Introduction to Soft-Collinear Effective Theory

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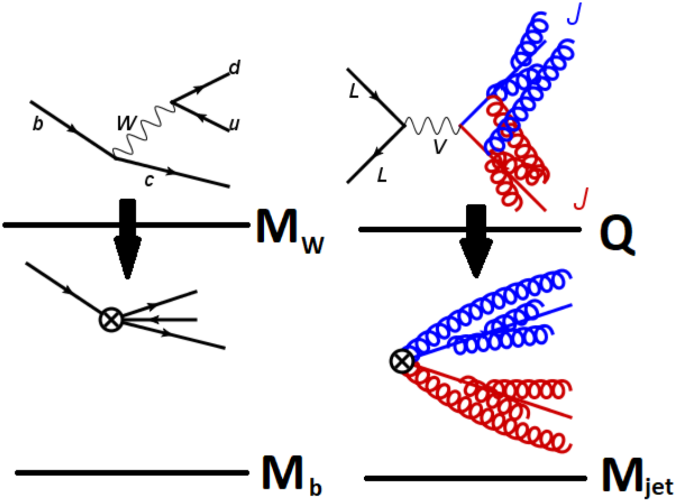
Motivation for EFTs

1) Phenomenology of the Standard Model
- What does the standard model actually predict?
- More precision means that we can more easily see BSM physics in our experiments

2) Model-independent predictions of high-energy BSM physics
- Don't need to know intimate details of BSM models to predict their influence on experiment
- Effective operators are contructed out of SM fields

When EFTs are used

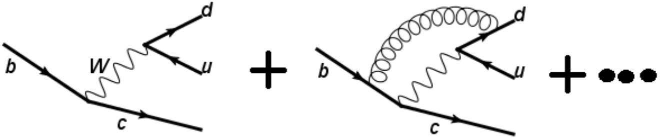
Examples: Non-leptonic B to C decay, $e^+e^- \rightarrow$ *jets* with small mass



Whenever there are multiple widely separated scales relevant to the problem

The problem EFTs solve

- Large logs in full theory prediction, spoiling perturbative expansion



$$\frac{\sigma}{\sigma_0} \supset 1 + \alpha_s \log \frac{M_W}{M_b} + \alpha_s^2 \log^2 \frac{M_W}{M_b} + \ldots$$

In this case $\log \frac{M_W}{M_b} \sim 3$ but in other processes the logarithm can be much larger

How EFTs help

$$\frac{\sigma}{\sigma_0} \supset 1 + \alpha_s \log \frac{M_W}{M_b} + \alpha_s^2 \log^2 \frac{M_W}{M_b} + \dots$$

- EFT factorizes the logarithms, each of which can be minimized at their canonical scale

$$\frac{\sigma}{\sigma_0} \sim \left(1 + \alpha_s \log \frac{M_W}{\mu}\right)\left(1 + \alpha_s \log \frac{\mu}{M_b}\right) = C\left(\frac{M_W}{\mu}\right) F\left(\frac{\mu}{M_b}\right)$$

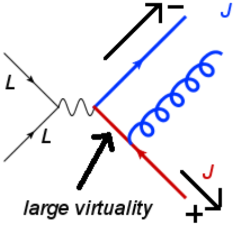
- Need to translate between multiple scales, which can be done using the renormalization group equation
- Renormalization group resums the logarithms, taming the large logarithms and predicting higher loop contributions

$$C\left(\frac{M_W}{M_b}\right) = \exp\left(\int_{M_W}^{M_b} d \log\mu \; \gamma_C(\mu)\right) C\left(\frac{M_W}{M_W}\right)$$

$$C\left(\frac{M_W}{M_b}\right) = \exp\left(\alpha_s \log \frac{M_W}{M_b}\right) C\left(\frac{M_W}{M_W}\right)$$

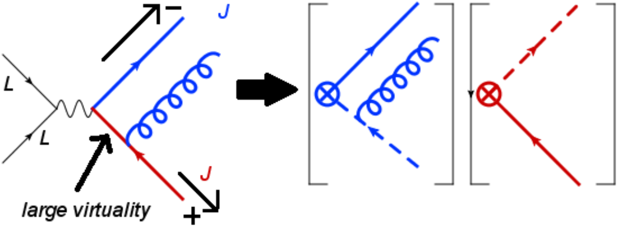
Translating to SCET

In $b \to c$ we remove the massive boson from the theory, because the propagator is far off-shell:
$1/(p^2 - M^2) \to -1/M^2 + \ldots$, which results in a point interaction
In SCET we have instead a large interaction between particles which we remove from the theory



$1/[(p+q)^2 - m^2] \to 1/(\bar{q}^+ g^-) + \ldots$, which results in an eikonal interaction
The common language we use is that below some cutoff scale we are removing highly offshell degrees of freedom from the theory

The eikonal interaction, a.k.a Wilson lines



$$L_{\mathrm{QCD}}^{\mathrm{weak}} = \bar{\psi}\Gamma\psi = C_2 O_2 + \ldots \sim C_2[\bar{\psi}W_+]_{blue}\hat{P}\Gamma\hat{P}[W_-^\dagger\psi]_{red}$$

Particles emitted into the blue jet only interact with the fields in the blue sector, etc.

Wilson lines are static, lightlike colour source – the blue jet can't resolve the dynamics of the red jet, and only sees a source of colour recoiling in the opposite direction

$$W_+ = \overline{\mathcal{P}}\exp\left(- ig_s \int_0^\infty ds\, n_+ \cdot A(x + n_+ s)\right)$$

Why "Soft" and "Collinear"

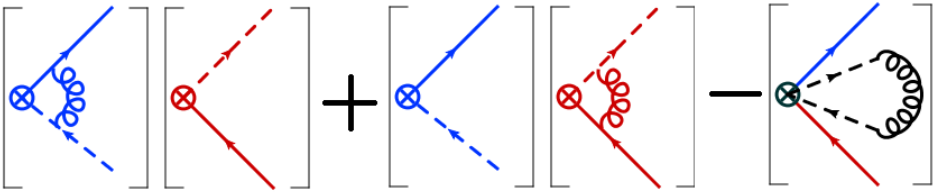
Two types of IR divergences in QCD – soft $(1/\epsilon)$ and collinear $(1/\epsilon)$
Look at vertex correction in QCD with massless quarks. Both types of poles contribute, so we get $1/\epsilon^2$ divergences

$$V_{\mathrm{QCD}}^{1-\mathrm{loop}} \sim \alpha_s \left(-\frac{2}{\epsilon_{IR}^2} + \frac{2\log\frac{Q^2}{\mu^2} - 3}{\epsilon_{IR}} - 2\log^2\frac{Q^2}{\mu^2} + 3\log\frac{Q^2}{\mu^2} \right)$$

Features to note:
1) The double pole with dim. reg. $(\mu^2/Q^2)^\epsilon$ gives double logs
2) This IR structure needs to be reproduced by the EFT (O_2) below the scale Q
3) SCET vertex correction to O_2 is scaleless, so the UV precisely cancels the IR
4) The counterterm/anom. dim. of O_2 contains a log (not seen in other EFTs)

Interesting structure/subtleties

Should be able to find counterterms of O_2 with whichever IR regulator we like (since UV physics is independent of IR physics)
Try introducing a "gluon" mass – find there is double counting between sectors, and need an overlap subtraction

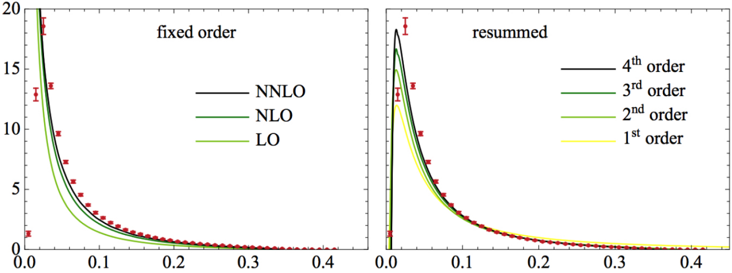


$$V_{\mathrm{SCET}}^{1-\mathrm{loop}} = I_{blue} + I_{red} - I_{sub}$$

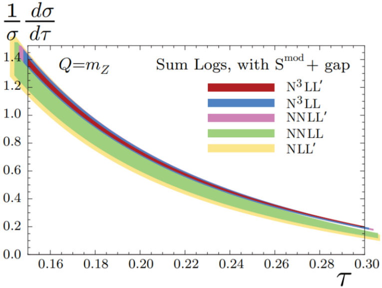
(Also find that I_{blue} and I_{red} are unregulated due to rapidity divergences – need subtraction to make the sum well-defined)

Successes of SCET

Using SCET, event shape measurements can be predicted at N^3LL' order

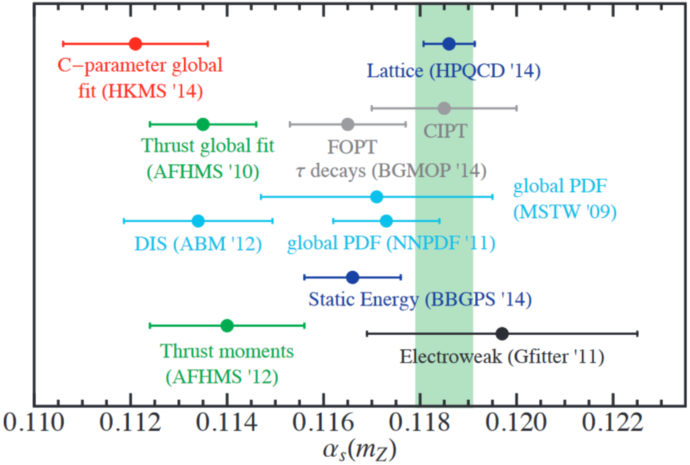


[Becher, Schwartz 0803.0342]



[Abbate, Fickinger, Hoang, Mateu, Stewart 1006.3080]

Yes, event shape measurements can be predicted at N^3LL' order
But there are possible descrepancies (opportunities for higher precision) when measuring α_s using jet observables



[Hoang, Kolodrubetz, Mateu, Stewart 1501.04111]

Our current work: subleading powers

Next thing to do is look at power corrections

$$\frac{d\sigma}{d\tau} \sim 1 + \alpha_s(\log^2\tau + \dots) + \alpha_s^2(\log^4\tau + \dots)$$
$$+ \frac{1}{Q^2}(\alpha_s\tau Q^2 \log\tau)[1 + \alpha_s(\log^2\tau + \dots) + \alpha_s^2(\log^4\tau + \dots)]$$
$$+ \dots$$

First line taken care of by LP resummation. Second line is left unsummed (until recently). Need higher dimension operators in SCET

$$L_{\mathrm{QCD}}^{\mathrm{weak}} = C_2 O_2 + \frac{1}{Q} C_2^{1i} O_2^{(1i)} + \frac{1}{Q} C_2^{2i} O_2^{(2i)} + \dots$$

Higher power operators in SCET expansion have interesting structure, e.g.

$$O_2^{(2A_1)} = \int dt[\bar{\psi} W^+(0, n_+t) iD^\mu(n_+t) iD^\nu(n_+t) W_+(n_+t, \infty)]_{blue}$$
$$\hat{P}\Gamma\hat{P}\gamma_\mu^\perp\gamma_\nu^\perp[W_-^\dagger\psi]_{red}$$

Thanks for listening!