Signals in Particle Detectors 2/4

Academic Training Lectures
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Signals in particle detectors

Main Auditorium, Mon. 2 Dec.

Lecture 1:
- Electrostatics
- Principles
- Reciprocity
- Induced currents
- Induced voltages
- Ramo-Shockley theorem
- Mean value theorem
- Capacitance matrix
- Equivalent circuits

Council Chamber, Tue. 3 Dec.

Lecture 2:
Signals in
- Ionization chambers
- Liquid argon calorimeters
- Diamond detectors
- Silicon detectors
- GEMs (Gas Electron Multiplier)
- Micromegas (Micromesh gas detector)
- APDs (Avalanche Photo Diodes)
- LGADS (Low Gain Avalanche Diodes)
- SiPMs (Silicon Photo Multipliers)
- Strip detectors
- Pixel detectors
- Wire Chambers
- Liquid Argon TPCs

TH conference room (4/3-006), Wed. 4 Dec.

Lecture 3:
- Media with conductivity
- Quasi-static approximations
- Signal theorem extensions
- Time dependent weighting fields
- Resistive plate chambers (RPCs)
- Un-depleted silicon sensors
- Monolithic pixel sensors

Filtration Plant (222/R-001), Thu. 5 Dec.

Lecture 4:
- Signal propagation
- Transmission lines
- Termination
- Linear signal processing
- Noise
- Optimum filters

Main Auditorium, Fri. 6 Dec.

Lecture 5:
- Possible overflow, wrap-up and Q&A session
Parallel plate geometry

Weighting potential, electrode 1

\[ \psi_1(y) = V_w \left( 1 - \frac{y}{d} \right) \]

\[ E_1(y) = \frac{V_w}{d} \]

Weighting potential, electrode 2

\[ \psi_2(y) = V_w \frac{y}{d} \]

\[ E_2(y) = -\frac{V_w}{d} \]

Induced charge

\[ Q_{1\text{ind}}^\text{ind}(y) = -\frac{q}{V_w} \psi_1(y) = -q \left( 1 - \frac{y}{d} \right) \]

\[ Q_{2\text{ind}}^\text{ind}(y) = -\frac{q}{V_w} \psi_2(y) = -q \frac{y}{d} \]

Sum of the weighting potentials

\[ \psi_1(y) + \psi_2(y) = V_w \]

Sum of induced charges

\[ Q_{1\text{ind}}^\text{ind} + Q_{2\text{ind}}^\text{ind} = -q \]
Parallel plate geometry

\[ y(t) = y_0 - vt \quad \frac{dy(t)}{dt} = -v \quad 0 < t < \frac{y_0}{v} \]

\[ I_1(t) = -\frac{q}{V_w} E_1(y(t)) \frac{dy(t)}{dt} = q \frac{v}{d} \]

\[ I_2(t) = -\frac{q}{V_w} E_2(y(t)) \frac{dy(t)}{dt} = -q \frac{v}{d} \]

\[ I_1(t) + I_2(t) = 0 \]

Total induced charge on electrode 1

\[ Q_{1\text{tot}}^{\text{ind}} = \int_0^{y_0/v} I_1(t)dt = \frac{q}{V_w} [\psi_1(0) - \psi_1(y_0)] = q \frac{y_0}{d} \]
The Heaviside step function is defined as:

\[ \Theta(t) = \begin{cases} 
0 & \text{for } t < 0 \\
1 & \text{for } t \geq 0 
\end{cases} \]

In case we have a constant current signal with amplitude \( I_0 \) starting at \( t=0 \) and lasting until \( t=t_1 \) we have:

\[ I(t) = I_0 \Theta(t_1 - t) \]

In case we have a constant current signal with amplitude \( I_0 \) starting at \( t=t_1 \) and lasting until \( t=t_2 \) we have:

\[ I(t) = I_0 [\Theta(t_2 - t) - \Theta(t_1 - t)] \]
Two charges $+q$, $-q$ moving from $y_0$ to the electrodes with velocities $v_1$ and $v_2$, arriving at the electrodes at times $t_1$ and $t_2$

\[
t_1 = \frac{y_0}{v_1}, \quad t_2 = \frac{d - y_0}{v_2}
\]

Induced currents

\[
I_1(t) = q \frac{v_1}{d} \Theta(t_1 - t) + q \frac{v_2}{d} \Theta(t_2 - t) \quad I_2(t) = -I_1(t)
\]

Total induced charges

\[
Q_{1\text{tot}}^{\text{ind}} = \int_0^\infty I_1(t) dt = \frac{q}{V_w} [\psi_1(0) - \psi_1(y_0)] - \frac{q}{V_w} [\psi_1(d) - \psi_1(y_0)] = q
\]

In all physics processes, pairs of charges with opposite sign are produced at the same position, which results in the fact that the total induced charge is equal to the charge that has arrived at the electrode, once ALL charges have arrived at the electrodes.
Charge particles passing sensors leave a trail of positive and negative charges (electrons/ions, electrons/holes). Assuming a uniform distribution along the track we have two ‘line charges’ $\pm \lambda$ (C/cm), moving with velocities $v_1$ and $v_2$, with the last charges arriving at the electrode at $t_1$ and $t_2$.

The induced current due to the movement of these charges is the sum of two ‘triangles’, $Q = \lambda d$

\[
I_1(t) = \lambda v_1 \left(1 - \frac{t}{t_1}\right) \Theta(t - t_1) + \lambda v_2 \left(1 - \frac{t}{t_2}\right) \Theta(t - t_2)
\]

\[
= \frac{Q}{t_1} \left(1 - \frac{t}{t_1}\right) \Theta(t - t_1) + \frac{Q}{t_2} \left(1 - \frac{t}{t_2}\right) \Theta(t - t_2)
\]

The total induced charge on electrode 1 is

\[
Q_{1\text{tot}}^{\text{ind}} = \int_0^\infty I_1(t)\,dt
\]

\[
= \frac{\lambda d}{2} + \frac{\lambda d}{2}
\]

\[
= \lambda d
\]

\[
= Q
\]
Charge Deposit
A charged particle passing through a piece of material will produce a trail of ionization.

Since the individual interactions with the atomic electrons are independent, the number of primary interactions follows a Poisson distribution.

The probability \( f(E) \) for transferring an energy \( E \) to the atomic electron in an interaction is given by the Rutherford cross-section at large energy transfers and by the atomic atomic shell structure at low energy transfers.

Landau distribution \( L(x) \) according to

\[
L(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp \left[ -x + x \ln a \right] dx
\]

\[
= \frac{1}{\pi} \int_{-\infty}^{\infty} \exp(-\pi t^2) \cos(x + t \ln a) dt
\]

\[
= \frac{1}{\pi} \int_{-\infty}^{\infty} \exp[-tx - t \ln a] \sin(\pi t) dt
\]

Expression (A.2) is well suited for evaluation for \( x < 0 \), while eq. (A.3) is well suited for evaluation for \( x > 0 \). For large values of \( x \) the Landau distribution approximates to

\[
L(x) = \frac{1}{x^2}
\]

\[
x = \frac{E}{\bar{\eta}e} + C_\gamma - 1 - \ln \bar{\eta}
\]

\[
\bar{\eta} = \frac{N_A \rho Z_2 k D}{A e}
\]

\[
\ln \epsilon = \ln \left( \frac{I^2}{I_{max}^2} + 2\beta^2 \right)
\]

\[
E_{MP} = \bar{\eta}(x_0 + 1 - C_\gamma) \approx \bar{\eta} \ln(0.2 + \ln \bar{\eta})
\]

\[
\Delta E_{FWHM} = 4.02 \bar{\eta} = 4.02 N_A \rho Z_2 k D
\]

\[
\frac{\Delta E_{FWHM}}{E_{MP}} = \frac{4.02}{0.2 + \ln \bar{\eta}}
\]
Charge deposit in silicon

Number of e-h pairs in silicon for 50um and 200um silicon.

PAI model Monte Carlo and Landau distribution superimposed.

The Landau Theory overestimates the fluctuations (FWHM/Most Probable) by 25-35%.

<table>
<thead>
<tr>
<th>d(μm)</th>
<th>clusters (PAI)</th>
<th>n_{MP} (PAI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>236</td>
<td>3160</td>
</tr>
<tr>
<td>100</td>
<td>472</td>
<td>8710</td>
</tr>
<tr>
<td>200</td>
<td>943</td>
<td>14200</td>
</tr>
<tr>
<td>300</td>
<td>1415</td>
<td>21900</td>
</tr>
</tbody>
</table>
The solid lines represent measurements from [27] F. Rieke, W. Prepejchal, Phys. Rev. A 6 (1972) 1507. The points represent results from a PAI model

Average distance between clusters, $\sim 1000x$ larger than silicon:

- Argon: 0.41mm
- Neon: 0.83mm
- Isobutane: 0.12mm

For single particle detection, internal gain is used in gas detectors
Transport of electrons and ions in gases
Electrons are completely ‘randomized’ in each collision. The actual drift velocity $v$ along the electric field is quite different from the average velocity $u$ of the electrons i.e. $\rightarrow$ about 100 times smaller.

The velocities $v$ and $u$ are determined by the atomic crosssection $\sigma(\varepsilon)$ and the fractional energy loss $\Delta(\varepsilon)$ per collision ($N$ is the gas density i.e. number of gas atoms/m$^3$, $m$ is the electron mass.):

$$ v = \sqrt{\frac{eE}{mN\sigma}} \sqrt{\frac{\Delta}{2}} \quad u = \sqrt{\frac{eE}{mN\sigma}} \sqrt{\frac{2}{\Delta}} \quad \frac{u}{v} = \sqrt{\frac{2}{\Delta}} $$

Because $\sigma(\varepsilon)$ and $\Delta(\varepsilon)$ show a strong dependence on the electron energy in the typical electric fields, the electron drift velocity $v$ shows a strong and complex variation with the applied electric field.

$v$ is depending on $E/N$: doubling the electric field and doubling the gas pressure at the same time results in the same electric field.
Typical electron drift velocities are $v=20\text{–}140\text{um/ns}$ ($20\text{,000}\text{–}140\text{,000m/s}$). The microscopic velocity $u$ is about ca. 100mal larger.

Typical ion drift velocities are around 1000 times smaller than electron drift velocities.

<table>
<thead>
<tr>
<th>Gas</th>
<th>Ion</th>
<th>Mobility ($\text{cm}^2\text{V}^{-1}\text{s}^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>He</td>
<td>He$^+$</td>
<td>$1.040\pm0.10$</td>
</tr>
<tr>
<td>Ne</td>
<td>Ne$^+$</td>
<td>$4.14\pm0.21$</td>
</tr>
<tr>
<td>Ar</td>
<td>Ar$^+$</td>
<td>$1.535\pm0.007$</td>
</tr>
<tr>
<td>Kr</td>
<td>Kr$^+$</td>
<td>$0.96\pm0.09$</td>
</tr>
<tr>
<td>Xe</td>
<td>Xe$^+$</td>
<td>$0.57\pm0.05$</td>
</tr>
</tbody>
</table>

*Average over several measurements

- at low fields ($u \propto E$) and at high fields ($u \propto \sqrt{E}$)

Argon Ions at 2kV/cm $\rightarrow 0.03\text{um/ns}$ 1000x slower than electrons

Transport of Electrons and Ions in Gases
Ionization chamber

Used for the LHC beam loss monitors (3520 objects).

Alternating layers of HV and GND with 5mm distance, 1.5kV filled with Nitrogen. 300ns electron drift time, 80us ion drift time

The total current averaged over times of 40us to 2.5ms is read out in order to measure the beam losses.

LHC UFO event 2010:

Single MIP in the ionization chamber, or many MIPs at the same time …

Signals in Particle Detectors, W. Riegler/CERN
## Transport of Electrons in Liquid Noble Gases

<table>
<thead>
<tr>
<th></th>
<th>Ar</th>
<th>Kr</th>
<th>Xe</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z$</td>
<td>18</td>
<td>36</td>
<td>58</td>
</tr>
<tr>
<td>$A$</td>
<td>40</td>
<td>84</td>
<td>131</td>
</tr>
<tr>
<td>$X_0$ (cm)</td>
<td>14</td>
<td>4.7</td>
<td>2.8</td>
</tr>
<tr>
<td>$R_M$ (cm)</td>
<td>7.2</td>
<td>4.7</td>
<td>4.2</td>
</tr>
<tr>
<td>Density (g/cm$^3$)</td>
<td>1.4</td>
<td>2.5</td>
<td>3.0</td>
</tr>
<tr>
<td>Ionization energy (eV/pair)</td>
<td>23.3</td>
<td>20.5</td>
<td>15.6</td>
</tr>
<tr>
<td>Critical energy $\epsilon$ (MeV)</td>
<td>41.7</td>
<td>21.5</td>
<td>14.5</td>
</tr>
<tr>
<td>Drift velocity at saturation (mm/μs)</td>
<td>10</td>
<td>5</td>
<td>3</td>
</tr>
</tbody>
</table>
**Liquid Argon Calorimeter**

ATLAS LArg calorimeter gap 2mm, 2kV → 10kV/cm
Electron velocity $v=4.5\text{um/ns} \rightarrow 450\text{ns for 2mm gap.}$

Ion mobility $8\times10^{-2}\text{mm}^2/\text{Vs} \rightarrow v=800\text{mm/s} = 2.5\text{ms for 2mm gap}$

Ions 5000 times slower than electrons.

Only the signal from the electrons plays a role for the energy measurement. The ions can just effect the movement of the electrons through spacecharge effects.
Transport of Charges in Silicon and Diamond

Electrons and holes in silicon

\[ v_{sat,e} = \mu_e E \left(1 + \frac{\mu_e E}{v_{th,e}}\right)^{1/\beta_e} \]

\[ v_{sat,h} = \mu_h E \left(1 + \frac{\mu_h E}{v_{th,h}}\right)^{1/\beta_h} \]  

(4.11)

where we chose \( \mu_e = 1417 \text{ cm}^2/\text{Vs}, \mu_h = 471 \text{ cm}^2/\text{Vs}, \beta_e = 1.109, \beta_h = 1.213 \) and \( v_{th,e} = 1.07 \times 10^7 \text{ cm/s} \) and \( v_{th,h} = 0.837 \times 10^7 \text{ cm/s} \) at 300K in accordance with the default models in Sentaurus Device [23]. The resulting drift velocity together with the time that the electrons and

Silicon:

\( \mu_e = 1417 \text{ cm}^2/\text{Vs}, \mu_h = 471 \text{ cm}^2/\text{Vs} \)

\( v_{sat,e} = 107 \text{ um/ns}, v_{sat,h} = 84 \text{ um/ns} \)

Diamond:

\( \mu_e = 1266 \text{ cm}^2/\text{Vs}, \mu_h = 1992 \text{ cm}^2/\text{Vs} \)

\( v_{sat,e} = 113 \text{ um/ns}, v_{sat,h} = 125 \text{ um/ns} \)
Edge TCT Diamond detector signals

Localized cluster of electrons and holes

Laser System

2.6V/μm

\( v_e = 84 \mu m/\text{ns} \)

\( v_h = 100 \mu m/\text{ns} \)

Signals in Particle Detectors, W. Riegler/CERN

R. Wallny, Vertex 2019
The band gap of Diamond/Silicon/Germanium is 5.5, 1.12, 0.66 eV.

The average energy to produce an electron/hole pair for Diamond/Silicon/Germanium is 13, 3.6, 2.9eV.

Solid state ionization chamber

\[ V = 1.4kV, \text{ 540um 2.6V/um} \]
\[ v_e = 84\text{um/ns} \]
\[ v_h = 100\text{um/ns} \]
Silicon detector with negligible depletion voltage

Solid state ionization chamber

1.4kV, 540um 2.6V/um=26kV/cm
\(v_e=87\text{um/ns}\)
\(v_h=56\text{um/ns}\)
Silicon Sensors
At the p-n junction the charges are depleted and a zone free of charge carriers is established.

By applying a voltage, the depletion zone can be extended to the entire diode \( \rightarrow \) highly insulating layer.

An ionizing particle produces free charge carriers in the diode, which drift in the electric field and induce an electrical signal on the metal electrodes.

As silicon is the most commonly used material in the electronics industry, it has one big advantage with respect to other materials, namely highly developed technology.
Under-Depleted Silicon Detector

Zone without free charge carriers positively charged. Sensitive Detector Volume.

Zone with free electrons. Conductive. Insensitive to particles.

Electric Field

$d_0$
Fully-Depleted Silicon Detector

Zone without free charge carriers positively charged sensitive detector volume.

Electric Field

\[ d \]
In contrast to the (un-doped) diamond detector where the bulk is neutral and the electric field is therefore constant, the sensitive volume of a doped silicon detector is charged (space charge region) and the field is therefore changing along the detector.

⇒ Velocity of electrons and holes is not constant along the detector.
The capacitance of the detector decreases as the depletion zone increases.
Silicon Detector Signals

\[ E(x) = -\left[ 2 \frac{d}{d^2} V_{dep} + \frac{V - V_{dep}}{d} \right] \]

\[ d_0 = d \frac{V + V_{dep}}{2V_{dep}} \]

\[ \tau_e = \frac{d^2}{2\mu_e V_{dep}} \]

\[ t_e = \tau_e \ln \frac{d_0 - x_0}{d_0 - d} \]

\[ \tau_h = \frac{d^2}{2\mu_h V_{dep}} \]

\[ t_h = -\tau_h \ln \left( 1 - \frac{x_0}{d_0} \right) \]

Solution of these differential equations:

\[ x_e(t) = d_0 - (d_0 - x_0)e^{-t/\tau_e} \quad 0 < t < t_e \]

\[ x_h(t) = d_0 - (d_0 - x_0)e^{t/\tau_h} \quad 0 < t < t_h \]

\[ v_e(t) = \frac{1}{\tau_e} e^{-t/\tau_e} (d_0 - x_0) \quad 0 < t < t_e \]

\[ v_h(t) = -\frac{1}{\tau_h} e^{t/\tau_h} (d_0 - x_0) \quad 0 < t < t_h \]

Weighting field of the bottom ‘electrode’ is calculated without considering the space charge.

Space charge is only affecting the movement of the charges!

\[ E_1(y) = \frac{V_w}{d} \]
Signals in Particle Detectors

Silicon Detector Signals

\[ E_1(y) = \frac{V_w}{d} \]

\[ I^{ind}(t, x_0) = \frac{e_0}{d} v_e(t) - \frac{e_0}{d} v_h(t) \]

\[ = \frac{e_0}{d} \left( \frac{1}{\tau_e} e^{-t/\tau_e} (d_0 - x_0) \Theta(t_e - t) + \frac{1}{\tau_h} e^{t/\tau_h} (d_0 - x_0) \Theta(t_e - t) \right) \]

Example: \( d = 300\mu m, V_{dep} = 58V, V = 1.2 \ V_{dep} = 68V \)

\( d_0 = 330\mu m, \tau_e = 5.6ns, \tau_h = 16.8ns \)

\[ E(x) = - \left[ 2 \frac{d - x}{d^2} V_{dep} + \frac{V - V_{dep}}{d} \right] \]

\[ d_0 = \frac{d V + V_{dep}}{2V_{dep}} \]

\[ \tau_e = \frac{d^2}{2 \mu_e V_{dep}} \]

\[ t_e = \tau_e \ln \left( \frac{d_0 - x_0}{d_0 - d} \right) \]

\[ \tau_h = \frac{d^2}{2 \mu_h V_{dep}} \]

\[ t_h = -\tau_h \ln \left( 1 - \frac{x_0}{d_0} \right) \]
Silicon Detector Signals

Electron and hole mobilities in a silicon detector:

- Electron mobility: $\mu_e = 3 \times 10^5 \text{cm}^2/\text{V} \cdot \text{s}$
- Hole mobility: $\mu_h = 1.2 \times 10^4 \text{cm}^2/\text{V} \cdot \text{s}$

Example: $d = 300 \mu\text{m}$, $V_{\text{dep}} = 58 \text{V}$, $V=1.2 V_{\text{dep}} = 68 \text{V}$

- $d_0 = 330 \mu\text{m}$, $\tau_e = 5.6 \text{ns}$, $\tau_h = 16.8 \text{ns}$

Mathematical expression for the electric field $E(x)$:

$E(x) = -\left[2 \frac{d-x}{d^2} V_{\text{dep}} + \frac{V - V_{\text{dep}}}{d}\right]$
GEM Detector
Gas Electron Multiplier GEM

Single electrons moving through a GEM hole are multiplied in the strong electric field.

Ions are moving to the top side of the GEM or back into the transfer gap.

Electrons are exiting the holes and move to the next amplification stage.

In the last gap the ‘Induction gap’ the electrons are moving to the readout electrodes and induce the signal.

The geometry is equivalent to a parallel plate chamber with only electrons moving through the entire gap.

The signal from a single electron starting in the conversion gap is a ‘box’ with a duration equivalent to the electron transit time in the induction gap approx. 10ns.
The conversion gap above the top GEM is the volume where charge particles deposit $e^+e^-$ pairs.

Typical thickness is 3-15mm (2.5m for the ALICE TPC !)

For a continuous charge deposit the signal has trapezoid shape where the length is dominated by the drift time in the conversion gap.
Parallel field avalanches
Gas avalanche multiplication

At sufficiently high electric fields (100kV/cm) the electrons gain energy in excess of the ionization energy \( \rightarrow \) secondary ionization etc. etc. \( \rightarrow \) exponential increase \( \rightarrow \) avalanche \( \rightarrow \) Townsend coefficient \( \alpha \)

\[
N_e(x) = e^{\alpha x} \quad N_e(t) = e^{\alpha v t} \Theta(d/v_e - t)
\]

The current induced by these moving electrons is

\[
I_{e\text{ind}}(t) = -\frac{e_0 v_e}{d} N_e(t)
\]

The ions move from the point of creation in opposite direction from the point of creation and the induced current is

\[
n_I(x)dx = \alpha e^{\alpha x} dx \quad dI_{I\text{ind}}(x,t)dx = -\frac{e_0 v_I}{d} \alpha e^{\alpha x} [\Theta(x/v_I + x/v_e - t) - \Theta(x/v_e - t)]dx
\]

\[
I_{I\text{ind}}(t) = \int_0^d dI_{I\text{ind}}(x,t)dx = -\frac{e_0 v_I}{d} \left[ (e^{\alpha d} - e^{\alpha v_e t/(v_e + v_I)})\Theta(d/v_e + d/v_I - t) - (e^{\alpha d} - e^{\alpha v_e t})\Theta(d/v_e - t) \right]
\]

The total charge induced by the electrons and the ions is

\[
Q_{e\text{ind}} = \int_0^{d/v_e} I_{e\text{ind}}(t)dt = -e_0 \frac{e^{\alpha d} - 1}{\alpha d}
\]

\[
Q_{I\text{ind}} = \int_0^{d/v_I} I_{I\text{ind}}(t)dt = -e_0 \frac{e^{\alpha d} (\alpha d - 1) + 1}{\alpha d}
\]

\[
Q_e = \frac{e^{\alpha d} - 1}{(\alpha d - 1)e^{\alpha d} + 1} \approx \frac{1}{\alpha d - 1} \quad Q_I \approx \frac{1}{\alpha d} \quad Q_e + Q_I \approx \frac{1}{\alpha d} \quad \text{for} \quad e^{\alpha d} \gg 1
\]
Electrons movement in the induction gap takes about $0.1\text{mm}/v_1 = 0.5\text{ns}$.

Collecting all electrons from the drift gap takes e.g. $3\text{mm}/v_1 = 60\text{ns}$.

The MICROMEGA electron signal has a length of about $60\text{ns}$.

Ion movement – e.g. Argon Ions take $130\text{ns}$ for $50\text{kV/cm}$ and $100\text{um}$ gap, so the total length of the ions component is around $180\text{ns}$.

When using ‘fast’ electronics one does not integrate the full charge $\rightarrow$ ballistic deficit.
A high field region is implemented in a silicon sensor by doping.

Electrons will produce an avalanche in this high field region.

While in the Micromega the high field region is produced by a metal mesh, in the APD and the LGAD the high field region is implemented by doping and related ‘spacecharge’ in the volume.

In the Micromega, the ions move just back to the mesh, while in the APD and LGAD the holes move through the entire sensor.

Electrons and holes move at similar velocities.

The sensor is operated in a region where there is electron multiplication but not yet hole multiplication.

For higher fields → electron+hole multiplication → avalanche divergence → quench resistor → SiPM
A high field region is implemented in a silicon sensor by doping.

Electrons will produce an avalanche in this high field region.
Strips and Pixels in Parallel Plate Geometries
Weighting field of a strip in a parallel plate geometry

\[ \psi_1(x, z) = \frac{V_w}{\pi} \left[ \arctan \left( \cot \left( \frac{z\pi}{2d} \right) \tanh \left( \frac{\pi x + w/2}{2d} \right) \right) - \arctan \left( \cot \left( \frac{z\pi}{2d} \right) \tanh \left( \frac{x - w/2}{2d} \right) \right) \right] \]

\[ E_{1z}(x, z) = \frac{V_w}{2d} \left[ \frac{\sinh \left( \frac{\pi x + w/2}{d} \right)}{\cosh \left( \frac{x + w/2}{d} \right) - \cos \left( \frac{z\pi}{d} \right)} - \frac{\sinh \left( \frac{\pi x - w/2}{d} \right)}{\cosh \left( \frac{x - w/2}{d} \right) - \cos \left( \frac{z\pi}{d} \right)} \right] \]
$I_1(x, t) = \frac{qU}{Vw} E_z(x, z_0 - vt) \Theta(z_0/v - t)$

For $w \gg d$ the signal is equal to the parallel plate geometry.

For $w = D$ the signal signal (weighting field) increases towards the strip. The signal on the neighbour strip is strictly bipolar.

The smaller the strip, the more peaked is the signal towards the end. The signal in the neighbour strip has also larger amplitude, but is still strictly bipolar.
Weighting field of a strip in a parallel plate geometry

$w \gg d$

$I_1(x, t) = \frac{qv_1}{V_w} E_z(x, z_0 - v_1 t) \Theta(z_0/v_1 - t)$

$+ \frac{qv_2}{V_w} E_z(x, z_0 + v_2 t) \Theta((d - z_0)/v_2 - t)$

$v_1 = 3v_2$

$w = d$

$w = d/2$
Weighting field of a strip in a parallel plate geometry

\[ I_1(x, t) = \frac{qv_1}{V_w} E_z(x, z_0 - v_1 t) \Theta(z_0/v_1 - t) + \frac{qv_2}{V_w} E_z(x, z_0 + v_2 t) \Theta((d - z_0)/v_2 - t) \]

\[ v_1 = \frac{v_2}{3} \]

\( w >> d \)

\( w = d \)

\( w = d/2 \)
A uniform charge distribution

Charge moving towards the strip:

\[ I_a(x, t) = \frac{\lambda v_1}{V_w} \int_0^d E_w(x, z - v_1 t) \Theta(z/v_1 - t) dz \]

\[ = \frac{\lambda v_1}{V_w} [\psi_1(x, 0) - \psi_1(x, d - v_1 t)] \Theta(d/v_1 - t) \]

Charge moving away from the strip:

\[ I_b(x, t) = \frac{\lambda v_2}{V_w} \int_0^d E_w(x, z + v_2 t) \Theta((d - z)/v_2 - t) dz \]

\[ = \frac{\lambda v_2}{V_w} \psi_1(x, v_2 t) \Theta(d/v_2 - t) \]

Total induced current on the strip

\[ I_1(x, t) = I_a(x, t) + I_b(x, t) \]

For very wide strips (parallel plate geometry) the induced charge from the movement of \(+\lambda\) and \(-\lambda\) is the same and equal to \(\lambda d/2\).

For small strips, the fraction of the signal due to the charges moving towards the strip increases.
Weighting field of a strip in a parallel plate geometry

\[ I_a(x, t) = \frac{\lambda v_1}{V_w} \int_0^d E_w(x, z - v_1 t) \Theta(z/v_1 - t) dz \]
\[ = \frac{\lambda v_1}{V_w} [\psi_1(x, 0) - \psi_1(x, 0 - v_1 t)] \Theta(d/v_1 - t) \]

\[ I_b(x, t) = \frac{\lambda v_2}{V_w} \int_0^d E_w(x, z + v_2 t) \Theta((d - z)/v_2 - t) dz \]
\[ = \frac{\lambda v_2}{V_w} \psi_1(x, v_2 t) \Theta(d/v_2 - t) \]

\[ I_1(x, t) = I_a(x, t) + I_b(x, t) \]

\( v_1 = v_2/3 \) e.g. holes moving to the strip and electrons moving away from the strip
The weighting potential of a pixel is given by

\[ \phi_w(x, y, z) = \frac{4V_w}{\pi^2} \int_0^\infty \int_0^\infty \cos(k_x x) \sin\left(\frac{k_x w_x}{2}\right) \cos(k_y y) \]
\[ \times \sin\left(\frac{k_y w_y}{2}\right) \sin(h(kd - z)) \frac{dk_x}{k_x} \frac{dk_y}{k_y} \]  

The expression can be written in the form

\[ \frac{\phi_w(x, y, z)}{V_w} \approx \frac{1}{2\pi} f(x, y, z) - \frac{1}{2\pi} \sum_{n=1}^{N} [f(x, y, 2nd - z) - f(x, y, 2nd + z)] \]

with

\[ f(x, y, u) = \int_{x-w_x/2}^{x+w_x/2} \int_{y-w_y/2}^{y+w_y/2} \frac{u}{(x^2 + y^2 + u^2)^{3/2}} \, dx \, dy \]

\[ = \arctan\left(\frac{x_1 y_1}{u \sqrt{x_1^2 + y_1^2 + u^2}}\right) + \arctan\left(\frac{x_2 y_2}{u \sqrt{x_2^2 + y_2^2 + u^2}}\right) \]
\[ - \arctan\left(\frac{x_1 y_2}{u \sqrt{x_1^2 + y_2^2 + u^2}}\right) - \arctan\left(\frac{x_2 y_1}{u \sqrt{x_2^2 + y_1^2 + u^2}}\right) \]

Terminating the series at \( N \) the error on the potential is

\[ |\Delta \phi_w| < \frac{V_w w_x w_y}{8\pi d^2} \frac{1}{N^2 d} \]

i.e. the error decreases quadratically with \( N \).
The z-component of the weighting field is given by

\[ \frac{E_w^z(x, y, z)}{V_w} \approx \frac{1}{2\pi} g(x, y, z) + \frac{1}{2\pi} \sum_{n=1}^{N} [g(x, y, 2nd + z) + g(x, y, 2nd - z)] \]

\[ g(x, y, u) = -\frac{\partial f(x, y, u)}{\partial u} \]

\[ = \frac{x_1 y_1 (x_1^2 + y_1^2 + 2u^2)}{(x_1^2 + u^2)(y_2^2 + u^2)\sqrt{x_1^2 + y_1^2 + u^2}} \]

\[ + \frac{x_2 y_2 (x_2^2 + y_2^2 + 2u^2)}{(x_2^2 + u^2)(y_2^2 + u^2)\sqrt{x_2^2 + y_2^2 + u^2}} \]

\[ - \frac{x_1 y_2 (x_1^2 + y_2^2 + 2u^2)}{(x_1^2 + u^2)(y_2^2 + u^2)\sqrt{x_1^2 + y_2^2 + u^2}} \]

\[ - \frac{x_2 y_1 (x_2^2 + y_1^2 + 2u^2)}{(x_2^2 + u^2)(y_1^2 + u^2)\sqrt{x_2^2 + y_1^2 + u^2}} \]

Terminating the series at \( N \), the error on the weighting field is

\[ |\Delta E_w^z| < \frac{V_w}{8\pi} \frac{W_x W_y}{d^2} \frac{1}{N^2} \]

in the entire volume, i.e. the error decreases quadratically with \( N \).
Bonus: Frisch Grid
Frisch Grid

\begin{align*}
E_w &= 1/d \\
Q_1 &+ Q_2 = -q
\end{align*}

\begin{align*}
Q_1 &+ Q_2 = -q
\end{align*}
Design and performance of an ionisation chamber for the measurement of low alpha-activities

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Abstract

A new ionisation chamber for alpha-spectroscopy has been built from radio-pure materials for the purpose of investigating long lived alpha-decays. The measurement makes use of pulse shape analysis to discriminate between signal and background events. The design and performance of the chamber is described in this paper. A background rate of \((10.9 \pm 0.6)\) counts per day in the energy region of 1 MeV to 9 MeV was achieved with a run period of 30.8 days. The background is dominantly produced by radon daughters.

Keywords:
Frisch grid; Ionization chamber; alpha-decay

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chamber diameter [cm]</td>
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</tr>
<tr>
<td>Grid wire radius [mm]</td>
<td>0.075</td>
</tr>
<tr>
<td>Grid separation distance [mm]</td>
<td>2.0</td>
</tr>
<tr>
<td>Distance from grid to anode [mm]</td>
<td>35</td>
</tr>
<tr>
<td>Distance from grid to cathode [cm]</td>
<td>10.0</td>
</tr>
<tr>
<td>GI [%]</td>
<td>2.429(7)</td>
</tr>
<tr>
<td>(\frac{V_{\text{anode}} - V_{\text{grid}}}{V_{\text{grid}} - V_{\text{cathode}}})</td>
<td>0.446</td>
</tr>
</tbody>
</table>

Table 1: The design parameters and measured grid inefficiency of the chamber. These parameters were chosen to maximise the energy resolution and minimise the grid inefficiency (GI).
From Rob Veenhof:

Just a reminder: The mobility of Ar+ ions, like the other ions you listed, play almost no role in quenched gases: quenchers have a lower IP and they pick up the charge. CH4 and other alkanes become heavier alkanes; CO₂ becomes CO₂+CO₂⁻n ...

In pure noble gases you get dimers, trimers etc. This is particularly relevant in Xe.

Further, the gain in GEMs happens more near the rims, less in the centre of the holes.

---

Understanding the gas gain in GEM detectors

All of us*

*Exchanges in the course
E-mail: gem@gains

ABSTRACT: Understanding the gas gain in GEM detectors.

KEYWORDS: GEM, gain.

---

Figure 7. Left: Effective gain $G_{\text{eff}}$ is defined as the number of electrons emerging from the GEM that travel at least 50 µm towards the anode. Short of being lost to attachment, which is unlikely in the induction field, these electrons reach the anode. The matching ions and on polyimide, a GEM electrode, or the cathode. If an ion ends on polyimide, the net charge induced by the electron-ion pair is smaller than 1 qₑ. On the other hand, some electrons do not emerge from the GEM. If such an electron is attached by CO₂ or polyimide, and if the accompanying ion reaches polyimide, then net charge is induced in the anode. It follows that the measured quantity, i.e. the net charge induced by an avalanche, can be both smaller and larger than $G_{\text{eff}}$. This figure compares the net charge estimated as the sum 2e⁻qₑ over the weighting potential differences between electrons and ion end points, with $G_{\text{eff}}$. The difference is typically less than 1%, with a 2e⁻qₑ > $G_{\text{eff}}$ tail extending to 1% at low GEM potentials. Right: Production points of electrons that do contribute to the effective gain (shades of blue) and that do not contribute (shades from green to red), per unit volume, per avalanche in an uncharged GEM with $V_{\text{GEM}} = 400$ V and $\eta = 0.7$. The effective gain is 57% of the total gain in this case. Although electrons are predominantly produced near the exit electrode (see figure 16), only electrons produced in the centre of the hole contribute to the effective gain.
Electron and hole mobility in solids

For a particle of mass $m$ that is randomized in each collision, the mobility is given by

$$\mu = \frac{q}{m^*} \bar{\tau}$$

where $m^*$ is the effective mass and $\tau$ is the average time between collisions.

In a gas, the mass corresponds to the true particle mass and $\tau$ depends on the scattering cross-sections and energy loss.

In a solid, also the effective mass depends on the crystal structure and even the direction of movement in the crystal. The cross-sections also depend significantly on impurities.
Wire Chambers
Wire Chambers

[ ... ]

**Game changer**
The invention of the Multi Wire Proportional Chamber (MWPC) by Georges Charpak in 1968 was a game changer, earning him the 1992 Nobel Prize in Physics. Suddenly, experimenters had access to large-area charged particle detectors with millimetre spatial resolution and staggering MHz-rate capability. Crucially, the emerging integrated-circuit technology could deliver amplifiers so small in size and cost to equip many thousands of proportional wires.

This ingenious and deceptively simple detector is relatively easy to construct. The workshops of many university physics departments could master the technology, attracting students and “democratising” particle physics. So compelling was experimentation with MWPCs that within a few years, large detector facilities with tens of thousands of wires were constructed [...]

An extension of this technique is the drift chamber, a MWPC-type geometry, with which the time difference between the passage of the particle and the onset of the wire signal is recorded, providing a measure of position with 100 μm-level resolution. The MWPC concept lends itself to a multitude of geometries and has found its “purest” application as the readout of time projection chambers (TPCs).
Physics Nobel Prices for Instrumentation

1927: C.T.R. Wilson, Cloud Chamber
1939: E. O. Lawrence, Cyclotron & Discoveries
1948: P.M.S. Blacket, Cloud Chamber & Discoveries
1950: C. Powell, Photographic Method & Discoveries
1954: Walter Bothe, Coincidence method & Discoveries
1960: Donald Glaser, Bubble Chamber
1968: L. Alvarez, Hydrogen Bubble Chamber & Discoveries
1992: Georges Charpak, Multi Wire Proportional Chamber

All Nobel Price Winners related to the Standard Model: 87 !
(personal and probably biased statistics by W. Riegler)

31 for Standard Model Experiments
13 for Standard Model Instrumentation and Experiments
3 for Standard Model Instrumentation
21 for Standard Model Theory
9 for Quantum Mechanics Theory
9 for Quantum Mechanics Experiments
1 for Relativity

56 for Experiments and instrumentation
31 for Theory
Wire Chamber Signals

Wire with radius (10-25\(\mu\)m) in a tube of radius \(b\) (1-3cm):

\[
\varphi(r) = \frac{V}{\ln(a/b)} \ln(r/b) \quad E_r(r) = \frac{V}{\ln(b/a)} \frac{1}{r}
\]

Electric field close to a thin wire (100-300kV/cm). E.g. \(V_0=1000\)V, \(a=10\mu\)m, \(b=10\)mm, \(E(a)=150\)kV/cm

Electric field is sufficient to accelerate electrons to energies which are sufficient to produce secondary ionization \(\rightarrow\) electron avalanche \(\rightarrow\) signal.
Wire Chamber Signals

The electrons are produced very close to the wire, so as a first approximation we can assume that the signal is only due to $N_{\text{tot}}$ ions moving from the wire surface to the tube wall:

$$\varphi(r) = \frac{V}{\ln(a/b)} \ln(r/b) \quad E_r(r) = \frac{V}{\ln(b/a)} \frac{1}{r}$$

Ions move with a velocity proportional to the electric field: $v(r) = \mu E(r)$

$$\frac{dr(t)}{dt} = \mu \frac{V}{\ln(b/a)} \frac{1}{r(t)} \quad \Rightarrow \quad r(t) = a \sqrt{1 + t/t_0} \quad 0 < t < t_{\text{max}}$$

$$t_0 = \frac{a^2 \ln(b/a)}{2\mu V} \quad t_{\text{max}} = t_0 \left( \frac{b^2}{a^2} - 1 \right)$$

Weighting field of the wire: Remove charge and set wire to $V_w$ while grounding the tube wall.

$$\psi_1(r) = -\frac{V_w \ln(r/b)}{\ln(b/a)} \quad E_1(r) = \frac{V_w}{r \ln(b/a)}$$

The induced current is therefore

$$I_{\text{ind}}^1(t) = -\frac{N_{\text{tot}} e_0}{V_w} E_1 [r(t)] \frac{dr(t)}{dt} = -\frac{N_{\text{tot}} e_0}{2 \ln(b/a)} \frac{1}{t + t_0}$$
Wire Chamber Signals

The signal from a single primary electron has $1/(t+t_0)$ shape with a very long tail …

Typically only a small fraction (e.g. 10%) of the total avalanche charge is induced during the electronics integration time.

Ballistic deficit. In Micropattern detectors one can integrate all the charge $\rightarrow$ 10 times lower gas gain for the same signal.

ATLAS muon drift tubes

$V_0$...voltage on the wire $\approx 3500V$

$a$...wire radius $= 25\mu m$

$b$...tube radius $= 1.46cm$

$t_0 = 11ns \; \; \; t_{max} = 3.73msec$
ATLAS muon system drift tubes

Tubes of 3cm diameter are assembled into chambers.

The first electrons arriving at the wire determine the distance of the track from the wire → drift tube.

The last arriving electrons are originating from r=1.5cm, so the last electrons always arrive at the same time.

The long signal tail needs dedicated electronics filtering to ensure limited deadtime.

80um position resolution over a few thousand m² area with only 330 000 channels!
The diameter of the wires in a wire chamber is very small compared to their distance to the neighbour wires and the cathode and anode metal planes.

We can therefore approximate the wire by an infinitely thin line charge.

The potential of a line charge at position $x_1$, $y_1$ is given by ($x_0$ is just the reference point where the potential is defined to be zero)

$$\varphi(x, y) = -\frac{\lambda}{2\pi \varepsilon_0} \ln \sqrt{(x - x_1)^2 - (y - y_1)^2} - \frac{\lambda}{2\pi \varepsilon_0} \ln \sqrt{(x_0 - x_1)^2 - (y_0 - y_1)^2}$$

For $N$ parallel wires at positions $x_n$, $y_n$ we therefore have the potential

$$\varphi(x, y) = -\frac{1}{4\pi \varepsilon_0} \sum_{n=1}^{N} \lambda_n \ln \left[ \frac{(x - x_n)^2 + (y - y_n)^2}{(x_0 - x_n)^2 + (y_0 - y_n)^2} \right]$$

We define the potentials $V_m$ on the surface of the wires

$$V_m = \varphi(x_m + r_m, y_m) \approx -\frac{1}{4\pi \varepsilon_0} \sum_{n \neq m=1}^{N} \lambda_n \ln \left[ \frac{(x_m - x_n)^2 + (y_m - y_n)^2}{(x_0 - x_n)^2 + (y_0 - y_n)^2} \right] - \frac{1}{4\pi \varepsilon_0} \lambda_m \ln \frac{r_m^2}{(x_0 - x_m)^2 + (y_0 - y_m)^2}$$

Which defines in turn the linear charge densities and the capacitance matrix

$$V_m = \sum_{n=1}^{N} a_{mn} \lambda_n \quad \lambda_m = \sum_{n=1}^{N} c_{mn} V_m \quad c_{mn} = a_{mn}^{-1}$$

$$a_{mn} = -\frac{1}{4\pi \varepsilon_0} \lambda_n \ln \left[ \frac{(x_m - x_n)^2 + (y_m - y_n)^2}{(x_0 - x_n)^2 + (y_0 - y_n)^2} \right] \quad m \neq n$$

$$a_{mn} = -\frac{1}{4\pi \varepsilon_0} \lambda_m \ln \left[ \frac{r_m^2}{(x_0 - x_m)^2 + (y_0 - y_m)^2} \right] \quad m = n$$
Multi Wire Proportional Chamber

If we have an infinite row of wires a distance \( s \) at position \( x = x_0 + ns, y=0 \) the potential is given by

\[
\varphi(x, y) = -\frac{\lambda}{4\pi\varepsilon_0} \ln \left[ \cosh \frac{2\pi y}{s} - \cos \frac{2\pi(x - x_0)}{s} \right] + c_2
\]

For values of \( \cosh(2\pi y/s) \gg 1 \) i.e. for \( y > s \) we can approximate

\[
\cosh(2\pi y/s) \approx 1/2 \exp 2\pi y/s
\]

and find

\[
\varphi(x, y) \approx -\frac{\lambda}{2\pi\varepsilon_0} \left( \frac{\pi|y|}{s} - 1/2 \ln 2 \right) + c_2 \quad E_y = \frac{\lambda}{2s\varepsilon_0}
\]

This means that the infinite row of wires looks like a uniform sheet of charge with surface charge density \( \lambda/s \). For small radii we can approximate the potential as

\[
\varphi(x_0, r) \approx -\frac{\lambda}{2\pi\varepsilon_0} \left( \ln \frac{\pi r}{s} + 1/2 \ln 2 \right) + c_2
\]

Defining the potential on the wire surface to be \( V_0 \) and defining the potential at \( y=h \) to be zero we have the two equations that define \( \lambda \).

\[
-\frac{\lambda}{2\pi\varepsilon_0} \left( \ln \frac{\pi r_0}{s} + 1/2 \ln 2 \right) + c_2 = V_0 \quad -\frac{\lambda}{2\pi\varepsilon_0} \left( \frac{\pi h}{s} - 1/2 \ln 2 \right) + c_2 = 0
\]

For the region close to the wire and for distances \( y > s \) we find the approximate relations

\[
\varphi(x, y) \approx \frac{V_0}{\ln \frac{r}{r_c}} \ln \frac{r}{r_c} \quad r \ll s \quad \varphi_1(x, y) \approx \frac{V_0}{s \ln \frac{r}{r_c}} (y - h) \quad e^{2\pi y/s} \gg 1 \quad r_c = \frac{s}{2\pi e^{2h/s}}
\]

In the vicinity of the wires, the electric field therefore looks like the one for the single wire of radius \( r_0 \) in the center of a tube with radius \( r_c \).
Multi Wire Proportional Chamber

Next we have to calculate the electric fields on the wires to find the ion movement and the weighting fields of the wires to find the induced signals.

Setting the individual wires to potentials $V_n$ defines the charge on the wires through the capacitance matrix as and the related electric field in the vicinity of the wires as

$$
\lambda_n = \sum_{n=1}^{N} c_{nm} V_m \quad \quad E_n(r) = \frac{\lambda_n}{2\pi \varepsilon_0 r} = \frac{\sum_{n=1}^{N} c_{nm} V_m}{2\pi \varepsilon_0 r}
$$

If the ions move only in the coaxial region of the wire, which is typically the case within the integration time of the electronics, the ion movement is given by

$$
r(t) = r_0 \sqrt{1 + t/t_0} \quad \quad t_0 = \frac{a^2 \pi \varepsilon_0}{\mu \sum_{m=1}^{N} c_{nm} V_m}
$$

To find the signal that this movement induces on the avalanche wire and the other wires we have to calculate the weighting fields. For the weighting field of the avalanche wire we put wire n to potential $V_w$ and ground the others, so we have

$$
E_n^w(r) = \frac{c_{nn} V_w}{2\pi \varepsilon_0 r} \quad \quad I_n(t) = -\frac{N_{tot} \varepsilon_0}{V_w} E_n^w(r(t)) \frac{dr(t)}{dt} = -\frac{N_{tot} \varepsilon_0}{4\pi \varepsilon_0} \frac{c_{nn}}{t + t_0}
$$

Since $c_{nn} > 0$ we know that the signal on the avalanche wire is always negative. The weighting field of a neighbouring wire m and the signal on this neighbouring wire are given by

$$
E_m^w(r) = \frac{c_{nm} V_w}{2\pi \varepsilon_0 r} \quad \quad I_n(t) = -\frac{N_{tot} \varepsilon_0}{V_w} E_m^w(r(t)) \frac{dr(t)}{dt} = -\frac{N_{tot} \varepsilon_0}{4\pi \varepsilon_0} \frac{c_{nm}}{t + t_0}
$$

Since $c_{nm} < 0$ we know that the signal induced on neighbouring wires is always positive!
Crosstalk to neighboring wires

The direct induction on the neighbor wire is opposite polarity, the electrical crosstalk from finite impedance of the readout electronics has the same polarity

\[
\frac{i_2}{i_1} = -\frac{C_{12}(1 + sZC_{12})}{C_{11} + sZ(C_{12}^2 - C_{11}^2 + C_{11}C_{12})} < 0 \quad \text{for} \quad C_{12} < C_{11}
\]

The resulting total crosstalk is always opposite polarity with respect to the signal on the wire where the avalanche happens. The capacitive (same polarity) crosstalk is cancelled by the direct induction signal (opposite polarity).

… which makes it ‘a simple matter to localize the wire, which is the seed of the avalanche, whatever the distance between the wires … ‘ [Charpak, Nobel lecture 1992].
Multi Wire Proportional Chamber

As long as the ions are moving in the coaxial region of the wire, the signal on ALL electrodes in the wire chamber have the same shape $1/(t+t_0)$ and their relative amplitudes are given by the capacitance matrix elements.

By considering the readout electrodes also to be made up from many parallel wires on which signal is summed, the same applies to the readout strips.

For crossed wires and readout strips and for readout pads, things are a bit more complex and ones needs a 3D treatment of the problem. And ‘effective’ pad response function (‘weighting field’) is used.

\[ \Gamma(\lambda) = K_1 \frac{1 - \tanh^2 K_2 \lambda}{1 + K_3 \tanh^2 K_2 \lambda}. \]
Liquid Argon TPCs

Prototypes for DUNE @ CERN

ICARUS
The ionization electrons created in the interaction of a charged particle with the Liquid Argon are drifting to a wire plane.

The potentials of the wires are set such that the electrons pass through the first three wire grids (induction planes) and arrive at the 4th wire plane (collection plane). There is no multiplication of electrons at the wires in LAr.

Even if the electrons do not arrive at the top three wire planes, they still induce a (fully bipolar) signal that can be read out.

The central two wire planes are at an angle with respect to the collection wire plane → 2D position resolution → 3D together with the drift time!
ProtoDUNE

Table 2.4: Baseline bias voltages for APA wire layers

<table>
<thead>
<tr>
<th>Anode Plane</th>
<th>Bias Voltage</th>
</tr>
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<tbody>
<tr>
<td>Grid (G)</td>
<td>-665 V</td>
</tr>
<tr>
<td>Induction (U)</td>
<td>-370 V</td>
</tr>
<tr>
<td>Induction (V)</td>
<td>0 V</td>
</tr>
<tr>
<td>Collection (X)</td>
<td>820 V</td>
</tr>
<tr>
<td>Mesh (M)</td>
<td>0 V</td>
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Table 2.5: APA design parameters

<table>
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<tr>
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<th>Value</th>
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<tbody>
<tr>
<td>Active Height</td>
<td>5.984 m</td>
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<tr>
<td>Active Walls</td>
<td>2.300 m</td>
</tr>
<tr>
<td>Wire Pitch (L/L)</td>
<td>4.669 mm</td>
</tr>
<tr>
<td>Wire Pitch (X/G)</td>
<td>4.700 mm</td>
</tr>
<tr>
<td>Wire Position Tolerance</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>Wire Plane Spacing</td>
<td>4.75 mm</td>
</tr>
<tr>
<td>Wire Angle (w.r.t. vertical) (L/L)</td>
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</tr>
<tr>
<td>Wire Angle (w.r.t. vertical) (X/G)</td>
<td>0°</td>
</tr>
<tr>
<td>Number Wires / APA</td>
<td>900 (X), 900 (G), 800 (L), 800 (V)</td>
</tr>
<tr>
<td>Number Electronic Channels / APA</td>
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</tr>
<tr>
<td>Wire Tension</td>
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<tr>
<td>Wire Material</td>
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<tr>
<td>Wire Diameter</td>
<td>150 μm</td>
</tr>
<tr>
<td>Wire Resistance</td>
<td>16.8 μΩ·cm @ 20°C</td>
</tr>
<tr>
<td>Wire Resistance/m</td>
<td>4.4 μΩ/m @ 20°C</td>
</tr>
<tr>
<td>Frame Flatness</td>
<td>5 mm</td>
</tr>
<tr>
<td>Photon Detector Skew</td>
<td>10°</td>
</tr>
</tbody>
</table>
Potential of two wires:
\[ \varphi(x, y) = -\frac{1}{4\pi\varepsilon_0} \lambda_1 \ln \left[ \frac{(x + x_1)^2 + (y)^2}{(x_0 + x_1)^2 + (y_0)^2} \right] - \frac{1}{4\pi\varepsilon_0} \lambda_2 \ln \left[ \frac{(x - x_1)^2 + (y)^2}{(x_0 - x_1)^2 + (y_0)^2} \right] \]

The potentials on the two wires define the line charge density
\[ \varphi(-x_1 + r_0, 0) = V_1 \quad \varphi(x_1 + r_0, 0) = V_2 \quad \rightarrow \quad \lambda_1, \lambda_2 \]

The weighting potential of wire at position \(-x_1\) is defined by setting it to \(V_1 = V_w\) and \(V_2 = 0\)

\[ \varphi^w_1(x, y) = \varphi(x, y) \quad V_1 = V_w, V_2 = 0 \]

The weighting field is
\[ E^w_1(x, y) = -\nabla \varphi^w_1(x, y) \]

For a charge moving along the y-axis with velocity \(-v\) we have
\[ I(t) = -\frac{q}{V_w} (E^w_{1x}[x(t), y(t)]\dot{x}(t) + E^w_{1y}[x(t), y(t)]\dot{y}(t)) \]
\[ = \frac{q}{V_w} E^w_{1y}[x_p, -vt]v \]

And the induced current is
\[ I(t) = c_1 t \left( \frac{\ln 4}{(x_1 - x_p)^2 + v^2 t^2} + \frac{2\ln(x_1/r_0)}{(x_1 + x_p)^2 + v^2 t^2} \right) \]
Signals in particle detectors

Lecture 1:
- Electrostatics
- Principles
- Reciprocity
- Induced currents
- Induced voltages
- Ramo-Shockley theorem
- Mean value theorem
- Capacitance matrix
- Equivalent circuits

Lecture 2:
- Signals in
  - Ionization chambers
  - Liquid argon calorimeters
  - Diamond detectors
  - Silicon detectors
  - GEMs (Gas Electron Multiplier)
  - Micromegas (Micromesh gas detector)
  - APDs (Avalanche Photo Diodes)
  - LGADS (Low Gain Avalanche Diodes)
  - SiPMs (Silicon Photo Multipliers)
  - Strip detectors
  - Pixel detectors
  - Wire Chambers
  - Liquid Argon TPCs

Lecture 3:
- Media with conductivity
  - Quasi-static approximations
  - Signal theorem extensions
  - Time dependent weighting fields
  - Resistive plate chambers (RPCs)
  - Un-depleted silicon sensors
  - Monolithic pixel sensors

Lecture 4:
- Signal propagation
- Transmission lines
- Termination
- Linear signal processing
- Noise
- Optimum filters

Lecture 5:
- Possible overflow, wrap-up and Q&A session

Main Auditorium, Mon. 2 Dec.
Council Chamber, Tue. 3 Dec.
TH conference room (4/3-006), Wed. 4 Dec.
Filtration Plant (222/R-001), Thu. 5 Dec.
Main Auditorium, Fri. 6 Dec.