The running of the top mass

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Quantum field theory

QCD

- Classical part of QCD Lagrangian

$$\mathcal{L} \quad = \quad -\frac{1}{4} F^a_{\mu\nu} F_b^{\mu\nu} + \sum_{\text{flavors}} \bar{q}_i \left(\mathrm{i}\not{D} - m_q\right)_{ij} q_j$$

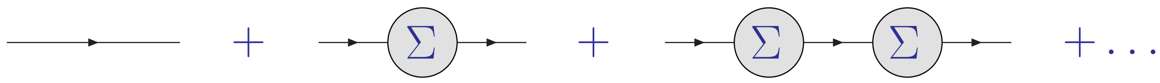
  - field strength tensor $F^a_{\mu\nu}$ and matter fields $q_i, \bar{q}_j$
  - covariant derivative $D_{\mu,ij} = \partial_\mu \delta_{ij} + \mathrm{i} g_s \left(t_a\right)_{ij} A^a_\mu$
- Formal parameters of the theory (no observables)
  - strong coupling $\alpha_s = g_s^2/(4\pi)$
  - quark masses m_q
- Parameters of Lagrangian have no unique physical interpretation
  - radiative corrections require definition of renormalization scheme

Challenge

- Suitable observables for measurements of α_s, m_q, ...
  - comparison of theory predictions and experimental data

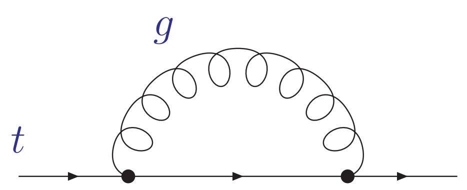
Quark mass renormalization

- Heavy-quark self-energy $\Sigma(p, m_q)$



$$= \frac{\mathrm{i}}{\not p - m_q - \Sigma(p, m_q)}$$

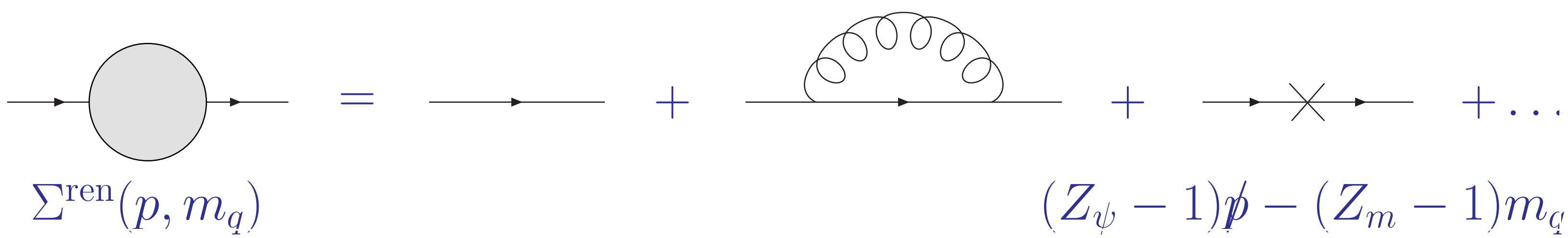
QCD



- QCD corrections to self-energy $\Sigma(p, m_q)$
  - dimensional regularization $D = 4 - 2\epsilon$
  - one-loop: UV divergence $1/\epsilon$ (Laurent expansion)

$$\Sigma^{(1),\mathrm{bare}}(p, m_q) = \frac{\alpha_s}{4\pi} \left(\frac{\mu^2}{m_q^2}\right)^{\epsilon} \left\{ (\not p - m_q) \left(-C_F \frac{1}{\epsilon} + \text{fin.} \right) + m_q \left(3C_F \frac{1}{\epsilon} + \text{fin.} \right) \right\}$$

- Relate bare and renormalized mass parameter $m_q^{\mathrm{bare}} = m_q^{\mathrm{ren}} + \delta m_q$



Mass renormalization scheme

Pole mass

- Based on (unphysical) concept of top-quark being a free parton
  - $m_q^{\rm ren}$ coincides with pole of propagator at each order

$$\not p - m_q - \Sigma(p, m_q)\Big|_{\not p = m_q} \rightarrow \not p - m_q^{\rm pole}$$

- Definition of pole mass ambiguous up to corrections $\mathcal{O}(\Lambda_{QCD})$
  - heavy-quark self-energy $\Sigma(p, m_q)$ receives contributions from regions of all loop momenta – also from momenta of $\mathcal{O}(\Lambda_{QCD})$
- Bounds:
  - lattice QCD $\Delta m_q \geq 0.7 \cdot \Lambda_{QCD} \simeq 200$ MeV Bauer, Bali, Pineda '11
  - perturbative QCD: $\Delta m_q \simeq 70$ MeV Beneke, Marquard, Nason, Steinhauser '16

$\overline{\rm MS}$ *scheme*

- $\overline{\rm MS}$ mass definition: for example one-loop minimal subtraction

$$\delta m_q^{(1)} = m_q \frac{\alpha_s}{4\pi} 3 C_F \left(\frac{1}{\epsilon} - \gamma_E + \ln 4\pi \right)$$

- $\overline{\rm MS}$ scheme induces scale dependence: $m(\mu)$

Running quark mass

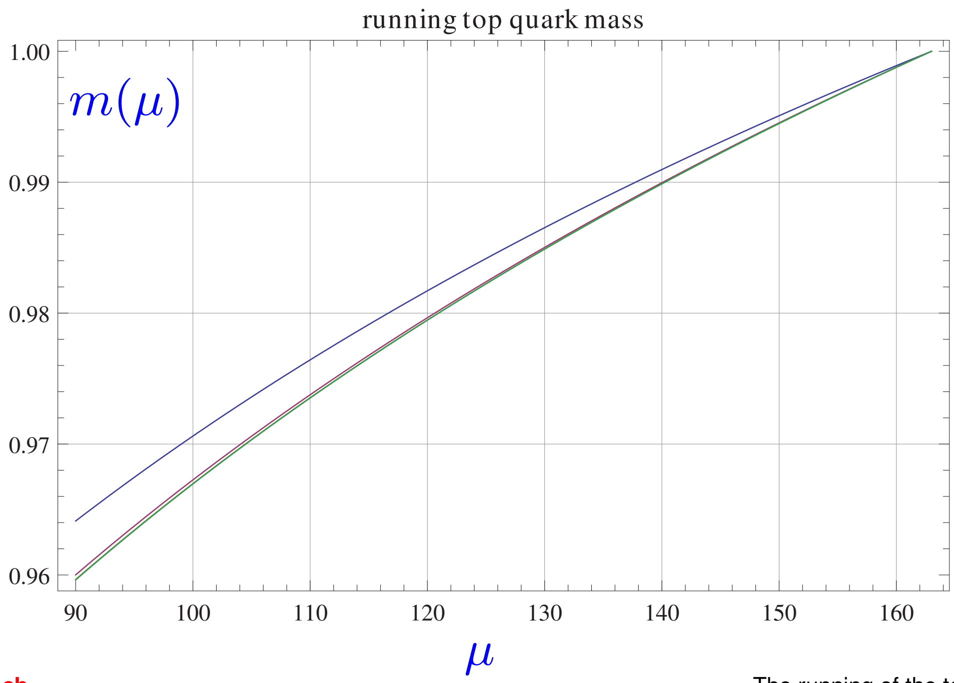
Scale dependence

- Renormalization group equation for scale dependence
  - mass anomalous dimension γ known to five loops

    Baikov, Chetyrkin, Kühn '14, Luthe, Maier, Marquard, Schröder '17

$$\left(\mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s}\right) m(\mu) \;=\; \gamma(\alpha_s) m(\mu)$$

- Plot mass ratio $m_t(163\text{GeV})/m_t(\mu)$



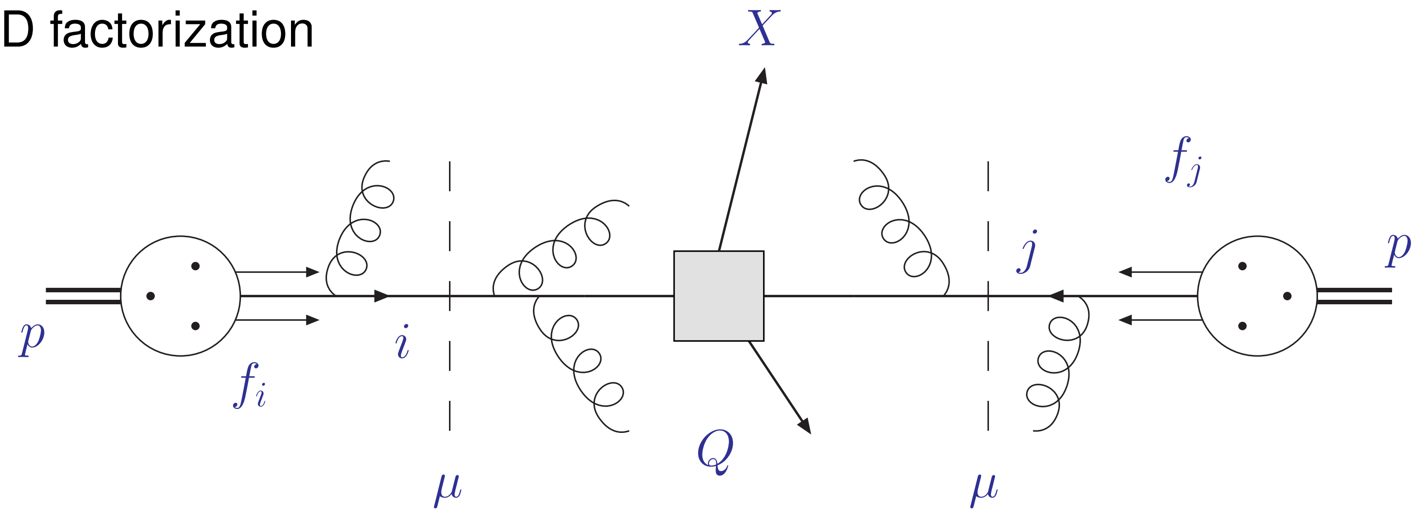
Scheme transformations

- Conversion between different renormalization schemes possible in perturbation theory
- Relation for pole mass and $\overline{\mathrm{MS}}$ mass
  - known to four loops in QCD Gray, Broadhurst, Gräfe, Schilcher '90; Chetyrkin, Steinhauser '99; Melnikov, v. Ritbergen '99; Marquard, Smirnov, Smirnov, Steinhauser '15
  - EW sector known to $\mathcal{O}(\alpha_{\mathrm{EW}}\alpha_{\mathrm{s}})$ Jegerlehner, Kalmykov '04; Eiras, Steinhauser '06
  - example: one-loop QCD

$$m^{\mathrm{pole}} = m(\mu)\left\{1 + \frac{\alpha_s(\mu)}{4\pi}\left(\frac{4}{3} + \ln\left(\frac{\mu^2}{m(\mu)^2}\right)\right) + \dots\right\}$$

Top-quark inclusive cross sections

- QCD factorization



$$\sigma_{pp\to X} \quad = \quad \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \underbrace{\hat{\sigma}_{ij\to X}\left(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2\right)}$$

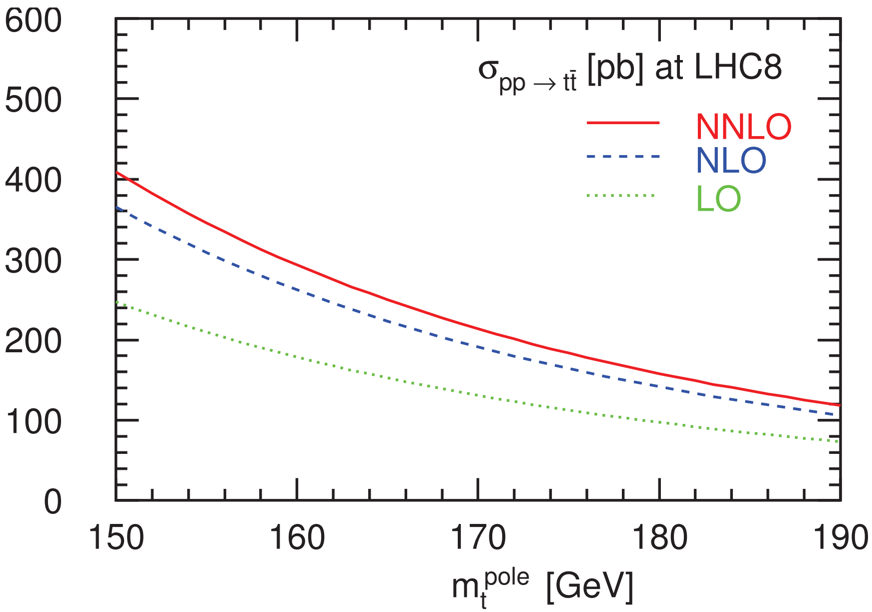
$$= \hat{\sigma}^{(0)}_{ij\to X} + \alpha_s\,\hat{\sigma}^{(1)}_{ij\to X} + \alpha_s^2\,\hat{\sigma}^{(2)}_{ij\to X} + \dots$$

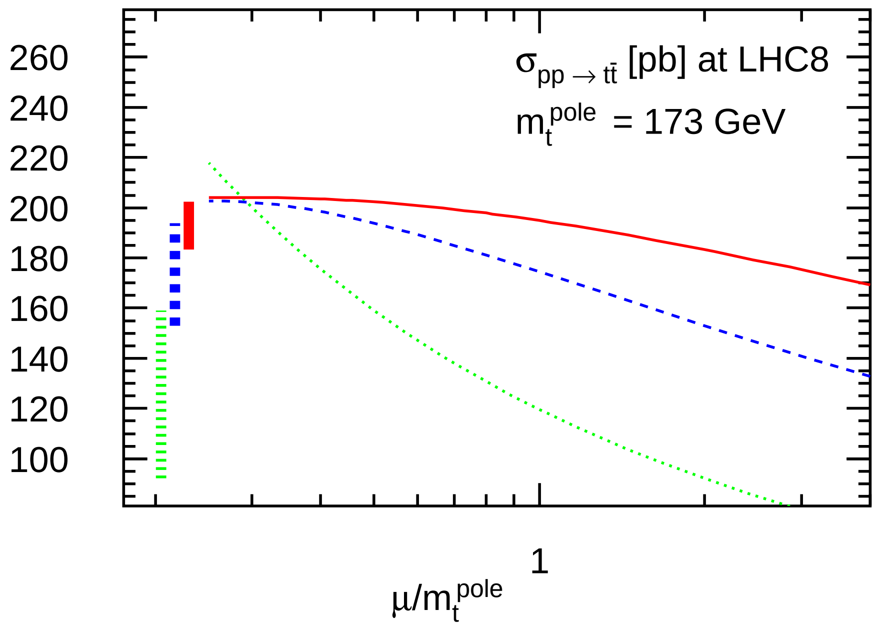
- Hard parton cross section $\hat{\sigma}_{ij\to X}$ calculable in perturbation theory
  - known to NLO, NNLO, . . . ($\mathcal{O}(\text{few}\%)$ theory uncertainty)
- Non-perturbative parameters: parton distribution functions f_i, strong coupling α_s, particle masses m_X
  - renormalization scheme for α_s and particle masses m_X

Total cross section

Exact result at NNLO in QCD

Czakon, Fiedler, Mitov ‘13



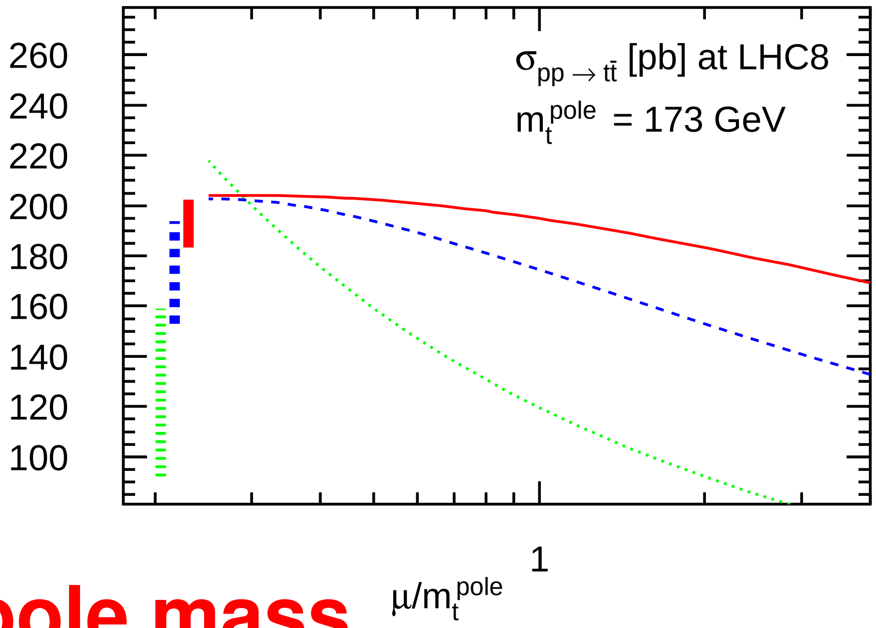


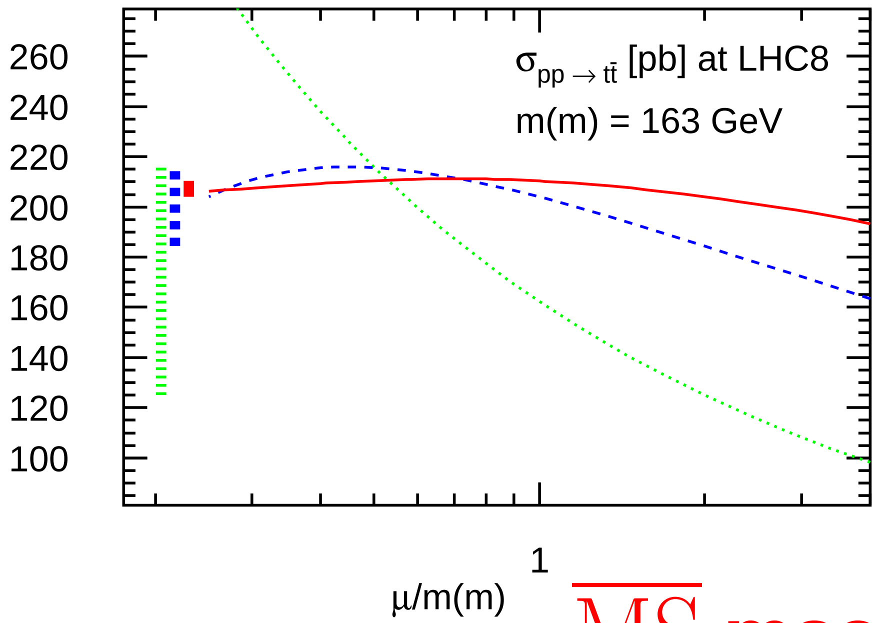
- NNLO perturbative corrections (e.g. at LHC with $\sqrt{s} = 8$ TeV)
- $\overline{\mathrm{MS}}$ renormalization scheme for α_s, on-shell scheme for m_t
  - K-factors: $K_{\mathrm{LO}\to\mathrm{NLO}} = 1.46$ and $K_{\mathrm{NLO}\to\mathrm{NNLO}} = 1.12$
  - scale stability at NNLO of $\mathcal{O}(\pm 5\%)$
  - point of minimal sensitivity at low scales $\mu \sim \mathcal{O}(m_t/4) \sim \mathcal{O}(45)$ GeV

Total cross section with running mass

Comparison pole mass vs. $\overline{\mathrm{MS}}$ mass

Dowling, S.M. ‘13





- NNLO cross section with $\overline{\mathrm{MS}}$ renormalization scheme for α_s and m_t
  - running mass with better apparent perturbative convergence
  - K-factors: $K_{\mathrm{LO}\to\mathrm{NLO}} = 1.26$ and $K_{\mathrm{NLO}\to\mathrm{NNLO}} = 1.03$
  - point of minimal sensitivity at natural hard scales $\mu \sim \mathcal{O}(m_t(m_t)) \sim \mathcal{O}(160)$ GeV

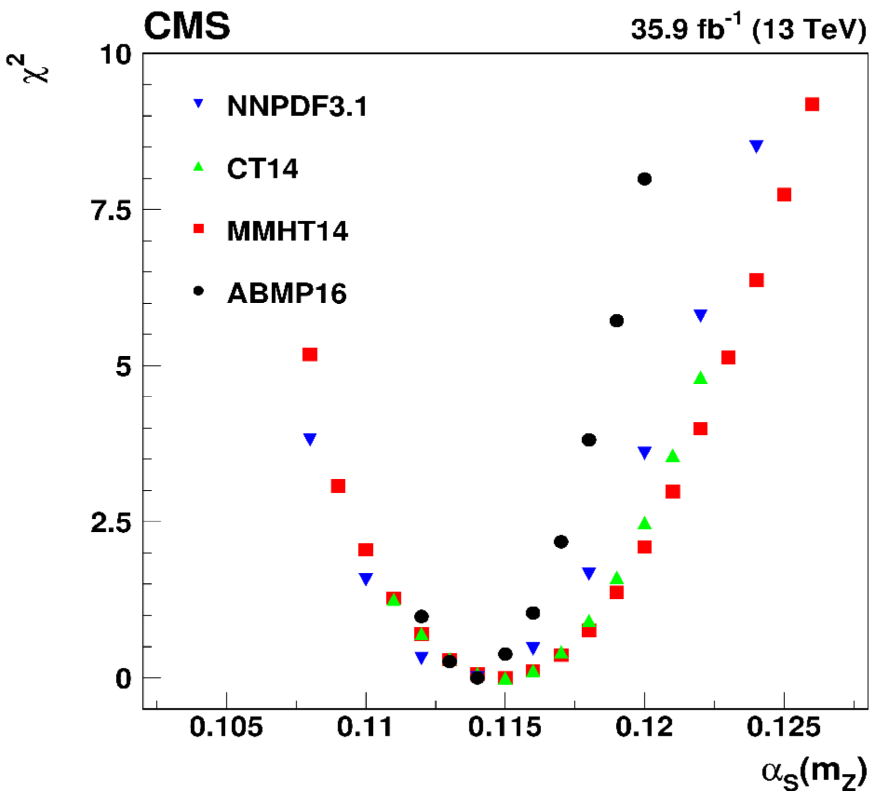
PDF landscape

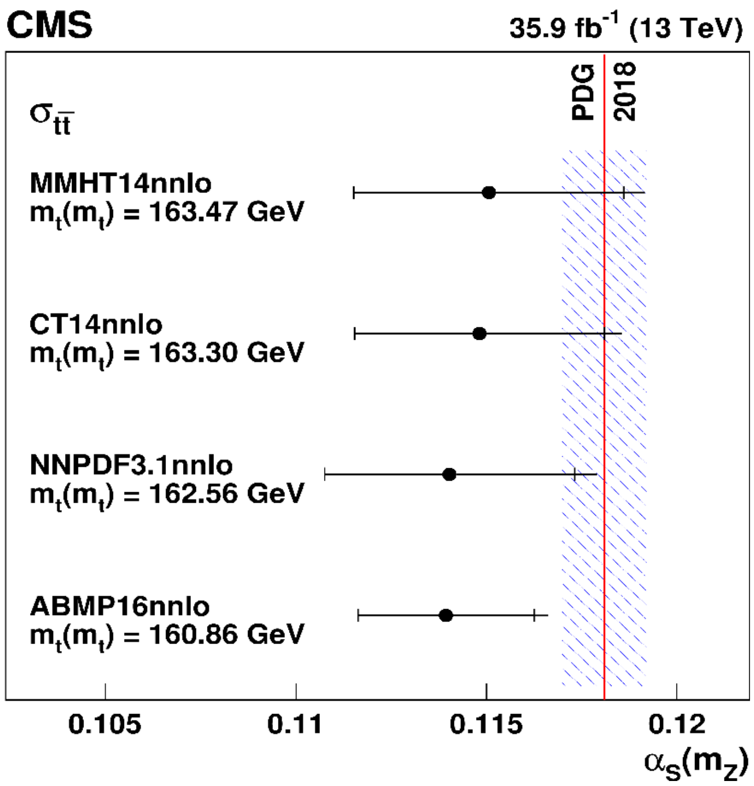
- Significant number of active groups ABMP16, CJ15, CT14, HERAPDF2.0, JR14, MMHT14, NNPDF3.1
  - PDFs accurate to NNLO in QCD, except for CJ15 (NLO)
  - different choices of data sets
  - different fitting procedures ($\Delta\chi^2$ criterium)

PDF sets	$\Delta\chi^2$ criterion	data sets used in analysis
ABMP16 arXiv:1701.05838	1	incl. DIS, DIS charm, DY, $t\bar{t}$, single t
CJ15 arXiv:1602.03154	1	incl. DIS, DY (incl. $p\bar{p} \to W^{\pm}X$), $p\bar{p}$ jets, γ+jet
CT14 arXiv:1506.07443	100	incl. DIS, DIS charm, DY, $p\bar{p}$ jets, pp jets
HERAPDF2.0 arXiv:1506.06042	1	incl. DIS, DIS charm, DIS jets
JR14 arXiv:1403.1852	1	incl. DIS, DIS charm, DY, $p\bar{p}$ jets, DIS jets
MMHT14 arXiv:1510.02332	$2.3\dots42.3$ (dynamical)	incl. DIS, DIS charm, DY, $p\bar{p}$ jets, pp jets, $t\bar{t}$
NNPDF3.1 arXiv:1706.00428	n.a.	incl. DIS, DIS charm, DY, $p\bar{p}$ jets, pp jets, $t\bar{t}$, $W+$charm, Zp_T

Top-quark mass determination

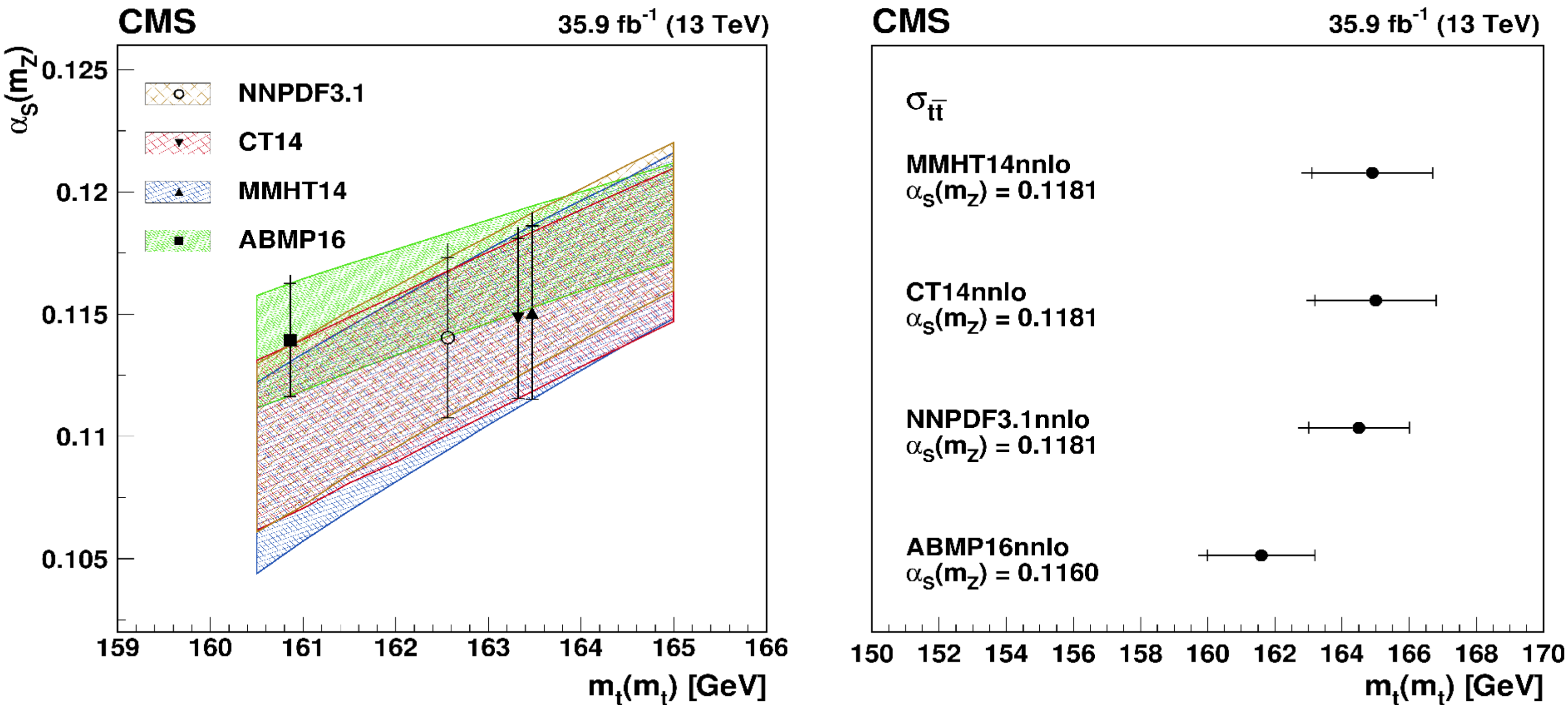
- Extraction of $\overline{\text{MS}}$ mass $m_t(m_t)$
  - data on $t\bar{t}$-production CMS coll. arXiv:1812.10505
- Quality of fit with goodness-of-fit estimator χ^2 for different PDFs and extracted $\alpha_s(M_Z)$ and $m_t(m_t)$ values





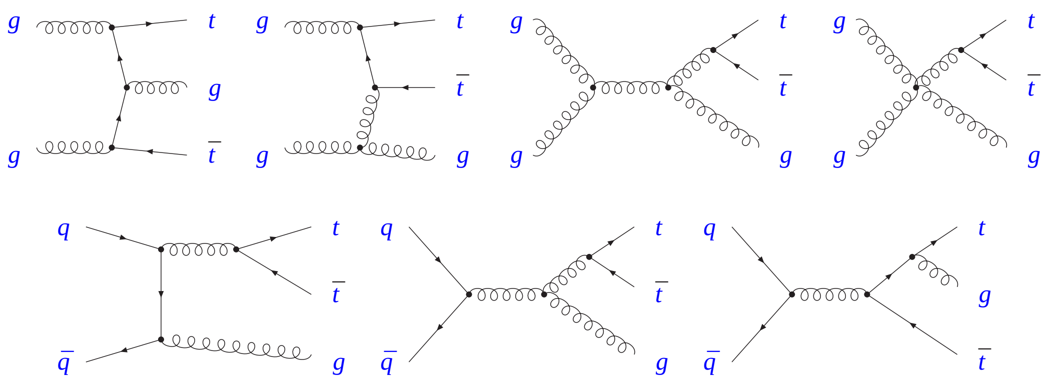
Correlations

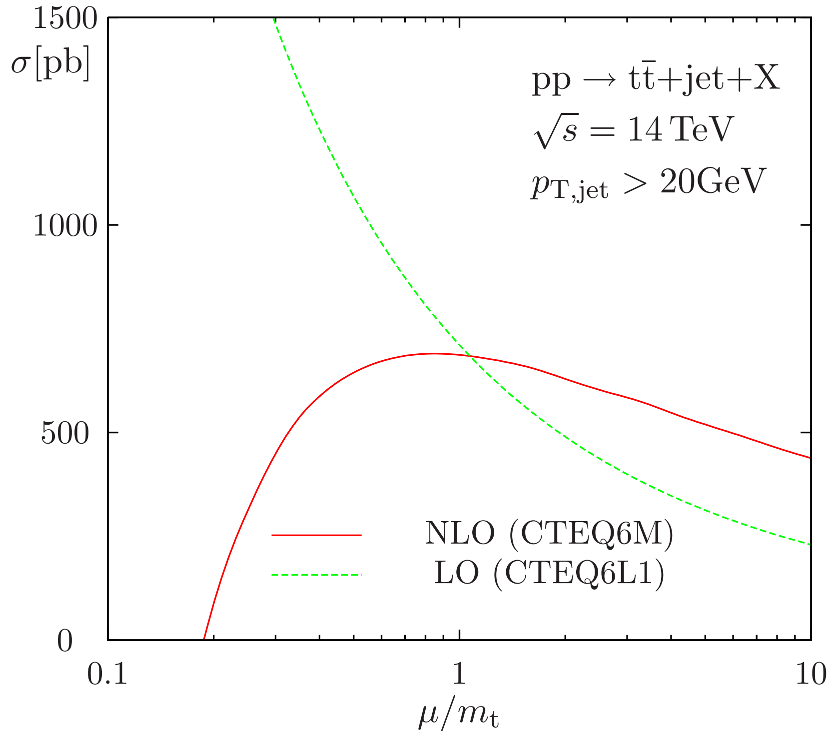
- Illustraton of correlations between gluon PDF $g(x)$, $\alpha_s(M_Z)$ and $m_t(m_t)$
  CMS coll. `arXiv:1812.10505`
  - fixed values of $\alpha_s(M_Z)$ affect value of $m_t(m_t)$



Top-quark pairs with one jet

- LHC: large rates for production of $t\bar{t}$-pairs with additional jets
- NLO QCD corrections for $t\bar{t} + 1\text{jet}$ Dittmaier, Uwer, Weinzierl '07-'08
  - scale dependence greatly reduced at NLO
  - corrections for total rate at scale $\mu_r = \mu_f = m_t$ are almost zero





- Additional jet raises kinematical threshold
  - invariant mass $\sqrt{s_{t\bar{t}+1\text{jet}}}$

Top-quark mass with $t\bar{t}$ + jet*-samples*

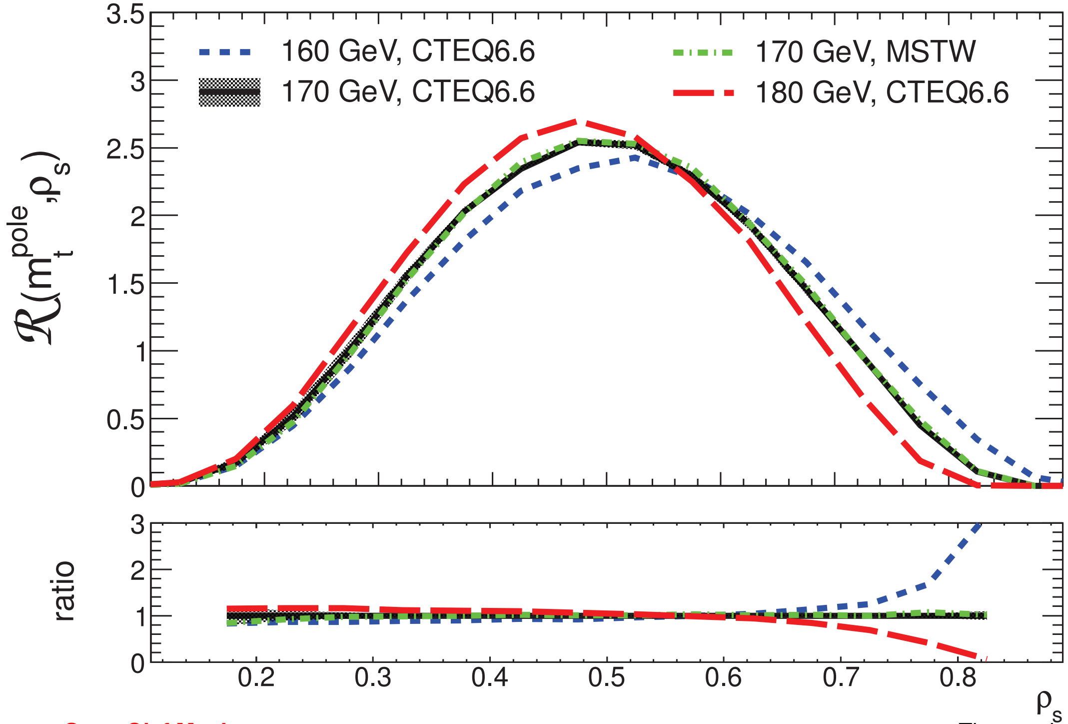
- Normalized-differential $t\bar{t}+\mathrm{jet}$ cross section

  Alioli, Fernandez, Fuster, Irles, S.M., Uwer, Vos '13

$$\mathcal{R}(m_t, \rho_s) = \frac{1}{\sigma_{t\bar{t}+1\mathrm{jet}}} \frac{d\sigma_{t\bar{t}+1\mathrm{jet}}}{d\rho_s}(m_t, \rho_s)$$

  - variable $\rho_s = \frac{2 \cdot m_0}{\sqrt{s_{t\bar{t}+1\mathrm{jet}}}}$ with invariant mass of $t\bar{t}+1\mathrm{jet}$ system and fixed scale $m_0 = 170\,\mathrm{GeV}$

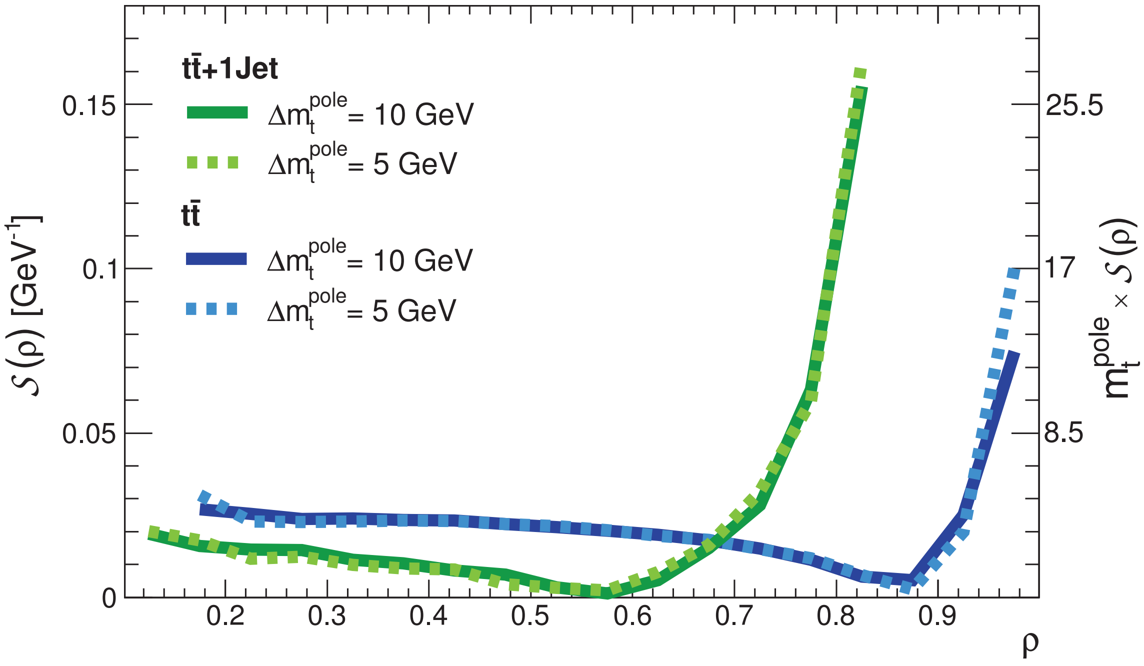
- Significant mass dependence for $0.4 \leq \rho_s \leq 0.5$ and $0.7 \leq \rho_s$



Mass sensitivity of $t\bar{t}$ + jet*-samples*

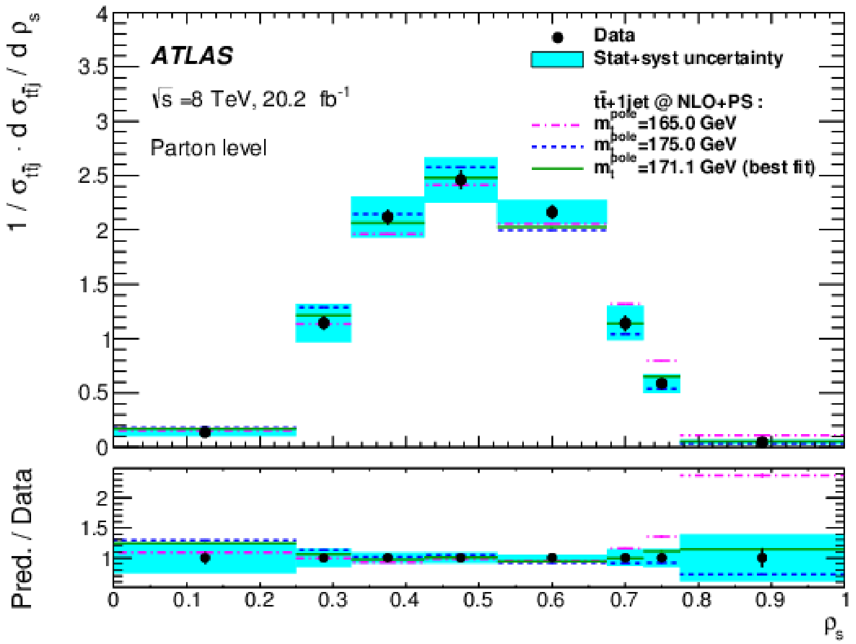
- Differential cross section $\mathcal{R}(m_t, \rho_s)$
  - good pertubative stability, small theory uncertainties, small dependence on experimental uncertainties, . . .
- Increased sensitivity for system $t\bar{t}$ + jet compared

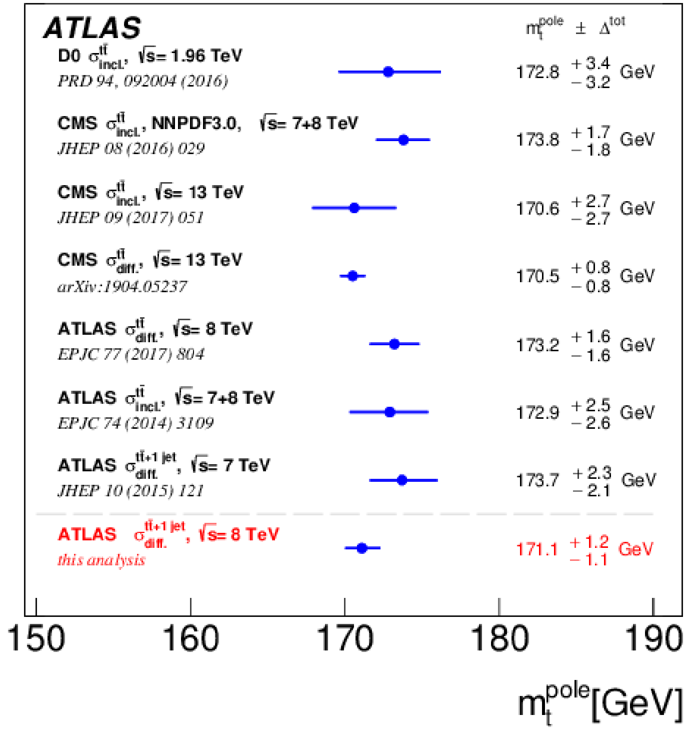
$$\left|\frac{\Delta\mathcal{R}}{\mathcal{R}}\right| \simeq (m_t\mathcal{S}) \times \left|\frac{\Delta m_t}{m_t}\right|$$



Top-quark mass determination (II)

- ATLAS measurement ATLAS coll. `arXiv:1905.02302`
  - high precision data in threshold region $0.7 \leq \rho_s$
- top-quark pole mass $m_t^{\rm pole} = 171.1 \pm 0.4({\rm stat}) \pm 0.9({\rm syst}) \, {}^{+0.7}_{-0.3}({\rm theo})$

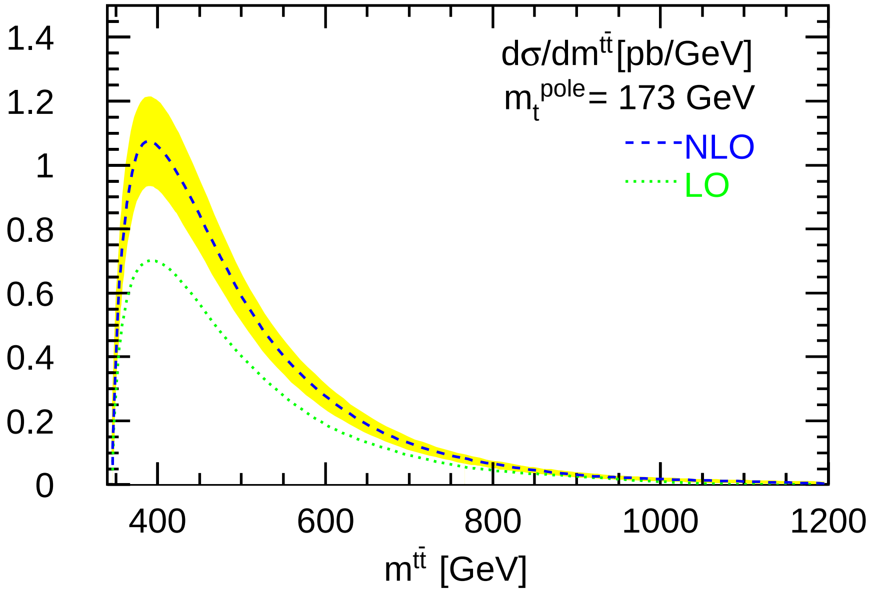


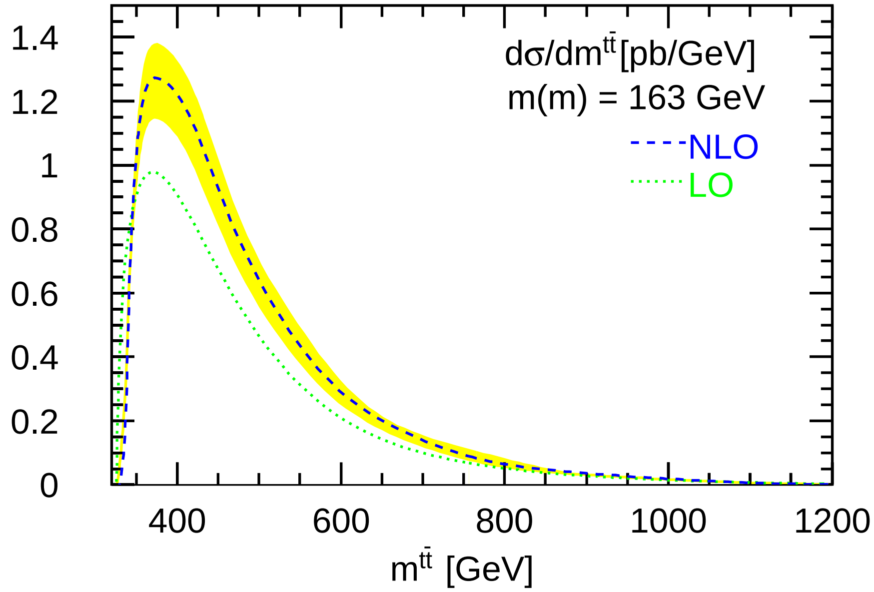


- Extraction of $\overline{\rm MS}$ mass $m_t(m_t) = 162.9 \pm 0.5({\rm stat}) \pm 1.0({\rm syst}) \, {}^{+2.1}_{-1.2}({\rm theo})$
  - larger scale uncertainity for $\overline{\rm MS}$ mass (sensitivity to threshold)
    Fuster, Irles, Melini, Uwer, Vos '17

Differential cross sections

NLO in QCD

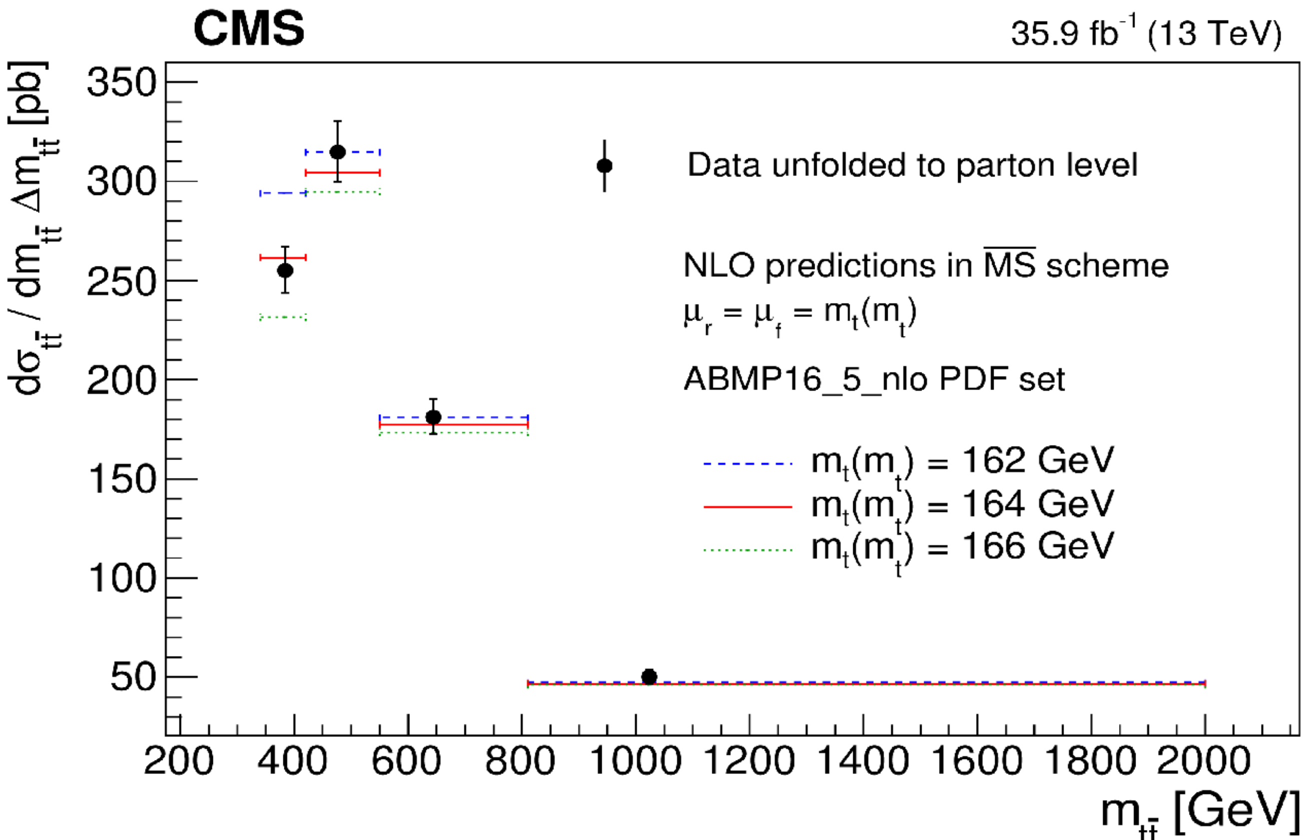




- Differential distributions for running mass show same features, e.g., $m_{t\bar{t}}$-distribution Dowling, S.M. '13
  - better convergence of perturbative expansion
  - smaller theoretical uncertainity from scale variation
- Measurement of $m(\mu = m_{t\bar{t}})$ with high statistics at $\sqrt{s} = 13$ TeV

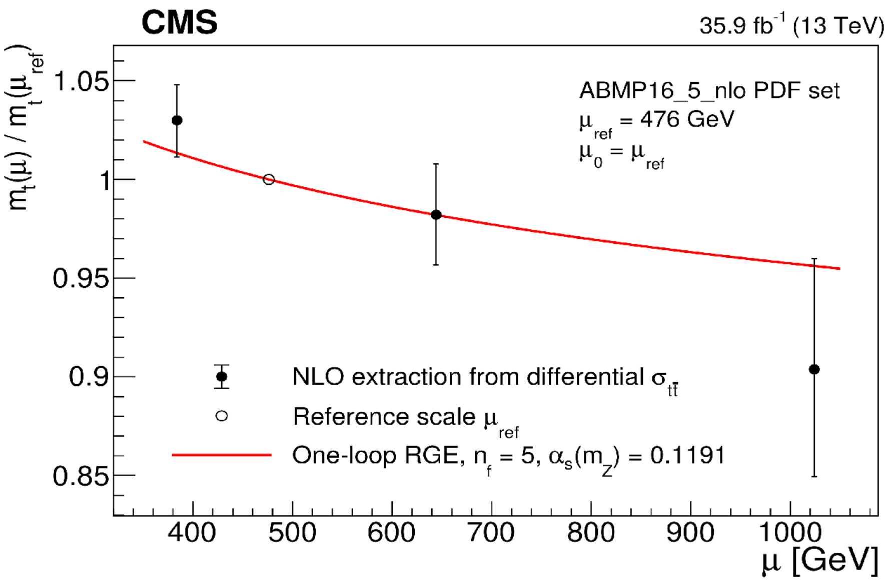
Top-quark mass determination (III)

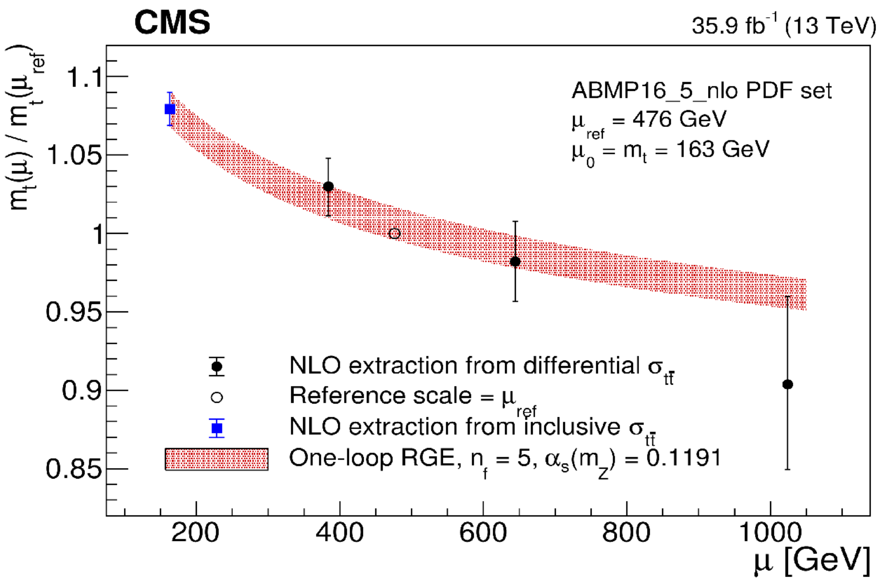
- Extraction of $\overline{\mathrm{MS}}$ mass $m_t(m_t)$
  - high statistics data on $t\bar{t}$-production
    CMS coll. arXiv:1909.09193
- Differential cross section $d\sigma/dm_{t\bar{t}}$
  - top-quark mass determintation in each bin



Running top-quark mass

- Running of top-quark mass
  CMS coll. `arXiv:1909.09193`
  - scale dependence of ttop-quark massop-quark mass $m(\mu = m_{t\bar{t}})$
  - ratio $m(\mu)/m(\mu_{\rm ref})$ cancels systematics





Summary

New extractions of top-quark mass

- $\overline{\mathrm{MS}}$ mass from data on $t\bar{t}$-production
  CMS coll. arXiv:1812.10505
  - NNLO accuracy in QCD
  - account of correlations with PDFs and value of $\alpha_s(M_Z)$
- Pole mass from data on $t\bar{t} + 1\mathrm{jet}$ production
  ATLAS coll. arXiv:1905.02302
  - cancellation of systematics in the normalized distribution
- Running top-quark mass from differential data on $t\bar{t}$-production
  CMS coll. arXiv:1909.09193
  - running over range of scales $\mu \in 400 \dots 1000$ GeV
  - systematics cancel in normalized mass ratio

Future tasks

- Joint effort theory and experiment
- Use of NNLO QCD predictions in analyses of differential distributions
- Push towards NNLO in QCD for $t\bar{t} + 1\mathrm{jet}$