New prospects in numerical relativity

Helvi Witek

Theoretical Particle Physics and Cosmology
Department of Physics
King's College London

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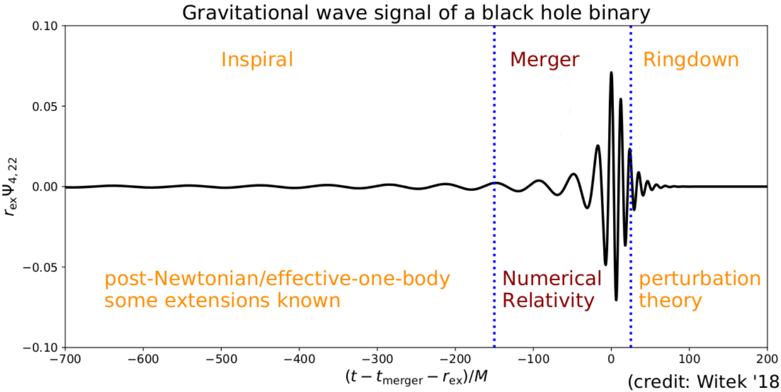
A need for theoretical models . . .

- Gravitational wave detections & interpretation: theory-driven!
- matched-filtering of data with templates



(credit: Witek '18)

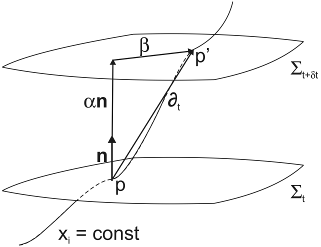
1. **Act I: Numerical (general) relativity in a nutshell**

2. **Act II: Numerical relativity beyond the standard model**

3. **Act III: Example: scalar Gauss–Bonnet gravity**

4. **Epilogue**

Act I: Numerical general relativity in a nutshell



Kinematics

- foliation of spacetime $\mathcal{M} = \Sigma_t \times R$
- 4D metric g_{ab}
  $\rightarrow$ 3D metric γ_{ij}, lapse α, shift β^i
- extrinsic curvature
  $K_{ij} = -\gamma^a{}_i \gamma^b{}_j \nabla_a n_b = -\frac{1}{2}\mathcal{L}_n \gamma_{ij}$

Dynamics: $G_{\mu\nu} = 16\pi T_{\mu\nu}$

Constraint equations
- elliptic PDEs
- initial conditions

Evolution equations
- hyperbolic (time evolution) PDEs
- supplement with gauge conditions

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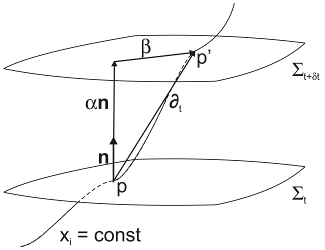

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Act I: Numerical general relativity in a nutshell

(SuperMUC Petascale system)

Technical implementation

- well-posed initial value formulation
- numerical schemes
  (e.g., method of lines with finite differences and RK time integrator)
- adaptive mesh refinement
- high performance computing

Looking for a public numerical relativity code? Check out

- Einstein Toolkit (einsteintoolkit.org)
- GRChombo (grchombo.org)
- Spectre
- NRPy+ (blackholesathome.org)
- ...

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Act II:

Numerical relativity beyond general relativity

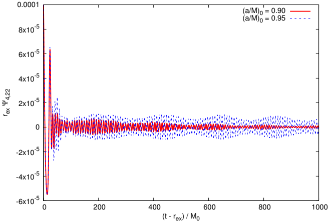
$$\begin{aligned}\mathcal{L} = \quad & f_0(\phi)R - \omega(\phi)(\nabla\phi)^2 - V(\phi) + \mathcal{L}_{\rm M}[\Psi, A^2(\phi)g_{ab}] & \text{scalar-tensor theory} \\ & + f_1(\phi)\left(Riem^2 - 4R_{ab}R^{ab} + R^2\right) + f_2(\phi)\,{}^*R_{abcd}R^{abcd} & \text{quadratic gravity} \\ & + \ldots \end{aligned}$$

CQG Focus Issue "Numerical relativity beyond GR" (eds. D. Gerosa, L. Stein, **HW** 2019)

Next workshop: "New frontiers in strong gravity" in Benasque (Spain) in July 2020

Black holes as cosmic lab for BSM particles

(Press & Teukolsky '72; Damour et al '76; Detweiler '80; Zouros & Eardley '79; Cardoso et al '05; Dolan '07; Rosa & Dolan '11; Barranco et al '11,'13; Pani et al '12; **HW** et al '12; Dolan '12; Shlapentokh-Rothman '14; Okawa, **HW** et al '14; Brito et al '15; Zilhao, **HW** et al '15; Moschidis '16; East '17, '18; Frolov et al '18; Dolan '18; Hannuksela et al '18; Ficarra, Pani, **HW** '19; Arvanitaki et al '19; Baumann et al '18, '19; Wong et al '19; ...)



(**HW** & Zilhão, in prep.)

(Single) black holes and light fields

- formation of scalar/vector condensates due to superradiant instability
- monochromatic gravitational radiation

$\Rightarrow$ search for beyond standard model particles

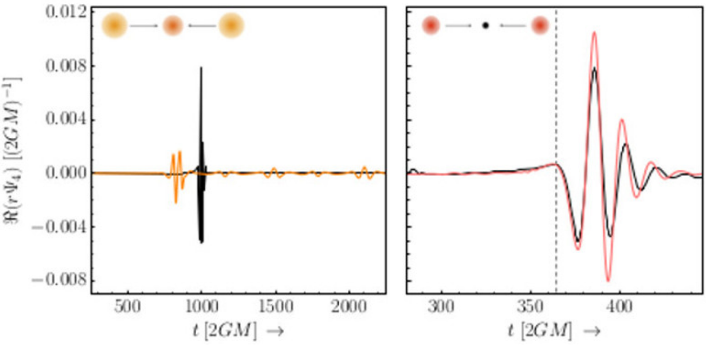
Detectors:	PTAs	LISA		LIGO/Virgo/3G	
BHs ($M_\odot$):	10^{10}	10^{6}	10^{3}	50	3
$m_B = \hbar\mu$ (eV):	10^{-21}	10^{-17}	10^{-14}	10^{-12}	10^{-11}

Note: code available in open-source library CANUDA (with M. Zilhão; `https://bitbucket.org/canuda/`)

Did we really detect the black holes of GR?

(see e.g. E. Lim's talk)

- (scalar) boson stars (Liebling & Palenzuela '12, Palenzuela et al '17, Helfer et al '18, Bezares et al '18, Alcubierre et al '19, ...),
- Proca stars (Sanchis-Gual et al '18)
- black holes with near-horizon fluctuations (Liebling et al '17)
- axion stars and black holes or neutron stars (Clough, Dietrich et al '18)



Example: boson stars (Helfer et al '18)

Is General Relativity THE theory of gravity?

black-hole formation

- gravitational collapse in Einstein-æther theory (Garfinkle et al '07)
- collapse in Einstein dilaton Gauss–Bonnet gravity (Benkel, Sotiriou, **HW** '16; Ripley & Pretorius '19)
- Spherical collapse in Horndeski gravity (Ripley & Pretorius '19)

compact binaries (see also M. Okounkova's talk)

- compact binaries in scalar-tensor theory (Barausse et al '12, Shibata et al '13, Healy et al '11, Berti et al '13)
- Einstein-Maxwell-Dilaton models (Hirschmann et al '17)
- dynamical Chern-Simons gravity (Okounkova et al '17, '19)
- scalar Gauss–Bonnet gravity (**HW** , L. Gualtieri, P. Pani, T. P. Sotiriou '18)

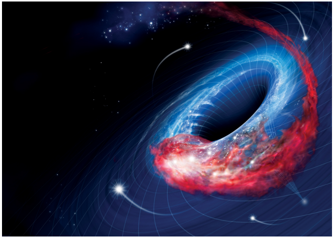
Act III: scalar Gauss–Bonnet gravity

[Benkel, **HW**, T. P. Sotiriou '16, '17;
HW, L. Gualtieri, P. Pani, T. P. Sotiriou '19]

Example: scalar Gauss–Bonnet gravity

High-energy physics

- higher curvature corrections
  relevant in strong-curvature regime
- low-energy limit of some string theories
  (Gross & Sloan '87, Kanti et al '95, Moura & Schiappa 06)
- compactification of Lovelock gravity
  (Charmousis '14)



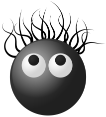
Action (e.g. Kanti et al '95, Alexander & Yunes '09, Yunes & Siemens '13, ...):

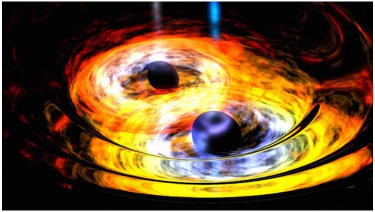
$$S = \frac{1}{16\pi}\int \mathrm{d}^4x\sqrt{-g}\left(R + 2\alpha_{\rm GB} f(\Phi)\mathcal{R}_{\rm GB} - \frac{1}{2}(\nabla\Phi)^2\right)$$

with $\mathcal{R}_{\rm GB} = R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd}$ and $f(\Phi) \sim e^{\Phi}$

Example: black holes in scalar Gauss-Bonnet gravity

- black holes have scalar hair (of the second kind)!
  (Kanti et al '95, Pani et al '09, '11, Stein et al '11, Sotiriou & Zhou '14, Ayzenberg & Yunes '14, Maselli et al '15, Okounkova '19, . . .)
- black holes can exceed the Kerr bound (Kleihaus et al '11, '14)
- scalar hair forms dynamically (Benkel et al '16, Witek et al '18)





Black hole binaries:

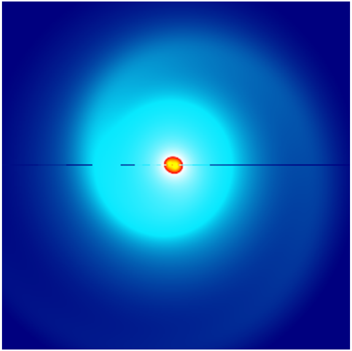
- scalar dipole radiation
  $\Rightarrow$ shift in binding energy
  $\Rightarrow$ shift in orbital frequency
- ringdown signature?
- effects in nonlinear regime?

Numerical implementation

- use effective-field theory-type reformulation
  (see Discussion on numerical relativity)
- implemented in Einstein Toolkit & Canuda
  (https://bitbucket.org/canuda/)
- background: BH binary with $q = 1, 1/2, 1/4$

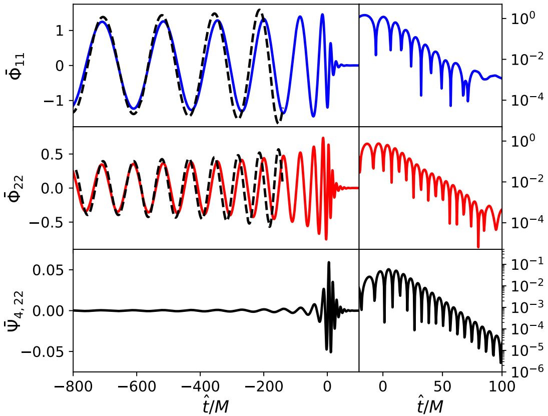
einsteintoolkit.org

Scalar field evolution



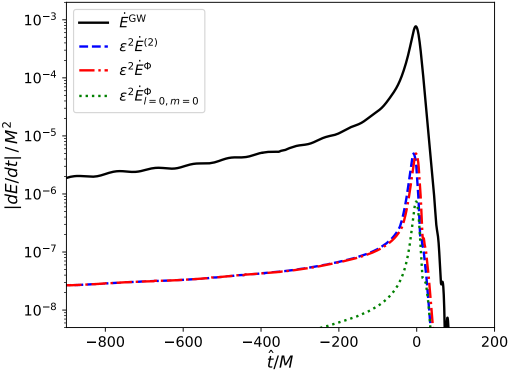
(**HW**, Gualtieri, Pani & Sotiriou '18)

Scalar radiation for $q = 1/2$ measured at $R_{\rm ex}/M = 100$



- early inspiral: agreement with PN prediction (Yagi et al '11)
- modulated ringdown (frequency domain so far only for Schwarzschild!)
  - superposition of scalar-led and gravitational-led modes
  - quality factors different for pure Kerr and Kerr + GWs

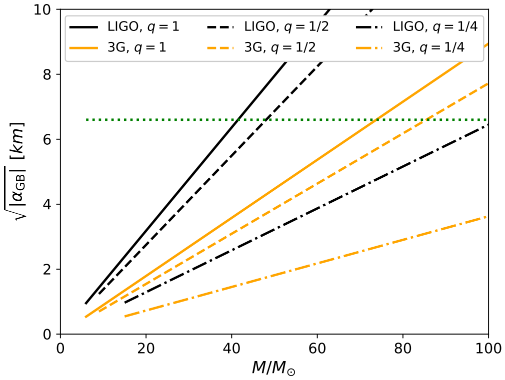
Energy flux at second order



- estimate second order effects: $G^{(2)}_{ab} = \frac{1}{2}\, T^{\rm eff}_{ab}\left(g^{(0)}_{ab}, \Phi^{(1)}\right)$
- inspiral: dominated by scalar dipole flux
- merger: dominated by scalar monopole flux, i.e., change in scalar charge

Observational constraints

non-detection of dephasing ⇒ bound on GB coupling: $|\epsilon| = \left|\frac{\alpha_{\rm GB}}{4M^2}\right| \lesssim \sqrt{\frac{\Delta\phi_{\rm det}}{\phi^{(2)}}}$



Epilogue:

A Shakespearian drama or comedy?

Epilogue

- GWs as new observational channel for beyond-standard model physics
- BUT: limited by our lack of knowledge of the merger phase in beyond-GR
  $\Rightarrow$ need novel analytic & numerical methods to model waveforms
- example: first binary black hole evolutions in scalar GB
  - numerical infrastructure publicly available in CANUDA
  - include merger $\rightarrow$ most stringent observational bounds so far
- next steps to connect theories with observations
  $\Rightarrow$ complete waveforms (PN+NR+perturbations)
  $\Rightarrow$ parametrized waveform model building
  $\Rightarrow$ towards "parametrized" NR within perturbative expansion

Thank you!

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