

The stochastic LapH approach to all-to-all quark propagation

John Bulava

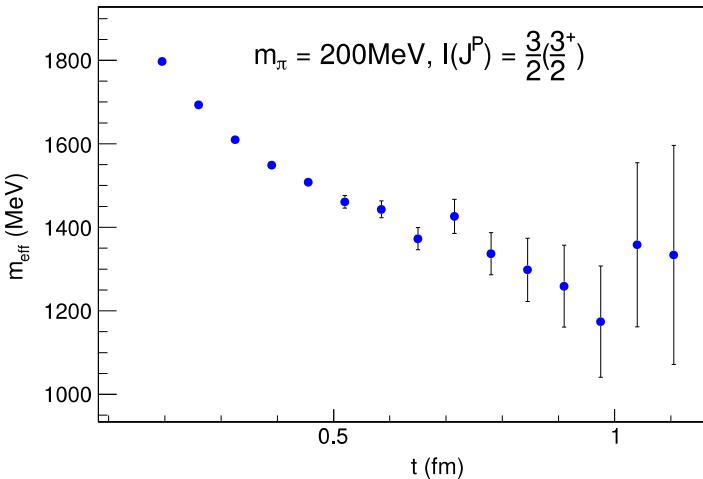
University of Southern Denmark
CP3-Origins



“Lattice Methods for the 2020’s”
Hamilton Mathematics Institute
Trinity College Dublin, IE
Dec. 2nd, 2019

Lattice QCD spectroscopy:

- Correlation matrix: $C_{ij}(t) = \langle \mathcal{O}_i(t)\bar{\mathcal{O}}_j(0) \rangle$
- Operators control which states appear, are irreducible under symmetries of Hamiltonian.
- Large-time asymptotics => energies (GEVP)
 $E_n = \lim_{t \rightarrow \infty} -\partial_t \ln \lambda_n(t, t_0)$ $C(t)v_n(t, t_0) = \lambda_n(t, t_0)C(t_0)v_n(t, t_0)$
- Signal-to-noise problem => ‘Teufelspakt’



=>



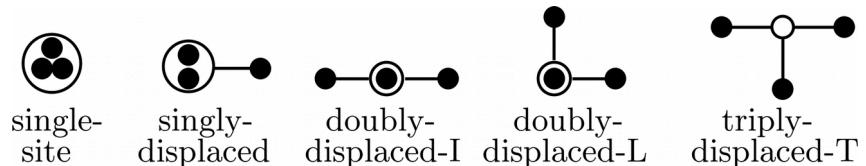
Data courtesy of C. W. Andersen, apologies to J. W. von Goethe

Example: excited baryons

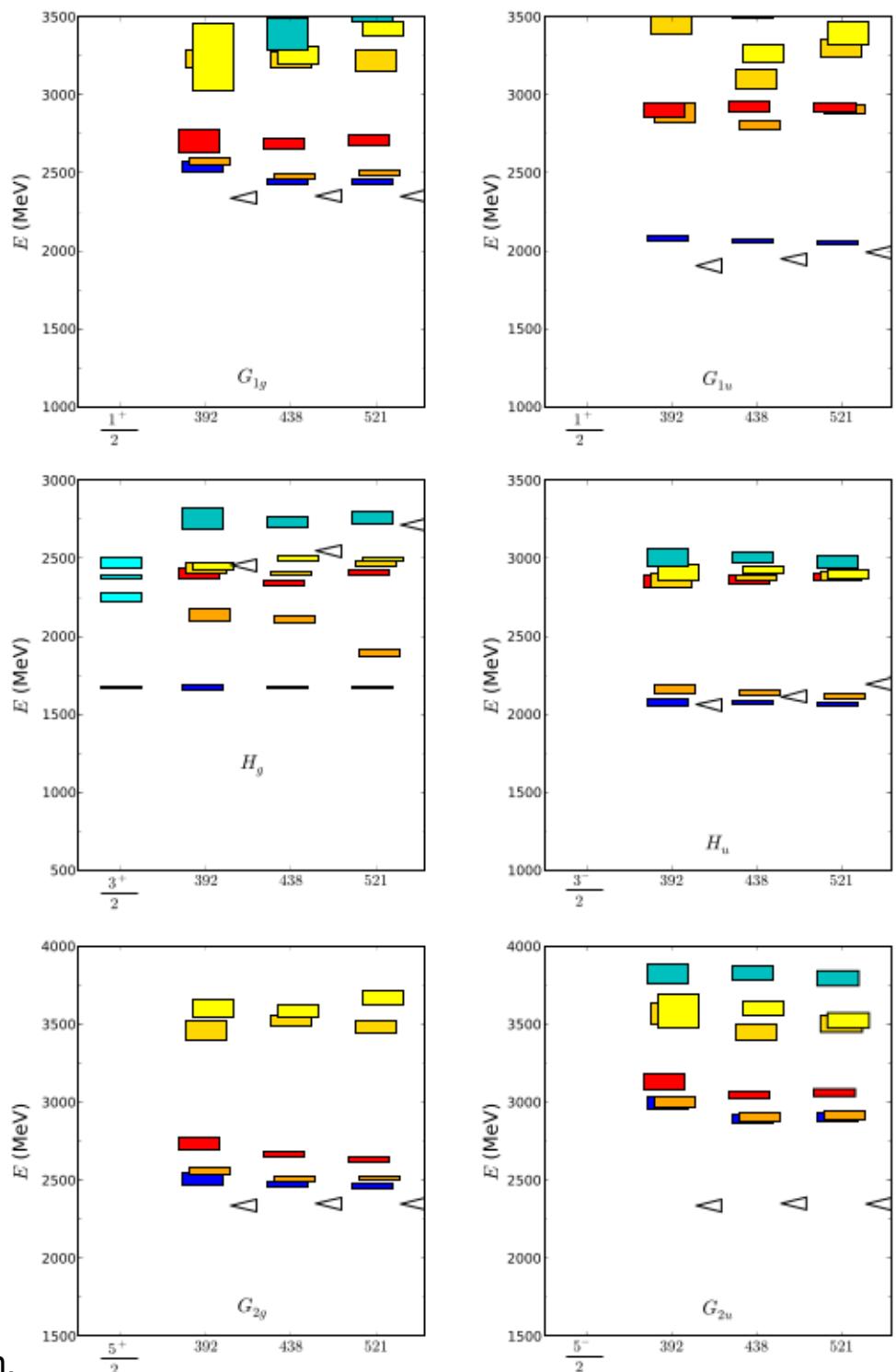
- $N_f = 2 + 1$, $m_\pi = 392, 438, 521\text{MeV}$

- Finite-volume spectra with strangeness = -3

- Large basis of interpolating operators:



- Triangles denote two-hadron thresholds



Determining Excited states: a ‘Folk theorem’

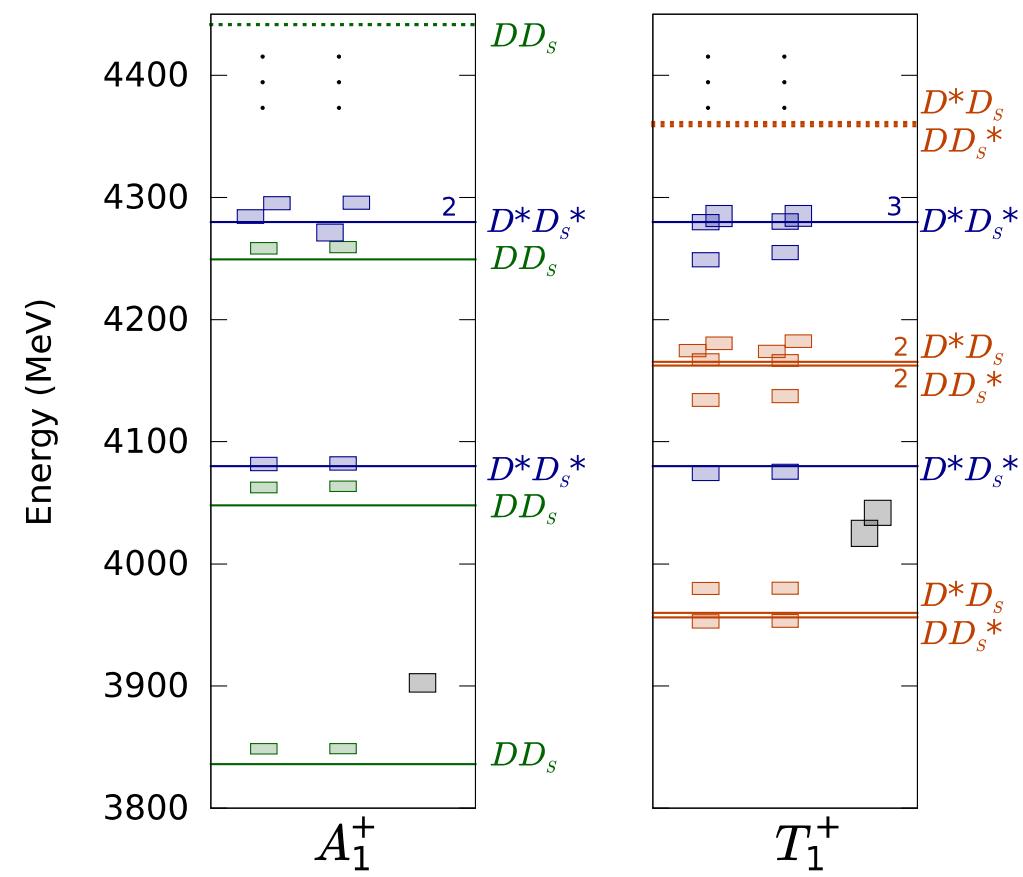
To reliably determine a finite-volume energy you must....

1) include operator(s) with good overlap onto it

2) reliably determine all states below it

Example of the Folk Theorem:

- Search for charm=2 tetraquarks*
- *hidden charm very hard because of 2)
- $I = 1/2, S = 1, m_\pi = 390\text{MeV}$
- 1st column: tetraquark + 2D ops.
- 2nd column: just 2D ops.
- 3rd column: just tetraquark ops.



Multi-hadron operators:

- Mesons:

$$\mathcal{O}_M(\vec{p}, \tau) = \sum_{\vec{x}} e^{i\vec{p} \cdot \vec{x}} \bar{\psi}(x) \Gamma \psi(x)$$

- Baryons:

$$\mathcal{O}_B(\vec{p}, \tau) = \sum_{\vec{x}} e^{i\vec{p} \cdot \vec{x}} \Gamma_{\alpha\beta\gamma} \psi_\alpha(x) \psi_\beta(x) \psi_\gamma(x)$$

- Two-hadron operators => momentum projection => all-to-all quark propagators!

Early idea for all-to-all propagators: stochastic estimators

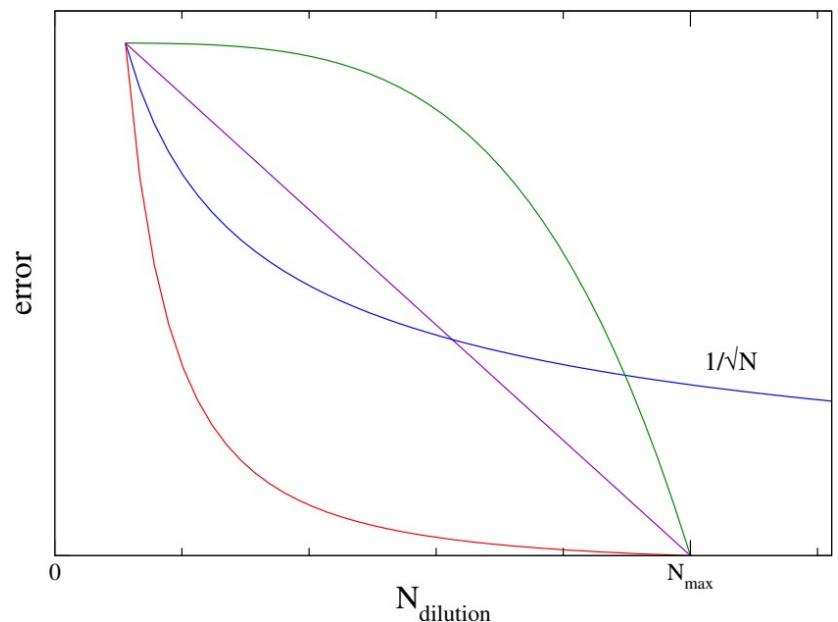
- ‘Noisy Unity’: $E[\eta_i \eta_j^*] = \delta_{ij}$

- Estimates of quark propagator:

$$M^{-1} = E[\psi \otimes \eta^\dagger], \quad \psi = M^{-1} \eta$$

- Variance reduction by dilution:

$$M^{-1} = \sum_{a=1}^{N_{\text{dil}}} E[\psi_a \otimes \eta_a^\dagger], \quad \eta_a = P_a \eta,$$



$$\sum_{a=1}^{N_{\text{dil}}} P_a = \mathbb{1}, \quad P_a P_b = \mathbb{1} \delta_{ab}$$

- Natural dilution schemes: time, spin, color, spatial partitioning
- Maximal dilution achieved at finite number of inversions.

Early idea for all-to-all propagators: stochastic estimators

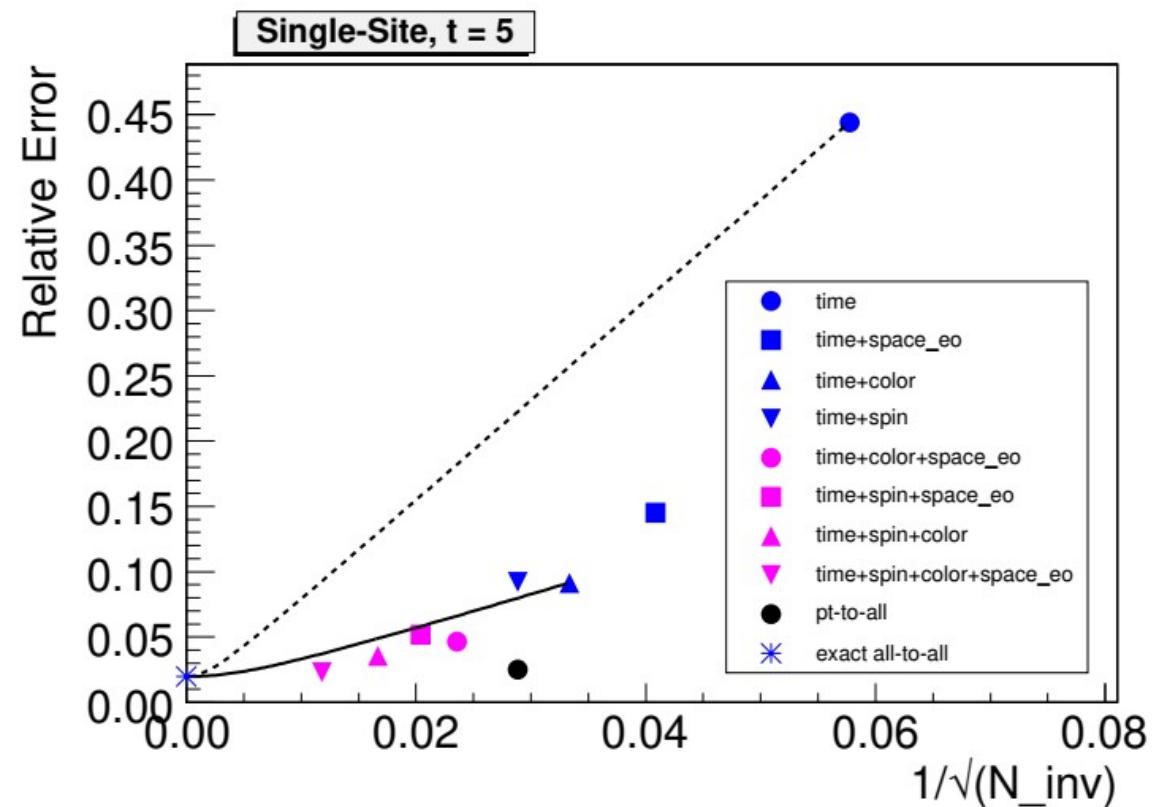
- test of standard dilution:

$$m_\pi = 700\text{MeV}, N_f = 0, a_s = 0.1\text{fm}, 12^3 \times 48$$

JB LATTICE08

- Relative error on a single-site nucleon correlator

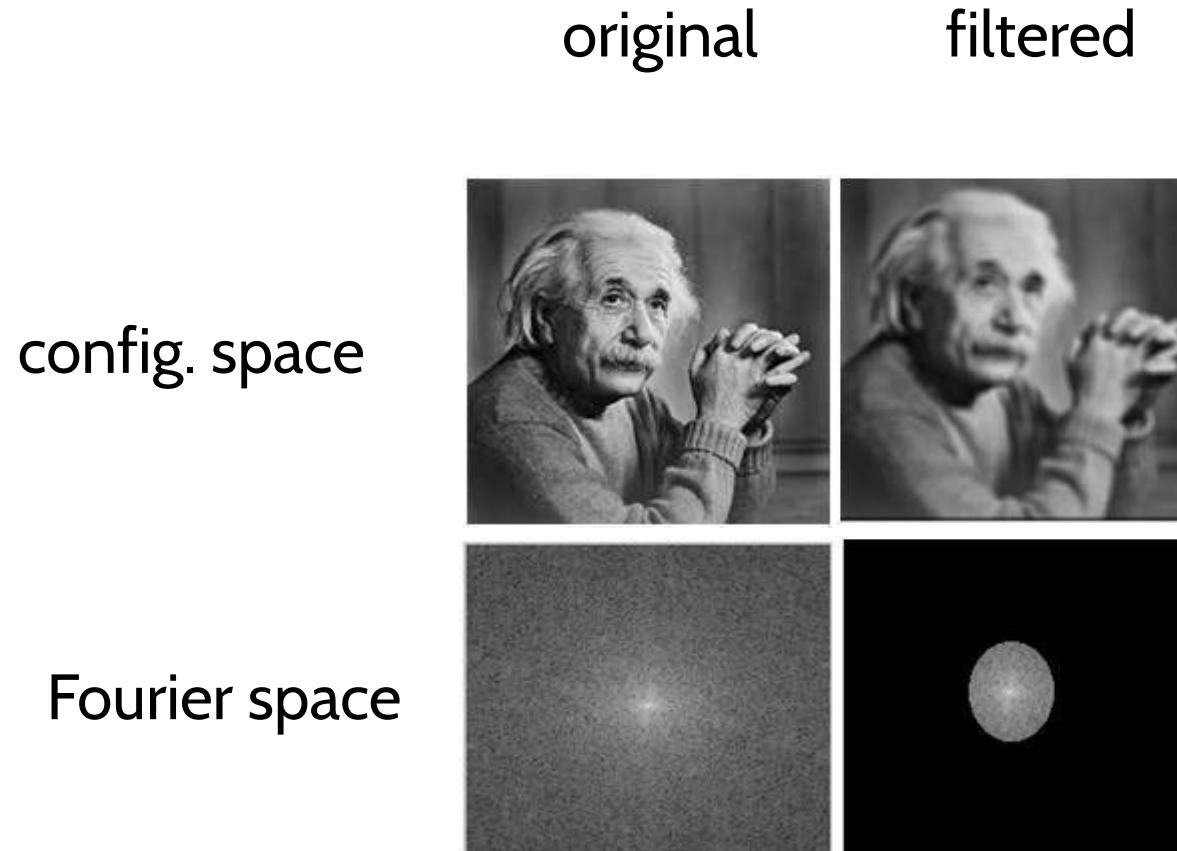
$$\sigma_{\text{tot}} = \sqrt{\sigma_g^2 + \sigma_{\text{stoch}}^2}$$



Laplacian-Heaviside smearing (distillation):

M. Peardon, JB, J. Foley, C. Morningstar, J. Dudek, R. Edwards, B. Joo, H.-W. Lin,
D. Richards, K. Juge, *Phys. Rev.* **D80** (2009) 054506

C. Morningstar, JB, J. Foley, K. Juge, D. Lenkner, M. Peardon, C. H. Wong, *Phys. Rev.* **D83** (2011) 114505

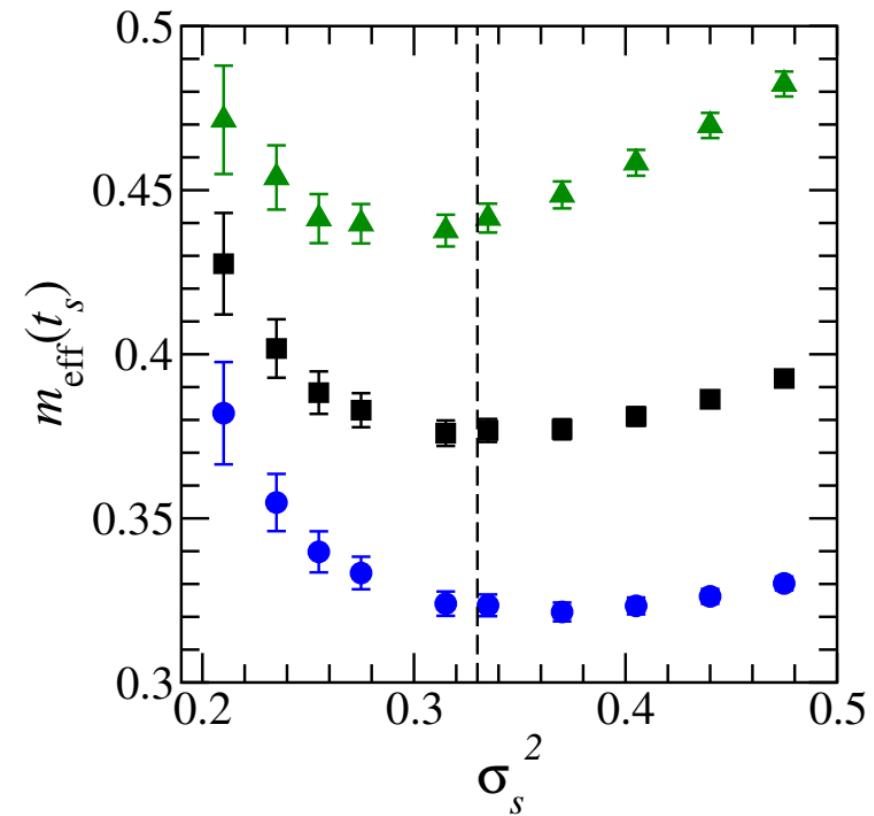
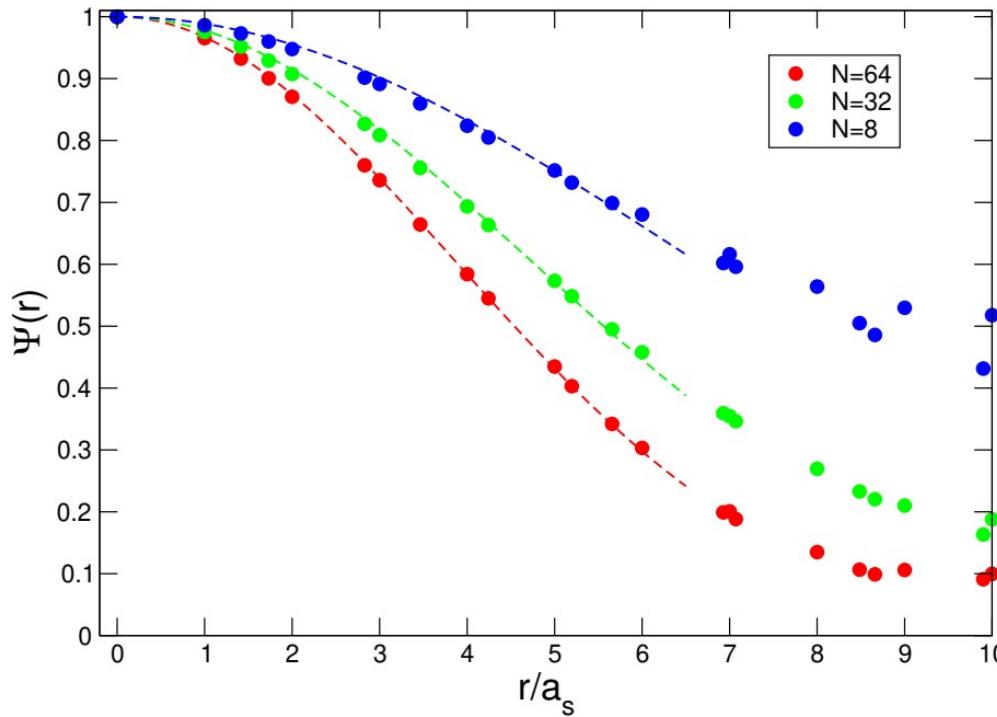


- Modes of (smeared) gauge-cov. 3-D laplacian: $\tilde{\Delta}[U]v_n = \lambda_n v_n$
- Smearing operator: $S = \Theta(\sigma_s^2 + \tilde{\Delta})$
- For fixed cutoff: $N_{\text{ev}} \propto L^3$

Laplacian-Heaviside smearing (distillation):

M. Peardon, JB, J. Foley, C. Morningstar, J. Dudek, R. Edwards, B. Joo, H.-W. Lin,
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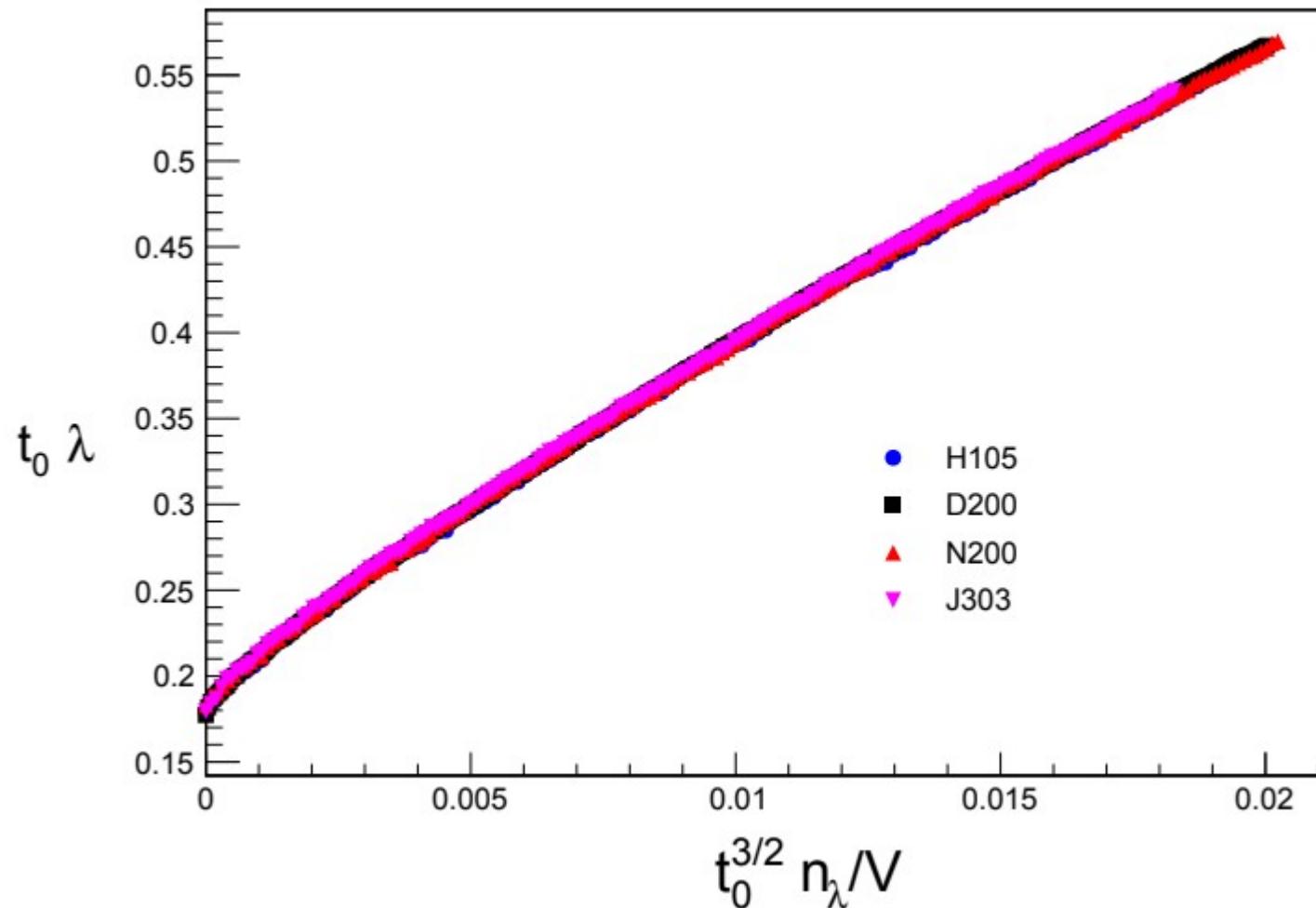
C. Morningstar, JB, J. Foley, K. Juge, D. Lenkner, M. Peardon, C. H. Wong, *Phys. Rev.* **D83** (2011) 114505



- Smearing width related to number of modes
- Optimal cutoff choice: $\sigma_s \approx 1\text{GeV}$

Laplacian-Heaviside smearing (distillation):

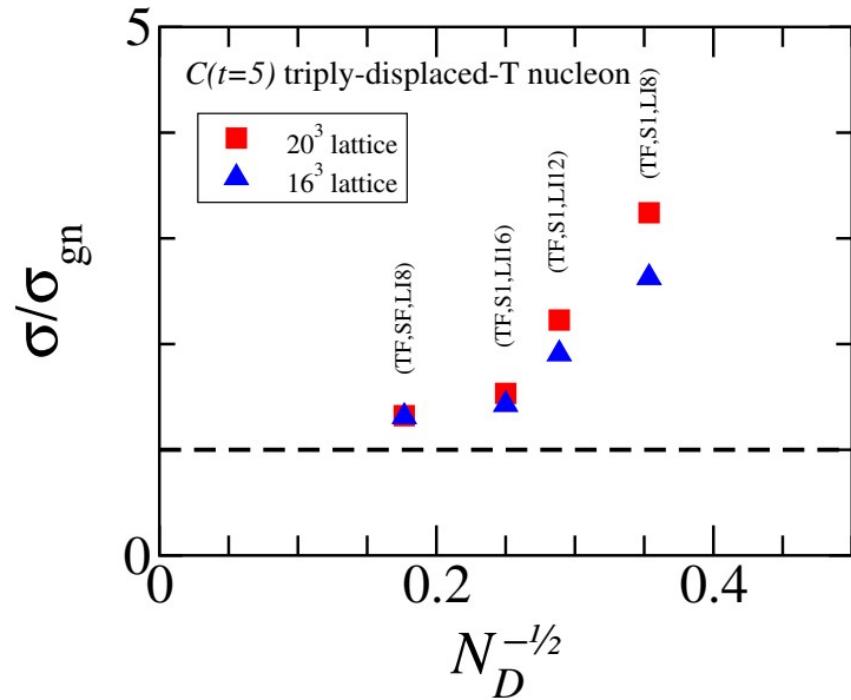
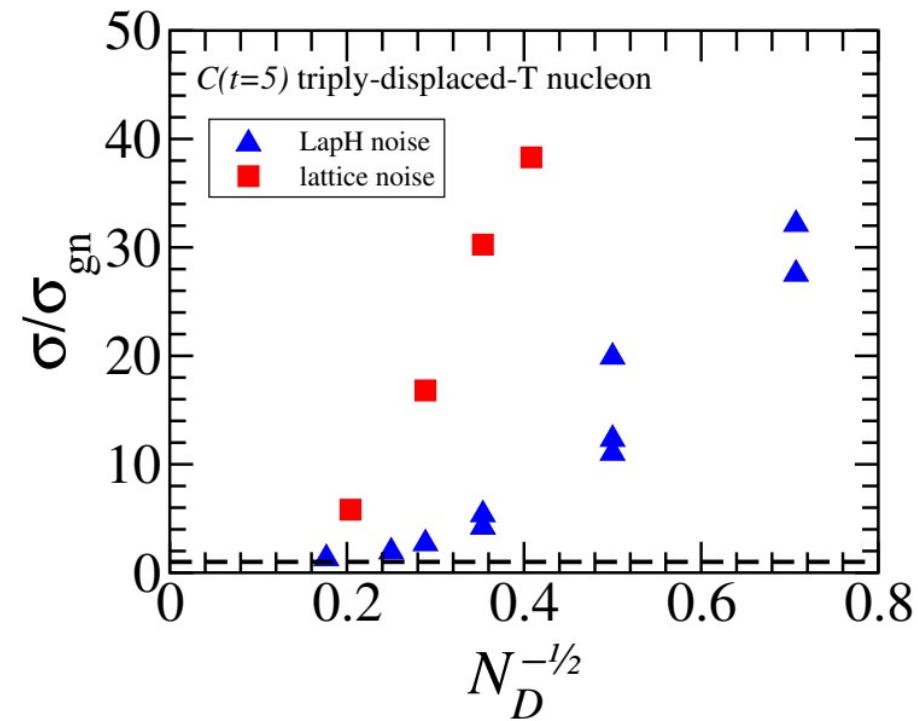
M. Peardon, JB, J. Foley, C. Morningstar, J. Dudek, R. Edwards, B. Joo, H.-W. Lin,
D. Richards, K. Juge, *Phys. Rev.* **D80** (2009) 054506
C. Morningstar, JB, J. Foley, K. Juge, D. Lenkner, M. Peardon, C. H. Wong, *Phys. Rev.* **D83** (2011) 114505



- Number of modes scales with physical volume
- Stout smearing iterations scaled appropriately $n_\rho \rho \propto a^{-2}$

Dilution and Laplacian-Heaviside smearing (distillation)

C. Morningstar, JB, J. Foley, K. Juge, D. Lenkner, M. Peardon, C. H. Wong, *Phys. Rev. D* **83** (2011) 114505



- Possible dilution schemes: either ‘full’ or ‘interlace’ in time, spin, and Laplacian eigenvector indices

$(TF, SF, LI16)$

$(TI8, SF, LI16)$

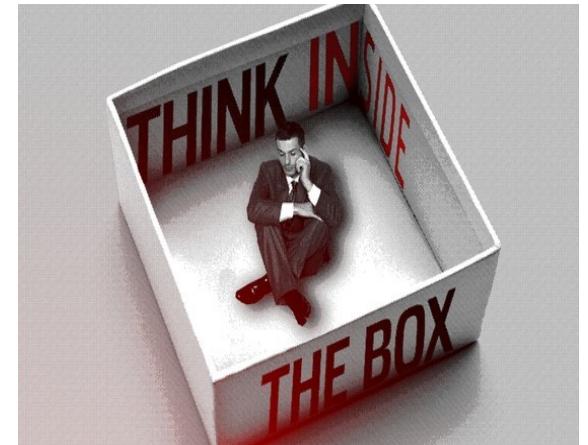
First test: scattering amplitudes

- In imaginary time, $\langle 0 | T[\hat{O}'(x') \dots \hat{O}^\dagger(x)] | 0 \rangle$ does not yield amplitudes asymptotically . L. Maiani, M. Testa, *Phys. Lett.* **B245** (1990) 585
- Finite volume method: below $n \geq 3$ hadron thresholds:

$$\det[1 - K(E_{\text{cm}})B(L\mathbf{q}_{\text{cm}})] + O(e^{-ML}) = 0$$

$$S = (1 - iK)^{-1}(1 + iK)$$

M. Lüscher, *Nucl. Phys.* **B354** (1991) 531



- Determinant over total angular momentum, channel, and total spin
- B-matrix mixes infinite-volume partial waves, truncation necessary
- Single-partial-wave approx.: one-to-one mapping $E_{\text{cm}} \rightarrow K(E_{\text{cm}})$
- Threshold expansion:

$$\Delta E = E_{2\pi}^{A_1} - 2m_\pi = -\frac{4\pi a_0}{m_\pi L^3} + O(L^{-4})$$

First Test/Results

- Anisotropic Wilson-clover lattice:
 - Dynamical light and strange quarks: $N_f = 2 + 1$
 - Large volume, fine temporal resolution: $a_s/a_t \approx 3.5$
 $32^3 \times 256$, $m_\pi \approx 240\text{MeV}$, $a_s \approx 0.12\text{fm}$, $L \approx 4\text{fm}$
 - Safe from 'thermal effects': $m_\pi T \approx 10$
- H.-W. Lin, S. Cohen, J. Dudek, R. Edwards, B. Joo, D. Richards, JB, J. Foley, C. Morningstar, E. Engelson, S. Wallace, K. J. Juge, N. Mathur, M. Peardon, S. Ryan, *Phys. Rev.* **D79** (2009) 034502
- Elastic pion-pion scattering:
 - Total Isospin: $I = 0, 1, 2$
 - Resonances: $\rho(770)$, $f_0(500)$, ...

Correlator Measurements

- Dilution Scheme:

N_{ev}	line type	N_r	scheme	N_{t_0}	N_{inv}
264	fixed	5	(TF, SF, LI8)	8	1280
	relative	2	(TI16, SF, LI8)	-	1024

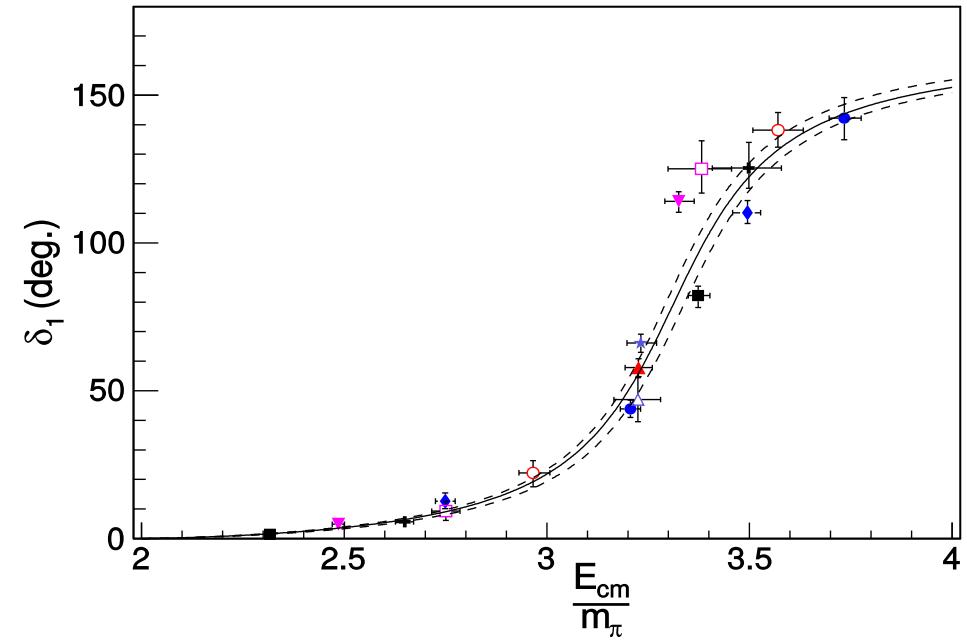
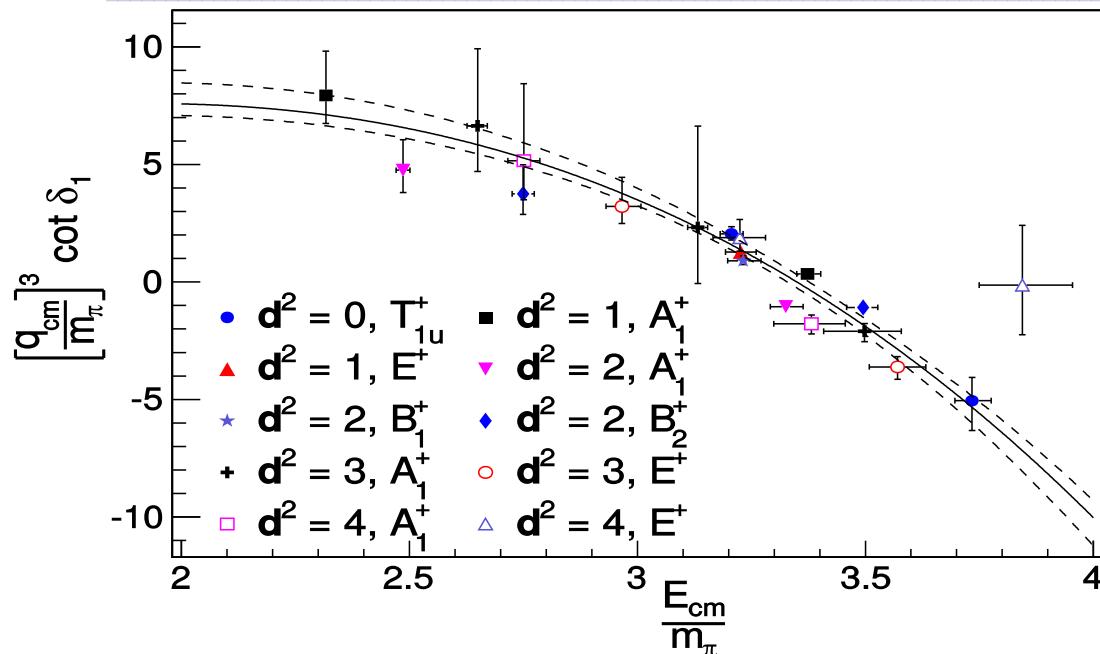
- 3-4 two-pion operators and 1 (local) rho-like operator (if applicable) in each irrep.

C. Morningstar, J. Bulava, B. Fahy, J. Foley, Y. C. Jhang, K. J. Juge, D. Lenkner, C. H. Wong,
Phys. Rev. D88 (2013) 014511

- All possible irreps with total momenta:

$$d^2 = 0, 1, 2, 3, 4$$

Isovector p-wave results

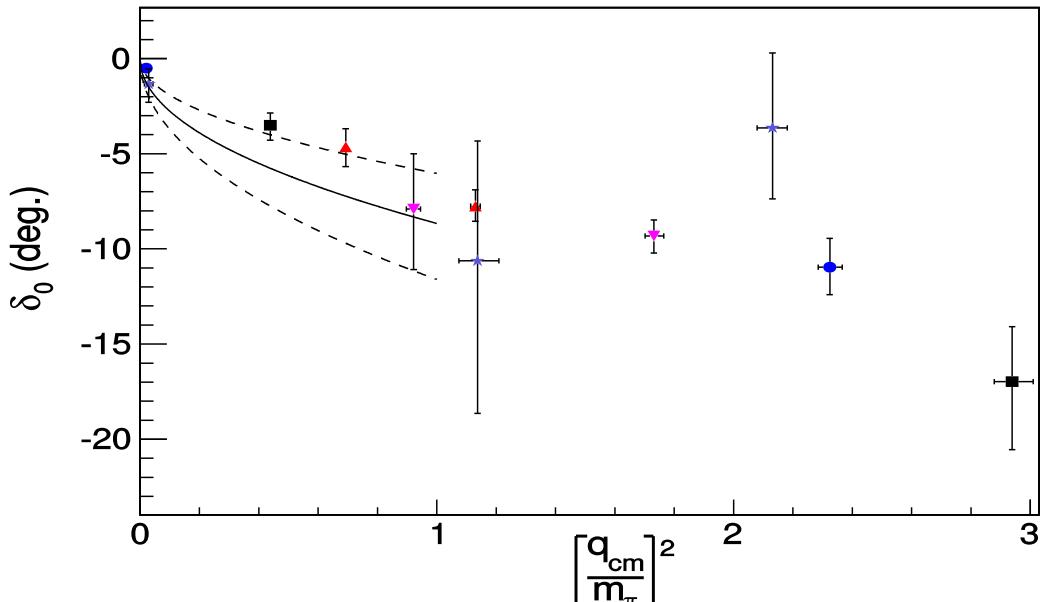
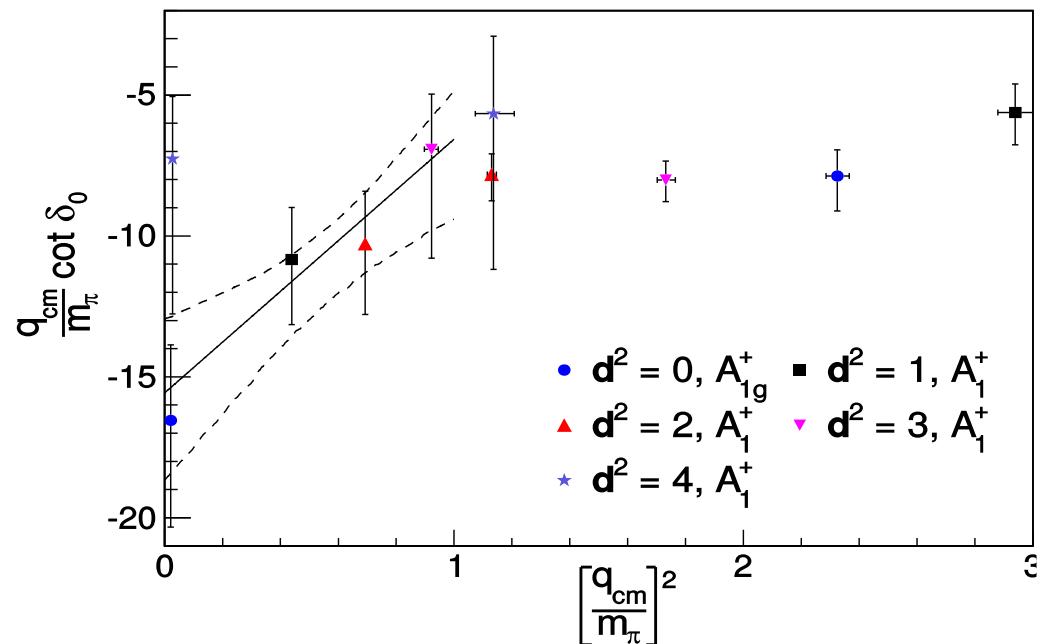


JB, B. Fahy, B. Hörz, K. J. Juge, C. Morningstar, C. H. Wong, *Nucl. Phys.* **B910** (2016) 114513

Breit-Wigner fit: $q_{\text{cm}}^3 \cot \delta_1 = (m_\rho^2 - s) \frac{6\pi\sqrt{s}}{g_{\rho\pi\pi}^2}$

$$\frac{m_\rho}{m_\pi} = 3.350(24), \quad g_{\rho\pi\pi} = 5.99(26), \quad \frac{\chi^2}{d.o.f} = 1.04$$

Isoquintet s-wave results



JB, B. Fahy, B. Hörz, K. J. Juge, C. Morningstar, C. H. Wong, *Nucl. Phys.* **B910** (2016) 114513

Effective range fit:

$$q_{\text{cm}} \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2} r p^2$$

$$m_\pi a_0 = -0.064(12), \quad m_\pi r = 18.1(8.4), \quad \frac{\chi^2}{d.o.f} = 0.19$$

Distillation comparison

Results from the JLab group on the same ensemble: $I = 1, \ell = 1$

D. Wilson, R. Briceno, J. Dudek, R. Edwards, C. Thomas, *Phys. Rev.* **D92** (2015) 094502

Ref.	N_{inv}	$a_t m_\pi$	$T_{1u}^+ a_t E_0$	$T_{1u}^+ a_t E_1$	$a_t m_\rho$	$g_{\rho\pi\pi}$
This work	2304	0.03939(19)	0.12625(94)	0.1470(16)	0.13190(87)	5.99(26)
JLab	393216	0.03928(18)	0.12488(40)	0.14534(52)	0.13175(35)	5.688(70)

- JLab (exact distillation):

$$N_{\text{inv}} = 4N_{\text{ev}}N_t = 4 \times 384 \times 256 = 393,216$$

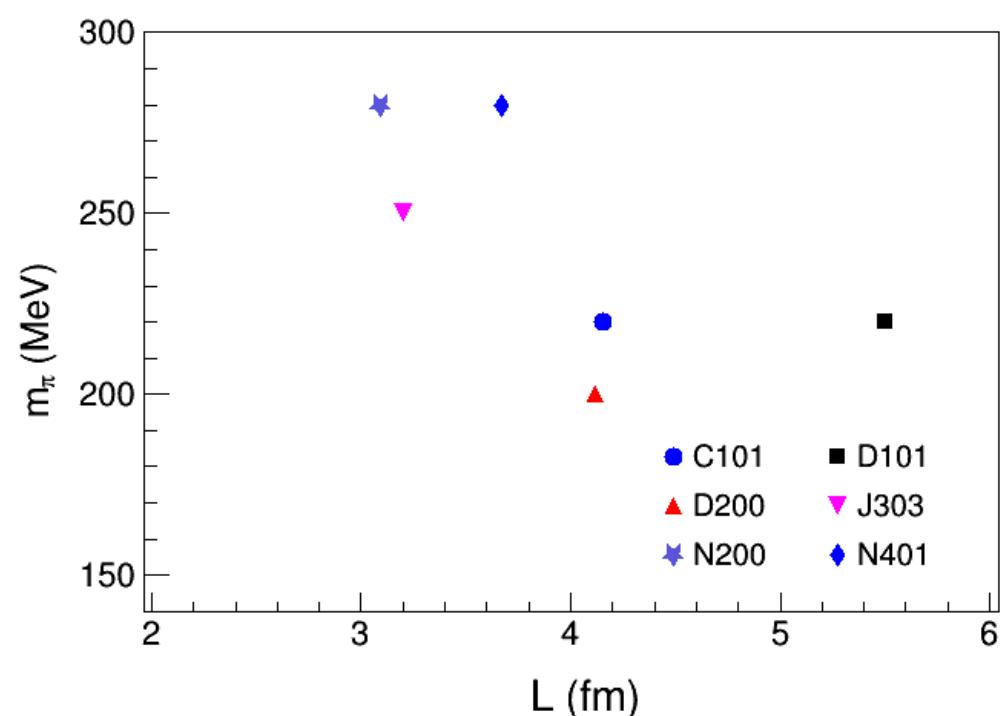
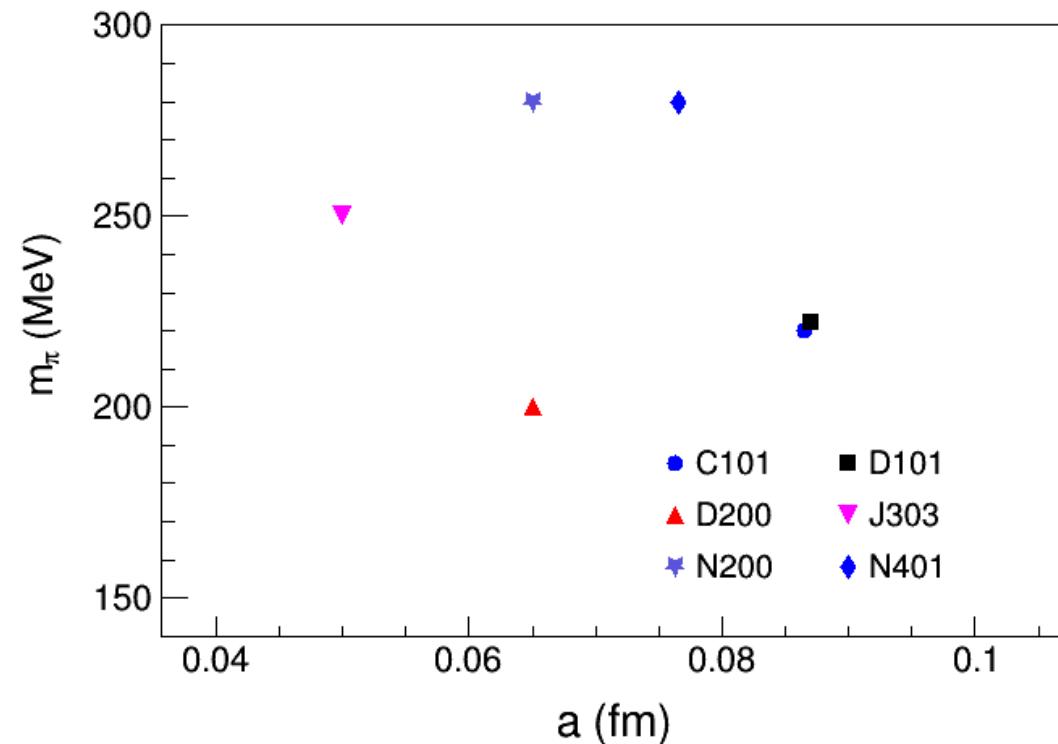
- This work (stochastic Laph):

$$N_{\text{inv}} = 5N_{\text{dil}}^{\text{conn}}N_{t_0} + 2N_{\text{dil}}^{\text{disc}} = 5 \times 32 \times 8 + 1024 = 2304$$

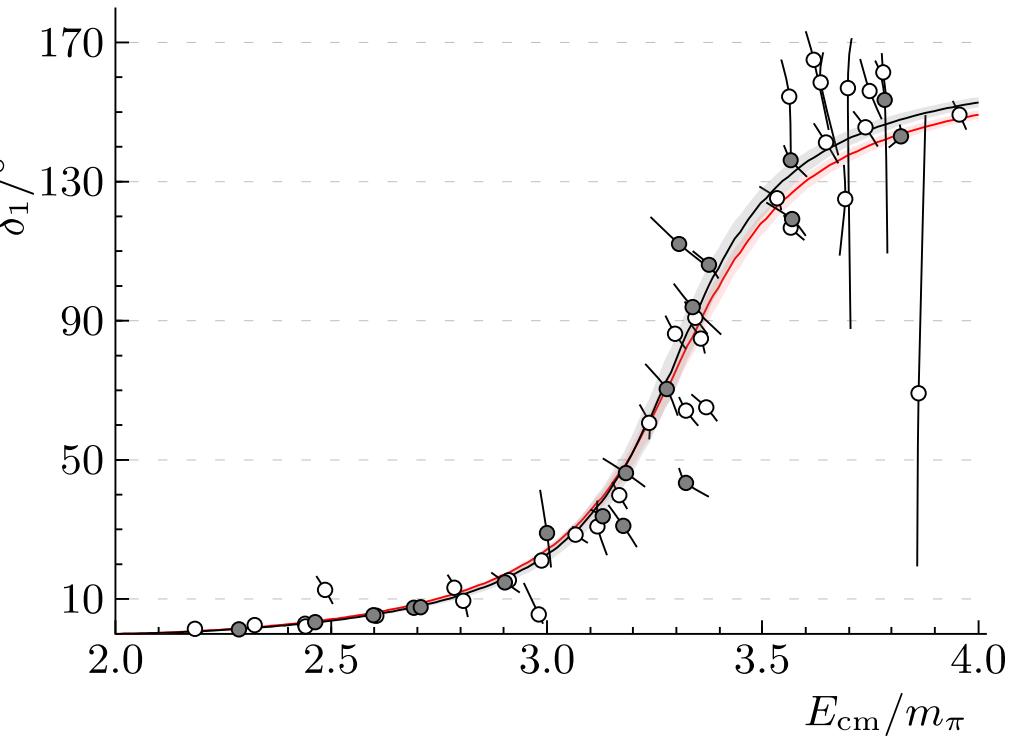
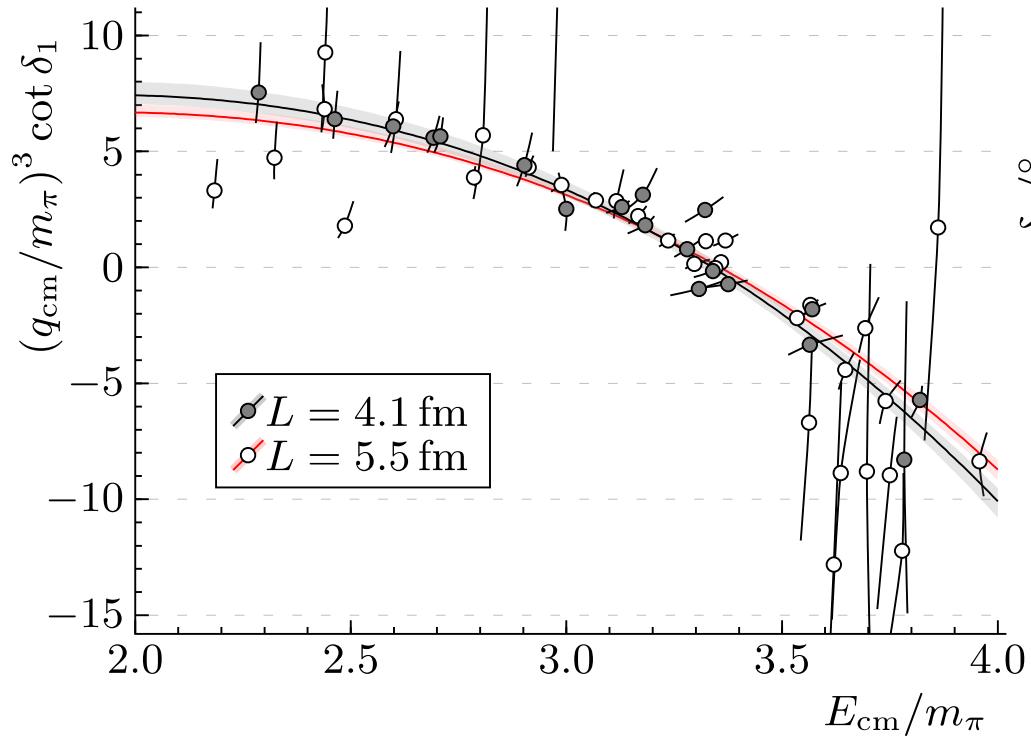
CLS ensembles

M. Bruno, D. Djukanovic, G. Engel, A. Francis, G. Herdoiza, H. Horch, P. Korcyl, T. Korzec, M. Papinutto, S. Schaefer, E. Scholz, J. Simeth, H. Simma, W. Söldner, JHEP **1502** (2015) 043

- **4 lattice spacings** $a \geq 0.05\text{fm}$, **pion masses** $m_\pi \gtrsim 200\text{MeV}$
- $N_f = 2 + 1$ **chiral limit**: $\text{Tr } M = 2m_{u,d} + m_s = \text{const.}$
- **Finite volume check**: $m_\pi L = 4.6, 6.1$



Results – finite volume check



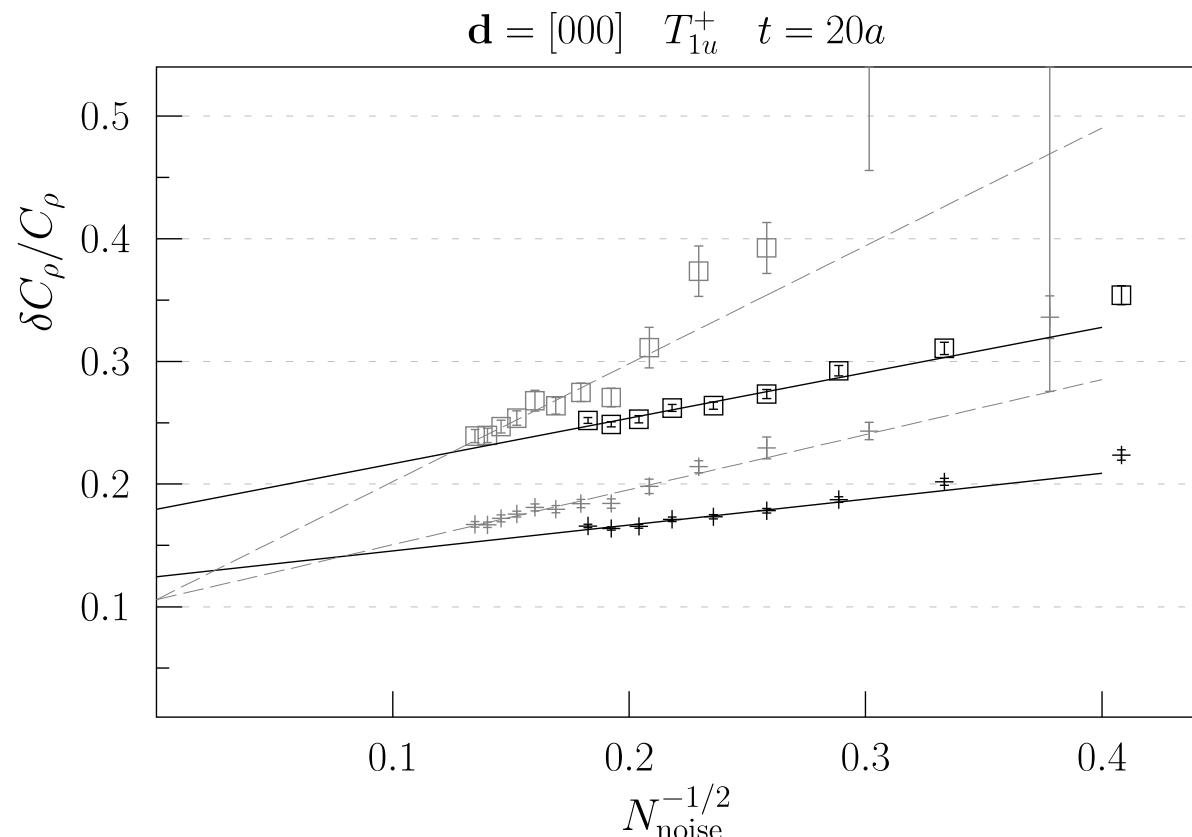
C101/D101: $m_\pi = 220 \text{ MeV}$, $a = 0.086 \text{ fm}$

Noise combinations

- Different permutations/combinations => additional stochastic estimate
- Horizontal axis: number of noise combinations

B. Hoerz, LATTICE2017

- Gray vs. black:
4.1 vs. 5.5fm
- Squares vs. crosses
LI8 v. LI16 dilution

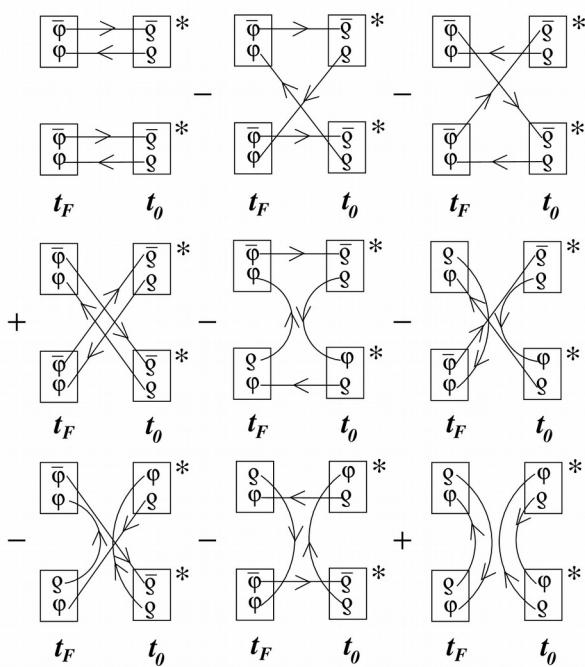


Difficulties in meson-baryon scattering

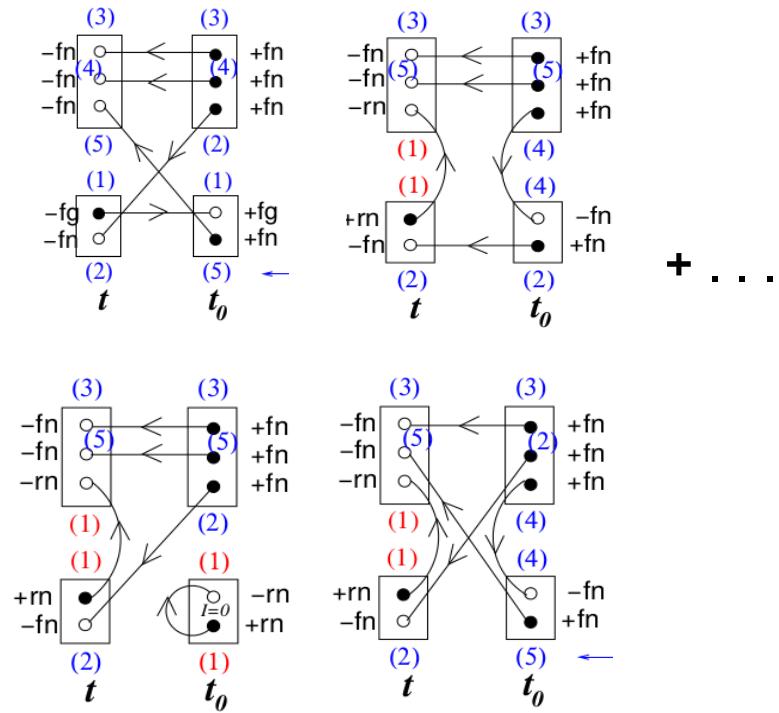
- Non-zero spin => additional partial wave mixing
- Increased (exponential) signal-to-noise degradation

$$C(\tau)/\sigma(\tau) \sim e^{-M\tau}$$

- Three quarks => costly (complicated) correlation function construction



VS.



Elastic $\Delta(1232) \rightarrow N\pi$ amplitude

- Both $\Delta(1232)$ and $N\pi$ interpolators. Ground and excited states:

$$E_{\text{cm}} \leq M_N + 2m_\pi$$

- Choose $I = 3/2$ irreps where $\ell(J^P) = 1(3/2^+)$ is the lowest partial wave

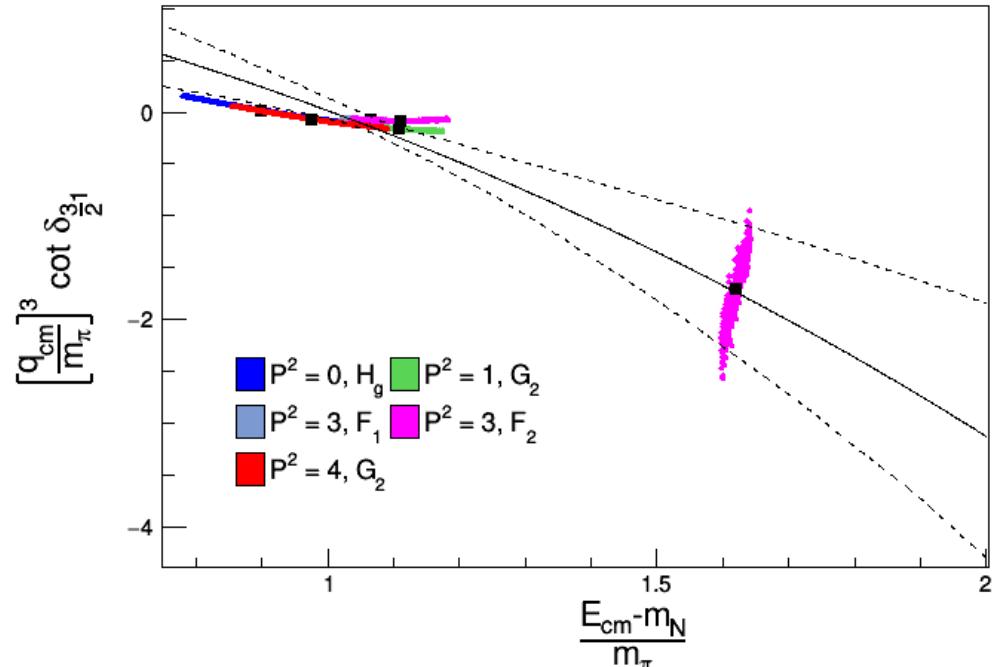
mom.	irrep	$\ell(J^p)$
$(0, 0, 0)$	H_g	$1(3/2^+), 3(5/2^+), \dots$
$(0, 0, n)$	G_2	$1(3/2^+), 2(3/2^-), 2(5/2^-), \dots$
(n, n, n)	F_1	$1(3/2^+), 2(3/2^-), 2(5/2^-), \dots$
	F_2	$1(3/2^+), 2(3/2^-), 2(5/2^-), \dots$

- Single-wave approx. relies on threshold suppression of d-wave.

$\Delta(1232) \rightarrow N\pi$ first results

($L = 3.6\text{fm}$, $a = 0.075\text{fm}$, $m_\pi = 280\text{MeV}$)

- Four total momenta, ground states and excited states
- Six levels in elastic region, near-threshold pole.
- Test of single partial wave approx: fit parameters insensitive to d -wave contributions: $J^P = 3/2^-$, $5/2^-$



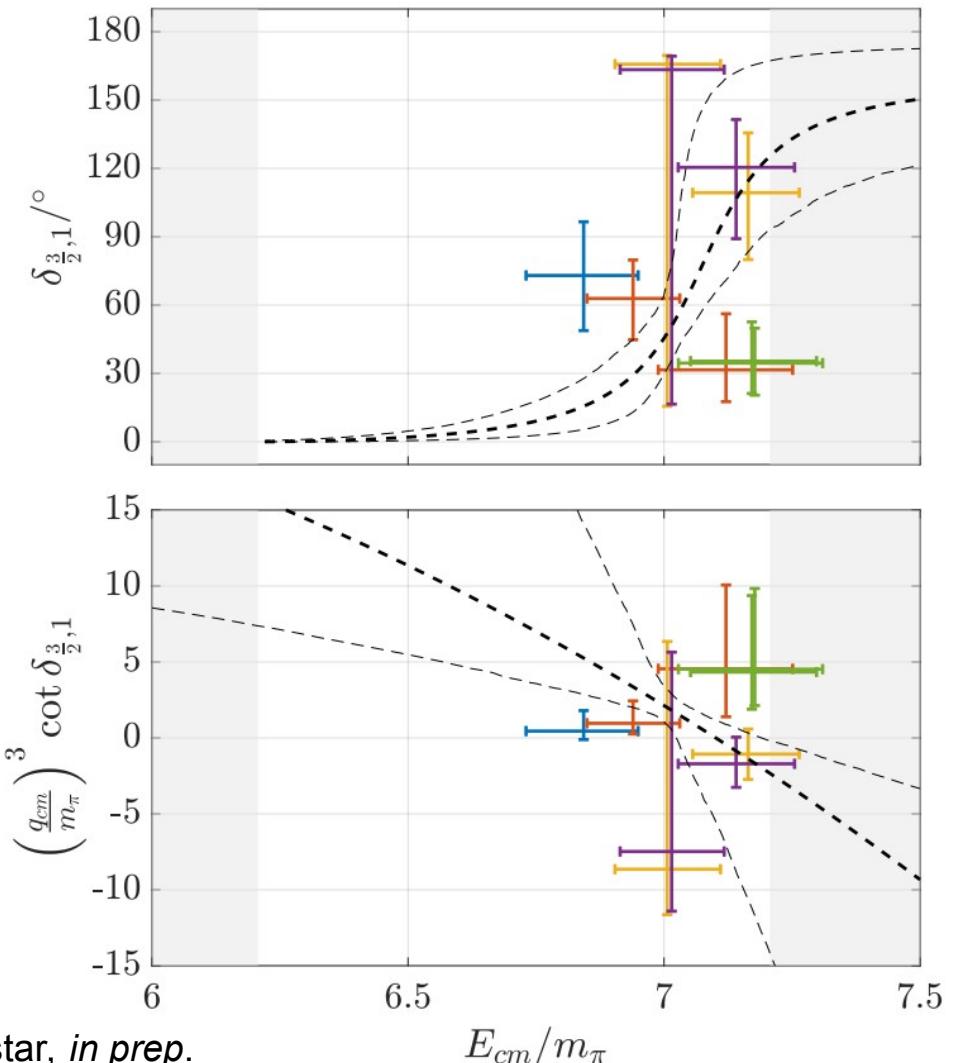
$$\frac{m_\Delta}{m_\pi} = 4.738(47), \quad g_{\Delta N\pi} = 19.0(4.7), \quad \frac{\chi^2}{d.o.f} = 1.11$$

$\Delta(1232) \rightarrow N\pi$ preliminary

$$L = 4.2\text{fm}, a = 0.065\text{fm}, m_\pi = 200\text{MeV}$$

- Five total momenta. Ground and excited states.
- Simultaneous fit to s- and p-waves.
(only p-wave irreps shown)
- Preliminary statistics: expect ~ 6 times smaller errors
- Light pion mass \rightarrow small elastic region

$$\frac{m_\Delta}{m_\pi} = 7.103(92), \quad g_{\Delta N\pi} = 9.5(6.7), \quad \frac{\chi^2}{d.o.f} = 0.86$$

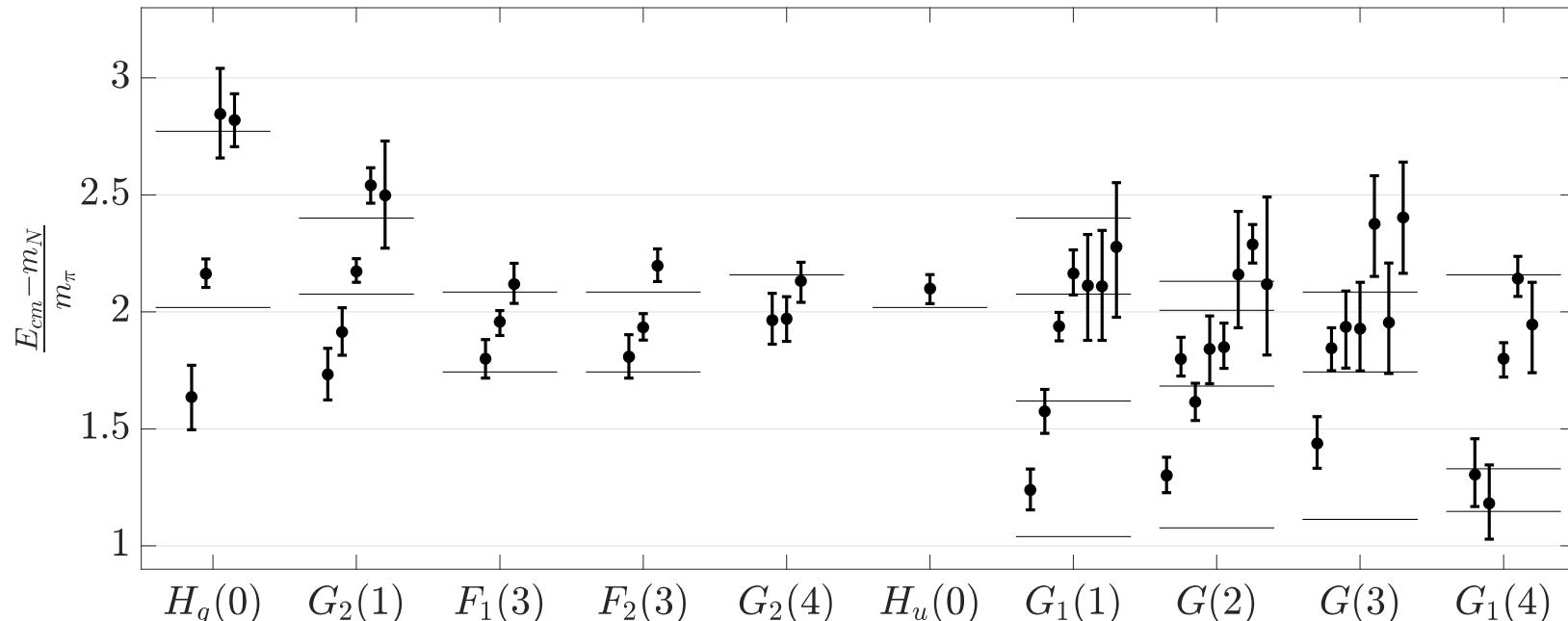


$\Delta(1232) \rightarrow N\pi$ preliminary

- Fits include irreps which mix s-wave and p-wave.
- Relies on automated determination of B -matrix elements.

$$\det[1 - K(E_{\text{cm}})B(L\mathbf{q}_{\text{cm}})] = 0 \quad \text{C. Morningstar, et al. '17. Freely available!}$$

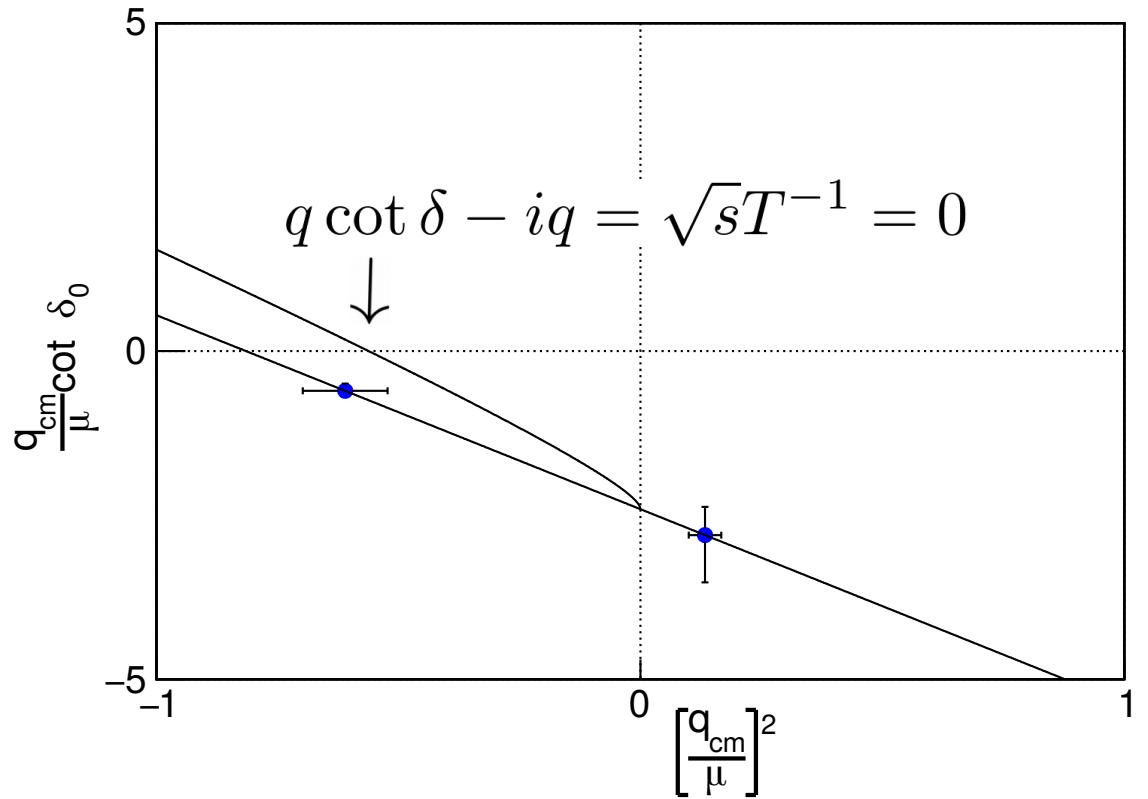
- Finite-volume spectrum:



$\Lambda(1405) \rightarrow \Sigma\pi$ preliminary

$\Lambda(1405) \rightarrow \Sigma\pi$ ($L = 3.12\text{fm}$, $a = 0.065\text{fm}$, $m_\pi = 280\text{MeV}$)

- $G_{1u}(0)$ below inelastic threshold only $\rightarrow J^P = 1/2^-$ (s-wave)



$$\frac{p_{\text{cm}}}{\mu} \cot \delta_0 = \frac{1}{a_0 \mu} + \frac{\mu r}{2} \left(\frac{p_{\text{cm}}}{\mu} \right)^2$$

$$\frac{m_R}{\mu} = 6.143(77), \quad \frac{1}{a\mu} = -2.41(57), \quad \frac{\mu r}{2} = -2.9(1.1)$$

$$m_R = 1399(24)\text{MeV}$$

B. Hörz, C. Andersen, JB, M. Hansen, D. Mohler, C. Morningstar, H. Wittig, *in prep.*

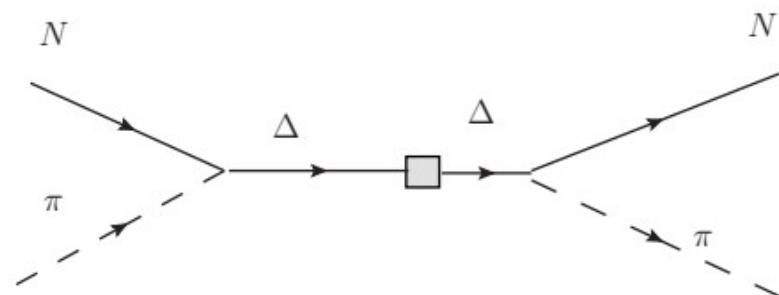
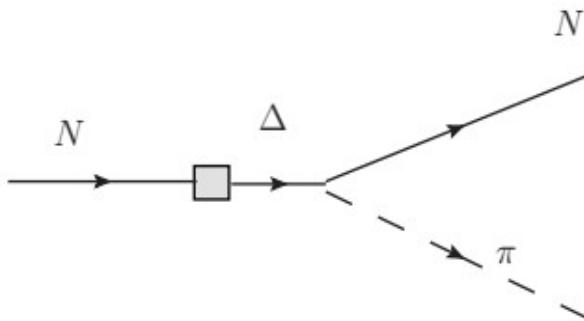
Meson-baryon scattering amplitudes: future prospects

- Low-lying resonances:
(two- and three-body)

$(2I, S)$	J^p	name
(1, 0)	$1/2^+$	$N(1440)$
	$3/2^-$	$N(1520)$
	$1/2^-$	$N(1535)$
(3, 0)	$3/2^+$	$\Delta(1232)$
	$1/2^-$	$\Lambda(1405)$
(0, -1)	$3/2^-$	$\Lambda(1520)$
	$3/2^+$	$\Xi(1530)$
(1, -2)	??	$\Xi(1690)$
	??	$\Omega(2012), \Omega(2250)$

- Form factors, e.g.

$$N + \hat{J}_{\text{ew}} \rightarrow \Delta(1232) \rightarrow N + \pi, \quad N + \pi + \hat{J}_{\text{ew}} \rightarrow N + \pi$$



- Interaction with EFT's:

Conclusions

- Distillation and Stochastic LapH enable precise finite-volume energies
→ scattering amplitudes below three-hadron (or more) thresholds
- Meson-baryon scattering just beginning: low-energy and elastic amplitudes.
- Inelastic structure to come: $N + J_{ew} \rightarrow N\pi$
in analogy with $\gamma^* \rightarrow \pi\pi$ and $\pi + \gamma^* \rightarrow \pi\pi$
- Main difficulties: signal-to-noise problem and inelastic thresholds.
- Distillation in the 2020's?: will volume scaling be overcome?