Standard-model prediction of ϵ_K with manifest CKM unitarity

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Based on 1911.06822 work with J. Brod, E. Stamou

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This talk

- Introduction to Flavour Physics and CP violation
- Defintion of ϵ_K
- CKM factors
- Effective Hamiltonian
- Results
- Conclusions

Flavour Symmetry

The standard model gauge sector is CP conserving and has a large global flavour symmetry

$$\mathcal{L}_g = \sum_f \bar{\psi}_f \not{D} \psi_f + \sum_i \tfrac{1}{4} g_i \vec{F}^i_{\mu\nu} \vec{F}^{i\,\mu\nu}$$

$$f \in \{u, d, e, Q, L\}$$

$$G_{\text{flavour}} = \prod_f SU(3)_f \times \prod_x U(1)_x$$

[Chivukula, Georgi `87]

Only Higgs Yukawa couplings break this symmetry in the SM

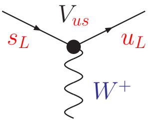
$$-\mathcal{L}_Y^q = \bar{u}_R Y_u \tilde{\varphi}^\dagger Q_L + \bar{d}_R Y_d \varphi^\dagger Q_L$$

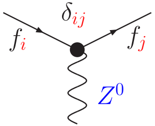
Mass eigenstates $\neq$ flavour eigenstates

$$\text{for diagonal } Y_d\text{:}\quad Y_u = \frac{1}{v} \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Neutral & Charged Current Interactions

Mass ≠ flavour eigenstates





SM: Only charged currents change the flavour ($\propto V_{us}$)

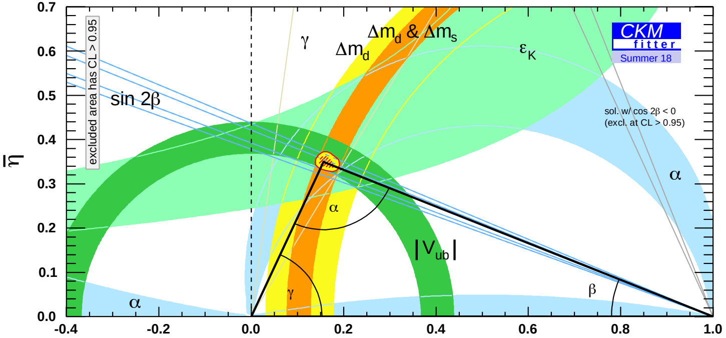
SM: Neutral currents do not change the flavour (i=j) at tree-level

$$V_{\text{CKM}} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho+i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

- CKM matrix parametrises CP and flavour violation in the SM
- Standard Model: Higgs sector is the source of flavour violation

Unitarity: $1 + (V_{ud}V_{ub}^* + V_{td}V_{tb}^*)/(V_{cd}V_{cb}^*) = 0$

- gives: $1 - [(\bar{\rho} + \bar{\eta}) + (1 - \bar{\rho} - \bar{\eta})] = 0$
- constraints from $\left|\frac{V_{ub}}{V_{cb}}\right|$ and $\frac{\Delta_{M_d}}{\Delta_{M_s}}$.



Flavour Problem

New physics like Supersymmetry, Extra Dimensions ...
will have new sources of flavour violation,

while flavour observables agree well with in current precision.

If we will have new physics at a scale Λ we will generate

$$\mathcal{L} = \mathcal{L}_{SM} + \Lambda^2 \phi^\dagger \phi + \frac{1}{\Lambda^2} (\bar{s} d_L)(\bar{s} d_R) + \ldots$$

From the Λ^2 term we expect $\Lambda = O(M_Z)$

From the $\frac{1}{\Lambda^2}$ term we expect $\Lambda >> M_Z$

New Physics Sensitivity

- The New Physics (NP) and the Standard Model (SM) compete

$$\delta L = \frac{C_{NP}}{\Lambda_{NP}^2}(\bar{s}d_L)(\bar{s}d_R) + \frac{C_{SM}}{v_{ew}^2}(\bar{s}\gamma_\mu d_L)(\bar{s}\gamma^\mu d_L)$$

- Since we have no particle physics evidence of new physics
  - Calculate the SM flavour violation as precisely as possible.
  - Understand the origin and correlation of NP flavour violation to be able to interpret small deviations.

Meson-antimeson mixing

Restricting to $\{|K^0\rangle, |\overline{K}^0\rangle\}$ basis we have

$$i\frac{d}{dt}\begin{pmatrix}|K^0\rangle \\ |\overline{K}^0\rangle\end{pmatrix} = \hat{H}\begin{pmatrix}|K^0\rangle \\ |\overline{K}^0\rangle\end{pmatrix}$$

a hermitian $\hat{M}$ and anti-hermitian $i\hat{\Gamma}$ contribution.

$$\hat{H} = \begin{pmatrix}H_{11} & H_{12} \\ H_{21} & H_{22}\end{pmatrix} = \begin{pmatrix}\langle K^0|T|K^0\rangle & \langle K^0|T|\overline{K}^0\rangle \\ \langle \overline{K}^0|T|K^0\rangle & \langle \overline{K}^0|T|\overline{K}^0\rangle\end{pmatrix} = \hat{M} - \frac{i}{2}\hat{\Gamma}$$

QCD generates $H_{11} = H_{22}$ (equality from CPT)
weak $\Delta F = 2$ flavour change: H_{12} and H_{21}

ϵ_K: Indirect CP violation

- If CP is conserved $K_L \not\to \pi\pi$, but mixing allows:

$$\epsilon_K \equiv \frac{\langle(\pi\pi)_{I=0}|K_L\rangle}{\langle(\pi\pi)_{I=0}|K_S\rangle} = e^{i\phi_\epsilon}\sin\phi_\epsilon \frac{1}{2}\arg\left(\frac{-M_{12}}{\Gamma_{12}}\right)$$

$$= e^{i\phi_\epsilon}\sin\phi_\epsilon\left(\frac{\mathrm{Im}(M_{12})^{Dis}}{\Delta M_K} + \xi\right)$$

- $\langle K^0|H^{|\Delta S|=2}|\bar{K}^0\rangle \to \mathrm{Im}(M_{12})^{Dis}$
- $\dfrac{\mathrm{Im}\langle(\pi\pi)_{I=0}|K^0\rangle}{\mathrm{Re}\langle(\pi\pi)_{I=0}|K^0\rangle} \to \xi$
- $\phi_\epsilon \equiv \arctan\dfrac{\Delta M_K}{\Delta\Gamma_K/2}$

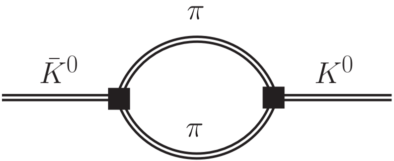
$\mathrm{Im}(M_{12})$

- We can factorise perturbatively calculated
  - short distance contributions at $\mu_t = m_t$,
  - from long distance effects calculated on Lattice

$$\langle H_{\text{eff}} \rangle = \langle Q^{|\Delta S=2|} \rangle (\mu_{\text{had}}) \quad U(\mu_{\text{had}}, \mu_c) \quad U(\mu_c, \mu_W) \quad C(\mu_W)$$

- factorising $U(\mu_{\text{had}}, \mu_c) = u^{-1}(\mu_{\text{had}}) u(\mu_c)$ we write:
- $\frac{2}{3} f_K^2 M_K^2 \hat{B}_K = \langle \bar{K}^0 | Q^{|\Delta S=2|} | K^0 \rangle u^{-1}(\mu_{\text{had}})$
- $\eta_{ij} S(x_i, x_j) = u(\mu_c) U(\mu_c, \mu_W) C(\mu_W)$
  is the short distance contribution
- $Q_{S2} = (\overline{s}_L \gamma_\mu d_L) \otimes (\overline{s}_L \gamma^\mu d_L)$

ξ: from $|\Delta S = 1|$ Hamiltonian



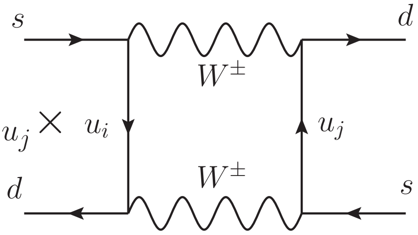
- ξ from the absorptive (imaginary) part of

$$\int d^4x \langle \bar{K}^0 | H^{|\Delta S=1|}(x) H^{|\Delta S=1|}(0) | K^0 \rangle$$

- Estimated from χ PT: +2.4% [Buras et. al. 1002.3612]
- Can also be extracted from Lattice and ϵ'/ϵ [Blum et. al. 1502.00263, 1505.07863]
- Paramterise $\phi_\epsilon \neq \pi/4$ and $\xi \neq 0$ by $\kappa_\epsilon = 0.94(2)$

CKM structure of $\Delta S = 2$ Hamiltonian

$$\sum_{u_i,u_j}^{u,c,t} \lambda_{u_i}\lambda_{u_j} \times$$



We define $\lambda_i = V_{id}V_{is}^*$

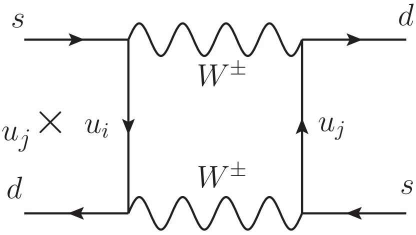
- Using $\lambda_u = -\lambda_c - \lambda_t$ we have

$$\begin{aligned} A &= \lambda_t^2(A_{tt} - 2A_{tu} + A_{uu}) + \\ & 2\lambda_t\lambda_c(A_{tc} - A_{tu} + A_{uu} - A_{cu}) \\ & \lambda_c^2(A_{uu} - 2A_{cu} + A_{cc}) \end{aligned}$$

- One could eliminate $\lambda_c = -\lambda_u - \lambda_t$.

CKM structure of $\Delta S = 2$ Hamiltonian

$$\sum_{u_i,u_j}^{u,c,t} \lambda_{u_i}\lambda_{u_j} \times$$



	Im	Re	$\mathcal{O}$
λ_t^2	$\sim \lambda^{10}$	$\sim \lambda^{10}$	m_t^2/M_W^2
$\lambda_c\ \lambda_t$	$\sim \lambda^6$	$\sim \lambda^6$	$m_c^2/M_W^2 \ln(m_t/m_c)$
λ_c^2	$\sim \lambda^6$	$\sim \lambda^2$	m_c^2/M_W^2
$\lambda_u\ \lambda_t$	$\sim \lambda^6$	$\sim \lambda^6$	$m_c^2/M_W^2 \ln(m_t/m_c)$
λ_u^2	0	$\sim \lambda^2$	m_c^2/M_W^2

Where $\lambda_i = V_{id}V_{is}^*$, $\lambda \equiv |V_{us}| \sim 0.2$ and we eliminated either: $\lambda_u = -\lambda_c - \lambda_t$ or $\lambda_c = -\lambda_u - \lambda_t$.

$\Delta S = 2$ Hamiltonian - Phase (In)Dependence

- Recall $\epsilon_K \propto \arg(-M_{12}/\Gamma_{12})$
- Trick: pull out λ_u^* and $(\lambda_u^*)^2$ from $H^{\Delta S=1}$ and $H^{\Delta S=2}$:
- Rephaseing invariant: $\lambda_i \lambda_j^* = V_{id} V_{is}^* V_{jd}^* V_{js}$
- $\Gamma_{12} \simeq A_0^* \bar{A}_0$ where $A_0 = \langle (\pi\pi)_{I=0} | K^0 \rangle$

$$\mathcal{H}_{f=3}^{\Delta S=2} = \frac{G_F^2 M_W^2}{4\pi^2 (\lambda_u^*)^2} Q_{S2} \Big\{ f_1 \mathcal{C}_1(\mu) + iJ \left[f_2 \mathcal{C}_2(\mu) + f_3 \mathcal{C}_3(\mu) \right] \Big\} + \text{h.c.}$$

- $J = \mathrm{Im}(V_{us} V_{cb} V_{ub}^* V_{cs}^*)$, f_1, f_2 and f_3 are rephasing invariant
- Real part $f_1 = |\lambda_u|^4$ is unique
- Splitting of f_2 and f_3 not

Traditional Form

Traditionally the effective Hamiltonian is written as:

$$\mathcal{H}_{f=3}^{\Delta=2} = \frac{G_F^2 M_W^2}{4\pi^2}\left[\lambda_c^2 C_{S2}^{cc}(\mu)+\lambda_t^2 C_{S2}^{tt}(\mu)+\lambda_c\lambda_t C_{S2}^{ct}(\mu)\right]Q_{S2}+\text{h.c.}$$

where $f_2 = \mathrm{Re}(\lambda_t\lambda_u^*)$, $f_3 = \mathrm{Re}(\lambda_c\lambda_u^*)$ and

$$C_{S2}^{cc} \equiv \mathcal{C}_1, \quad C_{S2}^{ct} \equiv 2\mathcal{C}_1 + \mathcal{C}_3, \quad 2C_{S2}^{tt} \equiv 2\mathcal{C}_1 + \mathcal{C}_2 + \mathcal{C}_3$$

- $\mathcal{C}_1 \leftarrow A_{uu} - 2A_{cu} + A_{cc}$ has bad short distance behaviour
- $\mathcal{C}_1$ determines ΔM_K via ReM_{12}
- But $\mathcal{C}_1$ contributes to ImM_{12} and hence ϵ_K

RGEs d6^2 → d8 for η_{ct}

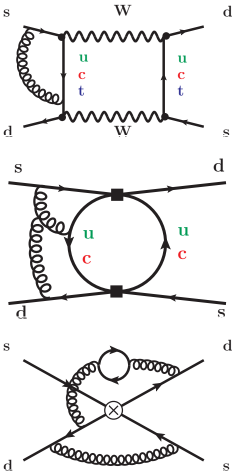
Renormalise H_{eff} at 3-loop

$$\mathcal{H}^{\text{eff}}_{f=5} = \frac{4G_F}{\sqrt{2}} \sum_{i=+,-,3}^{6} C_i \left[\sum_{j=+,-} Z_{ij} \sum_{k,l=u,c} V^*_{ks} V_{ld} Q^{kl}_j - \lambda_t \sum_{j=3}^{6} Z_{ij} Q_j \right] + 8G_F^2 \lambda_c \lambda_t \left[\sum_{k=+,-} \sum_{l=+,-,3}^{6} C_k C_l \hat{Z}_{kl,7} + \tilde{C}_7 \tilde{Z}_{77} \right] \tilde{Q}_7 + \text{h.c.}$$

to determine the relevant renormalisation group equations

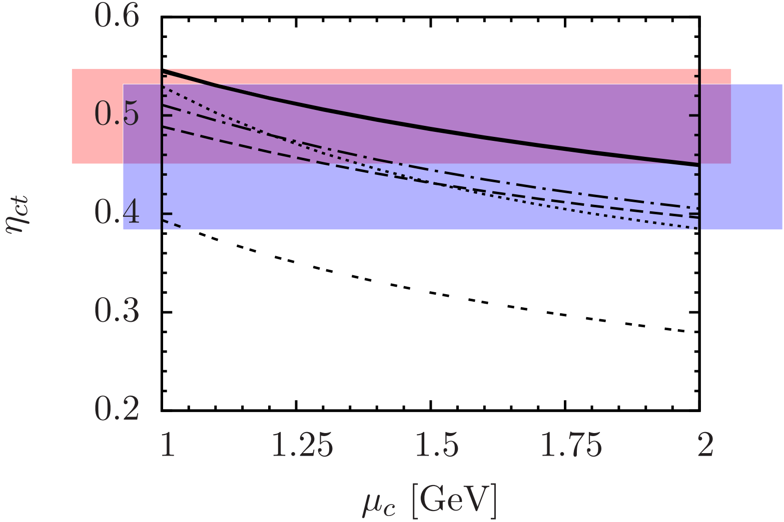
$$\mu \frac{d}{d\mu} \tilde{C}_7(\mu) = \tilde{C}_7(\mu) \tilde{\gamma}_{77} + \sum_{k=+,-} \sum_{n=+,-,3}^{6} C_k(\mu) C_n(\mu) \hat{\gamma}_{kn,7}$$

$\lambda_c \lambda_t$ Coefficient in Traditional Form



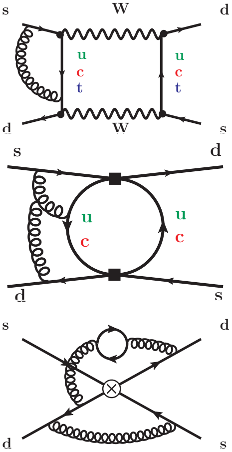
- Initial conditions: Matching at M_W
- Running to m_c
  - $\mathcal{O}(100\,000)$ Feynman diagrams
  - RGE for double insertion
  - Include threshold corrections at m_b
- Matching at m_c
- RGE in three-flavor EFT

Residual scale dependence



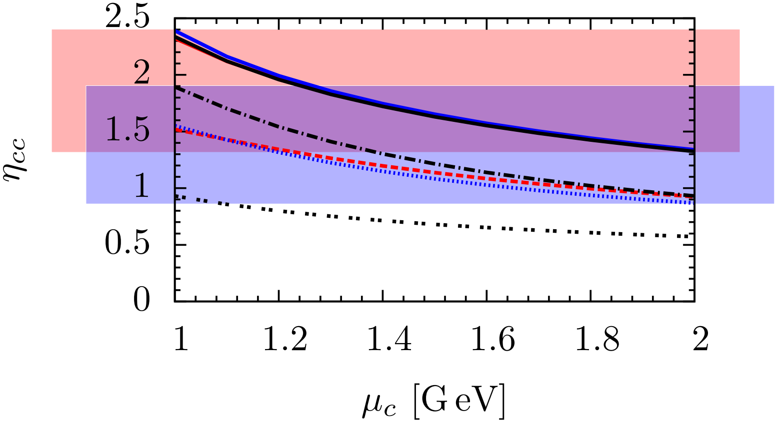
$$\eta_{ct} = 0.497(47)$$

λ_c^2 Coefficient in Traditional Form



- Initial conditions at M_W vanish by GIM
  [E. Witten, Nucl.Phys. B122 (1977) 109-143]
- Running to m_c
  - Only $|\Delta S| = 1$ operators contribute
  - Double insertions are finite (GIM)
- Matching at m_c
- $\mathcal{O}(100\,000)$ Feynman diagrams
- Including finite pieces

Residual scale dependence



$\eta_{cc} = 1.87(76)$

$\mathrm{Im}M_{12}$ without ΔM_K pollution

- Choose $f_2 = 2\mathrm{Re}(\lambda_t \lambda_u^*)$ and $f_3 = |\lambda_u|^2$

$$\mathcal{H}_{f=3}^{\Delta=2} = \frac{G_F^2 M_W^2}{4\pi^2}\left[\lambda_u^2 C_{S2}^{uu}(\mu) + \lambda_t^2 C_{S2}^{tt}(\mu) + \lambda_u \lambda_t C_{S2}^{ut}(\mu)\right] Q_{S2} + \mathrm{h.c.}$$

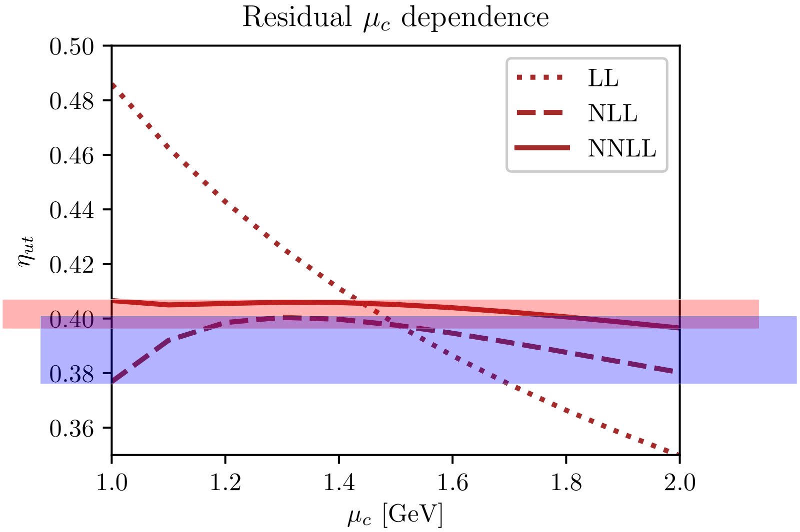
- Now real $\mathrm{Re}M_{12}$ and $\mathrm{Im}M_{12}$ are disentangled

$$C_{S2}^{uu} \equiv \mathcal{C}_1, \quad C_{S2}^{tt} \equiv \mathcal{C}_2, \quad C_{S2}^{ut} \equiv \mathcal{C}_3$$

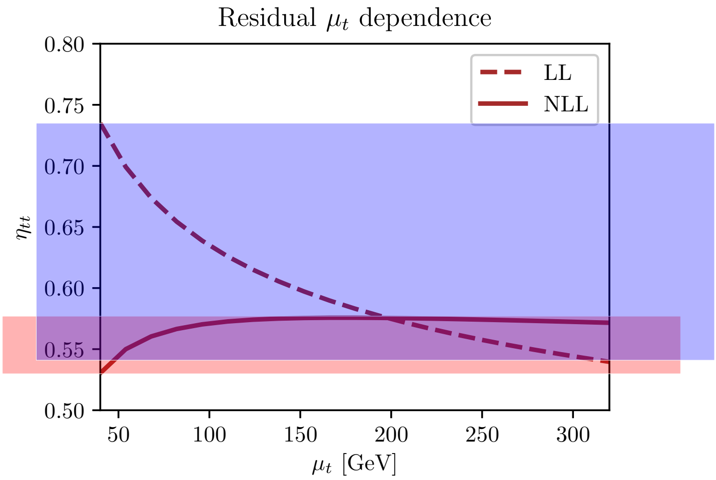
$$\begin{aligned} \mathcal{C}_3 \quad &\leftarrow \quad (A_{tu} - A_{tc} + A_{cc} - A_{cu}) \leftarrow \\ &\leftarrow \quad (A_{uu} - 2A_{cu} + A_{cc}) - (A_{tc} - A_{tu} + A_{uu} - A_{cu}) \end{aligned}$$

- Extract anomalous dimensions and matching from old calculation and incorporate matching from η_{cc}

Residual scale dependence



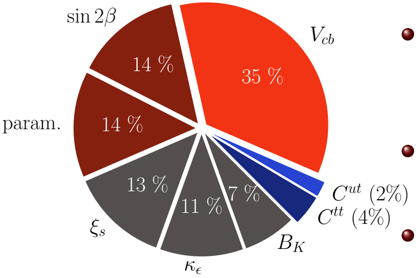
The top-quark: good convergence



Can be improved with NNLO calculation

SM prediction using PDG input

$$|\epsilon_K| = \kappa_\epsilon C_\epsilon \widehat{B}_K |V_{cb}|^2 \lambda^2 \bar{\eta} \times \Big[|V_{cb}|^2 (1-\bar{\rho}) \eta_{tt}(x_t) - \eta_{ut}(x_c, x_t) \Big]$$



- $\hat{B}_K = 0.7625(97)$
  [FLAG 2019, 1902.08191]
- $|\epsilon_K^{\rm SM}| = 2.16(18) \times 10^{-3}$
- $|\epsilon_K^{\rm exp}| = 2.228(11) \times 10^{-3}$

Using exclusive V_{cb} and lattice κ_ϵ

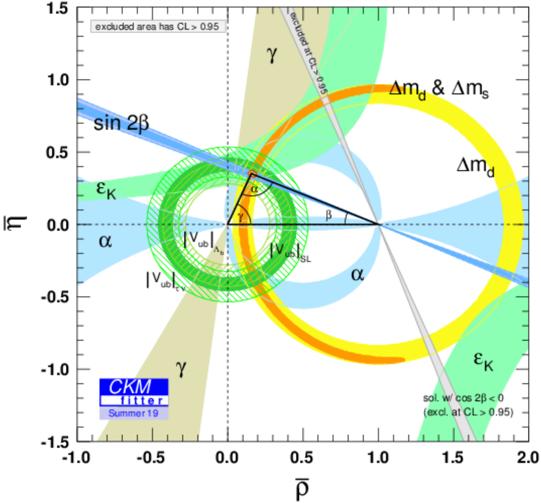
- exclusive $V_{cb} = 0.0403(8)$
- Lattice $\kappa_\epsilon = 0.923(6)$
- $|\epsilon_K(SM)| = 1.81(14) \times 10^{-3}$
- $|\epsilon_K(exp)| = 2.228(11) \times 10^{-3}$

Improvements can come from:

- V_{cb}
- NNLO calculation for η_{tt} [Brod, MG, Stamou, Yu in progress]
- NNLO matching of B_K to continuum [Kvedaraite, MG, Jäger in progress]
- Calculation of ξ without ϵ'/ϵ form Lattice

CKMfitter 2019 update

Incorporating new formalism shows reduced uncertainty, but $\bar{\rho}$ and $\bar{\eta}$ not the (only) dominant CKM factors.



ϵ_K-Konclusions

- Precise theory prediction of ϵ_K possible
  - bottelneck of bad pertubation
- Theory prediction can be systematically improved:
  - Matching Lattice $\leftrightarrow$ continuum
  - S_0 at NNLO.
- Measure
  - SM parameters: V_{cb} and m_t
  - or new physics