Theory issues on top quark mass measurement

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What we need to measure the top mass

- A top mass sensitive observable $O(m_t)$
- The ability to measure it.
- The ability to compute it.

Generally

$$O(m_t) = c_0 + c_1\alpha_s + c_2\alpha_s^2 \ldots + \left(\frac{\Lambda}{m_t}\right)^m .$$

with

$$\alpha_s = \frac{1}{b_0 \log \frac{m_t^2}{\Lambda^2}}$$

Precision is limited by missing HO ($c_k\alpha_s^k$, Higher Orders) and NP ($\left(\frac{\Lambda}{m_t}\right)^m$, Non-Perturbative) terms.

Precision

- Direct measurements precision near 500 MeV
- 500 MeV is not far from Λ_{qcd}
- Must worry about standard scale and scheme issues when extracting the top mass from a measurement, including top mass scheme issues.
- Must also worry about power corrections. For an m_t sensitive observable O we have

$$\begin{aligned} O(m_t) &= c_0 + c_1\alpha_s + \ldots + c_{\mathrm{np}} \left(\frac{\Lambda}{m_t}\right)^{m=1} \\ &\to \delta m_t \propto m_t \alpha_s^n,\ m_t \frac{\Lambda}{m_t} \end{aligned}$$

(What counts is $m = 1$ for top).

Selected Th. results relevant to top mass measurements

- Narrow width $t\bar{t}$ production and decay at NLO, Bernreuther,Brandenbourg,Si,Uwer 2004, Melnikov,Schulze 2009.
- $l\nu l\nu b\bar{b}$ final states with massive b, Frederix, 2013, Cascioli,Kallweit,Maierhöfer,Pozzorini, 2013.
- NNLO differential top decay, Brucherseifer,Caola,Melnikof 2013.
- NLO+PS in production and decay, Campbell,Ellis,Re,PN
- NNLO production, Czakon,Heymes,Mitov,2015.
- $l\nu l\nu b\bar{b} + \mathrm{jet}$ Bevilacqua,Hartanto,Kraus,Worek 2016.
- Approx. NNLO in production and exact NNLO in decay for $t\bar{t}$. Gao,Papanastasiou 2017.
- Resonance aware formalism for NLO+PS: Ježo,PN 2015;
- Off shell + interference effects+PS, Single top, Frederix,Frixione,Papanastasiou,Prestel,Torielli, 2016
- Off shell + interference effects+PS, $l\nu l\nu b\bar{b}$, Ježo,Lindert,Oleari,Pozzorini,PN, 2016.

Alternative mass-sensitive observables

- Butenschoen,Dehnadi,Hoang,Mateu,Preisser,Stewart,2016 Use boosted top jet mass + SCET.
- Agashe,Franceschini,Kim,Schulze,2016: peak of b-jet energy insensitive to production dynamics.
- Kawabata,Shimizu,Sumino,Yokoya,2014: shape of lepton spectrum. Insensitive to production dynamics and claimed to have reduced sensitivity to strong interaction effects.
- Frixione, Mitov: Selected lepton observables.
- Alioli, Fernandez, Fuster, Irles, Moch, Uwer, Vos ,2013; Bayu etal: M_t from $t\bar{t}j$ kinematics.
- $t\bar{t}$ threshold in $\gamma\gamma$ spectrum (needs very high luminosity), Kawabata,Yokoya,2015

Theory status: consensus in the theory community

- Top mass issues have been characterized by conflicting opinions among theorists.
- No point (and no time) to try to sort them out here.
- Some progress in understanding, and partially conciliate different viewpoints, has been made by
  G. Corcella, A. Hoang, H. Yokoya, P.N..
  in the HE-LHC Working Group report.
- Shall the LHCtopWG take it seriously as a starting point for a discussion on top mass theory issues, getting more theorists involved?

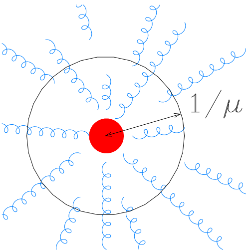
Theory status: consensus in the theory community

Some points where we reached agreement:

- As far as HO corrections are concerned (i.e. corrections of relative order $\alpha_s^k(m_t)$, to the top mass), direct measurements can be viewed as accessing the top pole mass.
- The MC mass scheme issue (advocated by Hoang and collaborators) has to do with short distance masses evaluated at low scales (of the order of the shower cut-off). These masses are very close to the pole mass.

The mass scheme

The mass of a heavy quark is also carried by its gluon field.



We can decide to include all the field accompanying the quark down to infinite distance. This is the POLE MASS.

Or we can cut it off, keeping only contributions at distance below some scale $1/\mu$ (i.e., keeping only momenta above μ.
These are the SHORT DISTANCE MASSES.

They are related in perturbation theory by a power expansion in α_s with well defined coefficients:

$$M_{\rm pole} = M(\mu)\left(1 + \sum_{i=1}^{\infty} c_i \alpha_s^i(\mu)\right)$$

Interplay between HO and NP corrections

The perturbative expansion is in general not convergent, and its high orders behaviour is related to the power of non-perturbative corrections:

$$
\begin{aligned}
c_k \alpha_s^k &\approx (m b_0)^k k! \alpha_s^k \\
\text{minimal term at} \quad k &= \frac{1}{m b_0 \alpha(m_t)} \quad \text{(typically 6-8)} \\
\text{at the minimum:} \quad c_k \alpha_s^k &= \left(\frac{\Lambda}{m_t}\right)^m
\end{aligned}
$$

where m is a positive integer.
For top, we worry only about $m = 1$, and the relation between the Pole and Short distance mass has a factorial growth corresponding to $m = 1$ power corrections.

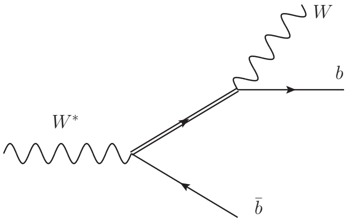
Linear Power Corrections

- There is an intuitive way to understand the renormalon presence: the “accompanying field” argument clearly breaks at distances of order $1/\Lambda_{\rm qcd}$, since at these scales $\alpha_s \approx 1$, non-perturbative effects become important, etc.
- This problem is not just for top. Bottom physics have the same problem, but it has been shown that there are observables in B decays that are calculable in terms of the short distance mass without linear power corrections.
- In top physics at colliders, linear renormalons are everywhere, not just in the mass. Every time we deal with jets, linear renormalons are there. Can we have a clearer view of their interplay, even in a simplified context, for the top mass measurements?

Linear Power Corrections: the Renormalon wisdom

Ferrario Ravasio, Oleari, P.N.2019

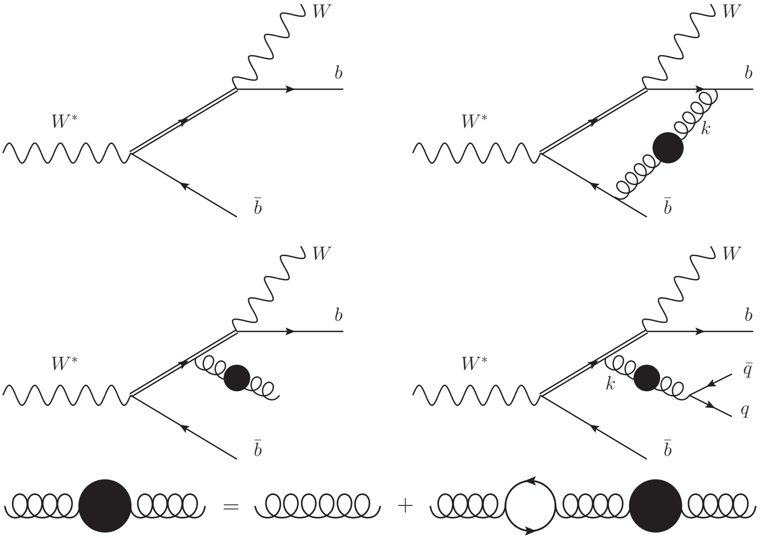
We consider a simplified production framework $W^* \to Wt\bar{b}$:



(i.e. no incoming hadrons). However:

- The b is taken massless, the W is taken stable, but the top is taken unstable, with a finite width.
- We can examine any infrared safe observable, no matter how complex.

Diagrams up to leading N_f one gluon correction:



Linear Power Corrections: the Renormalon wisdom

We find:

- The total cross section (in the simplified model!) does not have them. However, if cuts are present (even if only on the lepton!) they are there.
- Jets have large linear power corrections with coefficients of order $1/R$. These have some sort of universality, and may be controlled by calibration. However, power corrections with no $1/R$ enhancement are also there, and are not universal.

Linear Power Corrections: the Renormalon wisdom

- Leptonic observables have linear power corrections. These are seen to be absent for distributions defined in the top rest frame, consistently with the B decay example.
- In general, the top finite width screens the linear power corrections due to top emissions. Thus, for observables not involving jets, like the leptonic observables, we see that the linear corrections disappear for finite with.

Can this fact be exploited to perform top mass measurements free of linear power corrections?

At the moment the answer is not known.

Linear Power Corrections: the Renormalon wisdom

What do we learn from renormalons? A first example:

- A work of Smith,Willenbrock,1996 points out that the mass renormalon problem does not disappear even if we account for the finite width of the top. (mentioned also in Czakon talk at TOP2019).
- Our finding, that no renormalon is present in the physics thanks to the top finite width, has surprised some researchers, since it seems to contradict the Smith-Willenbrock result. This is not the case.
- In fact, there is no contradiction. The position of the pole in the top propagator is blurred by the mass renormalon. But since the finite width of the top has pushed this pole off the real axis, this has no impact on the physics.

We believe that the renormalon calculation has helped to clarify this point.

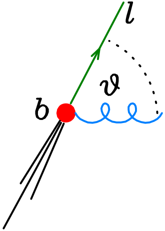
A second example:

- B physics experts have been surprised to see that (linear) renormalons are present in the leptonic spectrum of a heavy quark decay, in a frame where the heavy quark is not at rest.
- They are well aware of the fact that there are no renormalons if the leptonic spectrum is measured in the top rest frame (and the $\bar{M}S$ mass is used).

Can we understand this in a more intuitive way?

A tentative explanation:

- A b quark in a B meson undergoes Fermi motion, i.e. it has momentum of order Λ. But its kinetic energy is of order Λ^2/m_b, because it is non-relativistic. So, no linear power corrections there.
- The decay can take place in a time fraction when the b is in a virtual state associated with the emission of a soft gluon.



The decay product are boosted with velocity $v = k/m_b$, where k is the soft gluon momentum. The corresponding change in the lepton momentum is $\delta p_l \approx vp\cos\theta$. But this effect linear in v vanish under azimuthal average.

As a result, the semileptonic spectrum has no linear power corrections *if expressed in terms of a short distance mass*

(This explanation also holds for heavy quarks produced on-shell, since their soft radiation pattern does not depend upon its spin.)

A third example, from a totally different perspective:

- Hoang and collaborators (for the latesr see Hoang,Plätzer,Samitz 2018) have explored the relation of the MC mass parameter to short distance mass schemes.
- An intepretation in our simplified view: in the highly boosted limit the Coulomb field accompanying the quark can be view as a superposition of quanta (as in the method of virtual quanta, or Fermi-Weiszäker-Williams). The shower algorithm is fully consistent with this view, and the shower cutoff plays a role similar to the short-distance mass scale μ.
- Thus Hoang identifies the MC mass as a short distance mass evaluated at the scale of the shower cutoff (an identification that works in the highly boosted regime).

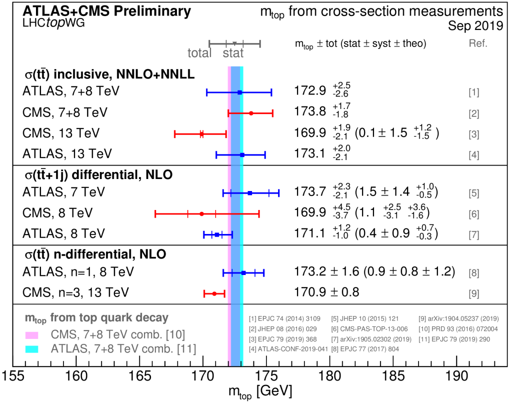
FEW OBSERVATIONS

REGARDING CURRENT MEASUREMENTS

2. Indirect measurements of m_t

[Pole mass summary plot from LHCtopWG]

Taking advantage of the mass dependence of the production cross section (and of differential cross sections) ATLAS/CMS improve D0's initial measurement of the pole mass



(5)

(6)

Warning: each measurement depends on the chosen PDF

According to our renormalon perspective, is there a substantial difference between the first three iterms and $m_{\rm top}$ from quark decay?

- In principle there are reason to believe that $\sigma(t\bar{t})$ could be free from linear renormalons. But we do not measure the full $\sigma(t\bar{t})$. We use tagging procedures (typically using jets) that introduce renormalons. For the remaining two items, we have no reasons to believe that linear renormalons should be absent.
- When comparing distributions to NLO calculations, we are limiting ourself to the first few orders of the perturbative expansion. The renormalon problems arise at much higher orders.
- Every time we do an unfolding to the top particle level, we are actually ignoring the renormalon problem.

Top mass from σ_{tt} (eμ) (4)

Inclusive cross section (13 TeV 2015-16, 36.1 fb^{-1})

ATLAS-CONF-2019-041

Selection:

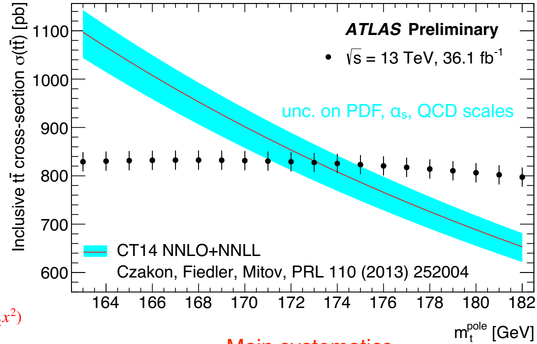
- 1 oppositely charged eμ pair
- 1 or 2 b-tagged jets

σ_{tt} dependence on m_t^{pole} parametrized as

$$\sigma_{tt}^{theo}(m_t^{pole}) = \sigma(172.5)\left(\frac{172.5}{m_t^{pole}}\right)^4 (1 + a_1 x + a_2 x^2)$$

$$x = (m_t^{pole} - 172.5)/172.5$$

m_t^{pole}=173.1 $^{+2.0}_{-2.1}$ GeV



Main systematics

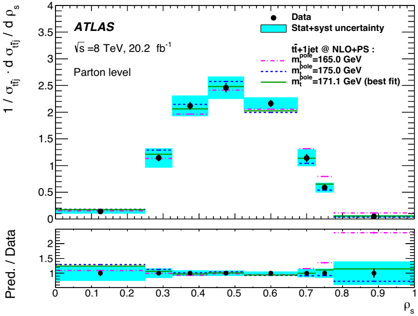
source	GeV
PDF+α_s	+1.5 −1.4
QCD scales	+1.0 −1.5
exp	±1.0

A. Castro @ TOP2019

Top mass with tt+1jet (5)

Comparing $1/\sigma\cdot d\sigma/d\rho_s$ to NLO+PS QCD predictions to derive the pole mass

Similar procedure to derive the $\overline{\text{MS}}$ mass



m_t^{pole}=171.1±0.4(stat)±0.9(syst)+0.7−0.3(theo) GeV (+1.2 −1.0) ⟵ dominated by exp. syst.

$m_t(m_t)$=162.9±0.5(stat)±1.0(syst)+2.1−1.2(theo) GeV (+2.4 −1.6) ⟵ larger scale variations

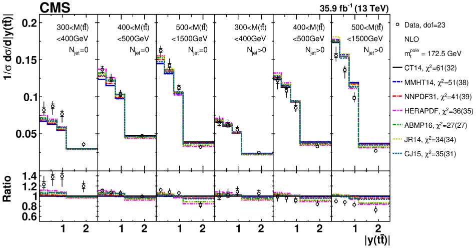
The two values are consistent when translated from one scheme to the other

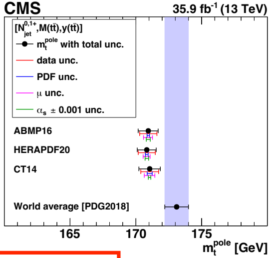
162.9 ⇒ ≃ 170.9 GeV

A. Castro @ TOP2019

Triple-differential cross section vs mass (dilepton) (6)

α_s and m_t^{pole} are extracted from comparison to fixed-order NLO predictions





Simultaneous α_s, m_t^{pole} and PDF fit *

m_t^{pole}=170.5±0.7(fit)±0.1(model)±0.1(param)±0.3(scale) GeV

m_t^{pole}=170.5±0.8 GeV (±0.47%)

Reached subGeV uncertainty

Similar precision as for direct measurements

* fixed PDF for the summary plot of slide 9

A. Castro @ TOP2019

Comments

- The error quoted from $\sigma(t\bar{t})$ has a $+1 - 1.5$ GeV from scale variation. It sounds reasonable. This error is larger than what one expect from non-perturbative (i.e. renormalon) effects.
- $\sigma(t\bar{t}j)$: theoretical error: $+0.7 - 0.3$. Here an error from an NLO calculation looks smaller than the one from an NNLO one. Suspicious?
- Triple differential: ± 0.3 from scale variation. More than suspicious?

Remember: scale variation can only give an indication. If it looks too small, one should try other methods to estimate the impact of missing HO effects. An easy one is to compare an NLO+PS result before and after shower. But variants of this can be very useful:

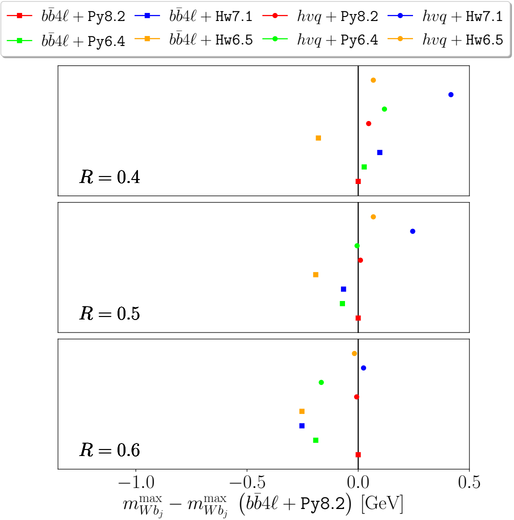
- Compare a LO calculation interfaced to a shower, stopping the shower after one radiation and after two radiations.
- Compare an NLO+PS interfaced to a shower with no radiation from the shower and one radiation from the shower.
- Try multijet samples.

This requires tinkering with the shower generators, but it is certainly something that shower authors can easily provide.

Direct Measurements

When we talk about linear power corrections, we do not really know if we are talking about 100 MeV or 1 GeV effects. Monte Carlo model can provide an estimate of the size of this effects.

- Ferrario Ravasio,Jezo,Oleari,P.N,2017-2019, The impact of the shower is assessed by comparing the Fortran and c++ versions of the Pythia and Herwig generators.
- Our results turned out to be quite disappointing if we consider a mass reconstructed observable smeared with some detector resolution effects.
- However, as far as the irreducible hadronization error is concerned, the result is quite encouraging.



Focus upon the groups of squares (our best generator) at fixed R. They span a range not larger than 250 MeV. This means that if you had a perfect detector, the intrinsic theoretical error would be $\pm$125 MeV.

- What if I add further variation on the shower side? (shower ending scale, colour reconnection, etc.?). The error band can only enlarge.
- However, data constraints can narrow it down (we are entitle to neglect shower parameter variations and models that do not fit relevant data).
- What about our results on the smeared distributions, yielding variations near a GeV? We acknowledge that ours was a very crude approach. Experimentalists can do a lot better, using the data itself to constrain parameter variations.
- Thus: insist on direct measurements! At the moment, nothing looks better than them!

Conclusions

- Top mass measurements at hadron colliders, when the precision approaches few hundred MeV's, pose difficult and profound theoretical problems, involving our understanding of non-perturbative corrections in QCD, and of how they are implemented in shower generators.
- The traditional method: aim at an observable, measure it, extract its value from a perturbative calculation, and estimate power corrections using a shower Monte Carlo, is still a valuable strategy to follow, as long as better ways of doing it are not in sight.
- Theoretical studies on the form of linear power corrections and to what extent they can/are implemented in shower Monte Carlo are at a primitive stage, but they are promising. They can help us to understand the limitations of current measurements, and they can help to identify better observables.

One last point:

- Isn't it time to wrap up the theoretical discussions on top mass measurements issues?
  An effort was made for the HiLumi report.
  Shall we take that as a starting point, and work out a TH wrap-up, as a guideline for the experimental collaborations?
  The LHCtopWG is the right framework to do this!

BACKUP

The mass scheme

The pictorial view of the definition of the short distance mass is more than just a suggestive picture. If we cut off long range radiation by giving a mass μ to the gluon, and compute the top self energy we get at order α_s

$$m_{\mathrm{pole}} = m(\mu)|_{\mu=0} = m(\mu) + \frac{2}{3}\mu\alpha_s. \tag{1}$$

One can check explicitly that this correction can be computed classically as the difference between the vacuum energies of the Coulomb field in the massless and massive (i.e. screened) case.

Top mass with tt+1jet (5)

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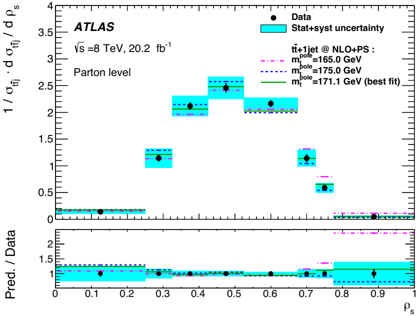

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The observation regards scheme translation: at leading order:

$$M_{\rm pole} = M(M) + 7.557 \quad 162.9 \to 170.46$$

If we do it up to the 4th order Marquard,Steinhauser

$$M_{\rm pole} = M(M) + 7.557 + 1.617 + 0.501 + 0.195 \quad 162.9 \to 172.8.$$

Which one is right? As of now, we do not know.

- If we could argue that the observable in question is free from renormalons when expressed in terms of $M(M)$, we should favour computing it using the $M(M)$ mass, and then translating to the pole mass using the full formula.
- If we could argue that the observable in question is free from renormalons when expressed in terms of $M_{\rm pole}$, it would be better to use the pole mass in the calculation, and translate it into the $M(M)$ mass using the full formula.

If neither of the two statements is true, or in case we do not know, any procedure is admissible.