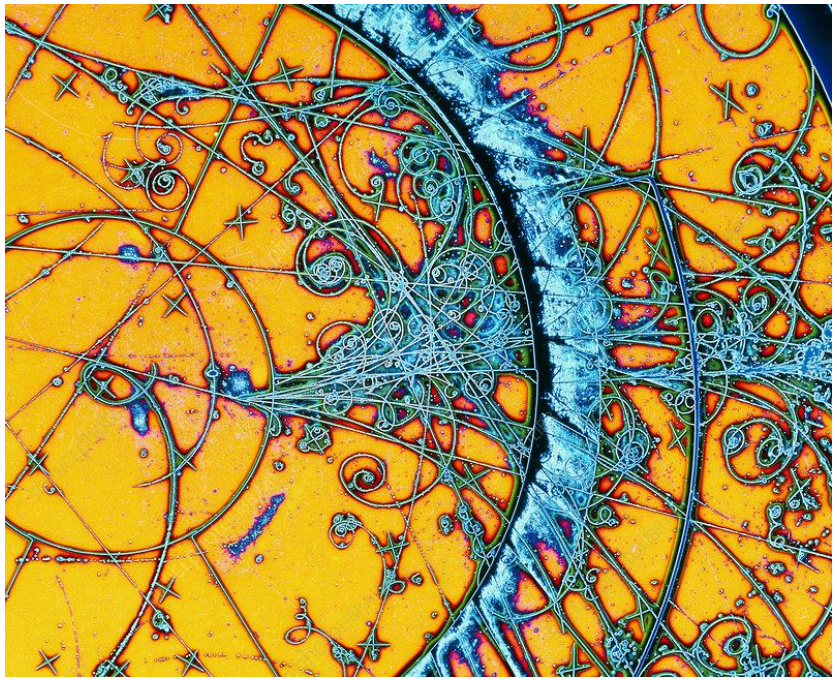

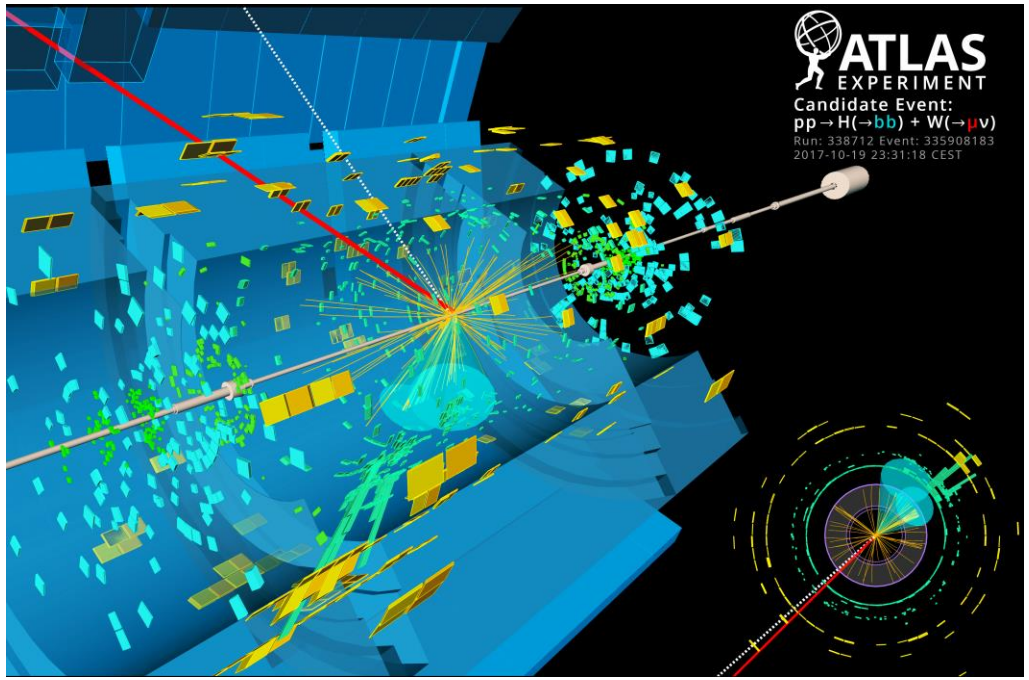
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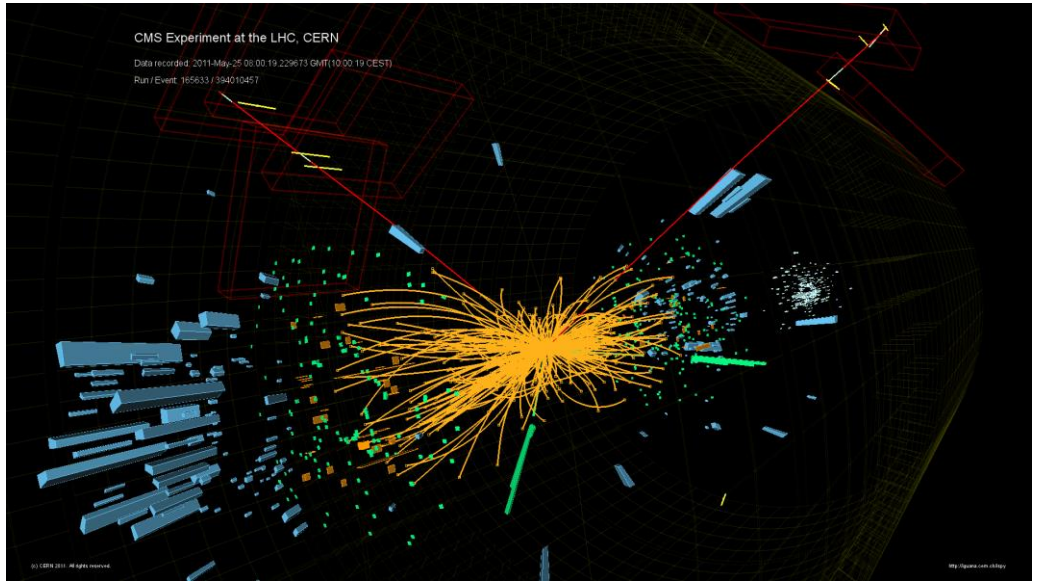

CMS Experiment at the LHC, CERN

Signals in Particle Detectors 4/4

Academic Training Lectures

Werner Riegler, CERN, werner.riegler@cern.ch

Signals in particle detectors

Main Auditorium, Mon. 2 Dec.

Lecture 1:

- Electrostatics
- Principles
- Reciprocity
- Induced currents
- Induced voltages
- Ramo-Shockley theorem
- Mean value theorem
- Capacitance matrix
- Equivalent circuits

Council Chamber, Tue. 3 Dec.

Lecture 2:

Signals in

- Ionization chambers
- Liquid argon calorimeters
- Diamond detectors
- Silicon detectors
- GEMs (Gas Electron Multiplier)
- Micromegas (Micromesh gas detector)
- APDs (Avalanche Photo Diodes)
- LGADs (Low Gain Avalanche Diodes)
- SiPMs (Silicon Photo Multipliers)
- Strip detectors
- Pixel detectors
- Wire Chambers
- Liquid Argon TPCs

TH conference room (4/3-006), Wed. 4 Dec.

Lecture 3:

- Media with conductivity
- Quasi-static approximations
- Signal theorem extensions
- Time dependent weighting fields
- Resistive plate chambers (RPCs)
- Un-depleted silicon sensors
- Monolithic pixel sensors

Filtration Plant (222/R-001), Thu. 5.Dec.

Lecture 4:

- Signal propagation
- Transmission lines
- Termination
- Linear signal processing
- Noise
- Optimum filters

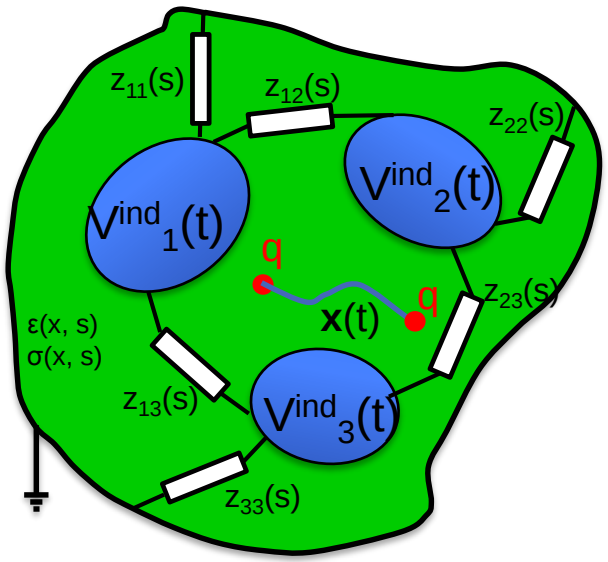
Main Auditorium, Fri. 6 Dec.

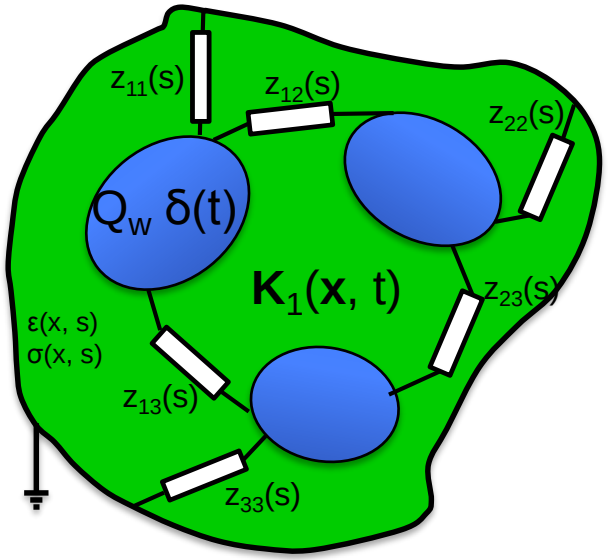
Lecture 5:

- Possible overflow, wrap-up and Q&A session

Summary on Signal Theorems

Medium with conductivity





A set of metal electrodes (perfect conductors) is embedded in a medium of position dependent permittivity and conductivity (resistivity).

The electrodes are in addition connected with discrete (linear) impedance elements.

A point charge q is moving along a trajectory **x**(t) in between these electrodes.

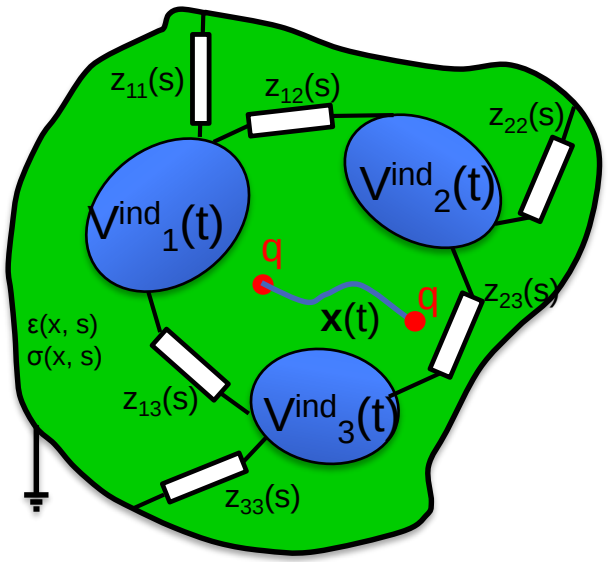
What are the voltage signals induced on these electrodes ?

Solution 1:

- Remove the point charge charge q
- Leave all other impedance elements in place
- Place a 'delta charge' $Q_w\,\delta(t)$ on the electrode in question, which defines the time dependent weighting field $\mathbf{K}_1(\mathbf{x}, t)$
- The induced voltage on this electrode is given by

$$V_1^{ind}(t) = -\frac{q}{Q_w}\int_0^t \mathbf{K_1}(\mathbf{x}(t'), t-t')\dot{\mathbf{x}}(t')dt'$$

Solution 2:



- Remove the point charge q and remove all discrete impedance elements
- Place a delta voltage pulse $V_w\,\delta(t)$ on electrode n, while grounding all other electrodes, which defines a time dependent weighting field $\mathbf{E}_n(\mathbf{x}, t)$
- Calculate all currents induced on the grounded electrodes by

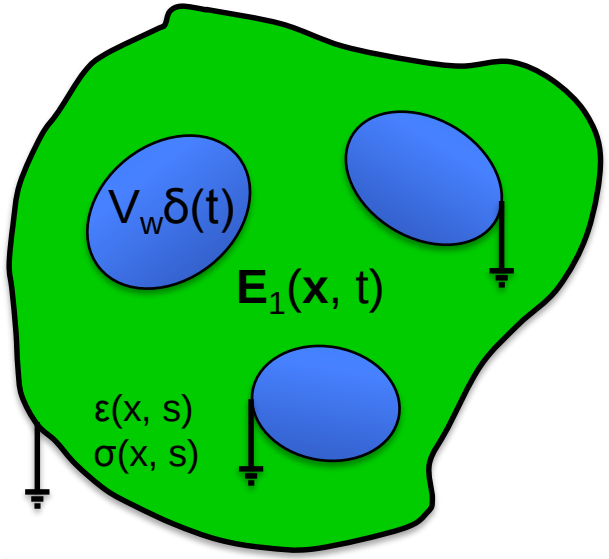
$$I_n^{ext}(t) = -\frac{q}{V_w}\int_0^t \mathbf{E}_n(\mathbf{x}(t'), t-t')\dot{\mathbf{x}}(t')dt'$$

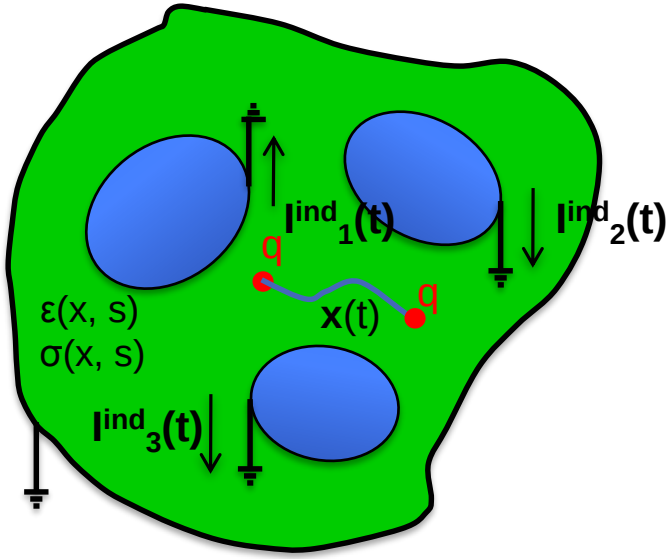
- Calculate the admittance matrix and the related impedance elements representing the medium

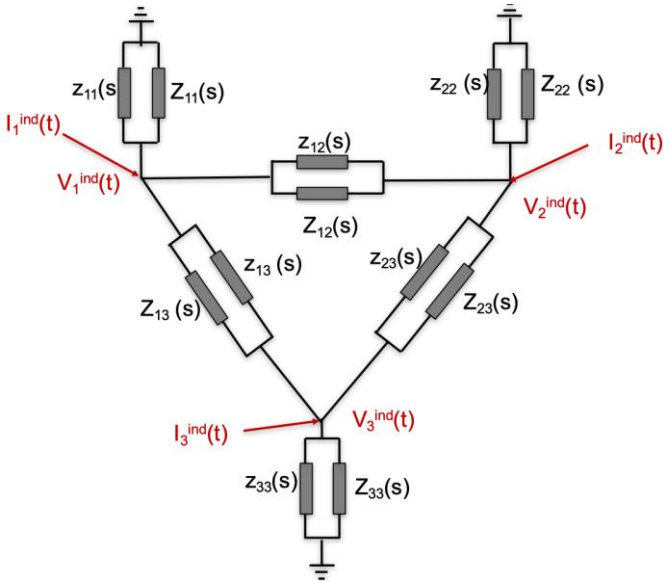
$$y_{mn}(s) = -\frac{s}{V_w}\oint_{\mathbf{A}_n} \varepsilon_{eff}(\mathbf{x}, s)\mathbf{E}_m(\mathbf{x}, s)d\mathbf{A}$$

$$Z_{nm}(s) = -\frac{1}{y_{nm}(s)} \quad n \neq m \qquad Z_{nn}(s) = \frac{1}{\sum_{m=1}^{N} y_{nm}(s)} = -\frac{1}{y_{0n}} \quad n = m$$

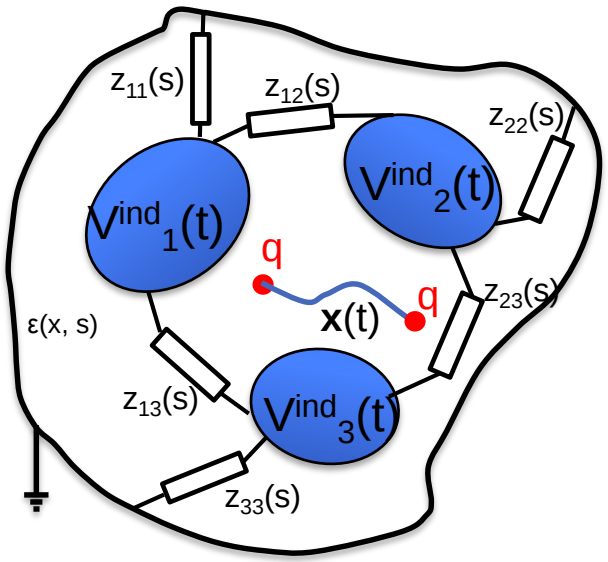
- Place these induced currents as ideal current sources on the equivalent circuit, where the discrete impedance elements are in parallel with the impedance representing the detector medium







Insulating medium



- Remove the point charge q and remove all discrete impedance elements
- Place a delta voltage pulse $V_w\,\delta(t)$ on electrode n, while grounding all other electrodes, which defines the static weighting field $\mathbf{E}_1(\mathbf{x})$
- Calculate all currents induced on the grounded electrodes by

$$I_n^{ind}(t) = -\frac{q}{V_w}\mathbf{E}_n(\mathbf{x}(t))\dot{\mathbf{x}}(t)$$

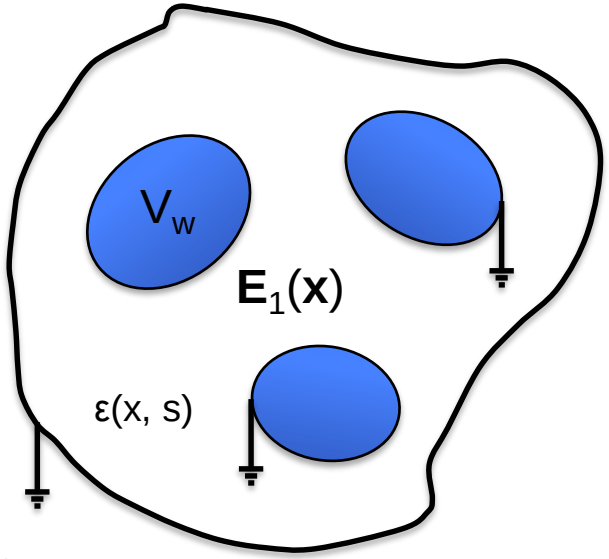
- Calculate the capacitance matrix and the related electrode capacitances

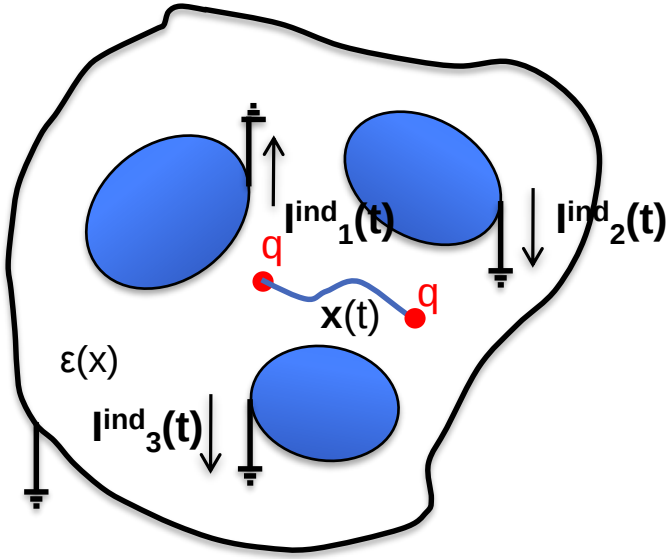
$$c_{mn} = -\frac{1}{V_w}\oint_{\mathbf{A}_n}\mathbf{E}_m(\mathbf{x})d\mathbf{A}$$

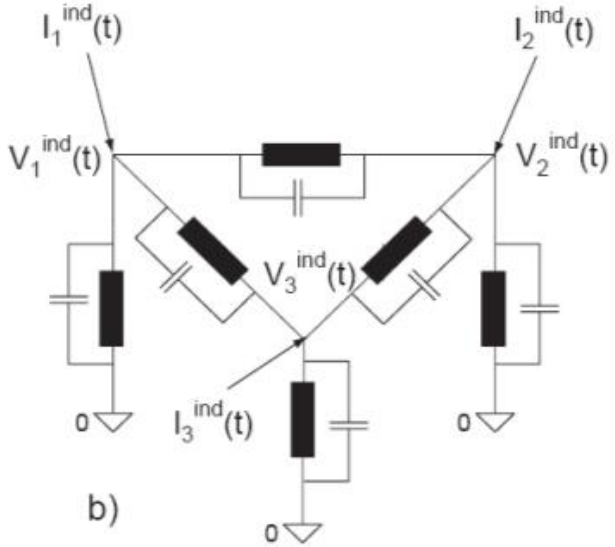
$$C_{nn} = \sum_{m=1}^{N} c_{mn} = -c_{n0} \qquad C_{mn} = -c_{mn} \quad m \neq n$$

- Place these induced currents as ideal current sources on the equivalent circuit, where the discrete impedance elements are in parallel with the electrode capacitances.

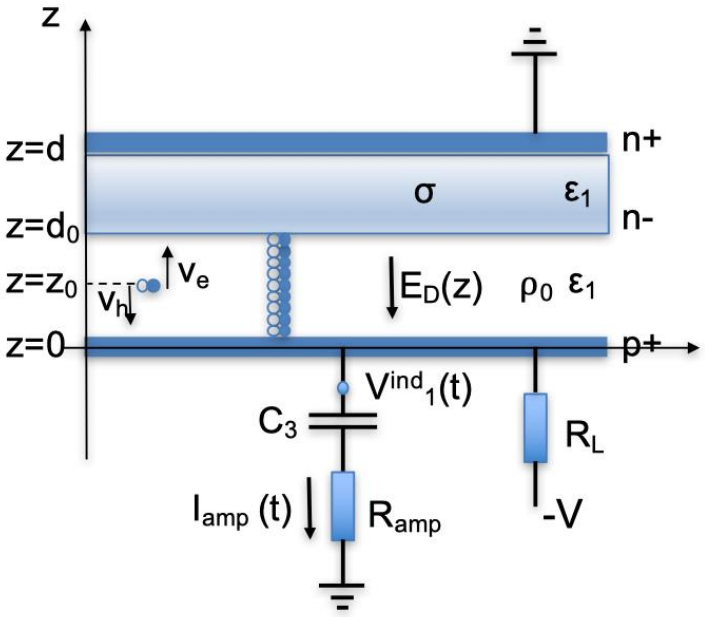
Solution 2:







Example un-depleted silicon sensor



$$V_{dep} = \frac{qN_D d^2}{2\varepsilon_1} \qquad d_0 = d\sqrt{\frac{V}{V_{dep}}} \quad \text{for} \quad 0<V<V_{dep} \tag{20}$$

where q is the elementary charge and $\varepsilon_1 = \varepsilon_r \varepsilon_0$ is the dielectric permittivity of silicon. The static space charge density ρ_0 of the depleted layer and the conductivity σ (the inverse of the volume resistivity) of the un-depleted bulk layer are given by

$$\rho_0 = qN_D = \frac{2V_{dep}\varepsilon_1}{d^2} \qquad \sigma = q\mu_e N_D \tag{21}$$

where μ_e is the electron mobility.

$$E_D(z) = -\frac{2V}{d_0}\left(1 - \frac{z}{d_0}\right) \qquad 0<z<d_0 \tag{22}$$

$$\frac{dz_e(t)}{dt} = -\mu_e E_D(z_e(t)) \qquad \frac{dz_h(t)}{dt} = \mu_h E_D(z_h(t)) \qquad z_e(0) = z_h(0) = z_0 \tag{23}$$

with the solution

$$z_e(t) = d_0 - (d_0 - z_0)e^{-t/\tau_e} \quad \tau_e = \frac{d^2}{2\mu_e V_{dep}} \quad 0 < t < \infty \tag{24}$$

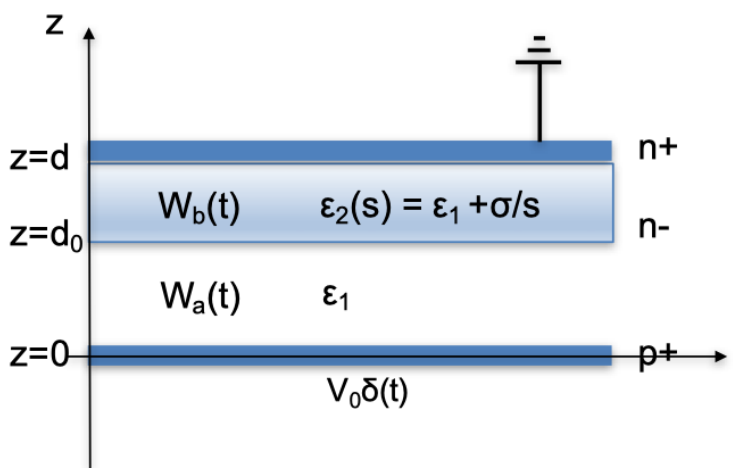
$$z_h(t) = d_0 - (d_0 - z_0)e^{t/\tau_h} \quad \tau_h = \frac{d^2}{2\mu_h V_{dep}} \quad 0 < t < t_h \tag{25}$$

The holes take the time $t_h(z_0) = -\tau_h \ln\left(1 - \frac{z_0}{d_0}\right)$ to arrive at $z = 0$, while the electrons take an infinite amount of time to arrive at $z = d_0$ since the electric field is zero at this position. The related velocities are:

$$v_e(t) = \frac{dz_e(t)}{dt} = \frac{d_0 - z_0}{\tau_e} e^{-t/\tau_e} \tag{26}$$

$$v_h(t) = \frac{dz_h(t)}{dt} = -\frac{d_0 - z_0}{\tau_h} e^{t/\tau_h}\,\Theta(t_h - t) \tag{27}$$

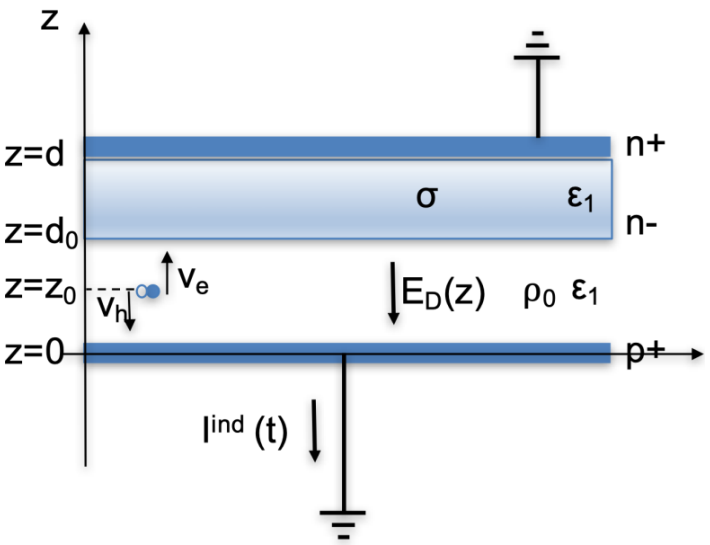
Weighting field and current induced by a single e-h pair



$$W_b(t) = \frac{V_0}{d}\left(\delta(t) - \frac{1}{\tau}e^{-t/\tau}\right) \qquad \tau = \frac{\varepsilon_1 d}{d_0\sigma}$$

$$W_a(t) = \frac{V_0}{d}\left(\delta(t) + \frac{d-d_0}{d_0}\frac{1}{\tau}e^{-t/\tau}\right)$$

$$\begin{aligned} i_e(t) &= \frac{q}{V_0}\int_0^t W_a(t-t')v_e(t')dt' \\ i_h(t) &= -\frac{q}{V_0}\int_0^t W_a(t-t')v_h(t')dt' \quad 0 < t < t_h \\ &= -\frac{q}{V_0}\int_0^{t_h} W_a(t-t')v_h(t')dt' \quad t > t_h \end{aligned}$$

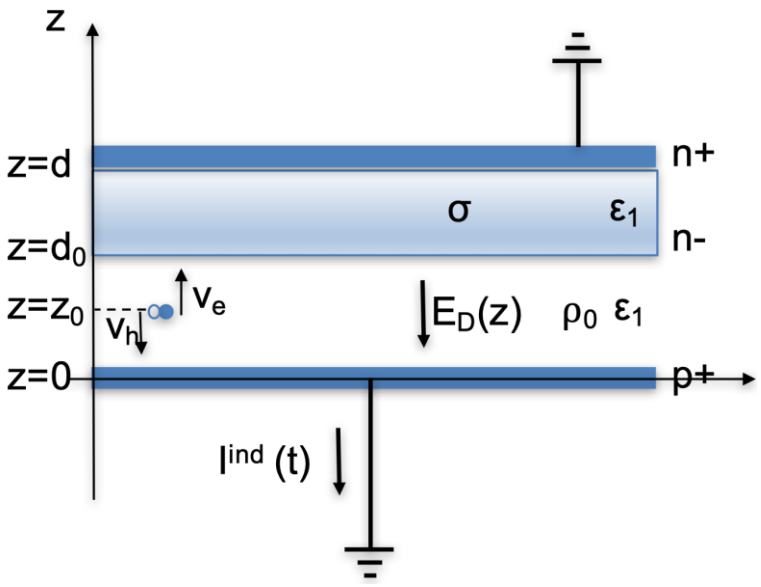


$$\begin{aligned} i_e(t,z_0) &= q\frac{d_0-z_0}{d}\left[\frac{d-d_0}{d_0(\tau-\tau_e)}\left(e^{-t/\tau}-e^{-t/\tau_e}\right)+\frac{1}{\tau_e}e^{-t/\tau_e}\right] \\ i_h(t,z_0) &= q\frac{d_0-z_0}{d}\left[\frac{d-d_0}{d_0(\tau+\tau_h)}\left(e^{t/\tau_h}-e^{-t/\tau}\right)+\frac{1}{\tau_h}e^{t/\tau_h}\right] \qquad t < t_h \\ &= q\frac{(d-d_0)(d_0-z_0)}{d_0 d(\tau+\tau_h)}\left(e^{t_h(z_0)/\tau+t_h(z_0)/\tau_h}-1\right)e^{-t/\tau} \qquad t > t_h \end{aligned}$$

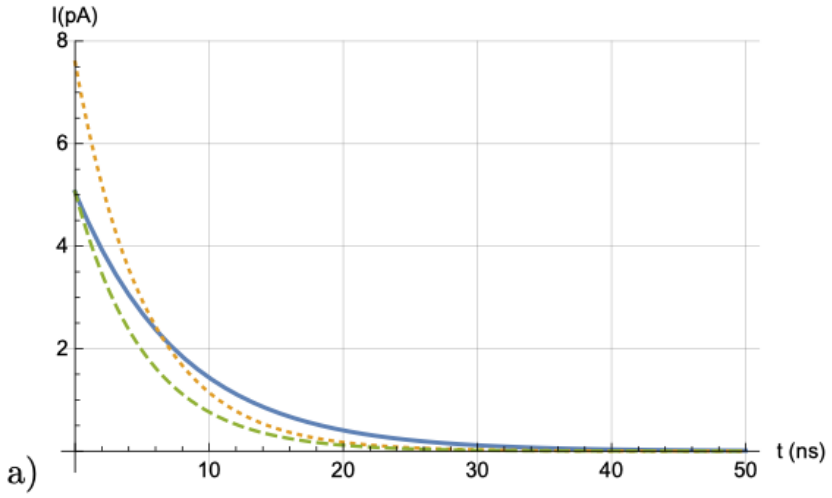
The charge induced by the electrons and the holes is given by

$$Q_e = \int_0^\infty i_e(t)dt = q\left(1-\frac{z_0}{d_0}\right) \qquad Q_h = \int_0^\infty i_h(t)dt = q\frac{z_0}{d_0}$$

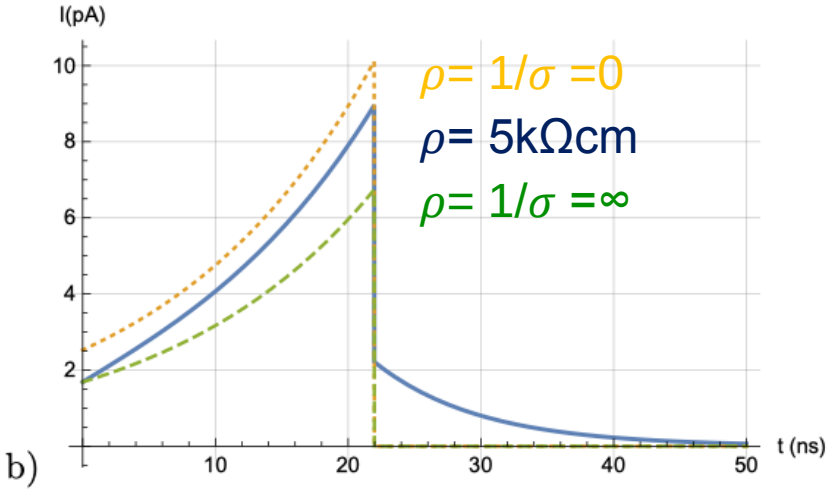
Current induced by a single e-h pair



Electrons



Holes



Electrons+Holes

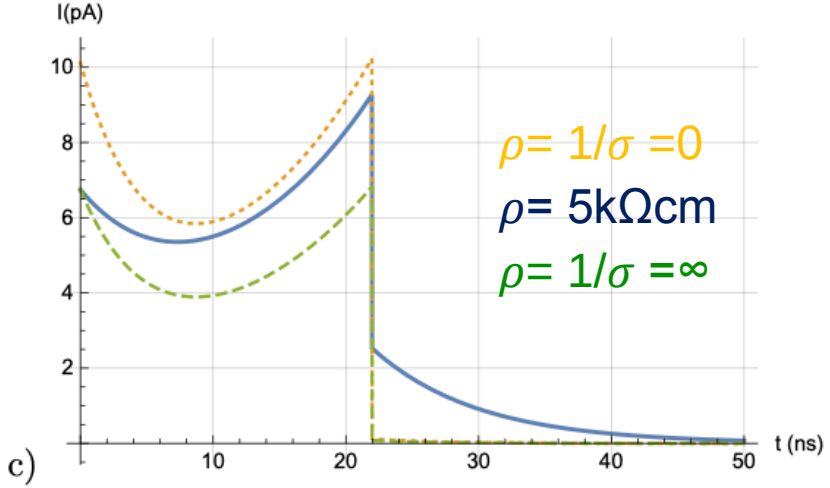


Figure 8: Induced currents from a) a single electron b) a single hole c) an single electron+hole pair, for a sensor of 300 μm thickness with a depletion voltage of 56.8 V. The applied voltage is V=-25.2 V resulting in a depleted region of $d_0 = 200\,\mu$m thickness. The e-h pair is deposited at $z_0 = 150\,\mu$m at $t = 0$. The dotted line assumes zero volume resistivity of the un-depleted layer and the dashed line assumes infinite volume resistivity.

This geometry can correspond to

- An un-depleted silicon sensor, a layer of low resistivity
- An irradiated un-depleted silicon sensor, a layer of high resistivity

Signal propagation in detectors with in long readout electrodes

ANALYSIS OF MULTICONDUCTOR TRANSMISSION LINES

Second Edition

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Mercer University
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Signal propagation, termination, crosstalk and losses in resistive plate chambers

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Signal coupling and signal integrity in multi-strip resistive plate chambers used for timing applications

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Frequency-domain formulation of signal propagation in multistrip Resistive Plate Chambers and its low-loss, weak-coupling analytical approximation

P. Fonte

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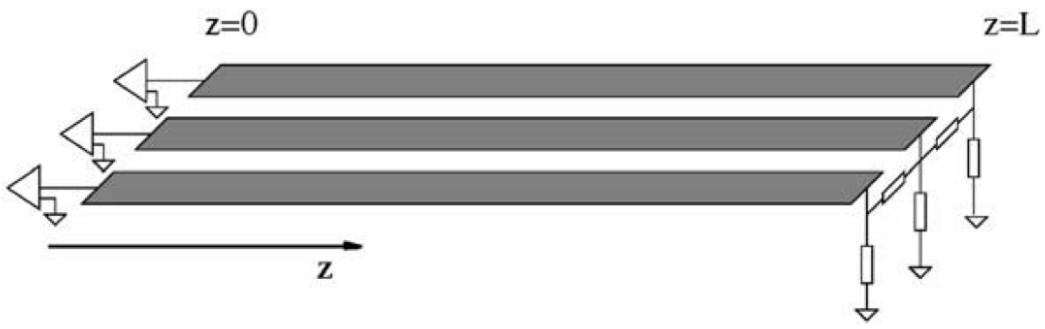
Multi-conductor transmission line

A 2 dimensional transmission line is defined by 4 ’per unit length’ parameter matrixes:

- The capacitance Matrix C
- The inductance Matrix L
- The resistance Matrix R
- The trans-conductance matrix G



The equations that determine the the currents and voltages on the transmission line are defined by the equations

$$\frac{\partial}{\partial z}\mathbf{V}(z,t) = -\hat{\mathbf{R}}\mathbf{I}(z,t) - \hat{\mathbf{L}}\frac{\partial}{\partial t}\mathbf{I}(z,t)$$

$$\frac{\partial}{\partial z}\mathbf{I}(z,t) = -\hat{\mathbf{G}}\mathbf{V}(z,t) - \hat{\mathbf{C}}\frac{\partial}{\partial t}\mathbf{V}(z,t)$$

$$\mathbf{I}(z,t) = \begin{pmatrix} I_1(z,t) \\ \cdot \\ I_N(z,t) \end{pmatrix} \qquad \mathbf{V}(z,t) = \begin{pmatrix} V_1(z,t) \\ \cdot \\ V_N(z,t) \end{pmatrix}$$

Multi-conductor transmission line

In case we can neglect 'losses' we have R=0 and G=0 and the equations become

$$\frac{d^2}{dz^2}\mathbf{V}(z,t) = \hat{\mathbf{L}}\hat{\mathbf{C}}\frac{d^2}{dt^2}\mathbf{V}(z,t)$$

$$\frac{d^2}{dz^2}\mathbf{I}(z,t) = \hat{\mathbf{C}}\hat{\mathbf{L}}\frac{d^2}{dt^2}\mathbf{I}(z,t)$$

With the general solution

$$\mathbf{I}(z,t) = \hat{\mathbf{T}}\left(\begin{pmatrix} I_1^+\left(t-\frac{z}{v_1}\right) \\ \cdot \\ I_N^+\left(t-\frac{z}{v_N}\right)\end{pmatrix} - \begin{pmatrix} I_1^-\left(t+\frac{z}{v_1}\right) \\ \cdot \\ I_N^-\left(t+\frac{z}{v_N}\right)\end{pmatrix}\right)$$

$$\mathbf{V}(z,t) = \hat{\mathbf{Z}}_C\hat{\mathbf{T}}\left(\begin{pmatrix} I_1^+\left(t-\frac{z}{v_1}\right) \\ \cdot \\ I_N^+\left(t-\frac{z}{v_N}\right)\end{pmatrix} + \begin{pmatrix} I_1^-\left(t+\frac{z}{v_1}\right) \\ \cdot \\ I_N^-\left(t+\frac{z}{v_N}\right)\end{pmatrix}\right)$$

$I_m^+(x)$ and $I_m^-(x)$ are 2N arbitrary functions and

$$\hat{\mathbf{T}}^{-1}\left(\hat{\mathbf{C}}\hat{\mathbf{L}}\right)\hat{\mathbf{T}} = \hat{\mathbf{v}}^{-2} \qquad \hat{\mathbf{Z}}_C = \sqrt{\hat{\mathbf{L}}/\hat{\mathbf{C}}} = \hat{\mathbf{L}}\hat{\mathbf{T}}\hat{\mathbf{v}}\hat{\mathbf{T}}^{-1}$$

T contains the normalized eigenvectors of **CL** and $1/v_n^2$ are the corresponding Eigenvectors.

$$\hat{\mathbf{v}}^{-2} = \begin{pmatrix} \frac{1}{v_1^2} & \cdot & 0 \\ \cdot & \cdot & \cdot \\ 0 & \cdot & \frac{1}{v_N^2}\end{pmatrix} \qquad \hat{\mathbf{v}} = \begin{pmatrix} v_1 & \cdot & 0 \\ \cdot & \cdot & \cdot \\ 0 & \cdot & v_N\end{pmatrix}$$

The matrix Z_C is called the characteristic impedance matrix.

Multi-conductor transmission line

A detector signal acts as an ideal current source $I^0(t)$ at a position z_0 somewhere along a conductor n, which defines the 2N functions. This defines the general solution

$$\mathbf{I}(z,t) = \frac{1}{2}\hat{\mathbf{T}}\left(\begin{pmatrix} t_{1n}^{-1} I^0\left(t - \frac{z-z_0}{v_1}\right) \\ \cdot \\ t_{Nn}^{-1} I^0\left(t - \frac{z-z_0}{v_N}\right) \end{pmatrix} - \begin{pmatrix} t_{1n}^{-1} I^0\left(t + \frac{z-z_0}{v_1}\right) \\ \cdot \\ t_{Nn}^{-1} I^0\left(t + \frac{z-z_0}{v_N}\right) \end{pmatrix}\right)$$

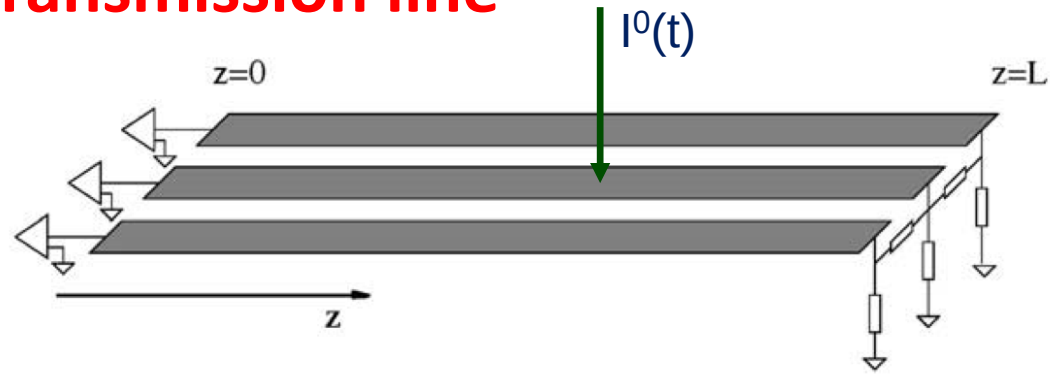
$$\mathbf{V}(z,t) = \frac{1}{2}\hat{\mathbf{Z}}_C\hat{\mathbf{T}}\left(\begin{pmatrix} t_{1n}^{-1} I^0\left(t - \frac{z-z_0}{v_1}\right) \\ \cdot \\ t_{Nn}^{-1} I^0\left(t - \frac{z-z_0}{v_N}\right) \end{pmatrix} + \begin{pmatrix} t_{1n}^{-1} I^0\left(t + \frac{z-z_0}{v_1}\right) \\ \cdot \\ t_{Nn}^{-1} I^0\left(t + \frac{z-z_0}{v_N}\right) \end{pmatrix}\right)$$

At $z=z_0$ we see that the boundary condition is satisfied, so this represents the correct solution. It can be written as

$$\mathbf{I}(z,t) = \mathbf{I}^+(z,t) - \mathbf{I}^-(z,t)$$

$$\mathbf{V}(z,t) = \hat{\mathbf{Z}}_C\left[\mathbf{I}^+(z,t) + \mathbf{I}^-(z,t)\right]$$

$$= \mathbf{V}^+(z,t) + \mathbf{V}^-(z,t).$$

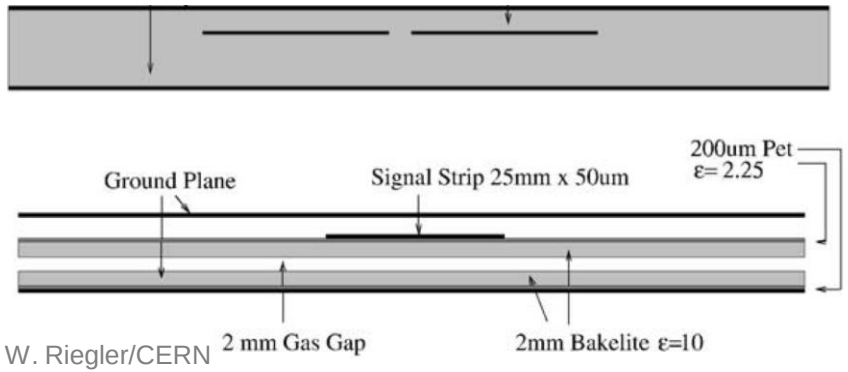


At $z=z_0$ we see that the boundary condition is satisfied.

This solution shows that there are pulses running symmetrically in the positive and negative direction from the point z_0.

The pulse running along one conductor is a superposition of N times the same pulse-shape $I^0(t)$ running with N different velocities v_i.

Therefore we find signal dispersion even for a lossless transmission line, which is called modal dispersion.



Homogeneous transmission line
→ One single velocity

Inhomogeneous transmission line
→ N different velocities

Multi-conductor transmission line

The pulses will travel until they hit the ends of the transmission line where they are reflected according to the connected networks.

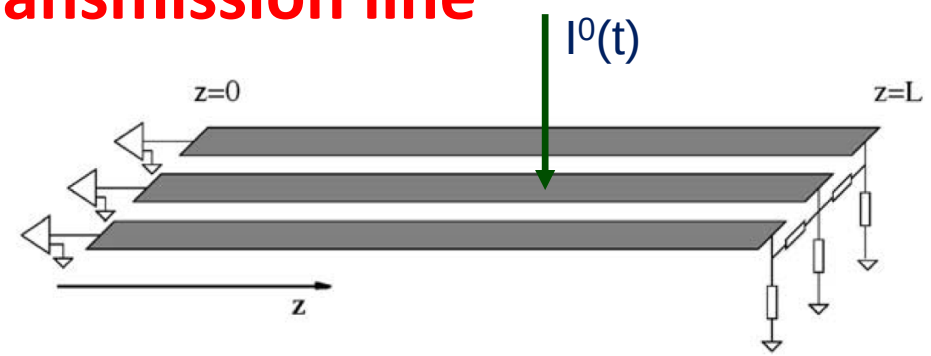
We assume an arbitrary interconnection of strips at z=0 and z=L with purely resistive loads. For z=L we define R_{ij} for the resistor between strip and strip j and R_{ii} for the resistor between strip i and ground.

The boundary condition is then given by

$$\mathbf{V}(L,t) = \hat{\mathbf{Z}}_T \mathbf{I}(L,t) \qquad \hat{\mathbf{Z}}_T = \hat{\mathbf{Y}}_T^{-1}$$

$$Y_{ij}^{\mathrm{T}} = -\frac{1}{R_{ij}} \quad i \neq j \qquad Y_{ii}^{\mathrm{T}} = \sum_{j=1}^{N} \frac{1}{R_{ij}}$$

Here we define Z_T as the 'load impedance' at z=L.
In the same way we define the impedance matrix Z_p on the amplifier side at z=0.



The effect of these boundary conditions is that voltage pulses are reflected according to

$$\mathbf{V}_{\mathrm{refl}}^{-} = \hat{\boldsymbol{\Gamma}}_T \mathbf{V}^{+} \quad \text{and} \quad \mathbf{V}_{\mathrm{refl}}^{+} = \hat{\boldsymbol{\Gamma}}_P \mathbf{V}^{-}$$

Where the reflection coefficient matrices at the line end are defines as

$$\hat{\boldsymbol{\Gamma}}_T = \left(\hat{\mathbf{Z}}_T - \hat{\mathbf{Z}}_C\right)\left(\hat{\mathbf{Z}}_T + \hat{\mathbf{Z}}_C\right)^{-1}$$

$$\hat{\boldsymbol{\Gamma}}_P = \left(\hat{\mathbf{Z}}_P - \hat{\mathbf{Z}}_C\right)\left(\hat{\mathbf{Z}}_P + \hat{\mathbf{Z}}_C\right)^{-1}$$

And the actual voltages at the line ends are given by

$$\mathbf{V}(L,t) = \mathbf{V}^{+} + \mathbf{V}_{\mathrm{refl}}^{-} = \left(\hat{\mathbf{1}} + \hat{\boldsymbol{\Gamma}}_T\right)\mathbf{V}^{+}$$

$$\mathbf{V}(0,t) = \mathbf{V}^{-} + \mathbf{V}_{\mathrm{refl}}^{+} = \left(\hat{\mathbf{1}} + \hat{\boldsymbol{\Gamma}}_P\right)\mathbf{V}^{-}.$$

If the transmission line is not terminated there will of course be multiple reflections.

Multi-conductor transmission line

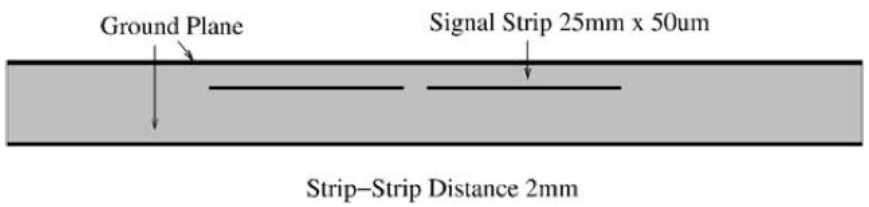
If we want to eliminate reflections at the line end at z=L, the reflection coefficient matrix has to be vanish

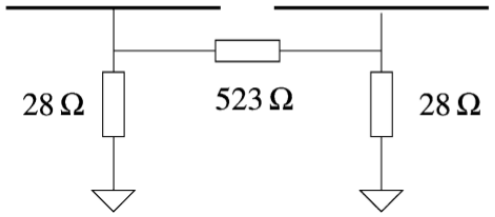
$$\hat{\boldsymbol{\Gamma}}_T = \left(\hat{\mathbf{Z}}_T - \hat{\mathbf{Z}}_C\right)\left(\hat{\mathbf{Z}}_T + \hat{\mathbf{Z}}_C\right)^{-1}$$

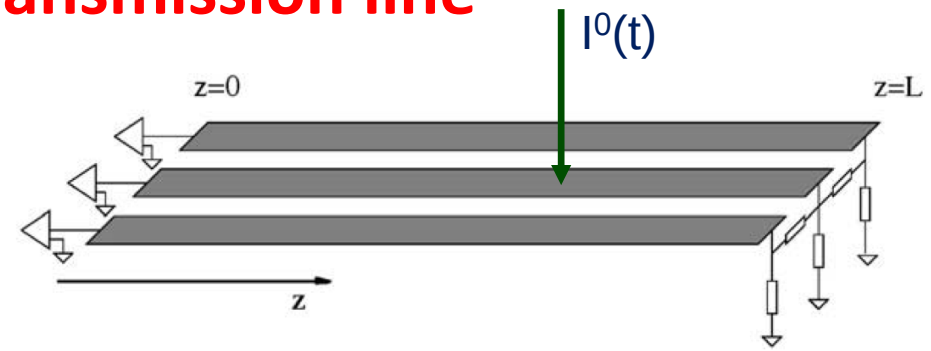
i.e. the impedance matrix Z_T has to be equal to the characteristic impedance matrix Z_C. The termination resistors are the calculated by inverting the relation shown before and we have

$$\hat{\mathbf{Y}}_C = \hat{\mathbf{Z}}_C^{-1} \quad R_{ij}^{\mathrm{T}} = -\frac{1}{Y_{ij}^C} \quad i \neq j \quad \frac{1}{R_{ii}^{\mathrm{T}}} = \sum_{j=0}^{N} Y_{ij}^C .$$

We find that in order to eliminate reflections we have to interconnect all the lines, i.e. we need N(N+1)/2 termination resistors.







Assuming that the transmission line is terminated at z=L and the other end is loaded by the impedance matrix Z_P , the voltage measured by the amplifiers is

$$\mathbf{V}_{\mathrm{meas}}(t) = \mathbf{V}(0,t) = \hat{\mathbf{Z}}_P \hat{\mathbf{Z}}_C \left(\hat{\mathbf{Z}}_P + \hat{\mathbf{Z}}_C\right)^{-1} \hat{\mathbf{T}} \begin{pmatrix} t_{1n}^{-1} I^0\left(t - \frac{z_0}{v_1}\right) \\ \cdot \\ t_{Nn}^{-1} I^0\left(t - \frac{z_0}{v_N}\right) \end{pmatrix}$$

If each strip is connected to an amplifier with input resistance R_{in} and if there is no interconnection we have

$$I_{\mathrm{meas}}(t) = \frac{1}{R_{\mathrm{in}}} \mathbf{V}_{\mathrm{meas}}(t) \neq \mathbf{I}(0,t)$$

Crosstalk

$$\mathbf{V}_{\text{meas}}(t) = \mathbf{V}(0,t) = \hat{\mathbf{Z}}_P \hat{\mathbf{Z}}_C \left(\hat{\mathbf{Z}}_P + \hat{\mathbf{Z}}_C\right)^{-1} \hat{\mathbf{T}} \begin{pmatrix} t_{1n}^{-1} I^0\left(t - \frac{z_0}{v_1}\right) \\ \cdot \\ t_{Nn}^{-1} I^0\left(t - \frac{z_0}{v_N}\right) \end{pmatrix}$$

$$I_{\text{meas}}(t) = \frac{1}{R_{\text{in}}} \mathbf{V}_{\text{meas}}(t) \neq \mathbf{I}(0,t)$$

In case the volume where the electro-magnetic waves are propagating, has uniform dielectric properties, all propagation velocities are equal and and it holds that

$$\hat{\mathbf{L}}\hat{\mathbf{C}} = \frac{1}{v^2}\hat{\mathbf{I}} \qquad \hat{\mathbf{Z}}_C = v\hat{\mathbf{L}}$$

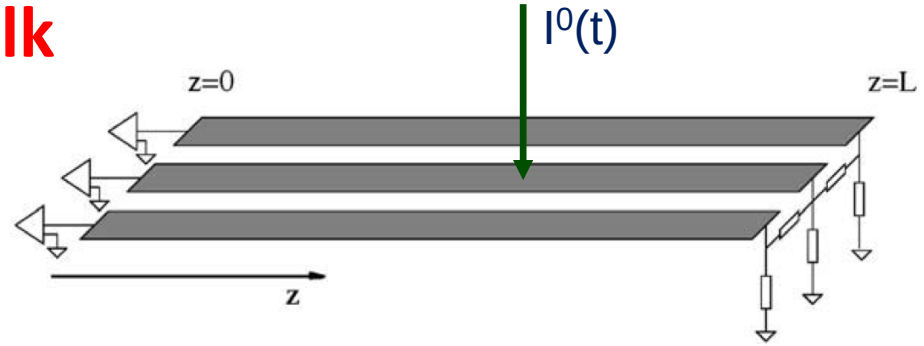
The measured voltages are given by

$$\mathbf{V}_{\text{meas}}(t) = \hat{\mathbf{Z}}_P \hat{\mathbf{Z}}_C \left(\hat{\mathbf{Z}}_P + \hat{\mathbf{Z}}_C\right)^{-1} \times \left(0, ., 0, I^0\left(t - \frac{z_0}{v}\right), 0, ., 0\right)^{\mathrm{T}}$$

Defining the matrix

$$\hat{\mathbf{M}} = \hat{\mathbf{Z}}_P \hat{\mathbf{Z}}_C \left(\hat{\mathbf{Z}}_P + \hat{\mathbf{Z}}_C\right)^{-1}.$$

The crosstalk from conductor m to conductor n is M_{mn}/M_{nn}.



In case we would interconnect the readout side such that there is no reflection, we have $Z_P=Z_C$ and the crosstalk is Z^C_{mn}/Z^C_{nn}.

To have small crosstalk, the off-diagonal elements of the impedance matrix i.e. the coupling between the electrodes, should therefore be small.

Terminating on the preamplifier side is however NOT the optimum in terms of collected charge and crosstalk.

If we do not interconnect the strips and connect each line to an amplifier with input resistance R_{in} we have

$$\hat{\mathbf{Z}}_P = \mathrm{Diag}(R_{\text{in}},\ R_{\text{in}},\ \ldots,\ R_{\text{in}})$$

And for $R_{in}=0$ we have

$$I_{\text{meas}}(t) = \left(0, 0, \ldots, I^0\left(t - \frac{z_0}{v}\right), \ldots, 0\right)^{\mathrm{T}}$$

So we measure exactly the pulse induced on line n and zero on all other line !

Crosstalk

In case the transmission line is long and inhomogeneous there will be dispersion and the crosstalk will increase. If the amplifier on the readout side is measuring the signal charge, i.e. if the amplifier peaking time is larger than the signal time and the dispersion time, the n expressions

$$\int I\left(t-(z-z_0)/v_n\right)\,\mathrm{d}t$$

will evaluate to the same value

$$q=\int I(t)\,\mathrm{d}t$$

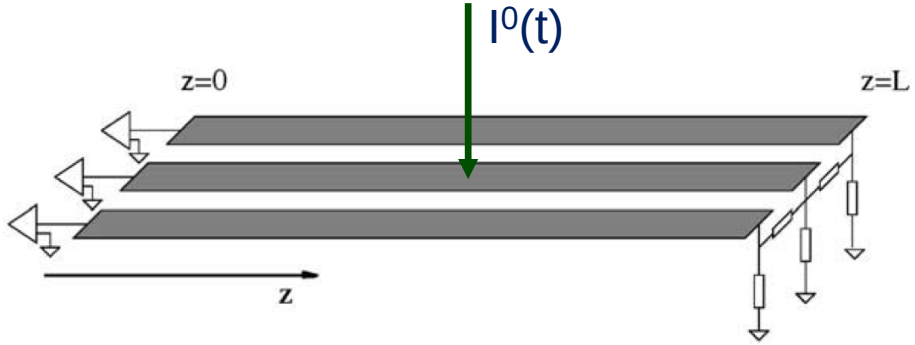
and the charge measured on each strips is

$$\mathbf{Q}=\int \mathbf{I}_{\mathrm{meas}}(t)\,\mathrm{d}t$$

$$=\frac{1}{R_{\mathrm{in}}}\hat{\mathbf{Z}}_P\hat{\mathbf{Z}}_C\left(\hat{\mathbf{Z}}_P+\hat{\mathbf{Z}}_C\right)^{-1}(0,.,0,q,0,.,0)^{\mathrm{T}}$$

And the crosstalk charge is as before given by M_{mn}/M_{nn}

For R_{in}=0 the 'crosstalk charge' is zero and the crosstalk signals are therefore strictly bipolar.

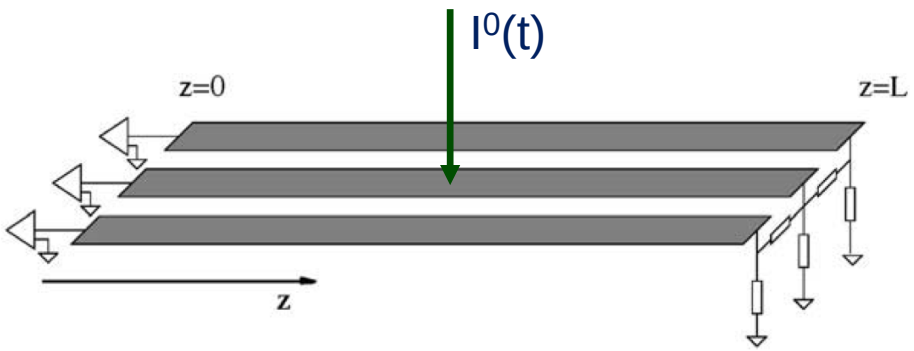


For a 'clean' signal and minimum crosstalk we want to

- terminate the transmission line on the 'far' end with an appropriate set of N(N+1)/2 resistors
- perform the strip readout with charge sensitive amplifiers that have minimum input resistance

Reflections

It will of course not be practical to place N(N+1)/2 resistors on the far end of a transmission line, so one has to approximate the situation with a given termination network



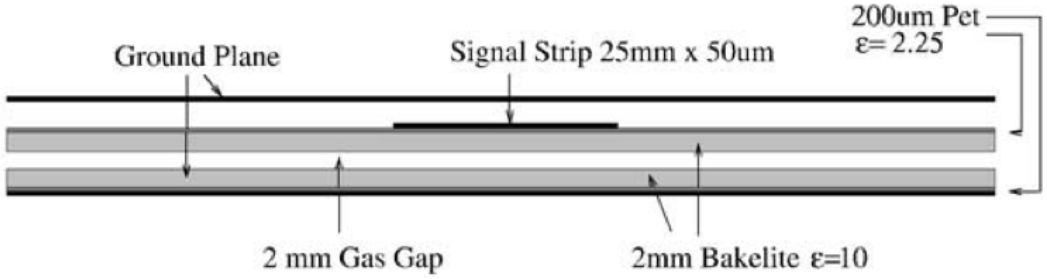
The measured signal is

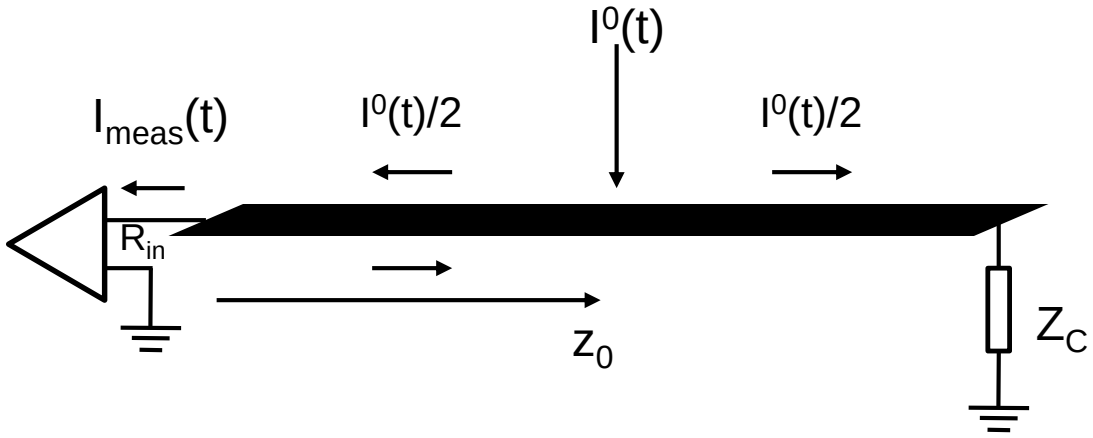
$$\mathbf{V}_{\text{meas}}(t) = \mathbf{V}(0,t) = \hat{\mathbf{Z}}_P \hat{\mathbf{Z}}_C \left(\hat{\mathbf{Z}}_P + \hat{\mathbf{Z}}_C\right)^{-1} \hat{\mathbf{T}} \begin{pmatrix} t_{1n}^{-1} I^0\left(t - \frac{z_0}{v_1}\right) \\ \cdot \\ t_{Nn}^{-1} I^0\left(t - \frac{z_0}{v_N}\right) \end{pmatrix}$$

And the reflected measured signal amounts to

$$\begin{aligned} \mathbf{V}_{ref}(0,t) &= (1 + \hat{\mathbf{\Gamma}}_P)\hat{\mathbf{\Gamma}}_T \frac{1}{2} \hat{\mathbf{Z}}_C \hat{\mathbf{T}} \begin{pmatrix} t_{1n}^{-1} I_0 \left(1 - \frac{2L - z_0}{v_1}\right) \\ \cdot \\ t_{Nn}^{-1} I_0 \left(1 - \frac{2L - z_0}{v_N}\right) \end{pmatrix} \\ &= \hat{\mathbf{Z}}_P (\hat{\mathbf{Z}}_P + \hat{\mathbf{Z}}_C)^{-1} (\hat{\mathbf{Z}}_T + \hat{\mathbf{Z}}_C)(\hat{\mathbf{Z}}_T + \hat{\mathbf{Z}}_C)^{-1} \hat{\mathbf{Z}}_C \hat{\mathbf{T}} \begin{pmatrix} t_{1n}^{-1} I_0 \left(1 - \frac{2L - z_0}{v_1}\right) \\ \cdot \\ t_{Nn}^{-1} I_0 \left(1 - \frac{2L - z_0}{v_N}\right) \end{pmatrix} \end{aligned}$$

Examples, single strip line





An 'inhomogeneous' transmission line with a single strip, representing a Resistive Plate Chamber.

Placing a current $I^0(t)$ at position $z=z_0$, one half of the pulse is moving towards the termination side and is absorbed.

The other half is moving to the amplifier side, is reflected and this reflection is also absorbed on the termination side.

The amount of current measured by the amplifier is defined by the input resistance of the amplifier.

$$C = 205\,\mathrm{pF/m} \quad L = 89.3\,\mathrm{nH/m} \Rightarrow Z_C = 20.87\,\Omega$$

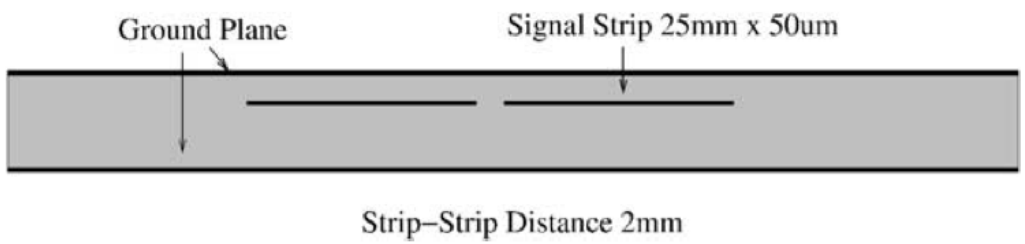
$$v = 2.34\times10^8\,\mathrm{m/s}$$

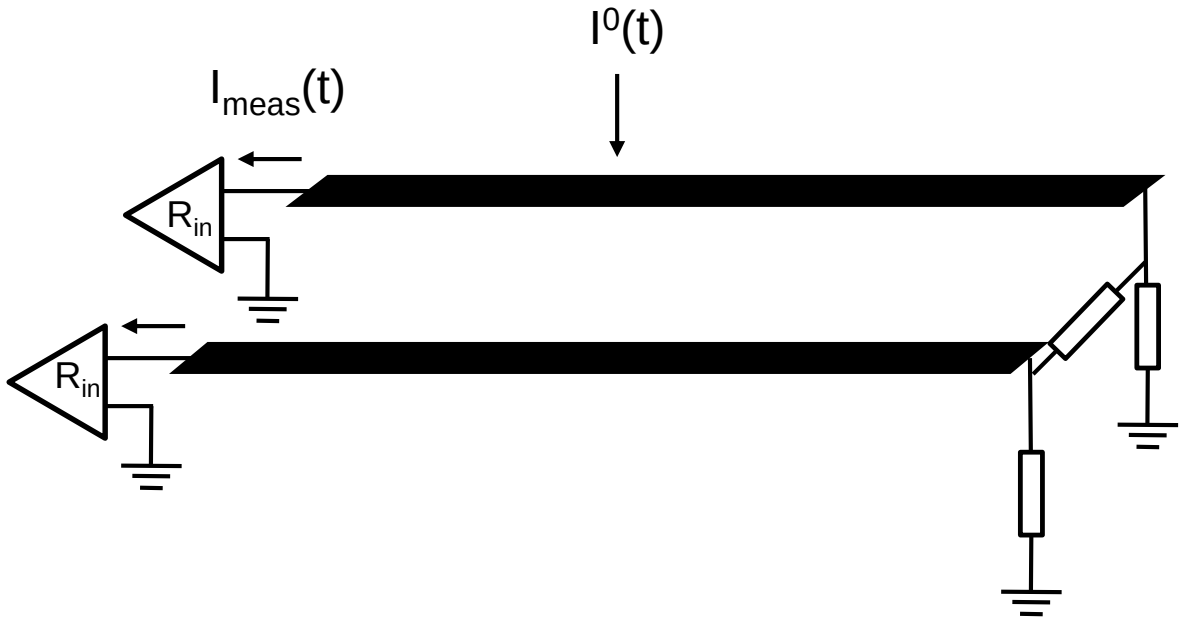
$$I_{\mathrm{meas}}(t) = \frac{Z_C}{Z_C + R_{\mathrm{in}}} I^0\left(t - \frac{z_0}{v}\right)$$

For $R_{in} = Z_C$ one measures only half the induced current and there is no reflection on the amplifier side.

For $R_{in} = 0$ one measures the full current, the pulse is negatively reflected at the amplifier side and then absorbed on the termination side.

Examples, homogeneous double single strip





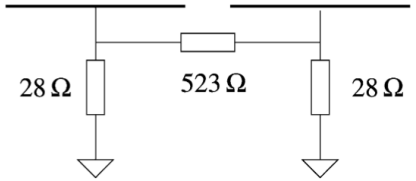
An 'inhomogeneous' transmission line with a single strip, representing a Resistive Plate Chamber. The capacitance and impedance matrix are given by

$$\hat{\mathbf{C}} = \begin{pmatrix} 126 & -6.4 \\ -6.4 & 126 \end{pmatrix} \mathrm{pF/m} \qquad \hat{\mathbf{L}} = \begin{pmatrix} 88.4 & 4.47 \\ 4.47 & 88.4 \end{pmatrix} \mathrm{nH/m}$$

The resulting characteristic impedance matrix, termination resistors and propagation velocity are

$$\hat{\mathbf{Z}}_C = \begin{pmatrix} 26.5 & 1.34 \\ 1.34 & 26.5 \end{pmatrix} \Omega \qquad \hat{\mathbf{R}}_T = \begin{pmatrix} 27.8 & 522.7 \\ 522.7 & 27.8 \end{pmatrix} \Omega \qquad v = 3\times10^8\ \mathrm{m/s}$$

There is a single propagation velocity and one needs 3 termination resistors.



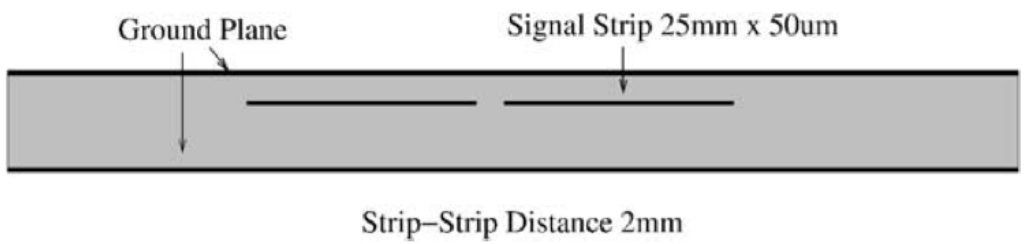
The signals on the two strip lines are given by

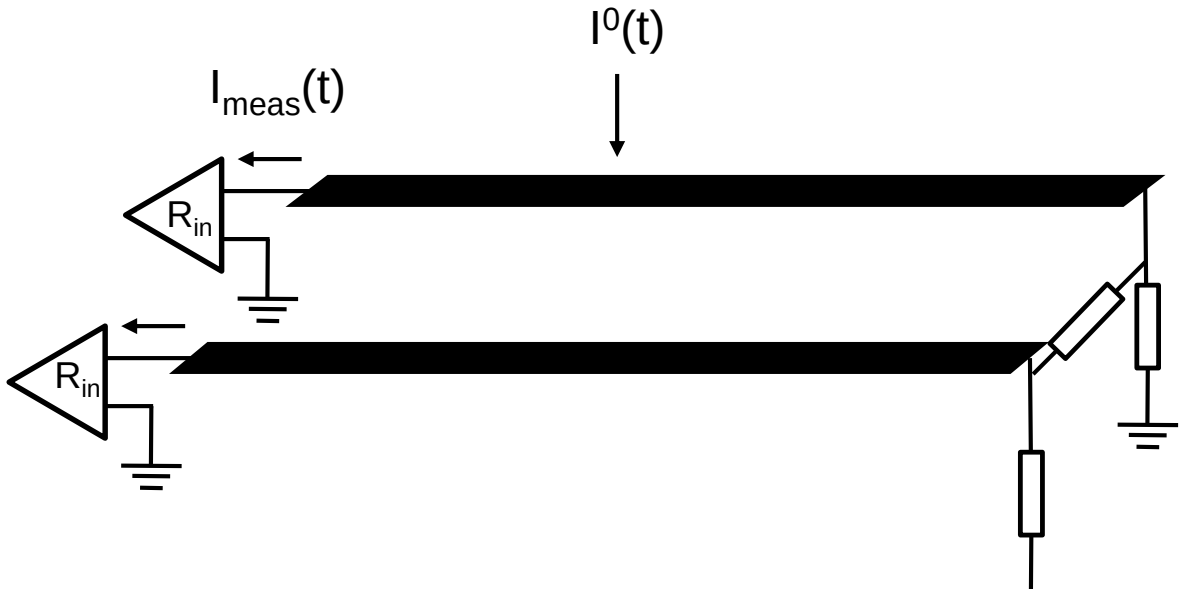
$$\begin{pmatrix} I^1_{\mathrm{meas}}(t) \\ I^2_{\mathrm{meas}}(t) \end{pmatrix} = \frac{1}{R^2_{\mathrm{in}} + 2R_{\mathrm{in}}Z_{11} + Z^2_{11} - Z^2_{12}} \times \begin{pmatrix} R_{\mathrm{in}}Z_{11} + Z^2_{11} - Z^2_{12} \\ R_{\mathrm{in}}Z_{12} \end{pmatrix} I^0\left(t - \frac{z_0}{c}\right)$$

And the crosstalk is therefore

$$\frac{I^2_{\mathrm{meas}}(t)}{I^1_{\mathrm{meas}}(t)} = \frac{R_{\mathrm{in}}Z_{12}}{R_{\mathrm{in}}Z_{11} + Z^2_{11} - Z^2_{12}}$$

Examples, homogeneous double single strip

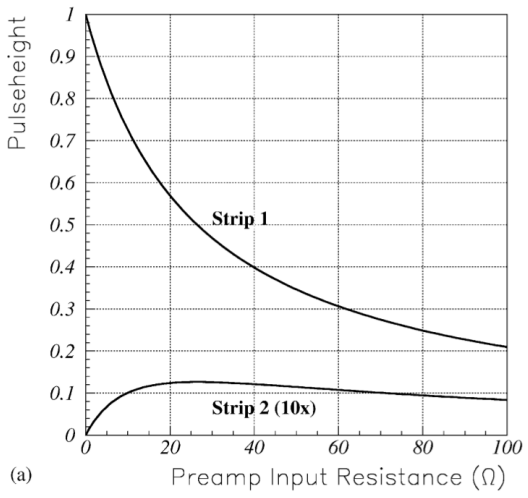


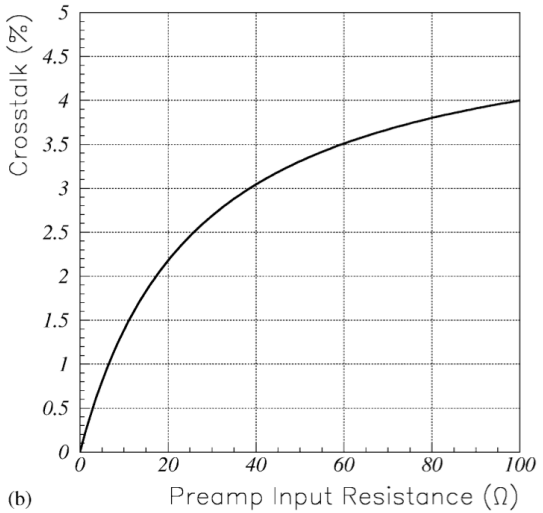


$$\hat{\mathbf{Z}}_C = \begin{pmatrix} 26.5 & 1.34 \\ 1.34 & 26.5 \end{pmatrix} \Omega \quad \hat{\mathbf{R}}_T = \begin{pmatrix} 27.8 & 522.7 \\ 522.7 & 27.8 \end{pmatrix} \Omega$$

$$\begin{pmatrix} I^1_{\text{meas}}(t) \\ I^2_{\text{meas}}(t) \end{pmatrix} = \frac{1}{R^2_{\text{in}} + 2R_{\text{in}}Z_{11} + Z^2_{11} - Z^2_{12}} \times \begin{pmatrix} R_{\text{in}}Z_{11} + Z^2_{11} - Z^2_{12} \\ R_{\text{in}}Z_{12} \end{pmatrix} I^0\left(t - \frac{z_0}{c}\right)$$

$$\frac{I^2_{\text{meas}}(t)}{I^1_{\text{meas}}(t)} = \frac{R_{\text{in}}Z_{12}}{R_{\text{in}}Z_{11} + Z^2_{11} - Z^2_{12}}$$





The measured signal decreases with the input resistance, 50% at $R_{in} \approx Z_{11} \approx R_{11}$

The crosstalk increases with the input resistance.

Examples, wire chamber

For a homogeneous transmission line it holds in general that there is only a single propagation velocity, so knowing the capacitance matrix and the propagation velocity we find all other parameters of the transmission line

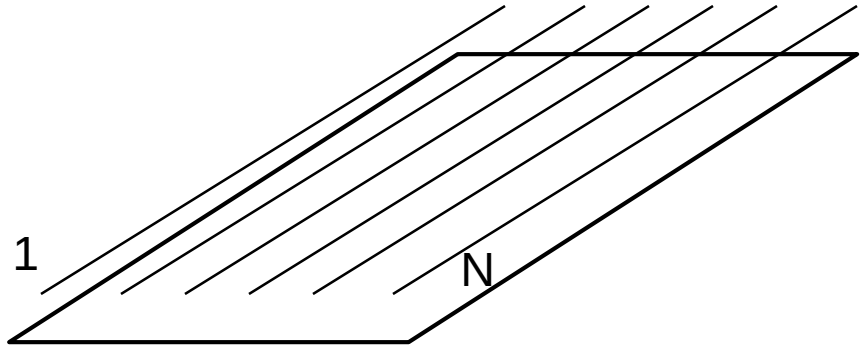
$$\hat{\mathbf{L}}\hat{\mathbf{C}} = \frac{1}{v^2}\hat{\mathbf{1}} \qquad \hat{\mathbf{Z}}_{\mathbf{C}} = \sqrt{\hat{\mathbf{L}}\hat{\mathbf{C}}^{-1}} = \frac{1}{v}\hat{\mathbf{C}}^{-1}$$

$$\hat{\mathbf{Y}}_{\mathbf{C}} = \hat{\mathbf{Z}}_{\mathbf{C}}^{-1} = v\hat{\mathbf{C}}$$

$$R_{ij}^T = -\frac{1}{Y_{ij}^C} = -\frac{1}{vc_{ij}} = \frac{1}{vC_{ij}}$$

$$R_{ii}^T = \frac{1}{\sum_j Y_{ij}^C} = \frac{1}{v\sum_j c_{ij}} = \frac{1}{vC_{ii}}$$

As an example we assume a plane of wires with r_a=15um radius, y_0 =3mm above a ground plane, wire separation of s=3mm.



$$\varphi(x,y) = \frac{1}{4\pi\varepsilon_0}\sum_{m=1}^{N}\lambda_m \ln\frac{(x-ms)^2+(y-y_0)^2}{(x-ms)^2+(y+y_0)^2}$$

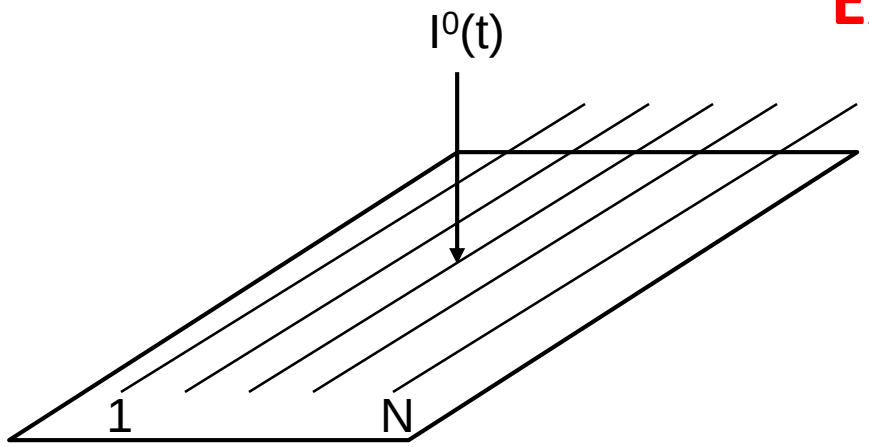
$$V_n = \varphi(ns+r_a, y_0) = \frac{1}{4\pi\varepsilon_0}\sum_{m=1}^{N}\lambda_m \ln\frac{(ns+r_a-ms)^2}{(ns+r_a-ms)^2+4y_0^2} = \sum_{m=1}^{N} a_{nm}\lambda_m$$

$$\hat{A} = a_{mn} = \frac{1}{4\pi\varepsilon_0}\ln\frac{(ns+r_a-ms)^2}{(ns+r_a-ms)^2+4y_0^2}$$

$$\lambda_n = \sum_{m=1}^{N} c_{nm}V_m \qquad \hat{C} = \hat{A}^{-1} \qquad \hat{Z}_C = \frac{1}{v}\hat{A}$$

$$\hat{\mathbf{C}} = \begin{pmatrix} 9.46 & -1.2 & -0.36 & -0.16 & -0.1 \\ -1.22 & 9.61 & -1.16 & -0.34 & -0.16 \\ -0.36 & -1.17 & 9.63 & -1.16 & -0.36 \\ -0.16 & -0.34 & -1.17 & 9.61 & -1.2 \\ -0.1 & -0.16 & -0.36 & -1.22 & 9.46 \end{pmatrix} \mathrm{pF/m}$$

Examples, wire chamber



As an example we assume a plane of wires with 15um radius, 3mm above a ground plane, wire separation 3mm.

$$\hat{\mathbf{Z}}_C = \begin{pmatrix} 359.6 & 48.1 & 20.7 & 11. & 6.7 \\ 48.5 & 359.6 & 48.1 & 20.7 & 11. \\ 20.9 & 48.5 & 359.6 & 48.1 & 20.7 \\ 11.1 & 20.9 & 48.5 & 359.6 & 48.1 \\ 6.7 & 11.1 & 20.9 & 48.5 & 359.6 \end{pmatrix} \Omega$$

Formulas for homogeneous transmission line

$$\hat{\mathbf{L}}\hat{\mathbf{C}} = \frac{1}{v^2}\hat{\mathbf{1}} \qquad \hat{\mathbf{Z}}_\mathbf{C} = \sqrt{\hat{\mathbf{L}}\hat{\mathbf{C}}^{-1}} = \frac{1}{v}\hat{\mathbf{C}}^{-1}$$

$$\hat{\mathbf{Y}}_\mathbf{C} = \hat{\mathbf{Z}}_\mathbf{C}^{-1} = v\hat{\mathbf{C}}$$

$$R_{ij}^T = -\frac{1}{Y_{ij}^C} = -\frac{1}{vc_{ij}} = \frac{1}{vC_{ij}}$$

$$R_{ii}^T = \frac{1}{\sum_j Y_{ij}^C} = \frac{1}{v\sum_j c_{ij}} = \frac{1}{vC_{ii}}$$

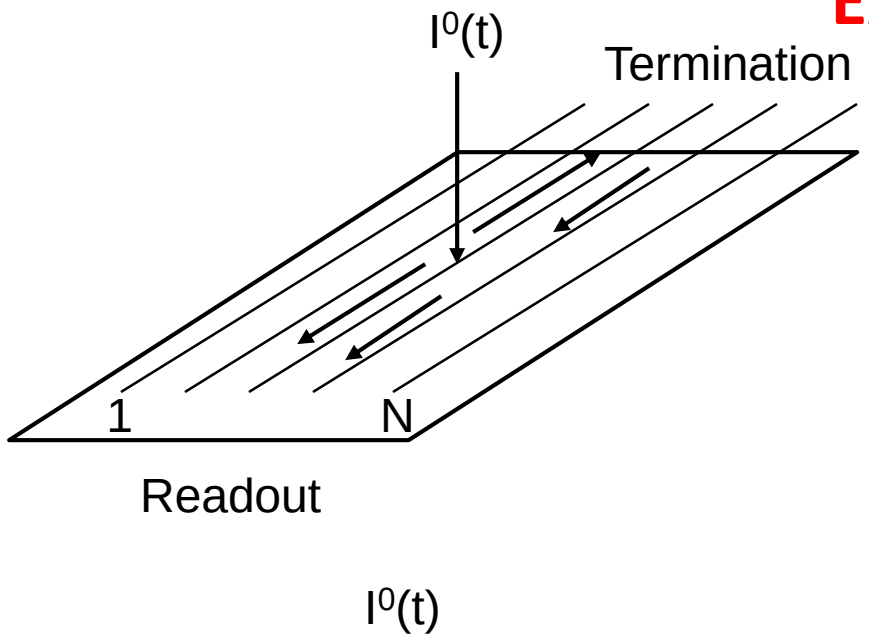
$$\hat{\mathbf{C}} = \begin{pmatrix} 9.46 & -1.2 & -0.36 & -0.16 & -0.1 \\ -1.22 & 9.61 & -1.16 & -0.34 & -0.16 \\ -0.36 & -1.17 & 9.63 & -1.16 & -0.36 \\ -0.16 & -0.34 & -1.17 & 9.61 & -1.2 \\ -0.1 & -0.16 & -0.36 & -1.22 & 9.46 \end{pmatrix} \mathrm{pF/m}$$

$$\hat{\mathbf{R}}_\mathbf{T} = \begin{pmatrix} 436.1 & 2768.8 & 9330.9 & 20928.2 & 34343.9 \\ 2739. & 494.7 & 2874.7 & 9791.9 & 20928.2 \\ 9308.8 & 2841.6 & 506.8 & 2874.7 & 9330.9 \\ 20938.1 & 9761.4 & 2841.6 & 494.8 & 2768.8 \\ 34422. & 20938.1 & 9308.8 & 2739. & 436.9 \end{pmatrix} \Omega$$

Let's assume we use very low input impedance on the preamp side, so there is negligible crosstalk.

In case we have imperfect termination on the other side there will be reflections that result in 'delayed' crosstalk.

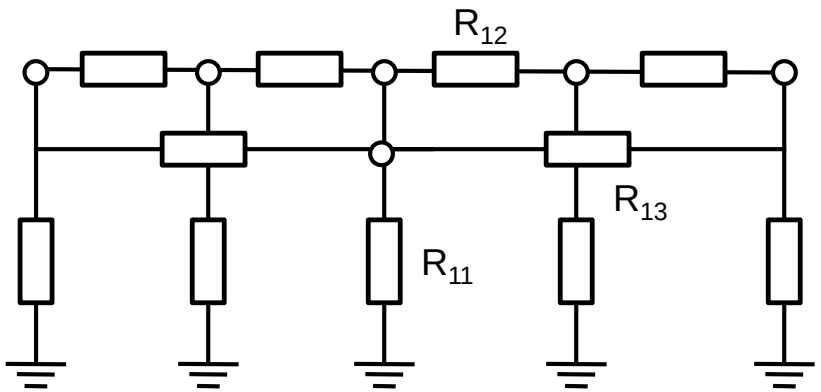
Examples, wire chamber



$$\hat{\mathbf{Z}}_C = \begin{pmatrix} 359.6 & 48.1 & 20.7 & 11. & 6.7 \\ 48.5 & 359.6 & 48.1 & 20.7 & 11. \\ 20.9 & 48.5 & 359.6 & 48.1 & 20.7 \\ 11.1 & 20.9 & 48.5 & 359.6 & 48.1 \\ 6.7 & 11.1 & 20.9 & 48.5 & 359.6 \end{pmatrix} \Omega$$

$$\hat{\mathbf{R}}_{\mathbf{T}} = \begin{pmatrix} 436.1 & 2768.8 & 9330.9 & 20928.2 & 34343.9 \\ 2739. & 494.7 & 2874.7 & 9791.9 & 20928.2 \\ 9308.8 & 2841.6 & 506.8 & 2874.7 & 9330.9 \\ 20938.1 & 9761.4 & 2841.6 & 494.8 & 2768.8 \\ 34422. & 20938.1 & 9308.8 & 2739. & 436.9 \end{pmatrix} \Omega$$

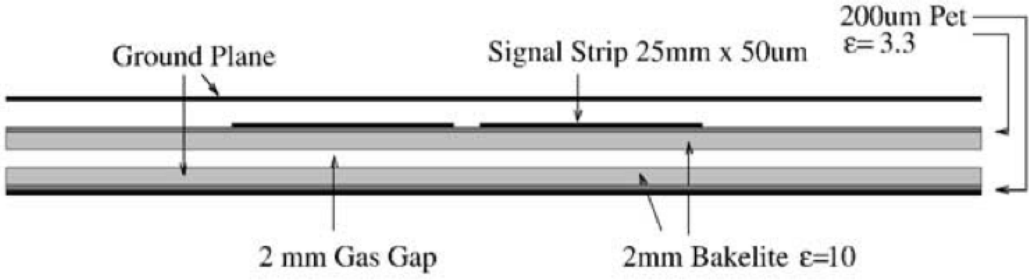
Termination Network

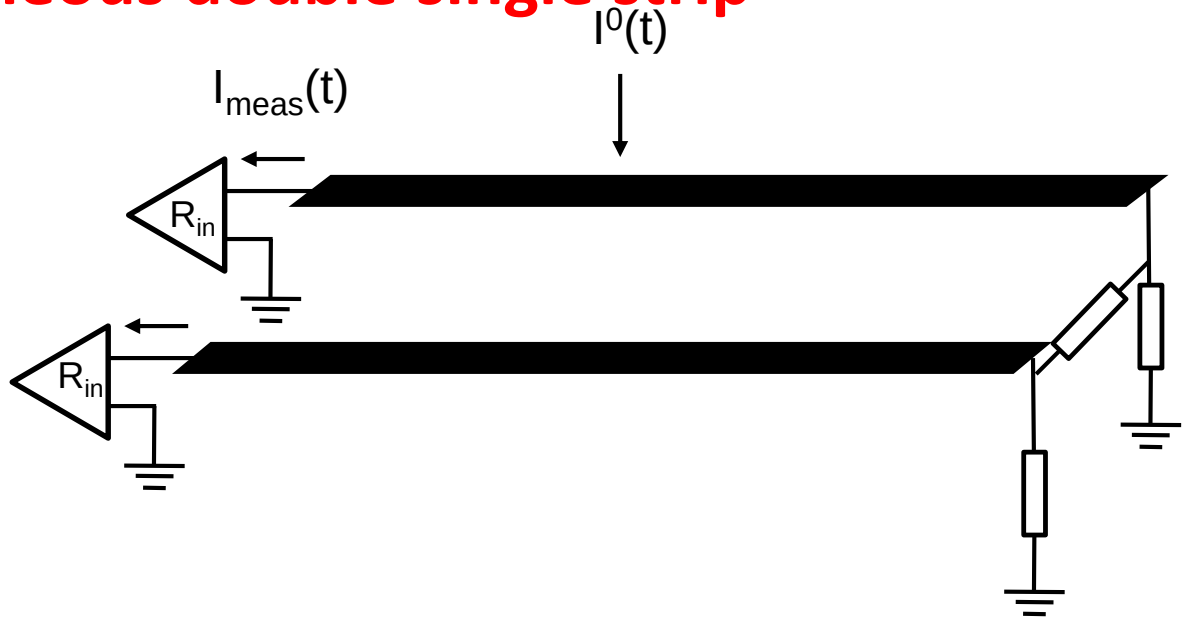


Fraction of measured reflected signal for imperfect termination:

R_{11} Ω	R_{12} Ω	R_{12} Ω	Cr11 %	Cr12 %	Cr13 %
506.8	∞	∞	17.8	-6.2	-2.4
359.6	∞	∞	1.0	-6.2	-2.4
352.4	∞	∞	0	-6.2	-2.4
506.8	2875	∞	3.8	0.3	-2.0
270	2800	∞	0.4	0	-2

Examples, inhomogeneous double single strip





The 'per unit length' parameters are

$$\hat{\mathbf{C}} = \begin{pmatrix} 216 & -30 \\ -30 & 216 \end{pmatrix} \mathrm{pF/cm} \qquad \hat{\mathbf{L}} = \begin{pmatrix} 88.4 & 4.47 \\ 4.47 & 88.4 \end{pmatrix} \mathrm{nH/m}$$

The resulting characteristic impedance matrix, termination resistors and propagation velocities are

$$\hat{\mathbf{Z}}_C = \begin{pmatrix} 20.4 & 1.93 \\ 1.93 & 20.4 \end{pmatrix} \Omega \qquad \hat{\mathbf{T}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

$$\hat{\mathbf{R}}_T = \begin{pmatrix} 22.3 & 213.7 \\ 213.7 & 22.3 \end{pmatrix} \Omega \qquad \hat{\mathbf{v}} = \begin{pmatrix} 2.2 & 0 \\ 0 & 2.4 \end{pmatrix} \times 10^8 \,\mathrm{m/s}$$

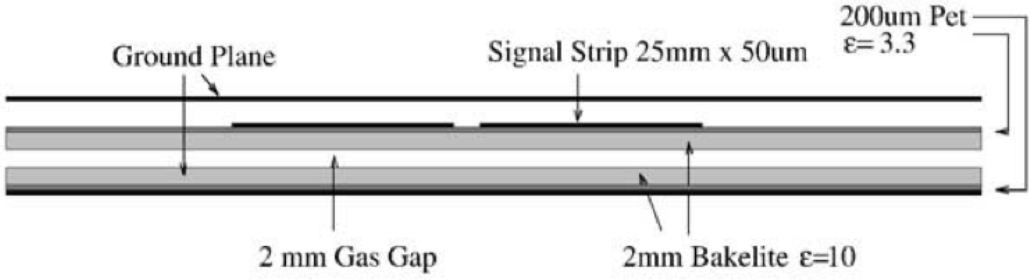
We see that the measured signals are a linear superposition of two pulses propagating with different velocities.

The measured pulse-height and crosstalk will therefore depend on the the distance of the induced current from the end of the transmission line.

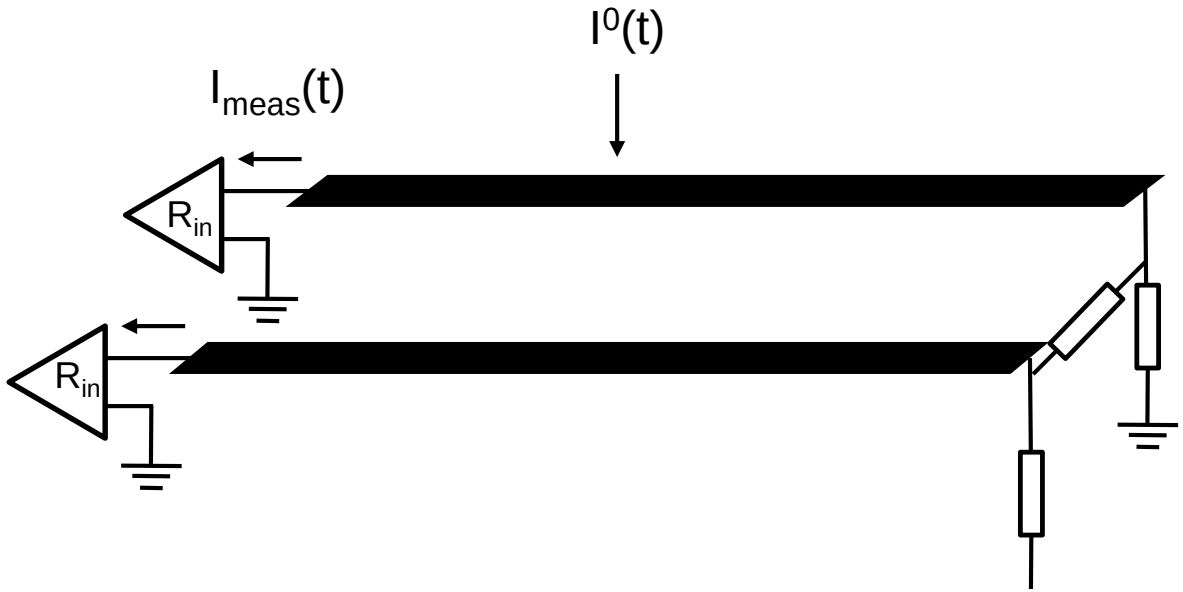
And the resulting measured signals are

$$\begin{pmatrix} I^1_{\mathrm{meas}}(t) \\ I^2_{\mathrm{meas}}(t) \end{pmatrix} = \frac{1}{2} c_1 \times \begin{pmatrix} R_{\mathrm{in}} Z_{11} + Z_{11}^2 - Z_{12}^2 & R_{\mathrm{in}} Z_{12} \\ R_{\mathrm{in}} Z_{12} & R_{\mathrm{in}} Z_{11} + Z_{11}^2 - Z_{12}^2 \end{pmatrix} \times \begin{pmatrix} I^0\left(t - \frac{z_0}{v_1}\right) + I^0\left(t - \frac{z_0}{v_2}\right) \\ -I^0\left(t - \frac{z_0}{v_1}\right) + I^0\left(t - \frac{z_0}{v_2}\right) \end{pmatrix} \qquad c_1 = \frac{1}{R_{\mathrm{in}}^2 + 2R_{\mathrm{in}} Z_{11} + Z_{11}^2 - Z_{12}^2}$$

Examples, inhomogeneous double single strip



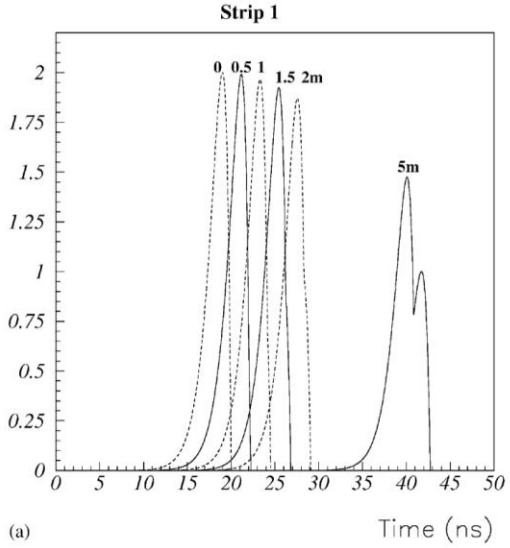
$$\hat{\mathbf{R}}_T = \begin{pmatrix} 22.3 & 213.7 \\ 213.7 & 22.3 \end{pmatrix} \Omega \qquad \hat{\mathbf{v}} = \begin{pmatrix} 2.2 & 0 \\ 0 & 2.4 \end{pmatrix} \times 10^8 \text{ m/s}$$

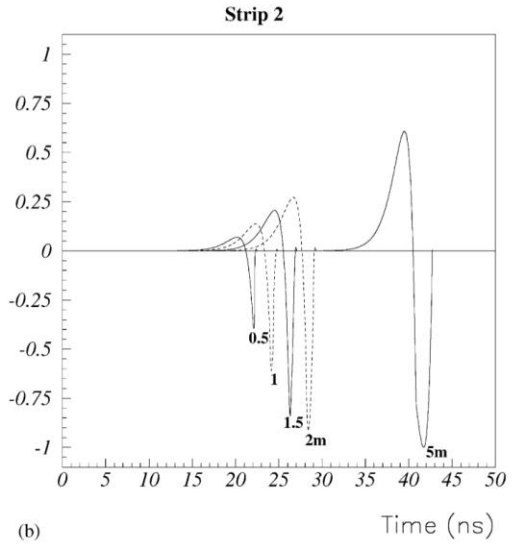


The further the position of the induced signal is from the amplifier side, the more the signal will disperse.

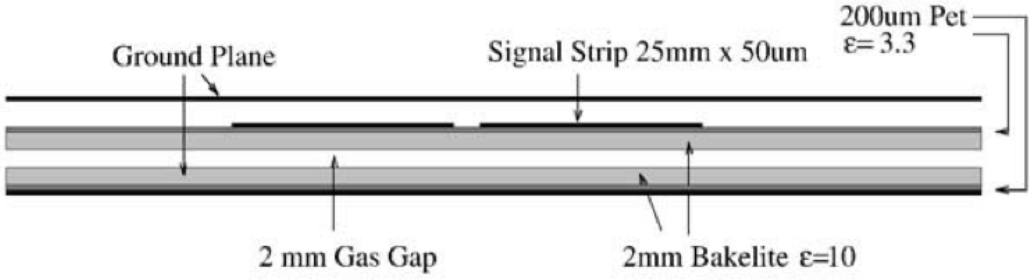
The crosstalk (assuming R_{in}=0) is zero if the pulse is induced close to the amplifier, but it increases as a function of distance.

The crosstalk signals are strictly bipolar for the case of R_{in}=0.

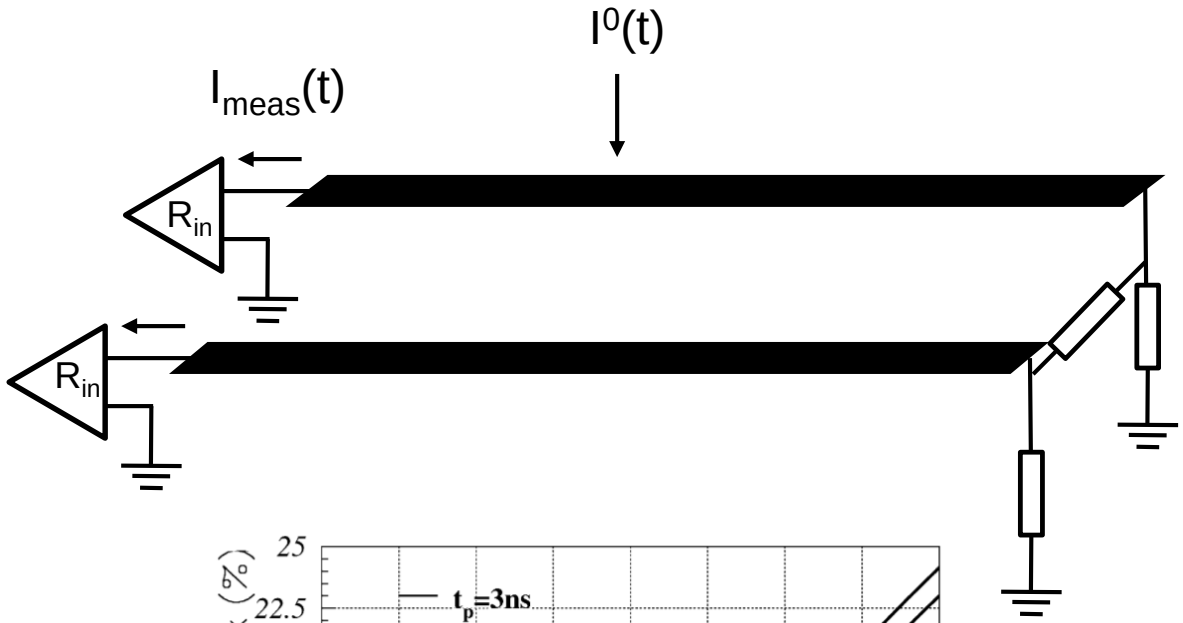


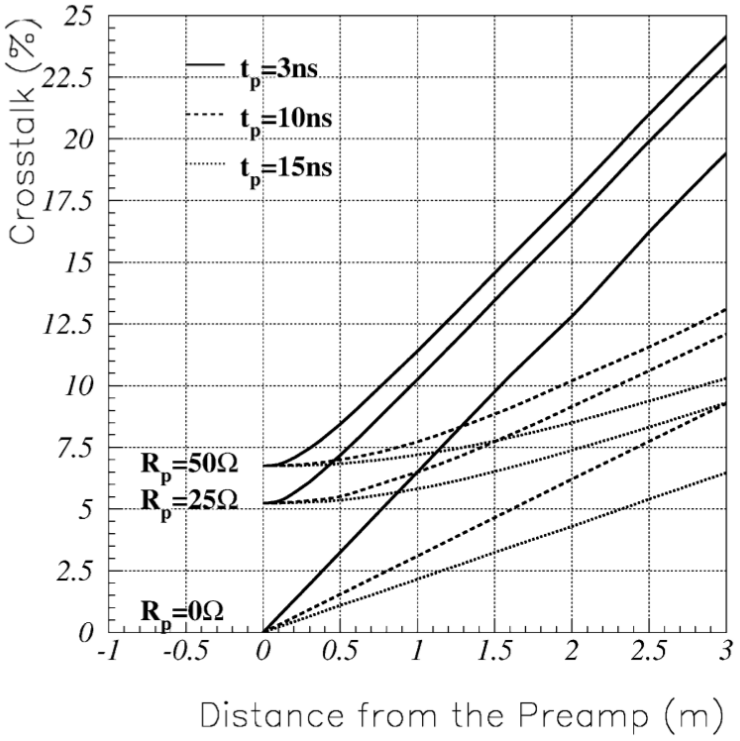


Examples, inhomogeneous double single strip



$$\hat{\mathbf{R}}_T = \begin{pmatrix} 22.3 & 213.7 \\ 213.7 & 22.3 \end{pmatrix} \Omega \qquad \hat{\mathbf{v}} = \begin{pmatrix} 2.2 & 0 \\ 0 & 2.4 \end{pmatrix} \times 10^8 \text{ m/s}$$

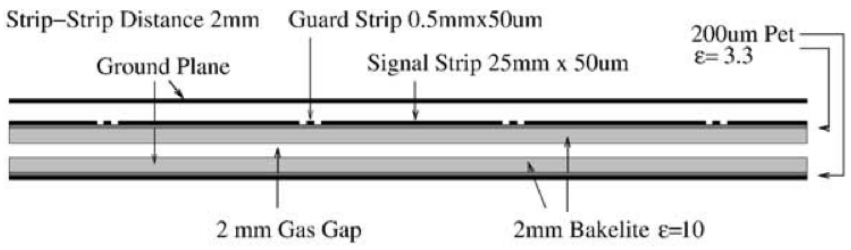


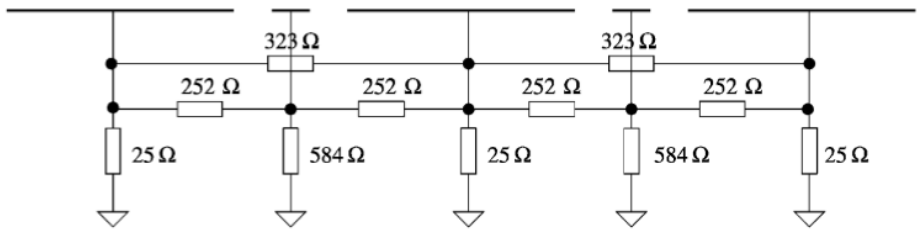


The crosstalk is therefore a function of the distance of the induced current from the preamp side and also a function of the signal shape and amplifier peaking time.

The ‘slower’ the amplifier, the more the signal is integrated and the smaller is the crosstalk.

Examples, multiple strips

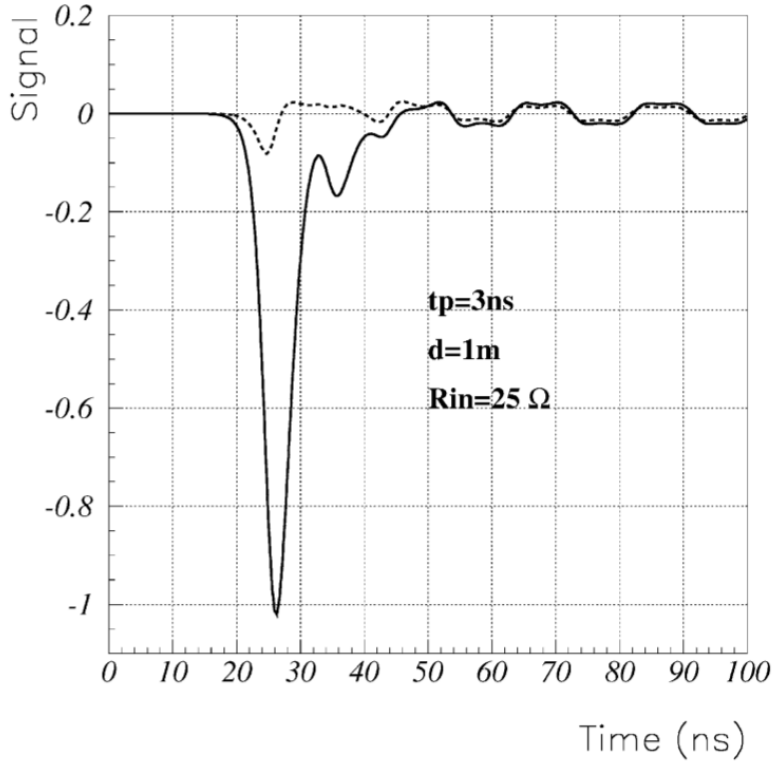




For a situation where we put guard strips in between the readout strips in order to reduce the coupling, one in principle also has to terminate these guard strips properly.

The plot shows the signal in case the strips are terminated with 25 Ohm and the guard strips are grounded.

As expected there are reflections.

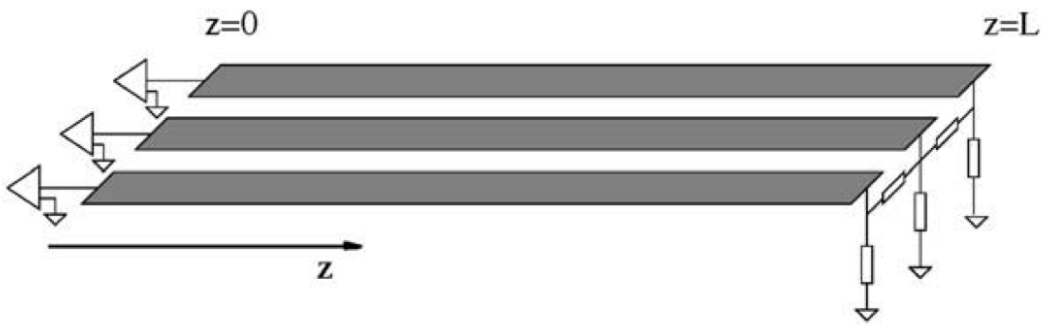


Lossy Transmission lines

A similar formalism exits for transmission lines where the losses due to resistance R and trans-conductance G cannot be neglected.

In that case the impedance matrix becomes frequency dependent and in turn all the discussed effects will become dependent on frequency.

To first order, the signals are exponentially attenuated as they propagate along the line with an attenuation coefficient given by



Transmission lines

Detectors with long readout strips constitute multi-conductor transmission lines.

If the medium surrounding the conductors in homogeneous (e.g. wires in a wire chamber) there is only a single propagation velocity (for lossless lines)

To eliminate reflections at the end of a transmission line with N conductors, one needs in principle N(N+1) termination resistors that interconnect the readout electrodes.

Using a sub-set of termination resistors will always result in some reflections.

If the medium surrounding the conductors in inhomogeneous (readout strips on a PCB) there are N different propagation velocities of the signal.

The crosstalk and signal shape depends on the position of the induced signal along the electrode – even for the lossless case.

Signals in particle detectors

Main Auditorium, Mon. 2 Dec.

Lecture 1:

- Electrostatics
- Principles
- Reciprocity
- Induced currents
- Induced voltages
- Ramo-Shockley theorem
- Mean value theorem
- Capacitance matrix
- Equivalent circuits

Council Chamber, Tue. 3 Dec.

Lecture 2:

Signals in

- Ionization chambers
- Liquid argon calorimeters
- Diamond detectors
- Silicon detectors
- GEMs (Gas Electron Multiplier)
- Micromegas (Micromesh gas detector)
- APDs (Avalanche Photo Diodes)
- LGADs (Low Gain Avalanche Diodes)
- SiPMs (Silicon Photo Multipliers)
- Strip detectors
- Pixel detectors
- Wire Chambers
- Liquid Argon TPCs

TH conference room (4/3-006), Wed. 4 Dec.

Lecture 3:

- Media with conductivity
- Quasi-static approximations
- Signal theorem extensions
- Time dependent weighting fields
- Resistive plate chambers (RPCs)
- Un-depleted silicon sensors
- Monolithic pixel sensors

Filtration Plant (222/R-001), Thu. 5.Dec.

Lecture 4:

- Signal propagation
- Transmission lines
- Termination
- Linear signal processing
- Noise
- Optimum filters

Main Auditorium, Fri. 6 Dec.

Lecture 5:

- Possible overflow, wrap-up and Q&A session

Thanks for your attention

Don’t hesitate to contact me (werner.riegler@cern.ch) for questions, corrections, comments.

There would be some interesting projects for implementing time dependent weighting fields for signals in multi-layer geometries with resistive elements (RPCs, Resistive Micromegas, Silicon sensors with resistive layers etc.) in Garfield++ …