FORWARD JETS WITH SMALL-X TMD FACTORIZATION

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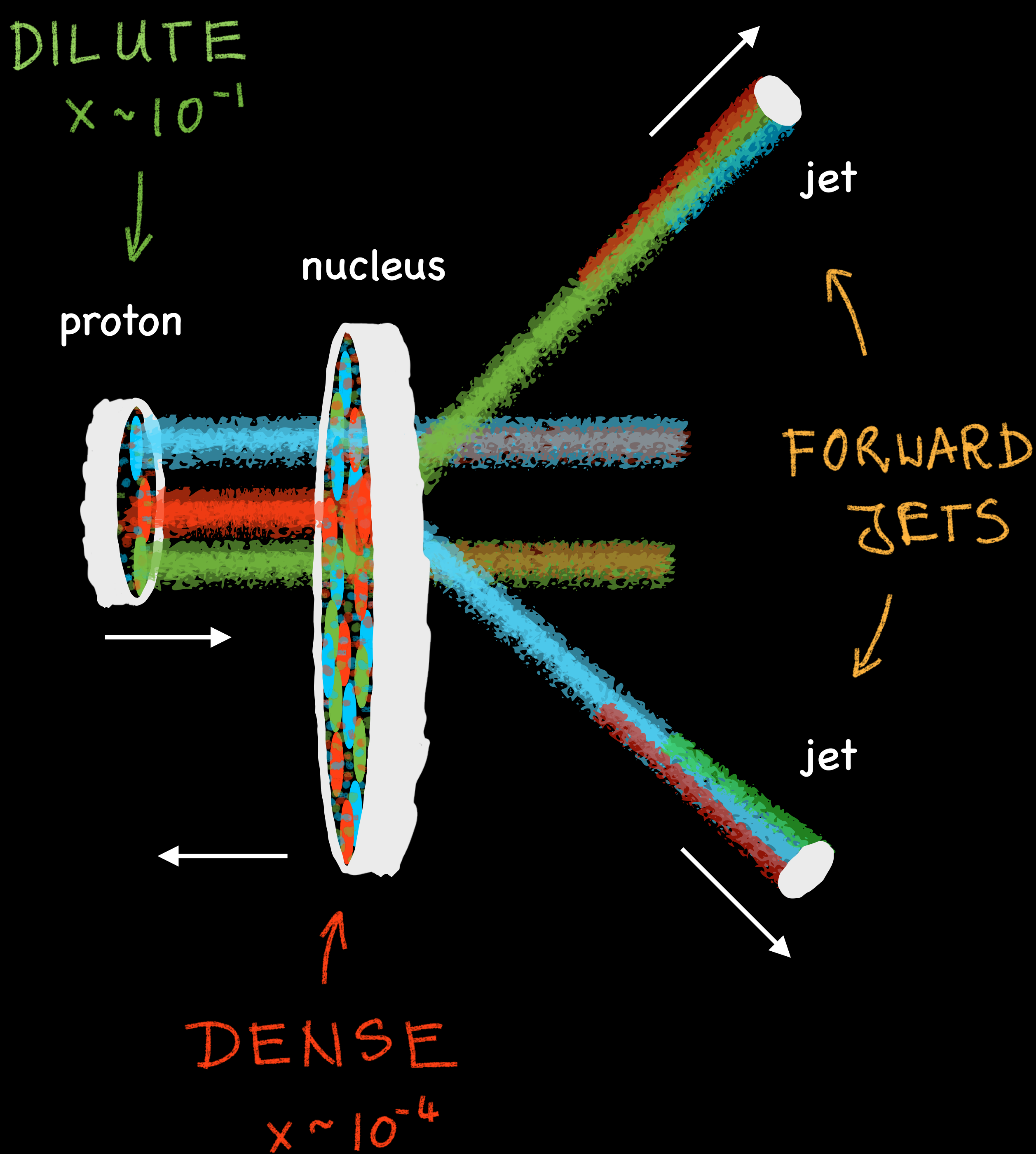
MOTIVATION



Study of high energy limit of QCD:

- small-x effects

  Resummation of large logs $\ln(1/x)$ to all orders.

  Evolution of Transverse Momentum Dependent (TMD) PDFs in x.

- k_T-factorization

  TMD factorization beyond leading power.

- saturation of gluon density

  Nonlinear evolution of TMD PDFs.

PLAN

1. Framework
    A. (Generalized) factorization formula
    B. Transverse Momentum Dependent (TMD) gluon distributions
    C. Off-shell hard factors

2. Phenomenology
    A. Obtaining TMD gluon distributions
    B. Forward dijets at LHC

3. Summary & Outlook

FRAMEWORK Small-x Improved TMD Factorization (ITMD)

Factorization formula for forward dijets in p-p and p-A collisions

[PK, K. Kutak, C. Marquet, E. Petreska, S. Sapeta, A. van Hameren, 2015]

$$\frac{d\sigma_{pA\to 2j+X}}{dy_1 dy_2 d^2p_{T1} d^2p_{T2}} \sim \sum_{a,c,d} f_{a/p}(x_1,\mu) \sum_{i=1,2} K^{(i)}_{ag\to cd}(k_T)\,\Phi^{(i)}_{ag\to cd}(x_2,k_T)$$

RAPIDITY — $x_2 \ll x_1$

TRANSVERSE MOMENTA — $|\vec{p}_{T1}+\vec{p}_{T2}| = k_T$

COLLINEAR PROTON PDF

GAUGE INVARIANT OFF-SHELL HARD FACTORS

TMD GLUON DISTRIBUTIONS (WITH OPERATOR DEFINITIONS)

TWO PER CHANNEL ($g^*q\to gq$, $g^*g\to gg$, $g^*g\to q\bar{q}$)

ITMD factorization formula has been proven from the Color Glass Condensate (CGC) theory.

⇒ RESUMMATION OF KINEMATIC TWISTS AND NEGLECTING GENUINE TWISTS.

[T. Altinoluk, R. Boussarie, PK, 2019]

$$\Lambda_{QCD} \ll Q_s \ll \mu$$

SATURATION SCALE

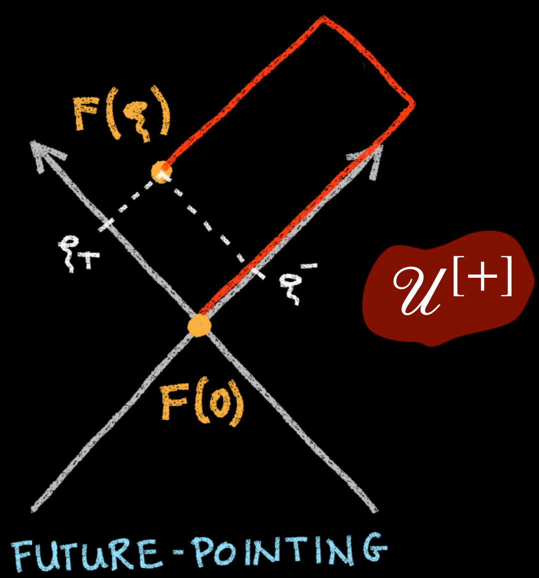
TMD gluon distributions

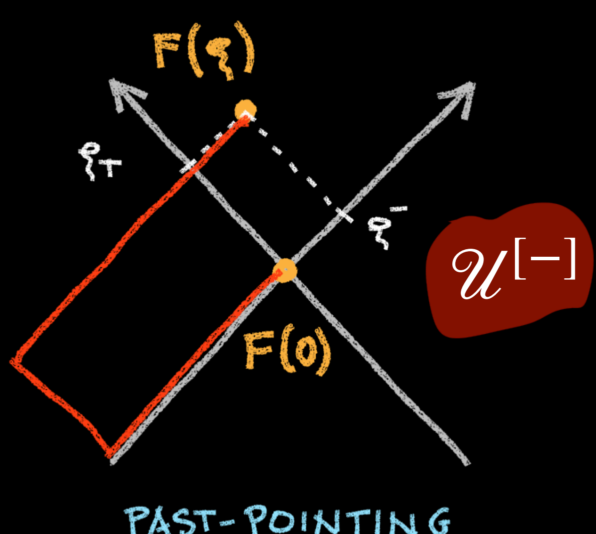
Generic operator definition (unpolarized)

$$\mathcal{F}_g(x,k_T) = 2\int \frac{d\xi^+ d^2\xi_T}{(2\pi)^3 P^-}\, e^{ixP^-\xi^+ - i\vec{k}_T\cdot\vec{\xi}_T}\,\langle P|\,\mathrm{Tr}\left[\hat{F}^{i-}\left(\xi^+,\vec{\xi}_T,\xi^-=0\right)\mathcal{U}_{C_1}\hat{F}^{i-}(0)\,\mathcal{U}_{C_2}\right]|P\rangle$$

GLUON FIELD $\hat{F} = F_a t^a$

WILSON LINES IN FUNDAMENTAL REPRESENTATION





Gauge links $\mathcal{U}_{C_1}, \mathcal{U}_{C_2}$ depend on the color structure of the hard process. They are build from two basic Wilson lines:

[C. Bomhof, P. Mulders, F. Pijlman, 2004]

$$\mathcal{U}^{[\pm]} = [0,(\pm\infty,\vec{0}_T,0)] \times [(\pm\infty,\vec{0}_T,0),(\pm\infty,\vec{\xi}_T,0)] \times [(\pm\infty,\vec{\xi}_T,0),(\xi^+,\vec{\xi}_T,0)]$$

$$[x,y] = \mathcal{P}\exp\left\{ig\int_{\overline{xy}} dz_\mu A^\mu_a(z)t^a\right\}$$

STRAIGHT LINE SEGMENT

Light-cone basis:

$$v^\pm = v^\mu n^\pm_\mu, \quad n^\pm = (1,0,0,\mp 1)$$

$$v^\mu = \frac{1}{2}v^+n^- + \frac{1}{2}v^-n^+ + v^\mu_T$$

FRAMEWORK   TMD gluon distributions: non-universality

All possible operators

$$\mathcal{F}_{qg}^{(1)} \sim \langle P|\,\mathrm{Tr}\left[\hat{F}^{i-}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i-}(0)\,\mathcal{U}^{[+]}\right]|P\rangle$$

$$\mathcal{F}_{qg}^{(2)} \sim \langle P|\,\frac{\mathrm{Tr}\,\mathcal{U}^{[\Box]}}{N_c}\mathrm{Tr}\left[\hat{F}^{i-}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i-}(0)\,\mathcal{U}^{[+]}\right]|P\rangle$$

$$\mathcal{F}_{qg}^{(3)} \sim \langle P|\,\mathrm{Tr}\left[\hat{F}^{i-}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i-}(0)\,\mathcal{U}^{[\Box]}\mathcal{U}^{[+]}\right]|P\rangle$$

$$\mathcal{F}_{gg}^{(1)} \sim \langle P|\,\frac{\mathrm{Tr}\,\mathcal{U}^{[\Box]\dagger}}{N_c}\mathrm{Tr}\left[\hat{F}^{i-}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i-}(0)\,\mathcal{U}^{[+]}\right]|P\rangle$$

$$\mathcal{F}_{gg}^{(2)} \sim \langle P|\,\mathrm{Tr}\left[\hat{F}^{i-}(\xi)\,\mathcal{U}^{[\Box]\dagger}\right]\mathrm{Tr}\left[\hat{F}^{i-}(0)\,\mathcal{U}^{[\Box]}\right]|P\rangle$$

$$\mathcal{F}_{gg}^{(3)} \sim \langle P|\,\mathrm{Tr}\left[\hat{F}^{i-}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i-}(0)\,\mathcal{U}^{[+]}\right]|P\rangle$$

$$\mathcal{F}_{gg}^{(4)} \sim \langle P|\,\mathrm{Tr}\left[\hat{F}^{i-}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i-}(0)\,\mathcal{U}^{[-]}\right]|P\rangle$$

$$\mathcal{F}_{gg}^{(5)} \sim \langle P|\,\mathrm{Tr}\left[\hat{F}^{i-}(\xi)\,\mathcal{U}^{[\Box]\dagger}\mathcal{U}^{[+]\dagger}\hat{F}^{i-}(0)\,\mathcal{U}^{[\Box]}\mathcal{U}^{[+]}\right]|P\rangle$$

$$\mathcal{F}_{gg}^{(6)} \sim \langle P|\,\frac{\mathrm{Tr}\,\mathcal{U}^{[\Box]}}{N_c}\frac{\mathrm{Tr}\,\mathcal{U}^{[\Box]\dagger}}{N_c}\mathrm{Tr}\left[\hat{F}^{i-}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i-}(0)\,\mathcal{U}^{[+]}\right]|P\rangle$$

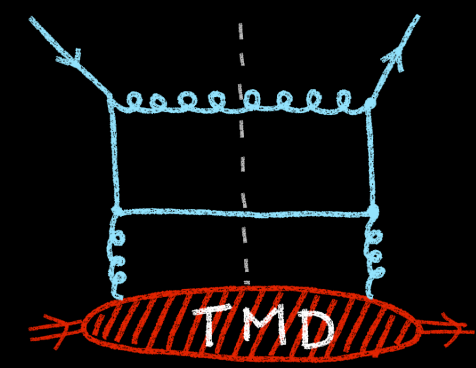
$$\mathcal{F}_{gg}^{(7)} \sim \langle P|\,\frac{\mathrm{Tr}\,\mathcal{U}^{[\Box]}}{N_c}\mathrm{Tr}\left[\hat{F}^{i-}(\xi)\,\mathcal{U}^{[\Box]\dagger}\mathcal{U}^{[+]\dagger}\hat{F}^{i-}(0)\,\mathcal{U}^{[+]}\right]|P\rangle$$

WILSON LOOP → $\mathcal{U}^{[\Box]} = \mathcal{U}^{[+]}\mathcal{U}^{[-]\dagger}$

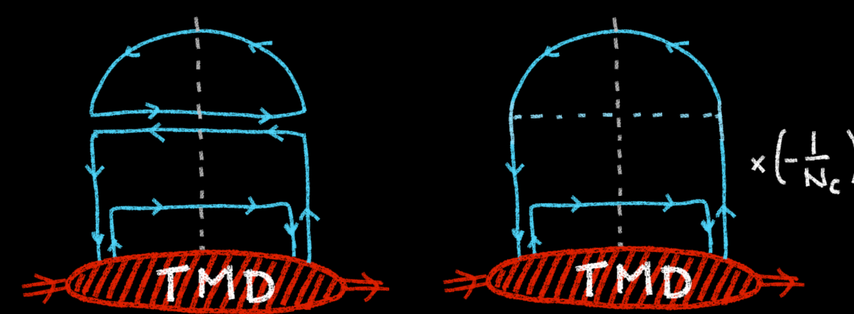
[M. Bury, PK , K. Kutak, 2018]

Example

What is the TMD gluon distribution for the following process:



Two independent color flows:



$$\rightsquigarrow \frac{N_c}{2C_F}\mathcal{F}_{qg}^{(2)} - \frac{1}{2N_cC_F}\mathcal{F}_{qg}^{(1)}$$

Gluon TMD for any multiparticle process is given by a linear combination.

TMD gluon distributions: small-x limit

Small-x limit of TMD gluon distributions

$$\int \frac{d\xi^+ d^2\xi_T}{(2\pi)^3 P^-} e^{ixP^-\xi^+ - i\vec{k}_T\cdot\vec{\xi}_T} \langle P|\,\mathrm{Tr}\left[\hat{F}^{i-}\left(\xi^+,\vec{\xi}_T,\xi^-=0\right)\mathcal{U}_{C_1}\hat{F}^{i-}(0)\,\mathcal{U}_{C_2}\right]|P\rangle$$

LIMIT
x → 0.

Dependence on x is only via the small-x evolution equations:
BFKL (Balitsky-Fadin-Kuraev-Lipatov), BK (Balitsky-Kovchegov),
JIMWLK (Balitsky-Jalilian-Marian-Iancu-McLerran-Weigert-Leonidov-Kovner), and variants.

Correspondence to CGC

DIPOLE GLUON DISTRIBUTION

Example:

$$\mathcal{F}_{qg}^{(1)} \sim \int \frac{d^2x_T d^2y_T}{(2\pi)^4} k_T^2 e^{-i\vec{k}_T\cdot(\vec{x}_T-\vec{y}_T)} \langle \mathrm{Tr}\left[U(\vec{x}_T)U^\dagger(\vec{y}_T)\right]\rangle_x$$

WILSON LINES

AVERAGE OVER CGC COLOR SOURCES

$$U(\vec{x}_T) = \mathscr{P}\exp\left\{ig\int_{-\infty}^{+\infty} dx^+\, A_a^-\left(x^+,\vec{x}_T\right)t^a\right\}$$

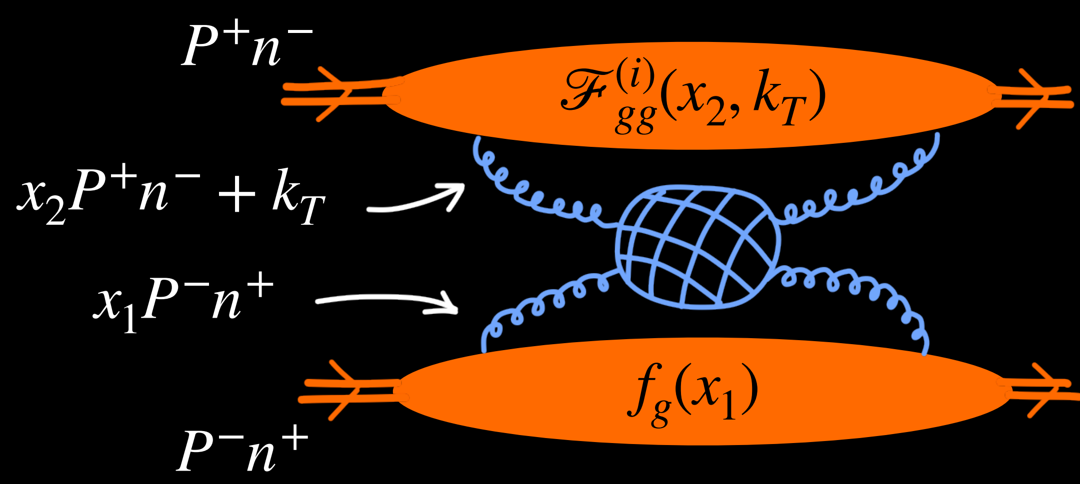
$$\langle \ldots \rangle_x \rightarrow \frac{\langle \rho_1 \ldots \rho_2 \rangle}{\langle \rho_1 \rho_2 \rangle}$$

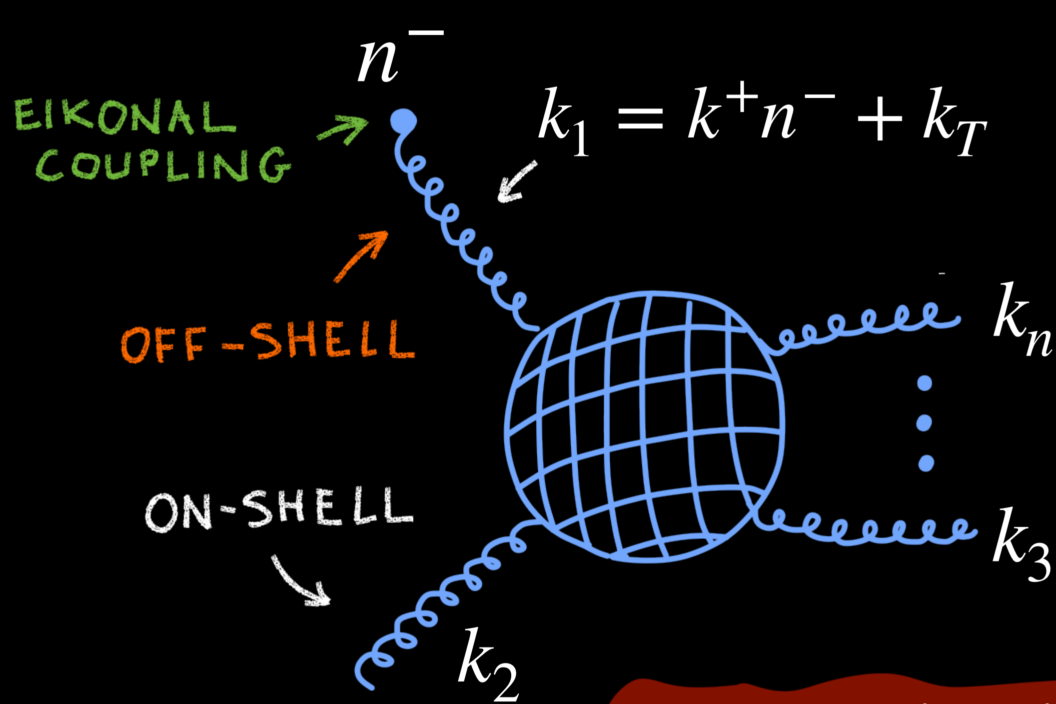
Intensively studied:

[D. Kharzeev, Y. Kovchegov, K. Tuchin, 2003]
[B. Xiao, F. Yuan, 2010]
[F. Dominguez, C. Marquet, B. Xiao, F. Yuan, 2011]
[A. Metz, J. Zhou, 2011]
[E. Akcakaya, A. Schafer, J. Zhou, 2012]
[C. Marquet, E. Petreska, C. Roiesnel, 2016]
[I. Balitsky, A. Tarasov, 2015, 2016]
[D. Boer, P. Mulders, J. Zhou, Y. Zhou, 2017]
[C. Marquet, C. Roiesnel, P. Taels, 2018]
[Y. Kovchegov, D. Pitonyak, M. Sievert, 2017,2018]
[T. Altinoluk, R. Boussarie, 2019]

FRAMEWORK Off-shell hard factors

Partonic amplitudes at high energy

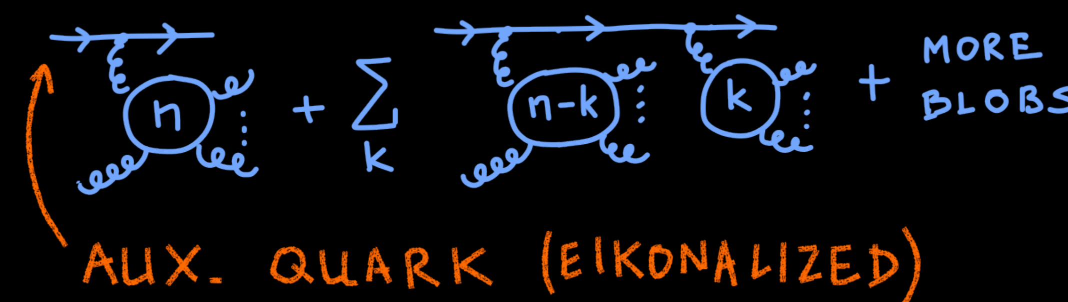




not gauge invariant

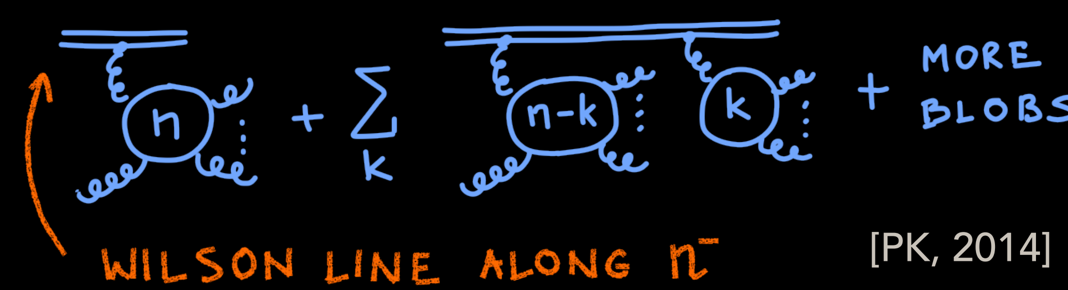
Tree-level (automatic) techniques

- "embedding"



[A. van Hameren, PK, K. Kutak, 2013]

- ME of straight infinite Wilson line



[PK, 2014]

- Berends-Giele + gauge inv. restoration

[A. Van Hameren, PK, K. Kutak, 2012]

- Off-shell BCFW

[A. Van Hameren, 2014]
[A. Van Hameren, M. Serino, 2014]

Consistent with the Lipatov's high energy effective action. [L. Lipatov, 1995]

FRAMEWORK Off-shell hard factors

Off-shell MHV tree amplitudes

$$\mathcal{M}\left(1^*,2^-,3^+,\ldots,n^+\right) \sim g^{n-2}\frac{\langle 1^* 2\rangle^4}{\langle 1^* 2\rangle\langle 23\rangle\langle 34\rangle\ldots\langle n1^*\rangle}$$

SPINOR PRODUCTS

$$\langle ij\rangle = \langle k_i - | k_j +\rangle$$

$$| k_j \pm\rangle = \frac{1}{2}(1 \pm \gamma_5)u(k_j)$$

FOR OFF-SHELL LEG:

$$\langle 1^* j\rangle = \langle k^+ n^- - | k_j +\rangle$$

gauge invariance is essential

[A. Van Hameren, PK, K. Kutak, 2012]

[A. Van Hameren, 2014]

Beyond tree level

See Maxim's talk on Lipatov's effective action approach.

[M. Nefedov, V. Saleev, 2017]

[M. Nefedov, 2019]

On going project towards automated one-loop corrections in "embedding" approach.

[A. Van Hameren, 2017]

[E. Blanco, A. Van Hameren, PK, K. Kutak, ongoing...]

PHENOMENOLOGY

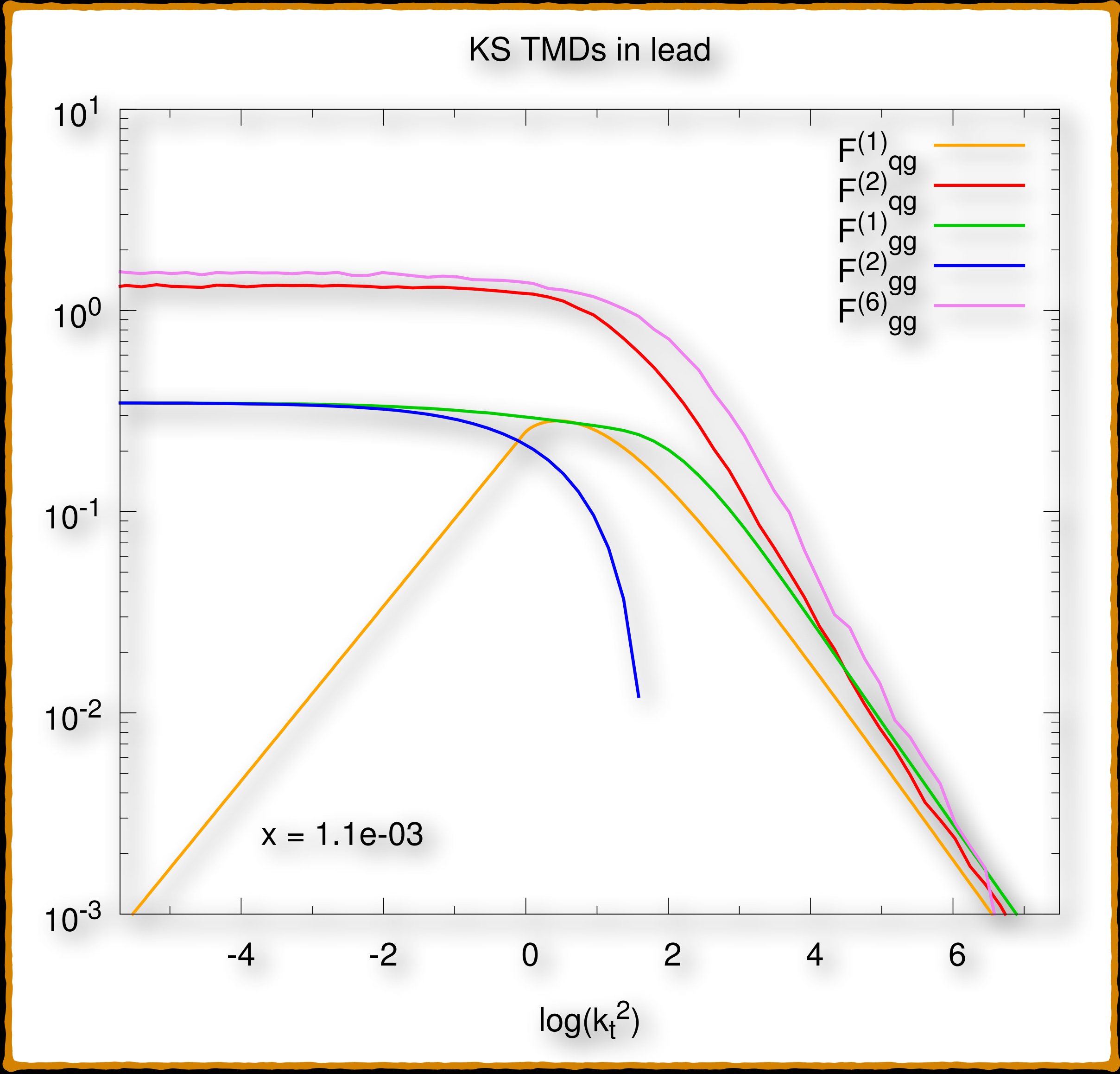
How to obtain the small-x TMD gluon distributions?

In CGC theory one can derive a relation between the TMDs (i) in the limit of large number of colors and (ii) assuming the Gaussian approximation.

All TMDs needed for dijet production can be calculated from the dipole gluon distribution.

CAN BE TAKEN FROM INCLUSIVE DIS DATA

[K. Kutak, S. Sapeta, 2012]



It is possible to relax the assumptions (i) and (ii) using the JIMWLK equation.

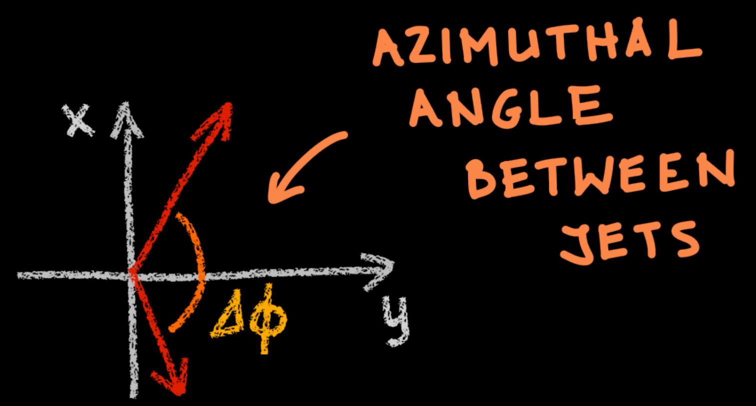
Prove of concept:

[C. Marquet, E. Petreska, C. Roiesnel, 2016]

[A. Van Hameren, PK, K. Kutak, C. Marquet, E. Petreska, S. Sapeta, 2016]

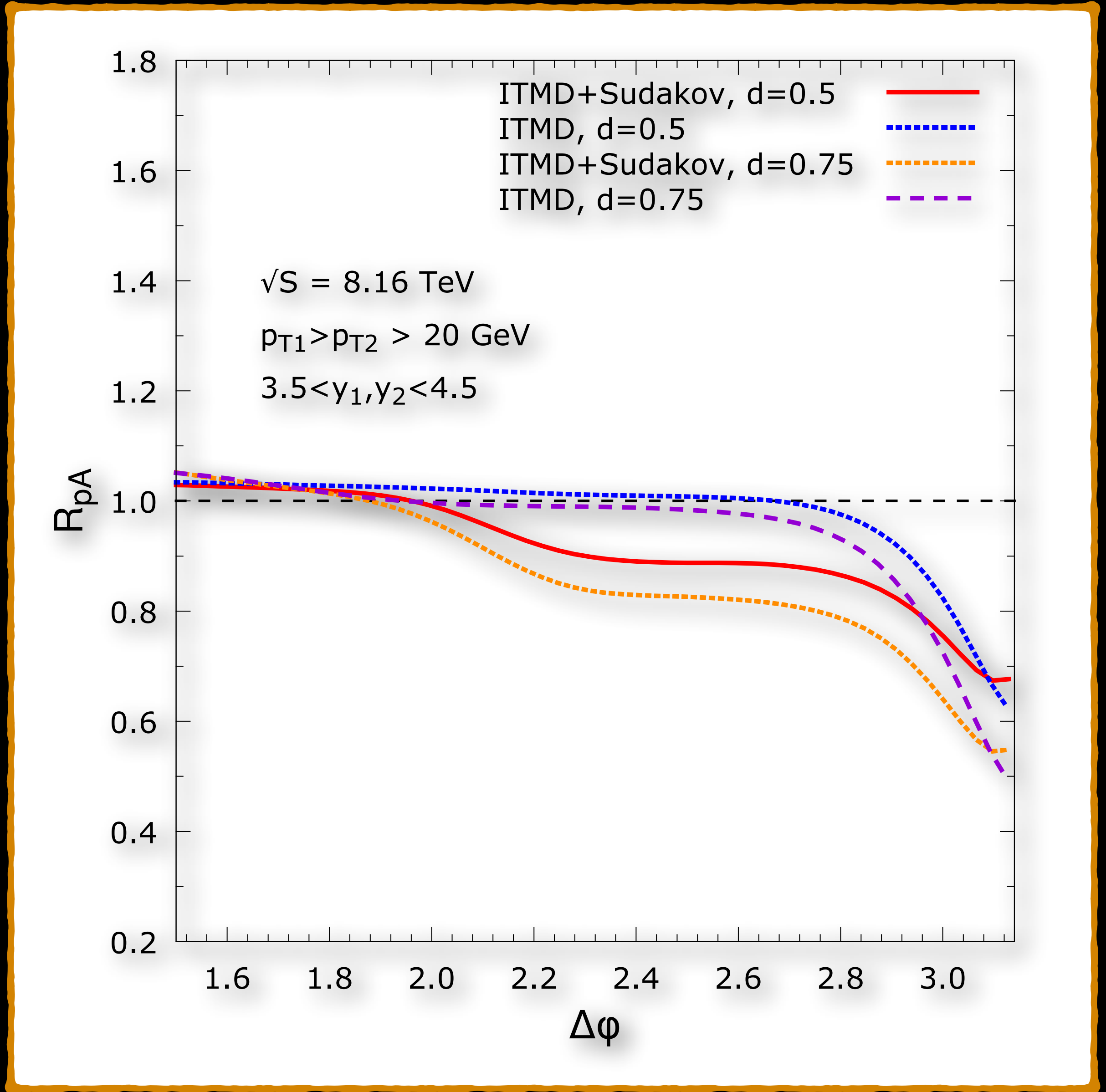
PHENOMENOLOGY

Forward dijet production in p-Pb collisions at LHC



Nuclear modification factor:

$$R_{pA} = \frac{\sigma_{pA}}{A\sigma_{pp}}$$



[A. Van Hameren, PK, K. Kutak, C. Marquet, E. Petreska, S. Sapeta, 2016]

PHENOMENOLOGY

ITMD vs new ATLAS data

Measurement of dijet azimuthal correlations in p+p and p+Pb. [ATLAS, Phys. Rev. C100 (2019)]

$\sqrt{S} = 5.02$ TeV   rapidity: $2.7 < y_1, y_2 < 4.5$

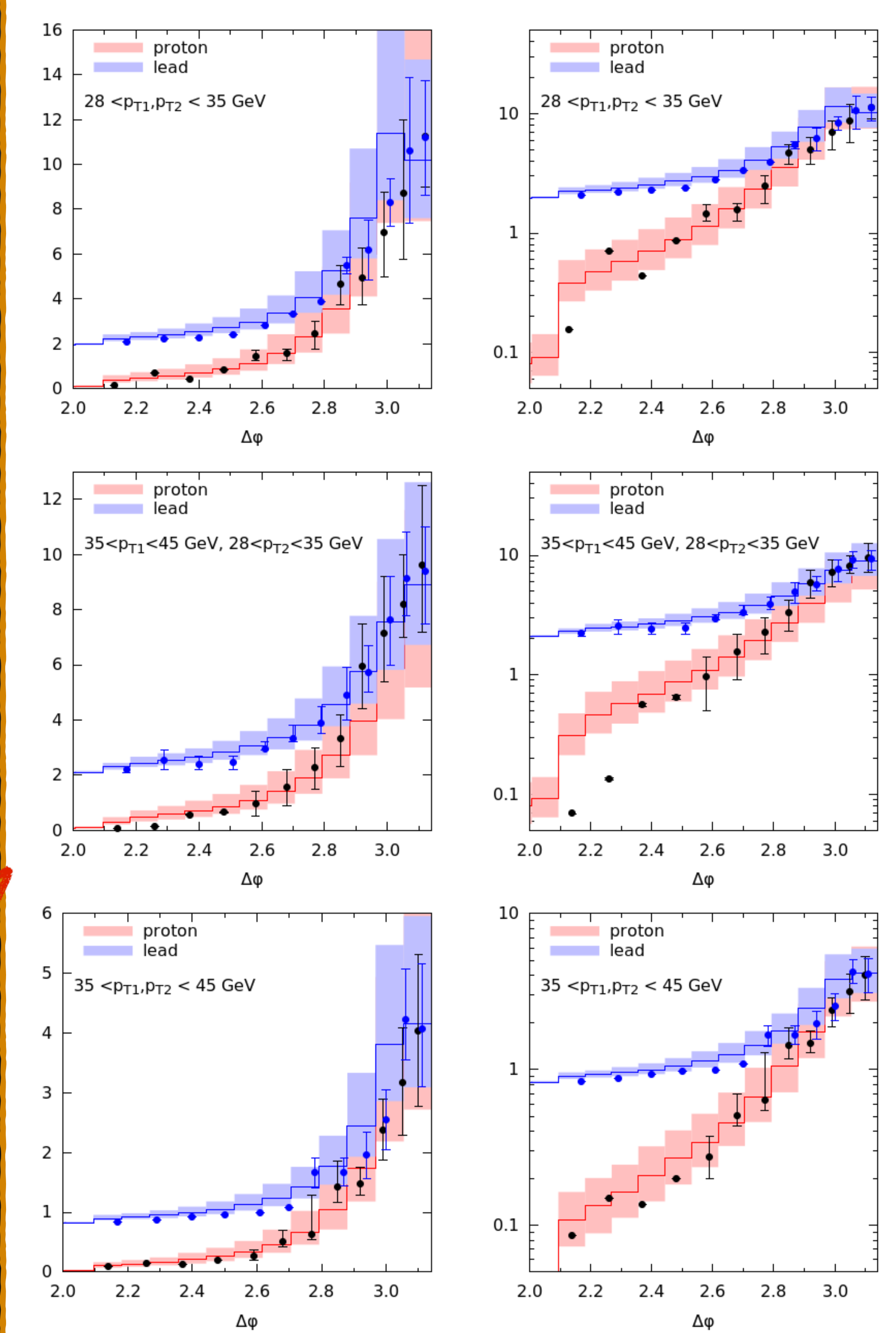
$$C_{12} = \frac{1}{N_1} \frac{dN_{12}}{d\Delta\phi}$$

← NUMBER OF DIJETS

↖ NUMBER OF LEADING JETS

We study an interplay of saturation and Sudakov resummation within ITMD vs the shape of C_{12}

Good description of the broadening effects



A. Van Hameren, P. Kotko, K. Kutak, S. Sapeta, Phys. Lett. B795 (2019) 511

SUMMARY

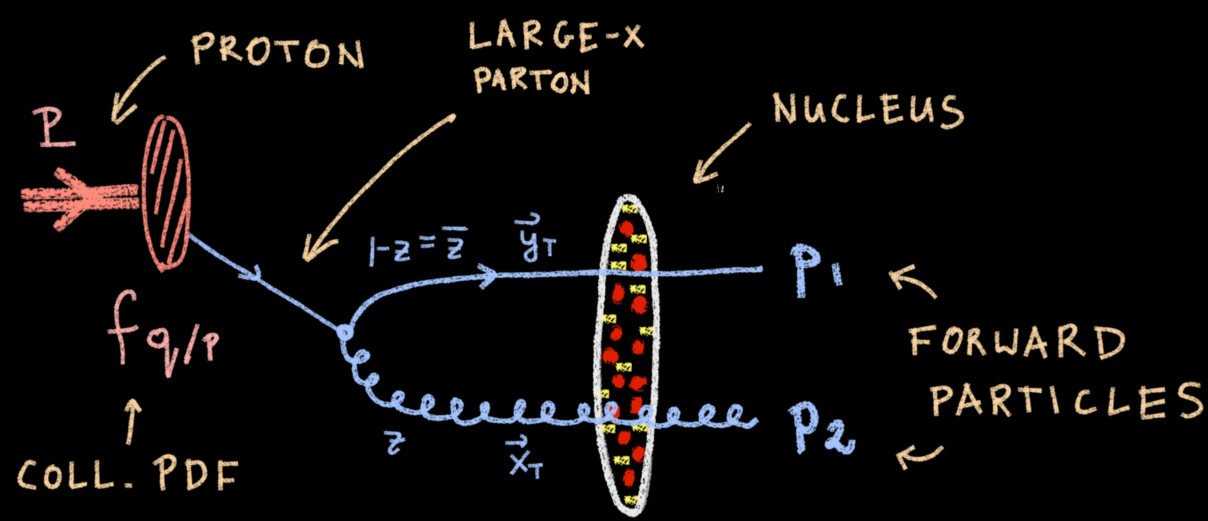
- The lack of universality of TMD gluon distributions is an inherent feature of QCD.
  - We should not reject it but accept.
  - It naturally appears in the high energy limit of QCD.
- Small-x-improved TMD factorization (ITMD) framework can be proven from the Color Glass Condensate theory.
- The framework is implemented in Monte Carlo generator KaTie for any process (however, not all small-x TMD gluon distributions are known) and the results are encouraging.

- Further research
  - Better Sudakov resummation
  - Automation in loop corrections for hard factors
  - Better small-x TMD gluon distributions

BACKUP

COLOR GLASS CONDENSATE

pA (dilute-dense) collisions within CGC



$$\frac{d\sigma_{qA\to 2j}}{d^3p_1 d^3p_2} \sim \int \frac{d^2x}{(2\pi)^2}\frac{d^2x'}{(2\pi)^2}\frac{d^2y}{(2\pi)^2}\frac{d^2y'}{(2\pi)^2}\, e^{-i\vec{p}_{T1}\cdot(\vec{x}_T-\vec{x}'_T)}e^{-i\vec{p}_{T2}\cdot(\vec{y}_T-\vec{y}'_T)}$$

$$\times \psi_z^*\left(\vec{x}'_T-\vec{y}'_T\right)\psi_z\left(\vec{x}_T-\vec{y}_T\right)$$ ← QUARK WAVE FUNCTION

$$\times\Big\{S_x^{(6)}\left(\vec{y}_T,\vec{x}_T,\vec{y}'_T,\vec{x}'_T\right)-S_x^{(4)}\left(\vec{y}_T,\vec{x}_T,\bar{z}\vec{y}'_T+z\vec{x}'_T\right)$$
$$-S_x^{(4)}\left(\bar{z}\vec{y}_T+z\vec{x}_T,\vec{y}'_T,\vec{x}'_T\right)-S_x^{(2)}\left(\bar{z}\vec{y}_T+z\vec{x}_T,\bar{z}\vec{y}'_T+z\vec{x}'_T\right)\Big\}$$

CORRELATORS OF WILSON LINES

$$S_x^{(2)}\left(\vec{y}_T,\vec{x}_T\right)=\frac{1}{N_c}\left\langle \mathrm{Tr}\,U(\vec{y}_T)U^\dagger(\vec{x}_T)\right\rangle_x$$

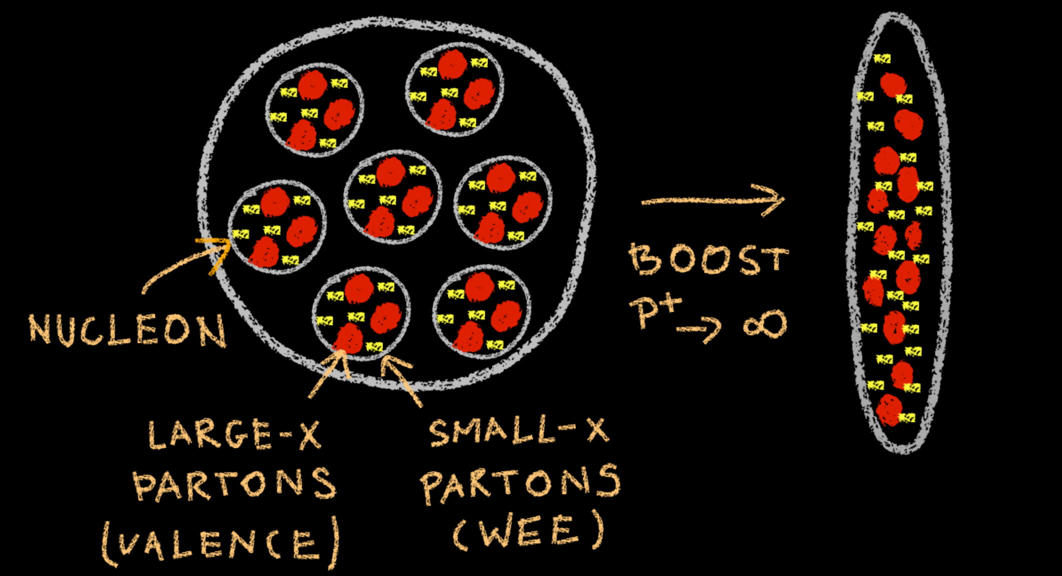
$$S_x^{(4)}\left(\vec{z}_T,\vec{y}_T,\vec{x}_T\right)=\frac{1}{2C_FN_c}\left\langle \mathrm{Tr}\left[U(\vec{z}_T)U^\dagger(\vec{y}_T)\right]\mathrm{Tr}\left[U(\vec{y}_T)U^\dagger(\vec{x}_T)\right]\right\rangle_x - S_x^{(2)}\left(\vec{z}_T,\vec{x}_T\right)$$

etc...

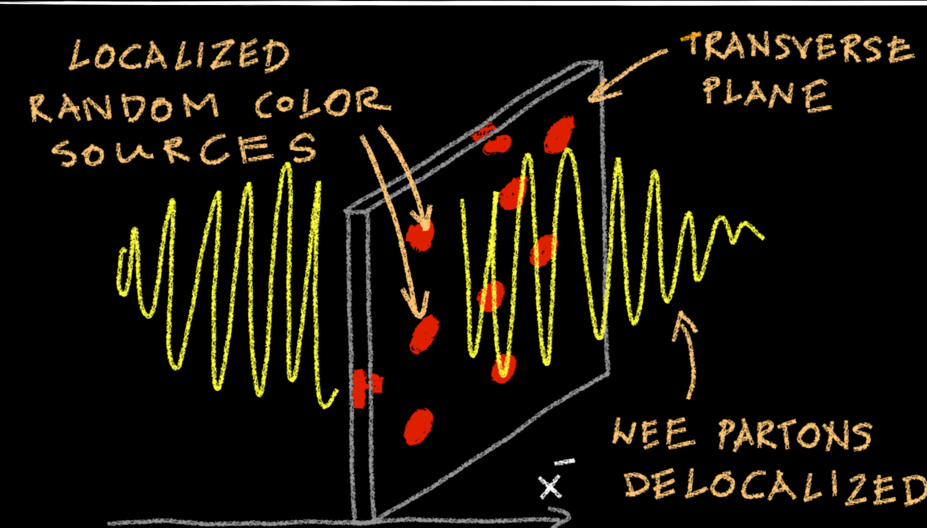
$$U(\vec{x}_T)=\mathscr{P}\exp\left\{ig\int_{-\infty}^{+\infty}dx^+\,A_a^-\left(x^+,\vec{x}_T\right)t^a\right\}$$

[C. Marquet, 2007]

COLOR FIELD OF THE NUCLEUS



[L. McLerran, R. Venugopalan, 1993]



Large-x partons — the color source for wee partons:

$$(D_\mu F^{\mu\nu})_a(x^-,\vec{x}_T)=\delta^{\nu+}\rho_a(\vec{x}_T)\delta(x^-)$$

RANDOM DISTRIBUTION OF COLOR SOURCES

AVERAGE OVER COLOR SOURCES

GAUSSIAN FUNCTIONAL → $\mathscr{W}_x[\rho]$

B-JIMWLK EVOLUTION IN X

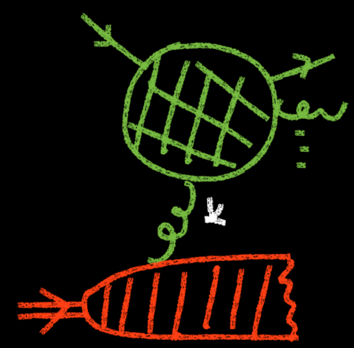
[Balitsky-Jalilian-Marian-Iancu-McLerran -Weigert-Leonidov-Kovner, 1996-2002]

Proof of the ITMD formula

[T. Altinoluk, R. Boussarie, PK, 2019]

TERMINOLOGY

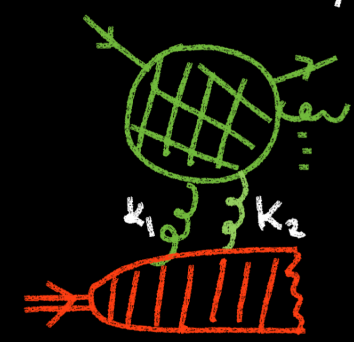
ONE-BODY



$\mathcal{O}_1 \sim \langle P | F_a^{-i}(x) | X \rangle$

$\mathcal{H}_1(k_T) \otimes \tilde{\mathcal{O}}_1(k_T)$

TWO-BODY, etc.



$\mathcal{O}_2 \sim \langle P | F_a^{-i}(x) F_b^{-j}(y) | X \rangle$

$\mathcal{H}_2(k_{T1}, k_{T2}) \otimes \tilde{\mathcal{O}}_2(k_{T1}, k_{T2})$

LEADING TWIST

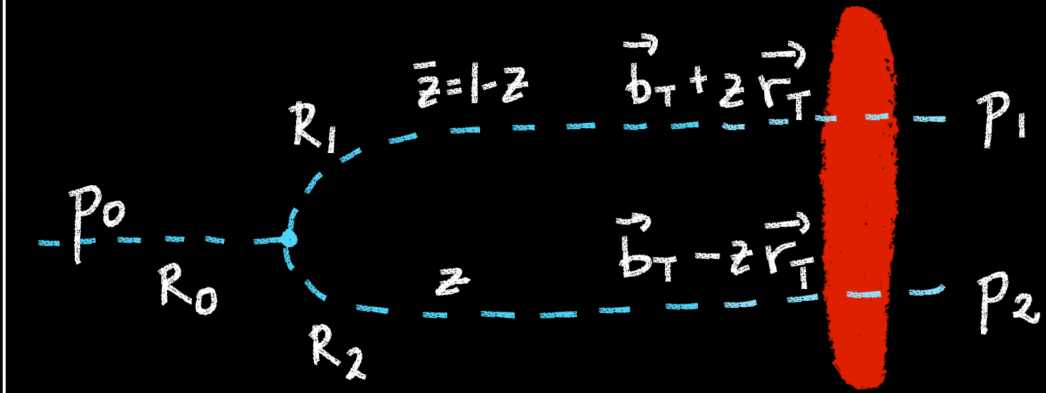
$$\mathcal{A}_{LT} = \mathcal{H}_1(0) \otimes \tilde{\mathcal{O}}_1(k_T)$$

NEXT-TO LEADING TWIST

$$\mathcal{A}_{NLT} = \vec{k}_T \cdot (\vec{\partial}_T \mathcal{H}_1)(0) \otimes \tilde{\mathcal{O}}_1(k_T) \quad \text{KINEMATIC TWIST}$$

$$+ \mathcal{H}_2(0,0) \otimes \tilde{\mathcal{O}}_2(k_{T1}, k_{T2}) \quad \text{GENUINE TWIST}$$

GENERIC 1→2 CGC AMPLITUDE



$\vec{k}_T = \vec{p}_{T1} + \vec{p}_{T2}$

$\vec{P}_T = \bar{z}\vec{p}_{T1} - z\vec{p}_{T2}$

R_i - COLOR REPRESENTATION

$$\mathcal{A} = \delta(p_1^+ + p_2^+ - p_0^+) \int d^2b_T d^2r_T \, e^{-i(\vec{P}_T\cdot\vec{r}_T + \vec{k}_T\cdot\vec{b}_T)}$$

$$\times \frac{r_T^\mu}{r_T^2} \Big\{ U^{R_1}(\vec{b} + \bar{z}\vec{r}) T^{R_0} U^{R_2}(\vec{b} - z\vec{r})$$

$$- U^{R_1}(\vec{b}) T^{R_0} U^{R_2}(\vec{b}) \Big\} \Gamma_\mu$$

WILSON LINE IN REPR. R_i

COLOR GENERATORS

DIRAC STRUCTURE

Proof of the ITMD formula

[T. Altinoluk, R. Boussarie, PK, 2019]

STEP #1 TAYLOR EXPANSION IN $\vec{r}_T$

$$\mathscr{A}^{(n)} = \delta(p_1^+ + p_2^+ - p_0^+)\int d^2b_T d^2r_T\, e^{-i(\vec{P}_T\cdot\vec{r}_T+\vec{k}_T\cdot\vec{b}_T)}\frac{r_T^\mu\Gamma_\mu}{r_T^2}$$
$$\frac{1}{n!}r_T^{\alpha_1}\ldots r_T^{\alpha_n}\sum_{i=0}^{n}\binom{n}{i}\bar{z}^i(-z)^{n-i}\left(\partial_{\alpha_1}\ldots\partial_{\alpha_i}U^{R_1}(\vec{b})\right)T^{R_0}\left(\partial_{\alpha_{i+1}}\ldots\partial_{\alpha_n}U^{R_2}(\vec{b})\right)$$

STEP #2 ISOLATION OF 1-BODY CONTRIBUTIONS

$$\mathscr{A}^{(n)}_{1-\text{body}} = \delta(p_1^+ + p_2^+ - p_0^+)\int d^2b_T d^2r_T\, e^{-i(\vec{P}_T\cdot\vec{r}_T+\vec{k}_T\cdot\vec{b}_T)}\frac{r_T^\mu\Gamma_\mu}{r_T^2}$$
$$\vec{r}_T^{\,\alpha}\sum_{j=0}^{n}\frac{(i\vec{k}_T\cdot\vec{r}_T)^j}{(j+1)!}\left\{\partial_\alpha U^{R_1}(\vec{b})T^{R_0}U^{R_2}(\vec{b})\bar{z}^{(j+1)} + U^{R_1}(\vec{b})T^{R_0}\partial_\alpha U^{R_2}(\vec{b})(-z)^{(j+1)}\right\}$$

STEP #3 RESUMMATION & INTEGRATION

$$\mathscr{A}_{1-\text{body}} = \delta(p_1^+ + p_2^+ - p_0^+)\int d^2b_T\, e^{-i\vec{k}_T\cdot\vec{b}_T}\frac{\Gamma_i}{k_T^2}(k_T^i\delta^{jl} + k_T^j\delta^{il} - k_T^l\delta^{ij})$$
$$\left\{\left(\frac{P_T^l}{P_T^2}+\frac{p_{2T}^l}{p_{2T}^2}\right)\partial_j U^{R_1}(\vec{b})T^{R_0}U^{R_2}(\vec{b}) + \left(\frac{P_T^l}{P_T^2}-\frac{p_{1T}^l}{p_{1T}^2}\right)U^{R_1}(\vec{b})T^{R_0}\partial_j U^{R_2}(\vec{b})\right\}$$

STEP #4 SQUARE THE AMPLITUDE

COLOR ALGEBRA $\Rightarrow$ ITMD