New constraints on heavy neutral leptons

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based on: Phys. Rev. D 98, 053001 , and arXiv:1910.01233

12.02.2020, Cracow ParticleFace COST Meeting

Motivation

[LEP, 2006]

$$N_\nu = 2.9840 \pm 0.0082$$

[P. Janot and S. Jadach, 2019]

$$N_\nu = 2.9975 \pm 0.0074$$

Theorem: [C. Jarlskog, 1990]
In the Standard Model with n left-handed lepton doublets and $N - n$ right-handed neutrinos, the effective number of neutrinos, N_ν, defined by

$$\Gamma(Z \to \nu' s) \equiv N_\nu \Gamma_0,$$

where Γ_0 is the standard width for one massless neutrino, satisfies

$$N_\nu \leq n.$$

Sterile neutrinos measurements

- Light neutrino oscillations
- Precision corrections
- Direct production in colliders

The simplest 3+1 model

$$\mathcal{L}_{\nu}^{M} = -\frac{1}{2}\bar{N}\begin{pmatrix} 0 & 0 & 0 & a_1 \\ 0 & 0 & 0 & a_2 \\ 0 & 0 & 0 & a_3 \\ a_1 & a_2 & a_3 & M \end{pmatrix} + H.c.$$

Four physical neutrino $\nu_1, \nu_2, \nu_P, \nu_F$, with ν_1, ν_2 massless and

$$m_P \backsim \mathcal{O}(eV) \ll m_F$$

This mass matrix is diagonalized by the following mixing matrix

$$\begin{pmatrix} c_\beta & -s_\beta s_\gamma & -s_\beta c_\gamma & 0 \\ 0 & c_\gamma & -s_\gamma & 0 \\ c_\alpha s_\beta & c_\alpha c_\beta s_\gamma & c_\alpha c_\beta c_\gamma & -s_\alpha \\ s_\alpha s_\beta & s_\alpha c_\beta s_\gamma & s_\alpha c_\beta c_\gamma & c_\alpha \end{pmatrix} \qquad s_\alpha \sim \sqrt{m_P/(m_F)}$$

[C. Jarlskog, 1990] [C.O. Escobar et al., 1993]

Number of neutrinos

Isolating non-standard contribution, we can write this as

$$N_\nu - 2 = \frac{1}{(x+y)^2}\left[x^2F(y) + y^2F(x) + 2xyG(x,y)\right],$$

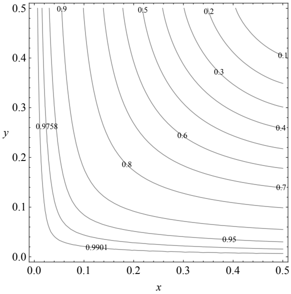
where

$$F(z) = (1-4z^2)^{\frac{3}{2}},$$
$$G(x,y) = \sqrt{1+(x^2+y^2)^2-2(x^2-y^2)}\left[1-\frac{x^2+y^2}{2}-3xy-\frac{(x^2-y^2)^2}{4}\right],$$

and

$$x = m_F/M_Z,\ y = m_P/M_Z.$$

$\sin\alpha$ estimation



We are looking for the line $y = x\tan^2\alpha$ lying below experimental limits

$$\text{LEP}: N_\nu = 2.9840 \pm 0.0082 \rightarrow \sum_i |U_{i4}| \equiv \sin\alpha < 0.174 \quad i = e, \mu, \tau$$

$$\text{NEW}: N_\nu = 2.9975 \pm 0.0074 \rightarrow \sum_i |U_{i4}| \equiv \sin\alpha < 0.115 \quad i = e, \mu, \tau$$

Neutrino mixing in the Standard Model

$$\nu_\alpha^{(f)} = (U_{\mathtt{PMNS}})_{\alpha i} \nu_i^{(m)}$$

Mixing matrix

$$U_{\mathtt{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Experimental values of mixing parameters

$$\theta_{12} \in [31.38^\circ, 35.99^\circ], \quad \theta_{23} \in [38.4^\circ, 53.0^\circ],$$
$$\theta_{13} \in [7.99^\circ, 8.91^\circ], \quad \delta \in [0, 2\pi]$$

Extended mixing - BSM models

Complete mixing

$$\begin{pmatrix} \nu^{(f)} \\ \hat{\nu}^{(f)} \end{pmatrix} = \begin{pmatrix} U_{\mathtt{PMNS}} & U_{lh} \\ U_{hl} & U_{hh} \end{pmatrix} \begin{pmatrix} \nu^{(m)} \\ \hat{\nu}^{(m)} \end{pmatrix} \equiv U \begin{pmatrix} \nu^{(m)} \\ \hat{\nu}^{(m)} \end{pmatrix}$$

Observable part

$$\nu_{\alpha}^{(f)} = \underbrace{(U_{PMNS})_{\alpha i} \nu_i^{(m)}}_{\text{SM part}} + \underbrace{(U_{lh})_{\alpha j} \hat{\nu}_j^{(m)}}_{\text{BSM part}}$$

Singular values

Singular values σ_i of a given matrix A are positive square roots of the eigenvalues λ_i of the matrix $AA^\dagger$

$$\sigma_i(A) = \sqrt{\lambda_i(AA^\dagger)}$$

Properties:

- generalization of eigenvalues
- always positive
- stable under perturbations

Unitary matrices

$$UU^\dagger = I = diag(1, 1, ..., 1) \Longrightarrow \text{all singular values equal to 1}$$

Contractions

$$\|A\| \leq 1$$

Operator norm (spectral norm)

$$\|A\| := \sup_{\|x\|=1} \|Ax\| = \sigma_{\max}(A)$$

Contractions as submatrices of the unitary matrix

$$\left\| \begin{pmatrix} U_{\texttt{PMNS}} & U_{lh} \\ U_{hl} & U_{hh} \end{pmatrix} \right\| = 1 \Longrightarrow \|U_{\texttt{PMNS}}\| \leq 1$$

Unitary dilation

BSM?

$$U_{\text{PMNS}} \xrightarrow{\text{dilation}} \begin{pmatrix} U_{\text{PMNS}} & U_{lh} \\ U_{hl} & U_{hh} \end{pmatrix} \equiv U \to UU^\dagger = I$$

CS decomposition

$$U \equiv \begin{pmatrix} U_{\text{PMNS}} & U_{lh} \\ U_{hl} & U_{hh} \end{pmatrix} = \begin{pmatrix} W_1 & 0 \\ 0 & W_2 \end{pmatrix} \left(\begin{array}{c|cc} C & -S & 0 \\ \hline S & C & 0 \\ 0 & 0 & I_{m-n} \end{array}\right) \begin{pmatrix} Q_1^\dagger & 0 \\ 0 & Q_2^\dagger \end{pmatrix}$$

where $C \geq 0$ and $S \geq 0$ are diagonal matrices satisfying $C^2 + S^2 = I_n$
$W_1, Q_1 \in M_{n\times n}$ and $W_2, Q_2 \in M_{m\times m}$ are unitary matrices.

Physical Region

$$\Omega := \mathrm{conv}(U_{\mathrm{PMNS}}) = \{\sum_{i=1}^{m} \alpha_i U_i \mid U_i \in U(3), \alpha_1, ..., \alpha_m \geq 0, \sum_{i=1}^{m} \alpha_i = 1, \theta_{12}, \theta_{13}, \theta_{23} \text{ and } \delta \text{ given by experimental values}\}$$

Ω is divided into four disjoint subsets

Ω_1 : 3+1 scenario: $\Sigma = \{\sigma_1 = 1.0, \sigma_2 = 1.0, \sigma_3 < 1.0\}$,

Ω_2 : 3+2 scenario: $\Sigma = \{\sigma_1 = 1.0, \sigma_2 < 1.0, \sigma_3 < 1.0\}$,

Ω_3 : 3+3 scenario: $\Sigma = \{\sigma_1 < 1.0, \sigma_2 < 1.0, \sigma_3 < 1.0\}$,

Ω_4 : PMNS scenario: $\Sigma = \{\sigma_1 = 1, \sigma_2 = 1, \sigma_3 = 1\}$.

α-parametrization and prescribed singular values

$$\not{U}_{\text{PMNS}} = (I - \alpha)W = TW.$$

where W is a unitary matrix and $T = I - \alpha$.

$$T = \begin{pmatrix} t_{11} & 0 & 0 \\ t_{21} & t_{22} & 0 \\ t_{31} & t_{32} & t_{33} \end{pmatrix}, \quad \Sigma = \{\sigma_1, \sigma_2, \sigma_3\}$$

Entry	(I): $m >$ EW	(II): $\Delta m^2 \gtrsim 100\ \mathrm{eV}^2$	(III): $\Delta m^2 \sim 0.1 - 1\ \mathrm{eV}^2$
$T_{11} = 1 - \alpha_{11}$	$0.99870 \div 1$	$0.976 \div 1$	$0.990 \div 1$
$T_{22} = 1 - \alpha_{22}$	$0.99978 \div 1$	$0.978 \div 1$	$0.986 \div 1$
$T_{33} = 1 - \alpha_{33}$	$0.99720 \div 1$	$0.900 \div 1$	$0.900 \div 1$
$T_{21} = \|\alpha_{21}\|$	$0.0 \div 0.00068$	$0.0 \div 0.025$	$0.0 \div 0.017$
$T_{31} = \|\alpha_{31}\|$	$0.0 \div 0.00270$	$0.0 \div 0.069$	$0,0 \div 0.045$
$T_{32} = \|\alpha_{32}\|$	$0.0 \div 0.00120$	$0.0 \div 0.012$	$0.0 \div 0.053$

[M. Blennow et al., 2017] (95% CL)

It is possible to construct lower triangular matrices with prescribed eigenvalues and singular values [C-K. Li and R. Mathias, 2004].

Estimation of the "light-heavy" mixing

Ω_1 : 3+1 scenario: $\Sigma = \{\sigma_1 = 1.0, \sigma_2 = 1.0, \sigma_3 < 1.0\}$

$$\begin{pmatrix} W_1 & 0 \\ 0 & W_2 \end{pmatrix} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c & -s \\ \hline 0 & 0 & s & c \end{array}\right) \begin{pmatrix} Q_1^\dagger & 0 \\ 0 & Q_2^\dagger \end{pmatrix}.$$

We are interested in the estimation of the light-heavy mixing sector which is given by

$$U_{lh} = W_1 S_{12} Q_2^\dagger,$$

where $W_1 \in \mathbb{C}^{3\times 3}$ is unitary, $S_{12} = (0, 0, -s)^T$ and $Q_2 = e^{i\theta}, \theta \in (0, 2\pi]$. Taking into account exact values of the W_1 we can estimate the light-heavy mixing by the analytical formula

$$|U_{i4}| = |w_{i3}| \cdot |\sqrt{1 - \sigma_3^2}|, \quad i = e, \mu, \tau.$$

Estimation of the "light-heavy" mixing

Estimation of the "light-heavy" mixing via CS decomposition

- (I): $m >$ EW.

  Ours : $|U_{e4}| \in [0, 0.021]$, $|U_{\mu 4}| \in [0.00013, 0.021]$, $|U_{\tau 4}| \in [0.0115, 0.075]$.
  Others : $|U_{e4}| \leq 0.041$, $|U_{\mu 4}| \leq 0.030$, $|U_{\tau 4}| \leq 0.087$ [J. de Blas, 2013]

- (II): $\Delta m^2 \gtrsim 100\ \mathrm{eV}^2$.

  Ours : $|U_{e4}| \in [0, 0.082]$, $|U_{\mu 4}| \in [0.00052, 0.099]$, $|U_{\tau 4}| \in [0.0365, 0.44]$.

- (III): $\Delta m^2 \sim 0.1 - 1\ \mathrm{eV}^2$.

  Ours : $|U_{e4}| \in [0, 0.130]$, $|U_{\mu 4}| \in [0.00052, 0.167]$, $|U_{\tau 4}| \in [0.0365, 0.436]$.
  Others : $|U_{e4}| \in [0.114, 0.167]$, $|U_{\mu 4}| \in [0.0911, 0.148]$, $|U_{\tau 4}| \leq 0.361$.
  [C. Giunti et al., 2017] [M. Dantler et al., 2018]

Comparisons

- From the Z decay:
  $\sum_i |U_{i4}|^2 < 0.0132, \quad i = e, \mu, \tau$
- From mixings (So far):
  $\sum_i |U_{i4}|^2 \leq 0.00867, \quad i = e, \mu, \tau$
- From mixings and singular values (Ours):
  $\sum_i |U_{i4}|^2 \leq 0.00651, \quad i = e, \mu, \tau$

Other use of singular values (details in backup)

- Introducing to neutrino physics the method of the inverse singular value problem. It allows to construct mixing matrices with encoded minimal number of additional neutrinos and to confront them with experimental data.
- Analysis of the amount of the space for additional neutrinos based on deviations of singular values from unity.
  Results: e.g. $\Delta m^2 \sim 0.1 - 1\ \mathrm{eV}^2 \rightarrow \sigma_3 = 0.889$
- A study of possible distinction between three scenarios with different number of additional neutrinos on the level of experimental data using singular values and corresponding division of the neutrino mixing space Ω.
  Results: 3+2 and 3+3 scenarios cannot be distinguished.
  3+1 scenario differs.

Summary and Outlook

- New prediction for the Z decay width significantly improves the light-heavy mixing estimation.
- New analytical formula for light-heavy mixing in the 3+1 scenario as a function of singular values has been derived.
- New estimations of upper bounds for the light-heavy mixings in the 3+1 scenario using CS decomposition are given.

Backup slides

Amount of space for n neutrinos: Analysis

- Construction of matrices with prescribed singular values, e.g., in 3+1 scenario we take $\sigma_1 = 1, \sigma_2 = 1, \sigma_3 < 1$, together with the requirement on the elements to stay within experimental limits.
- Go with the "free" singular values as low as possible, e.g., in the 3+1 scenario we take σ_3 the smallest possible.

Amount of space for n neutrinos: Results

		3 + 1	
			σ_3
$m >$ EW			0.9968
$\Delta m^2 \gtrsim 100\ \mathrm{eV}^2$			0.900
$\Delta m^2 \sim 0.1 - 1\ \mathrm{eV}^2$			0.889
		3 + 2	
		σ_2	σ_3
$m >$ EW		0.9987	0.9986
$\Delta m^2 \gtrsim 100\ \mathrm{eV}^2$		0.976	0.975
$\Delta m^2 \sim 0.1 - 1\ \mathrm{eV}^2$		0.986	0.985
		3 + 3	
	σ_1	σ_2	σ_3
$m >$ EW	0.9998	0.9996	0.9996
$\Delta m^2 \gtrsim 100\ \mathrm{eV}^2$	0.979	0.977	0.9773
$\Delta m^2 \sim 0.1 - 1\ \mathrm{eV}^2$	0.991	0.989	0.989

Error: 0.00003 (follows from Weyl's inequality, slides 26 and 27)

Distinction of the 3+1 scenario: Analysis

$$\sigma_1 = \sigma_2 = 1.$$

- In each massive scenario 10^8 matrices are produced, starting from σ_3 as large as possible and lowering it systematically to the smallest obtained value (previous slide).
- For each value of σ_3 the smallest and the largest values of produced matrix elements are taken.
- Repeating the procedure over possible σ_3 values, the allowed ranges of the 3×3 matrix elements are determined.

Distinction of the 3+1 scenario: Results

	(I): $m >$ EW	(II): $\Delta m^2 \gtrsim 100$ eV^2	(III): $\Delta m^2 \sim 0.1 - 1$ eV^2
(1, 1)	$0.99885 \div 0.99999$	$0.97641 \div 0.99996$	$0.99020 \div 0.99999$
Exp:	$0.99870 \div 1$	$0.976 \div 1$	$0.990 \div 1$
(2, 2)	$0.99980 \div 0.99999$	**0.99331** $\div 0.99999$	$0.98646 \div 0.99999$
Exp:	$0.99978 \div 1$	**0.978** $\div 1$	$0.986 \div 1$
(3, 3)	$0.99721 \div 0.99996$	$0.90040 \div 0.99985$	$0.90015 \div 0.99958$
Exp:	$0.99720 \div 1$	$0.900 \div 1$	$0.900 \div 1$
(2, 1)	$0.00001 \div 0.00062$	$0.00031 \div$ **0.02214**	$0.00014 \div 0.01615$
Exp:	$0.0 \div 0.00068$	$0.0 \div$ **0.025**	$0.0 \div 0.017$
(3, 1)	$0.00002 \div 0.00266$	$0.00048 \div 0.06892$	$0.00012 \div 0.04500$
Exp:	$0.0 \div 0.00270$	$0.0 \div 0.069$	$0.0 \div 0.045$
(3, 2)	$0.00008 \div 0.00113$	$0.00052 - 0.01196$	$0.00024 \div 0.05281$
Exp:	$0.0 \div 0.00120$	$0.0 \div 0.012$	$0.0 \div 0.053$

Similar results for 3+2 and 3+3.

So far no distinction among 3+n scenarios is possible. However,...

(I) Narrowing mixing spreads for individual sing. val.

- Generation of matrices with a prescribed set of singular values and with elements within experimental ranges.
- From the set of these matrices take the smallest and the largest value of each element.

E.g.: $\Delta m^2 \gtrsim 100 \ \mathrm{eV}^2, \Sigma = \{1, 1, 0.900\}$:

$$|A_{0.900}| = \begin{pmatrix} 0.999623 \div 0.999999 \ (1.5\%) & 0 & 0 \\ 0.000002 \div 0.000753 \ (3\%) & 0.999623 \div 0.999999 \ (2\%) & 0 \\ 0.000606 \div 0.011919 \ (16\%) & 0.000606 \div 0.011923 \ (94\%) & 0.900002 \div 0.900678 \ (1\%) \end{pmatrix}$$

Values in the brackets represent the percentage of the current experimental bounds.
For the other massive cases these values do not exceed 15%.

Matrix norm

A matrix norm is a function $\|\cdot\|$ from the set of all complex (real matrices) into $\mathbb{R}$ that satisfies the following properties

$$\|A\| \geq 0 \text{ and } \|\mathrm{A}\| = 0 \Longleftrightarrow \mathrm{A} = 0,$$
$$\|\alpha A\| = |\alpha|\|A\|, \alpha \in C,$$
$$\|A + B\| \leq \|A\| + \|B\|,$$
$$\|AB\| \leq \|A\|\|B\|$$

Examples of matrix norms

- spectral norm: $\|A\| = \max_{\|x\|_2=1} \|Ax\|_2 = \sigma_1(A)$
- Frobenius norm: $\|A\|_F = \sqrt{Tr(A^\dagger A)} = \sqrt{\sum_{i,j=1}^n |a_{ij}|^2} = \sqrt{\sum_{i=1}^n \sigma_i^2}$
- maximum absolute column sum norm: $\|A\|_1 = \max_{\|x\|_1=1} \|Ax\|_\infty = \max_j \sum_i |a_{ij}|$
- maximum absolute row sum norm: $\|A\|_\infty = \max_{\|x\|_\infty=1} \|Ax\|_\infty = \max_i \sum_j |a_{ij}|$

Weyl's inequality for singular values

Let A and B be a $m \times n$ matrices and let $q = \min\{m, n\}$. Then

$$\sigma_j(A + B) \leq \sigma_i(A) + \sigma_{j-i+1}(B) \text{ for } i \leq j$$

Error Estimation

Let us assume that the V matrix which realizes some *BSM* scenario includes an error matrix E which is of the form $V + E$. Using Weyl inequalities for decreasingly ordered pairs of singular values of V and $V + E$, the following relation takes place

$$|\sigma_i(V + E) - \sigma_i(V)| \leq ||E||.$$

A precision for elements of the A in the $m > \mathrm{EW}$ is 10^{-5}. In our analysis we keep the same precision for all massive cases. This does not contradict experimental results since we still work within experimentally established intervals. Thus, all entries of Error matrix can be taken as $E_{ij} \approx 0.00001$. Therefore, uncertainty of the calculated singular values is bounded by $||E|| = 0.00003$.

Algorithm

The following steps lead to a contraction settled by U_{PMNS} and then to its unitary dilation of a minimal dimension

1) Select a finite number of unitary matrices U_i, $i = 1, 2, ...m$, within experimentally allowed range of parameters θ_{13}, θ_{23} and δ.

2) Construct a contraction U_{11} as a convex combination of selected matrices U_i

$$V = \sum_{i=1}^{m} \alpha_i U_i, \quad \alpha_1, ..., \alpha_m \geq 0, \quad \sum_{i=1}^{m} \alpha_i = 1.$$

3) Find singular value decomposition of V, i.e.

$$V = W_1 \Sigma Q_1^\dagger$$

where W_1, Q_1 are unitary, Σ is diagonal, and determine number η of singular values strictly less than 1.

4) Use CS decomposition

$$U = \begin{pmatrix} U_{\mathrm{PMNS}} & U_{lh} \\ U_{hl} & U_{hh} \end{pmatrix} =$$

$$\begin{pmatrix} W_1 & 0 \\ 0 & W_2 \end{pmatrix} \left(\begin{array}{cc|c} I_r & 0 & 0 \\ 0 & C & -S \\ \hline 0 & S & C \end{array}\right) \begin{pmatrix} Q_1^\dagger & 0 \\ 0 & Q_2^\dagger \end{pmatrix}$$

to find the unitary dilation $U \in \mathbb{M}_{(3+\eta)\times(3+\eta)}$ of contraction U_{11}.

3+1 via CS decomposition

Thus we work with the following set of singular values $\sigma_1 = 1, \sigma_2 = 1, \sigma_3 < 1$ and the CS decomposition takes the form

$$\begin{pmatrix} W_1 & 0 \\ 0 & W_2 \end{pmatrix} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c & -s \\ \hline 0 & 0 & s & c \end{array}\right) \begin{pmatrix} Q_1^\dagger & 0 \\ 0 & Q_2^\dagger \end{pmatrix}. \tag{1}$$

We are interested in the estimation of the light-heavy mixing sector which is given by

$$U_{lh} = W_1 S_{12} Q_2^\dagger, \tag{2}$$

where $W_1 \in \mathbb{C}^{3\times3}$ is unitary, $S_{12} = (0, 0, -s)^T$ and $Q_2 = e^{i\theta}, \theta \in (0, 2\pi]$. Parametrizing the matrix W_1 as usual by Euler angles we get

$$U_{lh} = -(w_{e3}, w_{\mu3}, w_{\tau3})^T s e^{-i\theta} = -(-s_{12}e^{-i\theta_{13}}, s_{23}c_{13}, c_{23}c_{13})^T s e^{-i\theta} \tag{3}$$

3+1 via CS decomposition

We can see that if we want estimate just the absolute values for the elements of light-heavy sector we are left only with

$$|s| = |\sqrt{1 - c^2}| = |\sqrt{1 - \sigma_3^2}|. \tag{4}$$

Thus for each massive scenario we get

$$\begin{aligned} "m > \text{ EW}" &\equiv m_1 < |0.08359|, \\ "\Delta m^2 \gtrsim 100 \text{ eV}^2" &\equiv m_2 < |0.43795|, \\ "\Delta m^2 \sim 0.1 - 1 \text{ eV}^2" &\equiv m_3 < |0.43795|. \end{aligned} \tag{5}$$

Results on slide 21 have been obtained by taking exact maximal values of $w_{e3}, w_{\mu 3}$ and $w_{\tau 3}$, which follow from the singular value decomposition.

$$|U_{i4}| = |w_{i3}| \cdot |\sqrt{1-\sigma_3^2}|, \quad i = e, \mu, \tau.$$