

# Drell Yan production at NLO with the PB method

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Hannes Jung (DESY)

together with

A. Bermudez Martinez, P. Connor, D. Dominguez Damiani, L. Estevez Banos, F. Hautmann  
J. Lidrych, A. Lelek, M. Schmitz, S. Taheri Monfared, H. Yang, Q. Wang

- Recap of PB method
- Application of PB TMDs to
  - $Z_0$  production at the LHC  
(based on Phys Rev D.100.074027 (2019))
  - low mass DY at low energies  
(based on arXiv 2001.06488)

# Recap of Parton Branching method

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# DGLAP evolution – solution with parton branching method

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- differential form: 
$$\mu^2 \frac{\partial}{\partial \mu^2} f(x, \mu^2) = \int \frac{dz}{z} \frac{\alpha_s}{2\pi} P_+(z) f\left(\frac{x}{z}, \mu^2\right)$$

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$$\Delta_s(\mu^2) = \exp\left(-\int^{z_M} dz \int_{\mu_0^2}^{\mu^2} \frac{\alpha_s}{2\pi} \frac{d\mu'^2}{\mu'^2} P^{(R)}(z)\right)$$

- differential form using  $f/\Delta_s$  with

$$\mu^2 \frac{\partial}{\partial \mu^2} \frac{f(x, \mu^2)}{\Delta_s(\mu^2)} = \int \frac{dz}{z} \frac{\alpha_s}{2\pi} \frac{P^{(R)}(z)}{\Delta_s(\mu^2)} f\left(\frac{x}{z}, \mu^2\right)$$

# DGLAP evolution – solution with parton branching method

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- integral form

$$f(x, \mu^2) = f(x, \mu_0^2) \Delta_s(\mu^2) + \int \frac{dz}{z} \int \frac{d\mu'^2}{\mu'^2} \cdot \frac{\Delta_s(\mu^2)}{\Delta_s(\mu'^2)} P^{(R)}(z) f\left(\frac{x}{z}, \mu'^2\right)$$

  
 no – branching probability from  $\mu_0^2$  to  $\mu^2$

# DGLAP evolution – solution with parton branching method

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- solve integral equation via iteration:

$$f_0(x, \mu^2) = f(x, \mu_0^2) \Delta(\mu^2)$$

# DGLAP evolution – solution with parton branching method

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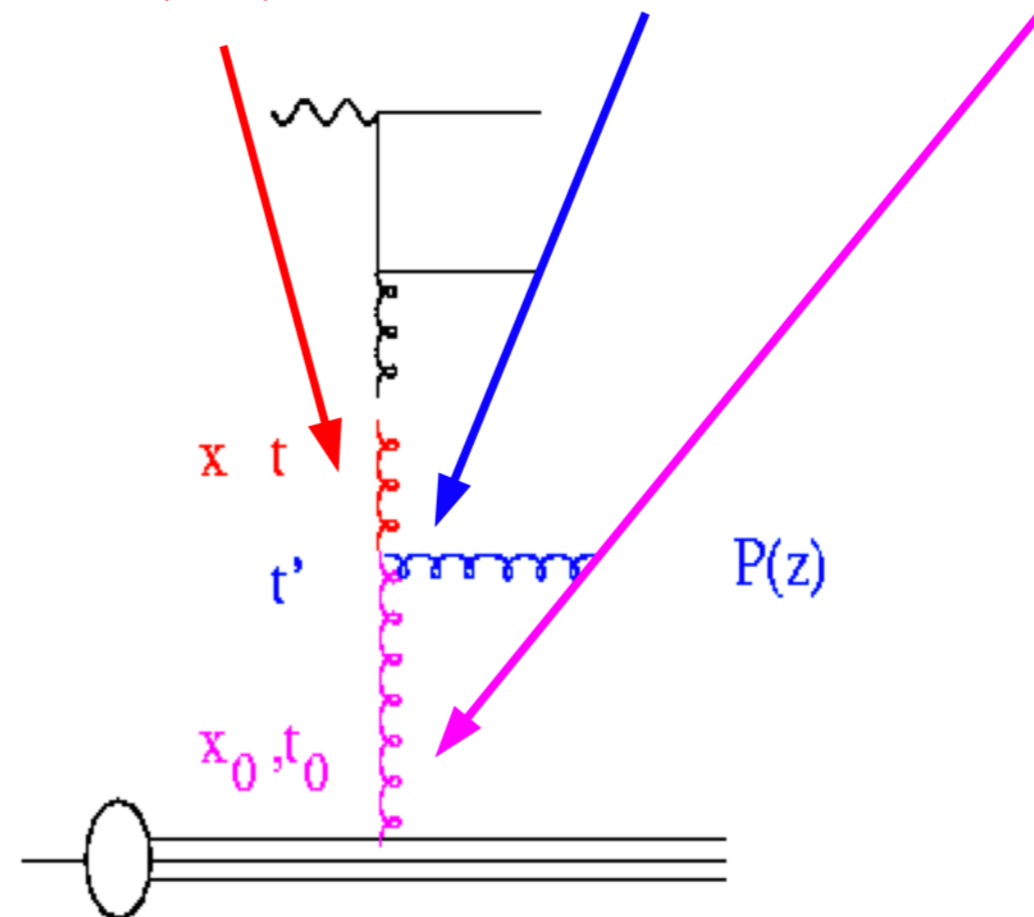
$$f_0(x, \mu^2) = f(x, \mu_0^2) \Delta(\mu^2)$$

$$f_1(x, \mu^2) = f(x, \mu_0^2) \Delta(\mu^2) + \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \frac{\Delta(\mu^2)}{\Delta(\mu'^2)} \int^{z_M} \frac{dz}{z} P^{(R)}(z) f(x/z, \mu_0^2) \Delta(\mu'^2)$$

from  $t'$  to  $t$   
w/o branching

branching at  $t'$

from  $t_0$  to  $t'$   
w/o branching



# DGLAP evolution – solution with parton branching method

$$f(x, \mu^2) = f(x, \mu_0^2) \Delta_s(\mu^2) + \int^{z_M} \frac{dz}{z} \int \frac{d\mu'^2}{\mu'^2} \cdot \frac{\Delta_s(\mu^2)}{\Delta_s(\mu'^2)} P^{(R)}(z) f\left(\frac{x}{z}, \mu'^2\right)$$

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from  $t'$  to  $t$   
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$$f_1(x, \mu^2) = f(x, \mu_0^2) \Delta(\mu^2) + \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \frac{\Delta(\mu^2)}{\Delta(\mu'^2)} \int^{z_M} \frac{dz}{z} P^{(R)}(z) f(x/z, \mu_0^2) \Delta(\mu'^2)$$

- with  $P_{ab}^{(R)}(z)$  real emission probability (without virtual terms)
  - $z_M$  introduced to separate real from virtual and non-emission probability
  - reproduces DGLAP up to  $\mathcal{O}(1 - z_M)$
- make use of momentum sum rule to treat virtual corrections
  - use Sudakov form factor for non-resolvable and virtual corrections

$$\Delta_a(z_M, \mu^2, \mu_0^2) = \exp \left( - \sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{z_M} dz z P_{ba}^{(R)}(\alpha_s), z \right)$$

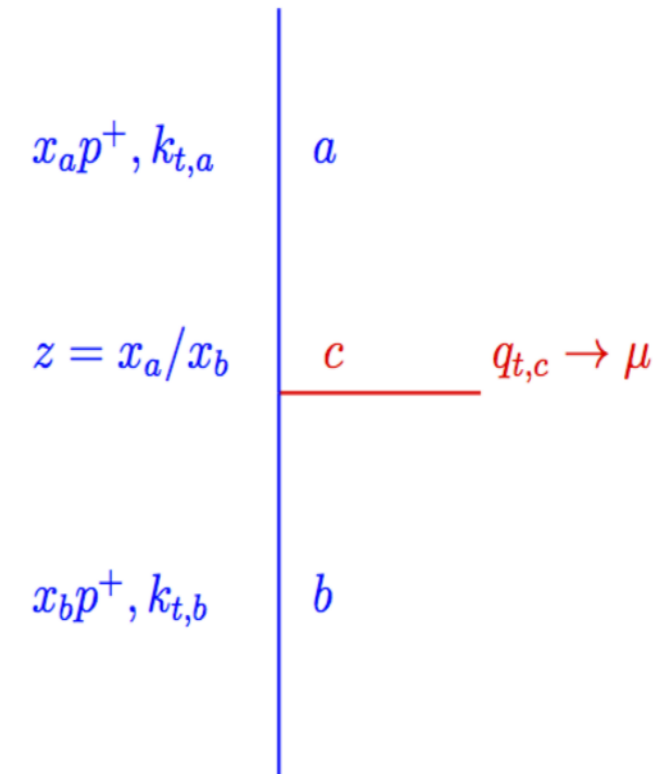


# Transverse Momentum Dependence

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- Parton Branching evolution generates every single branching:
  - kinematics can be calculated at every step
- Give physics interpretation of evolution scale:
  - angular ordering:

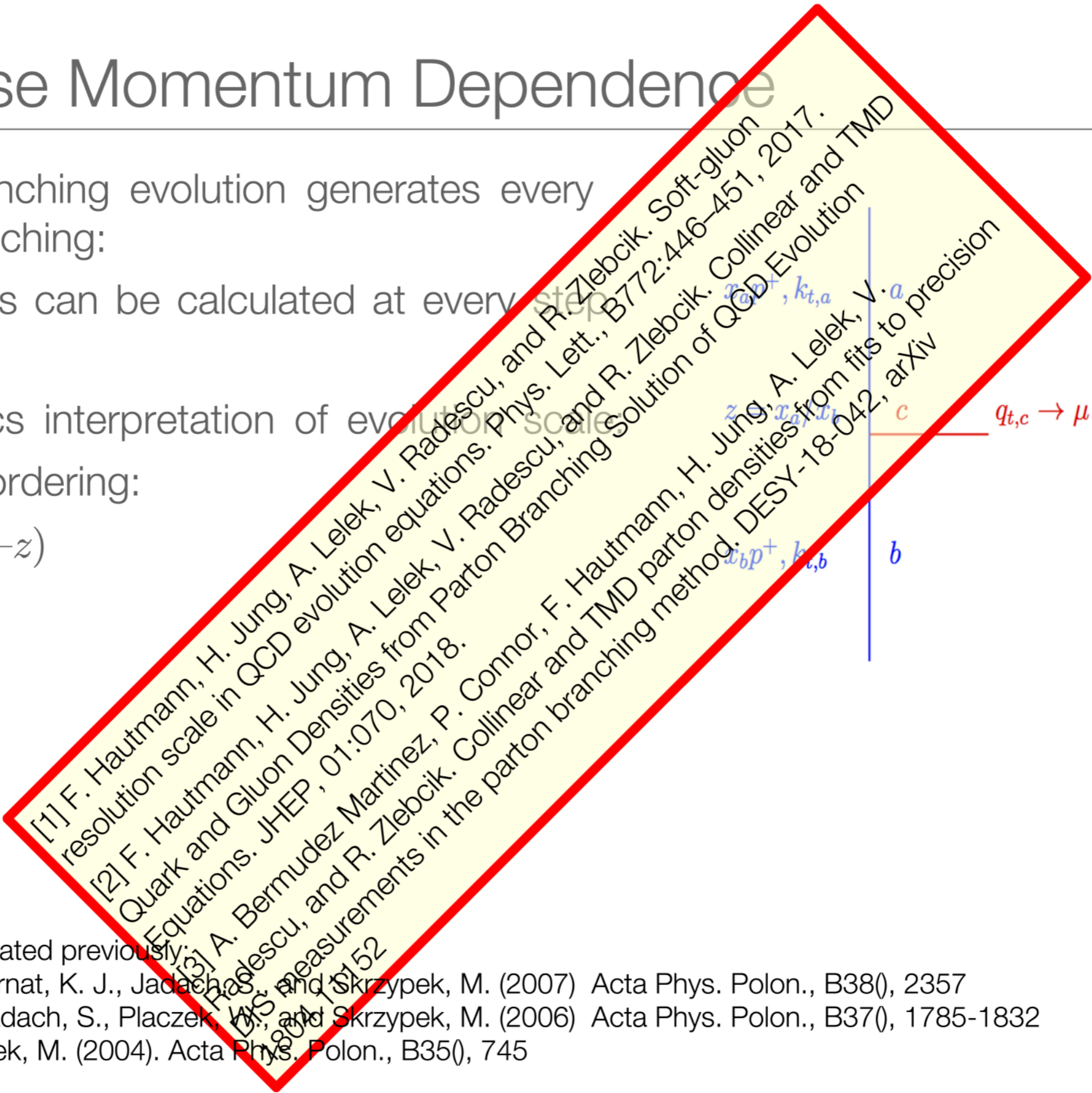
$$\mu = q_T / (1-z)$$



# Transverse Momentum Dependence

- Parton Branching evolution generates every single branching:
  - kinematics can be calculated at every step
- Give physics interpretation of evolution scale
  - angular ordering:

$$\mu = q_T / (1-z)$$



similar concept investigated previously:

Placzek, W., Golec-Biernat, K. J., Jadach, S., and Skrzypek, M. (2007) Acta Phys. Polon., B38(), 2357  
 Golec-Biernat, K. J., Jadach, S., Placzek, W., and Skrzypek, M. (2006) Acta Phys. Polon., B37(), 1785-1832  
 Jadach, S. and Skrzypek, M. (2004). Acta Phys. Polon., B35(), 745

# PDFs from Parton Branching method: fit to HERA data

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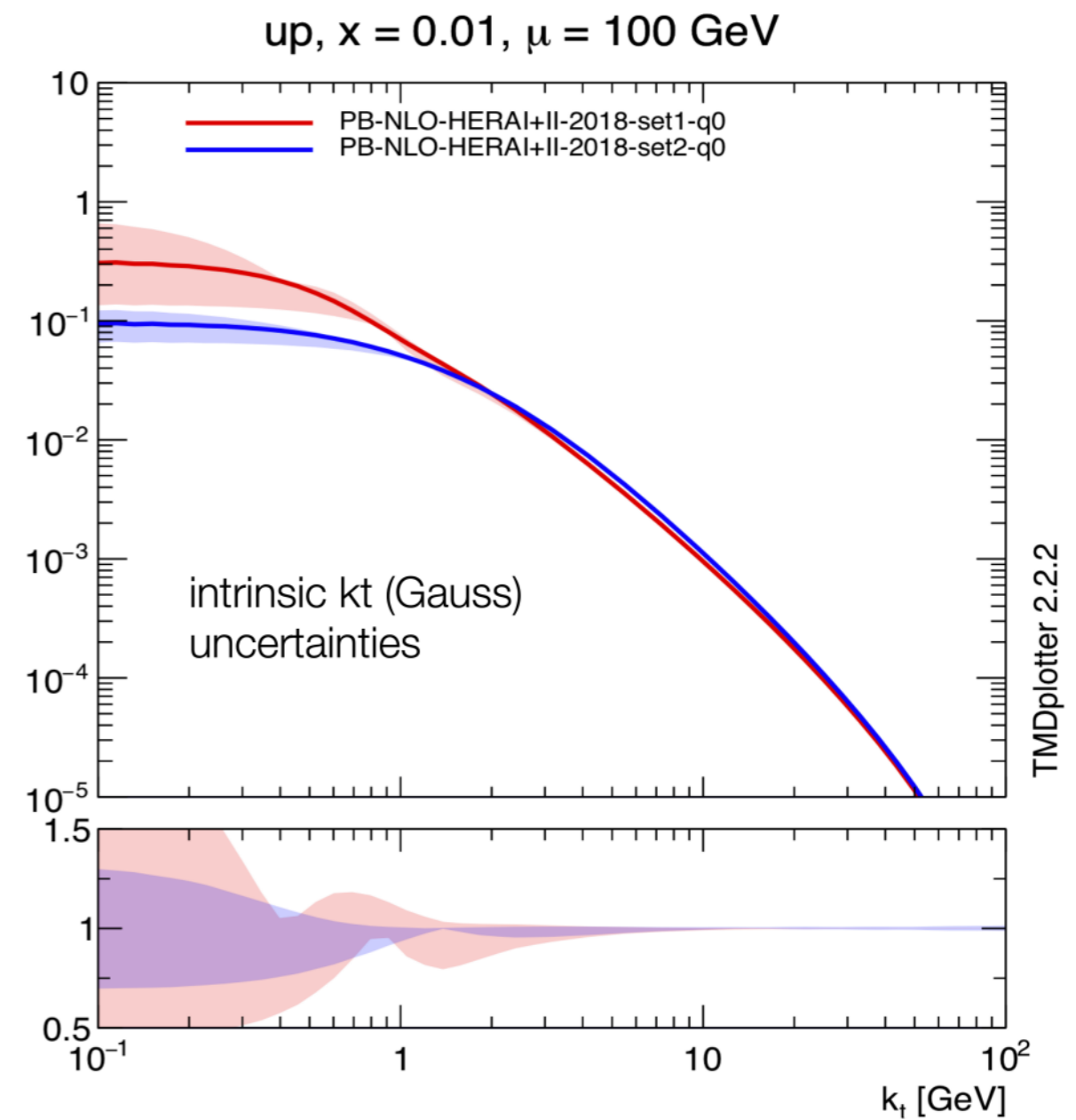
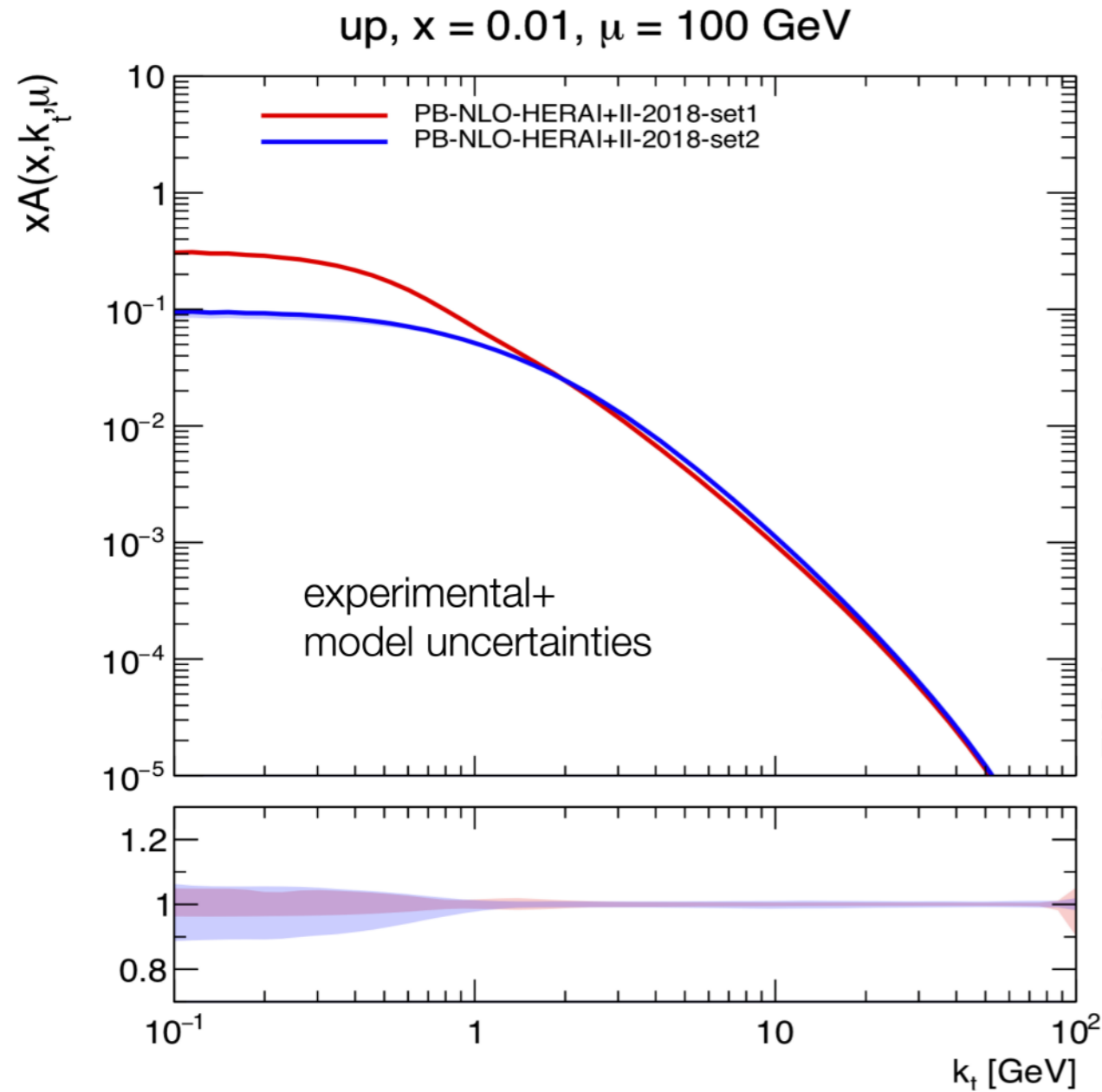
- Convolution of kernel with starting distribution

$$\begin{aligned} x f_a(x, \mu^2) &= x \int dx' \int dx'' \mathcal{A}_{0,b}(x') \tilde{\mathcal{A}}_a^b(x'', \mu^2) \delta(x'x'' - x) \\ &= \int dx' \mathcal{A}_{0,b}(x') \cdot \frac{x}{x'} \tilde{\mathcal{A}}_a^b\left(\frac{x}{x'}, \mu^2\right) \end{aligned}$$

- Fit performed using xFitter frame (with collinear Coefficient functions at NLO)
  - using full HERA 1+2 inclusive DIS (neutral current, charged current) data
    - in total 1145 data points
      - $3.5 < Q^2 < 50000 \text{ GeV}^2$
      - $4 \cdot 10^{-5} < x < 0.65$
    - using starting distribution as in HERAPDF2.0
    - $\chi^2/ndf = 1.2$

➔ Can be easily extended to include any other measurement for fit !

# TMD distributions



Differences essentially in low  $k_T$  region

- experimental+model uncertainties small
- at very low  $k_T$ , uncertainties from intrinsic  $k_T$  sizable

# Application to Drell – Yan production

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# Drell -Yan production: $q_T$ - spectrum

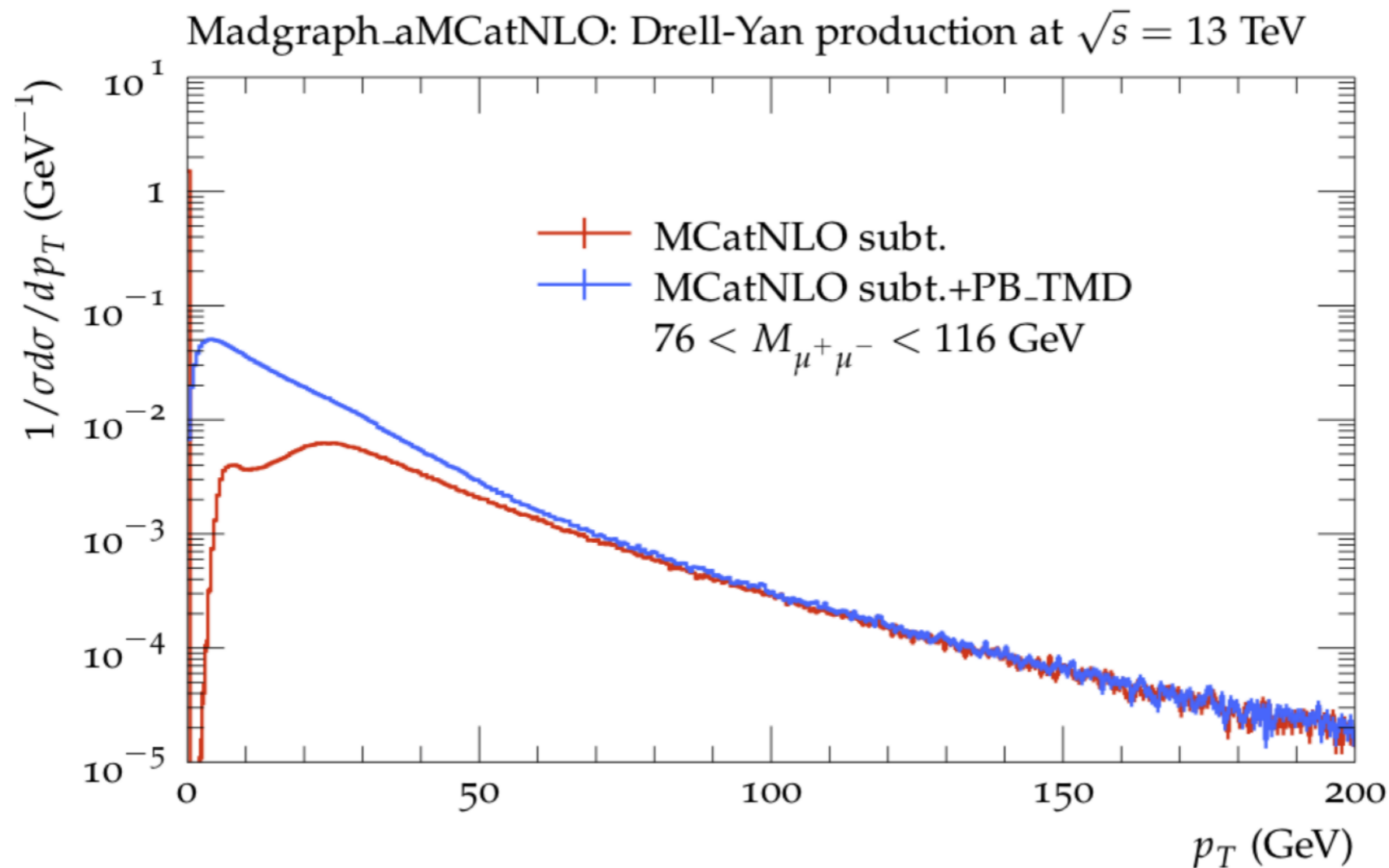
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- DY production
  - $q\bar{q} \rightarrow Z_0$
  - add  $k_t$  for each parton as function of  $x$  and  $\mu$  according to TMD
  - keep final state mass fixed:
    - $x_1$  and  $x_2$  (light-cone fraction) are different after adding  $k_t$
- use NLO calculations: MC@NLO

# Matching to hard process: MC@NLO method

- MC@NLO subtracts soft & collinear parts from NLO (added back by TMD and/or parton shower)
- MC@NLO without shower unphysical
  - use herwig6 subtraction terms

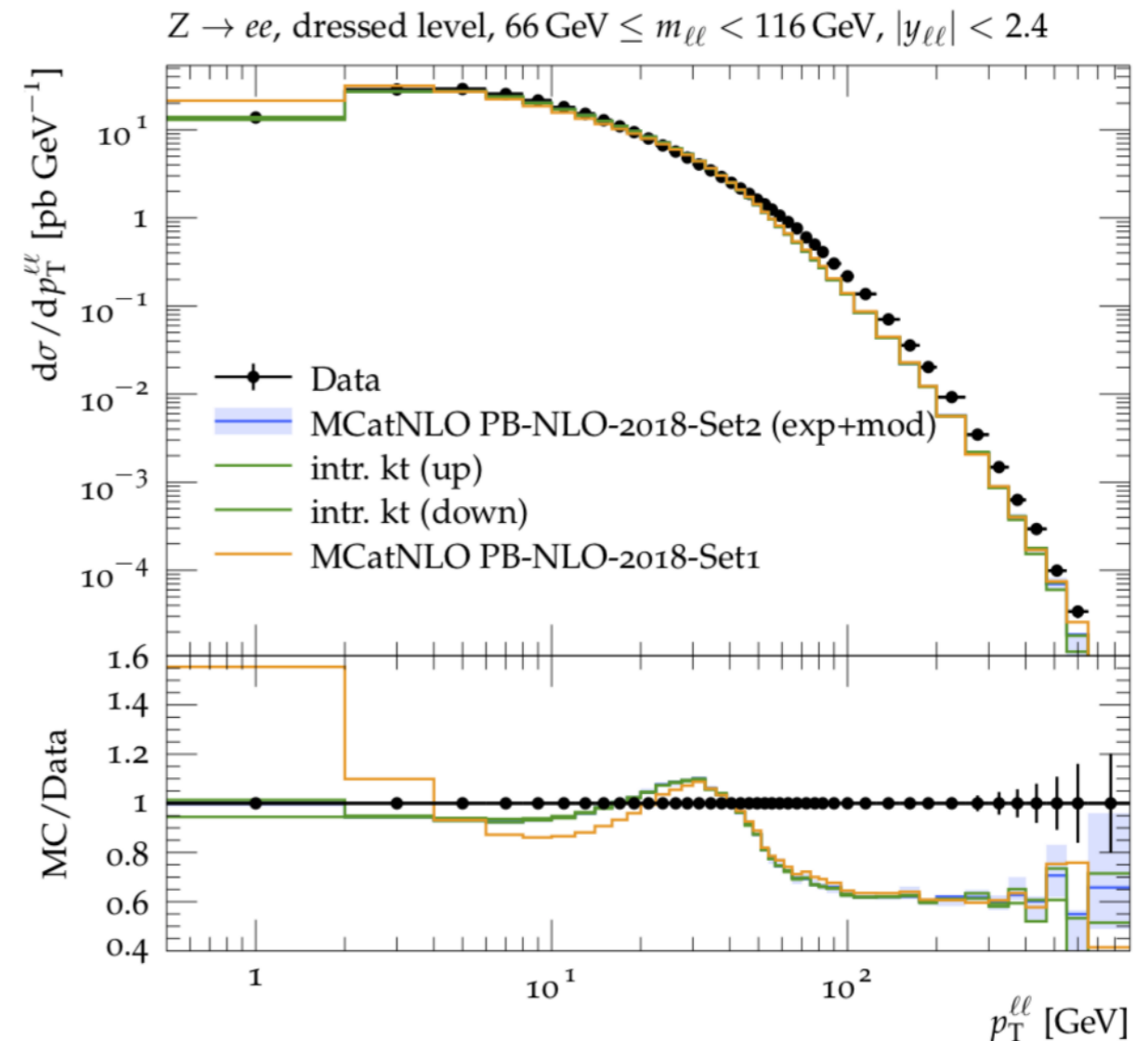
The transverse momentum spectrum of low mass Drell-Yan production at next-to-leading order in the parton branching method  
Bermudez Martinez, A. et al, arXiv 2001.06488



# Matching to hard process: MC@NLO method

- MC@NLO subtracts soft & collinear parts from NLO (added back by TMD and shower)
- low  $q_T$  region affected by subtraction of soft & collinear parts
  - filled by TMD ( + PS)
- DY production very well described by **TMD with MC@NLO**
  - TMD fills low  $q_T$  part
    - angular ordering with  $\alpha_s(q(1-z))$  is best
    - intrinsic Gauss plays little role

DY with PB TMDs: Bermudez Martinez, A. et al, arXiv 1906.00919, PhysRevD.100.074027



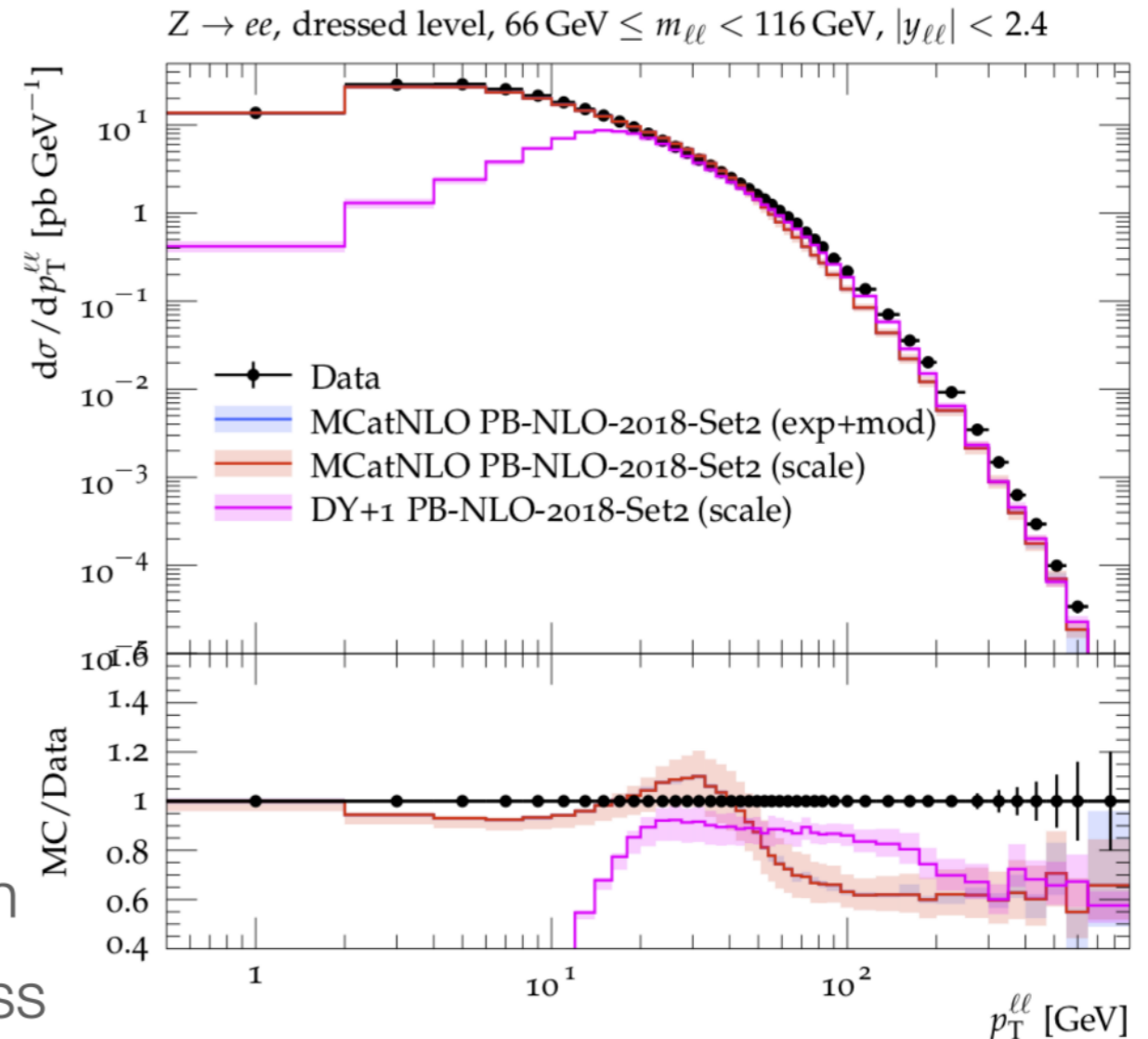
ATLAS (2016). DY at 8 TeV, EPJC76, 291, 1512.02192



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- DY production very well described by **TMD with MC@NLO**
  - TMD fills low  $q_T$  part
  - small uncertainties in small  $p_t$  region
  - scale uncertainties from hard process sizable !
  - at large  $q_T$  contribution from DY+1 jet significant

DY with PB TMDs: Bermudez Martinez, A. et al, arXiv 1906.00919, PhysRevD.100.074027

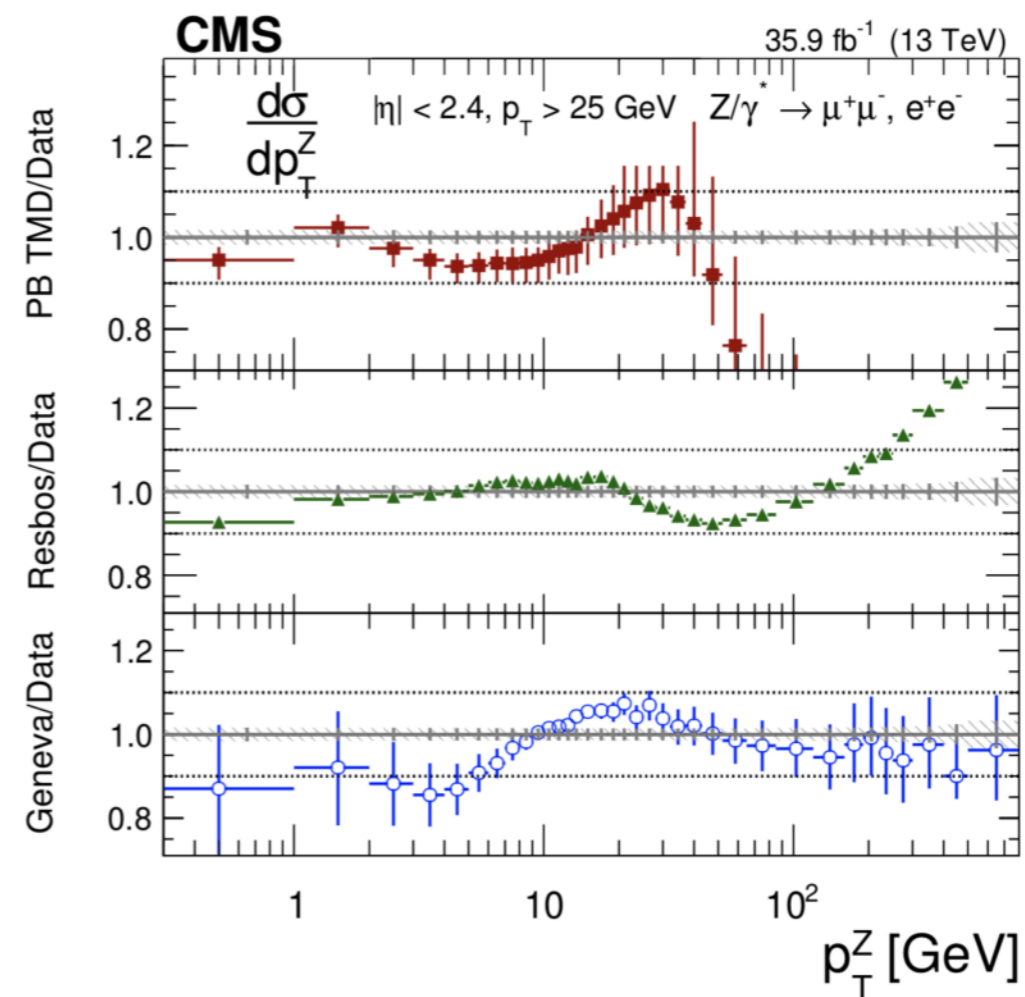
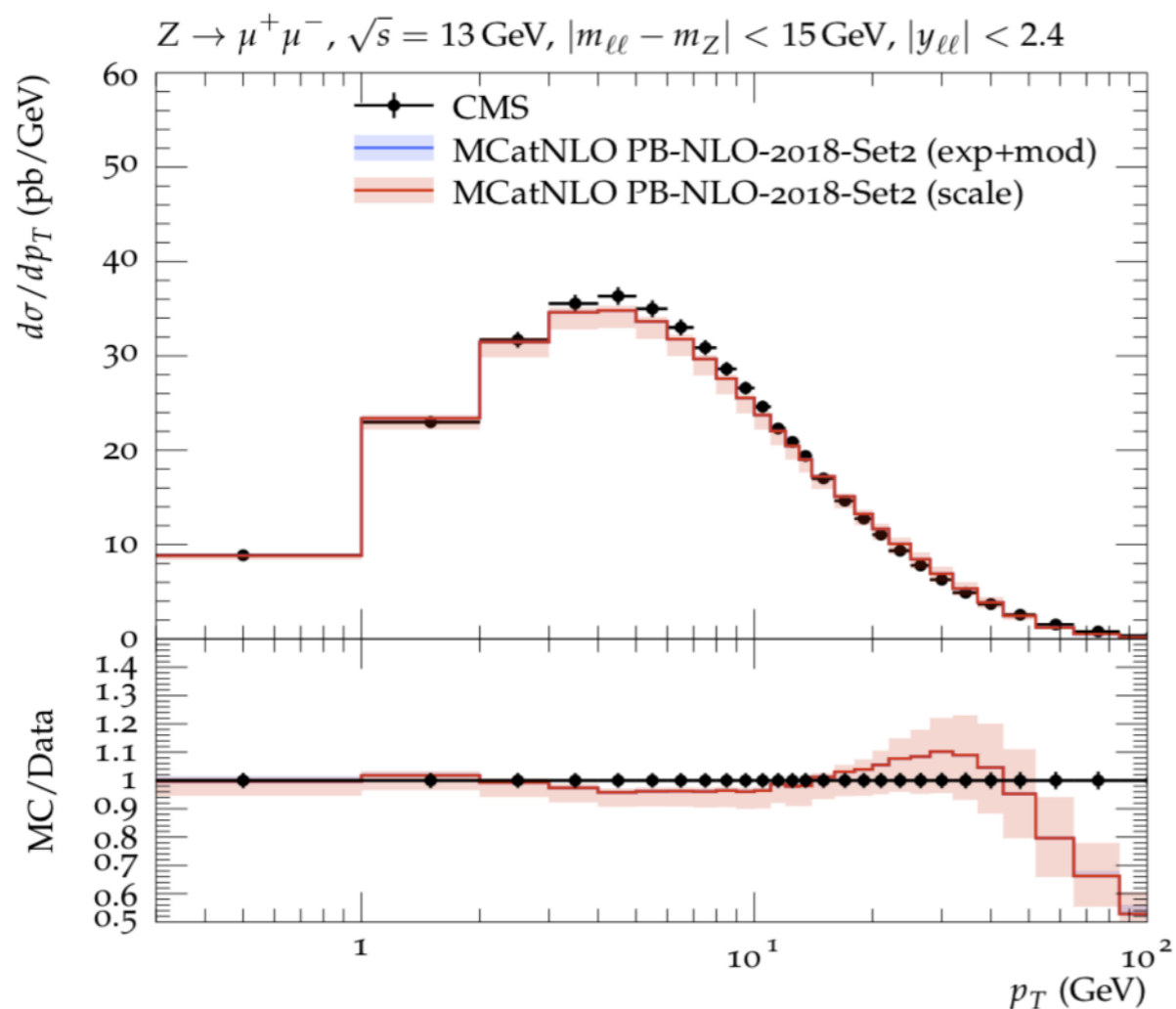


ATLAS (2016). DY at 8 TeV, EPJC76, 291, 1512.02192

# Z production at 13 TeV (CMS)

Bermudez Martinez, A. et al, arXiv 2001.06488

SMP-17-010, JHEP12 (2019) 061



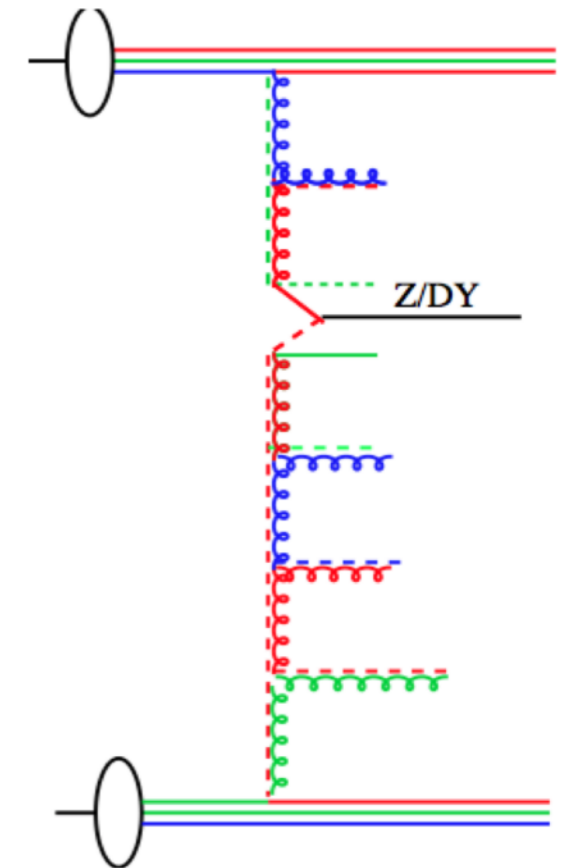
- very good description of low  $p_T$  region
  - at larger  $p_T$  contribution from higher order matrix elements important
- Uncertainties in PB method mainly from scale of **MC@NLO** matrix element

What happens at small  $m_{DY}$  and small  $\sqrt{s}$  ?

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# What happens at small $m_{DY}$ and small $\sqrt{s}$ ?

- at low mass
  - $p_T$  of DY is dominated by **intrinsic  $k_T$**  and by **soft gluons**, which need to be resummed :)
  - latest measurement: PHENIX (PhysRevD.99.072003) at  $\sqrt{s} = 200$  GeV for  $4.6 \leq m_{DY} \leq 8.2$  GeV
  - other measurements (older)
    - R209 (1982) PhysRevLett.48.302 at  $\sqrt{s} = 62$  GeV (data read from plot in paper)
    - NUSEA (2003) hep-ex/0301031 at  $\sqrt{s} = 38$  GeV (unpublished)
- Can PB method with MCatNLO + PB be applied to measurements at small  $\sqrt{s}$  and small  $m_{DY}$  ?
- Is there a small  $p_T$  crisis ?



# The difficulties at small $q_T$ and small $\sqrt{s}$

PHYSICAL REVIEW D **100**, 014018 (2019)

## Difficulties in the description of Drell-Yan processes at moderate invariant mass and high transverse momentum

Alessandro Bacchetta,<sup>1,2,\*</sup> Giuseppe Bozzi,<sup>1,2,†</sup> Martin Lambertsen,<sup>3,‡</sup> Fulvio Piacenza,<sup>1,2,§</sup>  
Julius Steiglechner,<sup>3,||</sup> and Werner Vogelsang<sup>3,¶</sup>

<sup>1</sup>*Dipartimento di Fisica, Università di Pavia, via Bassi 6, I-27100 Pavia, Italy*

<sup>2</sup>*INFN Sezione di Pavia, via Bassi 6, I-27100 Pavia, Italy*

<sup>3</sup>*Institute for Theoretical Physics, Tübingen University, Auf der Morgenstelle 14, D-72076 Tübingen, Germany*

(Received 30 January 2019; published 22 July 2019)

Both regimes,  $q_T \ll Q$  and  $q_T \sim Q$ , as well as their matching, must be under theoretical control in order to have a proper understanding of the physics of the Drell-Yan process. In the present work, we study the process at fixed-target energies for moderate values of the invariant mass  $Q$  and in the region  $q_T \lesssim Q$ . We focus on the predictions based on collinear factorization and examine their ability to describe the experimental data in this regime. **We find, in fact, that the predicted cross sections fall significantly short of the available data even at the highest accessible values of  $q_T$ .** We investigate possible sources of uncertainty in the predictions based on collinear factorization, and two extensions of the collinear framework: the resummation of high- $q_T$  threshold logarithms, and transverse-momentum smearing. None of these appear to lead to a satisfactory agreement with the data. **We argue that these findings also imply that the Drell-Yan cross section in the “matching regime”  $q_T \lesssim Q$  is presently not fully understood at fixed-target energies.**

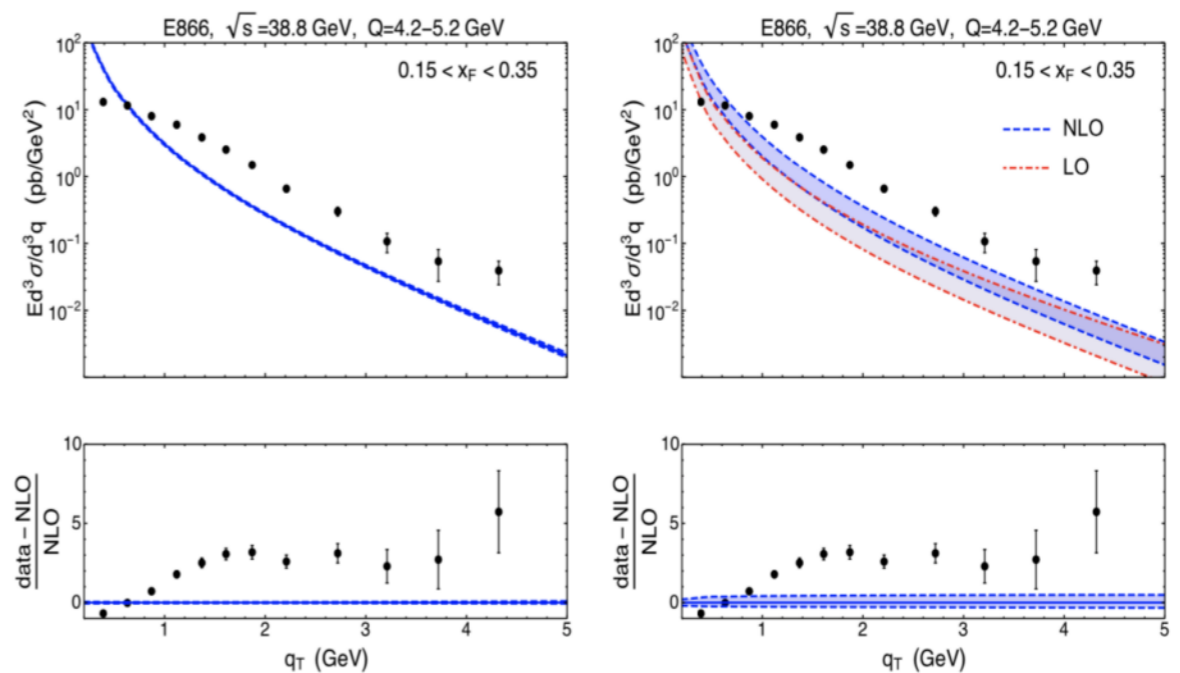
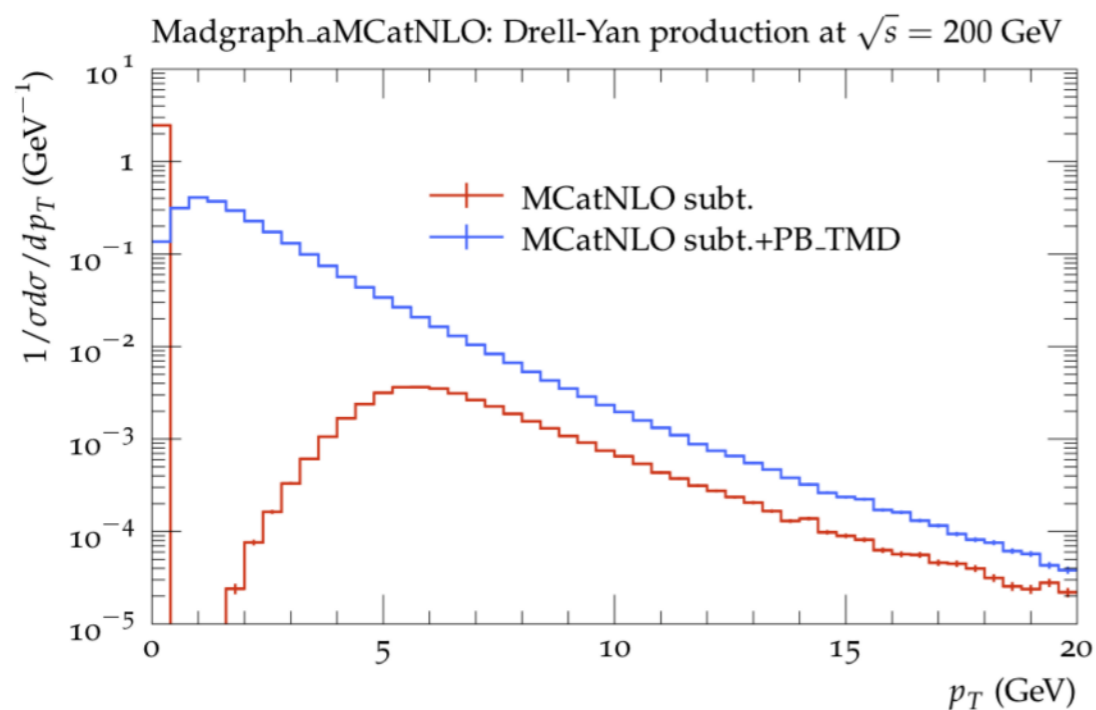
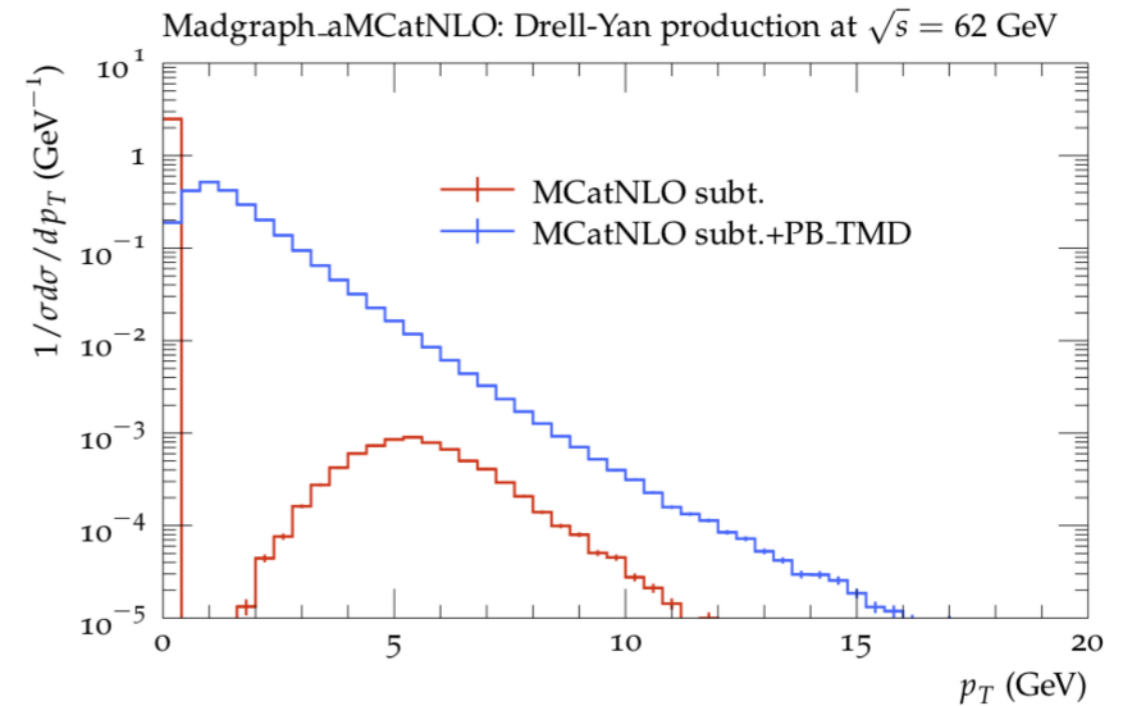
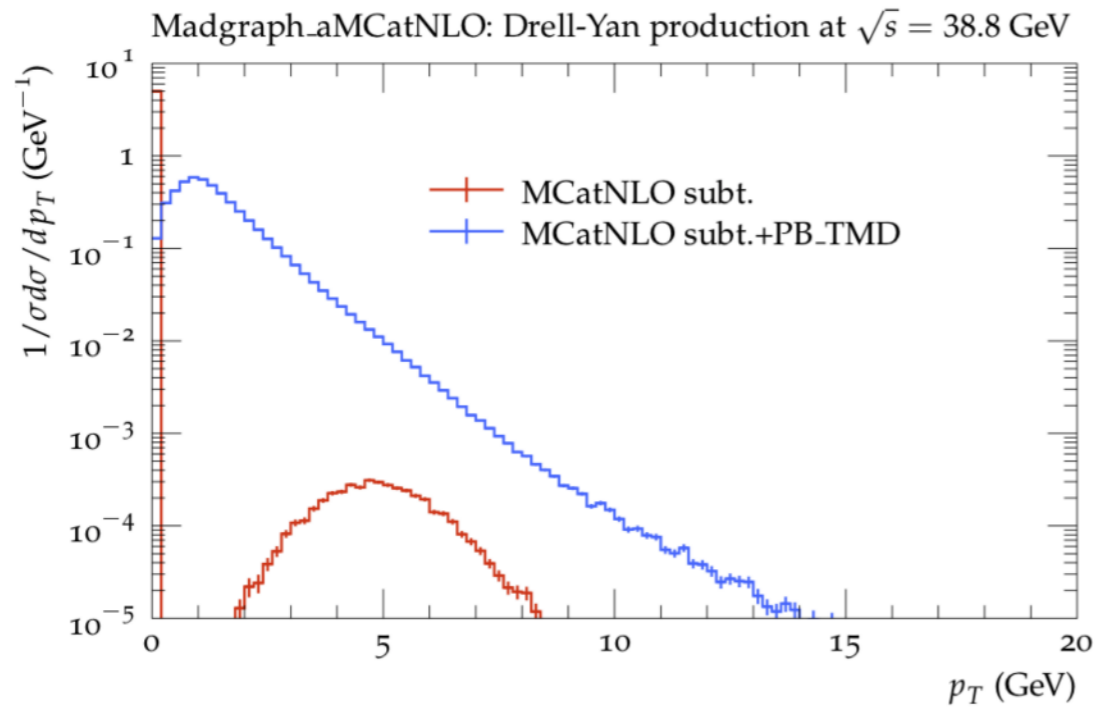


FIG. 2. Transverse-momentum distribution of Drell-Yan dimuon pairs at  $\sqrt{s} = 38.8$  GeV in a selected invariant mass range and Feynman- $x$  range: experimental data from Fermilab E866 (hydrogen target) [41] compared to LO QCD and NLO QCD results. (Left panels) NLO QCD [ $\mathcal{O}(\alpha_s^2)$ ] calculation with central values of the scales  $\mu_R = \mu_F = Q = 4.7$  GeV, including a 90% confidence interval from the CT14 PDF set [39]. (Right panels) LO QCD and NLO QCD theoretical uncertainty bands obtained by varying the renormalization and factorization scales independently in the range  $Q/2 < \mu_R, \mu_F < 2Q$ .

# MCatNLO for small $m_{DY}$ and small $\sqrt{s}$

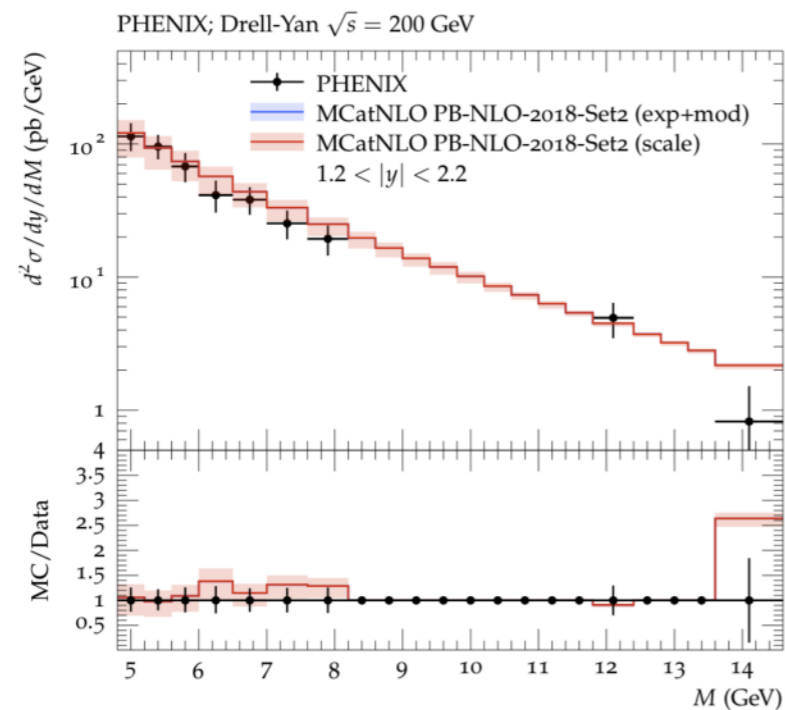
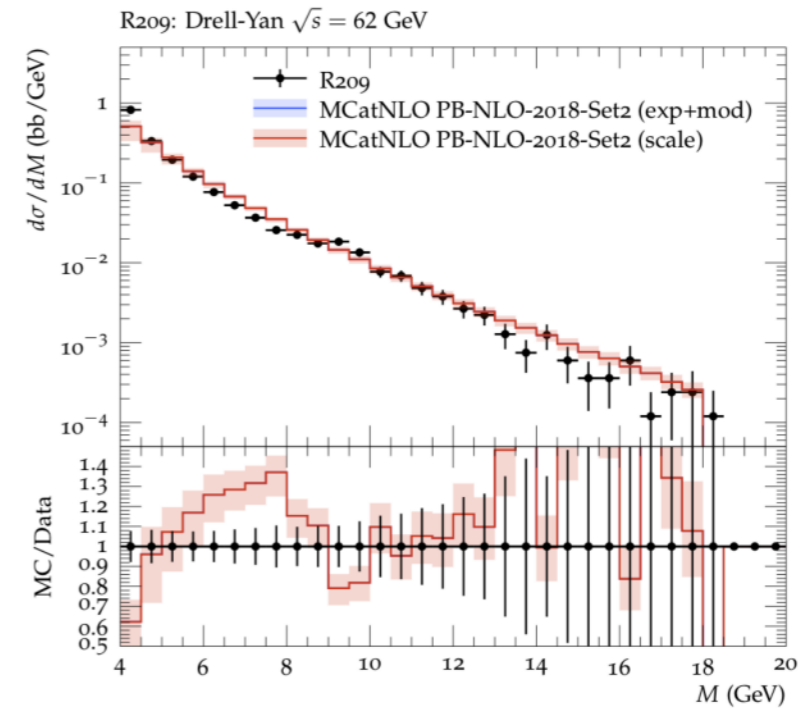
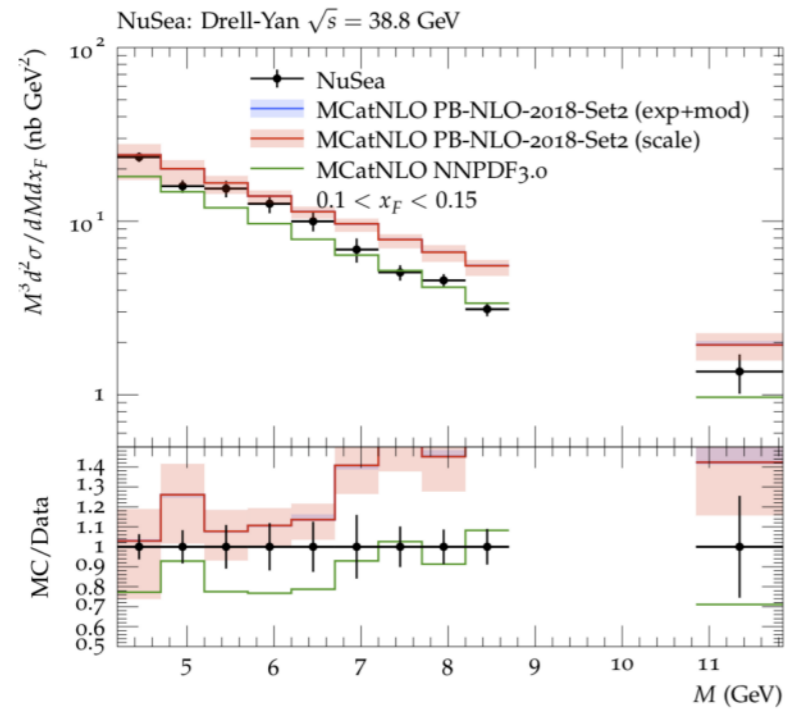
Bermudez Martinez, A. et al, arXiv 2001.06488



- Contribution of real 1 parton emission increases with  $\sqrt{s}$
- NLO corrections are large at small  $m_{DY}$  (factor of 2 or more) because scale ( $m_{DY}$ ) is small and  $\alpha_s(m_{DY})$  is large !

# Comparison with measurements

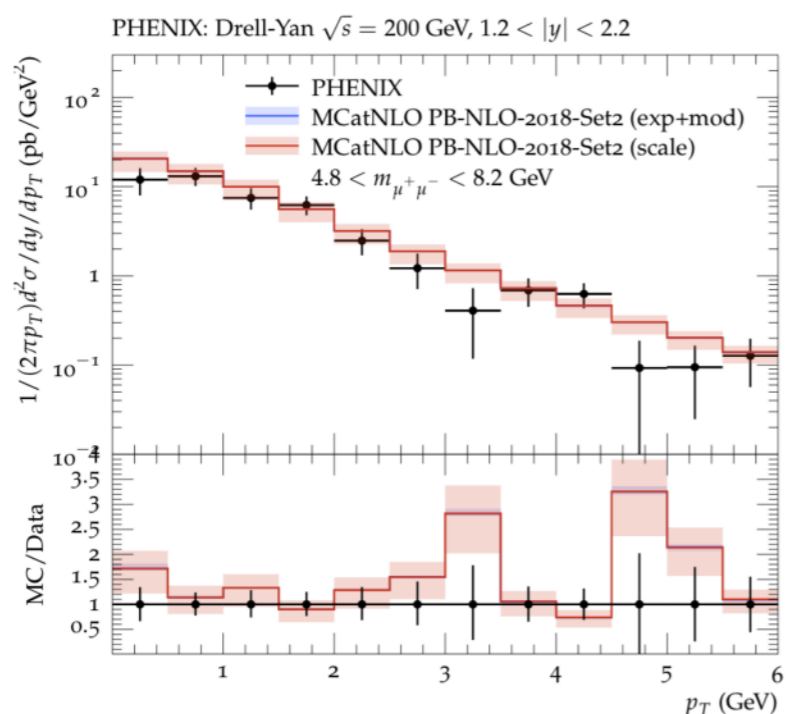
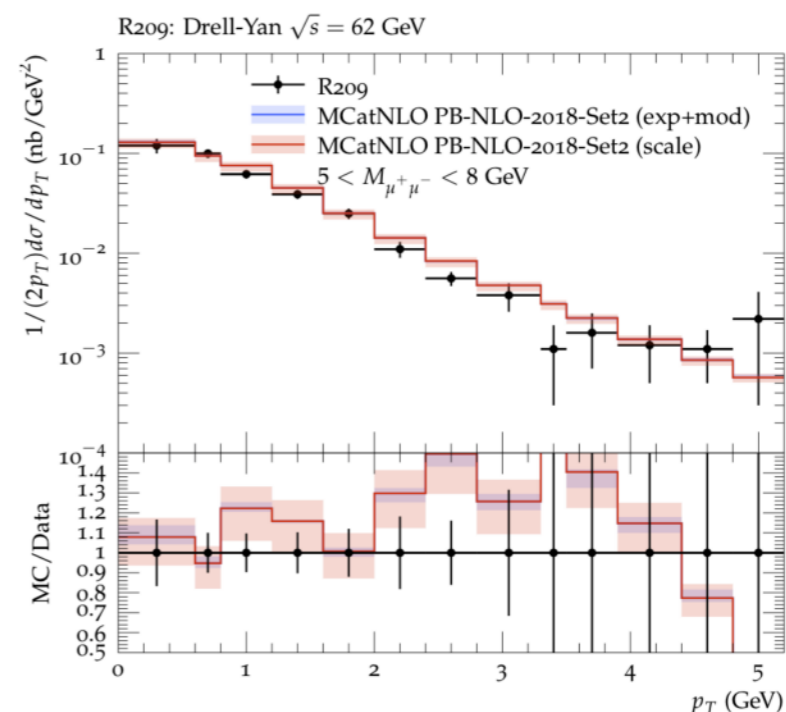
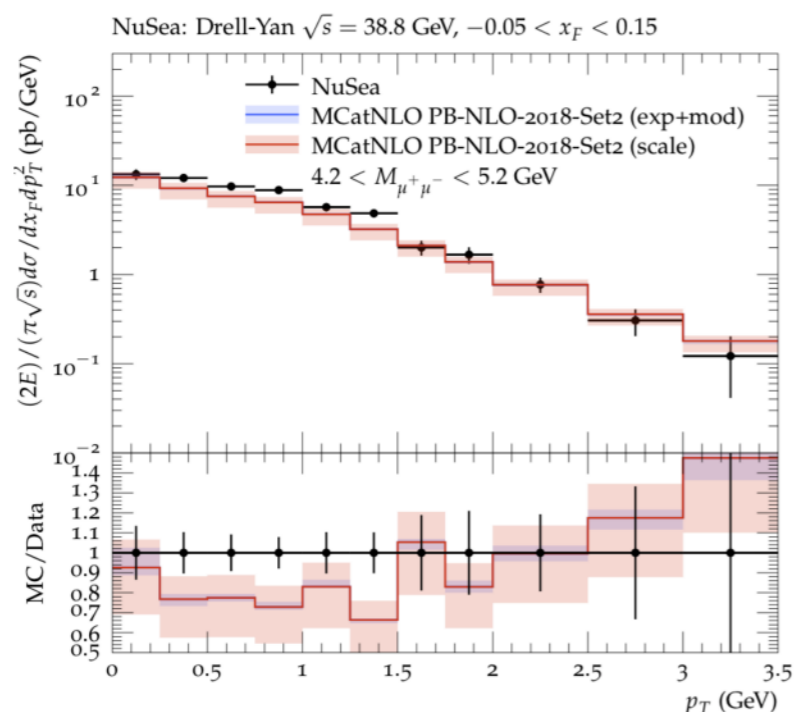
Bermudez Martinez, A. et al, arXiv 2001.06488



- Mass distribution well described with McatNLO + PB pdfs
- sensitive only to collinear pdf
  - at smallest  $\sqrt{s}$ , large  $x$  probed
  - pdfs are fitted to HERA data and not well constrained at large  $x$

# The DY $p_T$ - spectrum

Bermudez Martinez, A. et al, arXiv 2001.06488



- DY  $p_T$  -spectrum well described with MC@NLO+PB-TMDs
- good agreement within uncertainties:

	NuSea	R209	PHENIX
$\chi^2/ndf$	1.08	1.27	1.04

- no hint for  $p_T$  crisis !



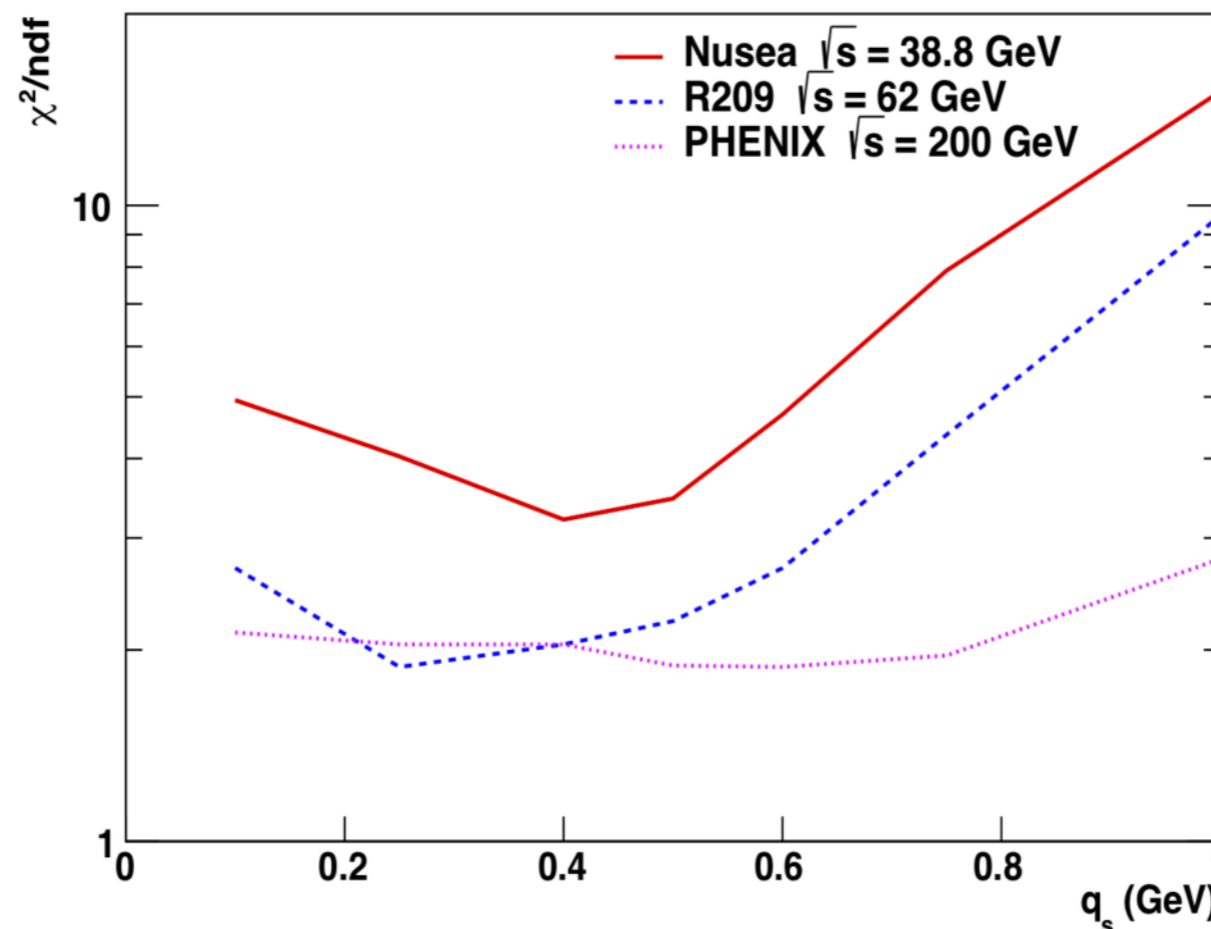
# Constraints on intrinsic $k_T$

Bermudez Martinez, A. et al, arXiv 2001.06488

- Intrinsic  $k_T$  is included in starting distribution, for simplicity Gauss is assumed

$$\mathcal{A}_{0,b}(x, k_T^2, \mu_0^2) = f_{0,b}(x, \mu_0^2) \cdot \exp(-|k_T^2|/2\sigma^2) / (2\pi\sigma^2)$$

- constrain width  $\sigma^2 = q_s^2/2$  of Gauss distribution (default  $q_s = 0.5 \text{ GeV}$ )

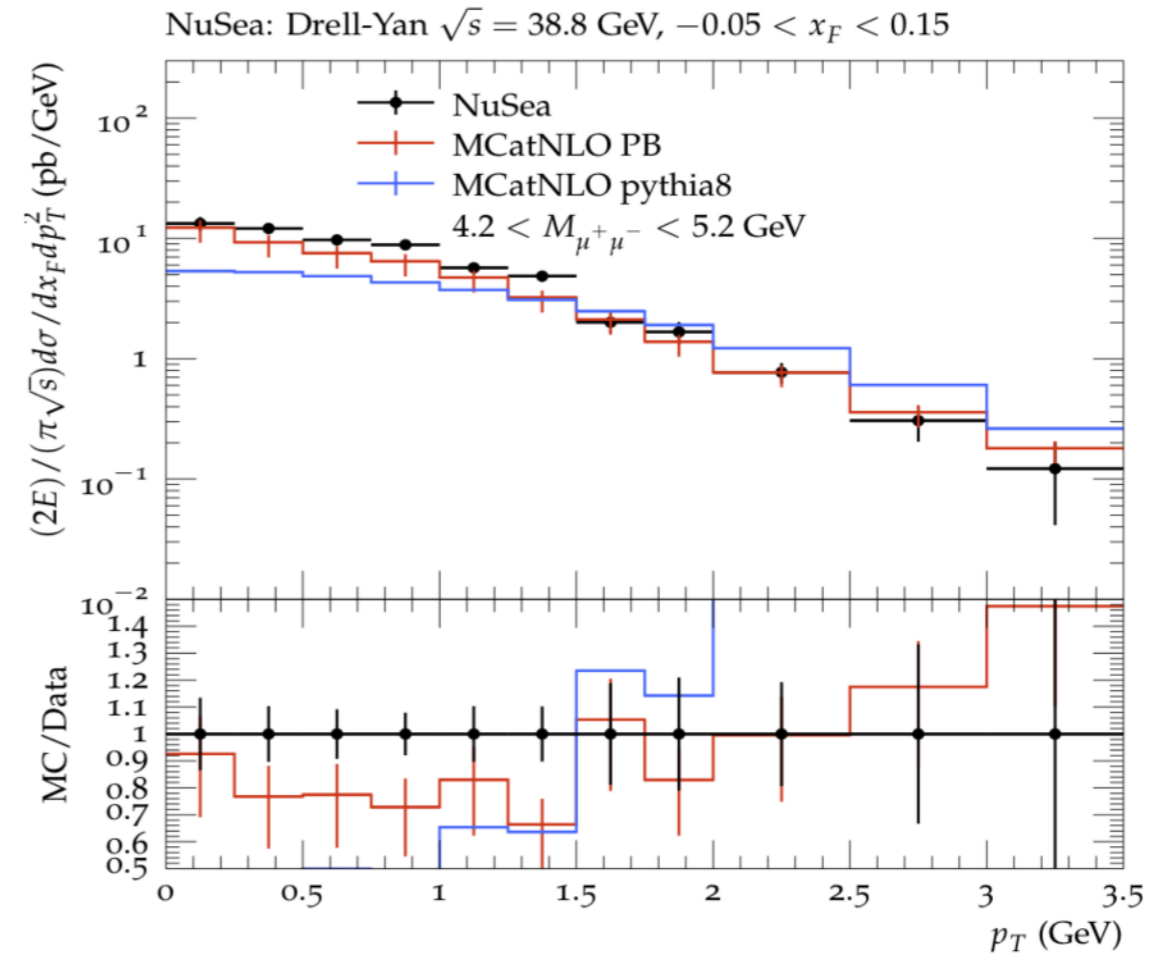
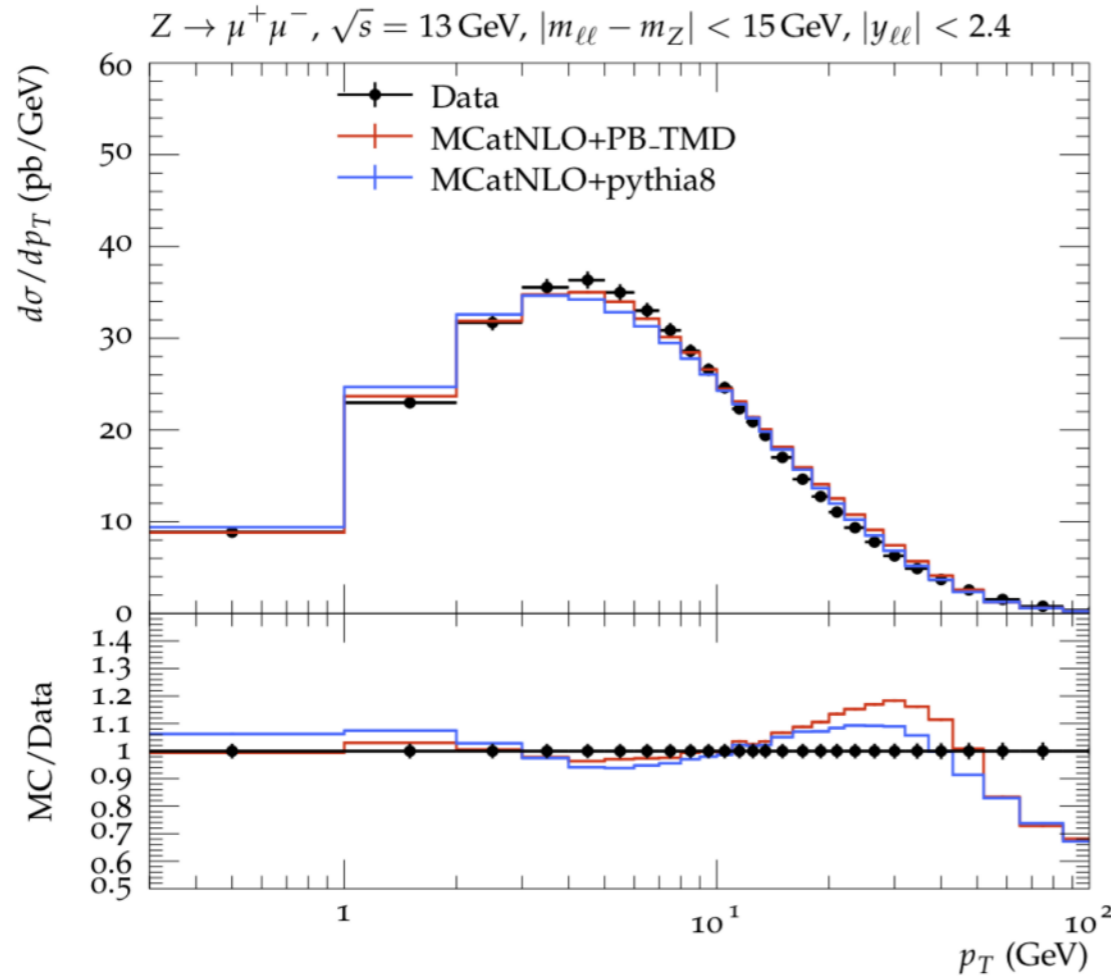


- Only at low energies, sensitivity to intrinsic Gauss observed....

# How do other approaches perform ?

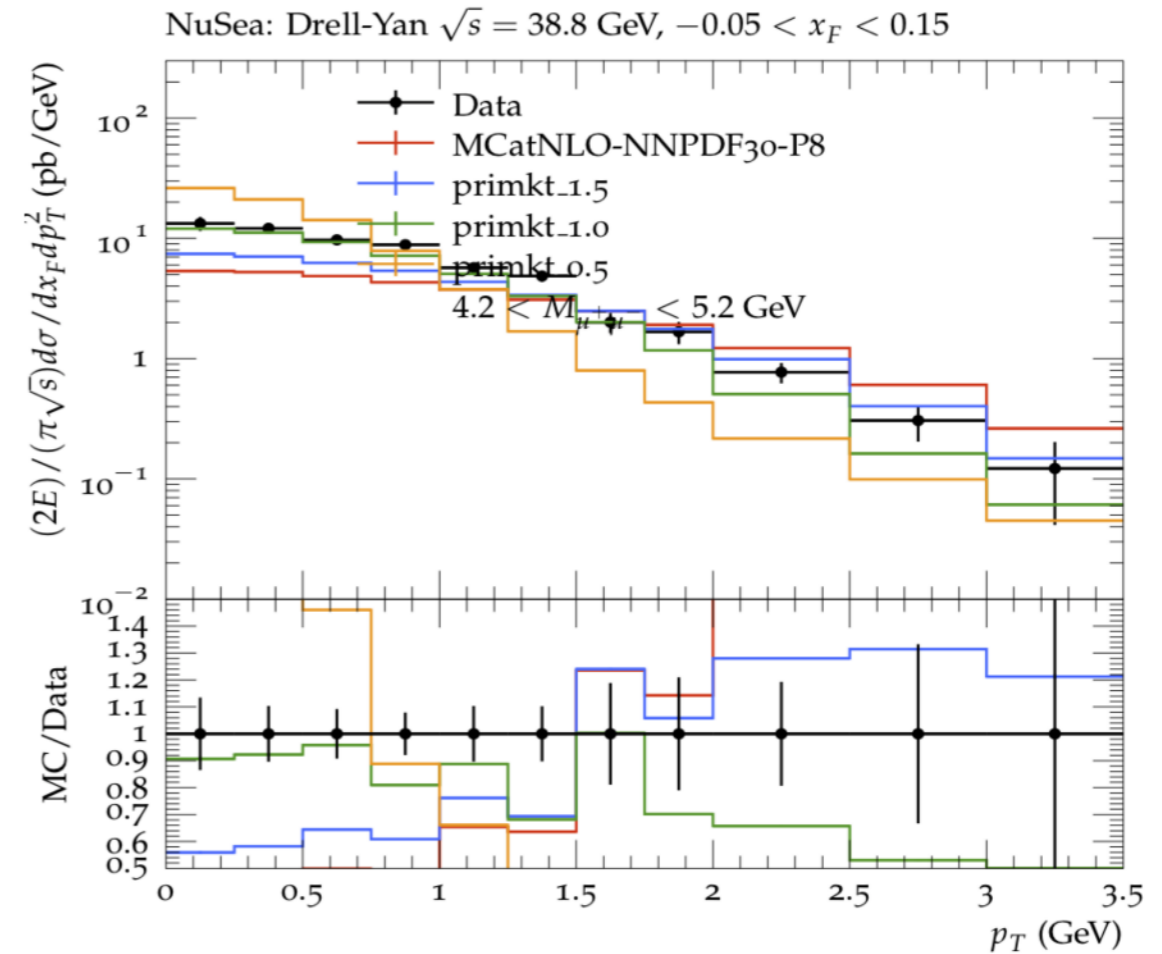
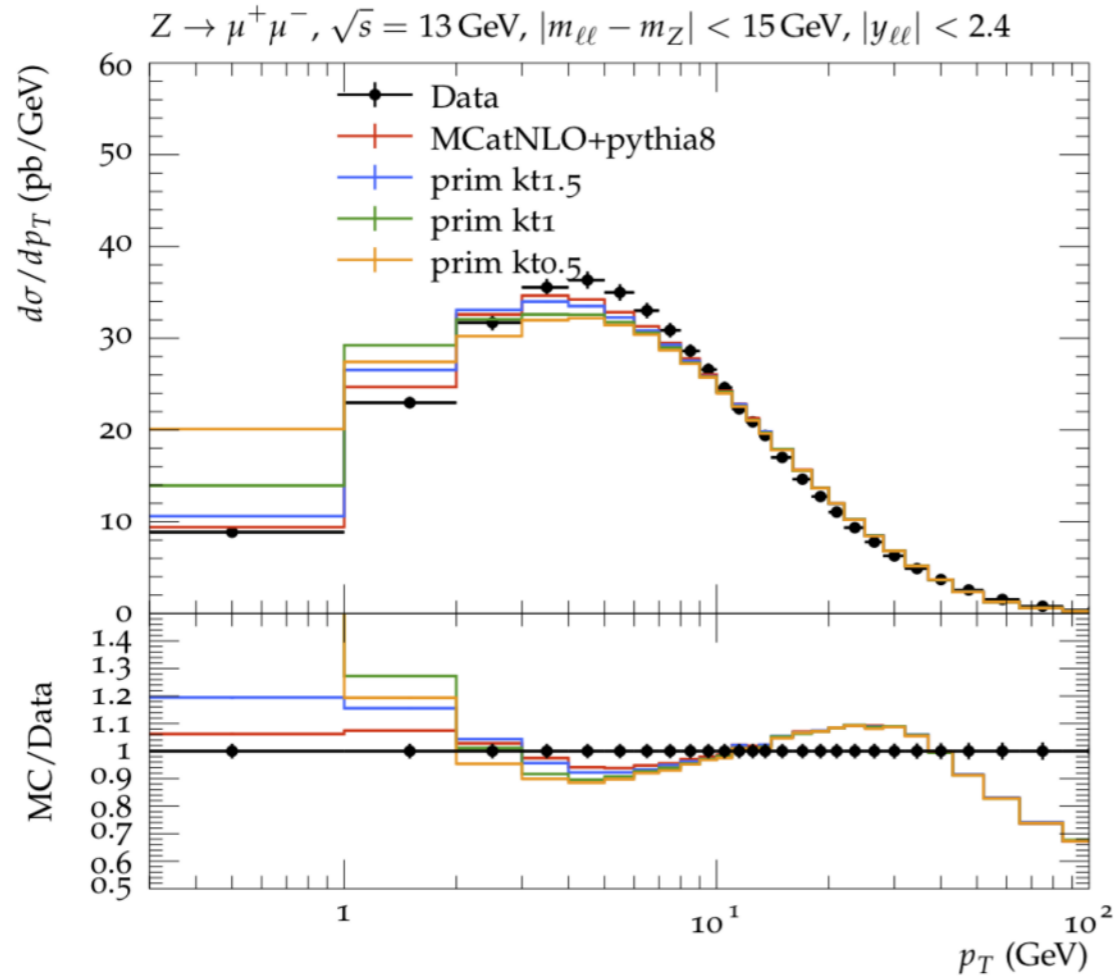
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# Predictions from MCatNLO+PYTHIA8



- differences observed with McatNLO using Monash tune in P8
  - too high at high energy
  - too low at low energy
    - can it be tuned ?

# Predictions from MCatNLO+PYTHIA8



- differences observed with McatNLO using Monash tune in P8
  - intrinsic  $k_T$  in P8 cannot be simply tuned to describe both high and low energy data

# Conclusion

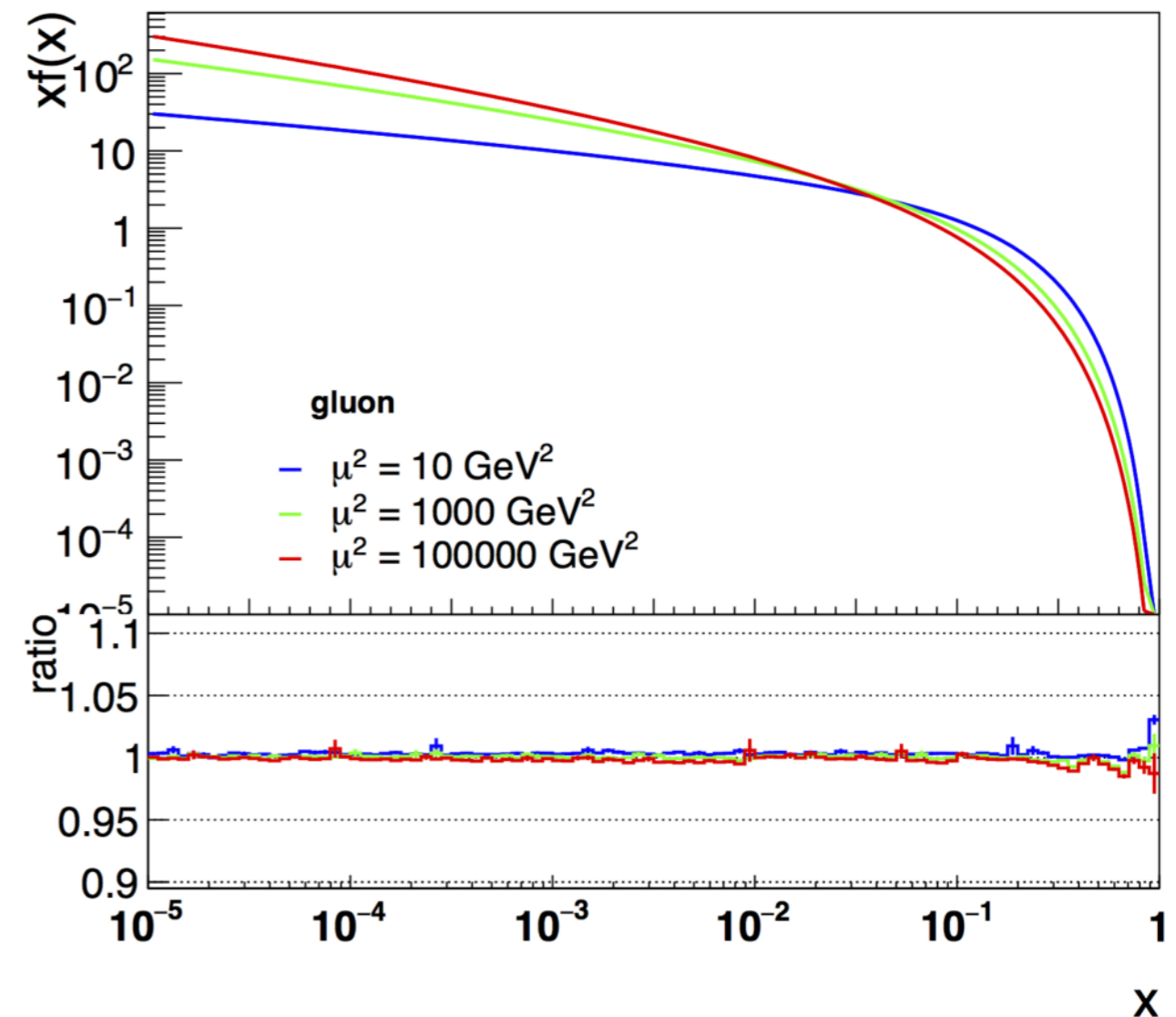
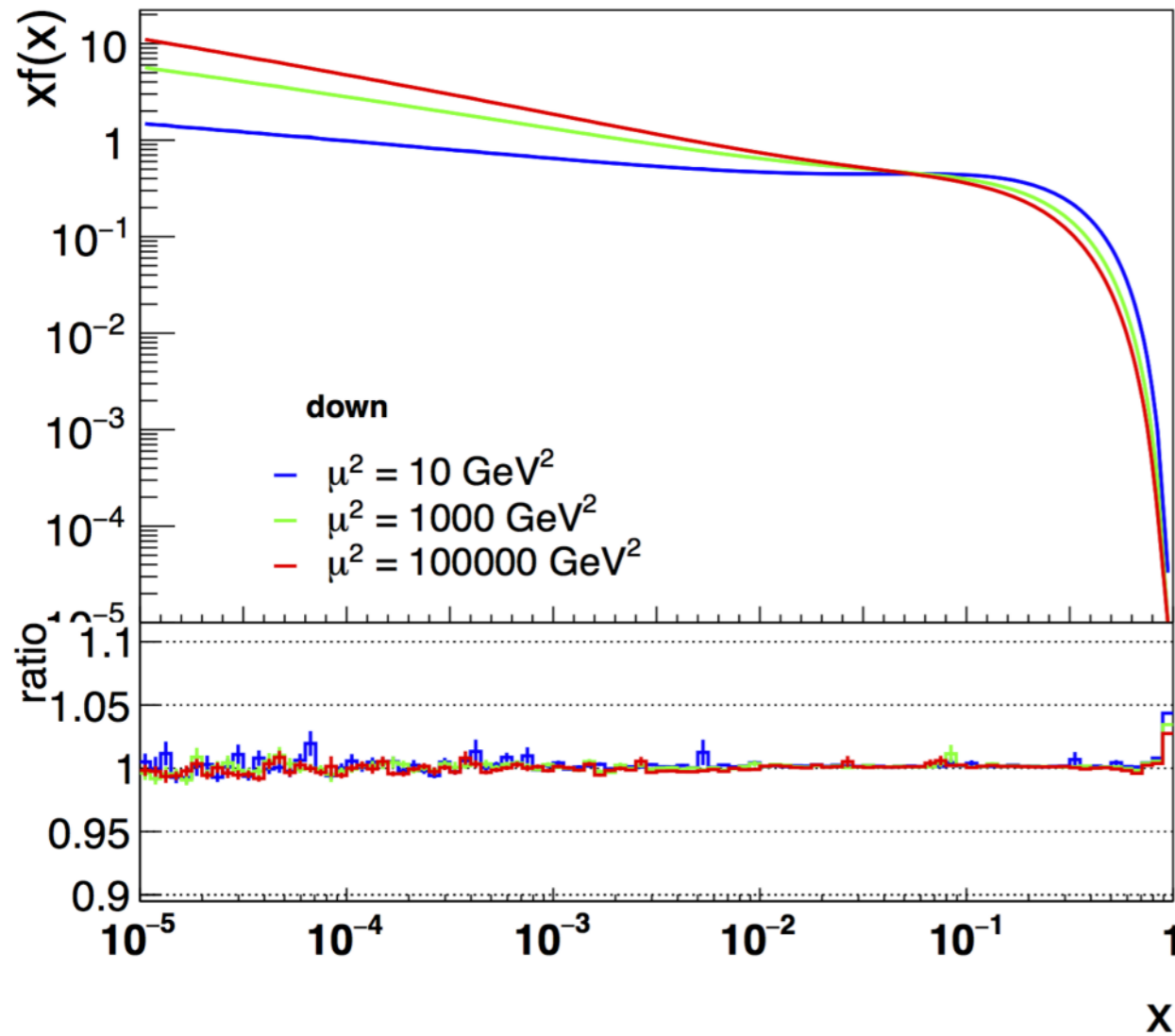
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- Parton Branching method to solve DGLAP equation at LO, NLO and NNLO
  - method directly applicable to determine  $k_t$  distribution (as would be done in PS)
  - ➔ TMD distributions for all flavors determined at LO and NLO
  - ➔ TMD evolution implemented in xFitter – fits to DIS processes at the moment
- Application to DY processes in pp:
  - ➔ DY  $q_T$  - spectrum without new parameters for Z and low mass DY
    - ➔ matching TMD with MC@NLO
  - DY  $q_T$  - spectrum at low mass and low energies well described
    - in contrast to prediction using pythia
  - Success of PB TMDs with McatNLO:
    - describe DY production over wide range
    - proper prediction of low  $p_T$  spectrum – needed for  $m_W$  determination

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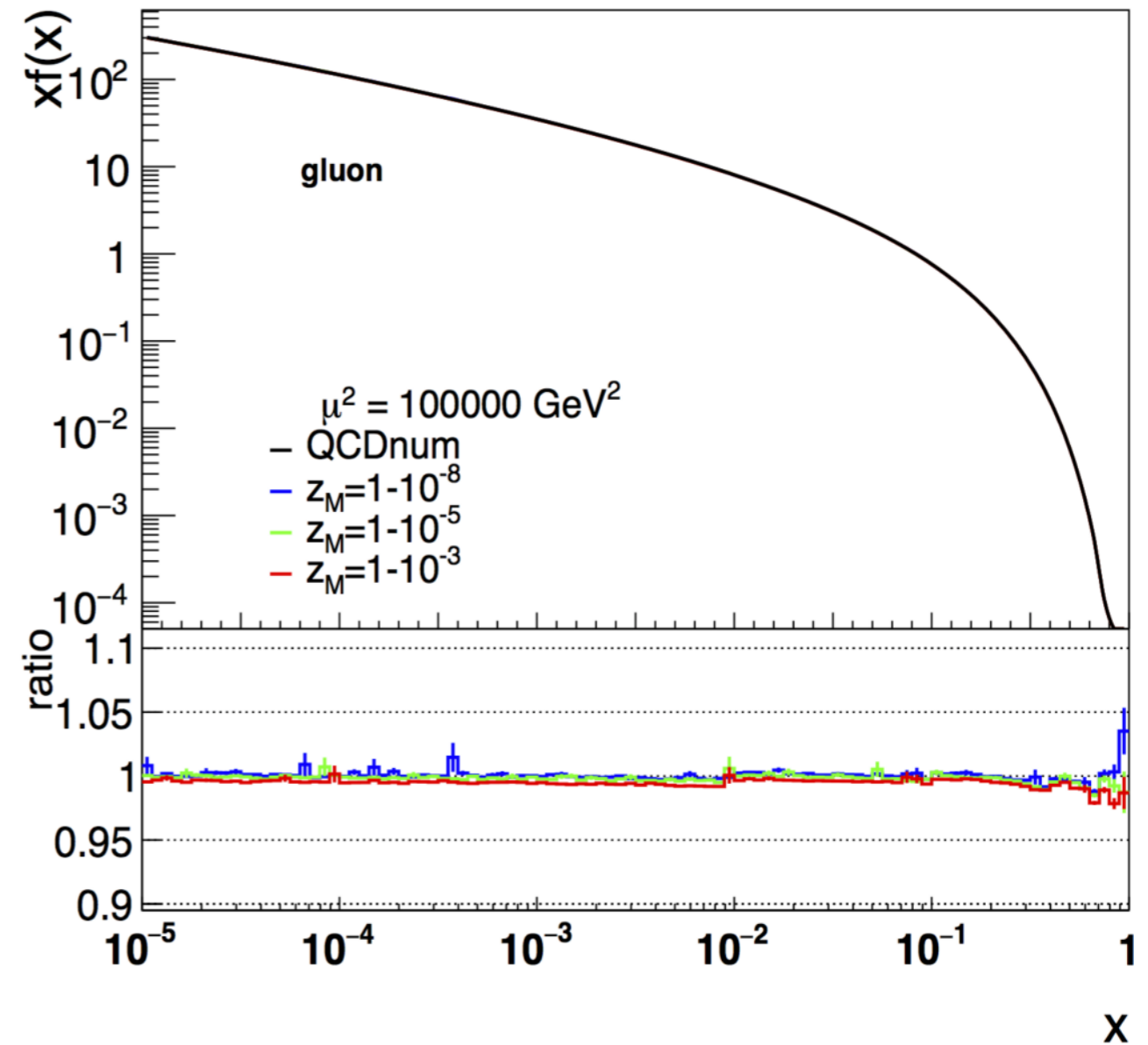
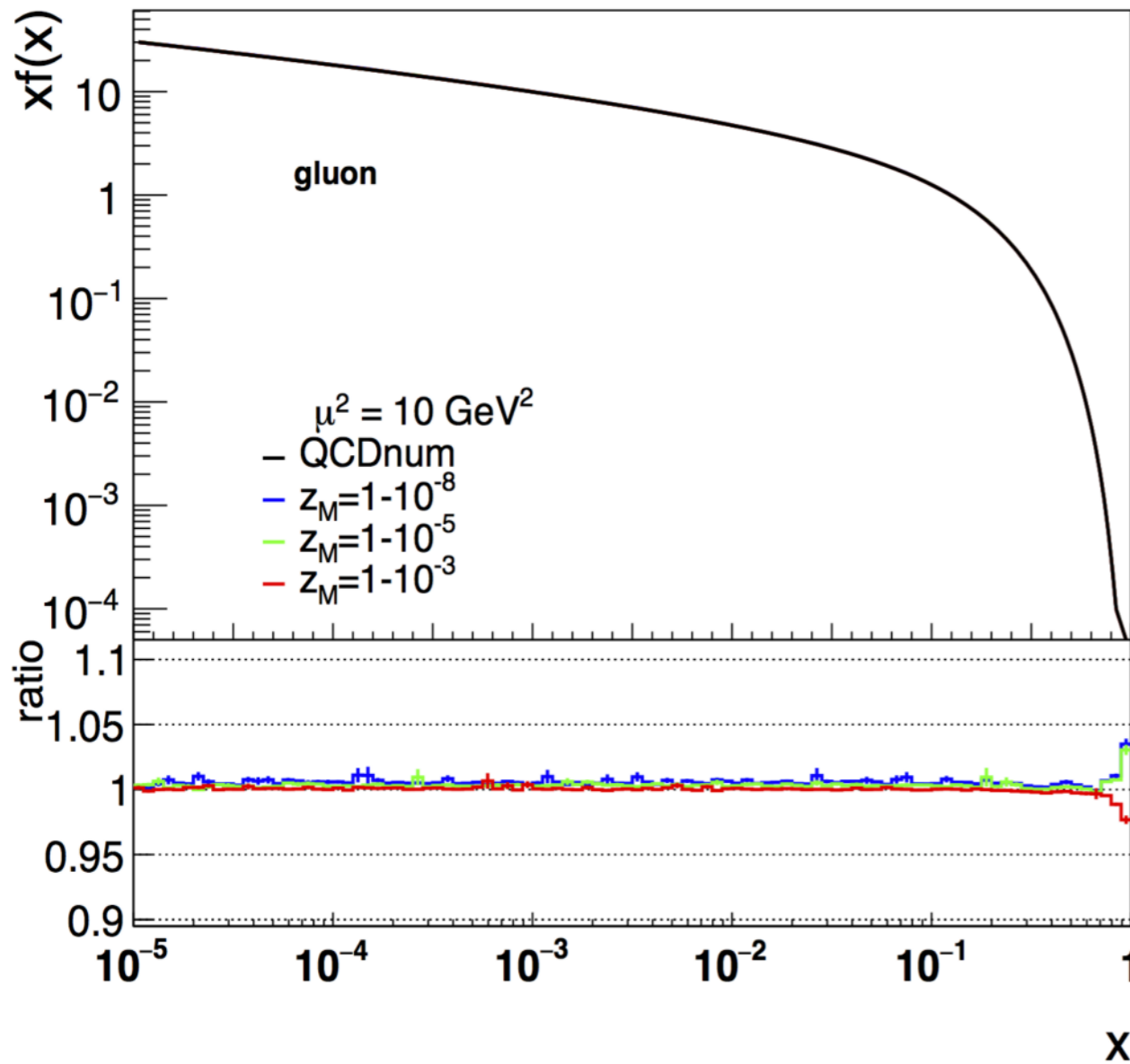
# Appendix

# Validation of method with QCDnum at **NLO**



- Very good agreement with **NLO** - QCDnum over all  $x$  and  $\mu^2$ 
  - the same approach works also at NNLO !

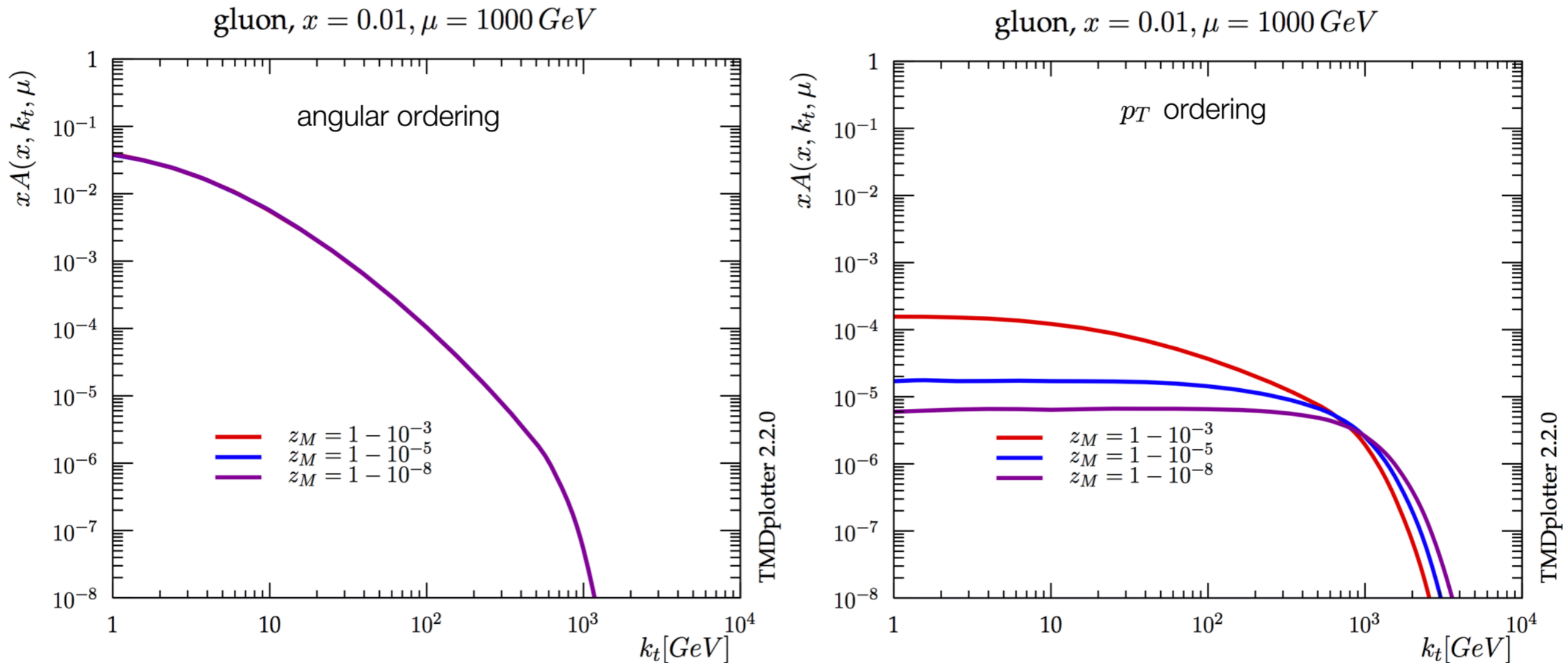
# Validation of method at **NLO**: $z_M$ - dependence



- No dependence on  $z_M$  if  $z_M$  is large enough:
  - approximation is of  $\mathcal{O}(1 - z_M)$
- Very good agreement with **NLO** - QCDnum

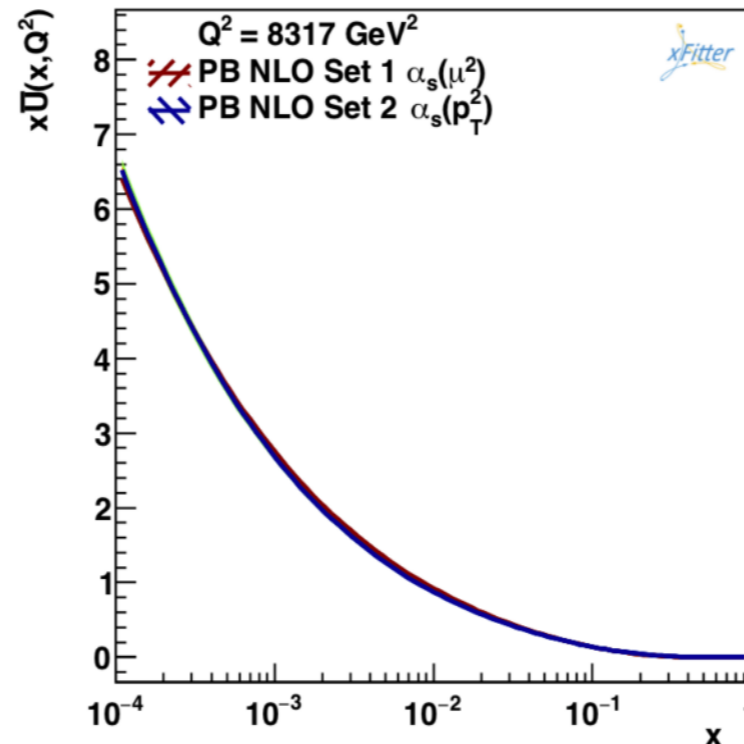
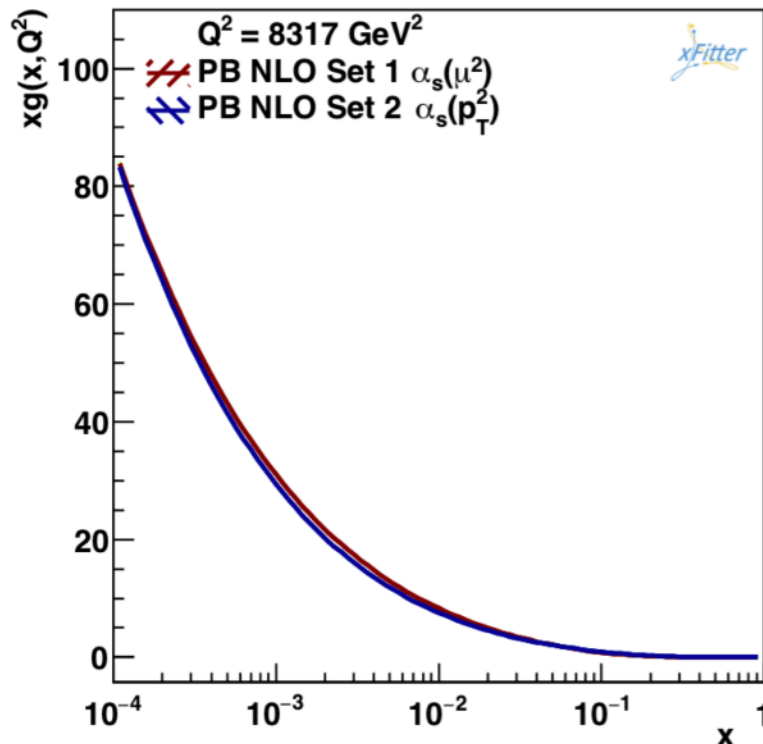
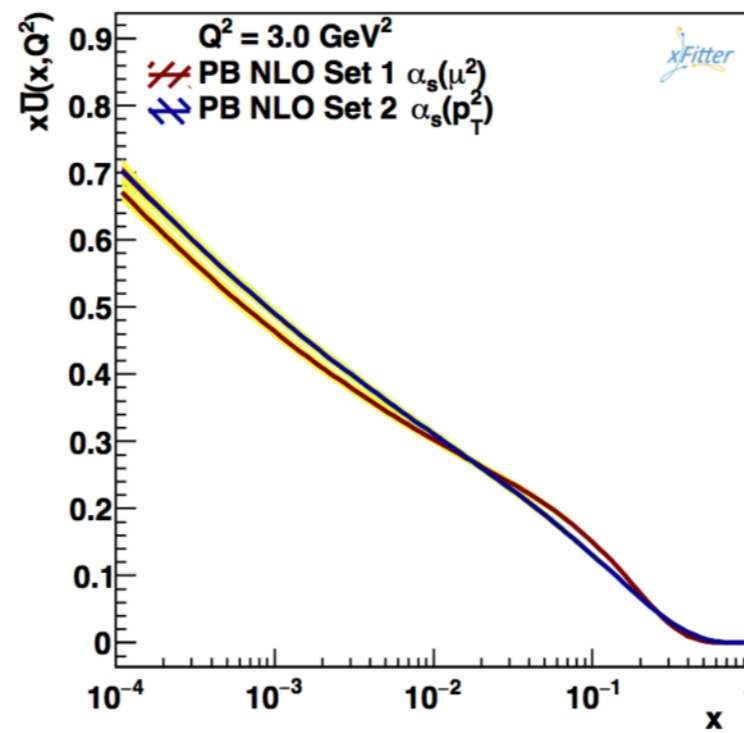
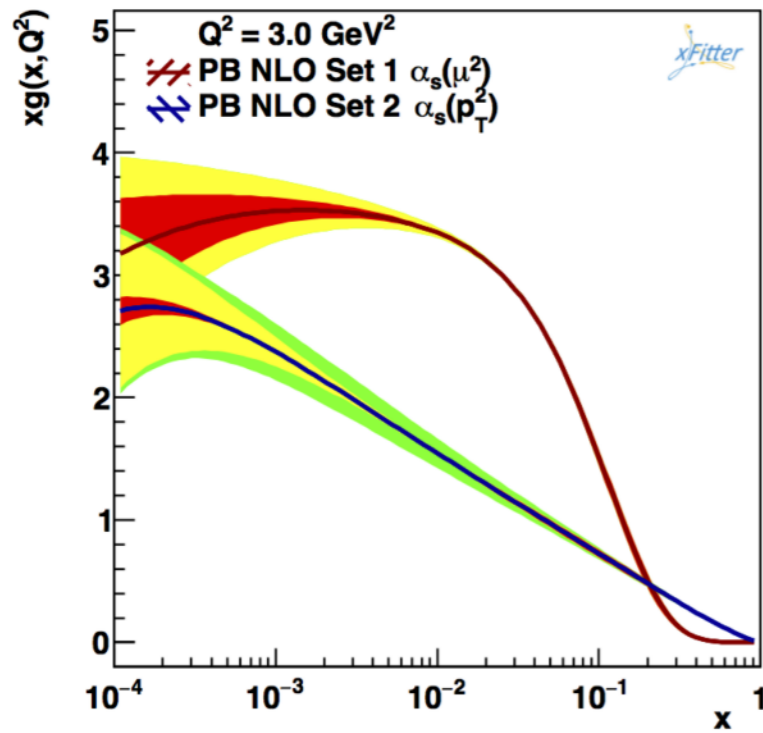


# Transverse Momentum: $z_M$ - dependence



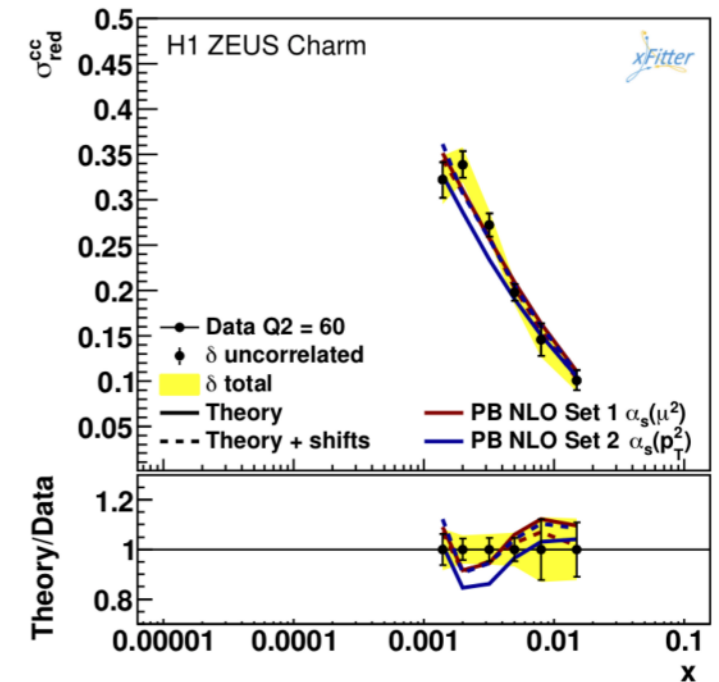
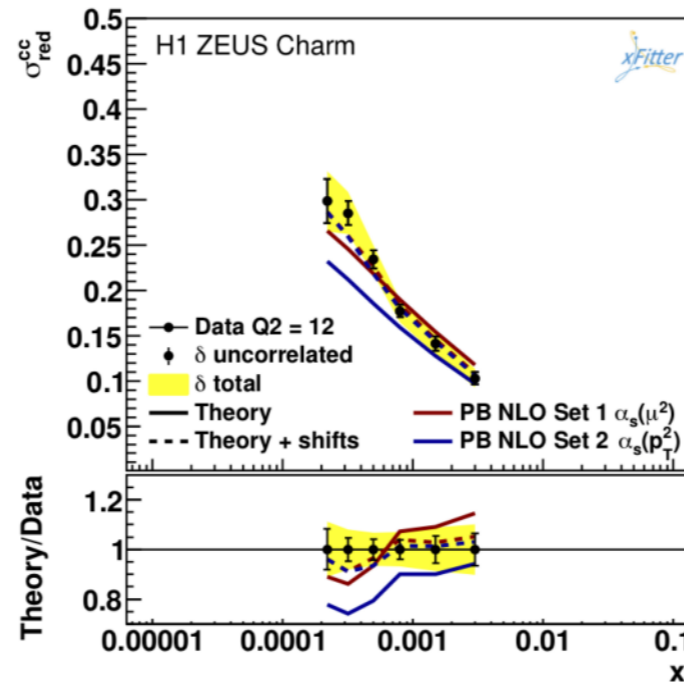
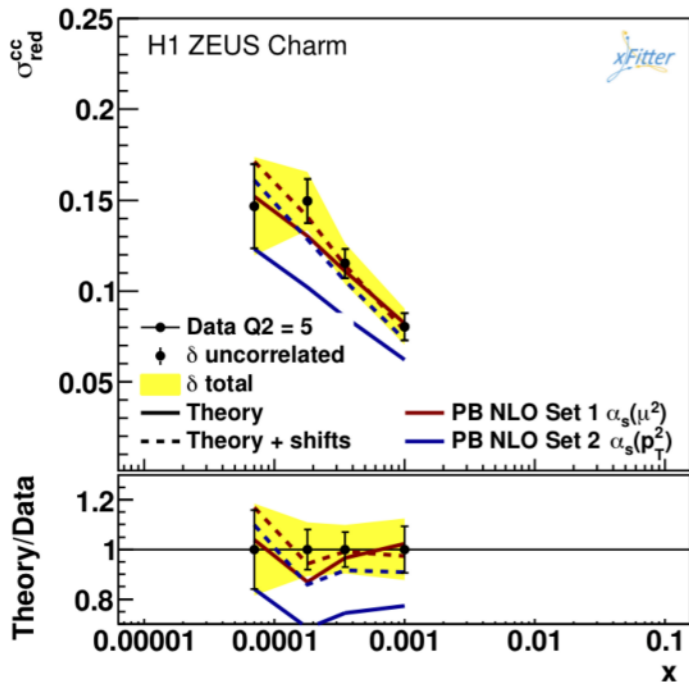
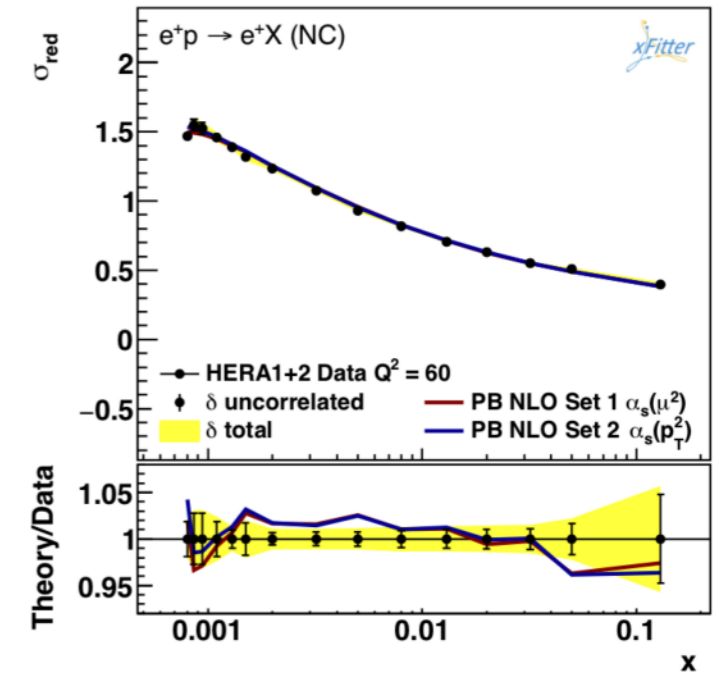
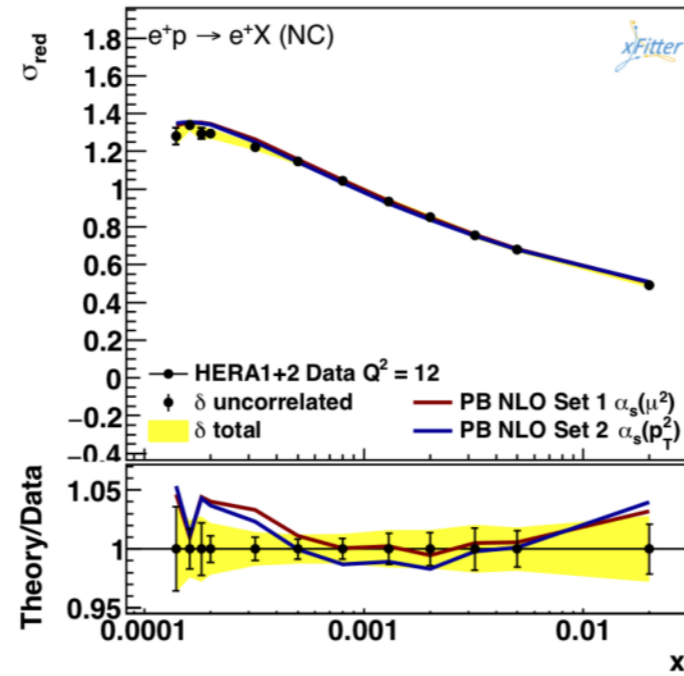
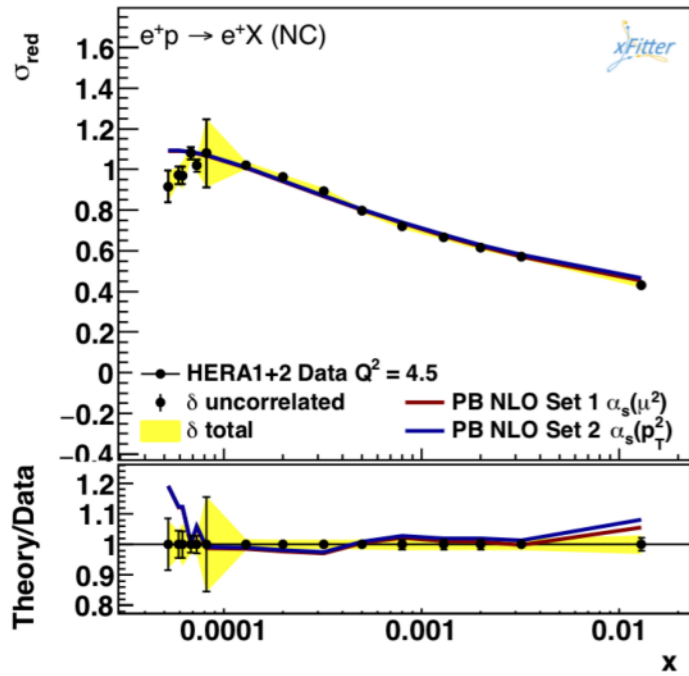
- $p_T$  ordering ( $\mu = q_T$ ) shows significant dependence on  $z_M$ : unstable result because of soft gluon contribution
- angular ordering ( $\mu = q_T/(1-z)$ ) is independent of  $z_M$ : stable results since soft gluons are suppressed

# Fit with different scale in $\alpha_s$



- fit 1 with  $\alpha_s(q)$ 
  - as good as HERAPDF2.0  
 $\chi^2/ndf = 1.2$
- fit 2 with  $\alpha_s(q(1-z))$ 
  - $\chi^2/ndf = 1.21$
- very different gluon distribution obtained at small  $Q^2$

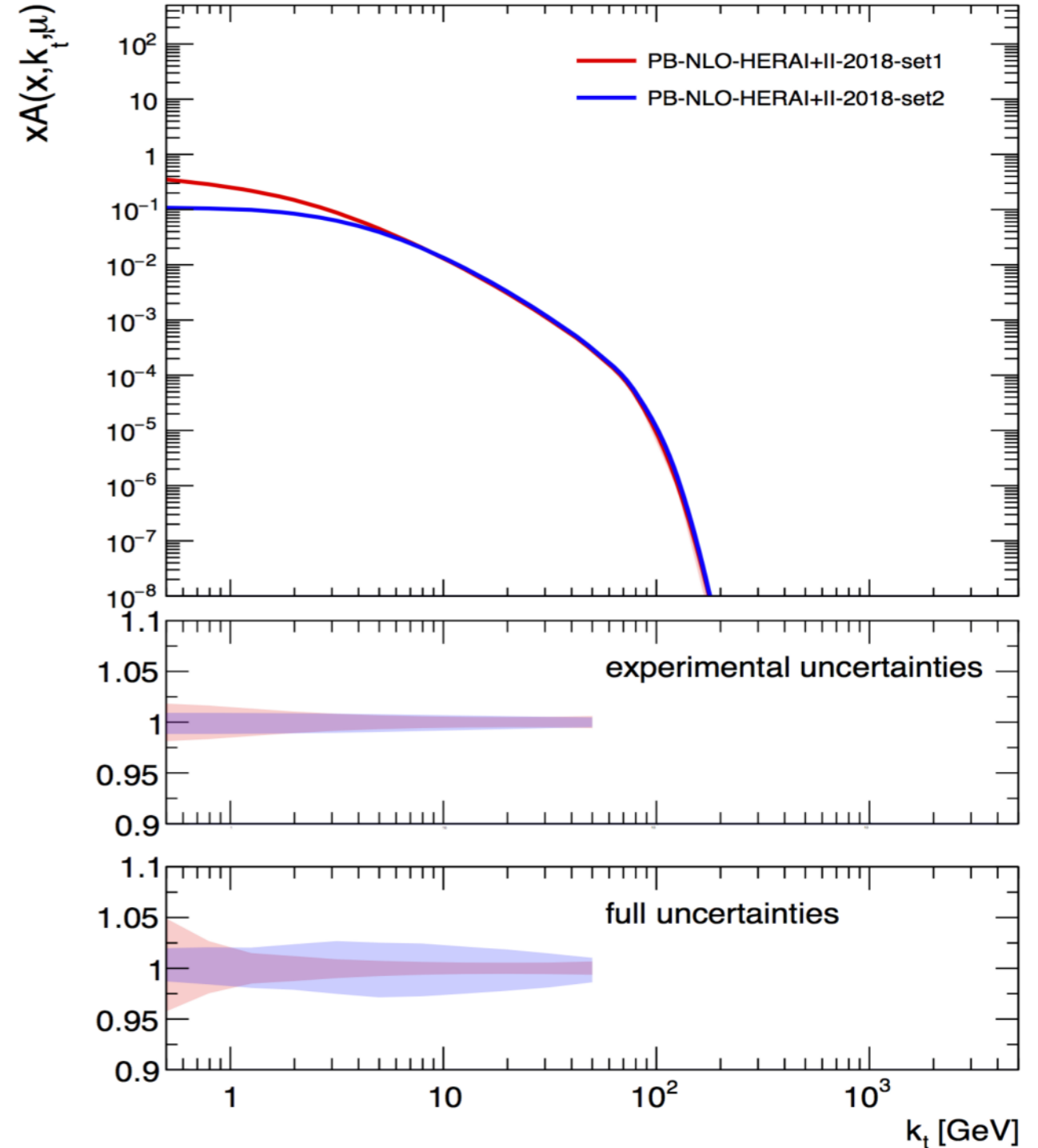
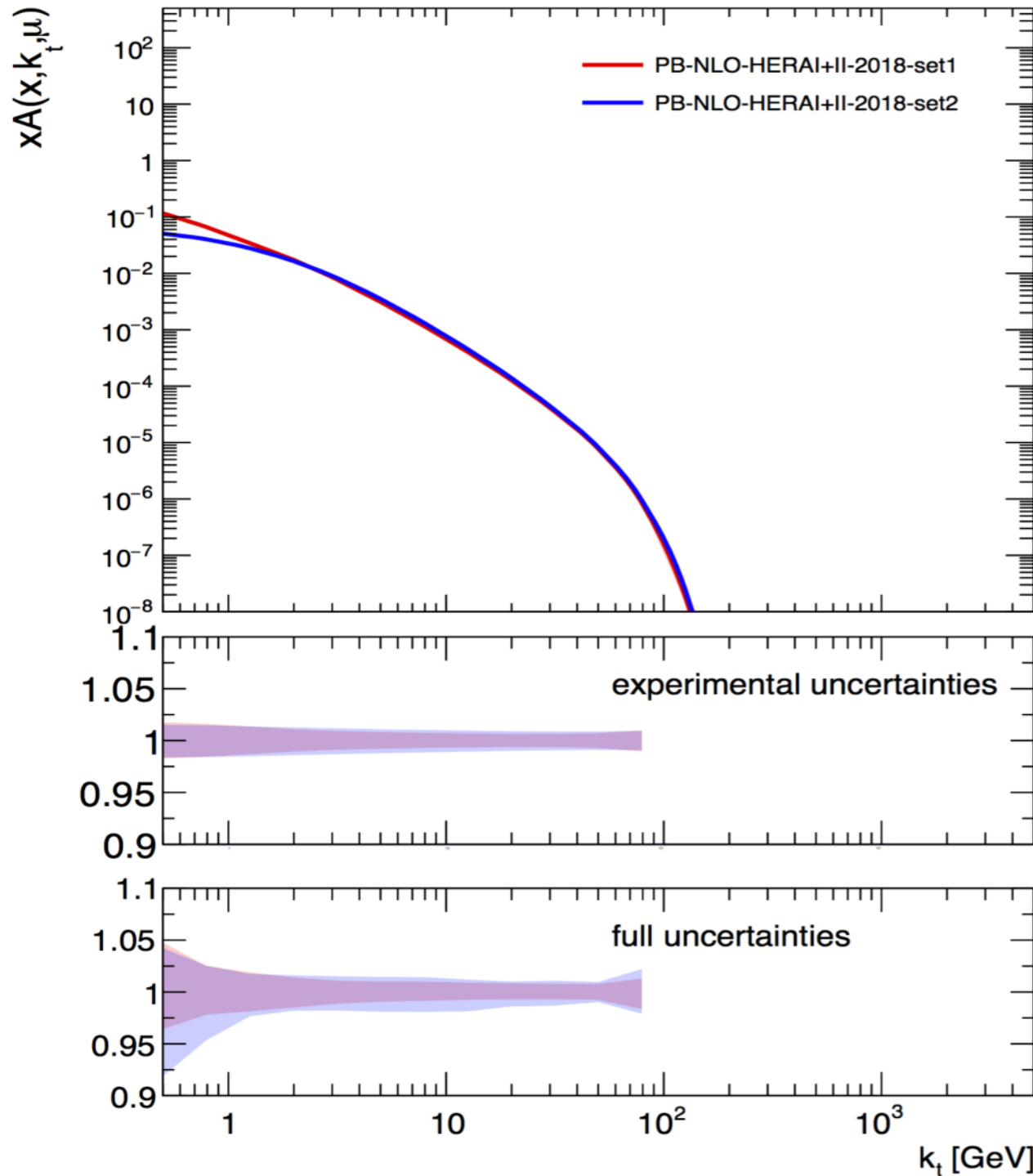
# Fits to DIS x-section at NLO: $F_2$ and $F_2^C$



# TMD distributions from fit to HERA data

anti-up,  $x = 0.01$ ,  $\mu = 100$  GeV

gluon,  $x = 0.01$ ,  $\mu = 100$  GeV



- model dependence larger than experimental uncertainties