V(\mu)) + \mathcal{D}(\mu_f,b)\ln\left(\frac{\mu_f^2}{\zeta_f}\right) - \mathcal{D}(\mu_i,b)\ln\left(\frac{\mu_i^2}{\zeta_i}\right)$$

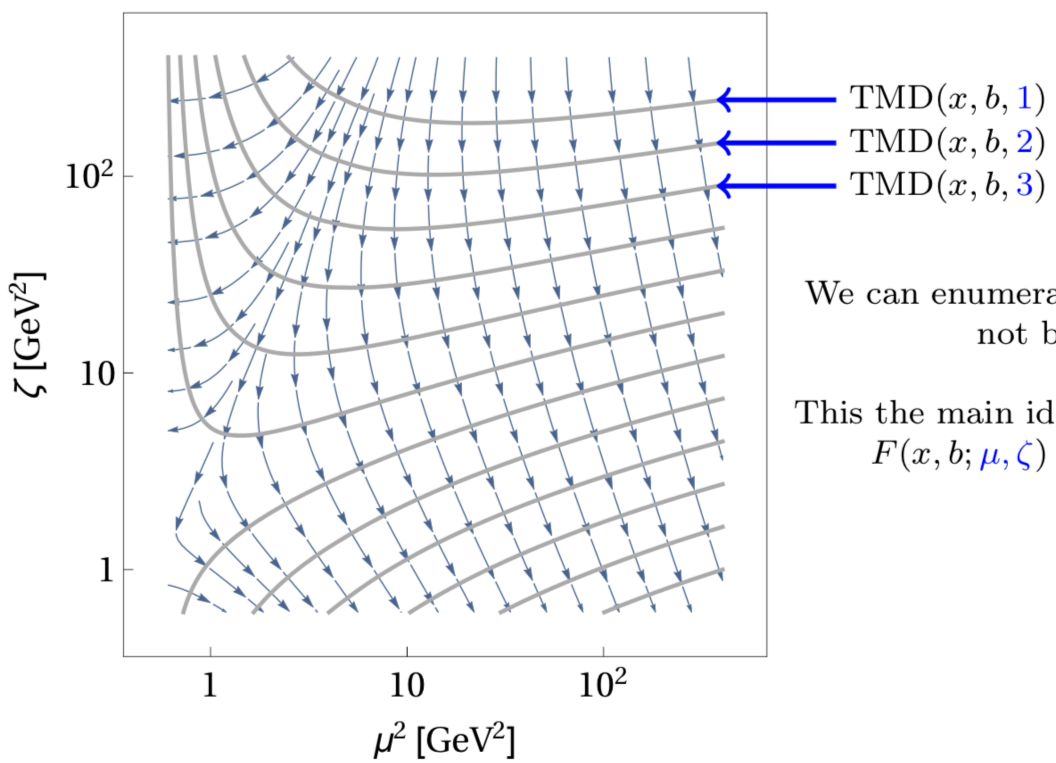
Clear advantages:

- No more the intermediate scale μ_0
- Path independence
- Simplicity
- We achieve a clear separation of evolution and nonperturbative part of the TMD

Equivalent TMDs: no evolution on equipotential lines

The 2-D evolution just connects TMDs on different equipotential lines

TMD distributions on the same equipotential line are equivalent.



We can enumerate them by a lines not by (μ, ζ)

This the main idea of ζ-prescription $F(x, b; \mu, \zeta) \to F(z, b; \text{line})$

Universität Regensburg

Left for Technical discussion

TMD on equipotential lines

The TMDs on equipotential lines are not evolved so one can define a TMD by a single parameter line

$$F(x,b;\vec{\nu}_B)=F(x,b;\vec{\nu}'_B),\qquad \vec{\nu}'_B\in\vec{\omega}(\vec{\nu}_B,b).$$

ONE CAN HAVE AN EVOLUTION ONLY WHEN MOVING BETWEEN DIFFERENT LINES

$$F(x,b;\vec{\nu}_B)=R[b;\vec{\nu}_B\to\vec{\nu}'_B]F(x,b;\vec{\nu}'_B)$$

Outcome: the modeling of the non-perturbative part of the TMD does not depend anymore on the relation between renormalization scale and impact parameter.

Question: Is there a preferred line?

Optimal TMD definition

IS THERE A PREFERRED INITIAL LINE?

$$\mathbf{E}(\boldsymbol{\nu}_{\text{saddle}}, b) = \mathbf{0}$$

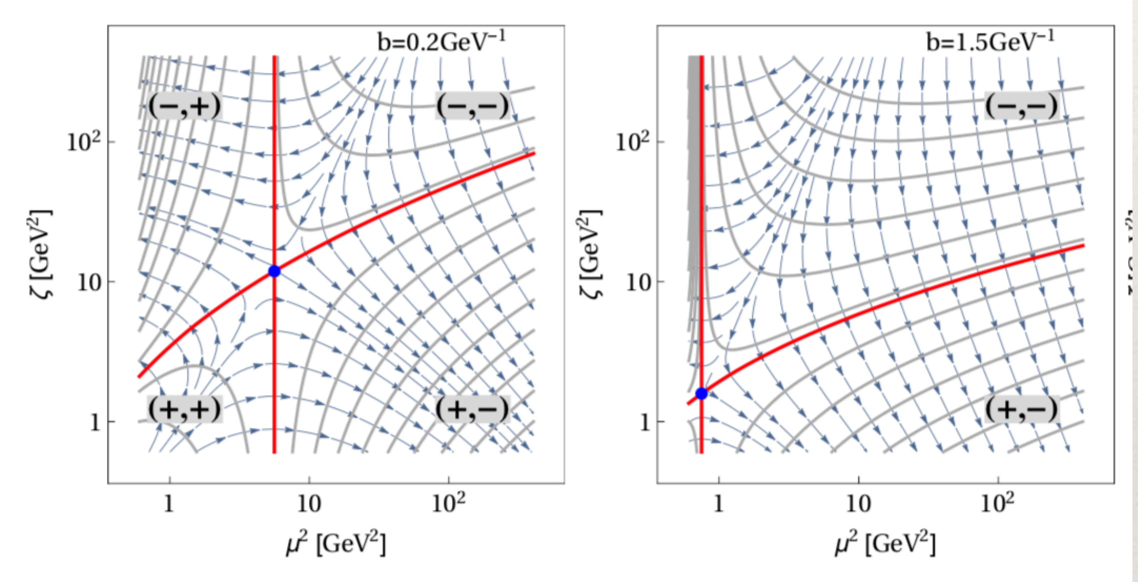
$$\mathcal{D}(\mu_{\text{saddle}}, b) = 0$$

$$\gamma_F(\mu_{\text{saddle}}, \zeta_{\text{saddle}}) = 0$$

The saddle point moves with *b*.

- *For every value of b one can find a saddle point*
- *The D-function should include also a nonperturbative part*

The saddle point!!



$$\mathcal{D}_{\text{NP}}(\mu, b) = \mathcal{D}(\mu, b^*), \qquad b^*(b) = \begin{cases} b, & b \ll \bar{b}, \\ b_{\max}, & b \gg \bar{b}, \end{cases}$$

$\bar{b}$ from renormalon estimate $\sim 3.5/\text{GeV}$

$$\delta\Gamma_{\text{NP}}(\mu, b) = \Gamma(\mu) - \mu \frac{d\mathcal{D}_{\text{NP}}(\mu, b)}{d\mu}$$

Optimal TMD definition

the scale μ_{saddle} is b-dependent, and defined by the equation

$$\mathcal{D}^f_{\text{NP}}(\mu_{\text{saddle}}, b) = 0.$$

The evolution does not depend on the initial point..

$$\begin{aligned} R^f[b;(\mu_f,\zeta_f)] &= \exp\Big\{-\int_{\mu_{\text{saddle}}}^{\mu_f}\frac{d\mu}{\mu}\Big(2\mathcal{D}^f_{\text{NP}}(\mu,b)+\gamma^f_V(\mu)\Big)+\mathcal{D}^f_{\text{NP}}(\mu_f,b)\ln\Big(\frac{\mu_f^2}{\zeta_f}\Big)\Big\} \\ &= \exp\Big\{-\mathcal{D}^f_{\text{NP}}(\mu_f,b)\ln\Big(\frac{\zeta_f}{\zeta_{\mu_f}(b)}\Big)\Big\}. \end{aligned}$$

AND THE TMD ARE TOTALLY SCALE INDEPENDENT!!

$$\frac{d\sigma}{dQ^2dydq_T^2} = \sigma_0\sum_{f_1,f_2}\int\frac{d^2\mathbf{b}}{4\pi}e^{i(\mathbf{b}\cdot\mathbf{q}_T)}H_{f_1f_2}(Q,Q)\{R[\mathbf{b};(Q,Q^2)]\}^2F_{f_1\leftarrow h_1}(x_1,\mathbf{b})F_{f_2\leftarrow h_2}(x_2,\mathbf{b}).$$

Left for Technical discussion

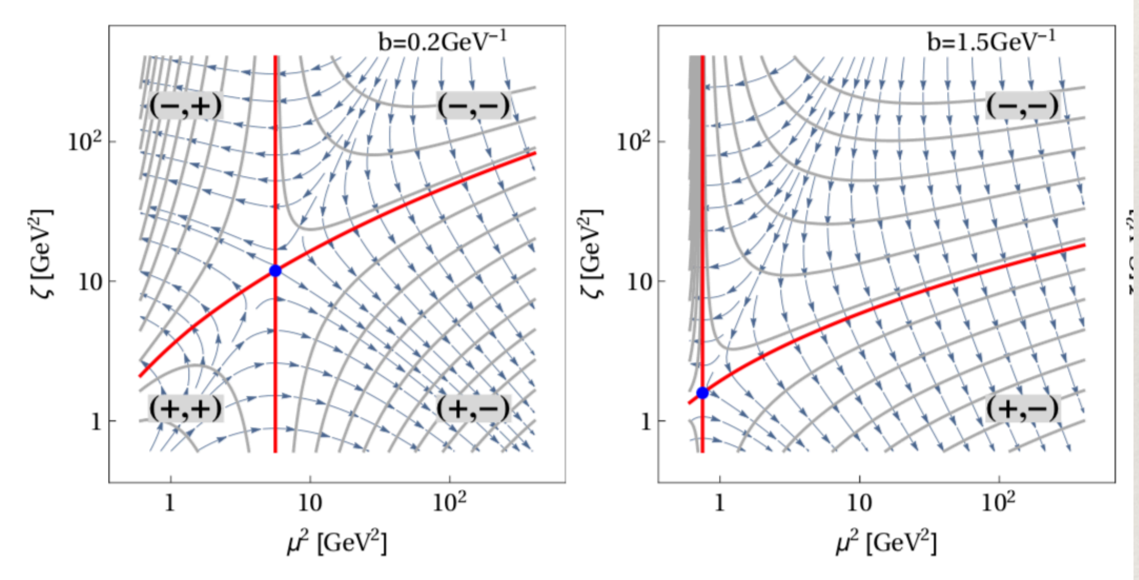
The optimal TMD distribution

There is a consistency constraint in the TMD matching to PDFs in small-b limit

$$\vec{\nu} = \left(\ln \frac{\mu^2}{1\ \mathrm{GeV}^2}, \ln \frac{\zeta}{1\ \mathrm{GeV}^2}\right)$$

$$F_{f\to k}(x,b;\vec{\nu}_B) = \sum_n \sum_{f'} C^{(n)}_{f\to f'}(x,b,\vec{\nu}_B,\mu_{\mathrm{OPE}}) \otimes f^{(n)}_{f'\to h}(x,\mu_{\mathrm{OPE}})$$

The values of μ_{OPE} are restricted to the values of μ taken along the null-evolution curve



$$\begin{aligned}
&\text{if } \nu_{B,1} < \ln \mu^2_{\mathrm{saddle}} \Rightarrow && \mu_{\mathrm{OPE}} < \mu_{\mathrm{saddle}},\\
&\text{if } \nu_{B,1} > \ln \mu^2_{\mathrm{saddle}} \Rightarrow && \mu_{\mathrm{OPE}} > \mu_{\mathrm{saddle}},\\
&\text{if } \vec{\nu}_B = (\ln \mu^2_{\mathrm{saddle}}, \ln \zeta_{\mathrm{saddle}}) \Rightarrow && \mu_{\mathrm{OPE}} \text{ unrestricted}
\end{aligned}$$

$$\mu = \frac{2e^{-\gamma_E}}{|\boldsymbol{b}|} + 2\,\mathrm{GeV}.$$