LUND UNIVERSITY

FACULTY OF SCIENCE

SPINOR-HELICITY DIAGRAMS AS FLOW DIAGRAMS

THE CHIRALITY-FLOW METHOD

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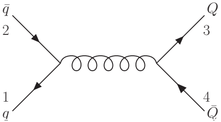
- This talk concerns the calculation of scattering amplitudes, and a new way to draw Feynman diagrams.
- We look at helicity diagrams in particular, and present an alternative graphical method to compute them.
  - The helicity diagrams are represented in terms of flow lines that connect between spinor indices.
  - They are expressed directly in terms of spinor inner products, without intermediate algebraic manipulations.
- This work is still in its early stages, and for now we look at tree-level diagrams with massless particles.
- Let us back up a little.
- Consider a very simple example (all momenta outgoing, Feynman gauge):

$$\mathcal{A}(q,\bar{q},Q,\bar{Q}) \quad = $$



$$= \quad i g_s^2 t^a_{12} t^a_{34} \; \frac{\big(\bar{u}(1)\gamma^\mu v(2)\big) g_{\mu\nu} \big(\bar{u}(3)\gamma^\nu v(4)\big)}{(p_1+p_2)^2}$$

- Textbook methods tell us to evaluate $|\mathcal{A}|^2$ by performing spin/polarization sums, using completeness relations for Dirac spinors and polarization vectors, using the Dirac equations, taking traces in Dirac space, using the Clifford algebra, etc.
- For more involved processes the algebraic manipuations become more involved, and one may resort to a more systmetic approach, e.g. by separating the kinematic quantities from the Dirac γ matrices and organizing the γ matrices in a canonical order, making extensive use of the Clifford algebra.
- For large strings of γ matrices, this is very cumbersome.
- And of course, nowadays things are done a bit different anyway.

- Dirac spinors of outgoing fermions and anti-fermions, $\bar{u}(p) = \left(\bar{u}_L(p)\,,\, \bar{u}_R(p) \right)$ and $v(p) = \begin{pmatrix} v_L(p) \\ v_R(p) \end{pmatrix}$.
- They transform under the $(1/2, 0) \oplus (0, 1/2)$ representation of the Lorentz group.
- Dirac matrix in the Weyl basis, $\gamma^\mu = \sqrt{2} \begin{pmatrix} 0 & \tau^\mu \\ \bar{\tau}^\mu & 0 \end{pmatrix}$, $\sqrt{2}\tau^{\mu,\dot{A}B} = \sigma^{\mu,\dot{A}B}$ and $\sqrt{2}\bar{\tau}^\mu_{A\dot{B}} = \bar{\sigma}^\mu_{A\dot{B}}$.
- In the massless limit, the Dirac equations decouple to the Weyl equations for massless Weyl spinors.
- In the (massless) spinor-helicity formalism:
  - Massless outgoing fermions and anti-fermions $\bar{u}^{+/-}$ and $v^{+/-}$ with positive/negative helicity:

$$\text{left-chiral} \quad \bar{u}^+(p) = \left([p|\,,\, 0 \right) = \left(\tilde{\lambda}_{p,\dot{A}}\,,\, 0 \right) \qquad v^+(p) = \begin{pmatrix} |p] \\ 0 \end{pmatrix} = \begin{pmatrix} \tilde{\lambda}^{\dot{A}}_p \\ 0 \end{pmatrix}$$

$$\text{right-chiral} \quad \bar{u}^-(p) = \left(0,\, \langle p| \right) = \left(0,\, \lambda^A_p \right) \qquad v^-(p) = \begin{pmatrix} 0 \\ |p\rangle \end{pmatrix} = \begin{pmatrix} 0 \\ \lambda_{p,A} \end{pmatrix}$$

  - Massless outgoing vector bosons (with q a light-like reference vector):

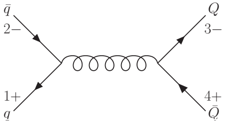
$$\epsilon^\mu_-(p,q) = \frac{\langle p|\bar{\tau}^\mu|q]}{[pq]} = \frac{\lambda^A_p \bar{\tau}^\mu_{A\dot{B}} \tilde{\lambda}^{\dot{B}}_q}{[pq]} \quad \text{or} \quad \epsilon^\mu_-(p,q) = \frac{[q|\tau^\mu|p\rangle}{[pq]} = \frac{\tilde{\lambda}_{q,\dot{B}} \tau^{\mu,\dot{B}A} \lambda_{p,A}}{[pq]}$$

$$\epsilon^\mu_+(p,q) = \frac{\langle q|\bar{\tau}^\mu|p]}{\langle qp\rangle} = \frac{\lambda^B_q \bar{\tau}^\mu_{B\dot{A}} \tilde{\lambda}^{\dot{A}}_p}{\langle qp\rangle} \quad \text{or} \quad \epsilon^\mu_+(p,q) = \frac{[p|\tau^\mu|q\rangle}{\langle qp\rangle} = \frac{\tilde{\lambda}_{p,\dot{A}} \tau^{\mu,\dot{A}B} \lambda_{q,B}}{\langle qp\rangle}$$

  - Contractions between Weyl spinors: $[ij] = \tilde{\lambda}_{i,\dot{C}} \tilde{\lambda}^{\dot{C}}_j = \epsilon_{\dot{C}\dot{D}} \tilde{\lambda}^{\dot{D}}_i \tilde{\lambda}^{\dot{C}}_i$ and $\langle ij\rangle = \lambda^C_i \lambda_{j,C} = \epsilon^{CD} \lambda_{i,D} \lambda_{i,C}$
    [here ϵ denotes the anti-symmetric Levi-Civita symbol]

- The calculation is simplified by the above; considering explicit helicities.
- Consider our simple example again, for a specific helicity configuration:

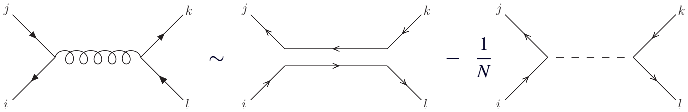
$$\mathcal{A}_{+--+}(q,\bar{q},Q,\bar{Q}) \quad =$$



$$= \quad i g_s^2 t^a_{12} t^a_{34} \, \frac{[14]\langle 32\rangle}{(p_1+p_2)^2}$$

- $|\mathcal{A}|^2 = \sum_h |\mathcal{A}_h|^2$.
- Expressed in terms of the spinor inner products $[ij]$ and $\langle ij\rangle$.
- Still requires the use of algebraic relations: $[i|\tau^\mu|j\rangle g_{\mu\nu}\langle k|\bar{\tau}^\nu|l] = [il]\langle kj\rangle$, $[i|\tau^\mu|j\rangle = \langle j|\bar{\tau}^\mu|i]$, etc.
- Explicit representations of the Pauli matrices may be chosen and the diagrams coded numerically, instead of reducing them algebraically to spinor inner products first.
- Either way, still left with intermediate algebraic manipulations and/or explicit matrix multiplications.
- For more involved processes it is not immediately obvious, which spinor inner products will result.

- Can we improve on this?
- Yes, using the Weyl-van-der-Waerden (WvdW) formalism, using that objects in higher-dimensional representations of the Lorentz group may be reduced to the irreducible $(1/2, 0)$ and $(0, 1/2)$ reps.
- Let's back up again.
  - The (restricted) Lorentz group is generated by spatial rotations $\vec{J}$ and Lorentz boosts $\vec{K}$.
  - Two copies of $SU(2)$ generators: $\vec{N}^{\pm} = \frac{1}{2}(\vec{J} \pm i\vec{K})$, with $[N_i^{\pm}, N_j^{\pm}] = i\epsilon_{ijk}N_k^{\pm}$ and $[N_i^{\pm}, N_j^{\mp}] = 0$.
  - $so(3,1) \simeq su(2)_{\text{left}} \oplus su(2)_{\text{right}}$.
  - Representations: $(0,0)$, scalars,
    $(1/2, 0)$ and $(0, 1/2)$, left- and right-chiral Weyl spinors,
    $(1/2, 0) \oplus (0, 1/2)$, Dirac spinors,
    $(1/2, 1/2)$, vectors.
- Can we use similar flow diagram methods as we use for $SU(N)$ amplitudes?
  - $SU(N)$ Fierz identity: $t^a_{ij} t^a_{kl} \sim \delta_{il}\delta_{kj} - \frac{1}{N} \delta_{ij}\delta_{kl}$



  - $SU(N)$: One set of $SU(N)$ generators $t^a \longrightarrow \delta$'s (e.g. color flow)
  - $so(3,1) \simeq su(2)_{\text{left}} \oplus su(2)_{\text{right}}$: τ^{μ}, $\bar{\tau}^{\mu}$, and external spinors $\longrightarrow [ij]$, $\langle kl \rangle$
- Short answer: Yes. But there are some steps involved.

- For Minkowski four-vectors the conversion to the $(1/2,1/2)$ representation is done by the τ matrices:

$$p_\mu\gamma^\mu = \begin{pmatrix} 0 & p_\mu\sigma^\mu \\ p_\mu\bar\sigma^\mu & 0 \end{pmatrix} = \begin{pmatrix} 0 & \not{p} \\ \bar{\not{p}} & 0 \end{pmatrix} \longleftrightarrow \sqrt{2}\begin{pmatrix} 0 & p_\mu\tau^{\mu,\dot{A}B} \\ p_\mu\bar\tau^\mu_{A\dot{B}} & 0 \end{pmatrix} = \sqrt{2}\begin{pmatrix} 0 & p^{\dot{A}B} \\ p_{A\dot{B}} & 0 \end{pmatrix}$$

For $p^2=0\ ,\quad \not{p} = |p]\langle p| \quad \longleftrightarrow \quad \sqrt{2}p^{\dot{A}B} = \tilde\lambda^{\dot{A}}_p\lambda^B_p$

For $p^2=0\ ,\quad \bar{\not{p}} = |p\rangle[p| \quad \longleftrightarrow \quad \sqrt{2}\bar{p}_{A\dot{B}} = \lambda_{p,A}\tilde\lambda_{p,\dot{B}}$

For $p=\sum_i c_ip_i\,,\ p_i^2=0\ ,\quad \not{p} = \sum_i c_i|p_i]\langle p_i| \quad \longleftrightarrow \quad \sqrt{2}p^{\dot{A}B} = \sum_i c_i\tilde\lambda^{\dot{A}}_{p_i}\lambda^B_{p_i}$

For $p=\sum_i c_ip_i\,,\ p_i^2=0\ ,\quad \bar{\not{p}} = \sum_i c_i|p_i\rangle[p_i| \quad \longleftrightarrow \quad \sqrt{2}\bar{p}_{A\dot{B}} = \sum_i c_i\lambda_{p_i,A}\tilde\lambda_{p_i,\dot{B}}$

- Also for the polarization vectors of massless outgoing vector bosons we have bispinor representations:

$$\epsilon^\mu_-\tau_\mu = \frac{|q]\langle p|}{[pq]} \leftrightarrow \epsilon_-^{\dot{B}A} = \frac{\tilde\lambda^{\dot{B}}_q\lambda^A_p}{[pq]} \quad \text{or} \quad \epsilon^\mu_-\bar\tau_\mu = \frac{|p\rangle[q|}{[pq]} \leftrightarrow \bar\epsilon_{-,A\dot{B}} = \frac{\lambda_{p,A}\tilde\lambda_{q,\dot{B}}}{[pq]}$$

$$\epsilon^\mu_+\tau_\mu = \frac{|p]\langle q|}{\langle qp\rangle} \leftrightarrow \epsilon_+^{\dot{A}B} = \frac{\tilde\lambda^{\dot{A}}_p\lambda^B_q}{\langle qp\rangle} \quad \text{or} \quad \epsilon^\mu_+\bar\tau_\mu = \frac{|q\rangle[p|}{\langle qp\rangle} \leftrightarrow \bar\epsilon_{+,B\dot{A}} = \frac{\lambda_{q,B}\tilde\lambda_{p,\dot{A}}}{\langle qp\rangle}$$

Switching notation a bit: $p_i \leftrightarrow i$ and $q \leftrightarrow r$

- What do we have so far?
- Spinors

fermion | anti-fermion

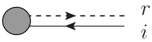
left-chiral $\tilde{\lambda}_{i,\dot{A}} \leftrightarrow [i| =$ 

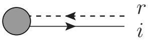
$\tilde{\lambda}_j^{\dot{A}} \leftrightarrow |j] =$ 

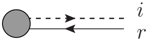
right-chiral $\lambda_i^{A} \leftrightarrow \langle i| =$ 

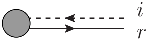
$\lambda_{j,A} \leftrightarrow |j\rangle =$ 

- Vector bosons (polarization bispinors)

$$\epsilon_-^{\dot{B}A} = \frac{\tilde{\lambda}_r^{\dot{B}}\lambda_i^{A}}{\tilde{\lambda}_{i,\dot{C}}\tilde{\lambda}_r^{\dot{C}}} \leftrightarrow \frac{|r]\langle i|}{[ir]} = \frac{1}{[ir]}$$ 

$$\bar{\epsilon}_{-,B\dot{A}} = \frac{\lambda_{i,B}\tilde{\lambda}_{r,\dot{A}}}{\tilde{\lambda}_{i,\dot{C}}\tilde{\lambda}_r^{\dot{C}}} \leftrightarrow \frac{|i\rangle[r|}{[ir]} = \frac{1}{[ir]}$$ 

$$\epsilon_+^{\dot{B}A} = \frac{\tilde{\lambda}_i^{\dot{B}}\lambda_r^{A}}{\lambda_r^{C}\lambda_{i,C}} \leftrightarrow \frac{|i]\langle r|}{\langle ri\rangle} = \frac{1}{\langle ri\rangle}$$ 

$$\bar{\epsilon}_{+,B\dot{A}} = \frac{\lambda_{r,B}\tilde{\lambda}_{i,\dot{A}}}{\lambda_r^{C}\lambda_{i,C}} \leftrightarrow \frac{|r\rangle[i|}{\langle ri\rangle} = \frac{1}{\langle ri\rangle}$$ 

- Spinor inner products

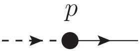
$[ij] =$ 

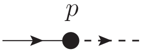
$[ji] =$ 

$\langle ij\rangle =$ 

$\langle ji\rangle =$ 

- Four-momenta (momentum bispinors; introducing a momentum-dot notation)

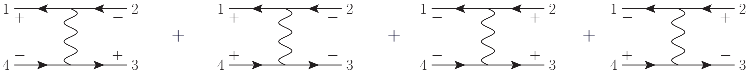
$\not{p} \leftrightarrow \sqrt{2}p^{\dot{A}B} =$ 

and $\bar{\not{p}} \leftrightarrow \sqrt{2}\bar{p}_{A\dot{B}} =$ 

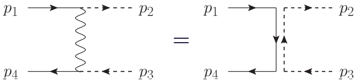
We are only interested in the Lorentz structure; use only a photon line.

- How does this look in our example?
- Look at all helicity configurations:

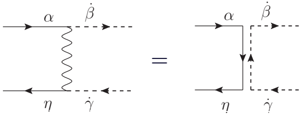
$$(\tilde\lambda_{1,\dot\alpha}\tau^{\mu,\dot\alpha\beta}\lambda_{2,\beta})(\tilde\lambda_{3,\dot\gamma}\tau_\mu^{\dot\gamma\eta}\lambda_{4,\eta}) + (\tilde\lambda_{1,\dot\alpha}\tau^{\mu,\dot\alpha\beta}\lambda_{2,\beta})(\lambda_3^\gamma\bar\tau_{\mu,\gamma\dot\eta}\tilde\lambda_4^{\dot\eta}) + (\lambda_1^\alpha\bar\tau^\mu_{\alpha\dot\beta}\tilde\lambda_2^{\dot\beta})(\tilde\lambda_{3,\dot\gamma}\tau_\mu^{\dot\gamma\eta}\lambda_{4,\eta}) + (\lambda_1^\alpha\bar\tau^\mu_{\alpha\dot\beta}\tilde\lambda_2^{\dot\beta})(\lambda_3^\gamma\bar\tau_{\mu,\gamma\dot\eta}\tilde\lambda_4^{\dot\eta})$$

$=$ 

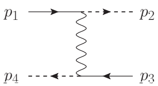
- The third term with (external) flow lines (remember, $+ \leftrightarrow$ dotted, $- \leftrightarrow$ undotted, flow arrows initially against fermion arrows):

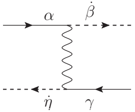


- This corresponds to: $\bar\tau^\mu_{A\dot B}\tau_\mu^{\dot C D}\lambda_1^A\tilde\lambda_2^{\dot B}\tilde\lambda_{3,\dot C}\lambda_{4,D} = \langle 1\,4\rangle[3\,2]$
- Without external spinors:

$\bar\tau^\mu_{A\dot B}\tau_\mu^{\dot C D} =$  $= \delta_A^{\ D}\delta^{\dot C}_{\ \dot B}$

- How about, say, the fourth term?

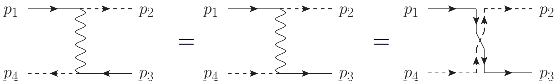


with $\bar{\tau}^{\mu}_{A\dot{B}}\bar{\tau}_{\mu,C\dot{D}}$ =


$= \epsilon_{AC}\epsilon_{\dot{B}\dot{D}}$

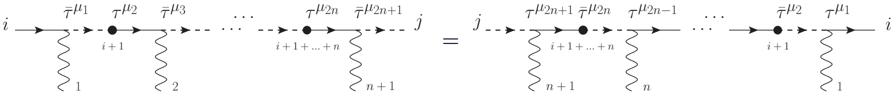
- Non-matching flow arrows: Inserting $g^{\mu\nu} = \mathrm{Tr}(\tau^{\mu}\bar{\tau}^{\nu})$, converting four-vectors to bispinors, one not always has contractions $\bar{\tau}^{\mu}\tau_{\mu} = \delta\delta$, but also $\tau^{\mu}\tau_{\mu} = \epsilon\epsilon$ and $\bar{\tau}^{\mu}\bar{\tau}_{\mu} = \epsilon\epsilon$.
- However, flow arrows at a fermion-photon vertex may be flipped, i.e. adjusted such that the diagram has matching flow arrows:

- Using $\lambda_i^{A}\bar{\tau}^{\mu}_{A\dot{B}}\tilde{\lambda}_j^{\dot{B}} = \tilde{\lambda}_{j,\dot{D}}\tau^{\mu,\dot{D}C}\lambda_{i,C}$, i.e. $\langle i|\bar{\tau}^{\mu}|j] = [j|\tau^{\mu}|i\rangle$.
- Trade a $\bar{\tau}$ for a τ, if squeezed between external spinors, and if the corresponding index lowering or raising operations are performed.
- We get, as expected,

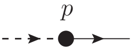


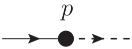
$= \langle 1\,3\rangle[4\,2]$

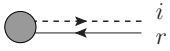
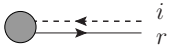
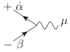
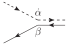
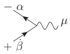
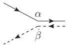
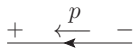
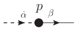
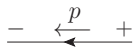
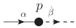
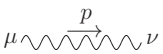
- In general we can show that $\lambda_i \bar{\tau}^{\mu_1} \tau^{\mu_2} \bar{\tau}^{\mu_3} \dots \tau^{\mu_{2n}} \bar{\tau}^{\mu_{2n+1}} \tilde{\lambda}_j = \tilde{\lambda}_j \tau^{\mu_{2n+1}} \bar{\tau}^{\mu_{2n}} \tau^{\mu_{2n-1}} \dots \bar{\tau}^{\mu_2} \tau^{\mu_1} \lambda_i$
  or $\langle i | \bar{\tau}^{\mu_1} \tau^{\mu_2} \bar{\tau}^{\mu_3} \dots \tau^{\mu_{2n}} \bar{\tau}^{\mu_{2n+1}} | j] = [j | \tau^{\mu_{2n+1}} \bar{\tau}^{\mu_{2n}} \tau^{\mu_{2n-1}} \dots \bar{\tau}^{\mu_2} \tau^{\mu_1} | i\rangle$
- Or diagrammatically

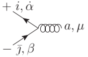
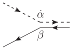
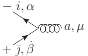
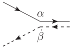
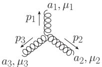
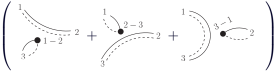
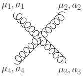
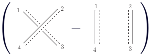
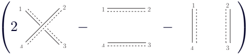
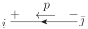
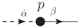
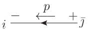
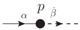
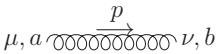


- In each multi-photon fermion line we may flip the chirality-flow arrows.
- We may connect one or more such objects to form QED tree-level diagrams.
- To match arrows:
  - For internal photons, we use that the flow arrows at fermion-photon vertices can be flipped.
  - For external photons, we use the flip of flow arrows between each polarization vector's two bispinor representations.

- For QCD diagrams non-abelian vertices appear.
- The four-gluon vertex only has $g^{\mu\nu}$ factors, the tripple-gluon vertex has terms with $g^{\mu\nu} p^{\kappa}$, where p^{κ} is a linear combination of external momenta (at the tree level).
- Essentially what we can show is that in each Feynman rule we may always represent:
  - Each metric $g^{\mu\nu}$ by a double-line  or 
  - Each four-momentum p^{μ} by 

or 

(Outgoing) Species	Dirac	Weyl	Bra-Ket	Feynman	Chirality-Flow
RH fermion	$\bar{u}(p_i)P_R = \left(0\,,\ (u_L(p_i))^\dagger\right)$	λ_i^α	$\langle i\|$		
RH anti-fermion	$P_R v(p_j) = \begin{pmatrix} 0 \\ v_R(p_j)\end{pmatrix}$	$\lambda_{j,\alpha}$	$\|j\rangle$		
LH fermion	$\bar{u}(p_i)P_L = \left((u_R(p_i))^\dagger\,,\ 0\right)$	$\tilde{\lambda}_{i,\dot\alpha}$	$[i\|$		
LH anti-fermion	$P_L v(p_j) = \begin{pmatrix} v_L(p_j) \\ 0\end{pmatrix}$	$\tilde{\lambda}_j^{\dot\alpha}$	$\|j]$		
photon	$\epsilon_-^\mu(p_i, r)$	$\frac{\lambda_i^\alpha \bar{\tau}^\mu_{\alpha\dot\beta}\tilde{\lambda}_r^{\dot\beta}}{[ir]}$ or $\frac{\tilde{\lambda}_{r,\dot\beta}\tau^{\mu,\dot\beta\alpha}\lambda_{i,\alpha}}{[ir]}$	$\frac{\langle i\|\bar{\tau}^\mu\|r]}{[ir]}$ or $\frac{[r\|\tau^\mu\|i\rangle}{[ir]}$		$\frac{1}{[ir]}$  or $\frac{1}{[ir]}$ 
photon	$\epsilon_+^\mu(p_i, r)$	$\frac{\lambda_r^\beta \bar{\tau}^\mu_{\beta\dot\alpha}\tilde{\lambda}_i^{\dot\alpha}}{\langle ri\rangle}$ or $\frac{\tilde{\lambda}_{i,\dot\alpha}\tau^{\mu,\dot\alpha\beta}\lambda_{r,\beta}}{\langle ri\rangle}$	$\frac{\langle r\|\bar{\tau}^\mu\|i]}{\langle ri\rangle}$ or $\frac{[i\|\tau^\mu\|r\rangle}{\langle ri\rangle}$		$\frac{1}{\langle ri\rangle}$  or $\frac{1}{\langle ri\rangle}$ 
Vertices	**Dirac**	**Weyl**	**Bra-Ket**	**Feynman**	**Chirality-Flow**
fermion-photon	$ieQ_f\gamma^\mu$	$ieQ_f\sqrt{2}\tau^{\mu,\dot\alpha\beta}$	$ieQ_f\sqrt{2}\tau^{\mu,\dot\alpha\beta}$		$ieQ_f\sqrt{2}$ 
fermion-photon	$ieQ_f\gamma^\mu$	$ieQ_f\sqrt{2}\bar{\tau}^\mu_{\alpha\dot\beta}$	$ieQ_f\sqrt{2}\bar{\tau}^\mu_{\alpha\dot\beta}$		$ieQ_f\sqrt{2}$ 
Propagators	**Dirac**	**Weyl**	**Bra-Ket**	**Feynman**	**Chirality-Flow**
fermion	$\frac{i}{\not{p}} = \frac{i\not{p}}{p^2}$	$i\frac{\tilde{\lambda}_p^{\dot\alpha}\lambda_p^\beta}{p^2}$	$i\frac{\|p]\langle p\|}{p^2}$		$\frac{i}{p^2}$ 
fermion	$\frac{i}{\not{p}} = \frac{i\not{p}}{p^2}$	$i\frac{\lambda_{p,\alpha}\tilde{\lambda}_{p,\dot\beta}}{p^2}$	$i\frac{\|p\rangle[p\|}{p^2}$		$\frac{i}{p^2}$ 
photon	$-i\frac{g_{\mu\nu}}{p^2}$				$-\frac{i}{p^2}$  or $-\frac{i}{p^2}$ 

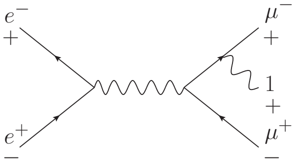
Vertices	Dirac	Weyl	Bra-Ket	Feynman	Chirality-Flow
quark-gluon	$i\frac{g_s}{\sqrt{2}}t^a_{i\bar{\jmath}}\gamma^\mu$	$ig_s t^a_{i\bar{\jmath}}\tau^{\mu,\dot{\alpha}\beta}$	$ig_s t^a_{i\bar{\jmath}}\tau^{\mu,\dot{\alpha}\beta}$		$ig_s t^a_{i\bar{\jmath}}$ 
quark-gluon	$i\frac{g_s}{\sqrt{2}}t^a_{i\bar{\jmath}}\gamma^\mu$	$ig_s t^a_{i\bar{\jmath}}\bar{\tau}^\mu_{\alpha\dot{\beta}}$	$ig_s t^a_{i\bar{\jmath}}\bar{\tau}^\mu_{\alpha\dot{\beta}}$		$ig_s t^a_{i\bar{\jmath}}$ 
three-gluon	$i\frac{g_s}{\sqrt{2}}(if^{abc})V_3^{\mu_1\mu_2\mu_2}$				$i\frac{g_s}{\sqrt{2}}(if^{abc})\frac{1}{\sqrt{2}}$ 
four-gluon	$i\left(\frac{g_s}{\sqrt{2}}\right)^2\sum_{Z(2,3,4)}(if^{a_1a_2b})(if^{ba_3a_4})\times$ $\times(g^{\mu_1\mu_3}g^{\mu_2\mu_4}-g^{\mu_1\mu_4}g^{\mu_2\mu_3})$				$i\left(\frac{g_s}{\sqrt{2}}\right)^2\sum_{Z(2,3,4)}(if^{a_1a_2b})(if^{ba_3a_4})$  $=i\left(\frac{g_s}{\sqrt{2}}\right)^2\sum_{S(2,3,4)}\mathrm{Tr}\left(t^{a_1}t^{a_2}t^{a_3}t^{a_4}\right)\times$ $\times$ 
Propagators	**Dirac**	**Weyl**	**Bra-Ket**	**Feynman**	**Chirality-Flow**
fermion	$\frac{i\delta_{i\bar{\jmath}}}{\not{p}}=\frac{i\delta_{i\bar{\jmath}}\not{p}}{p^2}$	$i\frac{\delta_{i\bar{\jmath}}\tilde{\lambda}^{\dot{\alpha}}_p\lambda^\beta_p}{p^2}$	$i\frac{\delta_{i\bar{\jmath}}\lvert p]\langle p\rvert}{p^2}$		$\frac{i\delta_{i\bar{\jmath}}}{p^2}$ 
fermion	$\frac{i\delta_{i\bar{\jmath}}}{\not{p}}=\frac{i\delta_{i\bar{\jmath}}\not{p}}{p^2}$	$i\frac{\delta_{i\bar{\jmath}}\lambda_{p,\alpha}\tilde{\lambda}_{p,\dot{\beta}}}{p^2}$	$i\frac{\delta_{i\bar{\jmath}}\lvert p\rangle[p\rvert}{p^2}$		$\frac{i\delta_{i\bar{\jmath}}}{p^2}$ 
gluon	$-i\frac{\delta^{ab}g_{\mu\nu}}{p^2}$				$-\frac{i\delta^{ab}}{p^2}$  or $-\frac{i\delta^{ab}}{p^2}$ 

1. Collect all powers of i, $\sqrt{2}$ and coupling constants from vertices and propagators, plus denominators from propagators and polarization vectors.

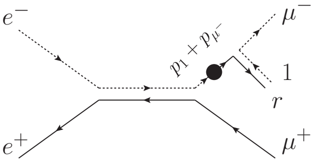
2. Assign chirality-flow lines, i.e. dotted and undotted lines, but ignore arrows as this step.
   - External fermions and anti-fermions a single dotted or undotted line, plus momentum label.
   - External vector bosons a double-line, one dotted and one undotted, and two momentum labels (the actual and the reference momentum).
   - Vector boson propagators a double-line, one dotted and one undotted.
   - Fermion propagators a pair of successive lines, turning from dotted to undotted (or vice versa), joined by a momentum dot.
   - Using the appropriate vertices, all lines are connected in the only possible way to form a chirality-flow diagram.

3. Assign chirality-flow arrows.
   - Start with any external chirality-flow line, and assign an arrow in an arbitrary direction.
   - Follow the line through the diagram, continuing through any potential momentum dot.
   - Assign the other arrows such that double-lines from vector bosons or non-abelian vertices have opposing arrows.
   - Non-abelian vertices give rise to disconnected pieces, i.e. lines not related to each other by either momentum dots or sharing double-lines; Apply the above for each disconnected piece independently.

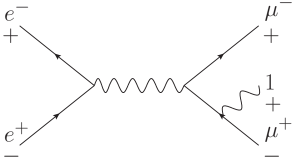
- Due to the sums of chirality flows from the non-abelian vertices, each Feynman diagram is now turned into a sum of chirality-flow diagrams.
- If it is desired to obtain the result in conventional form with spinor brackets, expand the momentum dots and translate the lines to spinor inner products.



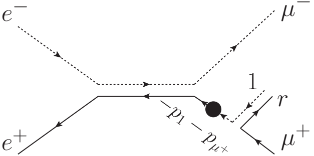
$$= \frac{-i2\sqrt{2}e^3}{s_{e^+e^-}s_{1\mu^-}\langle r1\rangle}$$



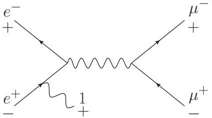
$$= \frac{-i2\sqrt{2}e^3}{s_{e^+e^-}s_{1\mu^-}\langle r1\rangle}\Big([e^-1]\langle 1r\rangle + [e^-\mu^-]\langle \mu^-r\rangle\Big)[1\mu^-]\langle \mu^+e^+\rangle$$



$$= \frac{-i2\sqrt{2}e^3}{s_{e^+e^-}s_{1\mu^+}\langle r1\rangle}$$



$$= \frac{-i2\sqrt{2}e^3}{s_{e^+e^-}s_{1\mu^+}\langle r1\rangle}[e^-\mu^-]\langle \mu^+r\rangle\Big(-0-[1\mu^+]\langle \mu^+e^+\rangle\Big)$$



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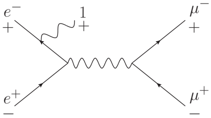
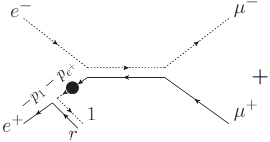
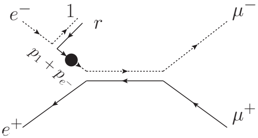


$$= \frac{-i2\sqrt{2}e^3}{s_{\mu^+\mu^-} s_{1e^+}\langle r1\rangle}$$

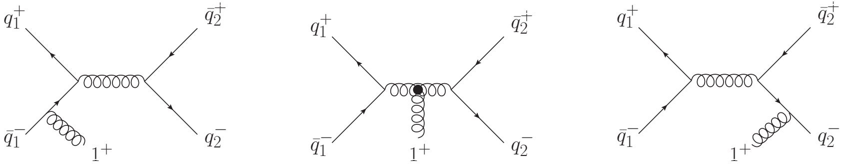


$$+ \frac{-i2\sqrt{2}e^3}{s_{\mu^+\mu^-} s_{1e^-}\langle r1\rangle}$$

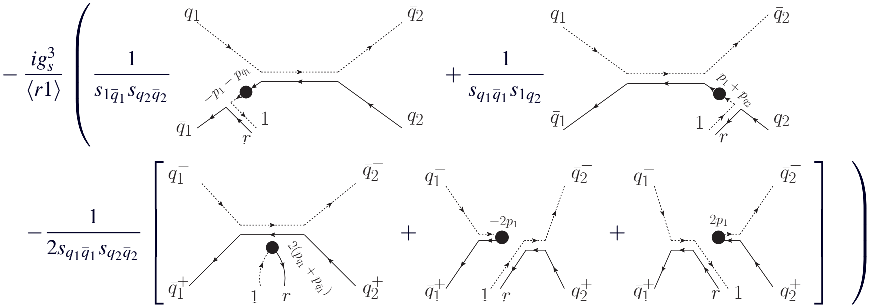


$$= \frac{i2\sqrt{2}e^3}{s_{\mu^+\mu^-} s_{1e^+}\langle r1\rangle}[e^-\mu^-]\langle\mu^+e^+\rangle[e^+1]\langle re^+\rangle + \frac{-i2\sqrt{2}e^3}{s_{\mu^+\mu^-} s_{1e^-}\langle r1\rangle}[e^-1]\Big(\langle r1\rangle[1\mu^-] + \langle re^-\rangle[e^-\mu^-]\Big)\langle\mu$$

- We have left the reference momentum unassigned.
- We may simplify the results by choosing it appropriately, but equal for all four diagrams.
- In the chirality-flow picture it's already transparent at the diagram level which choices are good choices.
- We did not need to perform a single algebraic manipulation, other than to expand momentum dots!



- Diagrams contributing to the color-ordered amplitude with color factor $t^{a_1}_{q_2\bar{q}_1}\,\delta_{q_1\bar{q}_2}$.



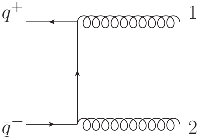
- Checked that this returns the same as with the ordinary spinor-helicity calculus.
- Here in one step, though!

- Diagrams contributing to the color-ordered amplitude with color factor $t^{a_1}_{qi}\, t^{a_2}_{i\bar{q}}$.

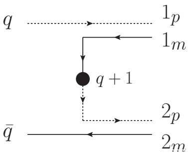
- In the following

$$h = \mp\,, \quad i_p \text{ and } i_m = \begin{cases} r \text{ and } i\,,\ h = - \\ i \text{ and } r\,,\ h = + \end{cases}, \quad f_h(i_p, i_m) = \{i_m i_p\} = \begin{cases} [i_m i_p] = [ir]\,,\ h = - \\ \langle i_m i_p \rangle = \langle ri \rangle\,,\ h = + \end{cases}$$

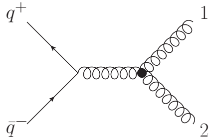
i_p/i_m denotes the positive-/negative-helicity spinor in gluon i.



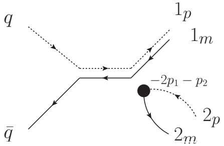
$$= \frac{-ig_s^2}{s_{q1} f_{h,1} f_{h,2}}$$



$$= \frac{-ig_s^2}{s_{q1} f_{h,1} f_{h,2}} \Big([q1_p]\big(\langle 1_m q\rangle [q2_p] + \langle 1_m 1\rangle [12_p]\big)\langle 2_m \bar{q}\rangle \Big)$$



$$= \frac{ig_s^2}{2s_{12}f_{h,1}f_{h,2}} \Bigg($$



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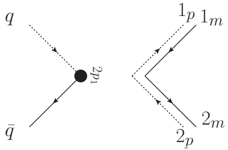

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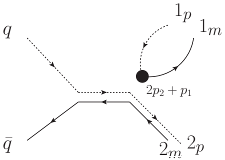

$\Bigg)$

$$= \frac{ig_s^2}{2s_{12}f_{h,1}f_{h,2}} \Big([q1_p]\langle 1_m\bar{q}\rangle(-2[2_p1]\langle 12_m\rangle + [2_p2]\langle 22_m\rangle)$$

$$+ 2\langle 1_m2_m\rangle[2_p1_p][q1]\langle 1\bar{q}\rangle) + [q2_p]\langle 2_m\bar{q}\rangle(2[1_p2]\langle 21_m\rangle - [1_p1]\langle 11_m\rangle) \Big)$$

The only non-zero case is MHV.
E.g. $h_1 = -$, $h_2 = +$, $(1_m, 1_p) = (1, r_1)$ and $(2_m, 2_p) = (r_2, 2)$. Choose $1_p = q$ and $2_m = \bar{q}$:

$$M(q^+, g_1, g_2, \bar{q}^-) = -\frac{ig_s^2\langle\bar{q}1\rangle^2[2q]}{\langle\bar{q}2\rangle\langle 12\rangle[21]}\frac{\langle\bar{q}q\rangle\langle q1\rangle}{\langle\bar{q}q\rangle\langle q1\rangle} = -\frac{ig_s^2\langle\bar{q}1\rangle^2\langle q1\rangle(-[21]\langle 1\bar{q}\rangle)}{\langle q1\rangle\langle 12\rangle\langle 2\bar{q}\rangle\langle\bar{q}q\rangle[21]} = -\frac{ig_s^2\langle\bar{q}1\rangle^3\langle q1\rangle}{\langle q1\rangle\langle 12\rangle\langle 2\bar{q}\rangle\langle\bar{q}q\rangle}$$

- We present an alternative way to draw Feynman diagrams.
  - Read spinor contractions directly from the chirality-flow diagrams; encoded in the diagrams themselves.
  - No intermediate algebraic manipulations are required.
- The double-line or flow character of the WvdW formalism has already been noted before. However, in order to arrive at flow diagrams, one needs to deal with the epsilon structures, which we do from a diagrammatic perspective with arrow flip identities. These turn fermion spinors to anti-fermion spinors and vice versa, but are nevertheless identites and easily applied in the chirality-flow picture.
- To get to the nice expressions for MHV amplitudes, for instance, still some work is required, which it is in any case. However, in the chirality-flow picture we can have a more transparent, more intuitive understanding of how certain spinor contractions arise from certain diagrams, as well as which choices of gluon reference vectors might be more sensible.
- An immediate application should be eg. to extend matrix-element programs using Feynman diagrams.
- Outlook:
  - Formulating the chirality-flow picture for massive particles and for the SM is work in progress; at this stage, though, it is clear that it will work.
  - Two things on the list afterwards:
    - Loop diagrams.
    - Amplitude-level description; gain new insights in the space-time structure of scattering amplitudes through the chirality-flow picture.

Thank you!