Summary Status and Issues for (not only NLO) QED/EW corrections for weak mixing angle and mW

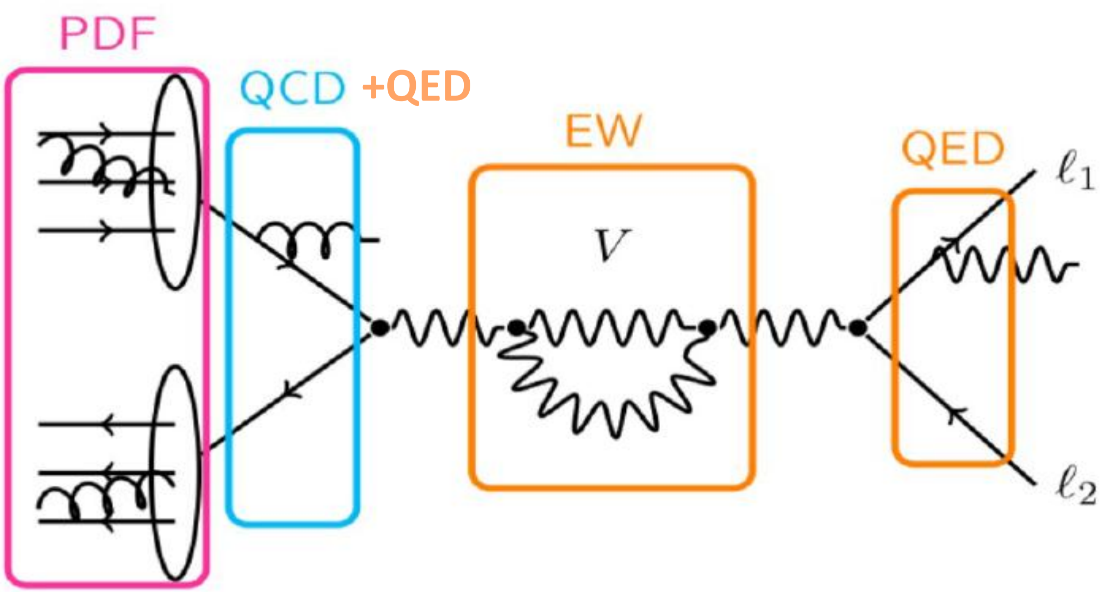
E. Richter-Was (IF UJ, Krakow)

- EW and QED corrections for $\sin^2{}_W$ measurement
  - EW schemes: from LEP to LHC
  - Observables, Pseudo-observables
  - Role of MC generators, weighting, semi-analytical calculations
- EW and QED corrections for m_W measurement

Please note:
NLO EW is insufficient for both $\sin^2\theta_{eff}$ and m_W !
We know from LEP times that fixed-order is not necessarily the right way to access corrections for precision measurements sensitive to QED/EW corrections.
For m_W, QED FSR beyond state-of-art YFS or Photos calculations may be needed.

Physics Modeling

- The Monte Carlo generator which does „all in one" with required precision is not existing yet:
  - NNLO QCD + resummation
  - Multi-photon QED emission with exponentiation and pair creation
  - EW corrections with theoretical precision below 5 10-5 for $\sin^2\theta_{eff}$
- We split modeling into components and make the best of existing tools:
  - Powheg_ew, MCSANC, Dizet form-factors, TauSpinner wt^{EW}, Photos, Winhac
  - HORACE, KKMC-hh



For overview of different tools see eg. talks at recent, „Ultimate Precision Workshop", indico.cern.ch/event/835066

Available codes (beyond LO)

wt^{EW} : TauSpinner + Dizet 6.21, 6.42, 6.45

↑ with updated $\alpha(M_Z)$

wt^{EW}

MC events
EW LO

Arbitrary
EW setup

- $\alpha(0)$ v0: LO, NLO+HO
- $\alpha(0)$ v1: LO
- G_μ: LO
- $\sin^2_{eff}$: LO

Powheg_ew: QCD LO, Z

- $\alpha(0)$ v0: LO
- $\alpha(0)$ v1: LO, NLO, NLO+HO
- G_μ: LO, NLO, NLO+HO
- $\sin^2_{eff}$: LO, NLO, NLO+HO

MCSANC: QCD LO, Z

- $\alpha(0)$ v1: LO, NLO, NLO+HO
- G_μ: LO, NLO, NLO+HO

New developments

EW schemes: input + renorm. counterterms

D. Bardin et al.
arXiv:9908433

- **LEP legacy: ($\alpha(0)$, G_μ, M_Z)**
  - Inputs are very precisely measured physics quantities
  - M_Z, M_W are on-shell masses
  - Genuine EW and lineshape corrections in form of (multiplicative) form-factors to LO couplings
  - Implemented in Dizet library.

S. Dittmaier, M. Huber
arXiv:0911.2329

- **LHC paradigm: (G_μ, M_Z, M_W).**
  - M_Z, M_W are pole-masses or complex masses.
  - Absorbs most of universal corrections into lowest-order couplings.
  - Higher-order corrections redefine couplings in non-multiplicative manner.

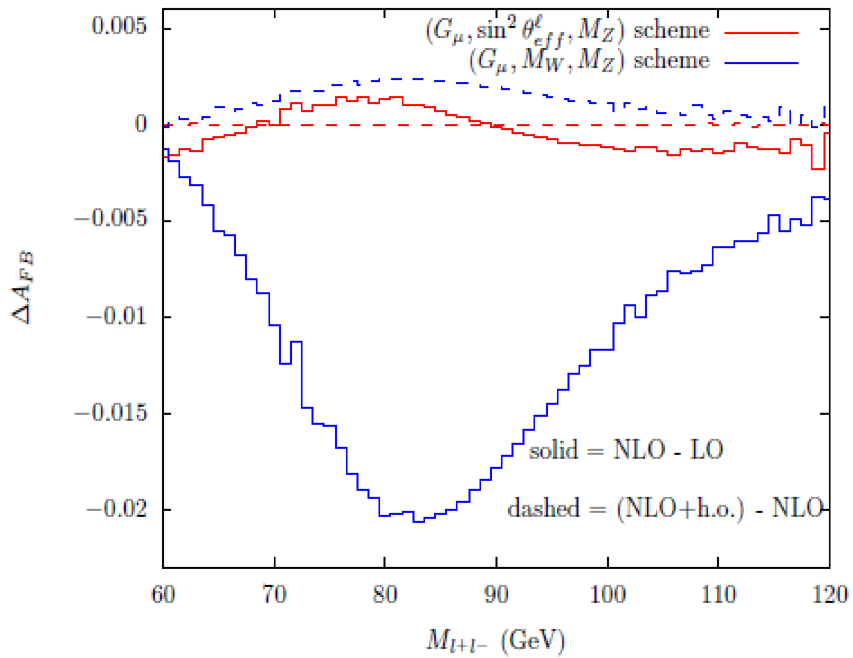
$$\boxed{s_W^2 \to \bar{s}_W^2 \equiv s_W^2 + \Delta\rho\, c_W^2\,, \qquad c_W^2 \to \bar{c}_W^2 \equiv 1 - \bar{s}_W^2 = (1 - \Delta\rho)\, c_W^2\,.}$$

Is it equivalent to $\sin^2\theta_{eff}$?

- **Recent development : (G_μ, $\sin^2\theta_{eff}$, M_Z)**
  - Proposed already in 90-ties, but not developed at that time
  - The weak mixing angle at Z-pole matches tree-level expression to all orders following LEP definition: real part of ratio of gV, and gA couplings.
  - Ones input $\sin^2\theta_{eff}{}^{l}$= measured value at LEP, corrections to A_{FB} become very small.
  - Implemented in Powheg_ew Monte Carlo.

M.Chiesa et al.,
arXiv:1906.11569

Predicting key observable: ΔA_{fb}



M.Chiesa, F. Piccinini, A. Vicini
arXiv:1906.11569 (PRD100, 071302 (2019))

- G_μ: G_μ, M_W, M_Z
- $\sin^2\vartheta^\ell_{eff}$: $\sin^2\vartheta^\ell_{eff} = 0.23147$, G_μ, M_Z

close to the value measured at LEP

M_{ll} = 89- 93 GeV
80-100 GeV

ΔA_{fb} = 1.6 10-2
1.6 10-2

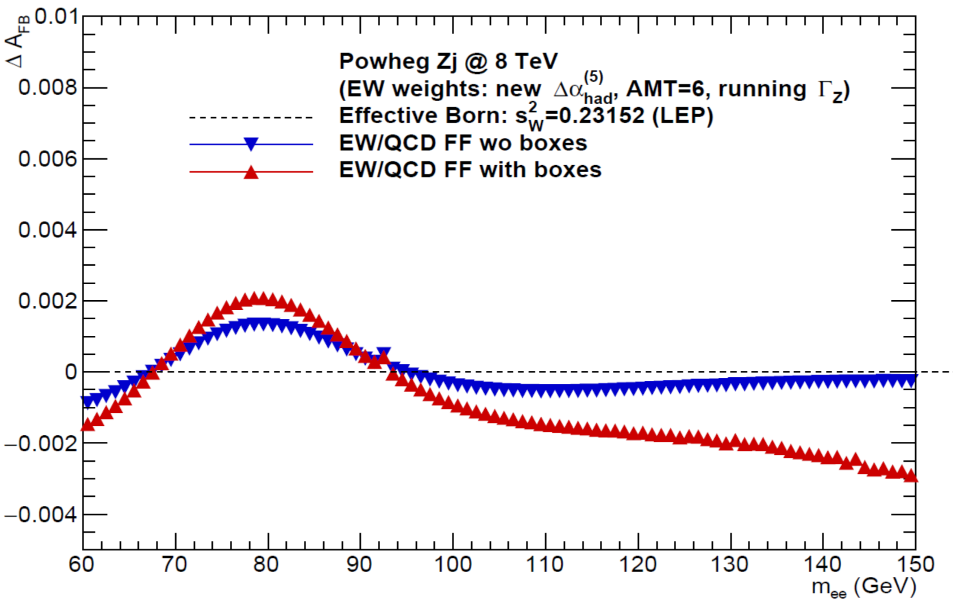
ΔA_{fb} = 2.0 10-4
1.7 10-4

A_{FB}	LO	NLO	NLO+HO
$\alpha0$ v1	0.046547(30)	0.030043(30)	0.030830(27)
	0.043445(26)	0.026908(27)	0.027702(22)
	0.042135(25)	0.025699(25)	0.026492(22)
G_μ	0.046548(30)	0.029058(28)	0.030903(27)
	0.043440(26)	0.025922(28)	0.027778(22)
	0.042130(25)	0.024719(26)	0.026569(22)
$\sin^2\vartheta^\ell_{eff}$ v1	0.030598(15)	0.030397(15)	0.030396(15)
	0.027437(13)	0.027267(13)	0.027267(13)
	0.026213(13)	0.026056(13)	0.026057(22)

Predicting key observable: ΔA_{fb}

Reference:
effective Born with LEP parametrisations

ERW and Z. Was, arXiv:1808.08616 (EPJ C79(2019), 480)



$\alpha(0)$, G_μ, M_Z

EW form-factors with Dizet 6.45

EW LO $\alpha(0)$ scheme	Effective Born *LEP*
$\alpha = 1/137.3599$ $s^2_W = 0.21215$ $\rho_{\ell f} = 1.0$	$\alpha = 1/128.8667$ $s^2_W = 0.23152$ $\rho_{\ell f} = 1.0$

Corrections to A_{FB}	$89 < m_{ee} < 93$ GeV	$80 < m_{ee} < 100$ GeV
A_{FB}(EW corr. m_W) - A_{FB}(EW LO $\alpha(0)$)	-0.02097	-0.02103
A_{FB}(EW corr. prop. $\chi(Z), \chi(\gamma)$) - A_{FB}(EW LO $\alpha(0)$)	-0.02066	-0.02098
A_{FB}(EW/QCD FF no boxes) - A_{FB}(EW LO $\alpha(0)$)	-0.03535	-0.03569
A_{FB}(EW/QCD FF with boxes) - A_{FB}(EW LO $\alpha(0)$)	-0.03534	-0.03567
A_{FB}(*LEP*) - A_{FB}(EW/QCD FF with boxes)	-0.00006	-0.00001
A_{FB}(*LEP with improved norm.*) - A_{FB}(EW/QCD FF with boxes)	-0.00005	-0.00002

Factor 3-20 (depending on mass window) smaller corrections than predicted by EW (G_μ, $\sin^2_{eff}$, M_Z) scheme. Needs to be understood.

EW schemes: benchmark input parameters

SM fundamental relation used to calculate EW LO parameters for different schemes. On-shell mass.

Parameter	$(\alpha(0), G_\mu, M_Z)$ $\alpha(0)$ v0	$(\alpha(0), M_W, M_Z)$ $\alpha(0)$ v1	(G_μ, M_Z, M_W) G_μ	$(\alpha(0), s_W^2, M_Z)$ $\sin^2_{eff}$ v1	(G_μ, s_W^2, M_Z) sin^2_{eff} v2
M_Z (GeV)	91.1876	91.1876	91.1876	91.1876	91.1876
Γ_Z (GeV)	2.4952	2.4952	2.4952	2.4952	2.4952
Γ_W (GeV)	2.085	2.085	2.085	2.085	2.085
$1/\alpha$	137.035999139	137.035999139	132.23323	137.035999139	128.744939484
α	0.007297353	0.007297353	0.007562396	0.007297353	0.007767296
G_μ (GeV^{-2})	$1.1663787 \cdot 10^{-5}$	$1.1254734 \cdot 10^{-5}$	$1.1663787 \cdot 10^{-5}$	$1.09580954 \cdot 10^{-5}$	$1.1663787 \cdot 10^{-5}$
M_W (GeV)	80.93886	80.385	80.385	79.93886984	79.93886984
s_W^2	0.2121517	0.2228972	0.2228972	0.231499	0.231499
$\frac{G_\mu \cdot M_Z^2 \cdot 16 c_W^2 s_W^2}{\sqrt{2} \cdot 8\pi \cdot \alpha} = 1.0$ $s_W^2 = 1 - m_W^2 / m_Z^2$	$\rightarrow s_W^2, M_W$	$\rightarrow G_\mu, s_W^2$	$\rightarrow \alpha, s_W^2$	$\rightarrow G_\mu, m_W$	$\rightarrow \alpha, m_W$
$\alpha_s(M_Z)$	0.120178900000	0.120178900000	0.120178900000	0.120178900000	0.120178900000

$$\boxed{s_W^2 = 1 - m_W^2 / m_Z^2}$$

$$\boxed{G_\mu = \frac{\pi\alpha}{\sqrt{2} M_W^2 s_W^2}}$$

Pseudo-observables at Z-pole: benchmarks

„Best predictions” in each EW scheme, i.e. EW NLO+HO

Dizet v6.45 | Powgeh_ew

Parameter	$(\alpha(0), G_\mu, M_Z)$ $\alpha(0)$ v0	$(\alpha(0), M_W, M_Z)$ $\alpha(0)$ v1	(G_μ, M_Z, M_W) G_μ	$(\alpha(0), s_W^2, M_Z)$ $\sin^2_{eff}$ v1	(G_μ, s_W^2, M_Z) $\sin^2_{eff}$ v2
M_Z (GeV)	91.1876	91.1876	91.1876	91.1876	91.1876
$1/\alpha(M_Z)$	0.0077549256				
$\alpha(M_Z)$	128.9503020				
G_μ (GeV^{-2})	$1.1663787 \cdot 10^{-5}$		$1.1663787 \cdot 10^{-5}$		$1.1663787 \cdot 10^{-5}$
M_W (GeV)	80.358935	80.385	80.385		
s_W^2	0.223401084	0.22289722	0.22289722		
$\sin^2\theta^\ell_{eff}$	0.231499			0.231499	0.231499
$\sin^2\theta^u_{eff}$	0.231392				
$\sin^2\theta^d_{eff}$	0.231265				
$\sin^2\theta^b_{eff}$	0.232733				

Experiments measure observables: cross-sections, asymmetries, distributions.
We need predictions to interpret these measurements. For now only Dizet provides predictions for LEP-style pseudo-observables.

Dizet 6.45

A. Arbuzov, L. Kalinovskaya, T. Riemann, S. Riemann, V. Yermolchyk

$\Delta\alpha^{(5)}_{hadr}$
- F. Jegerlehner, 2016
- F. Jegerlehner, 2017

$sin^2\theta_{eff}$
- $sin^2\theta^f_{eff}$, arXiv:0608099
  $sin^2\theta^b_{eff}$, arXiv:1607.08375
- $sin^2\theta_{eff}$, arXiv:1906.08815

IHVP	Realization	Result at M_Z
1	Jegerlehner(1995)	2.8039e-2
4	Jegerlehner(2016)	2.7586e-2
5	Jegerlehner(2019)	2.7604e-2

IAMT4	Description
6	$sin^2\theta^{lept}_{eff}$ with fermionic two-loop correction by Awramik, Czakon, Freitas, Weiglein (april 2004)
7	$sin^2\theta^f_{eff}$ & $sin^2\theta^b_{eff}$ with complete two-loop correction according arXiv:hep-ph/0608099 & arXiv:1607.08375
8	$sin^2\theta_{eff}$ with complete two-loop correction according arXiv:1906.08815

New best options:

IHVP=5

IAMT4=8

Dizet 6.45

```
PARAMETER(ALFAI=137.035999139D0,ALFA=1.D0/ALFAI,CONS=1.D0)
PARAMETER(ZMASS=91.1876D0,TMASS=173.0D0,HMASS=125.0D0)
PARAMETER(ALFAS=0.120178900000D0)
1 - old default, 4,5 - new options for dal5h
NPARD(1) = 5
6 - old default, 7,8 - new option for SIN2_EFF
NPARD(2) = 8
NPARD(3) = 3
NPARD(4) = 1
NPARD(5) = 0
NPARD(6) = 0
NPARD(7) = 3
NPARD(8) = 0
NPARD(9) = 0
NPARD(10)= 2
NPARD(11)= 1
NPARD(12)= 0
NPARD(13)= 0
NPARD(14)= 0
NPARD(15)= 3
NPARD(16)= 1
NPARD(17)= 1
NPARD(18)= 0
NPARD(19)= 3
NPARD(20)= 2
NPARD(21)= 1
NPARD(22)= 0
NPARD(23)= 1
NPARD(24)= 0
NPARD(25)= 0
```

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```
DIZET input parameters:
ZMASS    91.1876000000003       TMASS    173.000000000000
HMASS    125.000000000000       WMASS    0.00000000000000
DAL5H    0.00000000000000       ALQED5   137.035999139000001
ALFAS    0.120178900000001

DIZET results:
SIN2TW      0.22340108421408789
WMASSsin    80.358935565128405
WMASS       80.358935565128405
DAL5H       2.7576193213462830E-002
ALQED5      128.95030205596581
ALST        0.10941419165669332
ALPAS       0.120178900000001

CHANNEL          WIDTH         RHO_F_R       RHO_F_T        SIN2_EFF
-------        -------        --------      --------       --------
nu,nubar        167.203       1.007967      1.007967       0.231118
e+,e-            83.986       1.005223      1.005065       0.231499
mu+,mu-          83.985       1.005223      1.005065       0.231499
tau+,tau-        83.795       1.005223      1.005065       0.231499
u,ubar          300.206       1.005815      1.005760       0.231392
d,dbar          383.069       1.006736      1.006727       0.231265
c,cbar          300.149       1.005815      1.005760       0.231392
s,sbar          383.069       1.006736      1.006727       0.231265
t,tbar            0.000       0.000000      0.000000       0.000000
b,bbar          376.097       0.994186      0.994186       0.232733
hadron         1742.704
total          2496.078

W-widths
lept,nubar      678.985
down,ubar      1412.014
total          2090.999
*****************************************************
```

Control printout

In preparation: Dizet 7.0

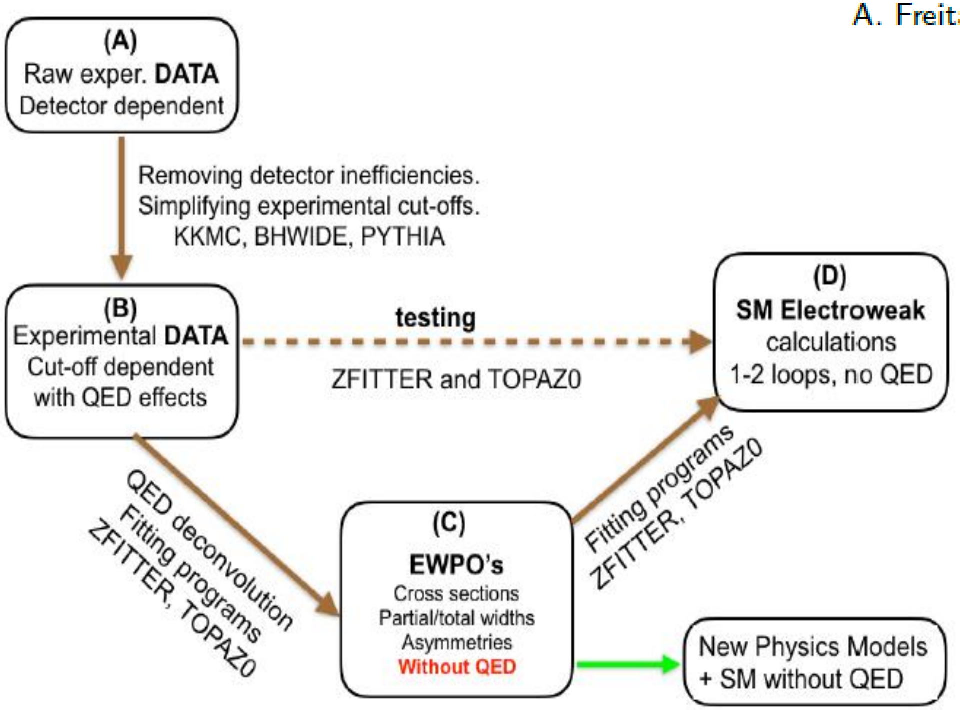
A. Arbuzov, L. Kalinovskaya, T. Riemann, S. Riemann, V. Yermolchyk

1) Implementation of the results from paper **'Electroweak pseudo-observables and Z-boson form factors at two-loop accuracy'** of Ievgen Dubovyk, Ayres Freitas, Janusz Gluza, Tord Riemann and Johann Usovitsch (arXiv:1906.08815), which provides complete fitting formula for **Z-boson decay widths**, **branching ratios** and **cross-section**.

2) Implementation of new electroweak schemes: (G_f, M_Z, M_W), $(G_f, \sin\theta_{eff}, M_Z)$.

Electroweak Pseudo-Observables at LEP: the meeting point between data and theory

A. Freitas, J. Gluza, S. Jadach, in arXiv:1809.01830



EWPOs at LEP1 (example OPAL)

$\chi^2/\mathrm{dof} = 155/194$	
m_Z [GeV]	91.1858 ± 0.0030
Γ_Z [GeV]	2.4948 ± 0.0041
σ^0_{had} [nb]	41.501 ± 0.055
R^0_e	20.901 ± 0.084
R^0_μ	20.811 ± 0.058
R^0_τ	20.832 ± 0.091
$A^{0,e}_{\mathrm{FB}}$	0.0089 ± 0.0045
$A^{0,\mu}_{\mathrm{FB}}$	0.0159 ± 0.0023
$A^{0,\tau}_{\mathrm{FB}}$	0.0145 ± 0.0030

Pseudo-observables:

cross-sections, ratios, asymmetries, corrected for QED and interpolated to the Z-pole.

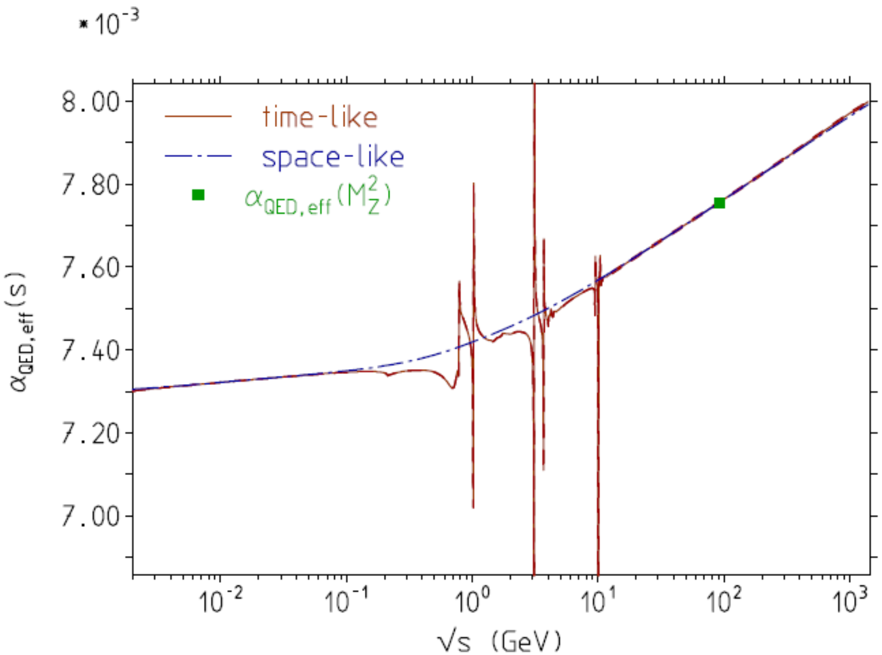
$\alpha_{QED}(s)$ and $\sin^2\theta_W(s)$

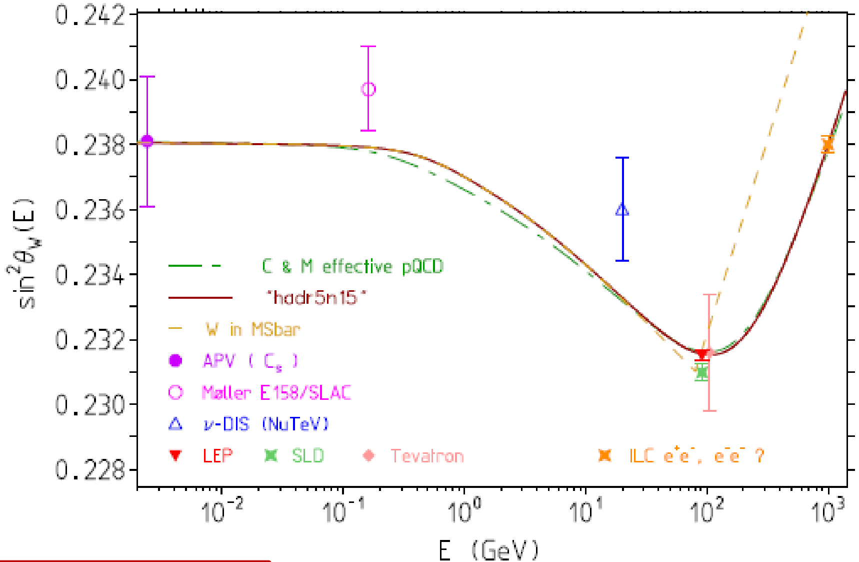
$$\alpha(s) = \frac{\alpha}{1-\Delta\alpha(s)}\ ; \quad \Delta\alpha(s) = \Delta\alpha_{\rm lep}(s) + \Delta\alpha^{(5)}_{\rm had}(s) + \Delta\alpha_{\rm top}(s)$$

F. Jegerlehner
FCCee WS 2019

$$\sin^2\Theta_i\ \cos^2\Theta_i\ = \frac{\pi\,\alpha}{\sqrt{2}\,G_\mu\,M_Z^2}\frac{1}{1-\Delta r_i}$$

$$\Delta r_i = \Delta r_i(\alpha,\ G_\mu,\ M_Z, m_H,\ m_{f\neq t},\ m_t)$$





$$\boxed{\Delta\alpha^{(5)}_{\rm hadrons}(M_Z^2) = 0.027738 \pm 0.000158\ [0.027523 \pm 0.000119]}$$

Open issue: Weak mixing angle @ LHC

D. Bardin et al.
arXiv:9908433

Using the formalism of Improved Born Approximation (Dizet)

LEP

$$\boxed{\sin^2\theta^f_{eff} = Re(K^f_Z) s^2_W + I^2_f}$$

$$g^f_Z = \frac{v_f}{a_f} = 1 - 4|q_f|(K^f_Z s^2_W + I^2_f)$$

$$s^2_W = 1 - M^2_W/M^2_Z$$

$$I^2_f = \alpha^2(s)\frac{35}{18}[1 - \frac{8}{3}Re(K^f_Z)s^2_W] = \sim 10^{-4}$$

We can extend definition outside the Z-pole

$$v_\ell = (2 \cdot T^\ell_3 - 4 \cdot q_\ell \cdot s^2_W \cdot \boxed{K_\ell(s,t)})/\Delta$$

$$v_f = (2 \cdot T^f_3 - 4 \cdot q_f \cdot s^2_W \cdot \boxed{K_f(s,t)})/\Delta$$

$$a_\ell = (2 \cdot T^\ell_3)/\Delta$$

$$a_f = (2 \cdot T^f_3)/\Delta$$

$$\Delta = \sqrt{16 \cdot s^2_W \cdot (1 - s^2_W)}$$

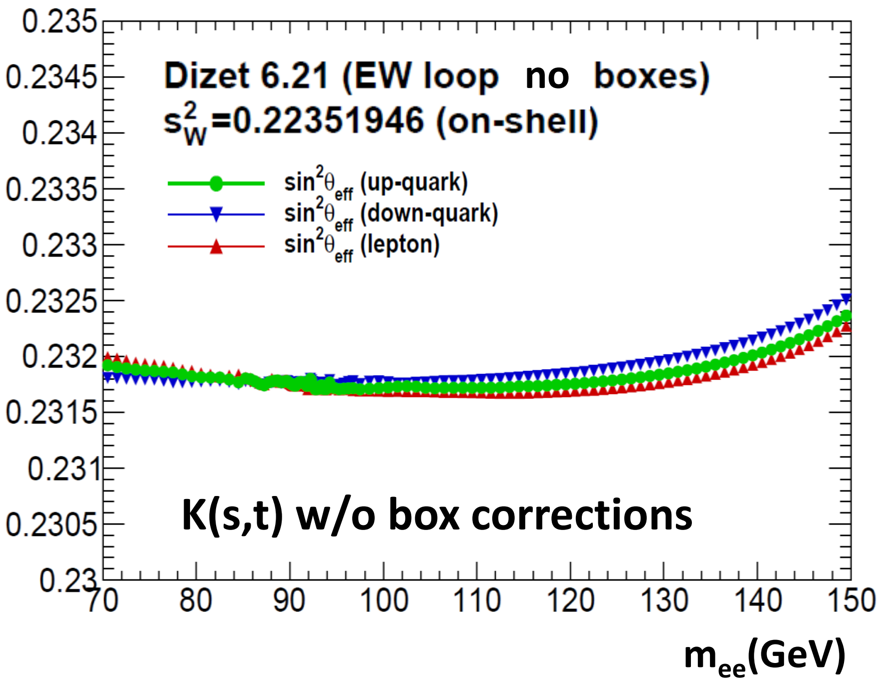
LHC

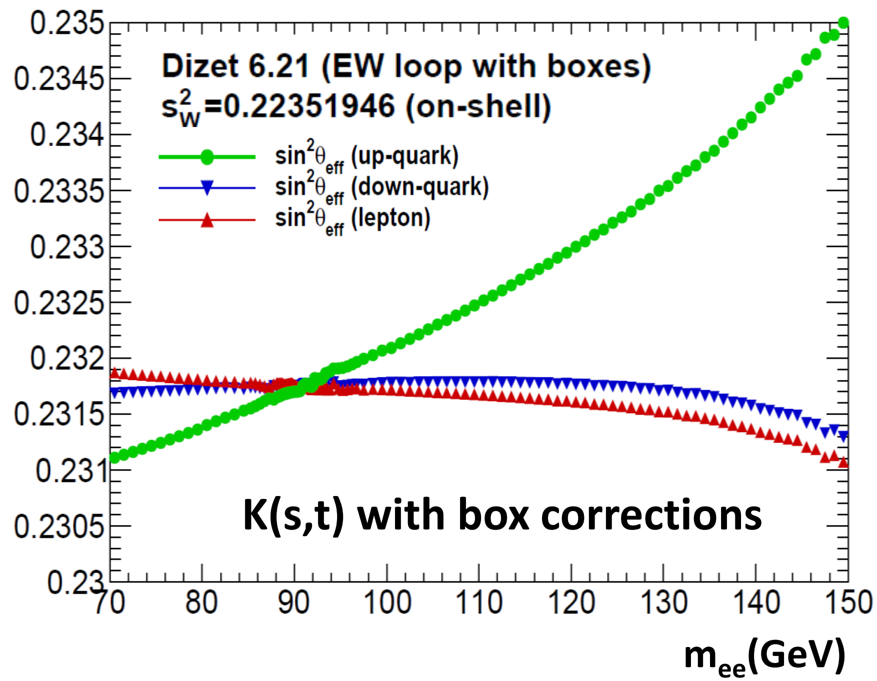
$$\boxed{\begin{aligned} \sin^2\theta^f_W(s,t) &= K^f(s,t) * s^2w = \\ &= K^f(s,t)/K^f_Z * \sin^2\theta^f_{eff} \end{aligned}}$$

EW form-factors , functions of $(s,t)=(m_{ll}, \cos\theta)$
Calculated with Dizet library.

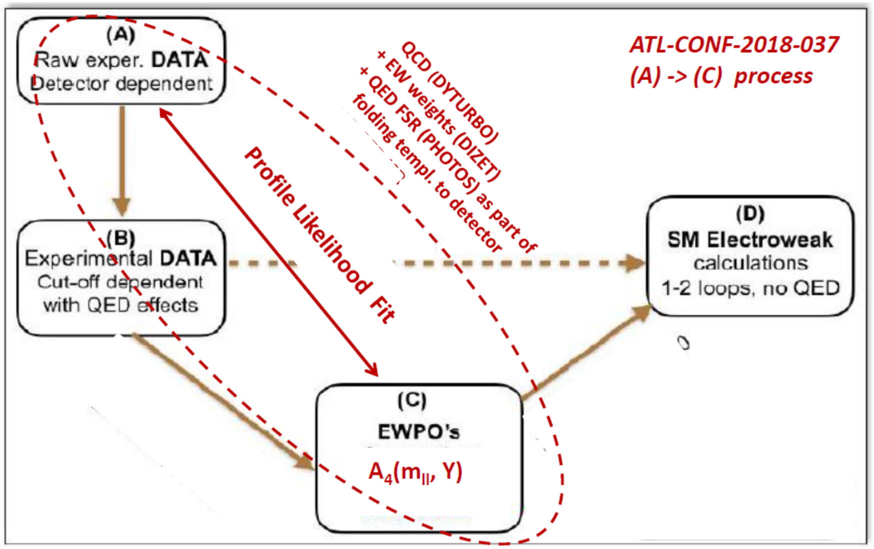
Effective weak mixing angle @ LHC

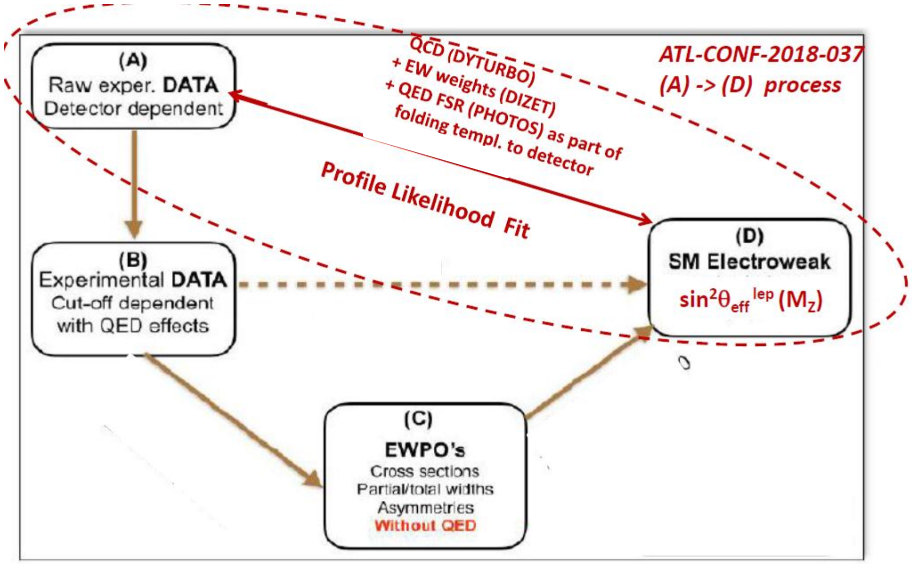
$$\sin^2\theta_W^f(s,t) = \mathrm{Re}(K^f(s,t))*s_W^2 = \mathrm{Re}(K^f(s,t)/K_Z^f) * \sin^2\theta^f_{eff}(M_Z)$$





Electroweak Pseudo-Observables at LHC: the meeting point between data and theory





$$\boxed{N^n_{\mathrm{exp}}(A,\sigma,\theta) = \left\{\sum_{j=0}^{\mathrm{N_{bins}}} \sigma_j \times L \times \left[t^n_{8j}(\beta) + \sum_{i=0}^{7} A_{ij} \times t^n_{ij}(\beta)\right]\right\} \times \gamma^n + \sum_{B}^{\mathrm{bkgs}} T^n_B(\beta),}$$

$$\boxed{A_{4,j}(\sin^2\theta^\ell_{\mathrm{eff}},\ \theta) = a_j(\theta) \times \sin^2\theta^\ell_{\mathrm{eff}} + b_j(\theta)}$$

Electroweak Pseudo-Observables at LHC: the meeting point between data and theory

ATL-CONF-2018-037

	$70 < m^{\ell\ell} < 80$ GeV			$80 < m^{\ell\ell} < 100$ GeV				$100 < m^{\ell\ell} < 125$ GeV		
$\lvert y^{\ell\ell}\rvert$	$0-0.8$	$0.8-1.6$	$1.6-2.5$	$0-0.8$	$0.8-1.6$	$1.6-2.5$	$2.5-3.6$	$0-0.8$	$0.8-1.6$	$1.6-2.5$
Central value (NNLO QCD)	-0.0870	-0.2907	-0.5970	0.0144	0.0471	0.0928	0.1464	0.1045	0.3444	0.6807
ΔA_4 (NNLO - NLO QCD)	0.0003	0.0010	0.0021	-0.0001	-0.0005	-0.0009	-0.0015	-0.0007	-0.0022	-0.0041
ΔA_4 (EW)	0.0008	0.0028	0.0056	0.0002	0.0007	0.0015	0.0026	-0.0008	-0.0026	-0.0048
$\Delta \sin^2\theta^\ell_{\mathrm{eff}}$ (EW)	0.00129	0.00130	0.00133	0.00024	0.00024	0.00025	0.00026	-0.00120	-0.00123	-0.00119
	Uncertainties			Uncertainties				Uncertainties		
Total	0.0035	0.0094	0.0137	0.0007	0.0017	0.0021	0.0021	0.0040	0.	0.0140
PDF	.0034	0.0092	[illegible]	0.0007	0.0016	0.0020	0.0019	0.0039	0.0100	0.0131
QCD scales	0.0006	0.0019	0.0052	0.0003	0.0003	0.0004	0.0008	0.0005	0.0022	0.0049

Table 2: Predicted values of A_4 at NNLO in QCD for the ten measurement bins in $(m^{\ell\ell}, |y^{\ell\ell}|)$ space, together with their uncertainties from PDFs, as obtained from the MMHT14 PDF set, and from factorisation and renormalisation scale variations, as obtained from the NLO predictions. The impact, ΔA_4 (NNLO - NLO QCD), of moving from NLO to NNLO QCD predictions is shown for each measurement bin. The predictions include the EW form factor corrections discussed in the text. The impact, ΔA_4 (EW), in each bin of these corrections on the predictions is also shown, as well as the impact, $\Delta \sin^2\theta^\ell_{\mathrm{eff}}$ (EW), on the value of $\sin^2\theta^\ell_{\mathrm{eff}}$ inferred from the predicted A_4.

Pseudo-observables or observables:

cross-sections and asymmetries (Afb, A4) , (unfolded to truth level) in different Mll, Y bins.

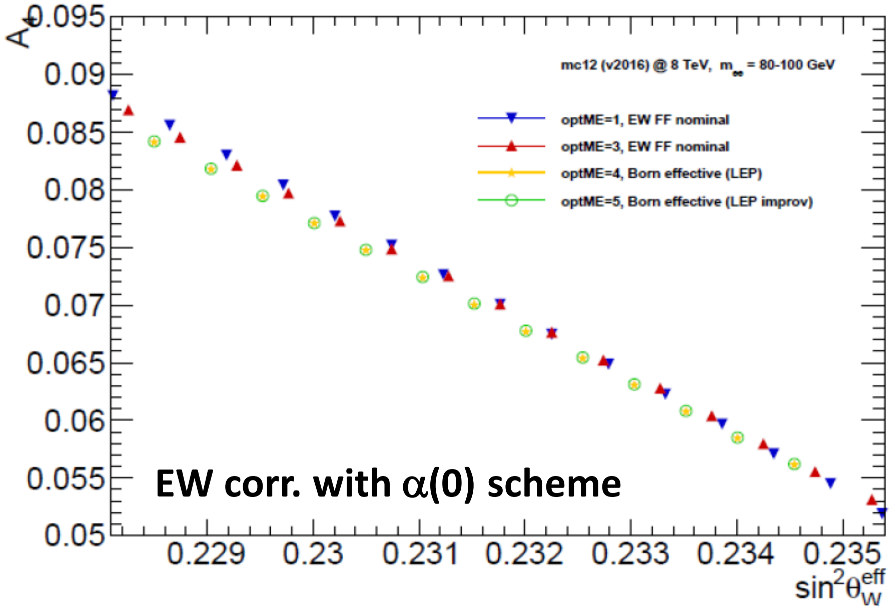
Δ $\sin^2{}_{\mathrm{eff}}{}^{\mathrm{lep}}$ (scan) -> ΔA_4(EW, QCD) predicted -> A4 (measured/fitted) –> $\sin^2{}_{\mathrm{eff}}{}^{\mathrm{lep}}$ (best)

$\sin^2\theta_{eff}$ scan for A_{FB} and/or A_4

Can be done with codes predicting $\sin^2\theta_{eff}$ explicitly

- **It is available with Dizet FF + wt^{EW} with $\alpha(0)$ scheme**
- **Main motivation for Powheg_ew to devellop new EW schemes was to have $\sin^2\theta_{eff}$ as an input parameter. We should now complete the comparison benchmark. Needs at least two additional $\sin^2\theta_W$ points, eg. $\sin^2\theta_{eff}$ = 0.231499 +- 0.00050 and then predicted slope of the A_{FB}.**

Example for A_4 with wt^{EW}

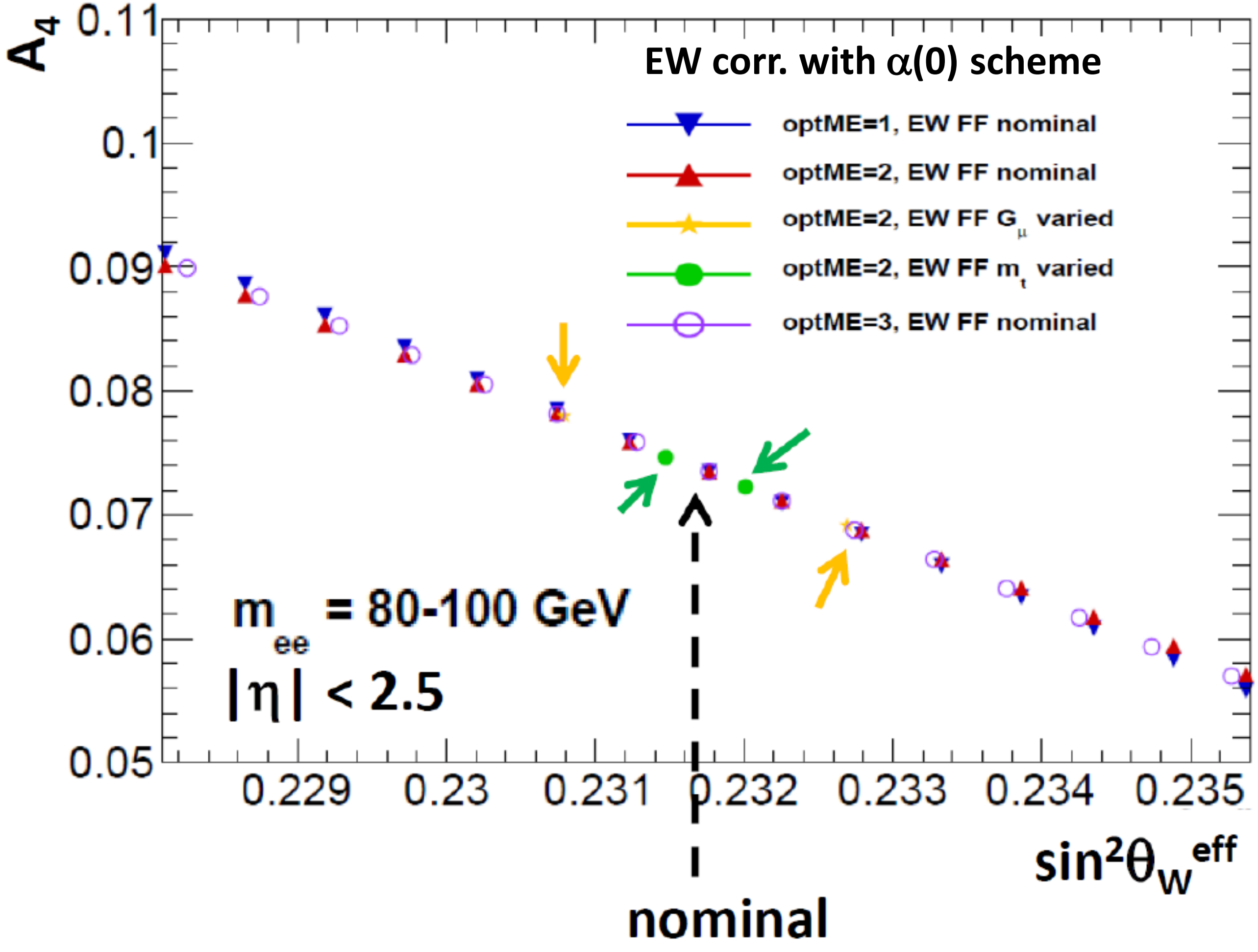


<u>Formulas used for this plot, varied δV</u>

$$
\begin{aligned}
v_\ell = &\quad (2 \cdot T_3^\ell - 4 \cdot q_\ell \cdot (s_W^2 \cdot K_\ell(s,t) + \delta_V))/\Delta \\
v_f = &\quad (2 \cdot T_3^f - 4 \cdot q_f \cdot (s_W^2 \cdot K_f(s,t) + \delta_V))/\Delta \\
vv_{\ell f} = &\quad \frac{1}{v_\ell \cdot v_f}[(2 \cdot T_3^\ell)(2 \cdot T_3^f) \\
&\quad -4 \cdot q_\ell \cdot (s_W^2 + \cdot K_f(s,t) + \delta_V)(2 \cdot T_3^\ell) \\
&\quad -4 \cdot q_f \cdot (s_W^2 \cdot K_\ell(s,t) + \delta_V)(2 \cdot T_3^f) \\
&\quad +(4 \cdot q_\ell \cdot s_W^2)(4 \cdot q_f \cdot s_W^2)K_{\ell f}(s,t) \\
&\quad +2 \cdot (4 \cdot q_\ell))(4 \cdot q_f \cdot) \cdot s_W^2 \cdot K_{\ell f}(s,t) \cdot \delta_V] \, \frac{1}{\Delta^2}
\end{aligned}
$$

$\sin^2\theta_{eff}$ scan for A_{FB} and/or A_4

Example for A_4 with wt^{EW}



Open Issues: choice of EW scheme

- **At LEP-times we used to have at least two independent calculations (Zfitter and TOPAZO), used/cross-checked by independent experimental groups + theory community brainstorming on every tiny detail.**
- **We should not be less prudent today when analysing LHC data. Fact that PDF uncertainties will dominate should not be the an excuse to be less strict for validating EW part.**
- **Each EW scheme used should be proven (numerically) to be compatible with the LEP-style definition when addressing the same physics observables. It might be the case already but we need to see it quantified: tables and plots.**

This work is ongoing now, but should involve more people, even if not directly providing numbers then formulating meanfull benchmarks.

Z-boson propagator

A. Freitas

Z lineshape

- Deconvolution of initial-state QED radiation:

$$\sigma[e^+e^- \to f\bar{f}] = \mathcal{R}_{\text{ini}}(s,s') \otimes \sigma_{\text{hard}}(s')$$

- Subtraction of γ-exchange, γ–Z interference, box contributions:

$$\sigma_{\text{hard}} = \sigma_Z + \sigma_\gamma + \sigma_{\gamma Z} \; ; \; \sigma_{\text{box}}$$

- Z-pole contribution:

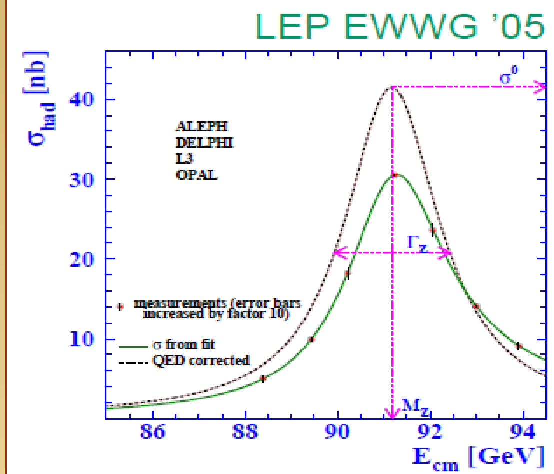
$$\sigma_Z = \frac{R}{(s-\overline{M}_Z^2)^2 + \overline{M}_Z^2\overline{\Gamma}_Z^2} + \sigma_{\text{non-res}}$$

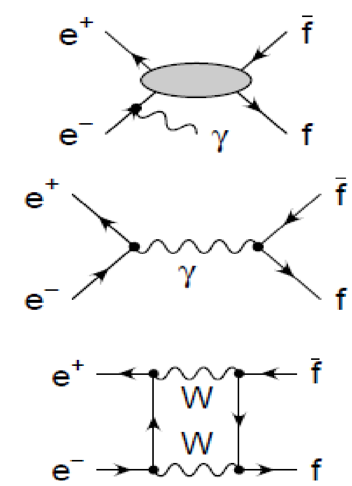
- In experimental analyses: No clear definition of $\sigma_{\text{non-res}}$ in running width scheme

$$\sigma \sim \frac{1}{(s-M_Z^2)^2 + s^2\Gamma_Z^2/M_Z^2}$$

$$\overline{M}_Z = M_Z/\sqrt{1+\Gamma_Z^2/M_Z^2} \approx M_Z - 34 \text{ MeV}$$

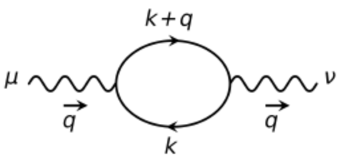
$$\overline{\Gamma}_Z = \Gamma_Z/\sqrt{1+\Gamma_Z^2/M_Z^2} \approx \Gamma_Z - 0.9 \text{ MeV}$$





EW LO schemes: running/fixed width

Running Z-boson width in the propagator is taking taking into account photon-loop corrections to Γ_Z



- **Fixed width** $$\chi_Z(s) = \frac{1}{s - M_Z^2 + i \cdot \Gamma_Z \cdot M_Z}.$$

- **Running width (LEP legacy)** $$\chi'_Z(s) = \frac{1}{s - M_Z^2 + i \cdot \Gamma_Z \cdot s/M_Z}$$

$$\begin{aligned}
\chi'_Z(s) &= \frac{1}{s(1 + i \cdot \Gamma_Z/M_Z) - M_Z^2} \\
&= \frac{(1 - i \cdot \Gamma_Z/M_Z)}{s(1 + \Gamma_Z^2/M_Z^2) - M_Z^2(1 - i \cdot \Gamma_Z/M_Z)} \\
&= \frac{(1 - i \cdot \Gamma_Z/M_Z)}{(1 + \Gamma_Z^2/M_Z^2)} \frac{1}{s - \frac{M_Z^2}{1+\Gamma_Z^2/M_Z^2} + i \cdot \frac{\Gamma_Z M_Z}{1+\Gamma_Z^2/M_Z^2}} \\
&= N_Z \frac{1}{s - {M'_Z}^2 + i\Gamma'_Z M'_Z} \\
M'_Z &= \frac{M_Z}{\sqrt{1 + \Gamma_Z^2/M_Z^2}} \\
\Gamma'_Z &= \frac{\Gamma_Z}{\sqrt{1 + \Gamma_Z^2/M_Z^2}}
\end{aligned}$$

$$\boxed{N_Z = \frac{(1 - i \cdot \Gamma_Z/M_Z)}{(1 + \Gamma_Z^2/M_Z^2)} = \frac{(1 - i \cdot \Gamma'_Z/M'_Z)}{(1 + {\Gamma'_Z}^2/{M'_Z}^2)}}$$

Both equivalent if redefined parameters m_Z, Γ_Z, N_Z (normalization).

Keeping N_Z is important for getting right interference term with photon contributions outside Z – pole, impacts Afb and is numerically relevant for $\sin^2_{eff}$ measurement.

Open Issues: Z-boson propagator

- **Discussed since fall last year, problem in nutshell.**
  - **LEP1 legacy (Dizet+Zfitter, experiments):**
    - use running width in the Born propagator
    - form-factors calculated with pole-mass/fixed width (internally converted), applied to Born with on-shell mass/running width
    - see references: hep-ex/0509008, hep-ph/9908433
  - **LEP2, LHC standard**
    - use complex-mass scheme, pole masses, fixed width propagator
  - **Zfitter+Dizet v6.42, v6.45, FCCee standard**
    - stayed with LEP1 convention

Is that a concern for $\sin^2\theta_{eff}$ measurement at LHC ?

For now we took pragmatic approach: use defaults of each code:

- Powheg_ew and MCSANC: pole-mass and fixed width propagator
- wt^{EW} : calculated with on-shell masses and running width propagator, as it is standard used by Zfitter+Dizet

We should keep it in mind, that ones we reach precision of the comparisons which might be sensitive to the effect of $\chi(s)$ implementation. It should be discussed as component of theoretical uncertainties of the predictions.

Theoretical and parametric uncertainties

ALDO+ SLD + Tevatron, arXiv:1012.2367

- The remaining theoretical uncertainties were estimated to be 4 MeV on m_W and 0.000049 on $\sin^2\theta_{eff}^{lep}$. We can use this estimate @ Z-pole.
- The parametric uncertainties were dominated by $\Delta\alpha_{had}(M_Z^2)$. The uncertainty of ±0.00035 caused an error of ±0.00013 on $\sin^2\theta_{eff}^{lep}$.

EW weights from TauSpinner/DIZET, $\alpha(0)$ scheme:

- The same code as ALDO for calculating EW genuine corrections. We can use this estimate.

  Theoretical uncertainties (EW, @Z-pole): 5 10^{-5} on $\sin^2\theta_{eff}^{lep}$
- Parametric uncertainties: 4 10-5 [35 10-6 (from $\Delta\alpha_{had}$) and 16 10-6 (from m_t)]

Parametric uncertainties: $\Delta\alpha_{had}$, m_t

@ Z-pole, $\alpha(0)$ scheme:

Parameter	$\Delta\alpha_h^{(5)}(M_Z^2)$ - 0.0001	$\Delta\alpha_h^{(5)}(M_Z^2) = 0.0275762$	$\Delta\alpha_h^{(5)}(M_Z^2)$ + 0.0001	$\Delta/2$
$\alpha(M_Z^2)$	0.0077540999	0.0077549240	0.0077557482	
$1/\alpha(M_Z^2)$	128.9640328306	128.9503292550	128.9366256793	
s_W^2	0.22340146	0.22343647	0.22347148	0.000035
$sin^2\theta_W^{eff}(M_Z^2)$ (electron, muon)	0.23150412	0.23153917	0.23157421	0.000035
$sin^2\theta_W^{eff}(M_Z^2)$ (up-quark)	0.23139759	0.23143261	0.23146763	0.000035
$sin^2\theta_W^{eff}(M_Z^2)$ (down-quark)	0.23127052	0.23130551	0.23134049	0.000035
M_W	80.35892 GeV	80.35710 GeV	80.35529 GeV	1.8 MeV
Δr	0.03625683	0.036609	0.03647535	
Δr_{rem}	0.01170310	0.01170287	0.01170264	

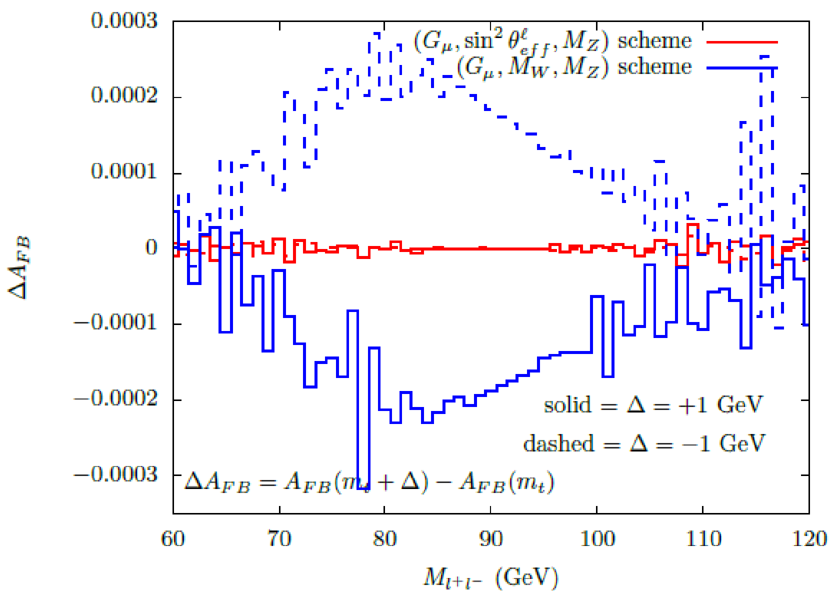
Parameter	m_t - 0.5 GeV	m_t = 173.2 GeV	m_t + 0.5 GeV	$\Delta/2$
$\alpha(M_Z^2)$	0.0077549205	0.0077549240	0.0077549274	
$1/\alpha(M_Z^2)$	128.9503873792	128.9503292550	128.9502716590	
s_W^2	0.22349450	0.22343647	0.22337836	0.000058
$sin^2\theta_W^{eff}(M_Z^2)$ (electron, muon)	0.23155486	0.23153917	0.23152344	0.000016
$sin^2\theta_W^{eff}(M_Z^2)$ (up-quark)	0.23144830	0.23143261	0.23141688	0.000016
$sin^2\theta_W^{eff}(M_Z^2)$ (down-quark)	0.23132119	0.23130551	0.23128979	0.000016
M_W	80.354102 GeV	80.35710 GeV	80.360111 GeV	3 MeV
Δr	0.03654697	0.036609	0.03618477	
Δr_{rem}	0.01169343	0.01170287	0.01171229	

Parametric uncertainties: 4 10-5 [35 10-6 (from $\Delta\alpha_{had}$) and 16 10-6 (from m_t)]

Parametric uncertainties: $\Delta\alpha_{had,}$ m_t

M.Chiesa, F. Piccinini, A. Vicini
arXiv:1906.11569 (PRD100, 071302 (2019))

m_t^2 dependence from $\Delta\rho$



EW $\sin^2$eff scheme:

- **Very small dependence of Afb on mt**
- **No parametric uncert. from mW**

EW G_μ scheme:

- **Large dependence of Afb on mt**
- **Large parametric uncert. from mW**

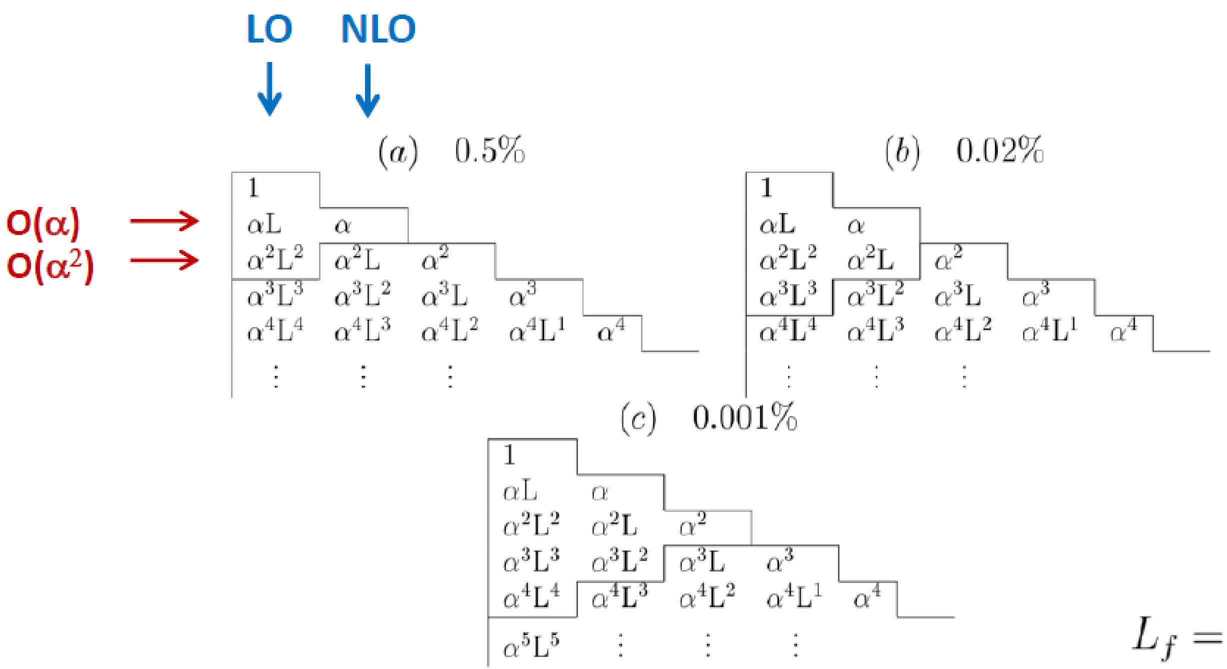
@ Z-pole, $\sin^2_{eff}$ scheme:

Parametric uncertainties: ??

QED corrections: LEP legacy

Precision: calculations are not done order-by-order

S. Jadach
FCCee WS 2019



$$L_f = \ln(s/m_f^2)$$

Figure 2: QED perturbative leading and subleading corrections. Rows represent corrections in consecutive perturbative orders – the first row is the Born contribution. The first column represents the leading logarithmic (LO) approximation and the second column depicts the next-to-leading (NLO) approximation. In the figure, terms selected for the same precision level (a) $5 \cdot 10^{-2}$ (b) $2 \cdot 10^{-3}$ and (c) $1 \cdot 10^{-5}$ are limited with the help of an additional line.

Residual QED corrections: LEP EWPO’s

After deconvoluting for QED effects, small residual dependence remains and added to experimental error.

Observable	Where from	Present
M_Z [MeV]	Z linesh. [2]	$91187.5 \pm 2.1\{0.3\}$
Γ_Z [MeV]	Z linesh. [2]	$2495.2 \pm 2.1\{0.2\}$
$R_l^Z = \Gamma_h/\Gamma_l$	$\sigma(M_Z)$ [3]	$20.767 \pm 0.025\{0.012\}$
σ^0_{had}	σ^0_{had} [2]	$41.541 \pm 0.037\{25\}$nb
N_ν	$\sigma(M_Z)$ [2]	$2.984 \pm 0.008\{0.006\}$
N_ν	$Z\gamma$ [4]	$2.69 \pm 0.15\{0.06\}$
$\sin^2\theta_W^{eff}$	$A_{FB}^{lept.}$ [3]	$0.23099 \pm 0.00053\{06\}$
$\sin^2\theta_W^{eff}$	$A^\tau_{pol.}$ [2, 3]	$0.23159 \pm 0.00041\{12\}$
M_W [MeV]	ADLO [5]	$80376 \pm 33\{7\}$
$A_{FB,\mu}^{M_Z \pm 3.5\text{GeV}}$	$\frac{d\sigma}{d\cos\theta}$ [2]	$\pm 0.020\{0.001\}$

S. Jadach
FCCee WS 2019

Bhabha scattering luminosity measurement

Table 1: Table of present experimental precision of electroweak observables, which are most sensitive to QED effects. The numbers in the braces {...} in the 3-rd column are component of the systematic error components due to QED calculation uncertainty.

QED ISR/IFI/FSR

- **QED FSR does not seem to be a concern.**
  - **Simulated with PHOTOS in the experimental analysis, now also option of including pair-creation**
  - **Tests performed in the past with KKMC, Powheg-ew or HORACE confirmed its addequatenes for LHC precision physics.**

- **QED IFI and ISR on the way to being estimated.**
  - **HORACE, Powheg_ew, MCSANC, KKMC-hh.**
  - **Good progress on convergence between different codes. As expected effect small and will be accounted for as systematics of the $\sin^2\theta_{eff}$ measurement.**

QED ISR/IFI/FSR

S.A.Yost et all.
KKMC-hh

KKMC-hh: Introduction

- KKMC-hh is an event generator for $pp \to f\bar{f} + n\gamma, f = e, \mu, \tau$, based on KKMC, which was used at LEP with a precision tag of 0.2% (LEP2).
- ISR and FSR γ emission are included to $\mathcal{O}(\alpha^2 L)$ including interference (IFI)
- The MC structure is based on CEEX (Coherent Exclusive Exponentiation), which is similar to YFS exponentiation but implemented at the level of spinor amplitudes.
- CEEX was introduced because traditional YFS exponentiation ("EEX" in KKMC-hh) suffers from a proliferation of interference terms, and is wll suited to calculating IFI.
- $\mathcal{O}(\alpha)$ electroweak corrections are added via DIZET 6.21.
- τ decay is implemented using TAUOLA.
- Events can be showered with HERWIG6.521 internally, or externally with any LHA-compatible shower.

QED ISR/IFI/FSR

Numerical Results

S.A.Yost et all.
KKMC-hh

Column 1 includes FSR only, with a non-QED PDF. Column 2 has FSR with LuxQED. Column 3 has KKMC-hh ISR + FSR with a non-QED PDF. Column 5 adds KKMC-hh IFI.

	1. No ISR	2. LuxQED	3. KKMC-hh ISR	4. %(ISR – no ISR)	5. With IFI	6. %(IFI – no IFI)
Uncut σ (pb)	939.86(1)	944.04(1)	944.99(2)	0.54597(2)%	944.91(2)	−0.083(4)%
Cut σ (pb)	439.10(1)	440.93(1)	442.36(1)	0.74223(3)%	442.33(1)	−0.083(2)%

KKMC-hh shows an ISR effect of a fraction of a percent. LuxQED shows a slightly smaller effect, about 0.4% for each cross section. KKMC-hh shows an IFI effect below 0.1%.

	1. No ISR	2. LuxQED	3. KKMC-hh ISR	4. ISR – no ISR	5. With IFI	6. IFI – no IFI
A_{FB}	0.01125(2)	0.01145(2)	0.01129(2)	$(3.9 \pm 2.8) \times 10^{-5}$	0.01132(2)	$(2.9 \pm 1.1) \times 10^{-5}$
A_4	0.06102(3)	0.06131(3)	0.06057(3)	$-(4.4 \pm 0.5) \times 10^{-5}$	0.06102(3)	$(4.5 \pm 0.3) \times 10^{-5}$

The ISR and IFI effects on the angular coefficients are both on the order of 10^{-5} in KKMC-hh. LuxQED gives a somewhat bigger ISR effect in this case, on the order of 10^{-4}.

QED ISR/IFI/ISR

Example table of ongoing comparisons.

Integrated A_{FB}(LO) and difference ΔA[QED]=[A_{FB}(LO+QED)-A_{FB}(LO)]
in green – the numbers from the report of F. Piccinini on Dec 12, 2018

	A_{FB}(LO)	ΔA[ISR]	ΔA[IFI]	ΔA[$q\gamma$]	ΔA[$\gamma\gamma$]
			[66-116]		
TV	0.03998(1)	-0.00004(1)	-0.00026(1)	0.00002(1)	-0.00018(1)
	0.03986(2)	-0.00001(3)			
FV	0.01813(1)	0.00002(1)	-0.00006(1)	0.00000(1)	-0.00002(1)
	0.01815(3)	0			
			[66-80]		
TV	-0.20465(6)	-0.00019(6)	-0.00076(6)	-0.00181(7)	0.01319(5)
FV	-0.06969(3)	0.00000(4)	-0.00017(3)	-0.00073(3)	0.00093(3)
			[80-102]		
TV	0.04496(1)	-0.00004(1)	-0.00018(1)	-0.00000(1)	-0.00008(1)
	0.04481(2)	0			
FV	0.01891(1)	-0.00003(1)	-0.00004(1)	-0.00000(1)	-0.00000(1)
	0.01895(3)	-0.00080(6)			
			[102-116]		
TV	0.21388(6)	0.00015(7)	-0.00215(6)	0.00087(7)	-0.00544(6)
FV	0.09327(4)	0.00004(4)	-0.00064(4)	0.00033(4)	-0.00055(4)

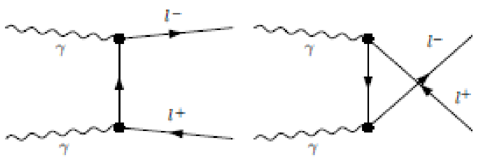
S. Bondarenko & L. Kalinovskaya

LHC EW Precision sub-group meeting, CERN, March 13, 2019

Open issues: Photon induced processes

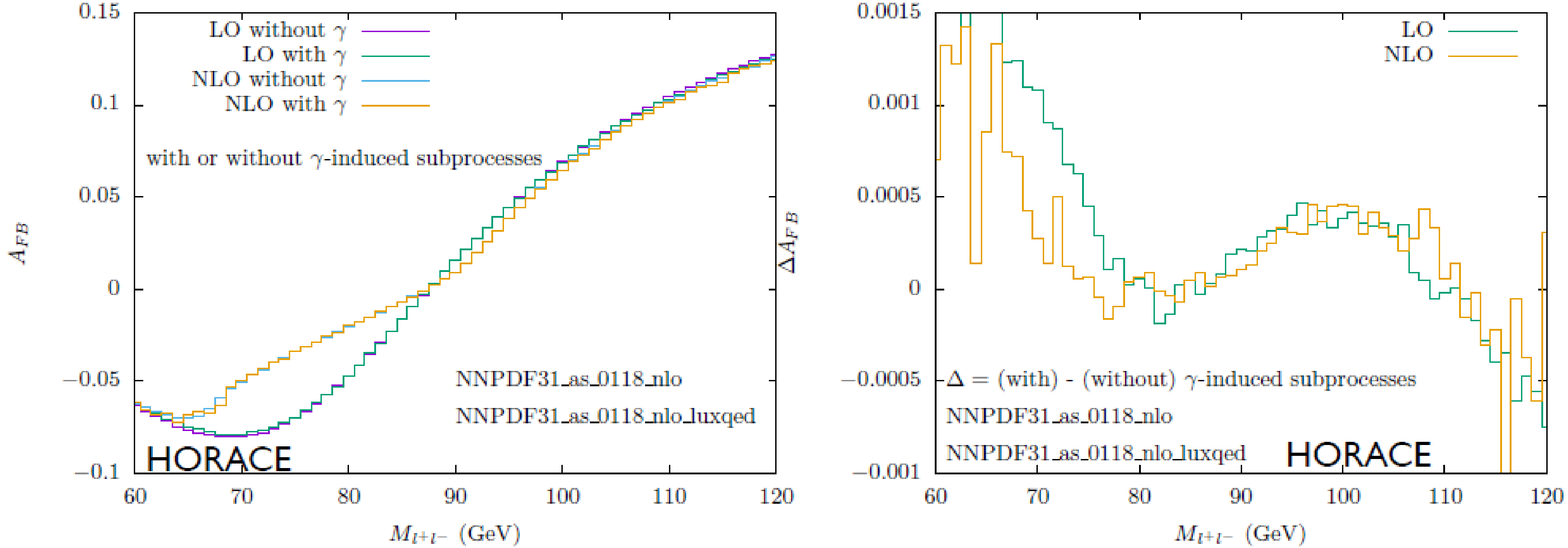
AFB distribution: photon-induced contributions

A. Vicini, L. Kalinovskaya, S. Bondarenko



simulation with γ-induced: NNPDF31_nlo_as_0118_luxqed and γ-induced subprocesses

simulation without γ-induced: NNPDF31_nlo_as_0118 and NO γ-induced subprocesses



preliminary results for γ-ind. contribution: effect on the asymmetry large

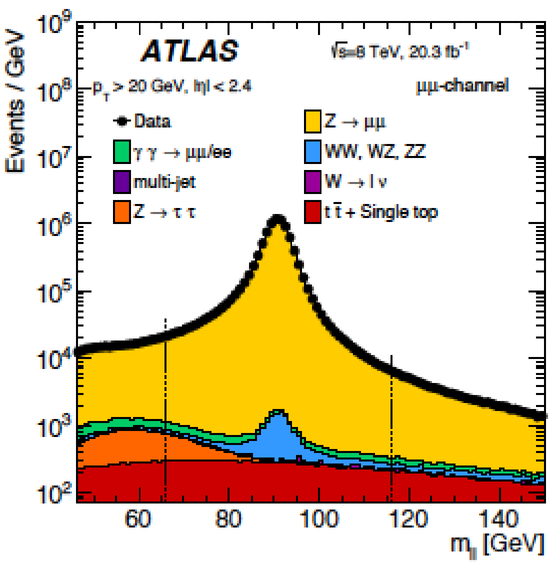
In experimental analysis can be subtracted with selection on primary vertex and leptons.

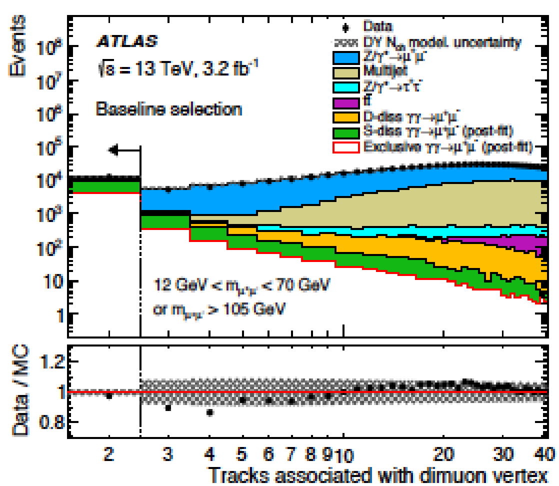
Treatment of photon-induced processes in experiments

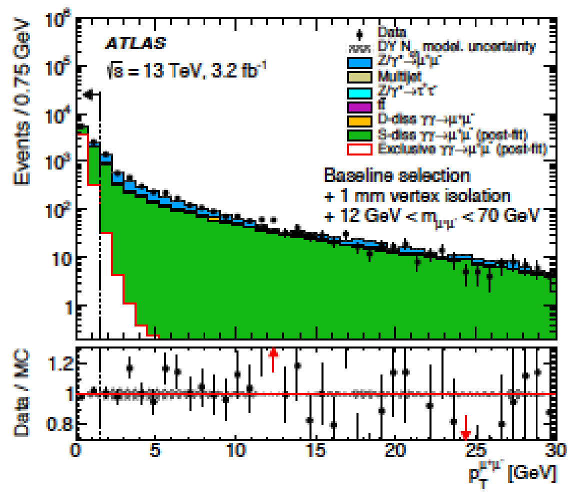
Properties of PI events

EPJC 76 (2016) 291, PLB 777 (2018) 303

- PI signature is not completely irreducible: can be used to select exclusive PI events: not resonant at $m_{\ell\ell} \sim m_Z$, no/little hadronic activity at vertex, $p_T^{\ell\ell}$ small
- In turn it is not guaranteed, that a DY-focused ATLAS analysis will retain PI events in the selection with the same efficiency as DY: a typical ATLAS Run 1 selection required three tracks at the vertex (i.e. 2 leptons + 1 hadronic track)



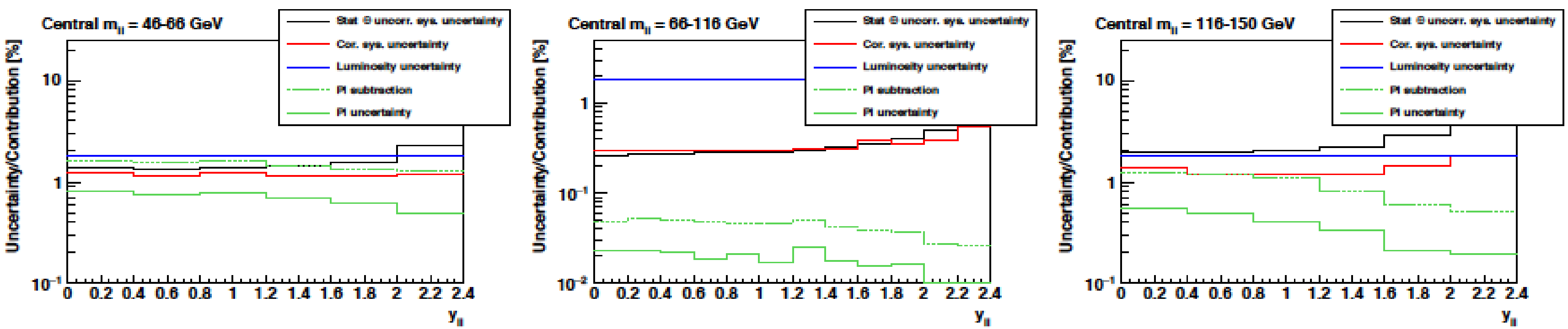




Treatment of photon-induced processes in experiments

- **HEPDATA should include amount of subtracted PI contribution in each measurement bin** **J. Kretzschmar**
- **Caveat: very few studies done including also quark-gamma processes which have significant p_T^{ll} and vertex multiplicity from extra jet**

- Plots below illustrate the amount of PI subtracted (*dashed*) and the corresponding PI uncertainty (**solid**) behaves compared to other measurement uncertainty (**stat.** and **corr. sys.**):
  - PI subtraction most important at $m_{\ell\ell} = 46 - 66$ GeV, where it is of similar order as measurement uncertainties; note that in this region also NNLO QCD uncertainties are not negligible
  - PI subtraction uncertainties subdominant everywhere



- These plots are basically made from public HEPDATA information, the exception is the exact amount of PI subtracted (*dashed*)

Treatment of photon-induced processes in experiments

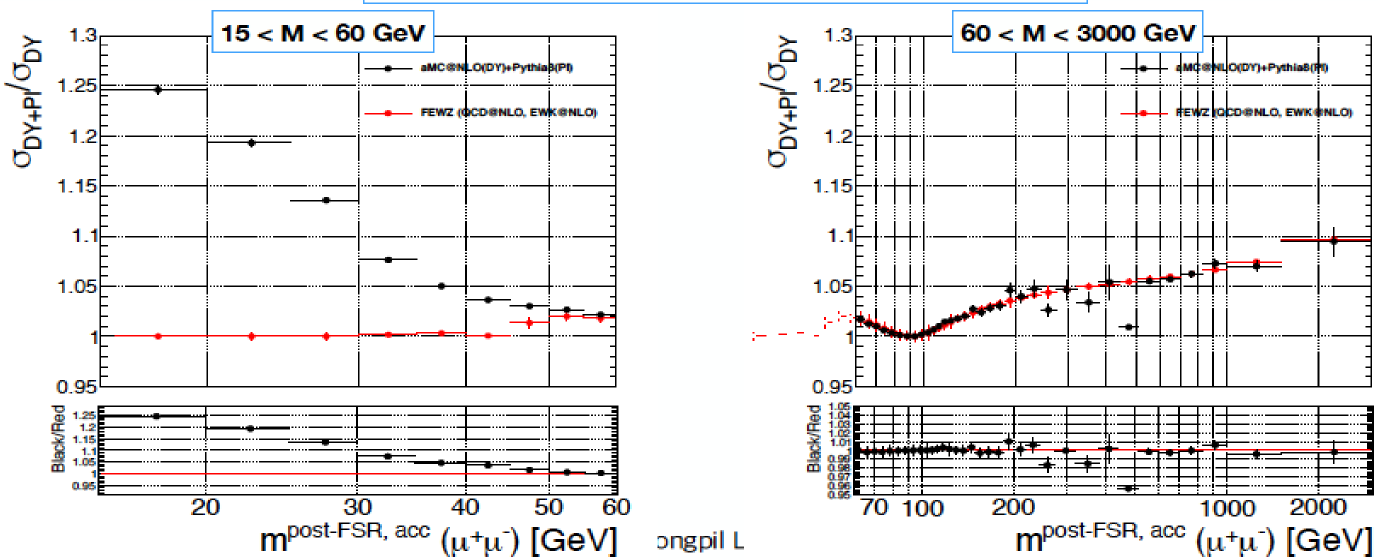
- **CMS see problem with Pythia8 simulation of PI-process ($\gamma\gamma$ to ll) for the 13 TeV analysis, so no subtraction is performed.**

Treatment of Photon-induced Contributions: **SMP-17-001 (13 TeV)**

K. Lee

- First try: subtract the background using the MC simulation
  - Simulate the dilepton events from PI process using pythia 8.212
    - Turn on **PhotonCollision:gmgm2mumu** option in pythia
    - PDF: MRST2004qed
  - Compare σ**(DY+PI) /** σ**(DY)** between MC(pythia8) and FEWZ calculation for a cross check
    - High mass region shows good agreement with FEWZ, but **MC predicts large (up to 25%) PI contribution** in the low mass region compared to FEWZ
    - Based on the experience from previous analyses, PI contribution is not that large in the low mass region
      → **MC is not reliable and not used for the subtraction**

σ**(DY+PI)/**σ**(DY) prediction as a function of dimuon mass**



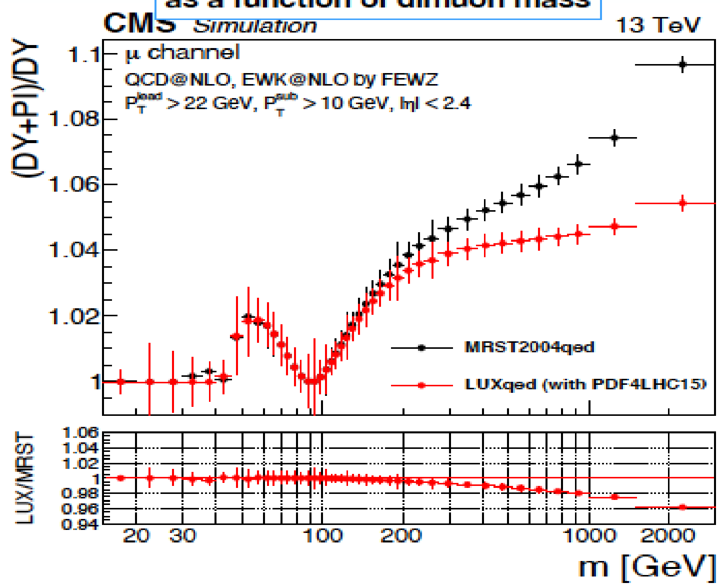
Treatment of photon-induced processes in experiments

- **So for 13 TeV CMS analysis, PI contribution is added to theory predictions for comparisons (this might induce small bias because efficiencies are not the same). This is done using LUXqed from PDF4LHC15 pdf set.**

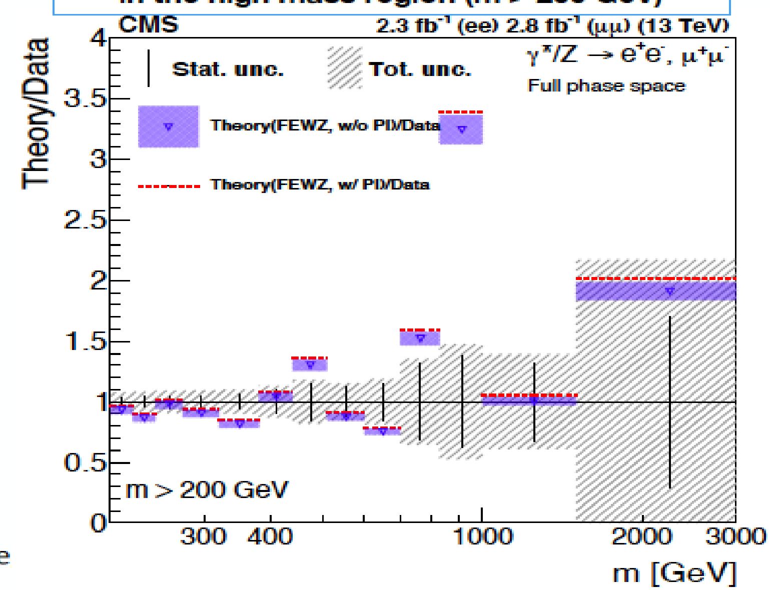
- Decision: **not to subtract** PI contribution from the data
  - PI contribution is negligible in most of the region compared to the other backgrounds
    - Its contribution becomes large in high mass region (go up to ~5% according to LUXqed), but the uncertainty on the cross section in this region is much larger than PI contribution (due to small statistics)
  - Instead, **a theory prediction with PI is added in the comparison with the data results**
    - The PI prediction is estimated with **LUXqed** PDF, which is one of the latest PDF with photons
      - ✓ Predict smaller contribution than MRST2004qed in the high mass region

K. Lee

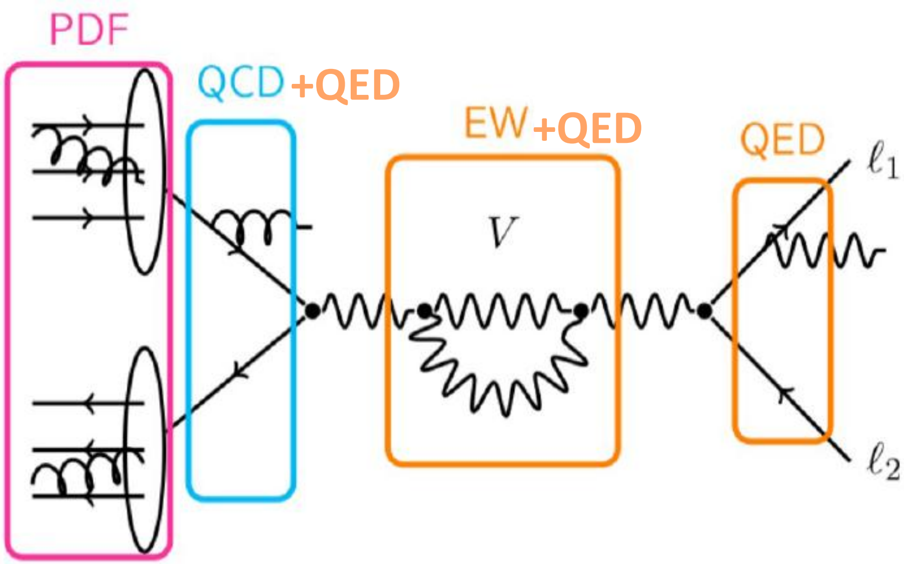
σ(DY+PI)/σ(DY) prediction as a function of dimuon mass



dσ/dm ratio of theory/data(e and μ combined) in the high mass region (m > 200 GeV)



m_W measurement: physics modeling



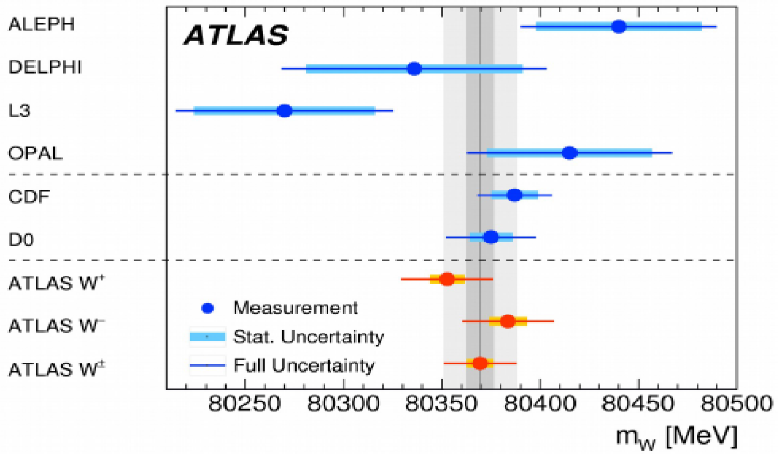
- The QCD aspects: discussed in talk by V. Bertone
- The QED/EW aspects:
  - No activity in the EW precision WG
  - Uncertainties published by ATLAS 7 TeV analysis, within the budget of total error.

From presentation at Ultimate precision WS

ATLAS 7 TeV result for m_W

The ATLAS result equals in precision the previous single-experiment best measurement of CDF

$$M_W = 80369.5 \pm 18.5 \text{ MeV}$$



$M_W = 80369.5 \pm 6.8 \text{ (stat)} \pm 10.6 \text{ (exp.syst.)} \pm 13.6 \text{ (model.syst.)}$ MeV

The dominant uncertainty is due to the physics modelling

Combined categories	Value [MeV]	Stat. Unc.	Muon Unc.	Elec. Unc.	Recoil Unc.	Bckg. Unc.	QCD Unc.	EW Unc.	PDF Unc.	Total Unc.	χ^2/dof of Comb.
m_T-p_T^ℓ, $W^\pm$, e-μ	80369.5	6.8	6.6	6.4	2.9	4.5	8.3	5.5	9.2	18.5	29/27

and the largest contributions are from QCD/PDF

Stefano Camarda 11

From presentation at Ultimate precision WS

Electroweak corrections

- QED FSR: dominant correction, included in the simulation with PHOTOS or others MC
- Other NLO electroweak corrections are usually estimated independently from QCD corrections, and applied as uncertainty

Decay channel	$W \to e\nu$		$W \to \mu\nu$	
Kinematic distribution	p_T^ℓ	m_T	p_T^ℓ	m_T
δm_W [MeV]				
FSR (real)	< 0.1	< 0.1	< 0.1	< 0.1
Pure weak and IFI corrections	3.3	2.5	3.5	2.5
FSR (pair production)	3.6	0.8	4.4	0.8
Total	4.9	2.6	5.6	2.6

- Many recent developments in higher order corrections, and benchmarking between different codes presented in the LPCC EW working group
- Main challenge for the m_W analyses: include electroweak corrections in the analyses, coherently combined with QCD corrections. Available tools are Powheg-EW, DIZET form factors, WINHAC, KKMC
- Open point: is the running-width propagator still the best definition for m_W, in view of higher order EW corrections?

Progress on QED FSR front relevant for W mass measurement

- **Recent work with Sherpa YFS formalism to reach NNLO QED + NLO EW accuracy. Impact is not large but not yet quantified at the level required for W mass measurement.**
-

J. Lindert, F. Krauss,
R. Linten, M. Schonherr

Improving the YFS formalisms

The YFS master formula"

$$2m\cdot\Gamma = \sum_{n_R}\frac{1}{n_R!}\int \mathrm{d}\Phi_{p_f}\mathrm{d}\Phi_k\,(2\pi)^4\,\delta^4\left(q-\sum_f p_f-\sum_{i=0}^{n_R}k_i\right)$$

$$\times\, e^{Y(\Omega,\{q\})}\prod_{i=1}^{n_R}\tilde{S}\left(k_i,\{q\}\right)\Theta\left(k_i,\Omega\right)\ \tilde{\beta}_0^0(\{q\})\,\mathcal{C}(\{p\},\{q\})\,\mathcal{J}(\{p\},\{q\})$$

YFS form-factor (resummed)

eikonal-factor

Born ME

Correction factor

Jacobean

$$\mathcal{C} = 1+\frac{1}{\tilde{\beta}_0^0}\left(\tilde{\beta}_0^1+\sum_{i=1}^{n_\gamma}\frac{\tilde{\beta}_1^1(k_i)}{\tilde{S}(k_i)}\right)+\frac{1}{\tilde{\beta}_0^0}\left(\tilde{\beta}_0^2+\sum_{i=1}^{n_\gamma}\frac{\tilde{\beta}_1^2(k_i)}{\tilde{S}(k_i)}+\sum_{\substack{i,j=1\\ i\neq j}}^{n_\gamma}\frac{\tilde{\beta}_2^2(k_i,k_j)}{\tilde{S}(k_i)\,\tilde{S}(k_j)}\right)+\frac{1}{\tilde{\beta}_0^0}\mathcal{O}\left(\alpha^3\right)$$

NLO: V, R

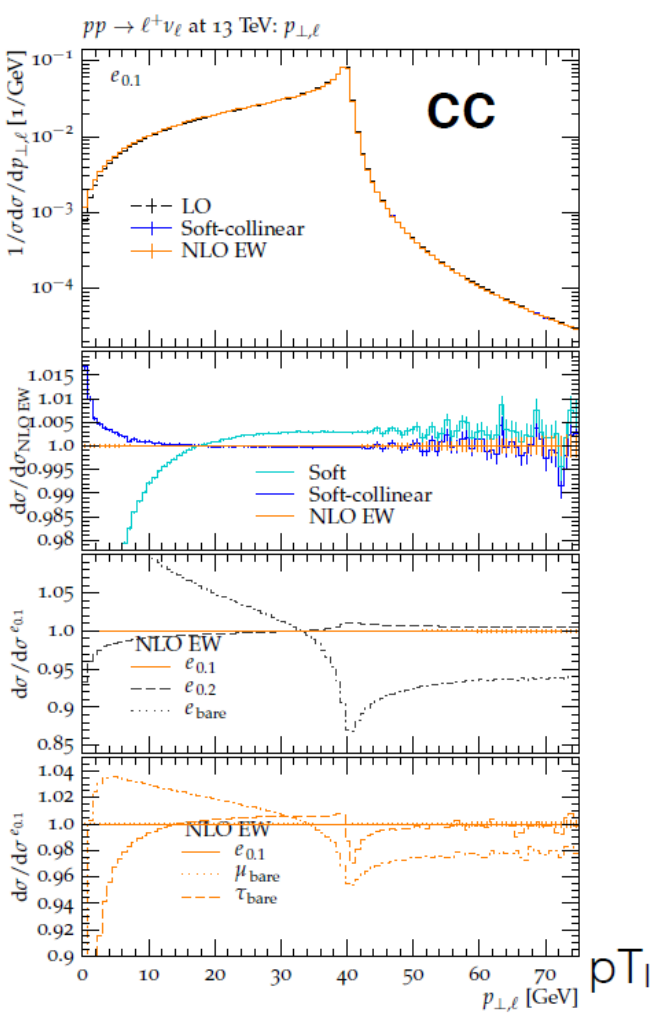
NNLO: VV, RV, RR

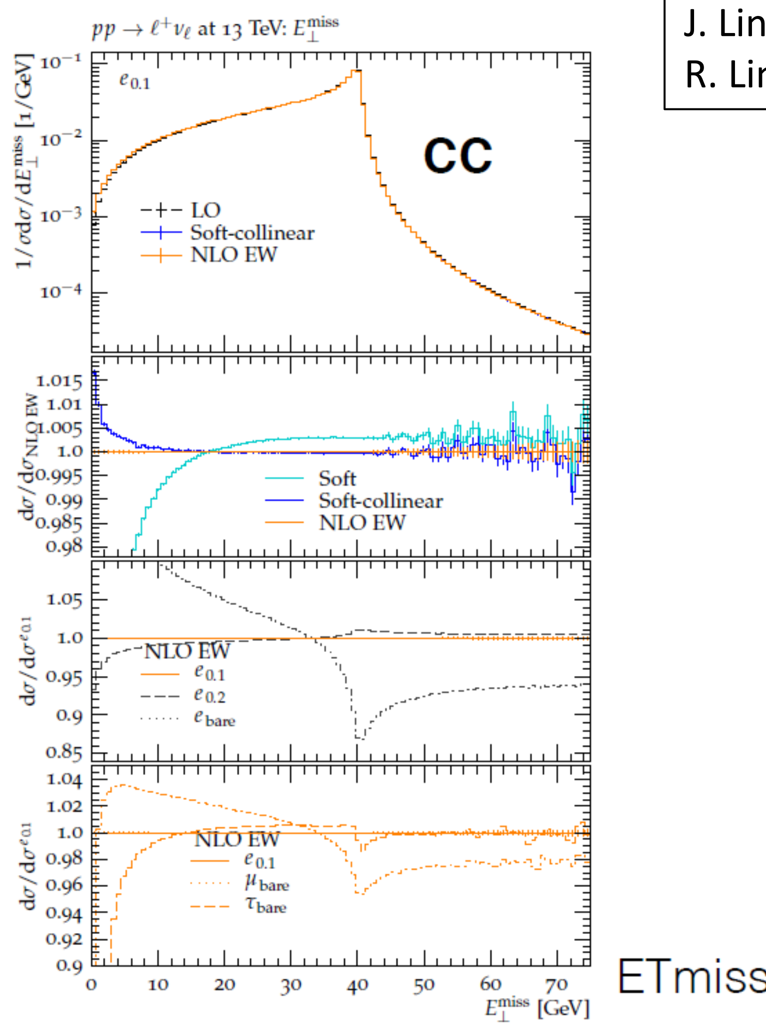
$\boxed{\text{NLO QED} \rightarrow \text{NLO EW+NNLO QED}}$

Progress on QED FSR front relevant for W mass measurement

- **Example of lepton pT spectrum shown below for W decays**

J. Lindert, F. Krauss, R. Linten, M. Schonherr





Summary

SPARES slides

EW schemes: input parameters

SM fundamental relation used to calculate EW LO parameters for different schemes (pole mass).

Completed since last meeting!

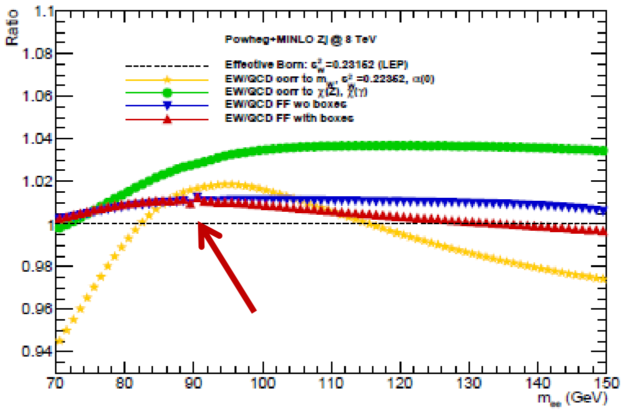
Parameter	$(\alpha(0), G_\mu, M_Z)$ $\alpha(0)$ v0	$(\alpha(0), M_W, M_Z)$ $\alpha(0)$ v1	(G_μ, M_Z, M_W) G_μ	$(\alpha(0), s_W^2, M_Z)$ $\sin^2_{eff}$ v1	(G_μ, s_W^2, M_Z) sin^2_{eff} v2
M_Z (GeV)	91.15348	91.15348	91.15348	91.15348	91.15348
Γ_Z (GeV)	2.494266	2.494266	2.494266	2.494266	2.494266
Γ_W (GeV)	2.085	2.085	2.085	2.085	2.085
$1/\alpha$	137.035999139	137.035999139	132.3572336357709	137.035999139	128.84133952
α	0.007297353	0.007297353	0.007555311	0.007297353	0.007761484
G_μ (GeV^{-2})	$1.1663787 \cdot 10^{-5}$	$1.126555497 \cdot 10^{-5}$	$1.1663787 \cdot 10^{-5}$	$1.09663005 \cdot 10^{-5}$	$1.1663787 \cdot 10^{-5}$
M_W (GeV)	80.91191	80.35797	80.35797	79.90895881	79.90895881
s_W^2	0.21208680	0.22283820939	0.22283820939	0.231499	0.231499
$\frac{G_\mu \cdot M_Z^2 \cdot 16 c_W^2 s_W^2}{\sqrt{2} \cdot 8\pi \cdot \alpha} = 1.0$ $s_W^2 = 1 - m_W^2/m_Z^2$	$\to s_W^2, M_W$	$\to G_\mu, s_W^2$	$\to \alpha, s_W^2$	$\to G_\mu, m_W$	$\to \alpha, m_W$
$\alpha_s(M_Z)$	0.120178900000	0.120178900000	0.120178900000	0.120178900000	0.120178900000

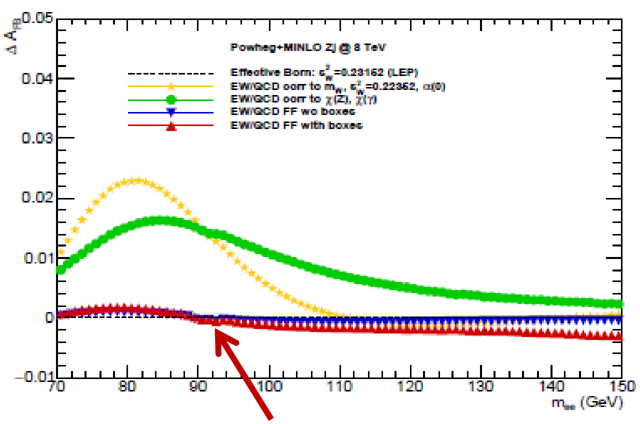
$$\boxed{s_W^2 = 1 - m_W^2/m_Z^2}$$

$$\boxed{G_\mu = \frac{\pi\alpha}{\sqrt{2} M_W^2 s_W^2}}$$

Effective Born and LEP param.

- TauSpinner + wt^{EW}: LEP and LEP imp. arXiv:1808.08616





EW LO $\alpha(0)$ scheme	Effective Born *LEP*	Effective Born *LEP with improved norm.*
$\alpha = 1/137.3599$ $s_W^2 = 0.21215$ $\rho_{\ell f} = 1.0$	$\alpha = 1/128.8667$ $s_W^2 = 0.23152$ $\rho_{\ell f} = 1.0$	$\alpha = 1/128.8667$ $s_W^2 = 0.23152$ $\rho_{\ell f} = 1.005$

Corrections to cross-section	$89 < m_{ee} < 93$ GeV	$80 < m_{ee} < 100$ GeV
σ(EW corr. to m_W)/σ(EW LO $\alpha(0)$)	0.97114	0.97162
σ(EW corr. to $\chi(Z),\chi(\gamma)$)/σ(EW LO $\alpha(0)$)	0.98246	0.98346
σ(EW/QCD FF no boxes)/σ(EW LO $\alpha(0)$)	0.96469	0.96602
σ(EW/QCD FF with boxes)/σ(EW LO $\alpha(0)$)	0.96473	0.96607
σ(*LEP*)/σ(EW/QCD FF with boxes)	1.01102	1.01093
σ(*LEP with improved norm.*)/σ(EW/QCD FF with boxes)	1.00100	1.00098

Corrections to A_{FB}	$89 < m_{ee} < 93$ GeV	$80 < m_{ee} < 100$ GeV
A_{FB}(EW corr. m_W) - A_{FB}(EW LO $\alpha(0)$)	-0.02097	-0.02103
A_{FB}(EW corr. prop. $\chi(Z),\chi(\gamma)$) - A_{FB}(EW LO $\alpha(0)$)	-0.02066	-0.02098
A_{FB}(EW/QCD FF no boxes) - A_{FB}(EW LO $\alpha(0)$)	-0.03535	-0.03569
A_{FB}(EW/QCD FF with boxes) - A_{FB}(EW LO $\alpha(0)$)	-0.03534	-0.03567
A_{FB}(*LEP*) - A_{FB}(EW/QCD FF with boxes)	-0.00006	-0.00001
A_{FB}(*LEP with improved norm.*) - A_{FB}(EW/QCD FF with boxes)	-0.00005	-0.00002

Size of corrections anticipated for $\sin^2\theta_{eff}$ EW scheme.

TauSpinner wt^{EW}: updates

- **Updated Dizet 6.21 -> Dizet 6.42 -> Dizet 6.45**
  - **Updated parametrisation of $\Delta\alpha_{had}^{(5)}(s)$ Jegerlehner 2017 (arXiv:1711.06089)**
  - **AMT4=6: Complete two-loop corrections to M_W and fermionic two-loop corrections to $\sin^2\theta_{eff}^{lep}$**

Parameter	September'18 m_{ll} = 89-93 GeV	September'19 m_{ll} = 89-93 GeV	
$\alpha(0)$ scheme			
σ NLO+HO/LO (no boxes)	0.96536	0.96503	0.03%
σ NLO+HO/LO (with boxes)	0.96536	0.96508	
A_{fb} NLO+HO - LO (no boxes)	-0.03529	-0.03496	+0.0003
A_{fb} NLO+HO - LO (with boxes)	-0.03526	-0.03495	
G_μ scheme			
σ NLO+HO/LO (no boxes)	0.99243	0.99204	0.04%
σ NLO+HO/LO (with boxes)	0.99244	0.99209	
A_{fb} NLO+HO - LO (no boxes)	-0.01553	-0.01514	+0.0004
A_{fb} NLO+HO - LO (with boxes)	-0.01550	-0.01512	

About + 0.00010 on $\sin^2_{eff}$

QED corrections: LEP legacy

QED Initial-final state Interference (IFI)

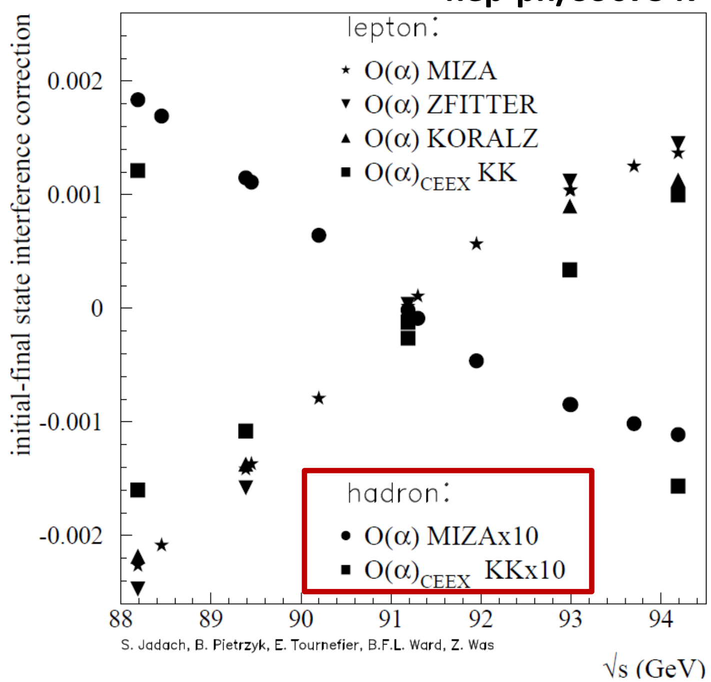
- Suppressed by the Γ_Z/M_Z factor , does not contain mass logarithms. Additional factor 10 of suppression comes from partial cancellation of contributions from different quark flavours
- Codes used at LHC should reproduce LEP benchmarks around Z-pole

$$\boxed{\frac{\sigma^{IFI}}{\sigma} \sim Q_e Q_f \frac{\alpha}{\pi} \mathrm{Max}\left\{ \frac{\Gamma_Z}{M_Z} \; ; \; \frac{s - M_Z^2}{M_Z^2} \right\}}$$

QED IFI: (arXiv:hep-ph/9907547)

- 0.02% corrections to cross-section σ_{had} at the Z peak
- 0.15 MeV for the Z mass shift at M_Z

hep-ph/9907547

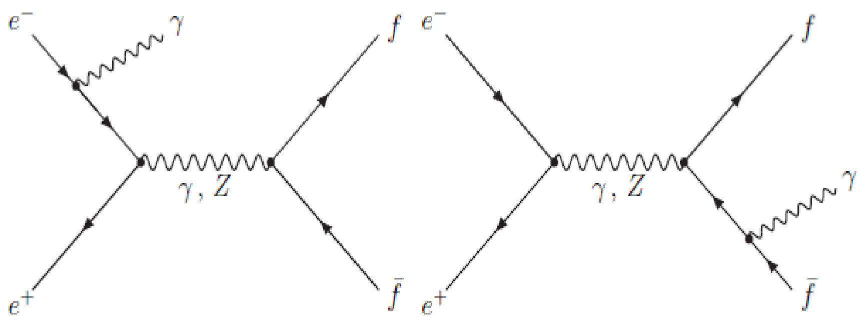


LEP legacy: QED (radiative) corrections

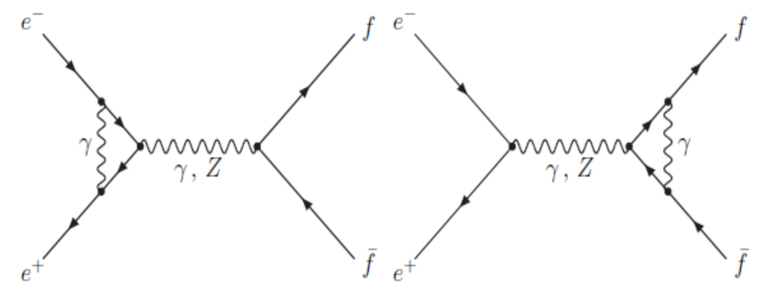
NOT discussed here.

QED FSR can be simulated by PHOTOS (exponentiated multi-photon emission) implemented as after-burner step on already generated event.

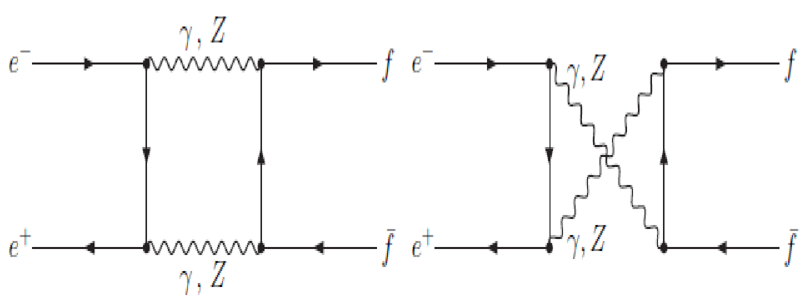
Real emission + pairs creation



Vertex corrections



$\gamma\gamma$ and γZ box diagrams



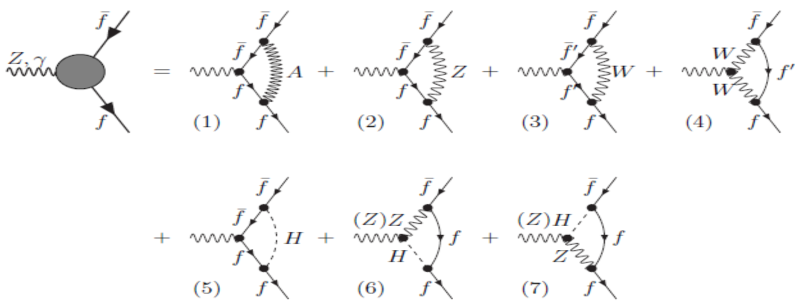
It is QED gauge-invariant set of diagrams (D. Bardin, hep-ph/9908433) which can be factorised out and/or convoluted with QCD corrections.

Calculated with fixed value of α_{QED}
α_{QED} = 1./137.0359895

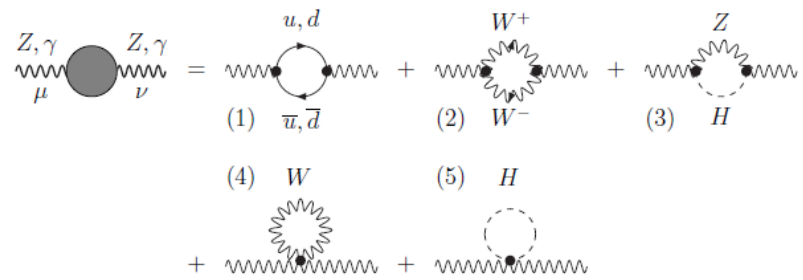
LEP legacy: Genuine EW and lineshape corrections

Also gauge-invariant set of diagrams. Calculated as form-factor corrections to couplings, propagators and masses. Eg. running $\alpha_{QED}(s)$, $\alpha_{QED}(M_Z) = 1./128.86674175$

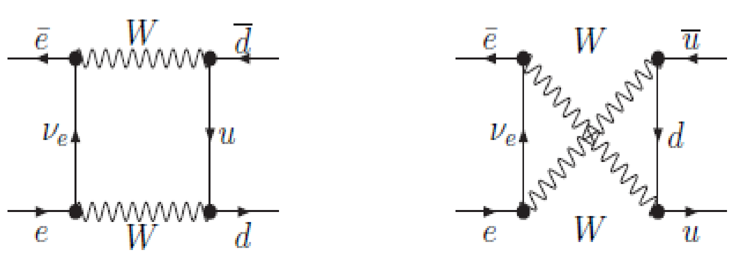
Zff and γ ff vertices



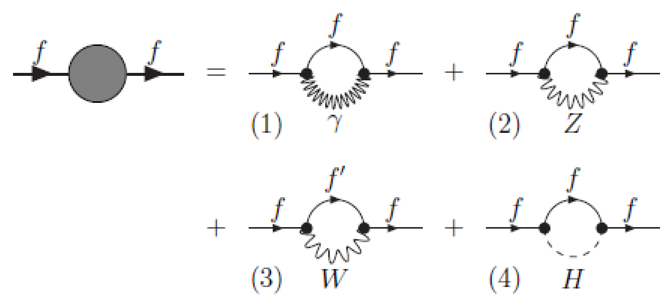
Bosonic self-energies



WW, ZZ boxes (shown only WW diagrams)



Fermionic self-energies



From Zfitter/Dizet documentation

D. Bardin et al.
arXiv:9908433

Zfitter is a **semi-analytical program** for calculating total cross-sections and pseudo-observables (eg. A_{fb}, $\sin^2\theta_W^{eff}$), used by LEP1, and to a lesser degree by LEP2.
DIZET is a library for calculating form-factors and some other corrections. Provides complete EW O(α) weak-loop corrections supplemented with selected higher order terms (eg. vacum polarisation, $\alpha_{QED}(Q^2)$).
For analyses at LEP1, LEP2 used aways in parallel with **MC generators (KoralZ, KoralW)** eg. to evaluate systematics of simplified cuts used in analysis integration.

$$\begin{aligned} \mathcal{A}_Z^{OLA}(s,t) = \; & i\sqrt{2}G_\mu I_e^{(3)} I_f^{(3)} M_Z^2 \chi_Z(s) \boxed{\rho_{ef}(s,t)} \Big\{ \gamma_\mu(1+\gamma_5)\otimes\gamma_\mu(1+\gamma_5) \\ & -4|Q_e|s_W^2 \boxed{\kappa_e(s,t)} \gamma_\mu\otimes\gamma_\mu(1+\gamma_5) - 4|Q_f|s_W^2 \boxed{\kappa_f(s,t)} \gamma_\mu(1+\gamma_5)\otimes\gamma_\mu \\ & +16|Q_eQ_f|s_W^4 \boxed{\kappa_{e,f}(s,t)} \gamma_\mu\otimes\gamma_\mu \Big\}. \qquad (A.4.75) \end{aligned}$$

one loop amplitude

$$A_\gamma^{OLA} = i\chi_\gamma(s)\boxed{\alpha(s)}\gamma_\mu\otimes\gamma_\mu\,. \qquad (2.2.36)$$

Dyson summation leads to the change of α into $\alpha(s)$:

$$\alpha(s) = \frac{\alpha(0)}{1-\Delta\alpha^{\mathrm{fer}}(s)} = \frac{\alpha(0)}{1-\boxed{\Delta\alpha^{(5)}(s)-\Delta\alpha^{t}(s)-\Delta\alpha^{\alpha\alpha_s}(s)}}, \qquad (2.2.37)$$

Vacuum polarisation corrections

LEP legacy: from Zfitter/Dizet documentation

After some trivial algebra one derives the final expressions:

$$\boxed{\rho_{ef}} = 1+\frac{g^2}{16\pi^2}\Big\{-\Delta\rho_z^F+\mathcal{D}_z^F(s)+\frac{5}{3}B_0^F(-s;M_w,M_w)-\frac{9}{4}\frac{c_w^2}{s_w^2}\ln c_w^2-6 \\ +\frac{5}{8}c_w^2\left(1+c_w^2\right)+\frac{1}{4c_w^2}\left(3v_e^2+a_e^2+3v_f^2+a_f^2\right)\mathcal{F}_z(s)+\hat{\mathcal{F}}_w^0(s)+\hat{\mathcal{F}}_w(s) \\ -\frac{r_t}{4}\left[B_0^F(-s;M_w,M_w)+1\right]-c_w^2\left(R_z-1\right)s\hat{\mathcal{B}}_{ww}^d(s,t)\Big\}, \tag{A.4.80}$$

$$\boxed{\kappa_e} = 1+\frac{g^2}{16\pi^2}\Big\{-\frac{c_w^2}{s_w^2}\Delta\rho^F-\Pi_{z\gamma}^F(s)-\frac{1}{6}B_0^F(-s;M_w,M_w)-\frac{1}{9}-\frac{v_e\sigma_e}{2c_w^2}\mathcal{F}_z(s) \\ -\hat{\mathcal{F}}_w^0(s)+\left(R_z-1\right)\Big[\frac{|Q_f|}{2}\left(1-4|Q_f|s_w^2\right)\mathcal{F}_z(s)+c_w^2\Big[\hat{\mathcal{F}}_{w_n}(s) \\ -|Q_{f'}|\mathcal{F}_{w_a}(s)+s\hat{\mathcal{B}}_{ww}^d(s,t)\Big]\Big]\Big\}, \tag{A.4.81}$$

$$\boxed{\kappa_f} = 1+\frac{g^2}{16\pi^2}\Big\{-\frac{c_w^2}{s_w^2}\Delta\rho^F-\Pi_{z\gamma}^F(s)-\frac{1}{6}B_0^F(-s;M_w,M_w)-\frac{1}{9}-\frac{v_f\sigma_f}{2c_w^2}\mathcal{F}_z(s) \\ -\hat{\mathcal{F}}_w(s)+\left(R_z-1\right)\Big[\frac{|Q_e|}{2}\left(1-4|Q_e|s_w^2\right)\mathcal{F}_z(s)+c_w^2\Big[\hat{\mathcal{F}}_{w_n}^0(s) \\ -|Q_{e'}|\mathcal{F}_{w_a}(s)+s\hat{\mathcal{B}}_{ww}^d(s,t)\Big]\Big]-\frac{r_t}{4}\left[B_0^F(-s;M_w,M_w)+1\right]\Big\}, \tag{A.4.82}$$

$$\boxed{\kappa_{ef}} = 1+\frac{g^2}{16\pi^2}\Big\{-2\frac{c_w^2}{s_w^2}\Delta\rho^F-2\Pi_{z\gamma}^F(s)-\frac{1}{3}B_0^F(-s;M_w,M_w)-\frac{2}{9} \\ -\frac{1}{4c_w^2}\left[\frac{\delta_e^2+\delta_f^2}{s_w^2}\left(R_w-1\right)+3v_e^2+a_e^2+3v_f^2+a_f^2\right]\mathcal{F}_z(s) \\ -\hat{\mathcal{F}}_w^0(s)-\hat{\mathcal{F}}_w(s)-\frac{r_t}{4}\left[B_0^F(-s;M_w,M_w)+1\right] \\ +c_w^2\left(R_z-1\right)\left[\frac{2}{3}-\hat{\Pi}_{\gamma\gamma}^{\mathrm{bos},F}(s)+s\hat{\mathcal{B}}_{ww}^d(s,t)\right]\Big\}. \tag{A.4.83}$$

interference

Fermionic loops in γ propagator

BOX

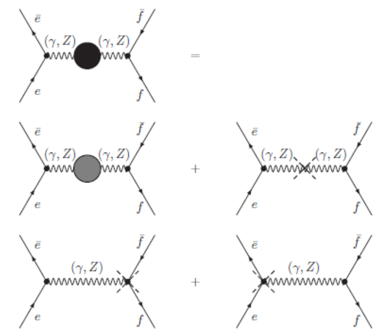


Figure A.11. *Bosonic self-energies and bosonic counter-terms for* $e\bar{e} \to (Z,\gamma) \to f\bar{f}$

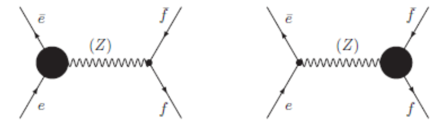


Figure A.10. *Electron (a) and final fermion (b) vertices in* $e\bar{e} \to (Z) \to f\bar{f}$

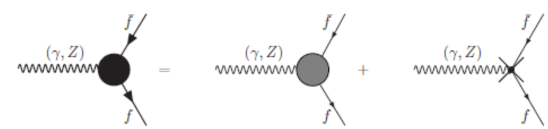


Figure A.6. *Off-shell* $Zf\bar{f}$ *and* $\gamma f\bar{f}$ *vertices*

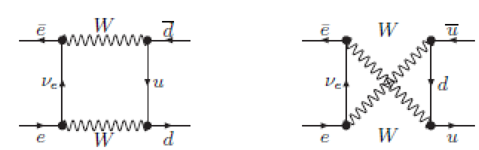


Figure A.7. *The WW boxes*

etc. etc.

Constructing wt^{EW}: EW Improved Born (IBA)

ERW and Z.Was, arXiv: 1808.08616

$$\mathscr{A}^{Born+EW} = \frac{\alpha}{s}\{ \quad [\bar{u}\gamma^{\mu}v g_{\mu\nu}\bar{v}\gamma^{\nu}u]\cdot(q_\ell \cdot q_f)\cdot \boxed{\Gamma_{V_\Pi}} \cdot \chi_\gamma(s)$$
$$+[\bar{u}\gamma^{\mu}v g_{\mu\nu}\bar{v}\gamma^{\nu}u\cdot(v_\ell\cdot v_f\cdot vv_{\ell f}) + \bar{u}\gamma^{\mu}v g_{\mu\nu}\bar{v}\gamma^{\nu}\gamma^{5}u\cdot(v_\ell\cdot a_f)$$
$$+\bar{u}\gamma^{\mu}\gamma^{5}v g_{\mu\nu}\bar{v}\gamma^{\nu}u\cdot(a_\ell\cdot v_f) + \bar{u}\gamma^{\mu}\gamma^{5}v g_{\mu\nu}\bar{v}\gamma^{\nu}\gamma^{5}u\cdot(a_\ell\cdot a_f)]\cdot \boxed{Z_{V_\Pi}} \cdot \chi_Z(s) \quad \}$$

$$\chi_\gamma(s) = 1$$

$$\chi_Z(s) = \frac{G_\mu \cdot M_z^2 \cdot \Delta^2}{\sqrt{2}\cdot 8\pi\cdot\alpha}\cdot\frac{s}{s - M_Z^2 + i\cdot\Gamma_Z\cdot M_Z}$$

$$v_\ell = (2\cdot T_3^\ell - 4\cdot q_\ell\cdot s_W^2 \cdot \boxed{K_\ell(s,t)})/\Delta$$
$$v_f = (2\cdot T_3^f - 4\cdot q_f\cdot s_W^2 \cdot \boxed{K_f(s,t)})/\Delta$$
$$a_\ell = (2\cdot T_3^\ell)/\Delta$$
$$a_f = (2\cdot T_3^f)/\Delta$$

$$\Delta = \sqrt{16\cdot s_W^2\cdot(1 - s_W^2)}$$

$$\boxed{Z_{V_\Pi}} = \rho_{e,f}(s,t)$$

EW form-factors , functions of $(s,t)=(m_{ll}, \cos\theta)$
Calculated with Dizet 6.21 library.

$$\boxed{\Gamma_{V_\Pi}} = \frac{1}{2-(1+\boxed{\Pi_{\gamma\gamma}(s)})}$$

$$vv_{\ell f} = \frac{1}{v_\ell\cdot v_f}[(2\cdot T_3^\ell)(2\cdot T_3^f) - 4\cdot q_\ell\cdot s_W^2\cdot\boxed{K_f(s,t)}(2\cdot T_3^\ell) - 4\cdot q_f\cdot s_W^2\cdot\boxed{K_\ell(s,t)}(2\cdot T_3^f)$$
$$+(4\cdot q_\ell\cdot s_W^2)(4\cdot q_f\cdot s_W^2)\boxed{K_{\ell f}(s,t)}]\frac{1}{\Delta^2}$$

Vacuum polarisation corrections, used low-energy experiment input.
Warning: problem for analytic continuation

EW schemes: details

EW schemes: come with „on-shell" or „pole" definitions!

Table 44: The EW parameters used at tree-level EW, with on-mass-shell definition (LEP convention).

Parameter	$\alpha(0)$ v0	$\alpha(0)$ v1	G_μ
M_Z	91.1876 GeV	91.1876 GeV	91.1876 GeV
Γ_Z	2.4952 GeV	2.4952 GeV	2.4952 GeV
Γ_W	2.085 GeV	2.085 GeV	2.085 GeV
α	1/137.03599	1/137.03599	1/132.23323
G_μ	$1.1663787 \cdot 10^{-5}$ GeV^{-2}	$1.1254734 \cdot 10^{-5}$ GeV^{-2}	$1.1663787 \cdot 10^{-5}$ GeV^{-2}
M_W	80.93886 GeV	80.385 GeV	80.385 GeV
s_W^2	0.2121517	0.2228972	0.2228972
$\frac{G_\mu \cdot M_z^2 \cdot \Delta^2}{\sqrt{2} \cdot 8\pi \cdot \alpha}$	1.0	1.0	1.0

Runing Γ_Z in Z-propagator

Shift:

- -30 MeV for M_Z
- change on Γ_Z
- -0.00006 for s^2w

Scaling

- 0.99906 for α

Table 45: The EW parameters used at tree-level EW, with pole definition of the Z, W masses.

Parameter	$\alpha(0)$ v0	$\alpha(0)$ v1	G_μ
M_Z	91.15348 GeV	91.15348 GeV	91.15348 GeV
Γ_Z	2.494266 GeV	2.494266	2.494266 GeV
Γ_W	2.085 GeV	2.085 GeV	2.085 GeV
α	1/137.03599	1/137.03599	1/132.3572336357709
G_μ	$1.1663787 \cdot 10^{-5}$ GeV^{-2}	$1.126555497 \cdot 10^{-5}$ GeV^{-2}	$1.1663787 \cdot 10^{-5}$ GeV^{-2}
M_W	80.91191 GeV	80.35797 GeV	80.35797 GeV
s_W^2	0.21208680	0.22283820939	0.22283820939
$\frac{G_\mu \cdot M_z^2 \cdot \Delta^2}{\sqrt{2} \cdot 8\pi \cdot \alpha}$	1.0	1.0	1.0

Fixed Γ_Z in Z-propagator

What we have so far .....

PowhegZj: QCD NLO, Z+j
wt^{EW} : TauSpinner + Dizet 6.21

wt^{EW}

EW LO → **$\alpha(0)$ v0: LO, NLO+HO**
EW LO → **$\alpha(0)$ v1: LO**
EW LO → **G_μ: LO**

Arbitrary EW setup

Powheg_ew: QCD LO, Z

- $\alpha(0)$ v0: LO
- $\alpha(0)$ v1: LO, NLO, NLO+HO
- G_μ: LO, NLO, NLO+HO

DYTURBO: QCD LO, NLO, Z

- $\alpha(0)$ 0: LO
- $\alpha(0)$ v1: LO
- G_μ : LO

MCSANC: QCD LO, Z

- $\alpha(0)$ v1: LO, NLO, HO
- G_μ: LO, NLO, HO

Constructing wt^{EW}: per-event weight

ERW and Z.Was, arXiv: 1808.08616

Define per event electroweak weight

$wt^{EW} = \sigma^{new}_{Born} / \sigma^{old}_{Born}$

$$wt^{EW} = \frac{d\sigma_{Born+EW}(x_1, x_2, \hat{s}, \cos\theta \;, s_W^2)}{d\sigma_{Born}(x_1, x_2, \hat{s}, \cos\theta \;, s_W^2)}$$

$$d\sigma_{Born}(x_1, x_2, \hat{s}, \cos\theta^*, s_W^2) = \sum_{q_f, \bar{q}_f} [f^{q_f}(x_1, ...) f^{\bar{q}_f}(x_2, ...) d\sigma^{q_f \bar{q}_f}_{Born}(\hat{s}, \cos\theta \;, s_W^2)$$
$$+ f^{q_f}(x_2, ...) f^{\bar{q}_f}(x_1, ...) d\sigma^{\bar{q}_f q_f}_{Born}(\hat{s}, -\cos\theta \;, s_w^2]$$

Approach developed in TauSpinner, arXiv:1802.05459

x_1, x_2, $\cos\theta$ (symmetrised) calculated using 4-momenta of outgoing leptons; asymmetry in sign of $\cos\theta$ from weighted average over PDFs

Allows to reweight MC event generated between different EW LO scheme and to Improved Born Approximation in EW scheme used for form-factors calculation.

Impact of $\Delta\alpha_{had}^{(5)}(M_Z^2)$

Predictions from Dizet 6.21 library

Parameter	$\Delta\alpha_h^{(5)}(M_Z^2) = 0.0280398$	$\Delta\alpha_h^{(5)}(M_Z^2) = 0.02753$	Ratio
$\alpha(M_Z^2)$	0.00775884	0.00775463	
$1/\alpha(M_Z^2)$	128.885224	128.95522	0.99932
s_W^2	0.22351946	0.22331458	1.00092
$sin^2\theta_W^{eff}(M_Z^2)$ (electron, muon)	0.23175990	0.23157062	1.00082
$sin^2\theta_W^{eff}(M_Z^2)$ (up-quark)	0.23164930	0.23146414	1.00080
$sin^2\theta_W^{eff}(M_Z^2)$ (down-quark)	0.23152214	0.23133715	1.00080
M_W	80.35281 GeV	80.36341 GeV	1.00013
Δr	0.03694272	0.03631342	1.01733
Δr_{rem}	0.01169749	0.01170244	0.99958
ρ_{eu}	1.005408	1.005426	0.99998
K_e	1.036649	1.036770	0.99988
K_u	1.036172	1.036293	0.99988
K_{eu}	1.074146	1.074397	0.99977
ρ_{ed}	1.005894	1.005906	0.99999
K_e	1.036649	1.036699	0.99995
K_d	1.035603	1.035719	0.99989
K_{ed}	1.073556	1.073859	0.99972

shift of about -0.00020 due to corrections to M_W

shift by +11 MeV

ATLAS measurement M_W = 80370 ± 19 MeV

$$M_W = \frac{M_Z}{\sqrt{2}}\sqrt{1+\sqrt{1-\frac{4A_0^2}{M_Z^2(1-\Delta r)}}}$$

$$\Delta r = \Delta\alpha(M_Z^2) + \Delta r_{EW}$$

$$A_0 = \sqrt{\frac{\pi\alpha(0)}{\sqrt{2}G_\mu}}$$

Impact of m_t

Parameter	m_t = 171 GeV	m_t = 173 GeV	m_t = 175 GeV
$\alpha(M_Z^2)$	0.00775882	0.00775884	0.00775885
$1/\alpha(M_Z^2)$	128.888558	128.885224	128.885079
s_W^2	0.22375411	0.22351946	0.22328310
$sin^2\theta_W^{eff}(M_Z^2)$ (electron, muon)	0.23181756	0.23175990	0.23169368
$sin^2\theta_W^{eff}(M_Z^2)$ (up-quark)	0.23171096	0.23164930	0.23169368
$sin^2\theta_W^{eff}(M_Z^2)$ (down-quark)	0.23158377	0.23152214	0.23145996
Δr	0.03766186	0.03694272	0.03621664
Δr_{rem}	0.01165959	0.01169749	0.01173500
ρ_{eu}	1.005229	1.005408	1.005589
K_e	1.035837	1.036649	1.037467
K_u	1.035361	1.036172	1.036990
K_{eu}	1.072465	1.074146	1.075843
ρ_{ed}	1.005714	1.005894	1.006075
K_e	1.035837	1.036649	1.037467
K_d	1.034792	1.035603	1.036420
K_{ed}	1.071876	1.073556	1.075252

±2 GeV shift in m_t corresponds to ±0.00005 shift in $\sin^2_{eff}{}^{lep}$