Theoretical Issues in Neutrino Physics

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고에너지 물리학 미래전략 회의

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Outline

- Current status of neutrino oscillations
- Theoretical Issues

  - Origin of neutrino mixing matrix
  - Origin of neutrino masses
  - New Physics in Neutrino Oscillations
- Prospect
- Conclusion

Current status of neutrino oscillations

- Two big discoveries over past two decades :

  - Neutrinos are massive

  - Leptons mix

T. Kajita

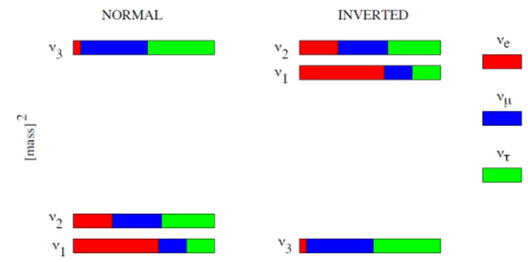
A. McDonald

- They have been achieved by the observation of neutrino oscillations

  (lots of sources: the sun, atmospheric, reactors and accelerators)

2015 Nobel Prize

• 3-flavor paradigm



$$\begin{pmatrix}\nu_e\\ e^-\end{pmatrix}_L \quad \begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}_L \quad \begin{pmatrix}\nu_\tau\\ \tau^-\end{pmatrix}_L \ :$$

weak eigenstates

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}=\begin{pmatrix}U_{e1} & U_{e2} & U_{e3}\\ U_{\mu1} & U_{\mu2} & U_{\mu3}\\ U_{\tau1} & U_{\tau2} & U_{\tau3}\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

mass eigenstates

- Specific parameterization of lepton mixing matrix

$$U_{PMNS}=\begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & -s_{23}\\ 0 & s_{23} & c_{23}\end{pmatrix}\begin{pmatrix}c_{13} & 0 & -s_{13}e^{-i\delta}\\ 0 & 1 & 0\\ s_{13}e^{i\delta} & 0 & c_{13}\end{pmatrix}\begin{pmatrix}c_{12} & -s_{12} & 0\\ s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}e^{i\alpha} & 0\\ 0 & e^{i\beta}\\ 0 & 0\end{pmatrix}$$

- How do we probe neutrino mixing ?

  ➔ neutrino oscillation

In vacuum, $\nu_\alpha \rightarrow \nu_\beta$ transition probability :

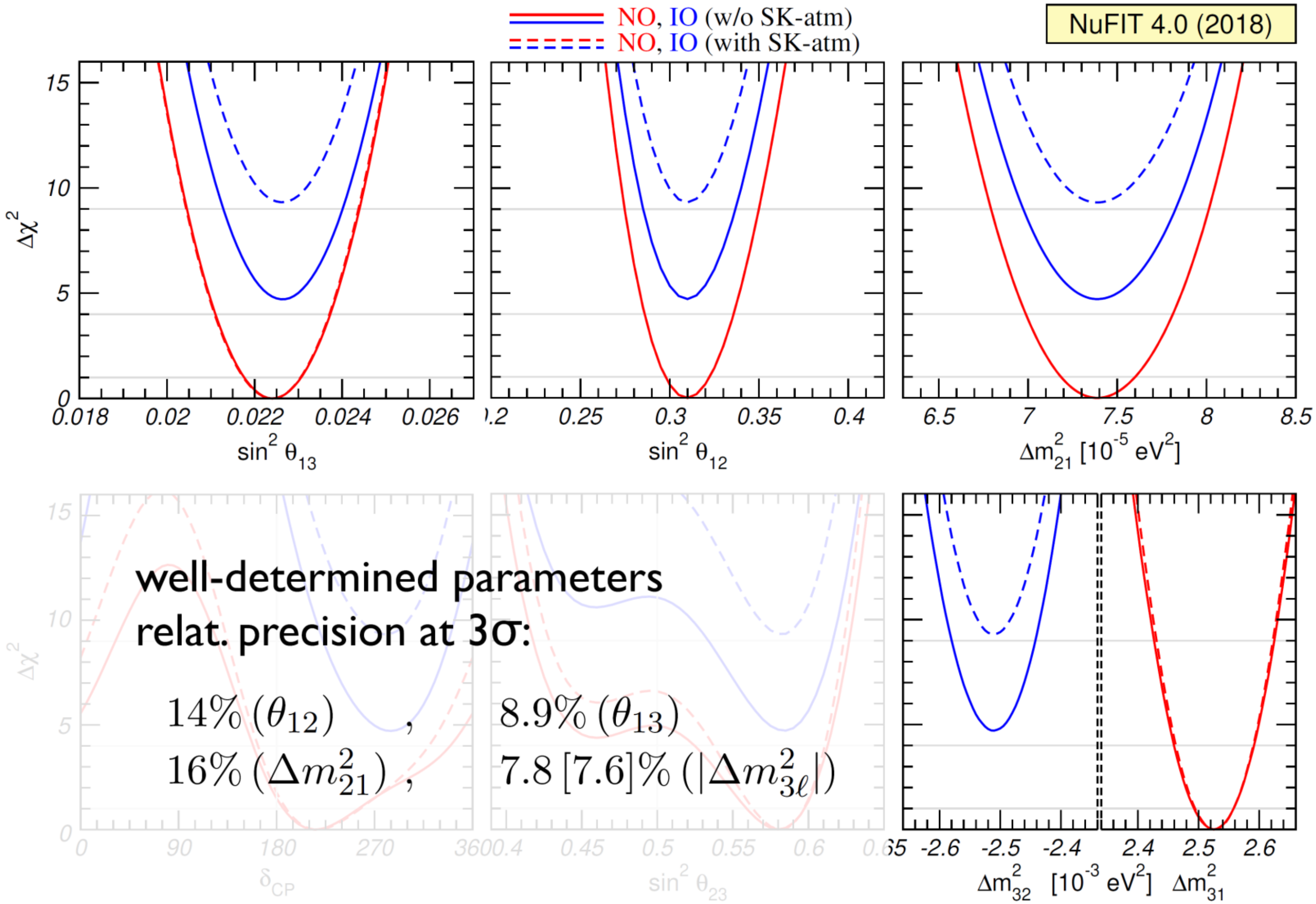
=0 if δ =0 in U, $\alpha = \beta$

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4\sum_{j\neq i}^{n} \mathrm{Re}[U_{\alpha i}^{\star} U_{\beta i} U_{\alpha j} U_{\beta j}^{\star}] \sin^2\left(\frac{\Delta_{ij}}{2}\right) + 2\sum_{j\neq i} \mathrm{Im}[U_{\alpha i}^{\star} U_{\beta i} U_{\alpha j} U_{\beta j}^{\star}] \sin(\Delta_{ij})$$

$$\frac{\Delta_{ij}}{2} = \frac{(E_i - E_j)L}{2} = 1.27 \frac{(m_i^2 - m_j^2)}{\mathrm{eV}^2} \frac{L/E}{\mathrm{Km/GeV}}$$

- From ν oscillation exps. we can determine
  - Δm_{21}^2 , Δm_{31}^2
  - θ_{12} , θ_{23} , θ_{13}
  - δ

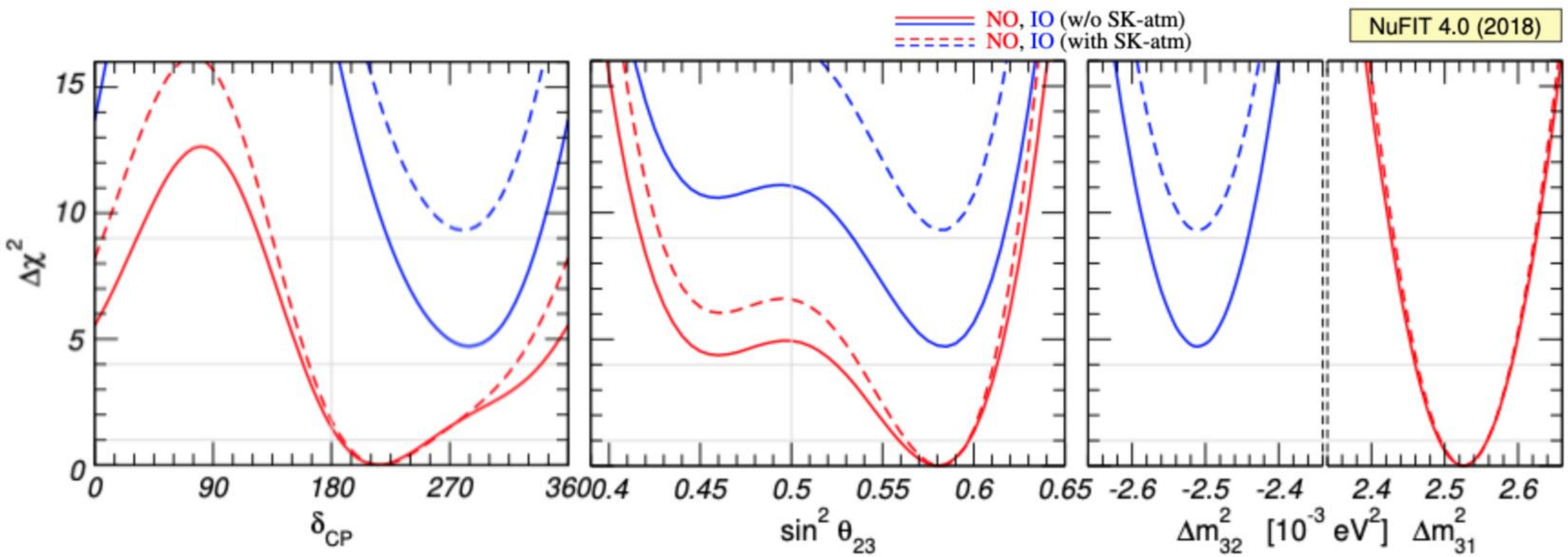
How precisely determined?



well-determined parameters
relat. precision at 3σ:

$14\%\,(\theta_{12})$, $8.9\%\,(\theta_{13})$
$16\%\,(\Delta m^2_{21})$, $7.8\,[7.6]\%\,(|\Delta m^2_{3\ell}|)$

Not-so-well measured



- CP conservation allowed at $\Delta\chi^2$ =1.8 but bf at $\delta = 217°$
- Octant of θ_{23} : 2nd octant preferred, bf at $sin^2\theta_{23} = 0.58$
- Mass ordering : NO is preferred over IO. (adding SK I-IV to the global fit → IO disfavored at 3σ)

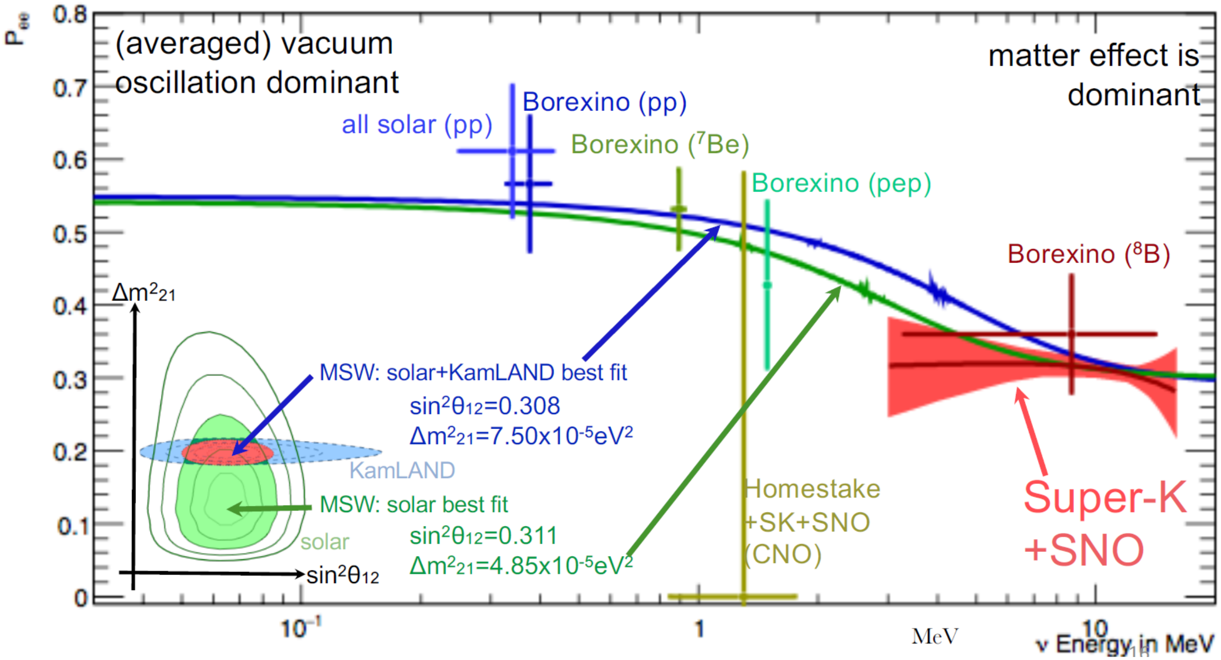
• MSW matter effect

(Minakata, Pena-garay, 2012)

$$P_{ee}^{D} = \cos^4\theta_{13}\left[1 - \frac{1}{2}\sin^2 2\theta_{12}(1 + \cos 2\theta_{12}\xi_S)\right] + \sin^4\theta_{13}, \quad \text{Low E}$$

$$P_{ee}^{D} = \cos^4\theta_{13}\left[\sin^2\theta_{12} + \frac{1}{4}\sin^2 2\theta_{12}\cos 2\theta_{12}\left(\frac{1}{\xi_S}\right)^2\right] + \sin^4\theta_{13} \quad \text{High E}$$

$$\xi_S \equiv \frac{l_\nu}{l_0} = 0.203 \times A_{MSW}\cos^2\theta_{13}\left(\frac{E}{1\,\mathrm{MeV}}\right)\left(\frac{\rho_S Y_e}{100\,g\,\mathrm{cm}^{-3}}\right)$$



BOREXINO (*Barbara Caccianiga* 2019)

Precision Measurements

parameter	best fit $\pm 1\sigma$	3σ range	relative 1σ uncertainty
Δm^2_{21} $[10^{-5}\mathrm{eV}^2]$	$7.55^{+0.20}_{-0.16}$	7.05–8.14	2.4%
$\|\Delta m^2_{31}\|$ $[10^{-3}\mathrm{eV}^2]$ (NO)	2.50 ± 0.03	2.41–2.60	1.3%
$\|\Delta m^2_{31}\|$ $[10^{-3}\mathrm{eV}^2]$ (IO)	$2.42^{+0.03}_{-0.04}$	2.31-2.51	
$\sin^2\theta_{12}/10^{-1}$	$3.20^{+0.20}_{-0.16}$	2.73–3.79	5.5%
$\sin^2\theta_{23}/10^{-1}$ (NO)	$5.47^{+0.20}_{-0.30}$	4.45–5.99	4.7%
$\sin^2\theta_{23}/10^{-1}$ (IO)	$5.51^{+0.18}_{-0.30}$	4.53–5.98	4.4%
$\sin^2\theta_{13}/10^{-2}$ (NO)	$2.160^{+0.083}_{-0.069}$	1.96–2.41	3.5%
$\sin^2\theta_{13}/10^{-2}$ (IO)	$2.220^{+0.074}_{-0.076}$	1.99–2.44	
δ/π (NO)	$1.32^{+0.21}_{-0.15}$	0.87–1.94	10%
δ/π (IO)	$1.56^{+0.13}_{-0.15}$	1.12–1.94	9%

(de Salas, Forero, Temes, Tortola, Valle, PLB782 , 1708.01186)

- Implications of global fit:

✓ $\theta_{12} + \theta_C = \pi/4$ satisfied within 2 $\boldsymbol{\sigma}$.

→ quark-lepton complementarity (Raidal, Smirnov, Minakata, SK, Kim,….'04)

✓ Non-maximal θ_{23} is favored at 2 (1.5) σ level for NO (IO)

→ could be related to $\sqrt{m_2/m_3}$ similar to Gatto-Sartoti-Tonin

✓ Zero θ_{13} is excluded at 10 σ. → test for flavor models

✓ Two large angles → hint for discrete flavor symmetry?

✓ $\delta \simeq 3\pi/2$ is favored by LBL exps.

→ could be related with mixing angles, flavor symmetries etc. ?

Theoretical Issues

- Origins of neutrino mixing & CP violation
  - flavor symmetry
  - predictions
- Origins of tiny neutrino mass
  - seesaw variants
  - radiative generation
- New physics in neutrino oscillation

I. Origin of mixing pattern

- From fit to neutrino data in 3-neutrino paradigm

$$|U_{PMNS}| = \begin{pmatrix} 0.800-0.844 & 0.515-0.581 & 0.139-0.155 \\ 0.229-0.516 & 0.438-0.699 & 0.614-0.790 \\ 0.249-0.528 & 0.462-0.715 & 0.595-0.776 \end{pmatrix}$$

Looks different from quark mixing matrix !!

$$|V_{CKM}| = \begin{pmatrix} 0.97434 & 0.22506 & 0.00357 \\ 0.22492 & 0.97351 & 0.0414 \\ 0.00875 & 00403 & 0.99915 \end{pmatrix}$$

PDG(2016)

- How do we understand ν mixing matrix ?

Before measuring θ_{13} , tri-bimaximal mixing hypothesis :

Harrison & Perkins & Scott (2002)

- $U^{TBM} = \begin{pmatrix} \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$

$\theta_{13} \approx 0; \quad \theta_{23} \approx 45^0; \quad \theta_{12} = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right) \approx 35.3^0$

- generates specific neutrino mass matrix

$$UM_\nu^D U^T = \begin{pmatrix} m_1 & m_2 & m_2 \\ \cdot & \frac{1}{2}(m_1+m_2+m_3) & \frac{1}{2}(m_1+m_2-m_3) \\ \cdot & \cdot & \frac{1}{2}(m_1+m_2+m_3) \end{pmatrix}$$

$$= \frac{m_1+m_3}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{m_2-m_1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_1-m_3}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

A_4 symmetric

– Integer matrix elements suggest non−Abelian discrete symmetry

- Mixing understanding from discrete symmetries

-setting $U_{PMNS} = (\vec{u}_1, \vec{u}_2, \vec{u}_3)$, we construct group generators:

$$S_1 = \vec{u}_1 \overrightarrow{u_1}^+ - \vec{u}_2 \overrightarrow{u_2}^+ - \vec{u}_3 \overrightarrow{u_3}^+$$

$$S_2 = -\vec{u}_1 \overrightarrow{u_1}^+ + \vec{u}_2 \overrightarrow{u_2}^+ - \vec{u}_3 \overrightarrow{u_3}^+$$

(CSLam'06)

$$S_3 = -\vec{u}_1 \overrightarrow{u_1}^+ - \vec{u}_2 \overrightarrow{u_2}^+ + \vec{u}_3 \overrightarrow{u_3}^+$$

- $S_1 = \frac{1}{3}\begin{pmatrix} 1 & -2 & -2 \\ -2 & -2 & -1 \\ 2 & -1 & -2 \end{pmatrix}$ $S_2 = \frac{1}{3}\begin{pmatrix} -1 & 2 & -2 \\ 2 & -1 & -2 \\ -2 & -2 & -1 \end{pmatrix}$ $S_3 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

  then, $S^T \bar{M}_\nu S = \bar{M}_\nu$

C.S.Lam, PRD98(2008)
arXiv:0809.1185

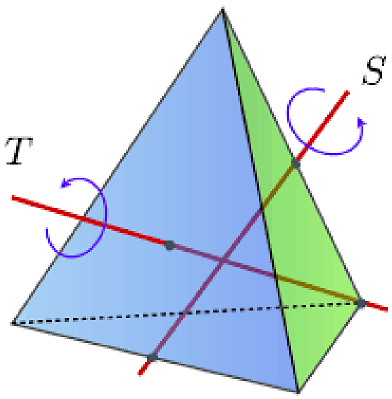
- For charged lepton, $T^+ \bar{M}_e T = \bar{M}_e$ $with$ $\bar{M}_e = {M_e}^+ M_e$ & $T^n = 1$

$\{S, T\}$ forms a discrete group (flavor symmetry)

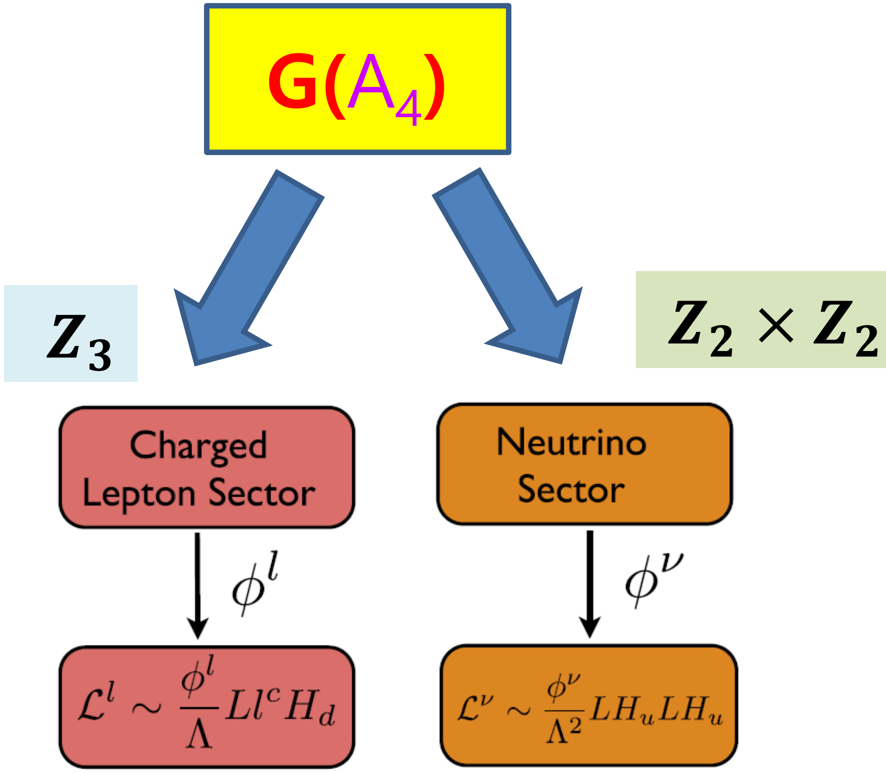
(Mixing matrices diagonalize M_ν and $\bar{M}_e$ also diagonalize S and T)

- Simplest group with a triplet representation: A_4

A_4 has subgroups: three Z_2, four Z_3, one $Z_2 \times Z_2$



- A_4 is spontaneously broken to subgroups:
  Neutrino sector preserves,
  $Z_2 \times Z_2$:
  Charged lepton sector preserves,
  Z_3:



arXiv: 1402.4271 King, Merle, Morisi, Simizu, Tanimoto

- Many discrete groups reproducing TBM mixing

Group	d	Irr. Repr.'s	Presentation
$D_3 \sim S_3$	6	$1,\ 1',\ 2$	$A^3 = B^2 = (AB)^2 = 1$
D_4	8	$1_1, ...1_4, 2$	$A^4 = B^2 = (AB)^2 = 1$
D_7	14	$1,\ 1',\ 2,\ 2',\ 2''$	$A^7 = B^2 = (AB)^2 = 1$
A_4	12	$1,\ 1',\ 1'',\ 3$	$A^3 = B^2 = (AB)^3 = 1$
$A_5 \sim PSL_2(5)$	60	$1,\ 3,\ 3',\ 4,\ 5$	$A^3 = B^2 = (BA)^5 = 1$
T'	24	$1,\ 1',\ 1'',\ 2,\ 2',\ 2'',\ 3$	$A^3 = (AB)^3 = R^2 = 1,\ \ B^2 = R$
S_4	24	$1,\ 1',\ 2,\ 3,\ 3'$	$BM : A^4 = B^2 = (AB)^3 = 1$ $TB : A^3 = B^4 = (BA^2)^2 = 1$
$\Delta(27) \sim Z_3 \rtimes Z_3$	27	$1_1, ...1_9, 3, \bar{3}$	
$PSL_2(7)$	168	$1, 3, \bar{3}, 6, 7, 8$	$A^3 = B^2 = (BA)^7 = (B^{-1}A^{-1}BA)^4 = 1$
$T_7 \sim Z_7 \rtimes Z_3$	21	$1, 1', \bar{1}', 3, \bar{3}$	$A^7 = B^3 = 1,\ \ AB = BA^4$

(Altarelli, Feruglio, 1002.0211)

Each group has many models!

Type	L_i	ℓ_i^c	ν_i^c	Δ
A1	$\underline{3}$	$\underline{1},\ \underline{1}',\ \underline{1}''$	$\cdots$	$\cdots$
A2				$\underline{1},\ \underline{1}',\ \underline{1}'',\ \underline{3}$
B1	$\underline{3}$	$\underline{1},\ \underline{1}',\ \underline{1}''$	$\underline{3}$	$\cdots$
B2				$\underline{1},\ \underline{3}$
C1	$\underline{3}$	$\underline{3}$	$\cdots$	$\cdots$
C2				$\underline{1}$
C3				$\underline{1},\ \underline{3}$
C4				$\underline{1},\ \underline{1}',\ \underline{1}'',\ \underline{3}$
D1	$\underline{3}$	$\underline{3}$	$\underline{3}$	$\cdots$
D2				$\underline{1}$
D3				$\underline{1}'$
D4				$\underline{1}',\ \underline{3}$
E	$\underline{3}$	$\underline{3}$	$\underline{1},\ \underline{1}',\ \underline{1}''$	$\cdots$
F	$\underline{1},\ \underline{1}',\ \underline{1}''$	$\underline{3}$	$\underline{3}$	$\underline{1}$ or $\underline{1}'$
G	$\underline{3}$	$\underline{1},\ \underline{1}',\ \underline{1}''$	$\underline{1},\ \underline{1}',\ \underline{1}''$	$\cdots$
H	$\underline{3}$	$\underline{1},\ \underline{1},\ \underline{1}$	$\cdots$	$\cdots$
I	$\underline{3}$	$\underline{1},\ \underline{1},\ \underline{1}$	$\underline{1},\ \underline{1},\ \underline{1}$	$\cdots$
J	$\underline{3}$	$\underline{1},\ \underline{1},\ \underline{1}$	$\underline{3}$	$\cdots$

(Barry, Rodejohann, PRD81(2010)

- Modification of Tri-Bimaximal Mixing

  - Simple possible forms :

$$\begin{cases} U_{TBM}\, U_{ij}(\theta) \\ U_{ij}^{+}(\theta)\, U_{TBM} \end{cases}$$

  - θ possibly gives rise to non-zero θ_{13} and deviation from maximal θ_{23}

(He & Zee, PLB645(2007), SK & Kim PRD90(2014)
See also, Goswami, Petcov, Ray, Rodejohann,PRD80(2009))

  - Best fit achieved by (SK & Kim, PRD90(2014))

$$U_{TBM} \cdot U_{23} \sim \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}}\lambda \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}}+\frac{1}{\sqrt{2}}\lambda & \frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}\lambda \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}}-\frac{1}{\sqrt{2}}\lambda & \frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}\lambda \end{pmatrix} \qquad (c_{23}\sim 1,\ \ s_{23}\sim\lambda)$$

- $$U^{TBM} \cdot U_{12} = \begin{pmatrix} \blacksquare & \blacksquare & 0 \\ \blacksquare & \blacksquare & -\frac{1}{\sqrt{2}} \\ \blacksquare & \blacksquare & \frac{1}{\sqrt{2}} \end{pmatrix} \quad U^{TBM} \cdot U_{13} = \begin{pmatrix} \blacksquare & -\frac{1}{\sqrt{3}} & \blacksquare \\ \blacksquare & \frac{1}{\sqrt{3}} & \blacksquare \\ \blacksquare & \frac{1}{\sqrt{3}} & \blacksquare \end{pmatrix}$$

unchanged

- $$U^{TBM} \cdot U_{23} = \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} & \blacksquare & \blacksquare \\ \frac{1}{\sqrt{6}} & \blacksquare & \blacksquare \\ \frac{1}{\sqrt{6}} & \blacksquare & \blacksquare \end{pmatrix}$$

Unchanged columns may reflect the remnants of flavor symmetry → residual symmetry

Predictions of CP phase

- Any forms of neutrino mixing matrix should be equivalent to U_{PMNS} presented in the standard parameterization :
- $U_{PMNS} = U_{23}(\theta_{23})U_{13}(\theta_{13},\delta_D)U_{12}(\theta_{12})P_\phi$

$$= \begin{pmatrix} c_{12}c_{13} & -s_{12}c_{13} & s_{13}e^{i\delta_D^*} \\ * & * & -s_{23}c_{13} \\ * & * & c_{23}c_{13} \end{pmatrix} \cdot \begin{pmatrix} e^{i\phi_1} & & \\ & e^{i\phi_2} & \\ & & e^{i\phi_3} \end{pmatrix}$$

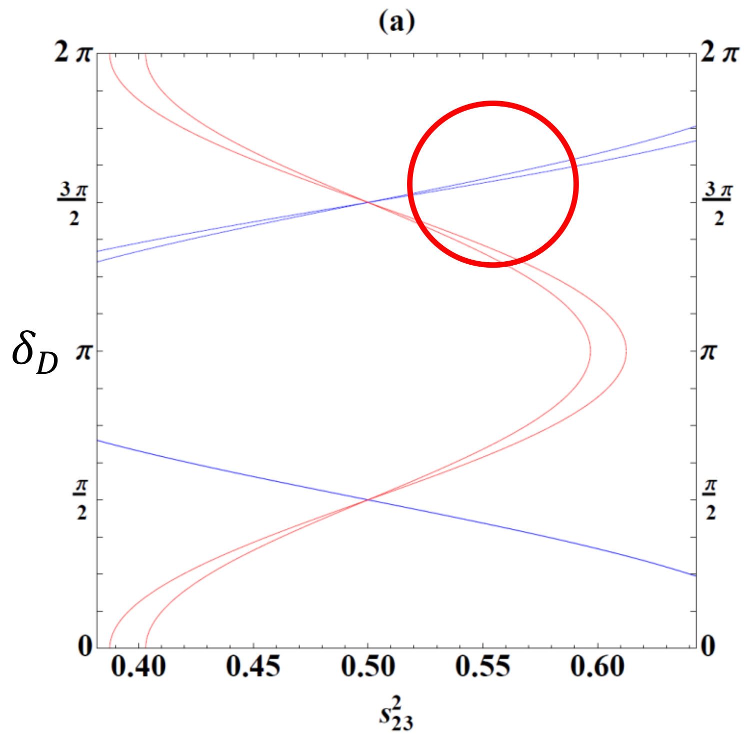
$$= P_\alpha \cdot V \cdot P_\beta \quad , V = U^{TBM} \cdot U_{23(13)}(\theta,\xi)$$

$$\Rightarrow \quad \boxed{V_{ij}e^{i(\alpha_i+\beta_j)} = (U_{PMNS})_{ij}}$$

- Predictions : (SK & CSKim, PRD90(2014), SK & Tanimoto, PRD91(2015))

$$s_{12}{}^2 = 1 - \frac{2}{3(1-s_{13}{}^2)} \qquad \cos\delta_D = \frac{1}{2\tan 2\theta_{23}} \cdot \frac{1-5s_{13}{}^2}{s_{13}\sqrt{2-6s_{13}{}^2}}$$

$$s_{12}{}^2 = \frac{1}{3(1-s_{13}{}^2)} \qquad \cos\delta_D = \frac{1}{\tan 2\theta_{23}} \cdot \frac{1-2s_{13}{}^2}{s_{13}\sqrt{2-3s_{13}{}^2}}$$



The relations can be obtained from A4 model

(SK & Tanimoto, PRD91 (2015))

.

Alternatively,

$$\begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{e^{-i\rho}\cos\theta}{\sqrt{3}} & -\frac{ie^{-i\rho}\sin\theta}{\sqrt{3}} \\ -\frac{e^{i\rho}}{\sqrt{6}} & \frac{\cos\theta}{\sqrt{3}} - \frac{ie^{-i\sigma}\sin\theta}{\sqrt{2}} & \frac{e^{-i\sigma}\cos\theta}{\sqrt{2}} - \frac{i\sin\theta}{\sqrt{3}} \\ \frac{e^{i(\rho+\sigma)}}{\sqrt{6}} & -\frac{e^{i\sigma}\cos\theta}{\sqrt{3}} - \frac{i\sin\theta}{\sqrt{2}} & \frac{\cos\theta}{\sqrt{2}} + \frac{ie^{i\sigma}\sin\theta}{\sqrt{3}} \end{bmatrix}$$

$$\sin^2\theta_{12} = \frac{\cos^2\theta}{\cos^2\theta + 2},$$

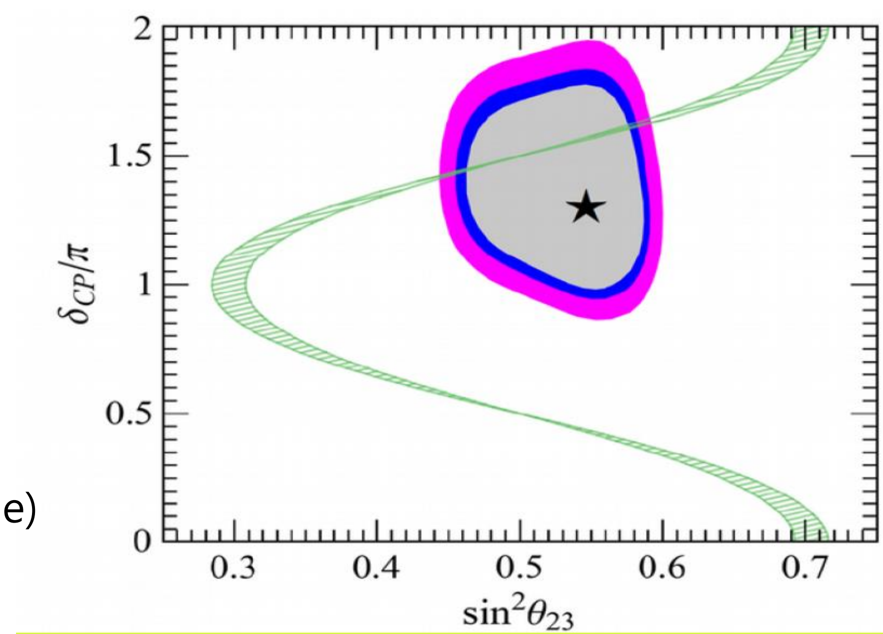
$$\sin^2\theta_{13} = \frac{\sin^2\theta}{3},$$

$$\sin^2\theta_{23} = \frac{1}{2} + \frac{\sqrt{6}\sin 2\theta \sin\sigma}{2\cos^2\theta + 4}$$

$$\tan\delta_{CP} = \frac{(\cos^2\theta + 2)\cot\sigma}{5\cos^2\theta - 2},$$

PHYSICAL REVIEW D **98,** 055019 (2018)

(Cheng, Chulia, Ding, Srivastava, Valle)



- A_4 model easily realizes non-vanishing θ_{13} & CPV

Ahn, SK, PRD86 (2012)
Ahn, SK, CSKim, PRD87 (2013)
SK, Shimizu, Takagi,Shunya Takahashi,Tanimoto, PTEP(2018)

Additional Matrix

$$M_\nu = a\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} + b\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + c\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix} + d\boxed{\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}}$$

$$a = \frac{y^\nu_{\phi_\nu}\alpha_\nu v_u^2}{\Lambda}, \qquad b = -\frac{y^\nu_{\phi_\nu}\alpha_\nu v_u^2}{3\Lambda}, \qquad c = \frac{y^\nu_{\xi}\alpha_\xi v_u^2}{\Lambda}, \qquad d = \frac{y^\nu_{\xi'}\alpha_{\xi'} v_u^2}{\Lambda} \qquad a = -3b$$

Both normal and inverted mass hierarchies are possible.

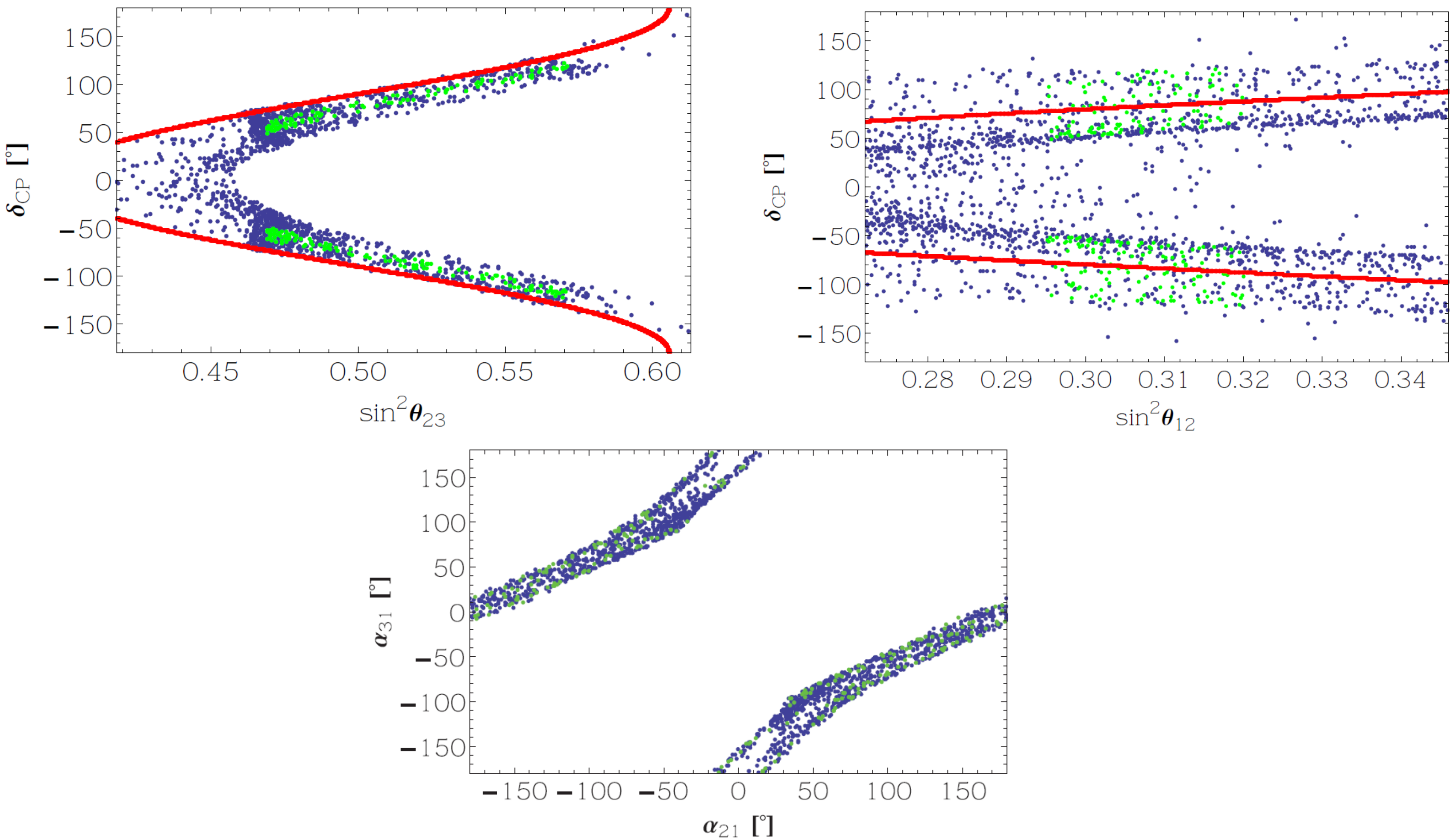
$$M_\nu = V_{\text{tri-bi}}\begin{pmatrix} a+c-\frac{d}{2} & 0 & \frac{\sqrt{3}}{2}d \\ 0 & a+3b+c+d & 0 \\ \frac{\sqrt{3}}{2}d & 0 & a-c+\frac{d}{2}\end{pmatrix} V^T_{\text{tri-bi}}$$

Tri-maximal mixing: TM2

$$\Delta m^2_{31} = -4a\sqrt{c^2+d^2-cd}\,, \qquad \Delta m^2_{21} = (a+3b+c+d)^2 - (a+\sqrt{c^2+d^2-cd})^2$$

- A_4 model easily realizes non-vanishing θ_{13} & CPV

Ahn, SK, PRD86 (2012)
Ahn, SK, CSKim, PRD87 (2013)
SK, Shimizu, Takagi,Shunya Takahashi,Tanimoto, PTEP(2018)



How to test Flavor Symmetry

- UV theories giving rise to flavor symmetry in lepton sector contains new scalars→ probe of signal be test of FlaSy.
- Mixing angle sum rules:

Example:

$$\sin^2\theta_{23} = \frac{1}{2}\frac{1}{\cos^2\theta_{13}} \geq \frac{1}{2}\,, \quad \sin^2\theta_{12} \simeq \frac{1}{\sqrt{3}} - \frac{2\sqrt{2}}{3}\sin\theta_{13}\cos\delta_{CP} + \frac{1}{3}\sin^2\theta_{13}\cos 2\delta_{CP}$$

$$\sin^2\theta_{12} = \frac{1}{3}\frac{1}{\cos^2\theta_{13}} \geq \frac{1}{3}\,, \qquad \cos\delta_{CP}\tan 2\theta_{23} \simeq \frac{1}{\sqrt{2}\sin\theta_{13}}\left(1 - \frac{5}{4}\sin^2\theta_{13}\right)$$

$$s_{12}{}^2 = 1 - \frac{2}{3(1-s_{13}{}^2)}$$

- Neutrino mass sum rules in FLaSy ⇔different $0\nu\beta\beta$

- Prediction of CP phase

(Girardi, Petcov, Titov, NPB894(2015))

(e.g.) $\cos\delta_D = \frac{1}{\tan 2\theta_{23}} \cdot \frac{1-5s_{13}{}^2}{s_{13}\sqrt{2-6s_{13}{}^2}}$

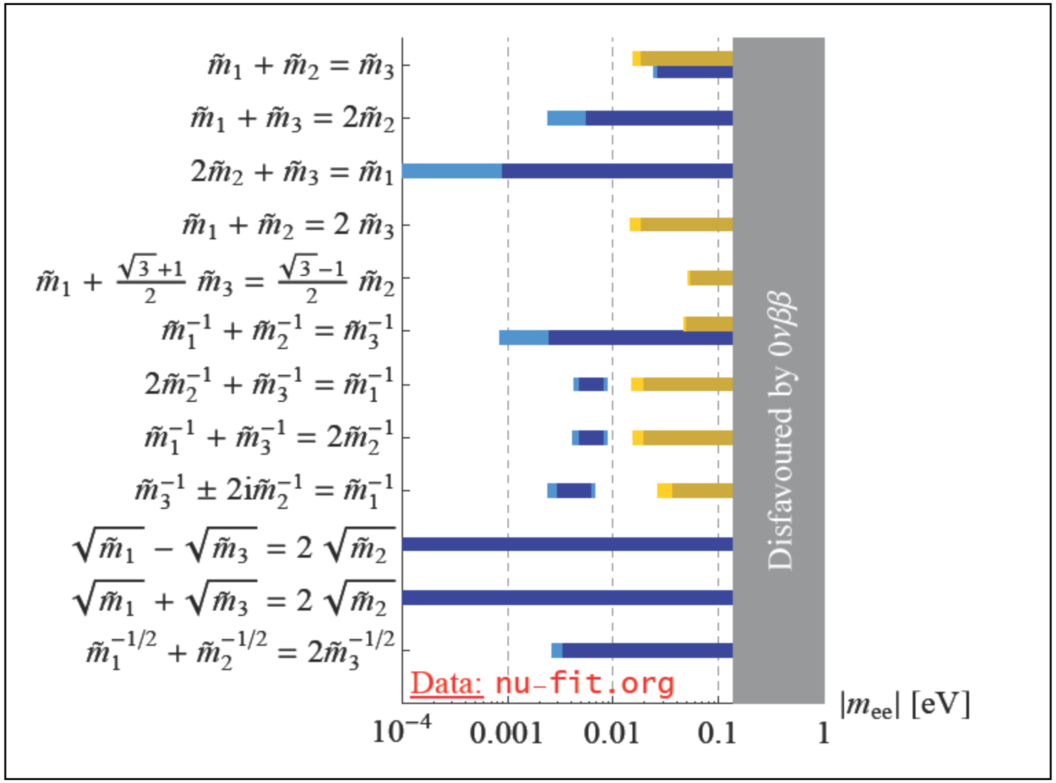
Neutrino mass sum rules

(King, Merle,Morisi, Shimizu, Tanimoto, New J. Phys. 2014)

Sum Rule	Group	Seesaw Type	Matrix
$\tilde{m}_1 + \tilde{m}_2 = \tilde{m}_3$	A_4[167]([175, 178–181]); S_4([182]); A_5[69][a]	Weinberg	m^{ν}_{LL}
$\tilde{m}_1 + \tilde{m}_2 = \tilde{m}_3$	$\Delta(54)$[183]; S_4([163])	Type II	M_L
$\tilde{m}_1 + 2\tilde{m}_2 = \tilde{m}_3$	S_4[120]	Type II	M_L
$2\tilde{m}_2 + \tilde{m}_3 = \tilde{m}_1$	A_4[165, 167] ([36, 37, 188–194, , , , , , , 178–181]) S_4([45, 124])[b]; T'[195, 196] ([46, 134, 197, 198]); T_7([199])	Weinberg	m^{ν}_{LL}
$2\tilde{m}_2 + \tilde{m}_3 = \tilde{m}_1$	A_4([200])	Type II	M_L
$\tilde{m}_1 + \tilde{m}_2 = 2\tilde{m}_3$	S_4[201][c]	Dirac[c]	m^D
$\tilde{m}_1 + \tilde{m}_2 = 2\tilde{m}_3$	$L_e - L_\mu - L_\tau$([202])	Type II	M_L
$\tilde{m}_1 + \frac{\sqrt{3}+1}{2}\tilde{m}_3 = \frac{\sqrt{3}-1}{2}\tilde{m}_2$	A_5'([203])	Weinberg	m^{ν}_{LL}
$\tilde{m}_1^{-1} + \tilde{m}_2^{-1} = \tilde{m}_3^{-1}$	A_4[167]; S_4([163, 175]); A_5[176, 177]	Type I	M_R
$\tilde{m}_1^{-1} + \tilde{m}_2^{-1} = \tilde{m}_3^{-1}$	S_4([163])	Type III	M_Σ
$2\tilde{m}_2^{-1} + \tilde{m}_3^{-1} = \tilde{m}_1^{-1}$	A_4[135, 164, 165, 167, 204] ([37, 137, 145, 205–211]); T'[196]	Type I	M_R
$\tilde{m}_1^{-1} + \tilde{m}_3^{-1} = 2\tilde{m}_2^{-1}$	A_4([212–214]); T'[215]	Type I	M_R
$\tilde{m}_3^{-1} \pm 2i\tilde{m}_2^{-1} = \tilde{m}_1^{-1}$	$\Delta(96)$[66]	Type I	M_R
$\tilde{m}_1^{1/2} - \tilde{m}_3^{1/2} = 2\tilde{m}_2^{1/2}$	A_4([162])	Type I	m^D
$\tilde{m}_1^{1/2} + \tilde{m}_3^{1/2} = 2\tilde{m}_2^{1/2}$	A_4([216])	Scotogenic	h_ν
$\tilde{m}_1^{-1/2} + \tilde{m}_2^{-1/2} = 2\tilde{m}_3^{-1/2}$	S_4[217]	Inverse	M_{RS}

Neutrino mass sum rules

Restrictions on $|m_{ee}|$ by mass sum rules



King, Merle, Stuart, JHEP2013
King, Merle,Morisi, Shimizu, Tanimoto, New J. Phys. 2014

II. Origin of Neutrino Mass

- Why neutrinos are massless in SM ?

  - no right-handed neutrinos
  - only SU(2) doublet Higgs scalars
  - prohibiting non-renormalizable terms

- How can neutrinos have mass ?

  -breaking those restrictions

Seesaw Origins

Type-I Seesaw

- Introducing L-conserving right-handed neutrinos

$$Y_\nu \Phi \bar{\nu}_L \nu_R \;\rightarrow\; Y_\nu < \phi^0 > \bar{\nu}_L \nu_R \quad \sim 0.2 \text{ eV}$$

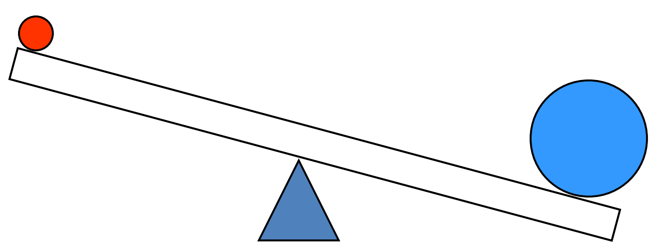
➔ $Y_\nu \sim 10^{-12}$ ： why so small?

- No principle prohibit $M_R \overline{\nu_R^C} \nu_R$
- Seesaw mechanism :

-ν_R can have large mass (L-violation: Type-I)(Minkowski '77 Gellman Ramond Slansky '80 Glashow, Yanagida '79 Mohapatra Senjanovic '80 Lazarides Shafi Weterrich '81 Schechter-Valle '80 & '82)

$$\boxed{L_{mass} = \frac{1}{2}(\nu_L^T N_R^T) C^{-1} \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix} \begin{pmatrix} \nu_L \\ N_R \end{pmatrix} + c.c.}$$

$$-\frac{m_D^2}{M}$$

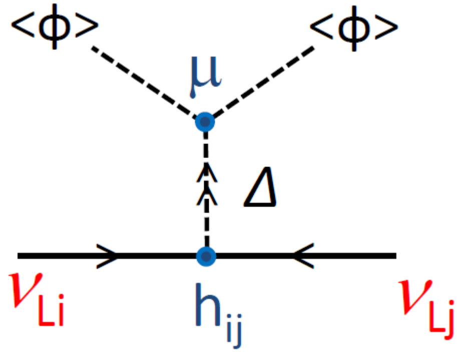


$$M$$

Type-II Seesaw

- Introducing SU(2) triplet Higgs (Δ) (type-II):
  $\mathrm{h}LL\Delta \leftarrow \langle\Delta\rangle$<8 GeV from ρ parameter.
  majorana mass

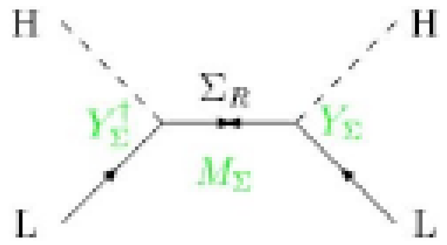
- Due to additional possible terms:
  $\mu\Phi\Delta^{+}\Phi + \mathrm{M}_{\Delta}{}^{2}\mathrm{Tr}[\Delta^{+}\Delta]$
  $\rightarrow \langle\Delta\rangle = \frac{\mu\langle\Phi^{0}\rangle^{2}}{\mathrm{M}_{\Delta}{}^{2}}$



(Magg, Wetterich; Lazarides, Shafi;
Mohapatra, Senjanovic; Schechter, Valle)

Type-III Seesaw

- Introducing SU(2) triplet fermions



$$m_\nu = Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma v^2$$

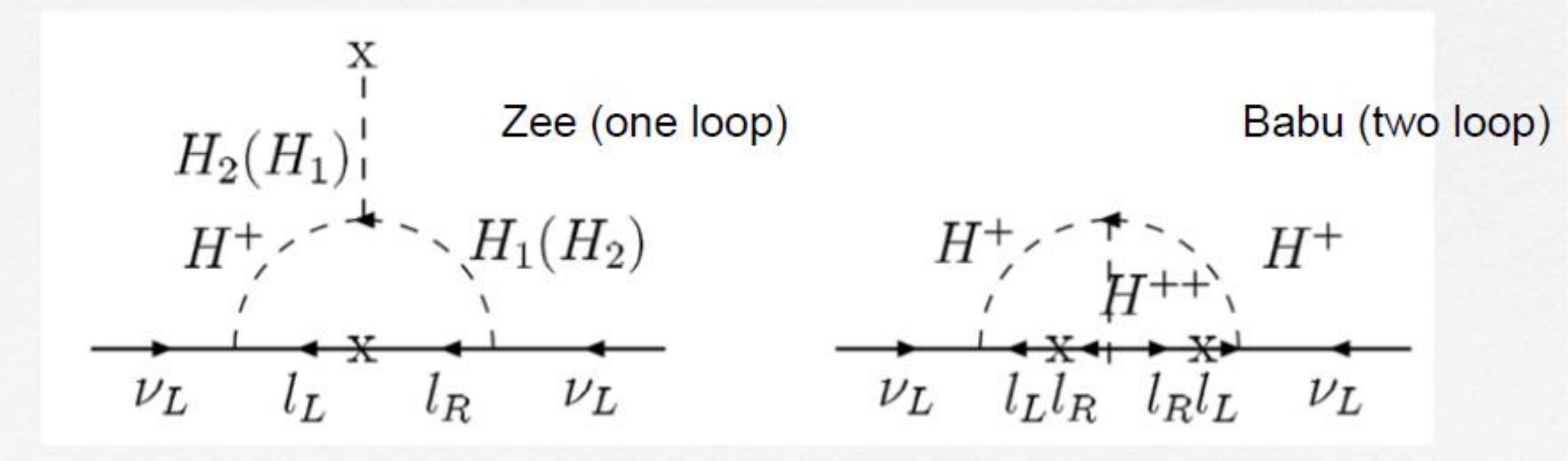
Foot, Lew, He, Joshi; Ma; Ma, Roy; T.H., Lin, Notari, Papucci, Strumia; Bajc, Nemevsek, Senjanovic; Dorsner, Fileviez-Perez;....

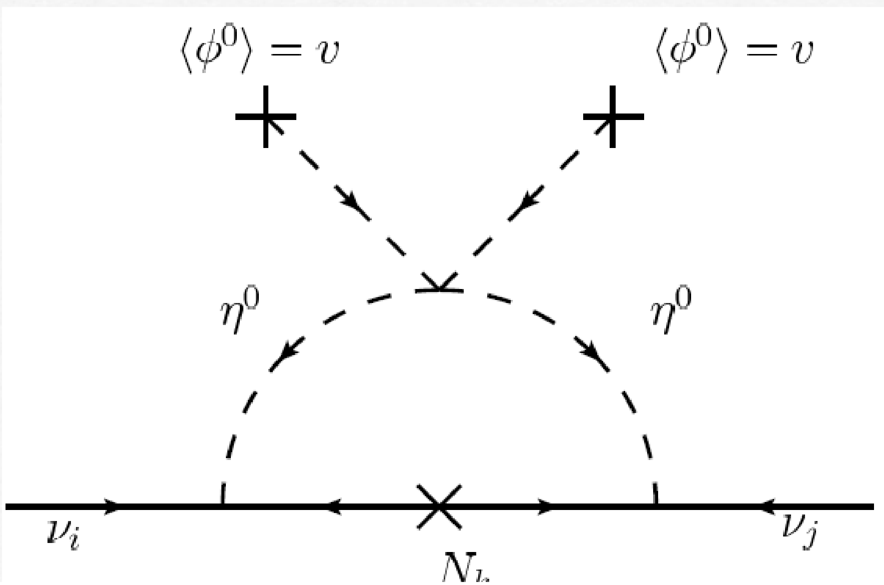
- Non-renormalizable term

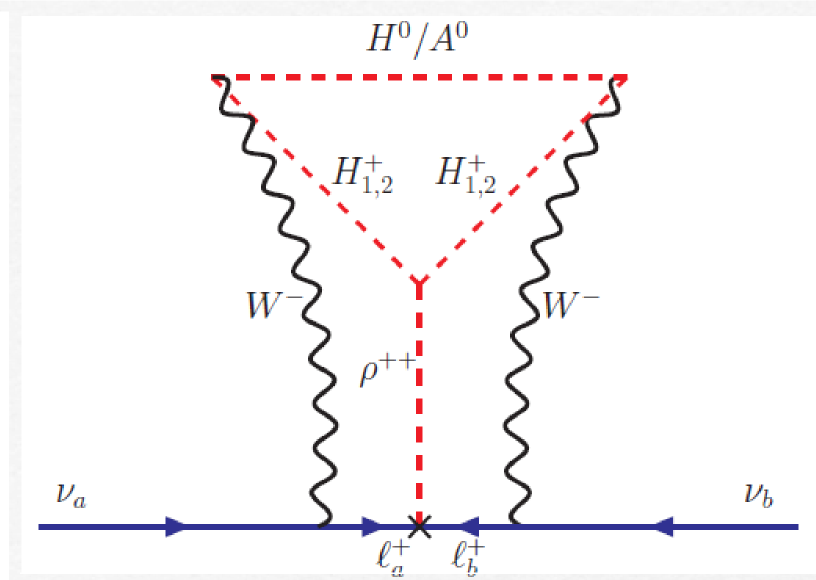
$$\frac{\lambda}{M} LL\Phi\Phi \rightarrow \frac{\lambda}{M} \langle \phi^0 \rangle^2$$ same as type-I seesaw

- Radiative generation of neutrino masses

talk by Ramond Volkas





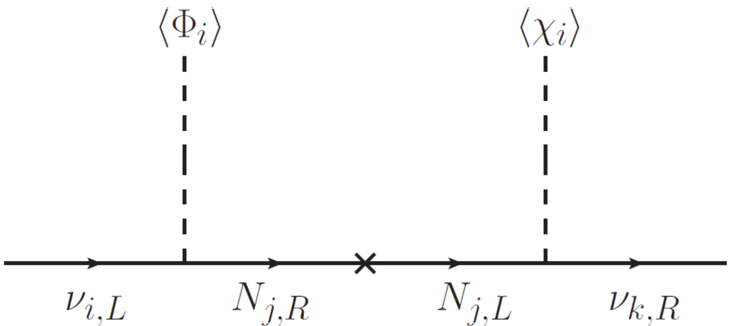


- Scotogenic (Ma) Cocktail (Gustafusson etal.)

- R-parity violating SUSY model

Seesaw for Dirac Neutrino

- Type-I Seesaw



Chulia, Srivastava, Valle, PLB761 (2016),
Chulia, Ma, Srivastava, Valle ,PLB767 (2017)

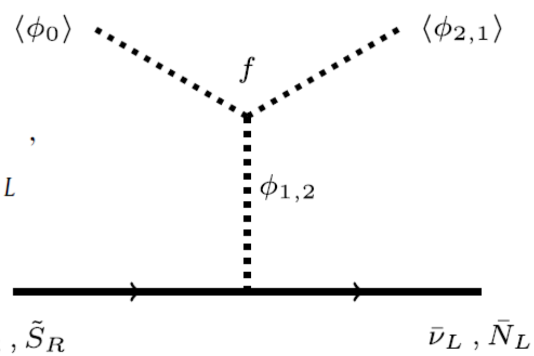
The Dirac type-I seesaw mechanism. Φ_i and χ_i are triplets under $\Delta(27)$

Type-II Seesaw

(Valle,Vaquera-Araujo, PLB755(2016),

Addazi et al PLB759 (2016))

$$\psi_L^\ell = \begin{pmatrix} \ell^- \\ \nu_\ell \\ N_\ell \end{pmatrix}_L ,$$



Anomaly free $SU(3)_C \times SU(3)_L \times U(1)_x$

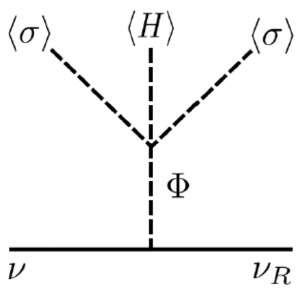
Matter content of the model, where $\hat{u}_R \equiv (u_R, c_R, t_R, U_R)$ and $\hat{d}_R \equiv (d_R, s_R, b_R, D_R, D'_R)$.

	ψ_L^ℓ	ℓ_R	S_R^ℓ, $\tilde{S}_R^\ell$	$Q_L^{1,2}$	Q_L^3	$\hat{u}_R$	$\hat{d}_R$	ϕ_0	ϕ_1	ϕ_2
$SU(3)_C$	**1**	**1**	**1**	**3**	**3**	**3**	**3**	**1**	**1**	**1**
$SU(3)_L$	$\mathbf{3}^*$	**1**	**1**	**3**	$\mathbf{3}^*$	**1**	**1**	$\mathbf{3}^*$	$\mathbf{3}^*$	$\mathbf{3}^*$
$U(1)_X$	$-\frac{1}{3}$	-1	0	0	$+\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$
$\mathcal{L}$	$-\frac{1}{3}$	-1	1	$-\frac{2}{3}$	$+\frac{2}{3}$	0	0	$+\frac{2}{3}$	$-\frac{4}{3}$	$-\frac{4}{3}$
$\mathbb{Z}_3^{aux}$	ω	ω	ω	ω^2	ω^2	ω^2	ω^2	1	1	1

Seesaw for Dirac Neutrino

- Type-II Seesaw

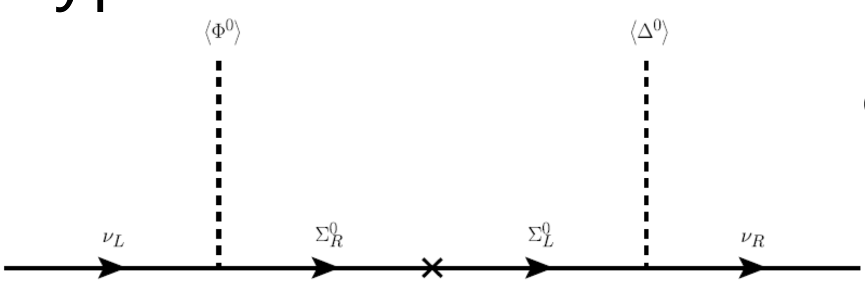
Bonilla, Valle, PLB762(2016)
Reig et al., PRD94(2016)



	$\overline{L}$	ℓ_R	ν_R	H	Φ	σ
$SU(2)_L$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$
$\mathbb{Z}_5$	ω	ω^4	ω	1	ω^3	ω
$\mathbb{Z}_3$	α^2	α	α	1	1	1

Neutrino mass generation in type-II Dirac seesaw mechanism

- Type-III Seesaw

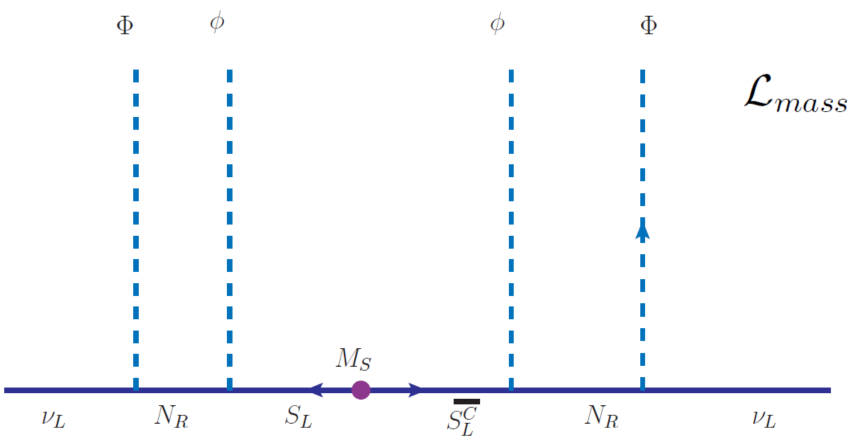


Chulia, Srivastava, Valle, PLB781(2018)

Neutrino mass generation in type-III Dirac seesaw
There can be d=5 op. leading to tiny Dirac mass.

Inverse seesaw

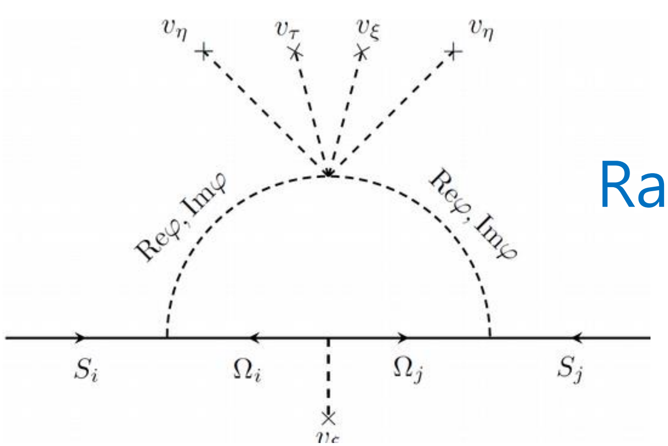
- Inverse seesaw



$$\mathcal{L}_{mass} = \left(\overline{\nu_L^C}\ \overline{N_R}\ \overline{S_L^C}\right) \begin{pmatrix} 0 & m_D^* & 0 \\ m_D^\dagger & 0 & m_{NS} \\ 0 & m_{NS}^T & M_S \end{pmatrix} \begin{pmatrix} \nu_L \\ N_R^C \\ S_L \end{pmatrix}$$

$$m_\nu = (m_D^* m_{NS}^{-1}) M_s (m_D^* m_{NS}^{-1})^T$$

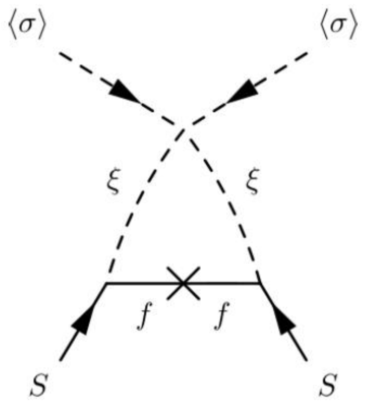
Mohapatra, PRL56(1986)
Mohapatra, Valle, PRD34(1986)



Radiative Inverse seesaw

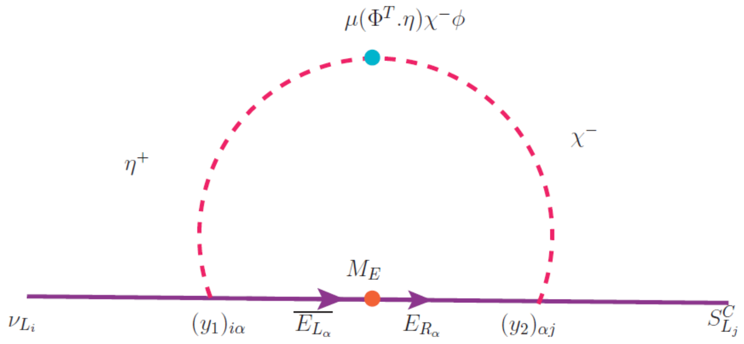
Carcamo Hernandez et al JHEP 1902 (2019)

- Scotogenic inverse seesaw



arXiv:1907.07728

- Inverse seesaw+1-loop (A. Das et al, 1704.02078)



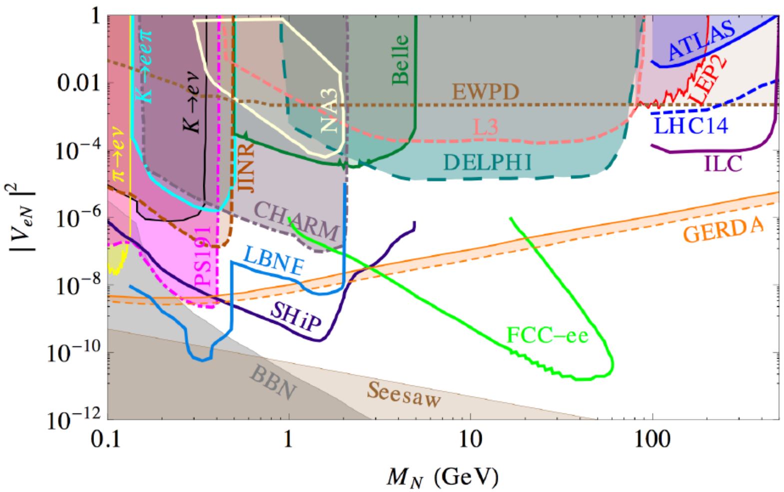
$$m_\nu^{\text{tree+1-loop}} = \begin{pmatrix} 0 & m_D^* & \delta_1^* \\ m_D^\dagger & 0 & m_{NS} \\ \delta_1^\dagger & m_{NS}^T & M_S \end{pmatrix}$$

- Dirac Inverse seesaw (Borah, Karmakar, PLB780(2018))

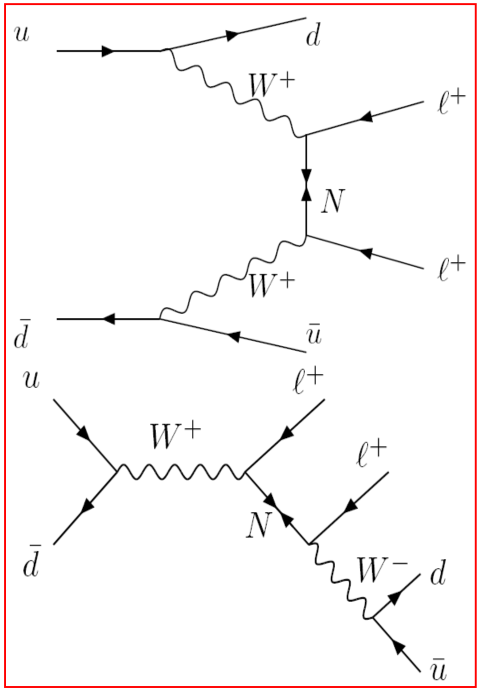
What is the Seesaw scale ?

- For $m_D \sim m_t$, neutrino mass of $m_\nu \leq 1$ eV implies $M_R \sim 10^{14}$ GeV
  - close to the scale of Grand Unification ~ 10^{16} GeV

- For $m_D \sim m_e$, neutrino mass of $m_\nu \leq 1$ eV implies $M_R \sim 1$ TeV.
  -potentially testable at collider



Deppisch, Dev, Pilaftsis, 1502.06541



What is the Seesaw scale ?

- νMSM (Asaka, Blanchet,Shaposhnikov, PLB631(2005)):
  - $M_{R1} \sim$ keV scale warm dark matter
  - $M_{R2(R3)}$ ~few GeV with tiny Yukawa couplings
- Minimal SM accommodating DM, baryogenesis at the price of fine tuning.

III.New Physics in ν Oscillation

- What causes deviation of standard oscillations
  - Non-standard Interactions(NSI)
  - Unitarity violation in U_{PMNS}
  - light sterile neutrinos
  - long-range forces
  - Lorentz/CPT violation
  - General neutrino interactions
  - decay etc.

NSI

- Existence of NSI indicates new physics beyond the SM

$$\delta\mathcal{L}_{\rm NSI} = -2\sqrt{2}\,G_F \sum_{f,P} \epsilon_{\alpha\beta}^{fP} \left(\overline{\nu_\alpha}\gamma^\mu P_L \nu_\beta\right)\left(\bar{f}\gamma_\mu P f\right)$$

- effect of NSI in propagation can be presented through modification of matter potential

$$\varepsilon_{\alpha\beta}^{f} = \varepsilon_{\alpha\beta}^{fL} + \varepsilon_{\alpha\beta}^{fR}$$

$$H_{\rm mat} = \sqrt{2}G_F N_e(x) \begin{pmatrix} 1+\epsilon_{ee}(x) & \epsilon_{e\mu}(x) & \epsilon_{e\tau}(x) \\ \epsilon_{e\mu}^*(x) & \epsilon_{\mu\mu}(x) & \epsilon_{\mu\tau}(x) \\ \epsilon_{e\tau}^*(x) & \epsilon_{\mu\tau}^*(x) & \epsilon_{\tau\tau}(x) \end{pmatrix}$$

- Even if no mixing in vacuum, $\nu_\alpha \to \nu_\beta$ can occur in matter
- Complex phases of off-diag. could be new source of CPV

Current conservative model independent bounds

$$\begin{pmatrix} |\epsilon_{ee}| < 4.2 & |\epsilon_{e\mu}| < 0.33 & |\epsilon_{e\tau}| < 3.0 \\ & |\epsilon_{\mu\mu}| < 0.07 & |\epsilon_{\mu\tau}| < 0.33 \\ & & |\epsilon_{\tau\tau}| < 21 \end{pmatrix}$$

Deepthi, Goswami, Nath, PLB936 (2018)

- NSI may affect neutrinos at the production point as well as detection point.
- To see those effects, we use different parameters ;

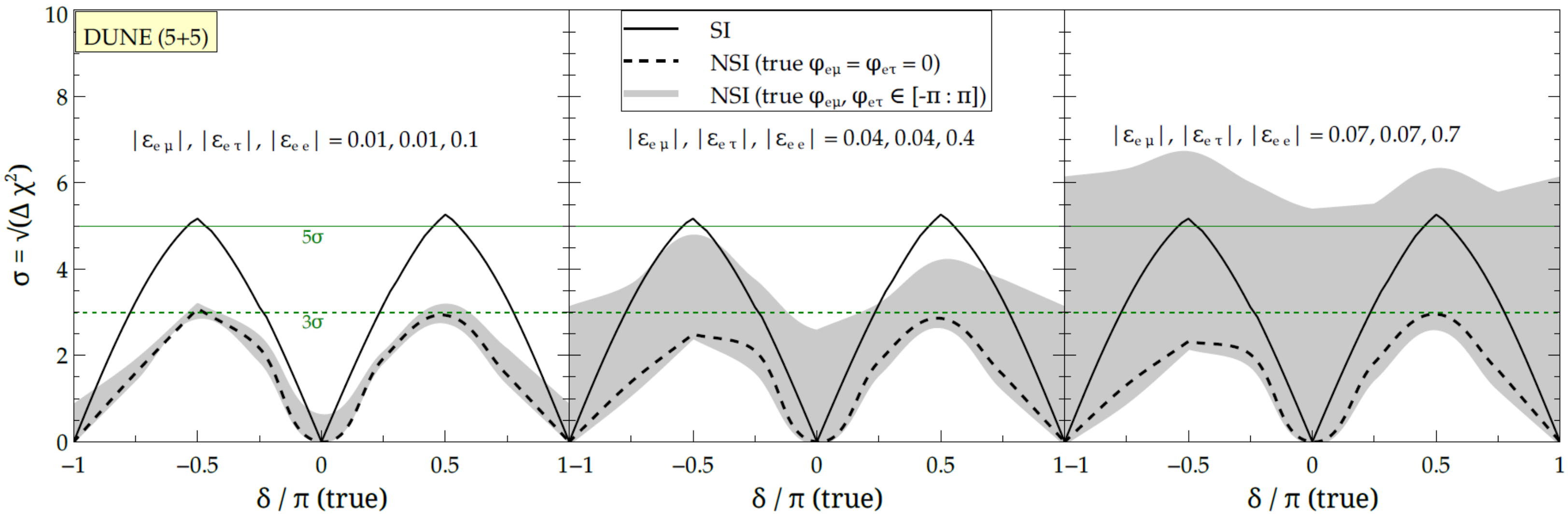
(ex) for production, $2G_F \sum_{\alpha} \varepsilon_{l\alpha}^{CC} \left[\bar{l} \left(1 - \gamma_5 \right) \gamma^{\rho} \nu_{\alpha} \right]$

Results from global fit to solar data and KamLAND

	LMA	LMA $\oplus$ LMA-D
$\varepsilon^u_{ee} - \varepsilon^u_{\mu\mu}$	$[-0.020, +0.456]$	$\oplus[-1.192, -0.802]$
$\varepsilon^u_{\tau\tau} - \varepsilon^u_{\mu\mu}$	$[-0.005, +0.130]$	$[-0.152, +0.130]$
$\varepsilon^u_{e\mu}$	$[-0.060, +0.049]$	$[-0.060, +0.067]$
$\varepsilon^u_{e\tau}$	$[-0.292, +0.119]$	$[-0.292, +0.336]$
$\varepsilon^u_{\mu\tau}$	$[-0.013, +0.010]$	$[-0.013, +0.014]$
$\varepsilon^d_{ee} - \varepsilon^d_{\mu\mu}$	$[-0.027, +0.474]$	$\oplus[-1.232, -1.111]$
$\varepsilon^d_{\tau\tau} - \varepsilon^d_{\mu\mu}$	$[-0.005, +0.095]$	$[-0.013, +0.095]$
$\varepsilon^d_{e\mu}$	$[-0.061, +0.049]$	$[-0.061, +0.073]$
$\varepsilon^d_{e\tau}$	$[-0.247, +0.119]$	$[-0.247, +0.119]$
$\varepsilon^d_{\mu\tau}$	$[-0.012, +0.009]$	$[-0.012, +0.009]$
$\varepsilon^p_{ee} - \varepsilon^p_{\mu\mu}$	$[-0.041, +1.312]$	$\oplus[-3.328, -1.958]$
$\varepsilon^p_{\tau\tau} - \varepsilon^p_{\mu\mu}$	$[-0.015, +0.426]$	$[-0.424, +0.426]$
$\varepsilon^p_{e\mu}$	$[-0.178, +0.147]$	$[-0.178, +0.178]$
$\varepsilon^p_{e\tau}$	$[-0.954, +0.356]$	$[-0.954, +0.949]$
$\varepsilon^p_{\mu\tau}$	$[-0.035, +0.027]$	$[-0.035, +0.035]$

(Esteban et al., 1805.04530)

NSI



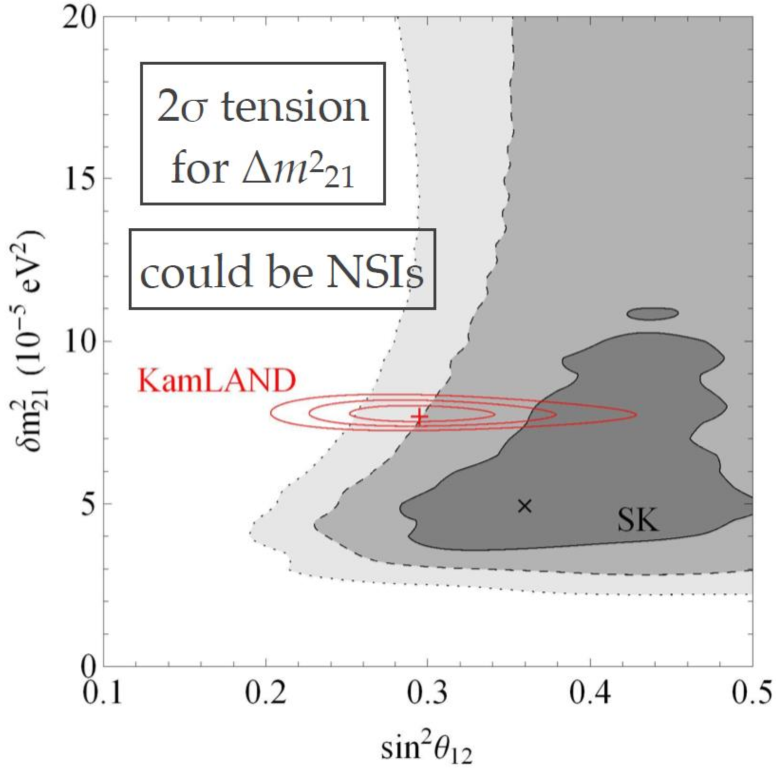
- NSI can prevent determination of CP violation

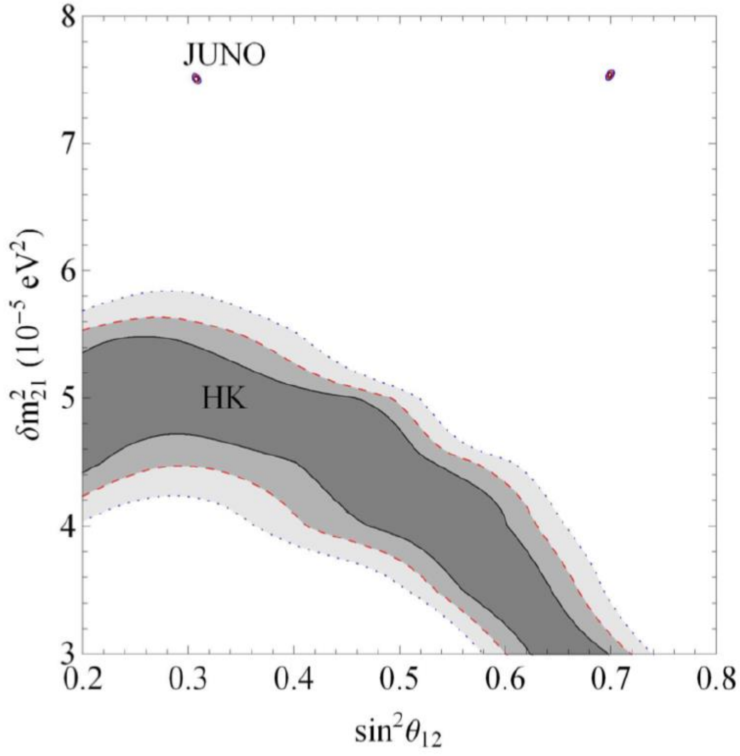
Masud, Mehta, PRD94(2016)

NSI

- 2σ tension for Δm^2_{21} could be due to NSI

Liao, Marfatia, Whisnant, 1704.04711





A(D/N) consistent with SK Δm^2_{21} is also Consistent with KL Δm^2_{21} and $\varepsilon^{u(d)}_{ee} \sim 0.1$

(JUNO and HyperK would reject no NSI-solution by 7σ)

Origin of NSI

- ε from integrating out scalar of type II seesaw: $\varepsilon_{\alpha\beta}^{e} \propto (m_\nu)_{\alpha\beta}$ (Malinsky, Ohlsson, Zhang, 0811.3346)
- ε from integrating out leptoquarks (Wise, Zhang, 1404.4663)
- ε from integrating out charge +1 scalar singlet:
- ε from loop effects, including secret neutrino interactions (Bischer, Rodejohann, Xu, 1807.08102)
- ε from higher dimensional operators (Gavela et al., 0809.3451); within flavor symmetry models have information on flavor symmetry (Wang, Zhou, 1801.05656)
- ε from integrating out Z′ (Heeck, Lindner, Rodejohann, Vogl, 1812.04067)

Non-unitarity

- Source of non-unitary : sterile neutrino, effective op… (minimal unitarity violation: Antusch et al, 2006)
- Parametrization (Z. Xing, PLB 2008, Escrihuela et al. PRD92(2015))

$$N = N^{NP} U = \begin{pmatrix} \alpha_{11} & 0 & 0 \\ \alpha_{21} & \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix} U$$

- Constraints from experimental data: ν oscillations, *W* & *Z* decays, rare lepton-flavor-violating decays, lepton universality tests, ……

- Sensitivity to CPV in LBL exps. can be affected by the presence of non-unitarity.

	One parameter (1 d.o.f.)		All parameters (6 d.o.f.)	
	90% C.L.	3σ	90% C.L.	3σ
Neutrinos + charged leptons				
$\alpha_{11} >$	0.9974	0.9963	0.9961	0.9952
$\alpha_{22} >$	0.9994	0.9991	0.9990	0.9987
$\alpha_{33} >$	0.9988	0.9976	0.9973	0.9961
$\lvert\alpha_{21}\rvert <$	1.7×10^{-3}	2.5×10^{-3}	2.6×10^{-3}	4.0×10^{-3}
$\lvert\alpha_{31}\rvert <$	2.0×10^{-3}	4.4×10^{-3}	5.0×10^{-3}	7.0×10^{-3}
$\lvert\alpha_{32}\rvert <$	1.1×10^{-3}	2.0×10^{-3}	2.4×10^{-3}	3.4×10^{-3}
Neutrinos only				
$\alpha_{11} >$	0.98	0.95	0.96	0.93
$\alpha_{22} >$	0.99	0.96	0.97	0.95
$\alpha_{33} >$	0.93	0.76	0.79	0.61
$\lvert\alpha_{21}\rvert <$	1.0×10^{-2}	2.6×10^{-2}	2.4×10^{-2}	3.6×10^{-2}
$\lvert\alpha_{31}\rvert <$	4.2×10^{-2}	9.8×10^{-2}	9.0×10^{-2}	1.3×10^{-1}
$\lvert\alpha_{32}\rvert <$	9.8×10^{-3}	1.7×10^{-2}	1.6×10^{-2}	2.1×10^{-2}

Escrihuela, Forero, Miranda, Tortola, Valle, New.J.Phys.19(2017)

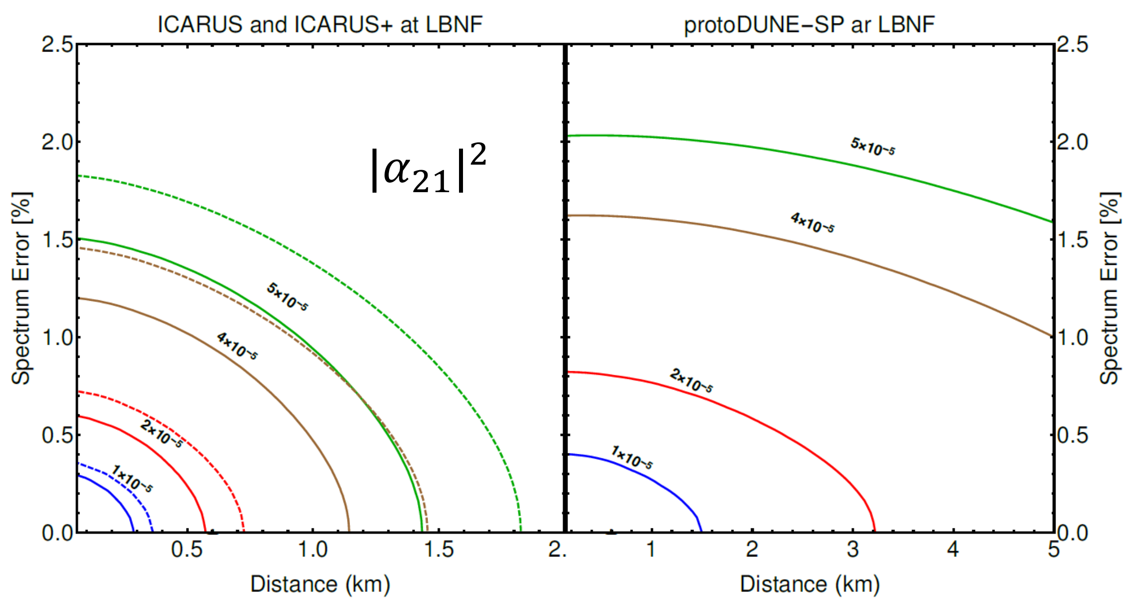
- Non-unitarity predicts "zero-distance effect"

$$P(\nu_\mu \to \nu_e) = \alpha_{11}|\alpha_{21}|^2$$

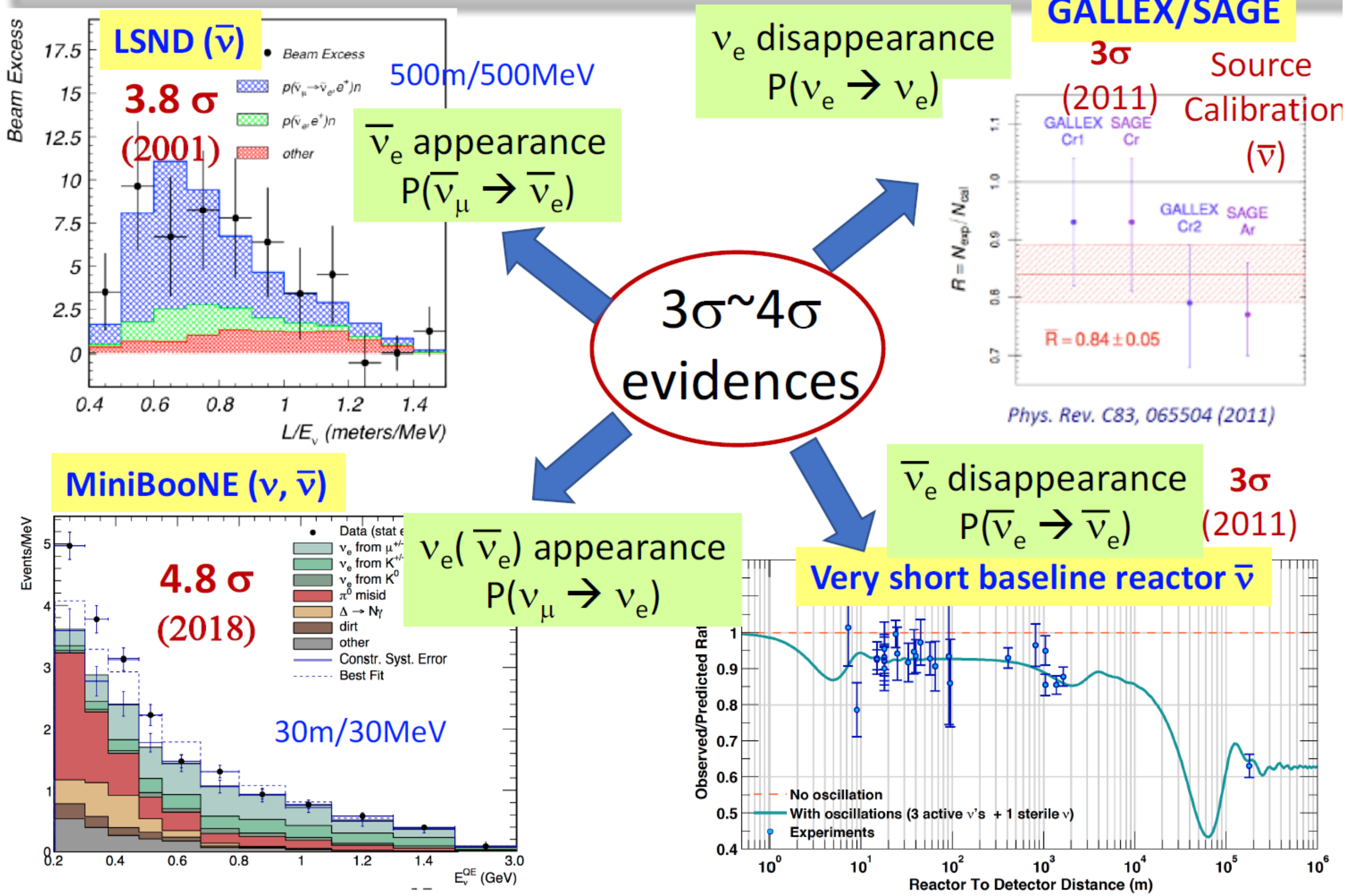
- Thus, at very short distances from the neutrino source, # of detected electron neutrinos, N_e, is given by

$$N_e = \phi^0_{\nu_e} + |\alpha_{21}|^2 \phi^0_{\nu_\mu}$$

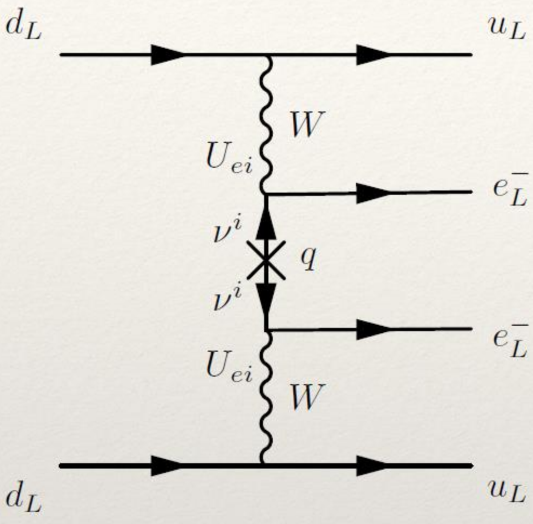
- Capabilities of SBL as well as LBL as a probe of the unitarity of lepton mixing :(Miranda et al. PRD97(2018); Escrihuela et al ,New.J.Phys.19(2017)

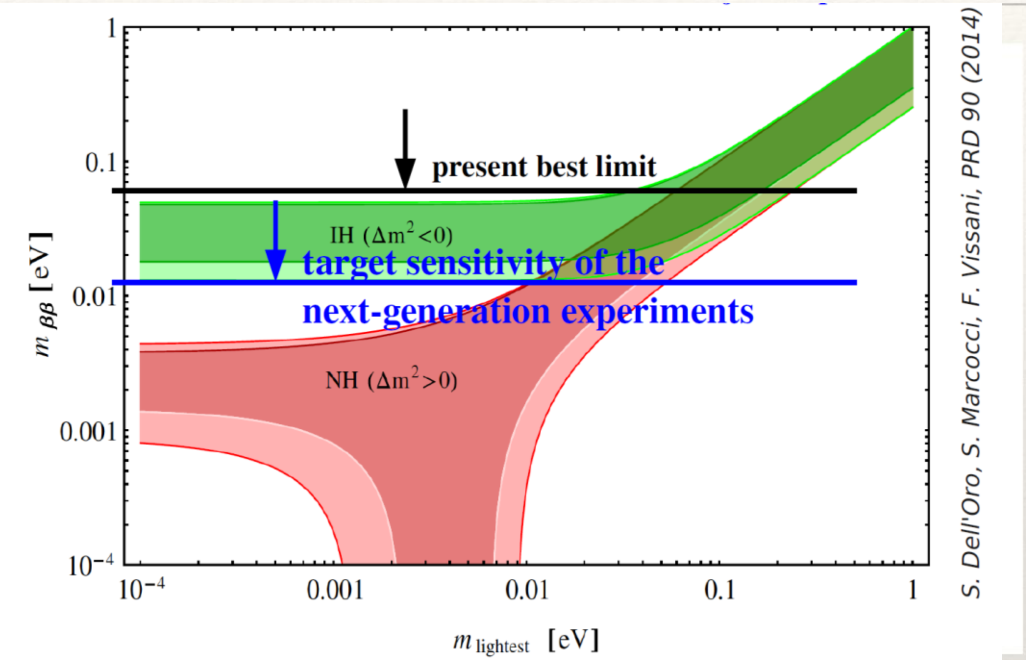


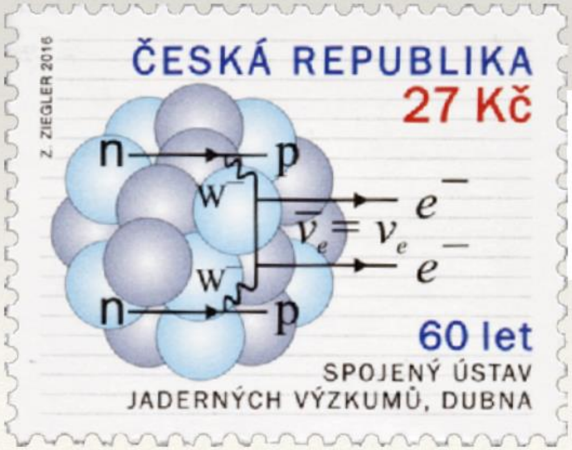
Sterile Neutrinos at ~eV ?



Double Beta Decay







$$|m_{ee}| = |\sum U_{ei}^2\, m_i| = |U_{e1}^2\, m_1 + U_{e2}^2\, m_2 e^{i\alpha} + U_{e3}^2\, m_3 e^{i\beta}|$$

$$= f\left(\theta_{12}, |U_{e3}|, m_i, \mathrm{sgn}(\Delta m_{\mathrm{A}}^2), \alpha, \beta\right)$$

known limits unknown

Prospects

- Precisely measuring PMNS mixing angles

  → test of neutrino models with flavor symmetry

  → some hint for grand unification or Q-L symmetry
- Determining mass ordering

  → implication on neutrinoless double beta decay

  → test of some of neutrino mass model
- Observing CP violation

  → implication on baryogenesis

  → hint for Q-L symmetry or grand unification
- Search for new physics beyond the SM
- Measuring neutrino mass scale
- Neutrino properties : Dirac vs. Majorana
- Search for sterile neutrinos

  light sterile → neutrino oscillation, low energy experiments

  heavy sterile → collider experiments

Conclusion

- Lots of progress in neutrino physics in the past years

  → PMNS parameters approach CKM precision
- Still lots to learn about neutrino

  → mass ordering, CP violation, Majorana or Dirac etc.
- Lots of theoretical idea proposed to understand our universe via neutrino

  → More idea will emerge in future
- Lots of experimental programs and proposals exist

  → New era of neutrino physics

Minimal Seesaw (type-I)

- 2 RH neutrinos : Frampton, Glashow, Yanagida, PLB548(2002), Endoh, SK, Kaneko, Morozumi, Tanimoto, PRL89(2002)
- Littlest Seesaw : Dirac texture zero & 2 RH ν (S.F. King, JHEP1307(2013))
- Littlest Seesaw from S_4 (Chen, Ding, King, Li : 1906.1141)

$$m_\nu = m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} + m_s e^{i\eta} \begin{pmatrix} 1 & 2-x & x \\ 2-x & (x-2)^2 & (2-x)x \\ x & (2-x)x & x^2 \end{pmatrix}$$

$$(x, \eta) = (-1/2, -\pi/2)$$

$$0.593 \le \sin^2\theta_{23} \le 0.609$$

$$-0.358 \le \delta_{CP}/\pi \le -0.348$$