= Exploring causality of loop amplitudes with the loop-tree duality =

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Outline

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  - Next-to-Next-to-Maximal Loop Topology
- Conclusions

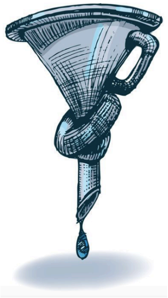
Motivation

Improve theoretical predictions

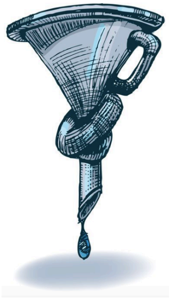
↳ Achieve higher perturbative orders

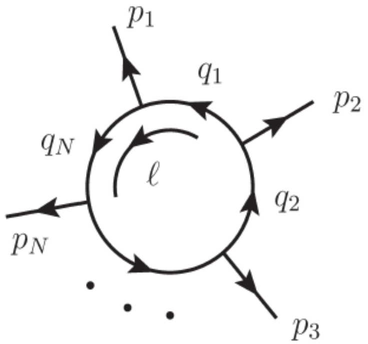
↳ Quantum fluctuations at high-energy scattering processes

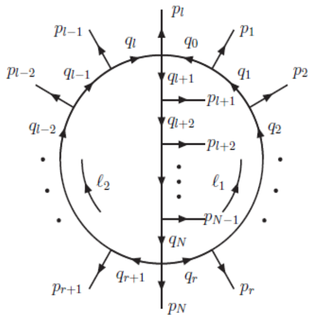
= Multiloop scattering amplitudes =

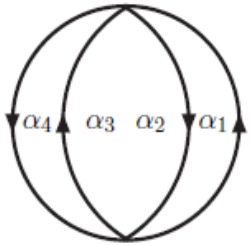


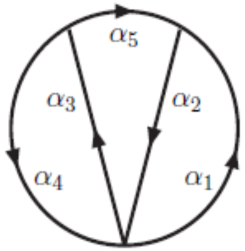
Loop diagrams











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Loop-Tree Duality

- S. Catani, T. Gleisberg, F. Krauss, G. Rodrigo and J. C. Winter, "From loops to trees by-passing Feynman's theorem," JHEP 0809 (2008) 065 [arXiv:0804.3170 [hep-ph]].
- I. Bierenbaum, S. Catani, P. Draggiotis and G. Rodrigo, "A Tree-Loop Duality Relation at Two Loops and Beyond," JHEP 1010 (2010) 073 [arXiv:1007.0194 [hep-ph]].
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Duality relation (LTD)

- S. Buchta, G. Chachamis, P. Draggiotis, I. Malamos and G. Rodrigo, "On the singular behaviour of scattering amplitudes in quantum field theory," JHEP 1411 (2014) 014 [arXiv:1405.7850 [hep-ph]].
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- R. J. Hernández-Pinto, G. F. R. Sborlini and G. Rodrigo, "Towards gauge theories in four dimensions," JHEP 1602 (2016) 044 [arXiv:1506.04617 [hep-ph]].

Singular behaviour at one-loop

- G. F. R. Sborlini, F. Driencourt-Mangin, J. Hernández-Pinto and G. Rodrigo, "Four dimensional unsubtraction from the loop-tree duality,"JHEP 1608 (2016) 160 [arXiv:1604.06699 [hep-ph]].
- G. F. R. Sborlini, F. Driencourt-Mangin and G. Rodrigo, "Four dimensional unsubtraction with massive particles", JHEP 1610 (2016) 162 [arXiv:1608.01584 [hep-ph]].

FDU

- F. Driencourt-Mangin, G. Rodrigo and G.F.R. Sborlini, "Universal dual amplitudes and asymptoticexpansions for $gg \to H$ and $H \to \gamma\gamma$", EPJC 78 (2018) no.3, 231[arXiv:1702.07581 [hep-ph]].

Applications

Loop-Tree Duality

- F. Driencourt-Mangin, G. Rodrigo, G.F.R. Sborlini and W.J. Torres Bobadilla, "Universal four-dimensional representation of $H \rightarrow \gamma\gamma$ at two loops through the Loop-Tree Duality", JHEP 1902 (2019) 143[arXiv:1901.09853 [hep-ph]].
- F. Driencourt-Mangin, G. Rodrigo, G. F. R. Sborlini and W. J. Torres Bobadilla, "On the interplay between the loop-tree duality and helicity amplitudes", [arXiv:1911.11125 [hep-ph]].

Applications

- J.J. Aguilera-Verdugo, F. Driencourt-Mangin, J. Plen-ter, S. Ramírez-Uribe, G. Rodrigo, G.F.R. Sborlini, W. J. Torres Bobadilla and S. Tracz, "Causality, unitarity thresholds, anomalous thresholds and infrared singularities from the loop-tree duality at higher orders", JHEP 1912 (2019) 163 [arXiv:1904.08389 [hep-ph]].

Causal and anomalous thresholds

- R. Runkel, Z. Szor, J. P. Vesga and S. Weinzierl, "Causality and loop-tree duality at higher loops", Phys. Rev. Lett. 122 (2019) no.11, 111603 Erratum: [Phys. Rev. Lett. 123 (2019) no.5, 059902] [arXiv:1902.02135 [hep-ph]].
- R.Runkel, Z.Szor, J.P. Vesga and S. Weinzierl, "Integrands of loop amplitudes", arXiv:1906.02218 [hep-ph]
- Z. Capatti, V. Hirschi, D. Kermanschah and B. Ruijl, "Loop Tree Duality for multi-loop numerical integration", Phys. Rev. Lett. 123 (2019) no.15, 151602 [arXiv:1906.06138 [hep-ph]].
- Z. Capatti, V. Hirschi, D. Kermanschah, A. Pelloni and B. Ruijl, "Numerical Loop-Tree Duality: contour deformation and subtraction", arXiv:1912.09291 [hep-ph].

Alternative dual representation

- J.J. Aguilera-Verdugo, F. Driencourt-Mangin, J. Hernández-Pinto, J. Plen-ter, S. Ramírez-Uribe, G. Rodrigo, G.F.R. Sborlini, W. J. Torres Bobadilla and S. Tracz, "**Open loop amplitudes and causality to all orders and powers from the loop-tree duality**", arXiv:2001.03564 [hep-ph].

Reformulation of LTD to all orders

Loop-Tree Duality

- **What** does LTD do?

  Opens any loop diagram to a forest of non-disjoint trees.

- **How** does it do?

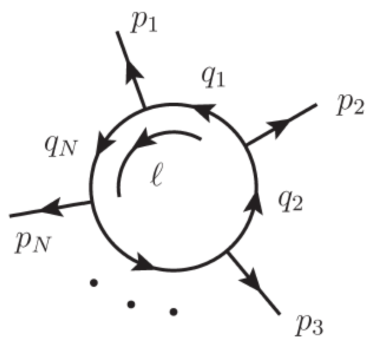
  Exploits the Cauchy residue theorem to reduce the dimension of the integration domain by one unit:

$$\int_{\boldsymbol{q}} \int dq_0 \prod_{i=1}^{N} G_F(q_i) = -2\pi i \int_{\boldsymbol{q}} \sum_{\boldsymbol{i}} Res_{\{Im\ q_0<0\}} \left[\prod_{i=1}^{N} G_F(q_i) \right]$$

Minkowski space → Euclidean space

Loop-Tree Duality

- At one loop



$$= -\sum_i \int_\ell \tilde{\delta}(q_i)\, \mathcal{N}\left(\ell, \{p_j\}_N\right) \prod_{j=1, j\neq i}^{N} G_D(q_i;\, q_j)$$

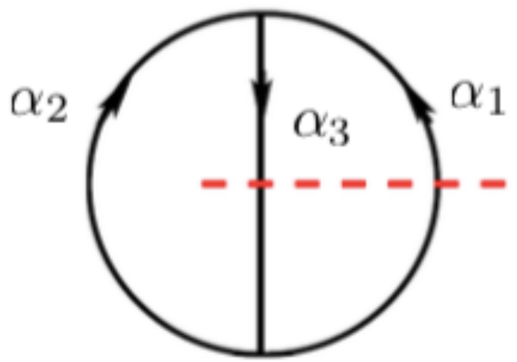
- $\tilde{\delta}(q_i) = i2\pi\theta(q_{i,0})\delta(q_i^2 - m_i^2)$ sets internal line on-shell, positive energy mode.
- $G_D(q_i;\, q_j) = \frac{1}{q_j^2 - m_j^2 - i0\eta k_{ji}}$ **dual propagator**, $k_{ji} = q_j - q_i$
- Modification of the customary $+i0$ Feynman's prescription. Only the sign matters.
- η a future-like vector, convenient to take $\eta^\mu = (\mathbf{1}, \mathbf{0})$.

Loop-Tree Duality

- Dual representation of the two-loop integral as a function of double-cut only.

$$\int_{\ell_1}\int_{\ell_2} \mathcal{N}\left(\{\ell_1,\ell_2\},\{p_j\}_N\right) G_F(\alpha_1 \cup \alpha_2 \cup \alpha_3) = \int_{\ell_1}\int_{\ell_2} \mathcal{N}\left(\{\ell_1,\ell_2\},\{p_j\}_N\right) [G_D(\alpha_1) G_D(\alpha_2 \cup \alpha_3) + G_D(-\alpha_1 \cup \alpha_2) G_D(\alpha_3) - G_D(\alpha_1) G_F(\alpha_2) G_D(\alpha_3)]$$

Reversing momentum flow

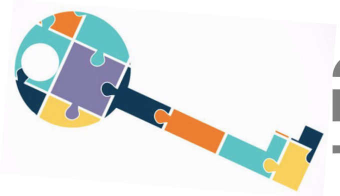


$$G_F(\alpha_k) = -\sum_{i\in\alpha_k} G_F(q_i)$$

$$G_D(\alpha_k) = -\sum_{i\in\alpha_k} \int_\ell \tilde{\delta}(q_i) \prod_{j\neq i} G_D(q_i;\, q_j)$$

New dual representation to all orders

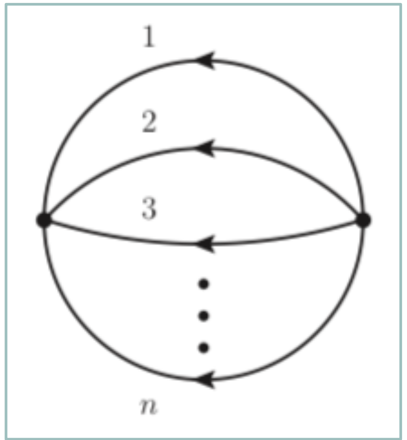
- Can we find explicit and more compact analytic expressions with the LTD formalism to all orders?



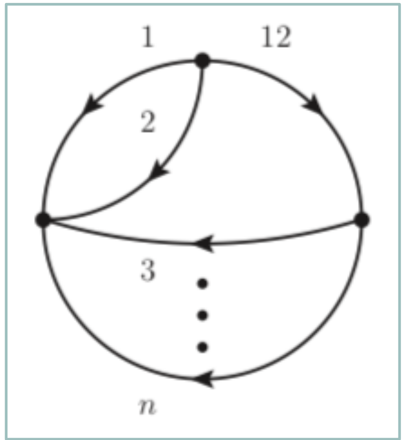
Causality

Powerful guide to identify the appropriated dual contributions to all orders

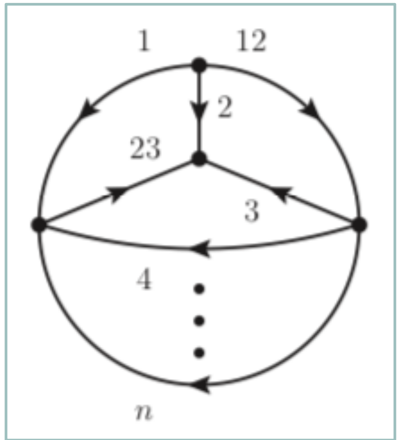
MLT



$NMLT$



N^2MLT



Notation

- s : set of internal propagators that depend on ℓ_s with momenta
$$q_{i_s} = \ell_s + k_{i_s}\,, \qquad i_s \in s$$
and k_{i_s} a linear combination of external momenta, $\{p_j\}_N$

- Feynman propagator
$$G_F(q_{i_s}) = \frac{1}{q_{i_s,0}^2 - \left(q_{i_s,0}^{(+)}\right)^2} \quad \text{where} \quad q_{i_s,0}^{(+)} = \sqrt{\boldsymbol{q}_{i_s}^2 + m_{i_s}^2 - i0}$$
$q_{i_s,0}$ and $\boldsymbol{q}_{i_s}$ time and spacial components of q_{i_s}, m_{i_s} its mass, and $i0$ the usual Feynman's infinitesimal imaginary prescription.

Notation

- Encoding in a compact way

$$G_F(1,\dots,n) = \prod_{i \in 1\cup\cdots\cup n} \left(G_F(q_i)\right)^{a_i},$$

a_i an arbitrary power.

- L-loop scattering amplitude is expressed as

$$\mathcal{A}_N^{(L)}(1,\dots,n) = \int_{\ell_1,\dots,\ell_L} \mathcal{N}\left(\{\ell_i\}_L, \{p_j\}_N\right) G_F(1,\dots,n),$$

in the Feynman representation and integration measure

$$\int_{\ell_i} = -i\mu^{4-d} \int \frac{d^d\ell_i}{(2\pi)^d} .$$

Notation

- Considering two internal lines that depend on the same loop momentum:

$$G_D(s;\ t) = -2\pi i \sum_{\boldsymbol{i_s \in s}} Res\left(G_F(s,t), Im\,\eta q_{i_s} < 0\right)$$

- Recursive residue involving several lines:

$$G_D(1,\dots,s;n) = -2\pi i \sum_{\boldsymbol{i_s \in s}} Res\left(G_D(\underbrace{1,\dots,s-1}_{\text{On-shell}};\underbrace{s,n}_{\text{Off-shell}}), Im\,\eta q_{i_s} < 0\right)$$

We also introduce numerators and define the corresponding unintegrated open dual amplitudes as

$$\mathcal{A}_D^{(L)}(1,\dots,r;n).$$

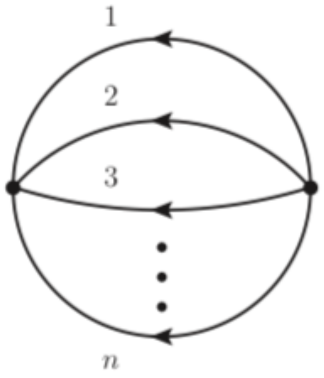
Maximal Loop Topology (*MLT*)

- MLT is a L-loop topology with $n = L + 1$ sets of propagators.

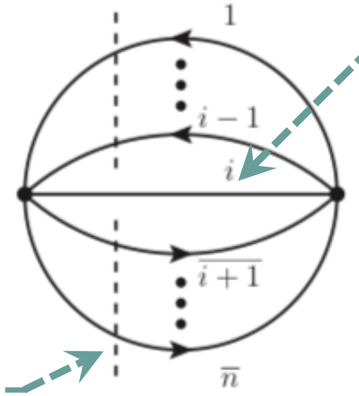
$$q_{i_s} = \begin{cases} \ell_s + k_{i_s}, & s \in \{1, \dots, L\} \\ -\sum_{s=1}^{L} \ell_s + k_{i_n}, & s = n \end{cases}$$

- Proven by induction:

$\mathcal{A}_{MLT}^{(L)}(1, \dots, n)$



$= \sum_{i=1}^{n}$



$= \int_{\ell_{1,\dots,}\ell_L} \sum_{i=1}^{n} \mathcal{A}_D^{(L)}(1, \dots, i-1, \overline{i+1}, \dots, \bar{n}; i)$

Off-shell

On-shell

Momentum flow reversed

$n - 1$ on-shell propagators

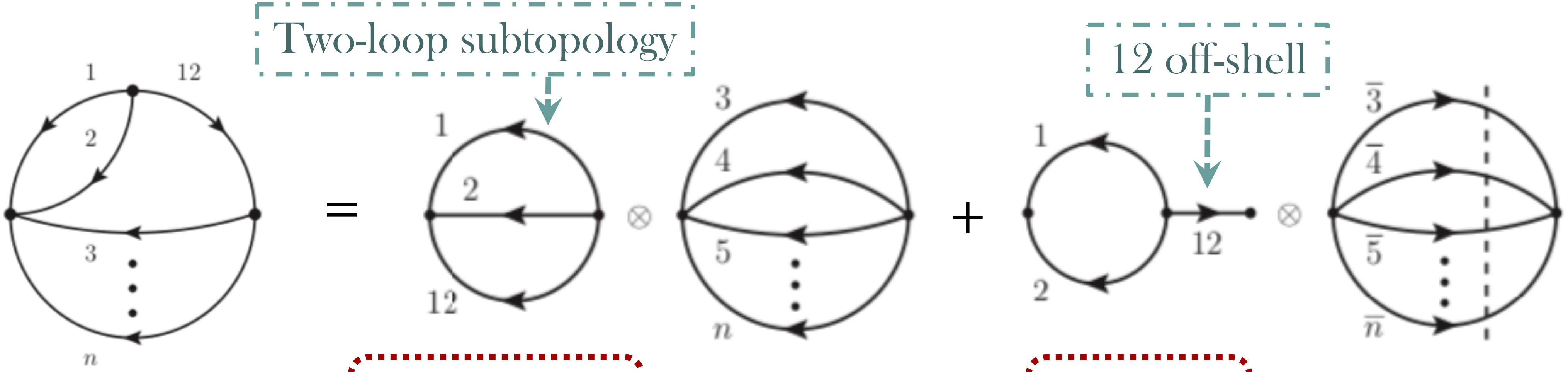
Maximal Loop Topology (*MLT*)

- We have the dual cancellations of unphysical or non-causal singularities, which are essential to support that all the causal and anomalous thresholds, and infrared singularities are restricted to a compact region (see Aguilera's talk).
- $\mathcal{A}_{MLT}^{(L)}(1,\ldots,n)$ can be written in terms of causal propagators only. We conjecture that this holds to all-orders/topologies.

Next-to-Maximal Loop Topology ($NMLT$)

- $NMLT$, the next multiloop topology in complexity contains an extra set of momenta, denoted by 12.

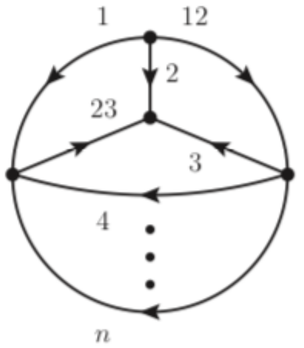
$$q_{i_s} = \begin{cases} \ell_s + k_{i_s} \ , & s \in \{1, \dots, L\} \\ -\sum_{s=1}^{L} \ell_s + k_{i_n} \ , & s = n \\ -\ell_1 - \ell_2 + k_{i_{12}} \ , & s = 12 \end{cases}$$



$$\mathcal{A}_{NMLT}^{(L)}(1, \dots, n, 12) = \mathcal{A}_{MLT}^{(2)}(1, 2, 12) \otimes \mathcal{A}_{MLT}^{(L-2)}(3, \dots, n) + \mathcal{A}_{MLT}^{(1)}(1, 2) \otimes \mathcal{A}^{(0)}(12) \otimes \mathcal{A}_{MLT}^{(L-1)}(\bar{3}, \dots, \bar{n})$$

Next-to-Next-to-Maximal Loop Topology (N^2MLT)

- N^2MLT, the next multiloop topology in complexity contains, besides the 12-set another set denoted 23.

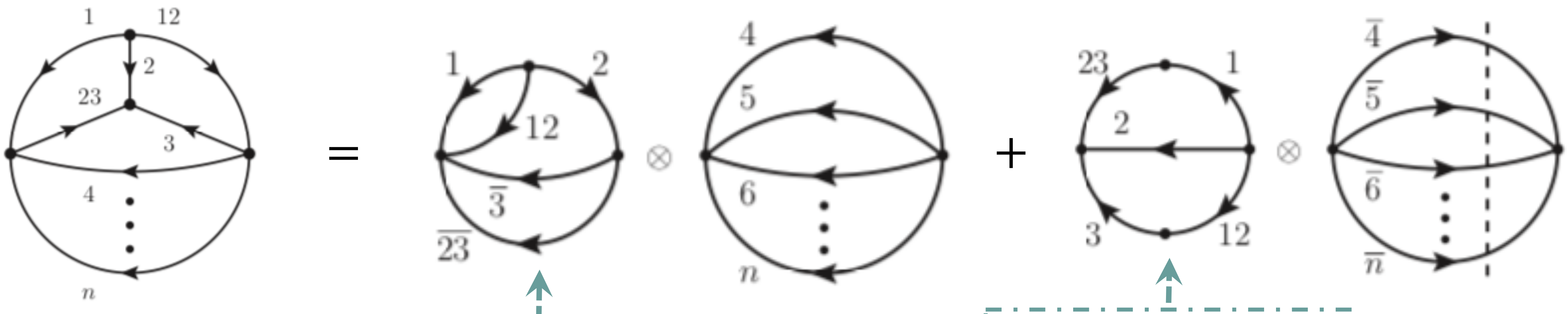


$$q_{i_s} = \begin{cases} \ell_s + k_{i_s} \;, & s \in \{1, \dots, L\} \\ -\sum_{s=1}^{L} \ell_s + k_{i_n} \;, & s = n \\ -\ell_1 - \ell_2 + k_{i_{12}} \;, & s = 12 \\ -\ell_2 - \ell_3 + k_{i_{23}} \;, & s = 23 \end{cases}$$

$$\mathcal{A}^{(L)}_{N^2MLT}(1, \dots, n, 12, 23) = \mathcal{A}^{(3)}_{NMLT}(1, 12, \bar{3}, \overline{23}, 2) \otimes \mathcal{A}^{(L-3)}_{MLT}(4, \dots, n)$$

$$+ \mathcal{A}^{(2)}_{MLT}(1 \cup 23, 2, 3 \cup 12) \otimes \mathcal{A}^{(L-2)}_{MLT}(\bar{4} \dots, \bar{n})$$

Next-to-Next-to-Maximal Loop Topology (N^2MLT)



NMLT subtopology

MLT subtopology

$$\mathcal{A}^{(2)}_{MLT}(1 \cup 23, 2, 3 \cup 12)$$

$$\mathcal{A}^{(3)}_{NMLT}(1, 12, \bar{3}, \overline{23}, 2) = \mathcal{A}^{(2)}_{MLT}(1, 12, 2) \otimes \mathcal{A}^{(1)}_{MLT}(\bar{3}, \overline{23})$$

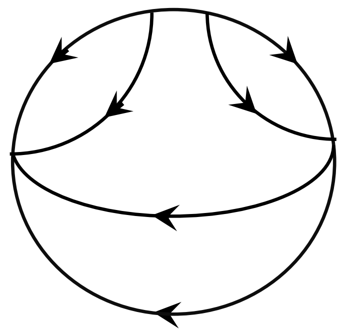
$$+ \mathcal{A}^{(1)}_{MLT}(1, 12) \otimes \mathcal{A}^{(0)}(2) \otimes \mathcal{A}^{(2)}_{MLT}(\bar{3}, \overline{23})$$

$(2, 3, 23)$ can not generate a disjoint sub-tree

Higher orders

- More complex topologies at higher orders.

For example the multiloop topology made of one MLT and two two-loop $NMLT$ subtopologies appears for the first time at four loops.



This topology is open to non-disjoint trees by leaving three loop sets off-shell and by introducing on-shell conditions in the others under certain conditions.

Conclusions

- We have obtained a very compact dual representation for selected loop topologies to all orders up to three loops. New topologies at four loops qualitatively described.
- These loop-tree dual representations exhibit a nested form in terms of simpler topologies. We conjecture that this factorization works at all orders.
- The explicit expressions presented allow us to describe any scattering amplitude up to three-loops.
- This reformulation of LTD based on the original dual relation is particularly advantageous to reveal formal aspects of multiloop scattering amplitudes.


Gracias