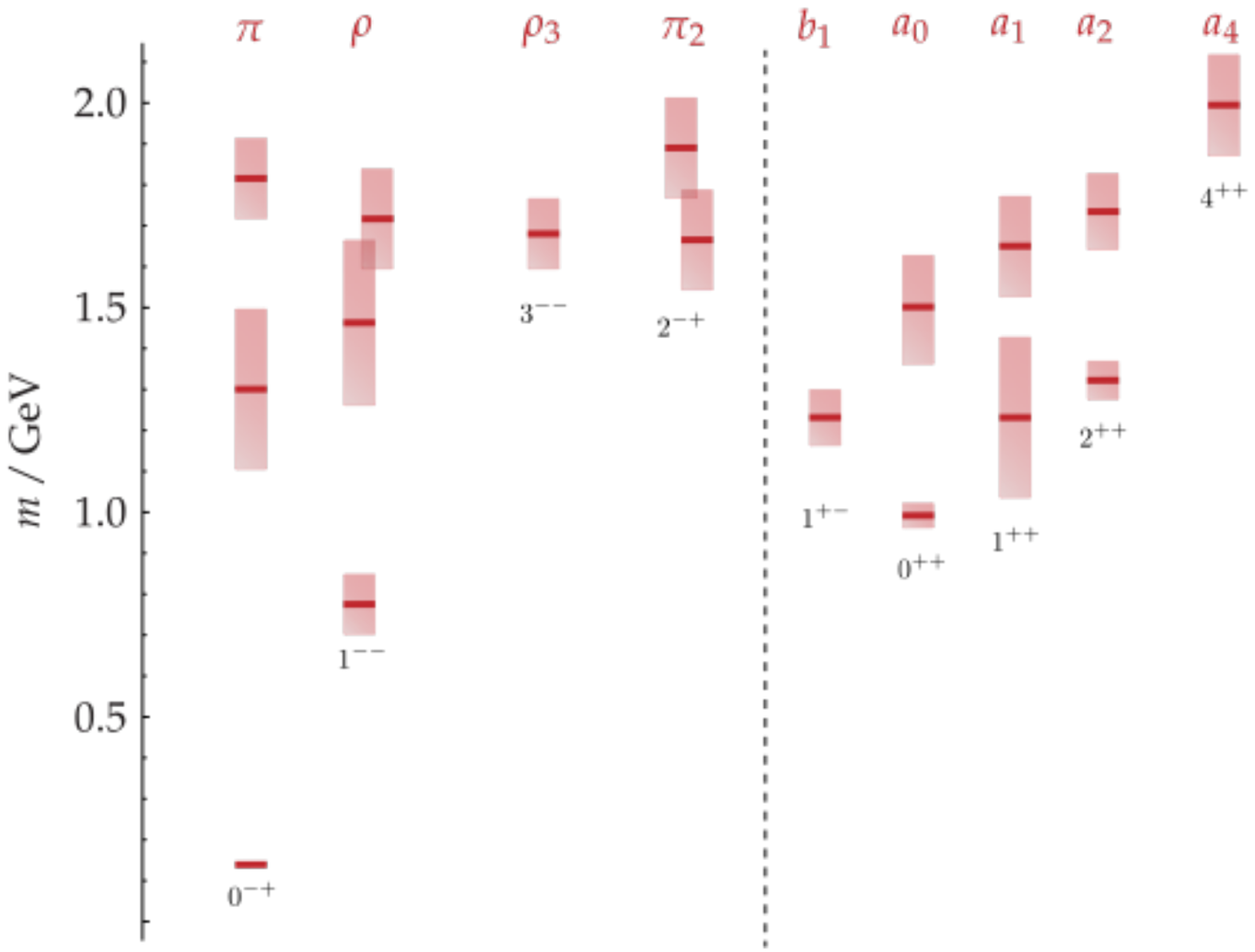
The power of distillation

Happy 10-th Birthday! Signed, Robert Edwards

Experimental meson spectrum

- Mesons classified by their conserved quantum numbers
  - Spin, isospin, charge-conjugation J^{PC}

ISOSPIN=1 MESON SPECTRUM



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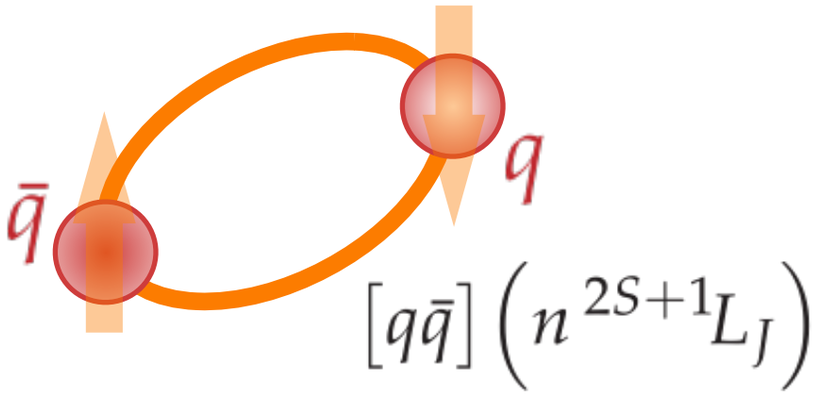

$$[q\bar{q}]\left(n\,^{2S+1}L_J\right)$$

$$L = 0 : 0^{-+}, 1^{--}$$

$$L = 1 : 1^{+-}, (0,1,2)^{++}$$

$$L = 2 : 2^{-+}, (1,2,3)^{--}$$

$$\vdots$$

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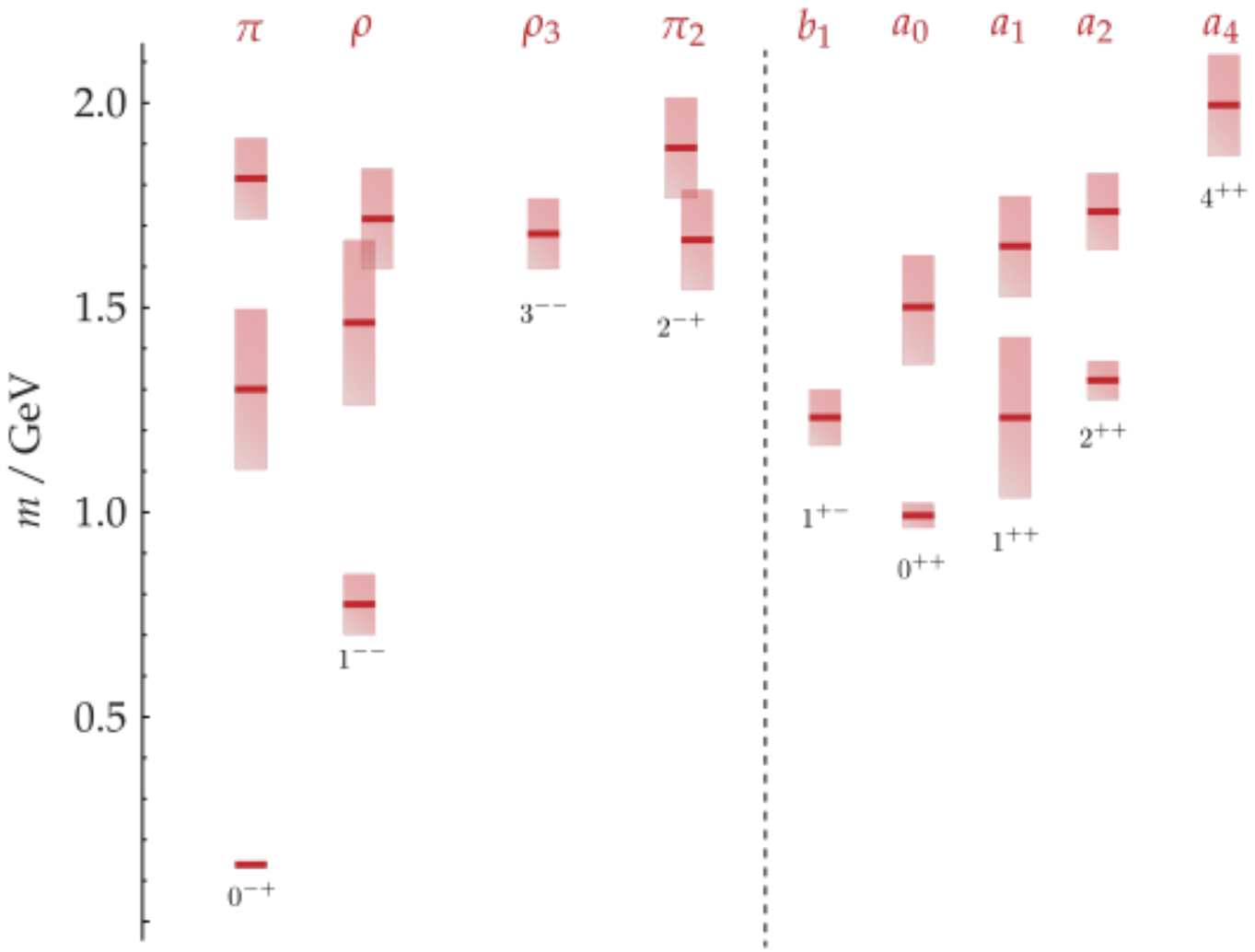

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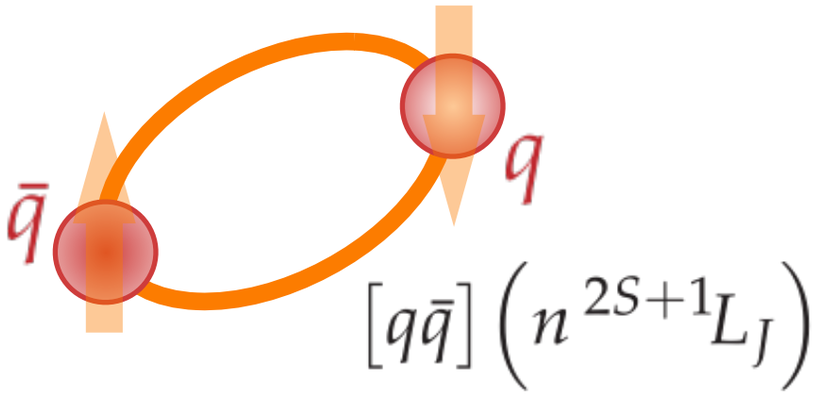

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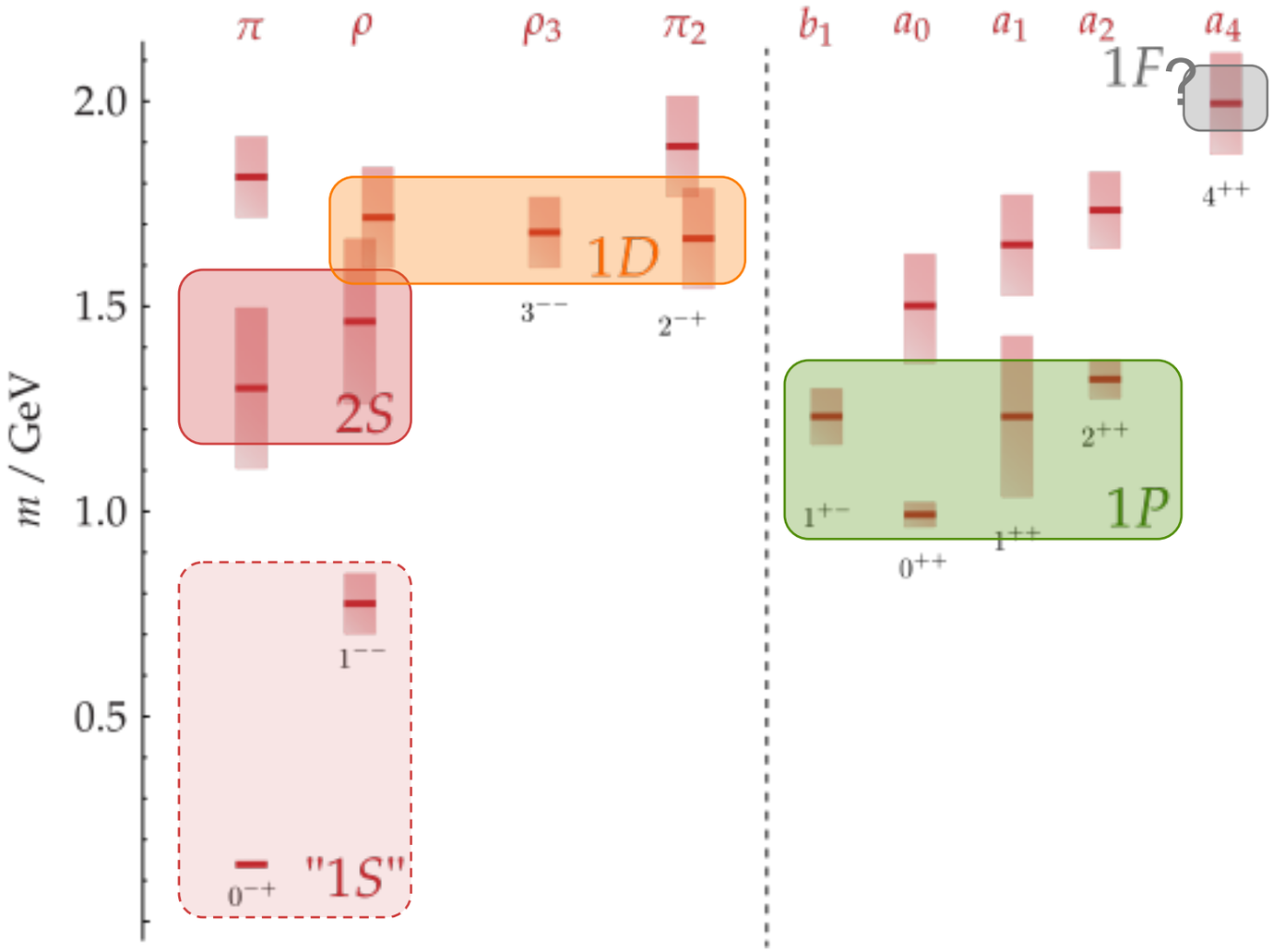

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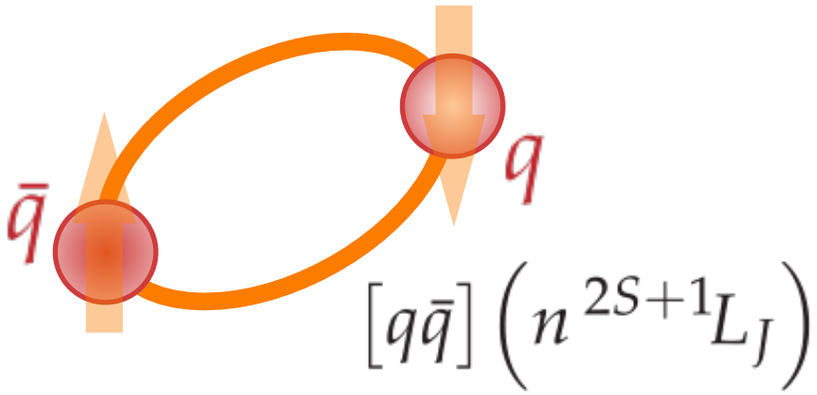

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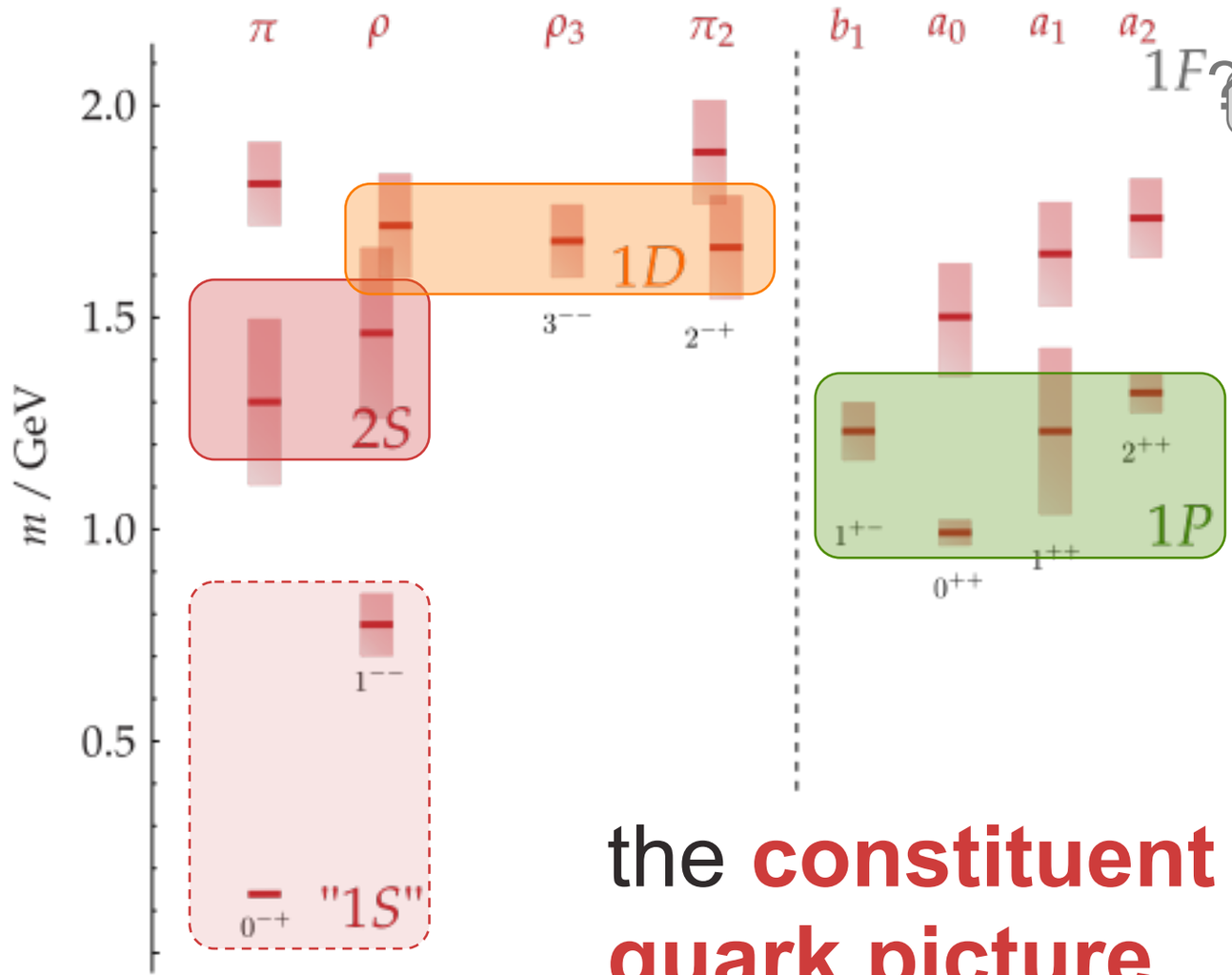
n.b. absent: $0^{--},\ 0^{+-},\ 1^{-+},\ 2^{+-}\ \ldots$

ISOSPIN=1 MESON SPECTRUM

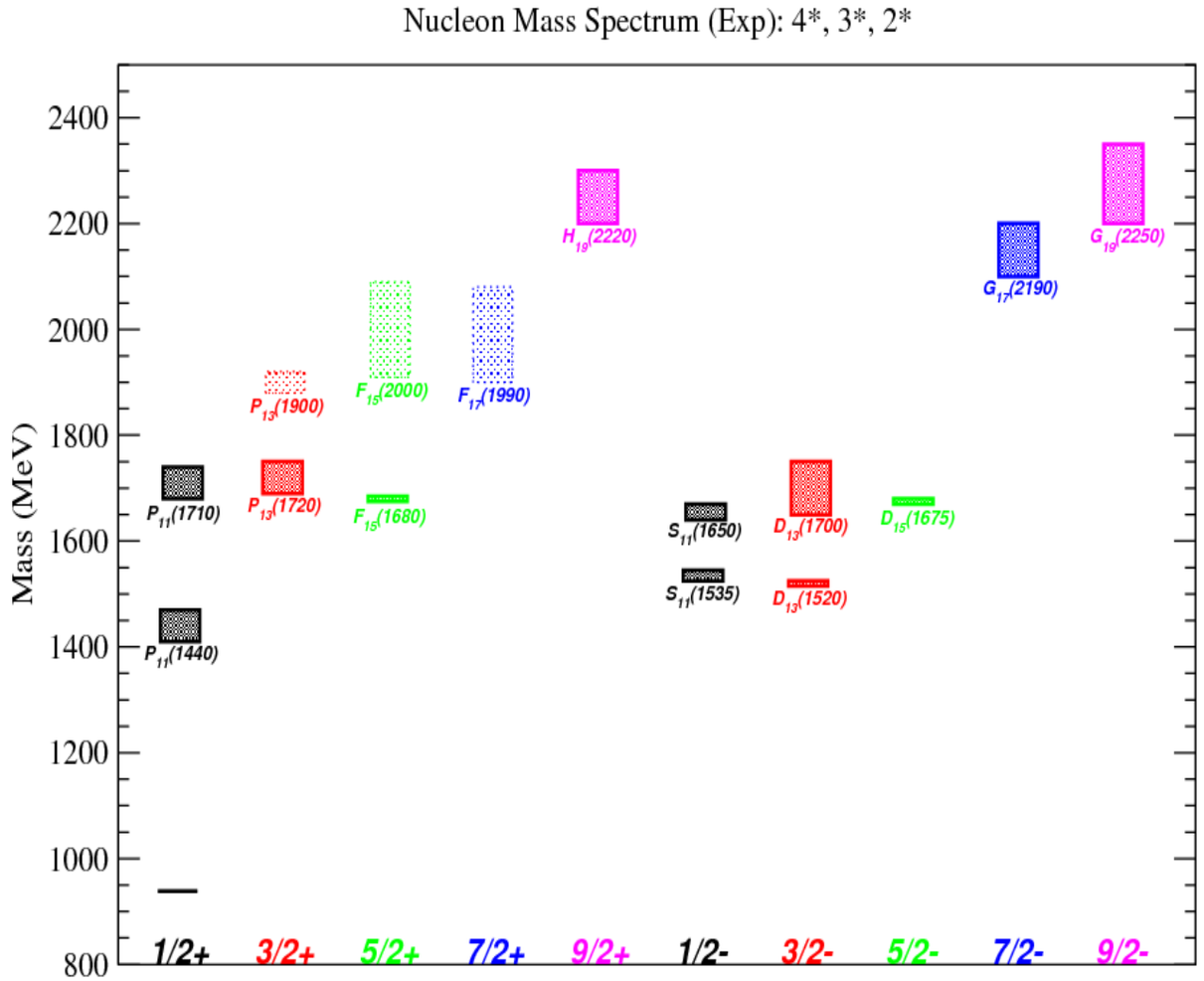


the **constituent quark picture**

Experimental baryon spectrum

- Baryons classified by their conserved quantum numbers
  - Spin, parity, isospin **J^P**

ISOSPIN=1/2 BARYON SPECTRUM



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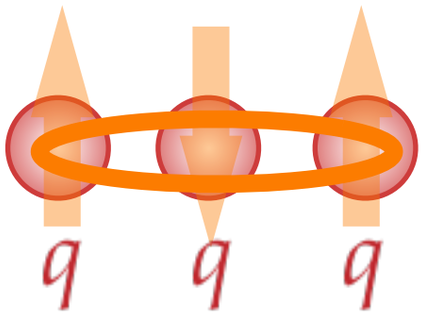


$$[qqq]\left(n^{2S+1}L_{\pi}\right)$$

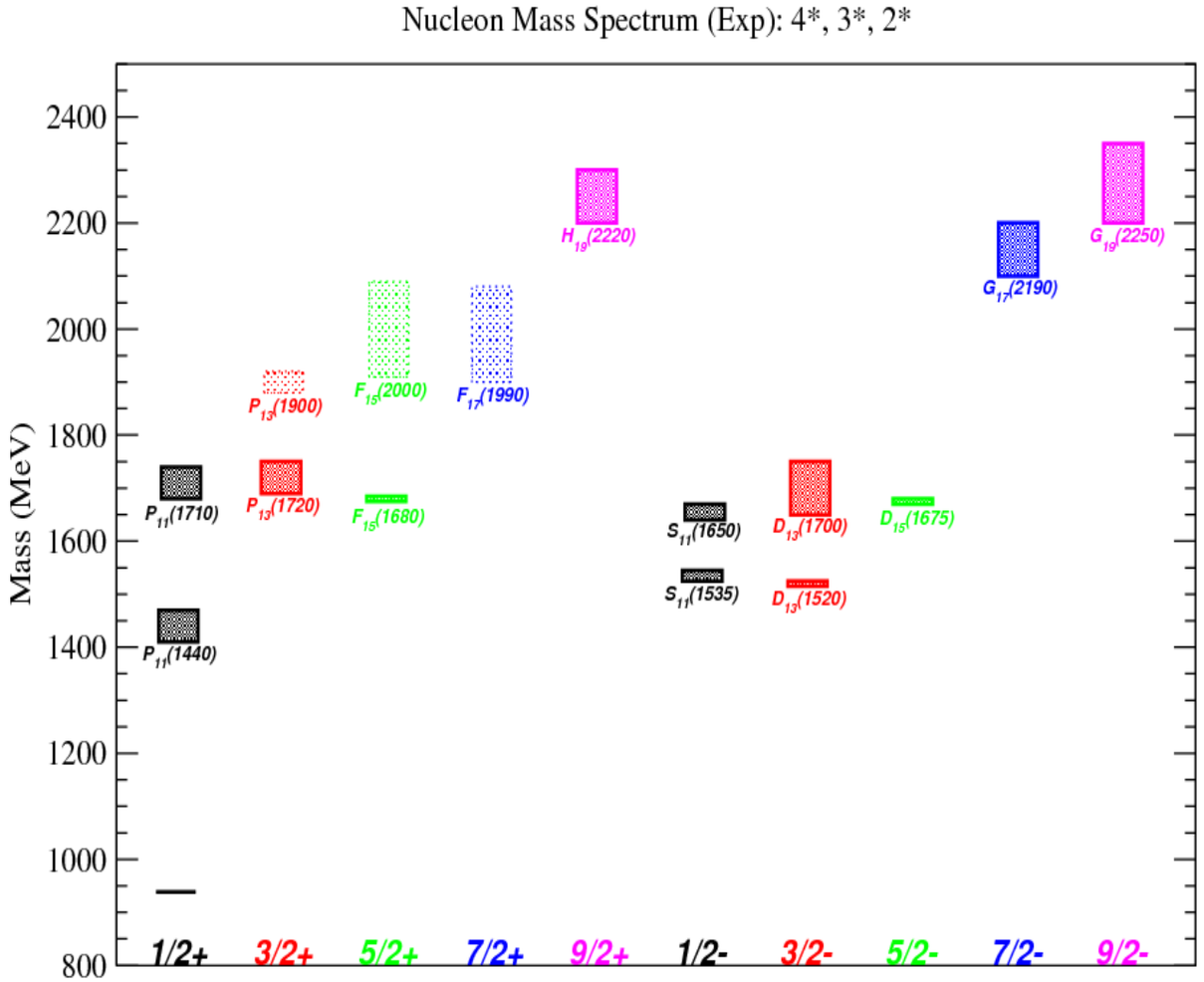
Antisymmetric under interchange

π = permutation of quarks in space

$$\mathrm{L} = \quad 0_S \quad : \quad \tfrac{1}{2}^+$$

$$\mathrm{L} = \quad 1_M \quad : \quad (\tfrac{1}{2},\tfrac{3}{2}\ ,\ \tfrac{1}{2},\tfrac{3}{2},\tfrac{5}{2})^-$$

ISOSPIN=1/2 BARYON SPECTRUM



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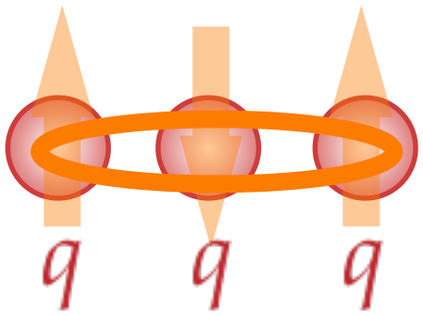

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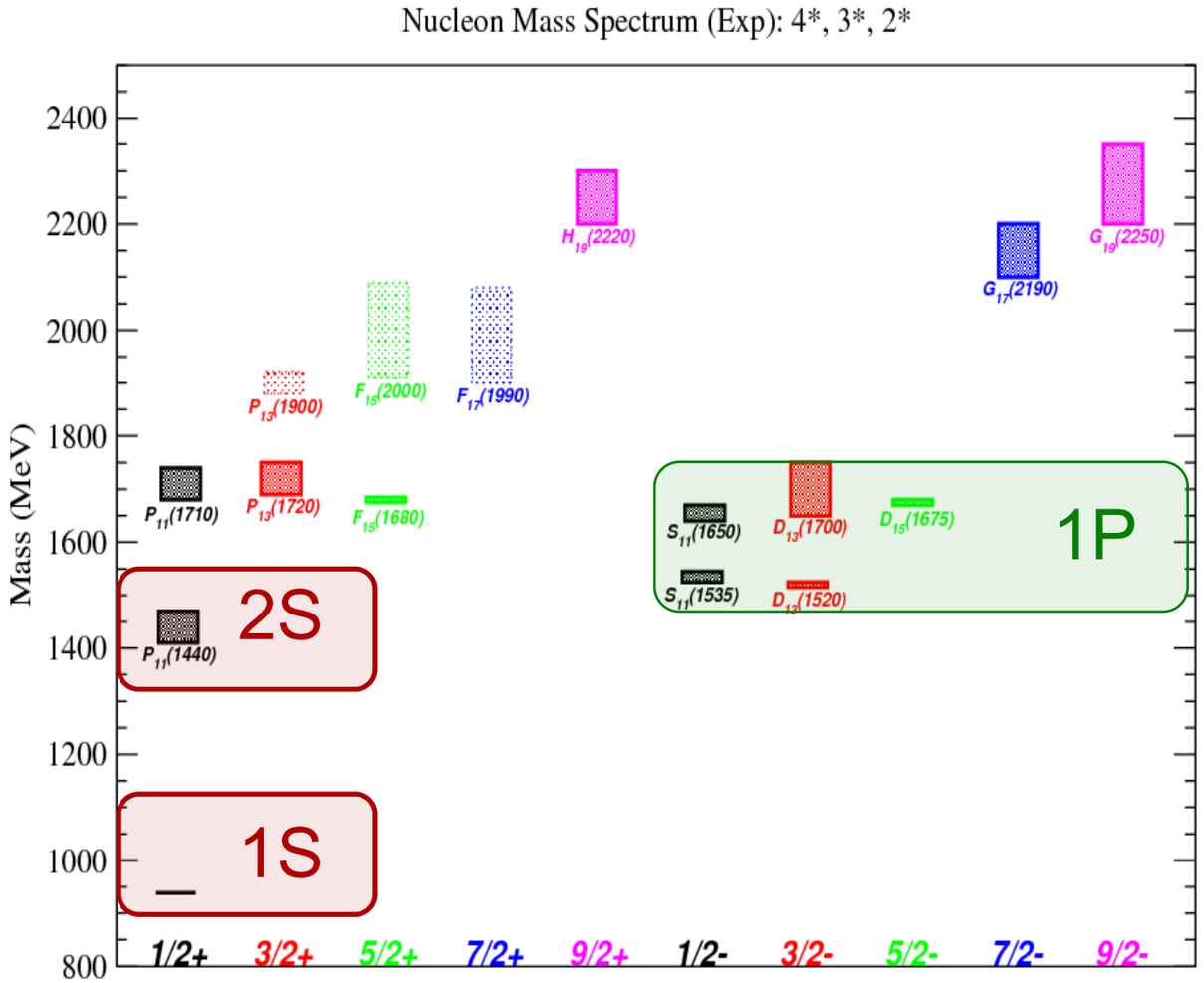
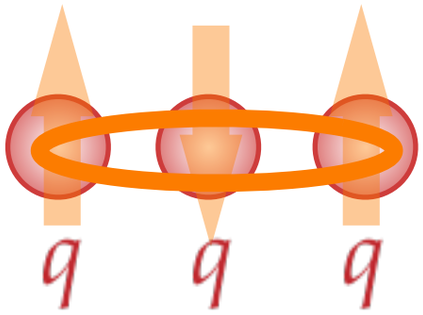


Experimental baryon spectrum

- Some states are “missing” ???



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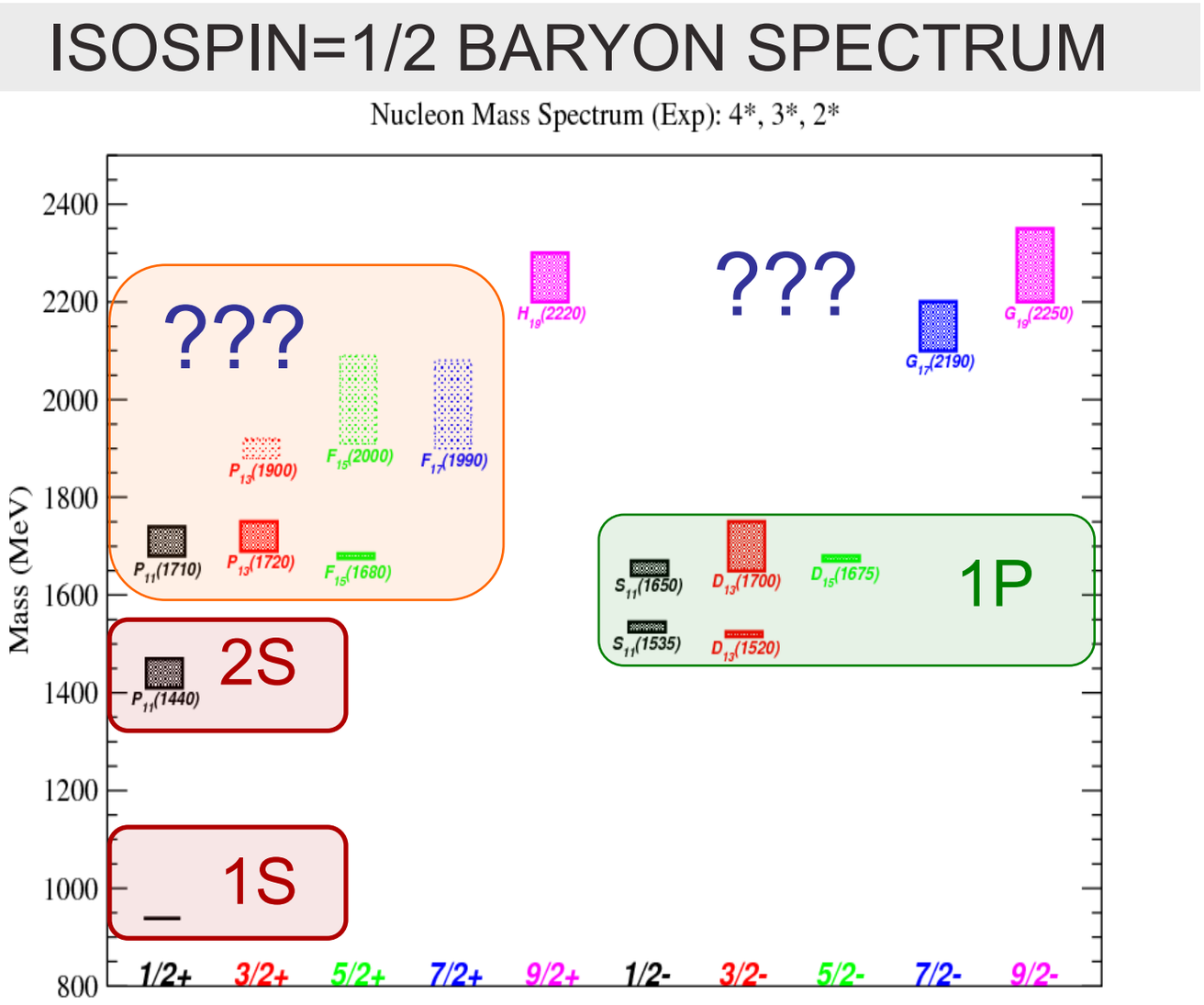
L =	2_M :	$(\frac{3}{2},\frac{5}{2})^+\ ,\ (\frac{1}{2},\frac{3}{2},\frac{5}{2})^+$
L =	1_A :	$(\frac{1}{2},\frac{3}{2})^+$
L =	0_M :	$(\frac{1}{2},\frac{3}{2})^+$

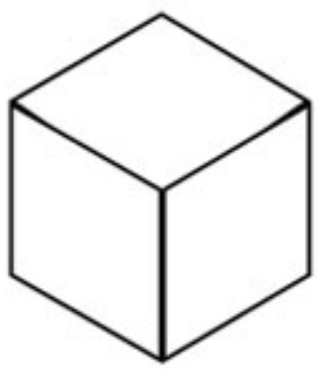


Finite volume QCD & the hadron spectrum

Compute **correlation functions** as an average over field configurations

$$\text{e.g.} \quad \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A_\mu \; \bar{\psi}\Gamma\psi(t)\, \bar{\psi}\Gamma\psi(0) \; e^{-\int d^4x\, \mathcal{L}_{\text{QCD}}(\psi,\bar{\psi},A_\mu)}$$

'sum' 'field correlation' 'probability weight'



Field integration within a finite, but continuous, hypercube

Need some kind of ultraviolet regulator….

Spectrum from two-point correlation functions

$$\begin{aligned} C(t) &= \langle 0|\mathcal{O}(t)\, \mathcal{O}^\dagger(0)|0\rangle \\ &= \sum_{\mathfrak{n}} e^{-E(\mathfrak{n})t} \langle 0|\mathcal{O}(0)|\mathfrak{n}\rangle \langle \mathfrak{n}|\mathcal{O}^\dagger(0)|0\rangle \end{aligned}$$

States $|\mathfrak{n}\rangle$ are finite-volume distorted

Lattice QCD & the hadron spectrum

Compute **correlation functions** as a Monte Carlo average over field configurations

$$\text{e.g.} \quad \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A_\mu \; \bar{\psi}\Gamma\psi(t)\, \bar{\psi}\Gamma\psi(0) \; e^{-\int d^4x\, \mathcal{L}_{\text{QCD}}(\psi,\bar{\psi},A_\mu)}$$

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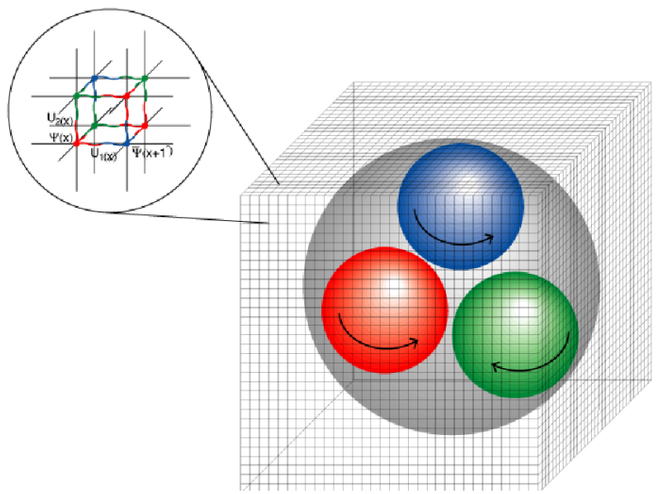

Discretize the action over sites

Serves as an ultraviolet regulator

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Excited states from correlators

- How to get at excited QCD eigenstates ?

  - optimal operator for state $|\mathfrak{n}\rangle$: $\Omega^\dagger_{\mathfrak{n}} \sim \sum_i v_i^{(\mathfrak{n})} \mathcal{O}_i^\dagger$

    for a basis of meson operators $\{\mathcal{O}_i\}$

  - can be obtained (in a variational sense) from the matrix of correlators

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t)\, \mathcal{O}_j^\dagger(0) | 0 \rangle$$

  - by solving a generalized eigenvalue problem

'diagonalize the correlation matrix'

$$C(t) v^{(\mathfrak{n})} = C(t_0) v^{(\mathfrak{n})}\, \lambda_{\mathfrak{n}}(t)$$

eigenvalues $\lambda_{\mathfrak{n}}(t) \sim e^{-E_{\mathfrak{n}}(t-t_0)}$

  - a large basis can be constructed using covariant derivatives :

$$\mathcal{O} \sim \bar{\psi} \Gamma \overleftrightarrow{D} \ldots \overleftrightarrow{D} \psi$$

Operators - quark bilinears

- Scalar/Vector Dirac structures & covariant derivatives

$$\bar{\psi}\gamma_5\psi \qquad \rightarrow J = 0$$
$$\bar{\psi}\gamma_i\psi \qquad \rightarrow J = 0$$
$$\bar{\psi}\overleftrightarrow{D_i}\psi \qquad \rightarrow J = 1$$

➡ Combine gamma & derivatives

$$\bar{\psi}\Gamma\overleftrightarrow{D}\overleftrightarrow{D}\psi$$

in terms of

$$\langle 1m_1\ 1m_2|Jm\rangle\bar{\psi}\overleftrightarrow{D}_{m_1}\overleftrightarrow{D}_{m_2}\psi \qquad \rightarrow J = 0,1,2$$
$$\langle 1s\ Ll|Jm\rangle\langle 1l_1\ 1l_2|Ll\rangle\bar{\psi}\Gamma_s\overleftrightarrow{D}_{l_1}\overleftrightarrow{D}_{l_2}\psi \qquad \rightarrow J = 0,1,2,3$$

- Subduction of continuum to cubic reps.

$$\mathcal{O}^{[J]}_{\Lambda\lambda} = \sum_M \mathcal{S}^{JM}_{\Lambda\mu}\mathcal{O}^{JM}$$

arxiv:1004.4930, 1104.5152, 1201.2349

Operators - three quarks

- Baryons operators are projectors acting on flavor, Dirac spin, and spatial indices

$$(\mathrm{Flavor}_{\pi_F} \otimes \mathrm{Spin}_{\pi_S} \otimes \mathrm{Space}_{\pi_D})\,\{\psi_1\psi_2\psi_3\}$$

- Symmetric under quark permutations π & color is antisymmetric

- Now CG-s in permutations π and coupling spin and derivatives/space

e.g., two derivatives

$$\mathcal{O}_i \sim (\mathrm{CGCs})_{i,j,k}\ \{\overrightarrow{D}\overrightarrow{D}\}_j\ \{\psi\psi\psi\}_k \qquad 1\times 1\times \mathcal{S} \rightarrow\ J=\tfrac{1}{2},\tfrac{3}{2},\tfrac{5}{2},\tfrac{7}{2}$$

arxiv:1004.4930, 1104.5152, 1201.2349

Distillation

- Define a low rank (spatial) smearing operator

  e.g., quark bilinear

$$C_{ij}(t) \;=\; \langle \bar{\psi}\Box\Gamma^i_t\Box\psi \;\cdot\; \bar{\psi}\Box\Gamma^j_0\Box\psi\rangle \qquad \Box_{xy} = V_x V_y^\dagger$$

- Factorize propagators and operator constructions

$$C_{ij}(t) \;=\; \mathrm{Tr}\left[\Phi^i(t)\; P(t,0)\; \Phi^j(0) P(0,t)\right]$$

$$\Phi^i_{\alpha\beta}(t) \;=\; V(t)^\dagger \Gamma^i_{\alpha\beta}(t) V(t)$$ **matrix rep. of operator**

$$P_{\alpha\beta}(t,0) \;=\; V(t)^\dagger \mathcal{M}^{-1}_{\alpha\beta}(t,0) V(0)$$ ***perambulator***

arxiv:0905.2160

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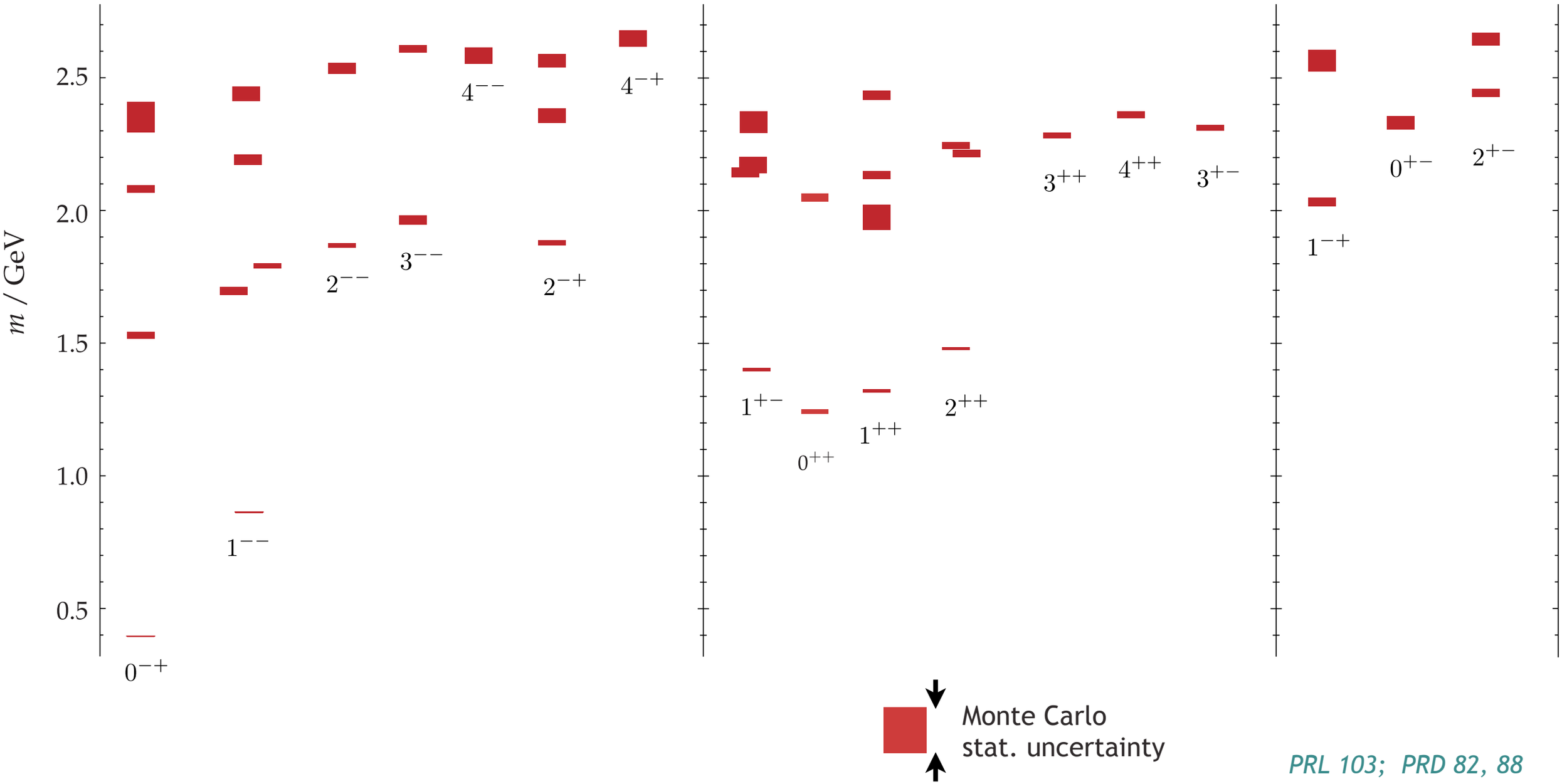
`and a powerful ally it is'

arxiv:0905.2160

Glimpse of meson spectrum from lattice QCD

- Appears to be some $q\bar{q}$-like near-degeneracy patterns - isovectors

$m_\pi \sim 391\,\mathrm{MeV}$



PRL 103; PRD 82, 88

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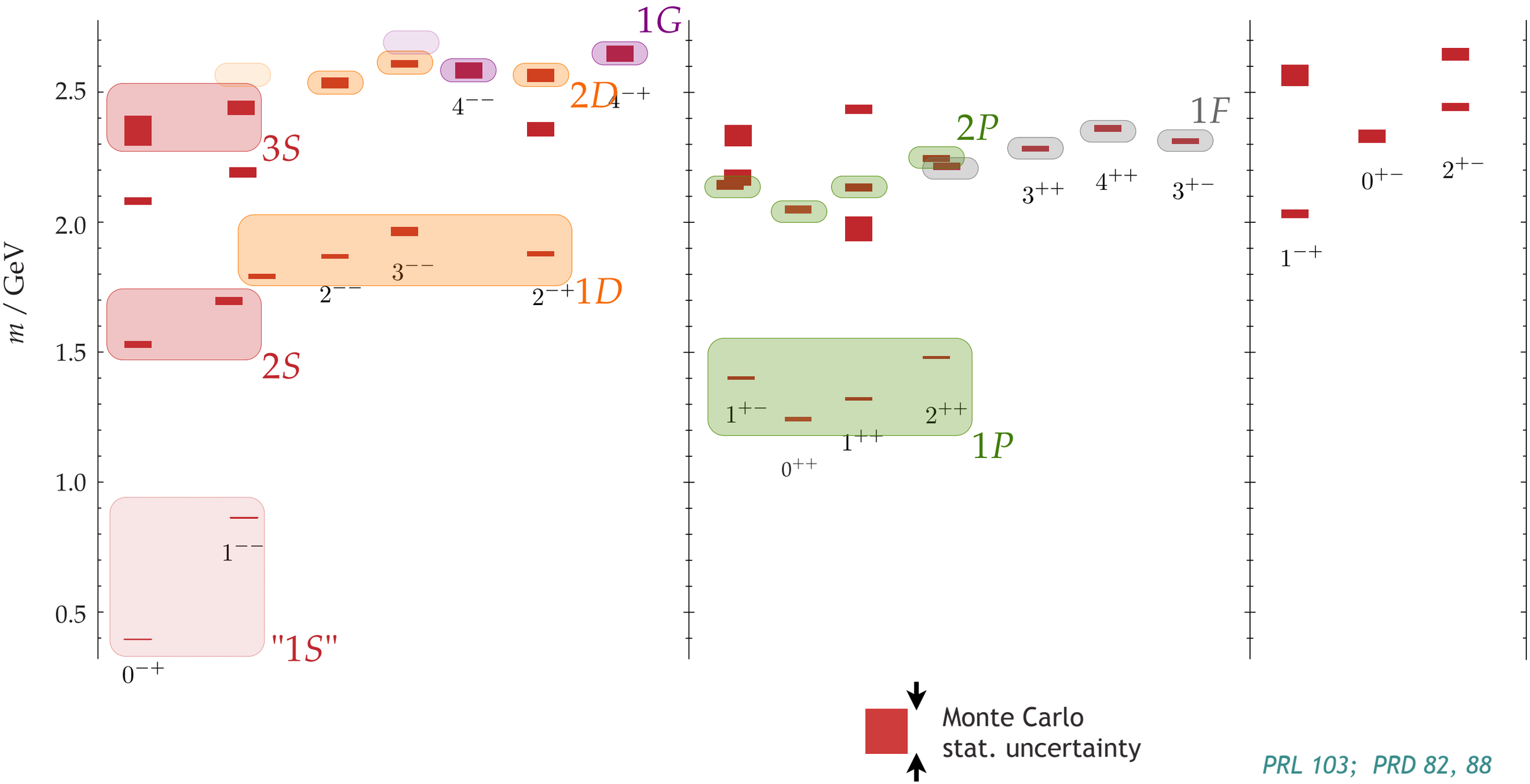
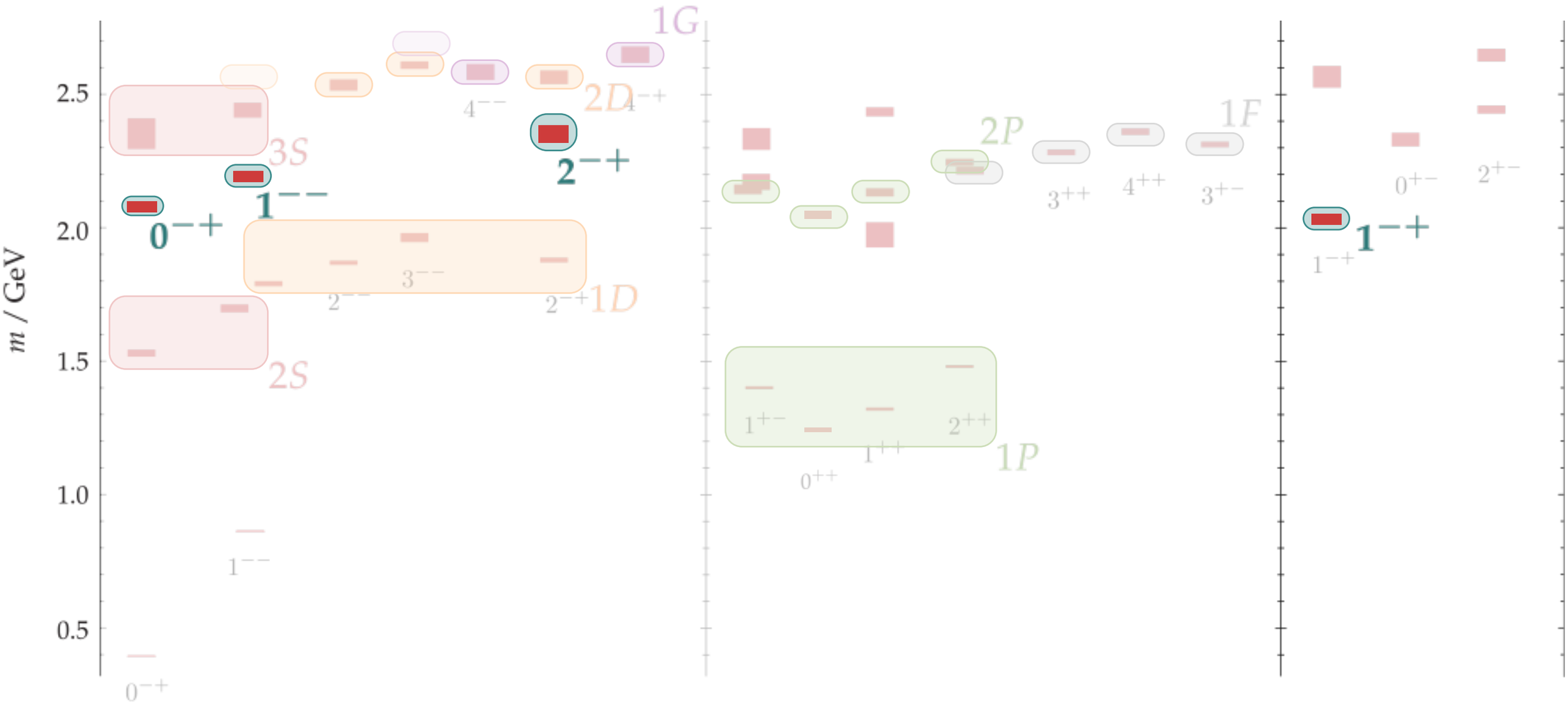


Monte Carlo
stat. uncertainty

PRL 103; PRD 82, 88

qq interpretation?

- "Extra" non-exotic states at same energy scale as lightest exotic?



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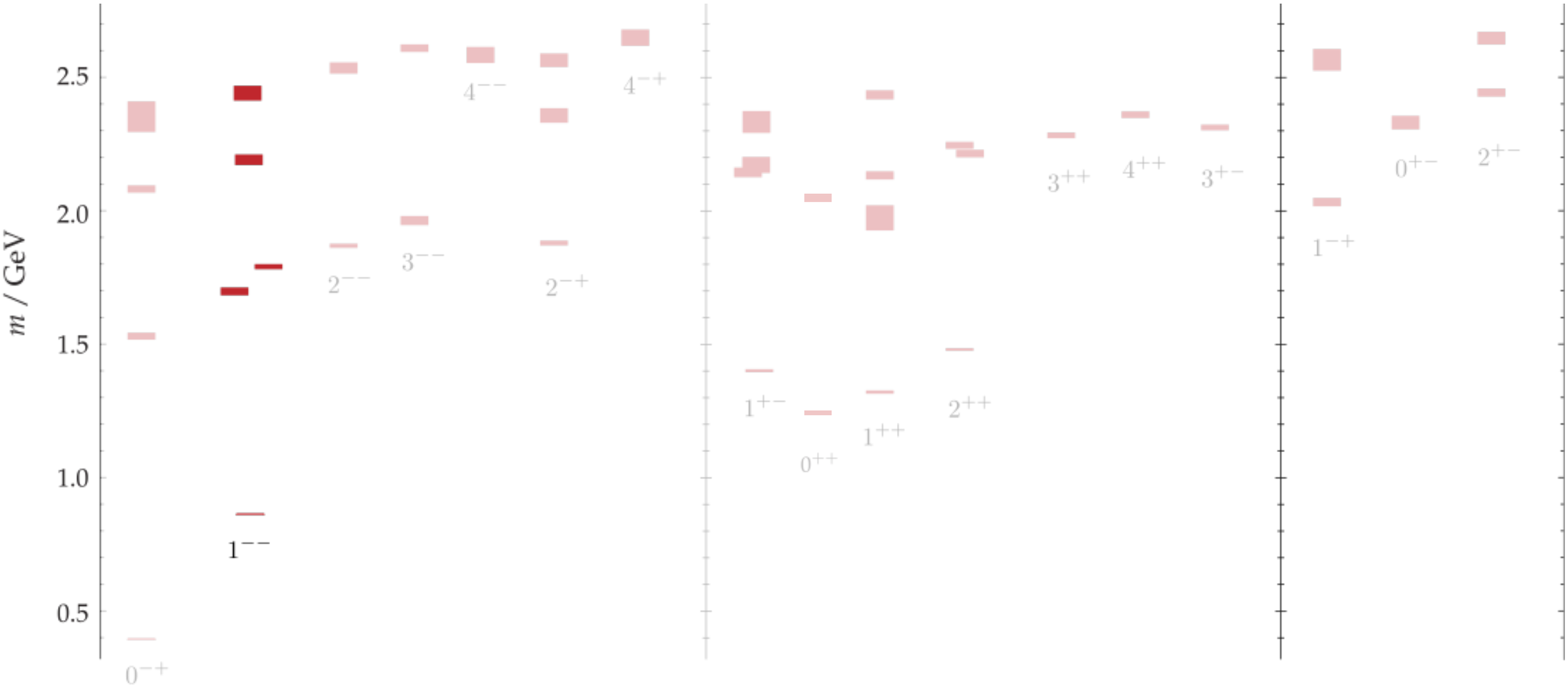
- Consider the relative size of operator overlaps $\langle \mathfrak{n} | \mathcal{O}_i^\dagger | \varnothing \rangle$



1^{--} operator overlaps

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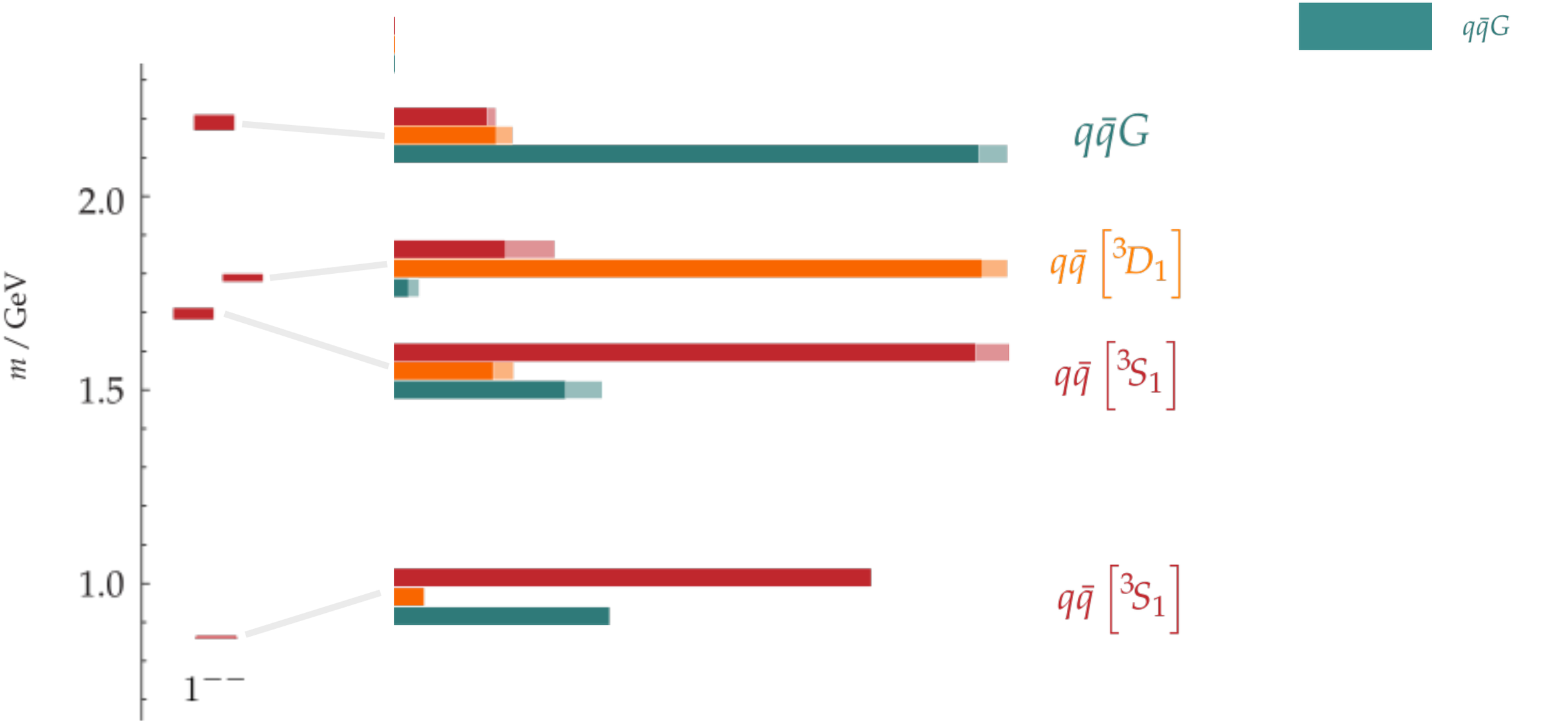
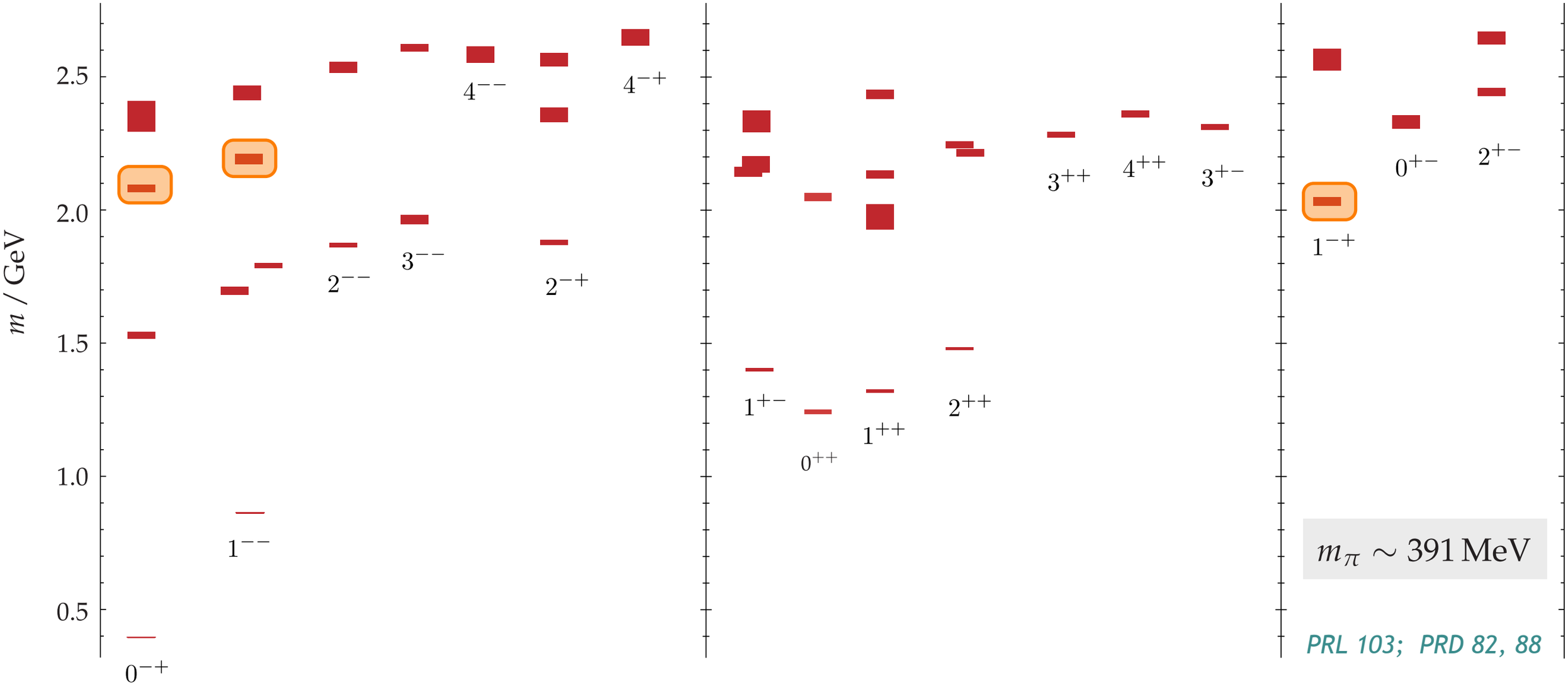


Glimpse of meson spectrum from lattice QCD

- ‘super’-multiplet of **hybrid mesons** roughly 1.2 GeV above the ρ $\quad (0,\mathbf{1},2)^{-+},\,1^{--}$



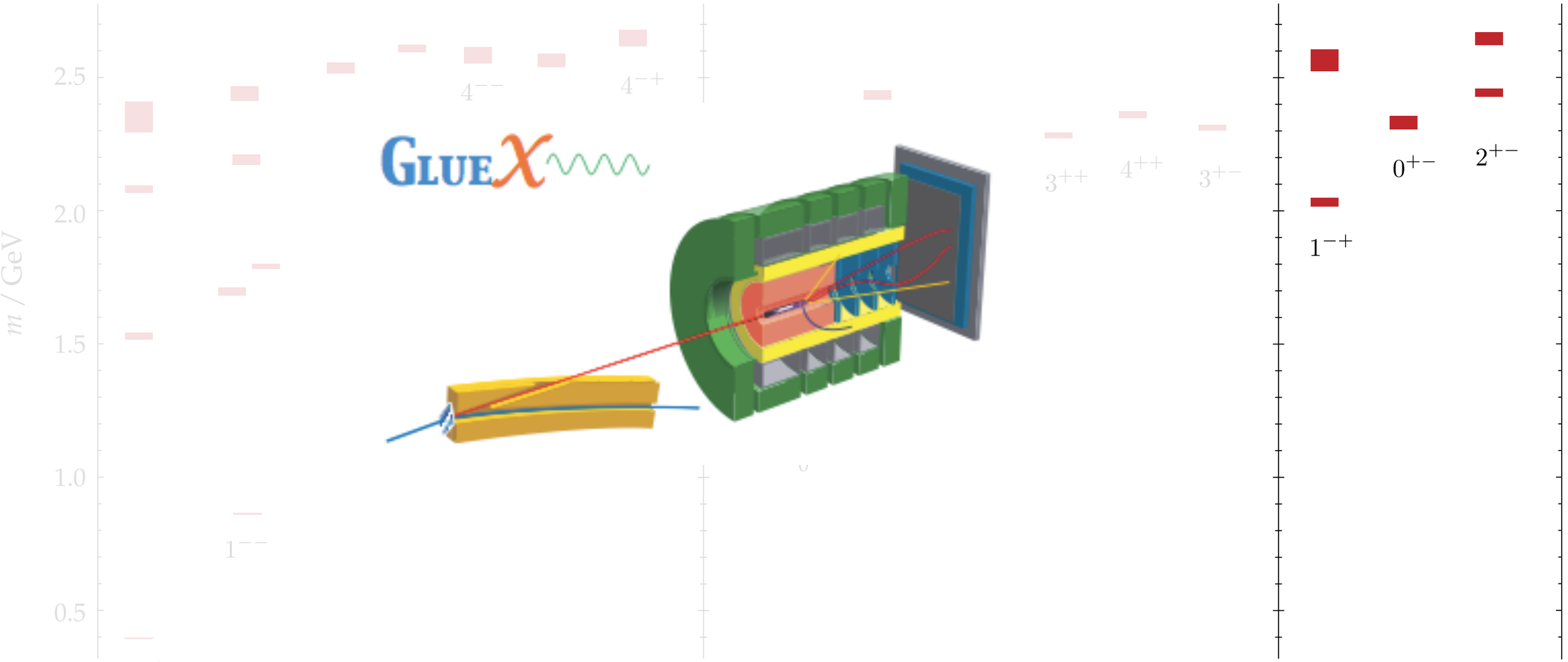
PRL 103; PRD 82, 88

- these states have a dominant overlap onto $\bar{\psi}\Gamma[D,D]\psi \;\sim [q\bar{q}]_{\mathbf{8}_c} \otimes B_{\mathbf{8}_c}$

Glimpse of meson spectrum from lattice QCD

Multiple exotic mesons within range of GlueX

EXOTIC MESONS

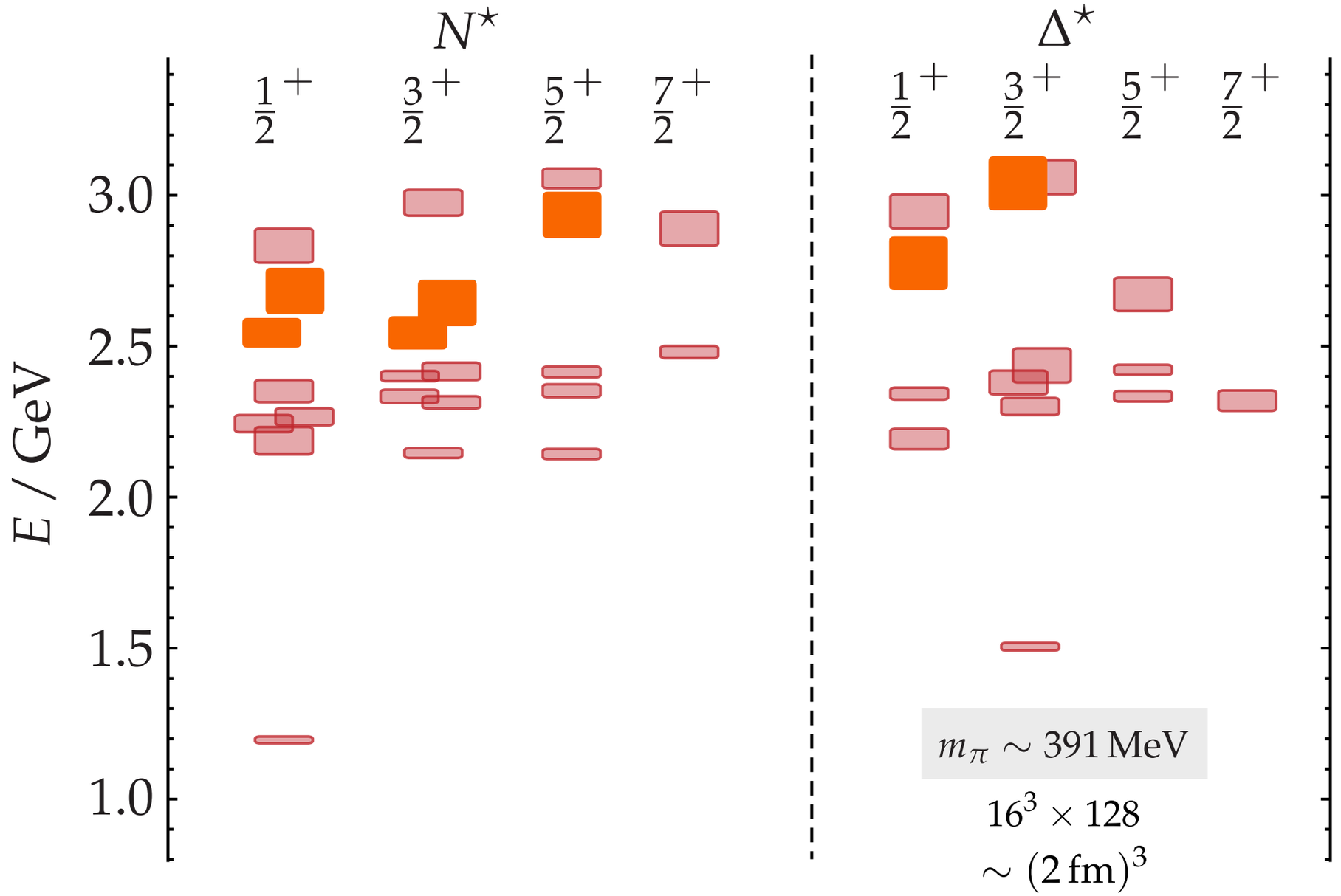


PRL 103; PRD 82, 88

Excited light quark baryons

- A ‘super’-multiplet of **hybrid baryons**

Searches in CLAS12



spectrum from large basis of baryon operators

PRD84 074508 (2011)
PRD85 054016 (2012)

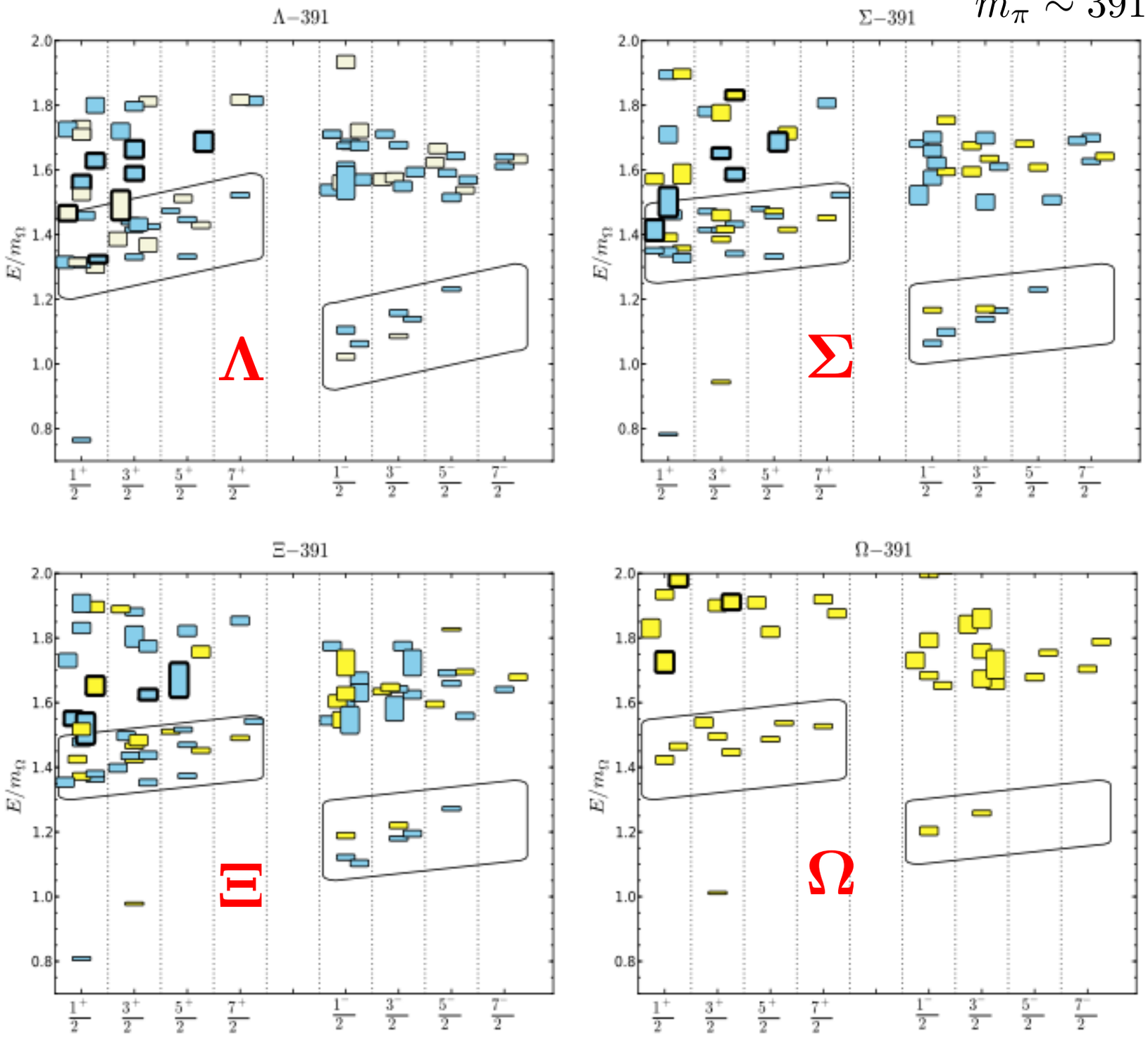
Excited strange (and charm) quark baryons

$m_\pi \sim 391$ MeV

Light quarks – SU(3) flavor broken

Full non-relativistic quark model counting

Some mixing of SU(3) flavor irreps

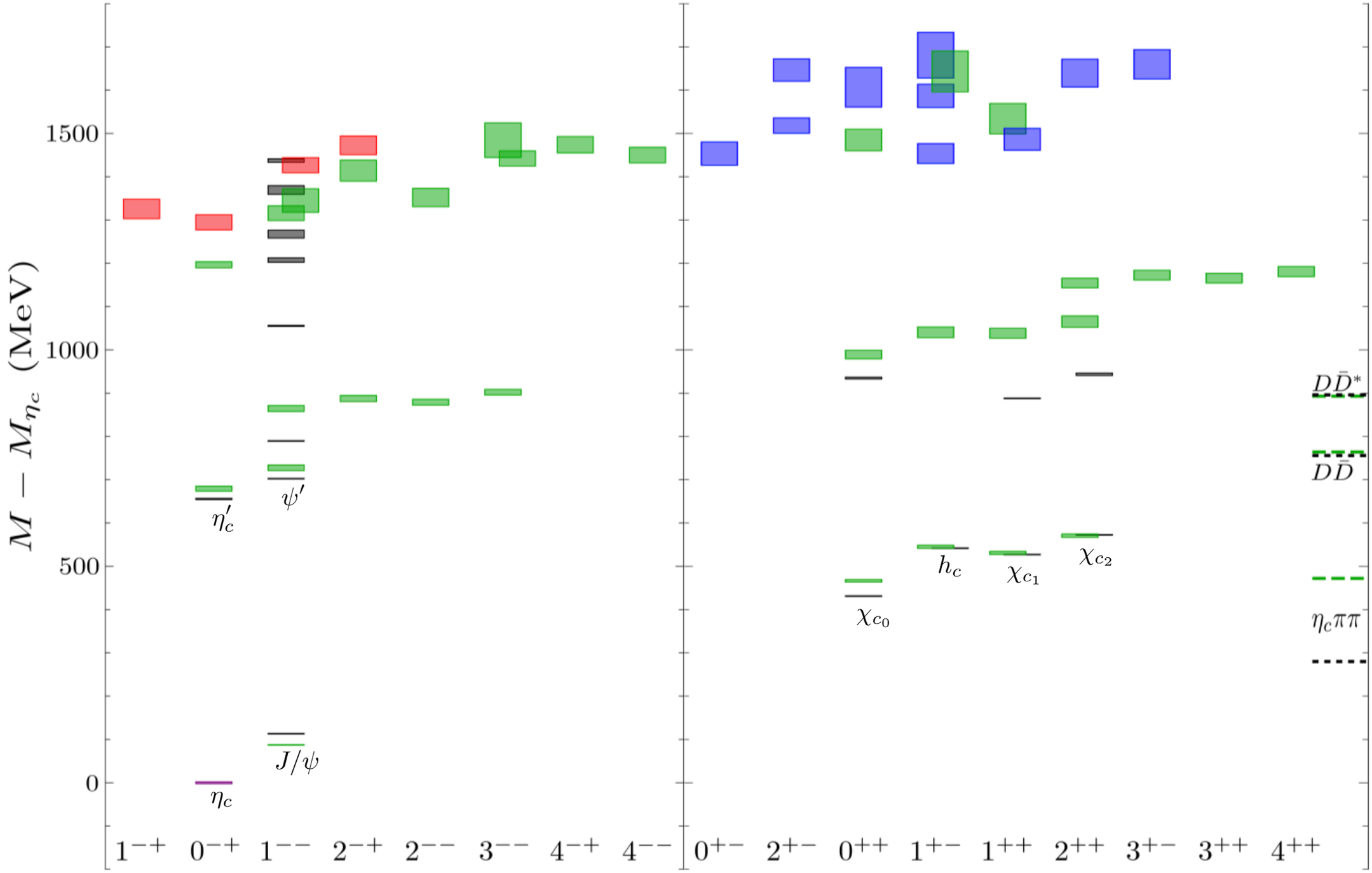


PRD87 054506 (2013)
PRD90 074504 (2014)
PRD91 054502 (2015)

Charmonium spectrum

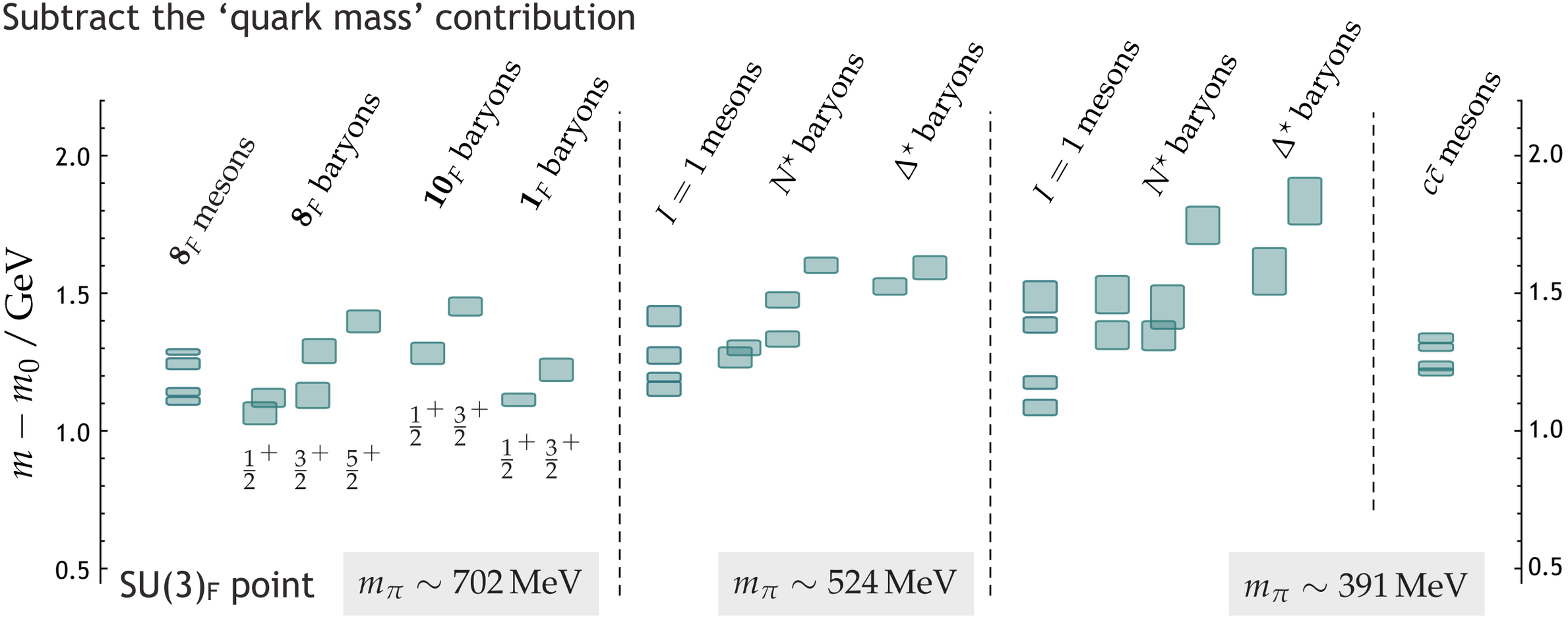
- A ‘super’-multiplet of **hybrid mesons**

Cheung, O’Hara, Tims, Moir, Peardon, Ryan, Thomas (2016)



Chromo-magnetic excitation

- Subtract the ‘quark mass’ contribution



HADRON SPECTRUM: PRD83 (2011); PRD88 (2013)

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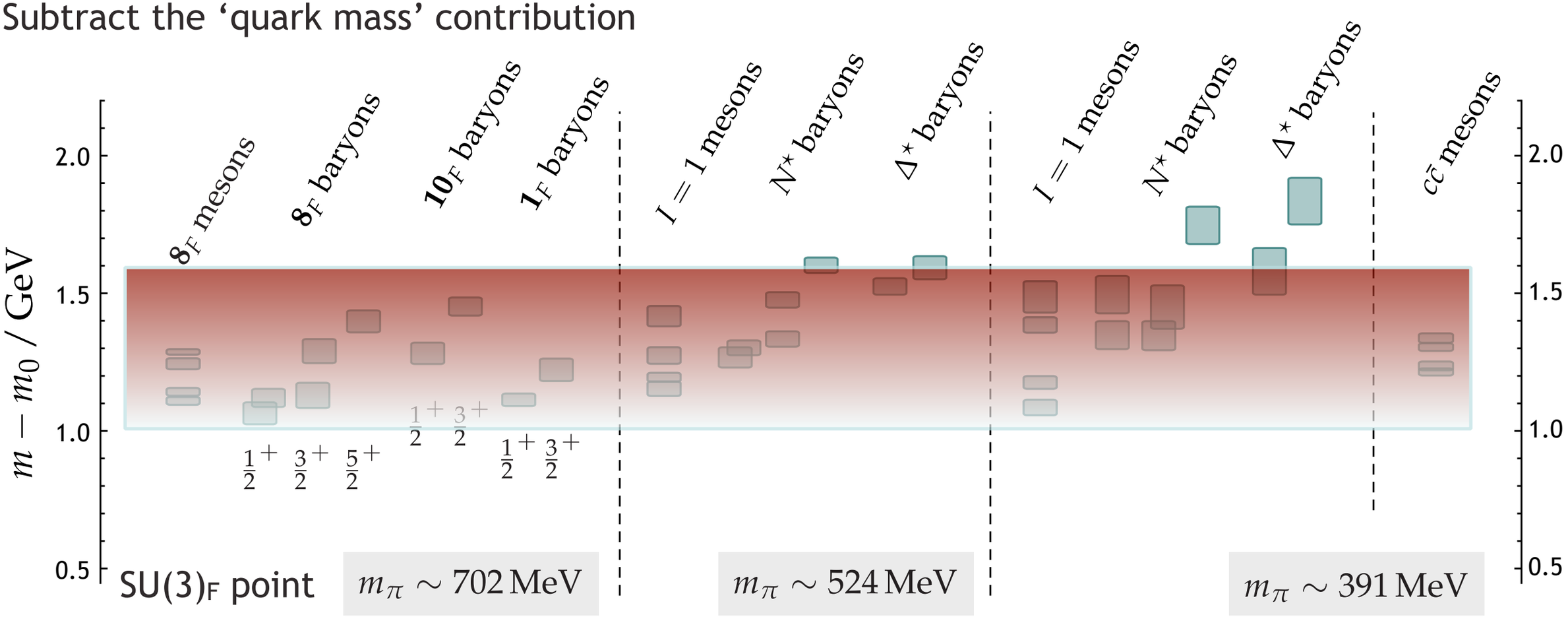

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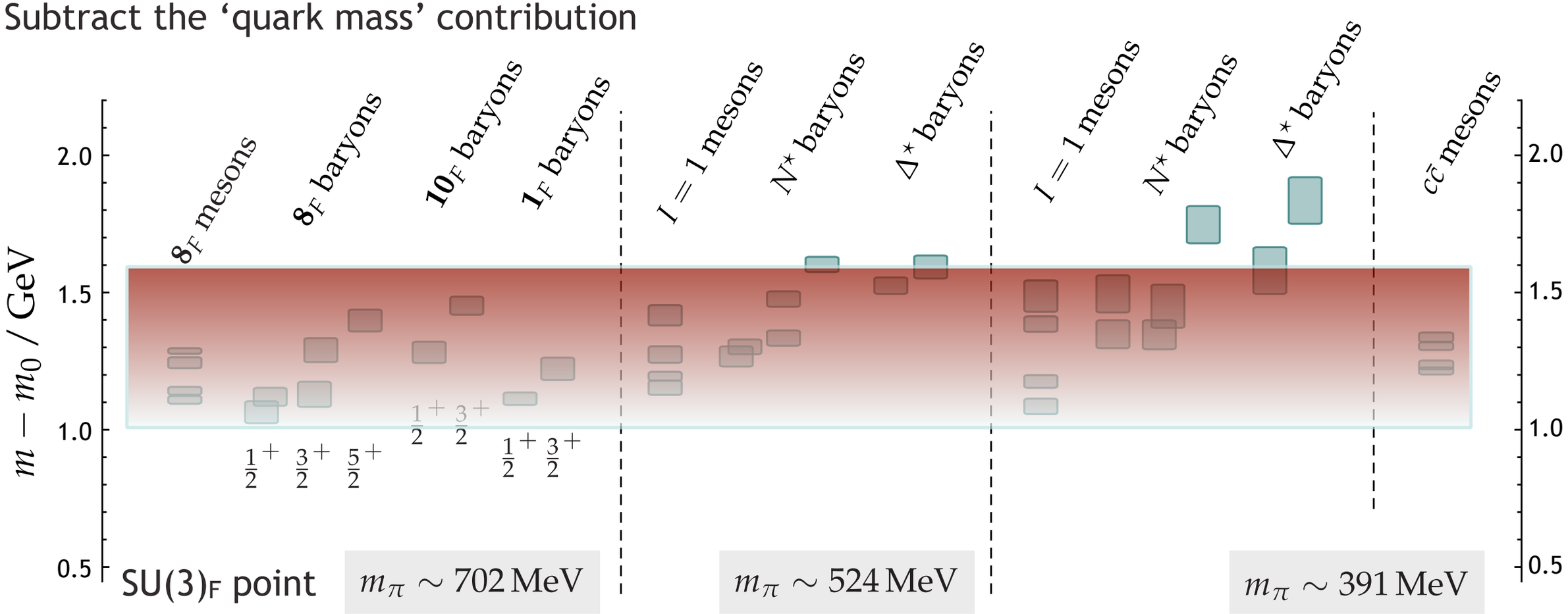
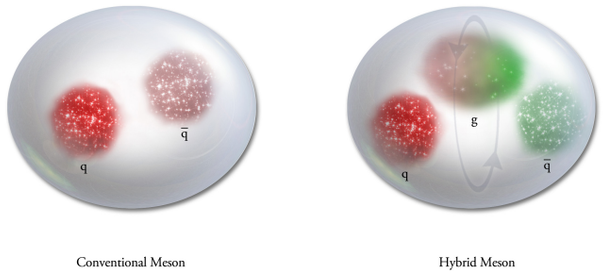


Pattern of states suggest gluonic excitations



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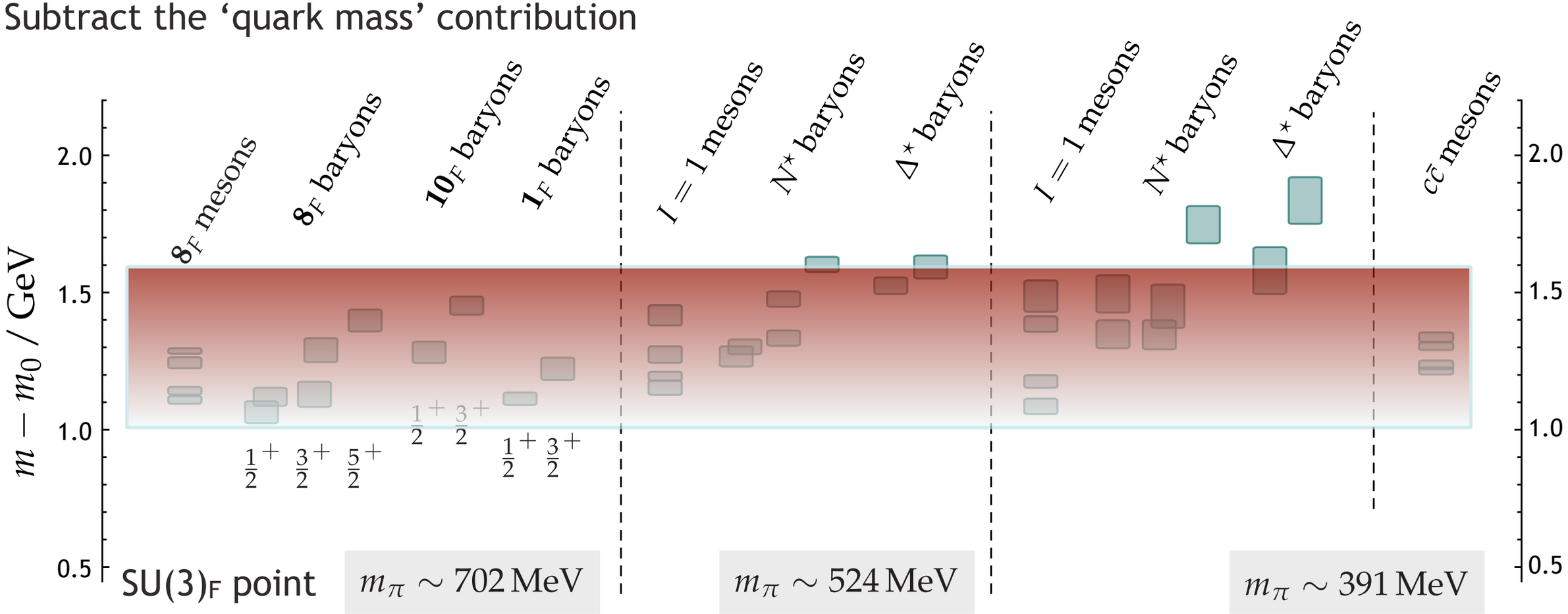
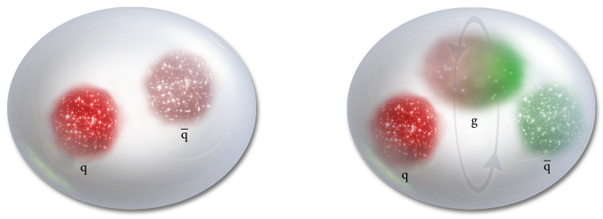
Chromo-magnetic excitation

- Subtract the ‘quark mass’ contribution



Pattern of states suggest gluonic excitations



Conventional Meson Hybrid Meson

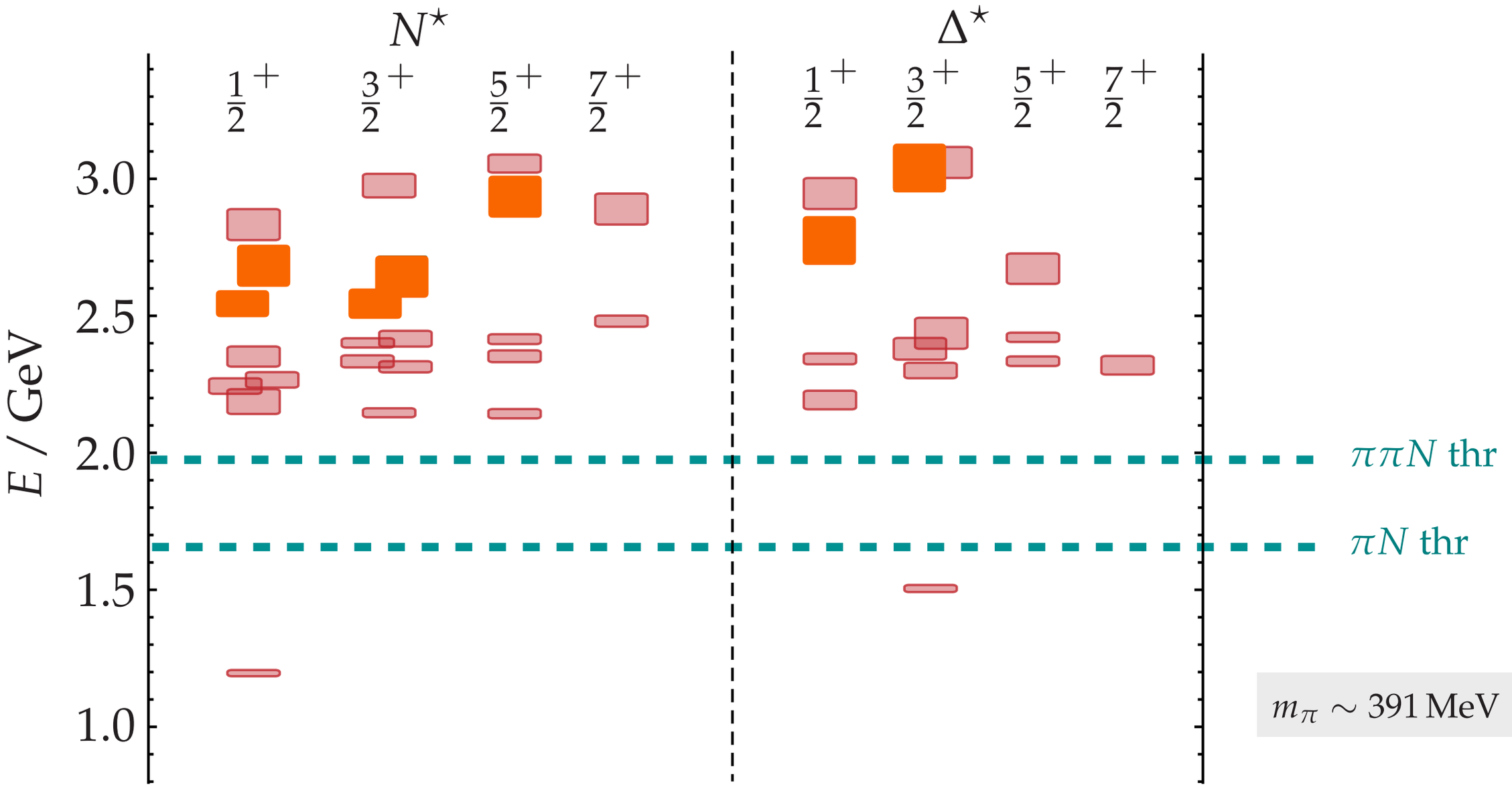
- Common energy scale of gluonic excitation $\sim 1.3\,\mathrm{GeV}$

➡ Need to know decay modes and rates to compare to expt.

HADRON SPECTRUM: PRD83 (2011); PRD88 (2013)

Excited states are resonances

- Initial determination of spectrum with only qqq style operators
  → missing scattering states

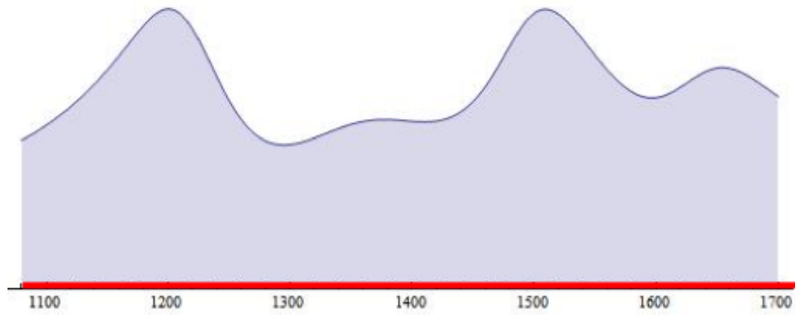


- Some initial results in S11 & P33 have appeared (Graz group)

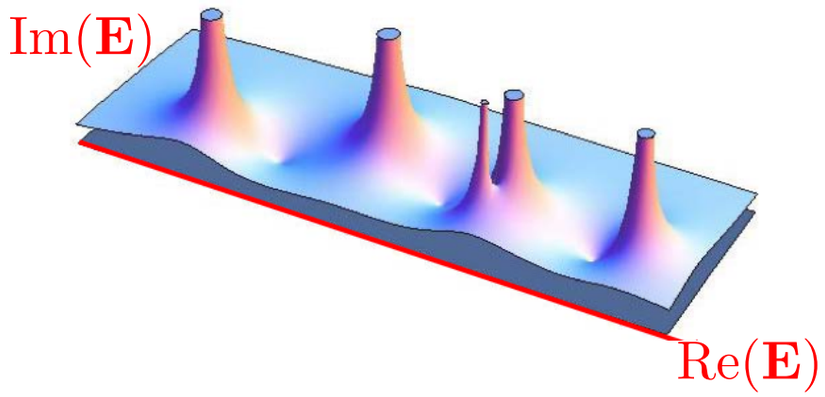
Resonances

Manifest as behavior of real scattering amplitudes

- E.g., πN πN



$\mathbf{E}$ (MeV)



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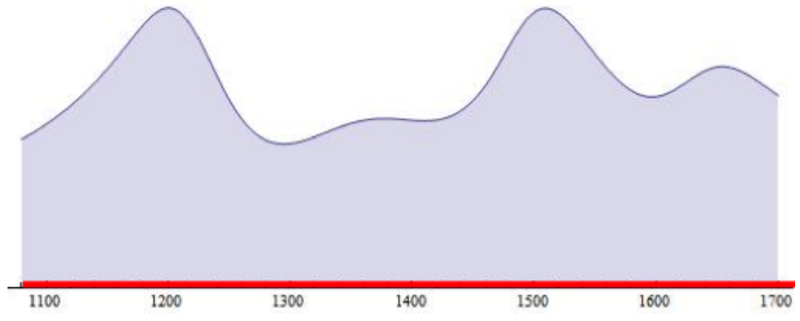

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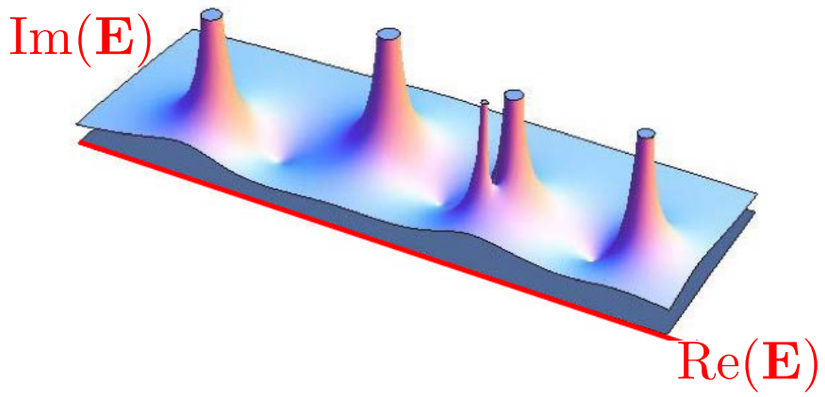
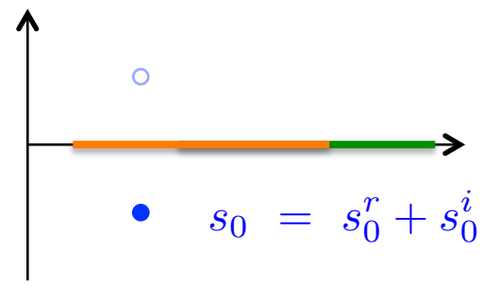


Formally defined as a pole in a partial-wave scattering amplitude

$$t_l(s) \sim \frac{R}{s_0 - s} + \ldots$$



Different channels should have same pole location

Pole structure (location and residue) gives decay information

QFT in a periodic cube

Lüscher (1986) : application to 3+1dim quantum field theories

subsequent extensions for moving frames, coupled-channel systems - will come back to this

quantization condition:

solutions, E_n, of

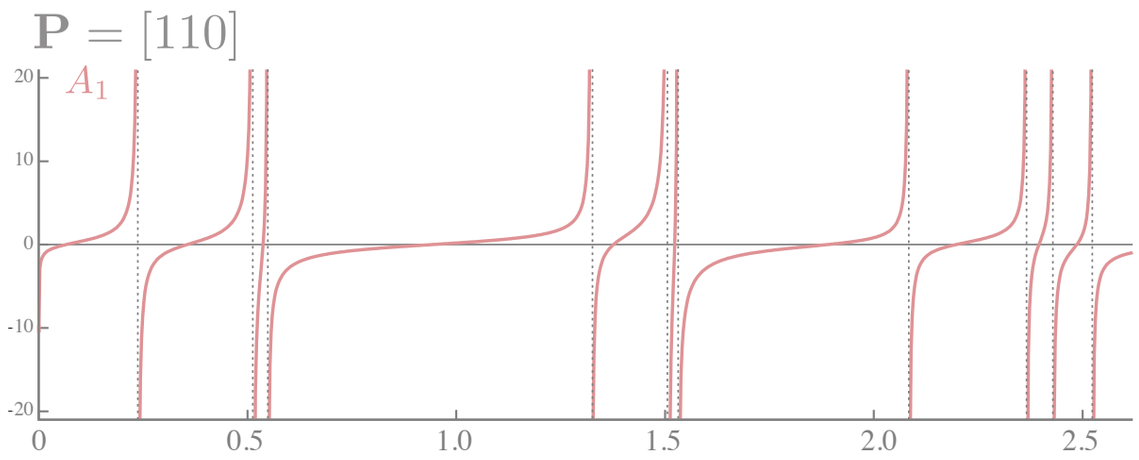
$$\det\left[\mathbf{1} + i\boldsymbol{\rho}\cdot\mathbf{t}\cdot(\mathbf{1}+i\boldsymbol{\mathcal{M}})\right] = 0$$

$\boldsymbol{\rho}(E)$ phase-space

$\mathbf{t}(E)$ scattering matrix $\quad\left(\mathbf{S} = \mathbf{1} + 2i\sqrt{\boldsymbol{\rho}}\cdot\mathbf{t}\cdot\sqrt{\boldsymbol{\rho}}\right)$

$\boldsymbol{\mathcal{M}}(E,L)$ finite-volume function

technicalities: partial-wave basis not ‘diagonal’ in a cube



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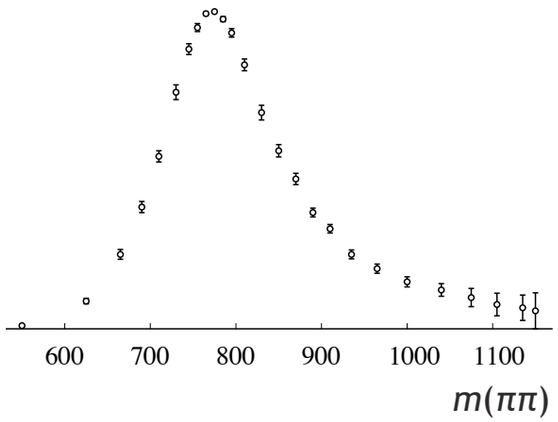
simplest case — elastic scattering of a single partial wave:

$$t^{(\ell)} = \frac{1}{\rho} e^{i\delta_\ell}\sin\delta_\ell \quad \Rightarrow \quad \cot\delta(E) = \mathcal{M}(E,L)$$

E_n value maps to $\delta(E_n)$

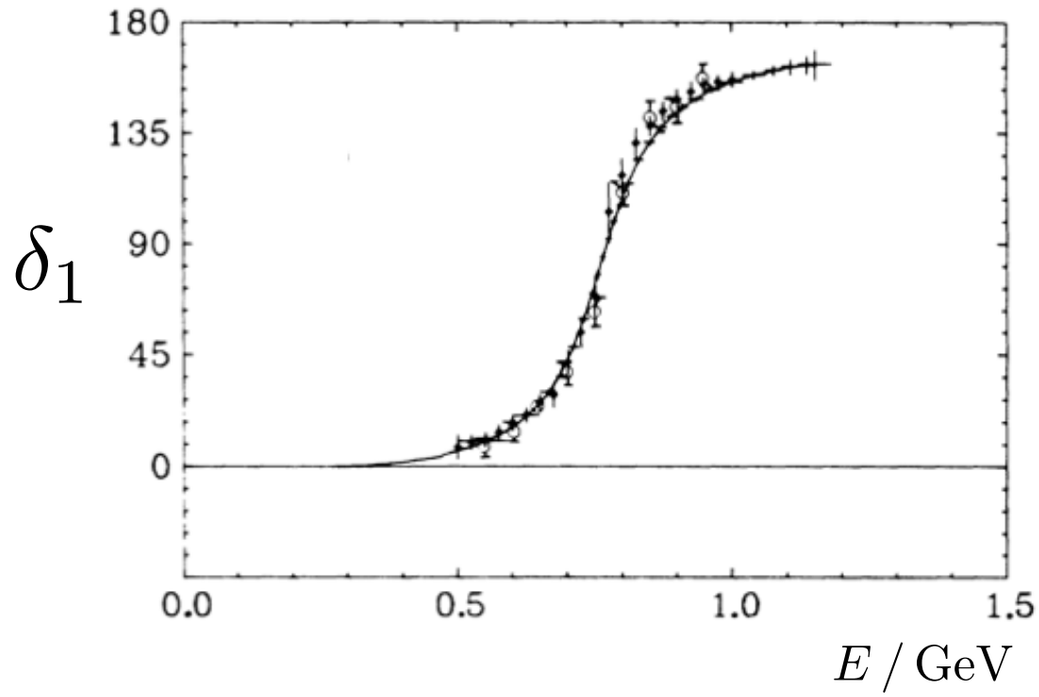
need to compute the spectrum …

An elastic resonance — the ρ in $\pi\pi$



canonical resonance ‘bump’
described by a rapidly rising phase-shift

scattering phase-shift



PHYSICAL REVIEW D VOLUME 7, NUMBER 5 1 MARCH 1973

$\pi\pi$ Partial-Wave Analysis from Reactions $\pi^+p \to \pi^+\pi^-\Delta^{++}$ and $\pi^+p \to K^+K^-\Delta^{++}$ at 7.1 GeV/c†

S. D. Protopopescu,* M. Alston-Garnjost, A. Barbaro-Galtieri, S. M. Flatté,‡
J. H. Friedman,§ T. A. Lasinski, G. R. Lynch, M. S. Rabin,‖ and F. T. Solmitz
Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720
(Received 25 September 1972)

Lattice QCD spectrum

Variational analysis of a matrix of correlation functions $C_{ij}(t) = \left\langle 0 \middle| \mathcal{O}_i(t) \mathcal{O}_j(0) \middle| 0 \right\rangle$

$$= \sum_{\mathfrak{n}} e^{-E_{\mathfrak{n}} t} \left\langle 0 \middle| \mathcal{O}_i \middle| \mathfrak{n} \right\rangle \left\langle \mathfrak{n} \middle| \mathcal{O}_j \middle| 0 \right\rangle$$

operator basis: ‘single-meson’ + ‘meson-meson’

$$\overline{\psi} \mathbf{\Gamma} \psi \qquad \sum_{\hat{\mathbf{p}}_1, \hat{\mathbf{p}}_2} C(\mathbf{p}_1, \mathbf{p}_2; \mathbf{p})\, M_1(\mathbf{p}_1)\, M_2(\mathbf{p}_2)$$

(& tetraquark & …)

$$\mathbf{p} = \tfrac{2\pi}{L} [n_x, n_y, n_z]$$

maximum momentum guided by non-interacting energies

$$\sqrt{m_1^2 + \mathbf{p}_1^2} + \sqrt{m_2^2 + \mathbf{p}_2^2}$$

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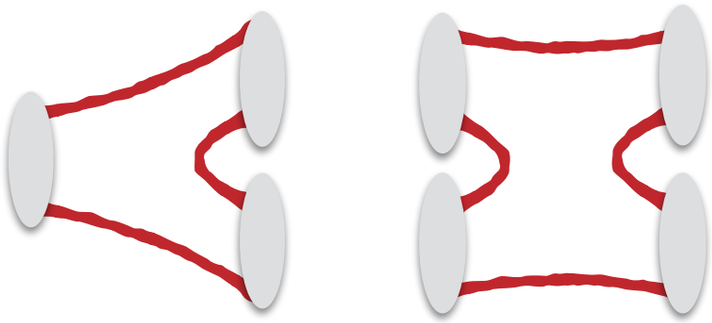
now need to evaluate diagrams like



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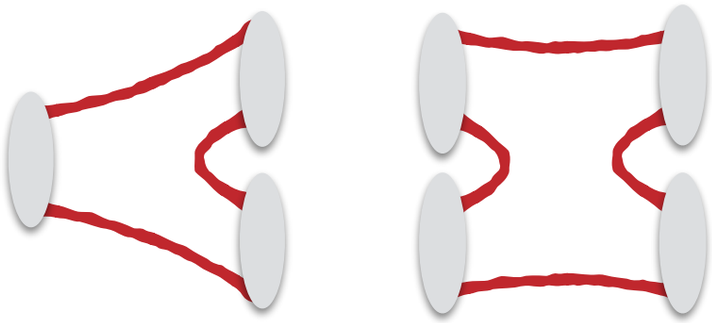

Distillation handles quark annihilation lines

Linear ops from KNLs+GPUs

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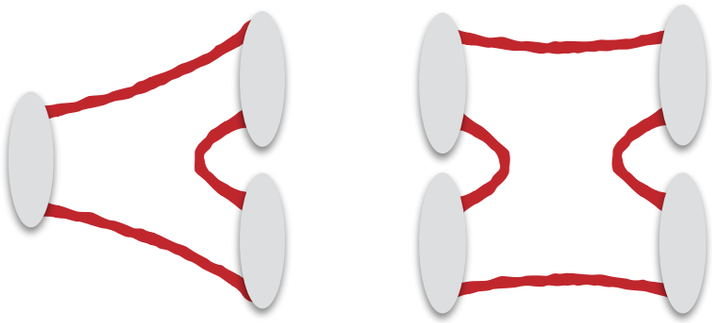

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`Don’t underestimate the power…’

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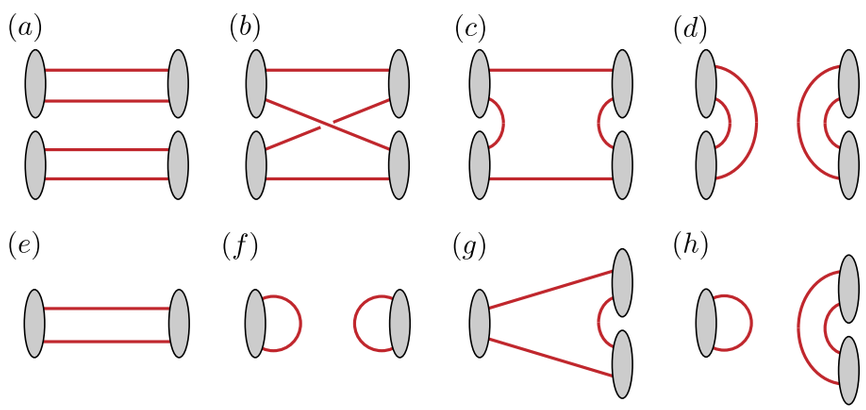
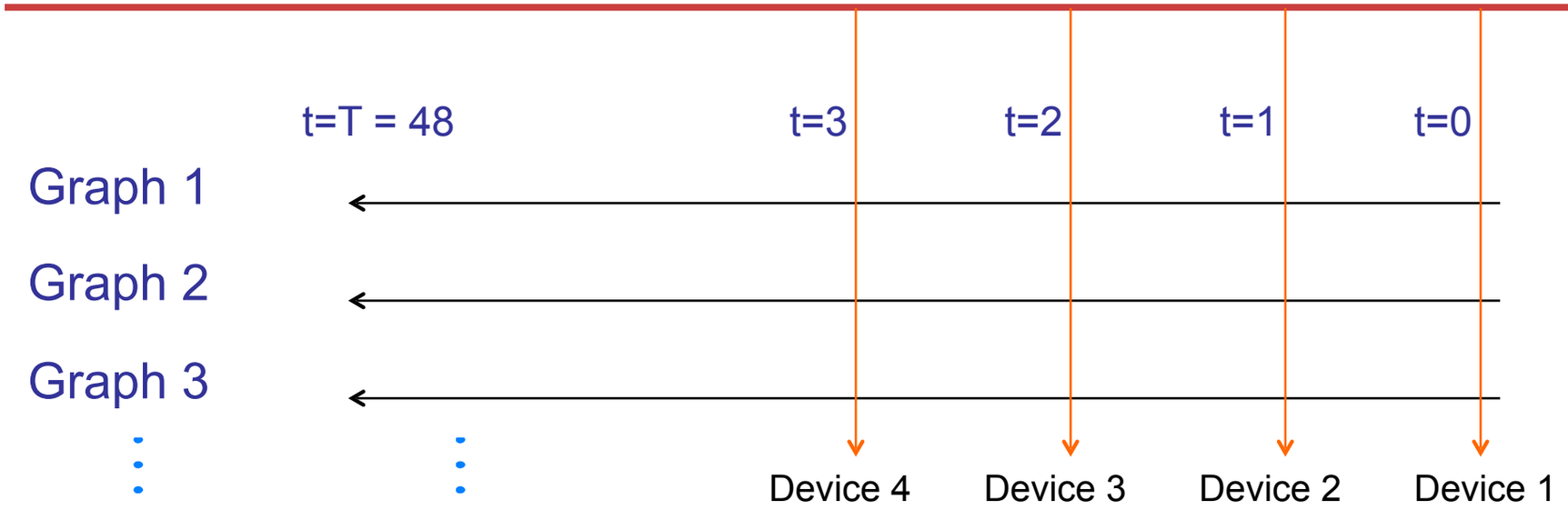
Can be lots of Wick contractions, and momentum projections

Worst case: rest-frame —> p=100 -> 6x, p=110 -> 12x, p=111 -> 8x



Optimize order of operations

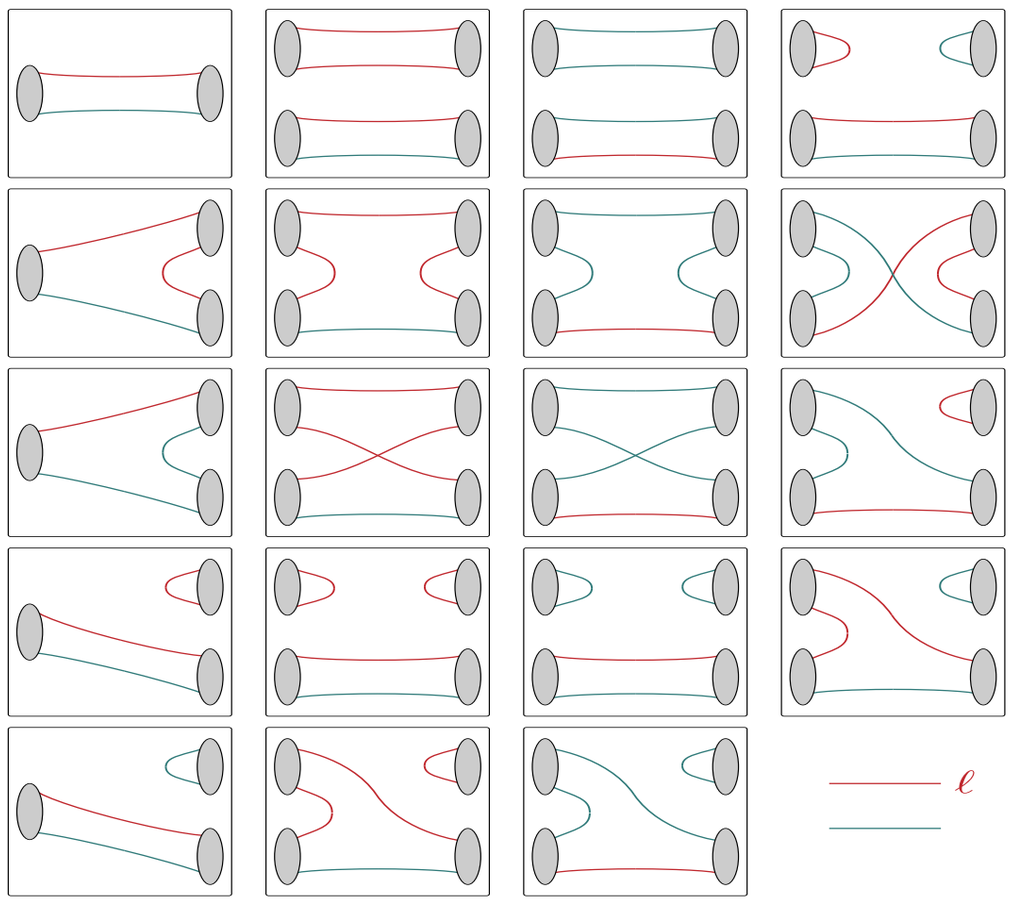


$$\mathrm{tr}\left(A(t) * B(t) * C(0) * D(0)\right)$$

$$\mathrm{tr}\left(A(t) * B(t) * E(0) * F(0)\right)$$

$$\mathrm{tr}\left(A(t) * B(t) * G(0)\right)$$

- Traverse graphs along a t-slice
  - 10,000's of graphs
- 3-particles and more…
- Common sub-expression elimination
- For fixed t-slice - 100's vertices



I=1/2 K*π arXiv:1406.4158

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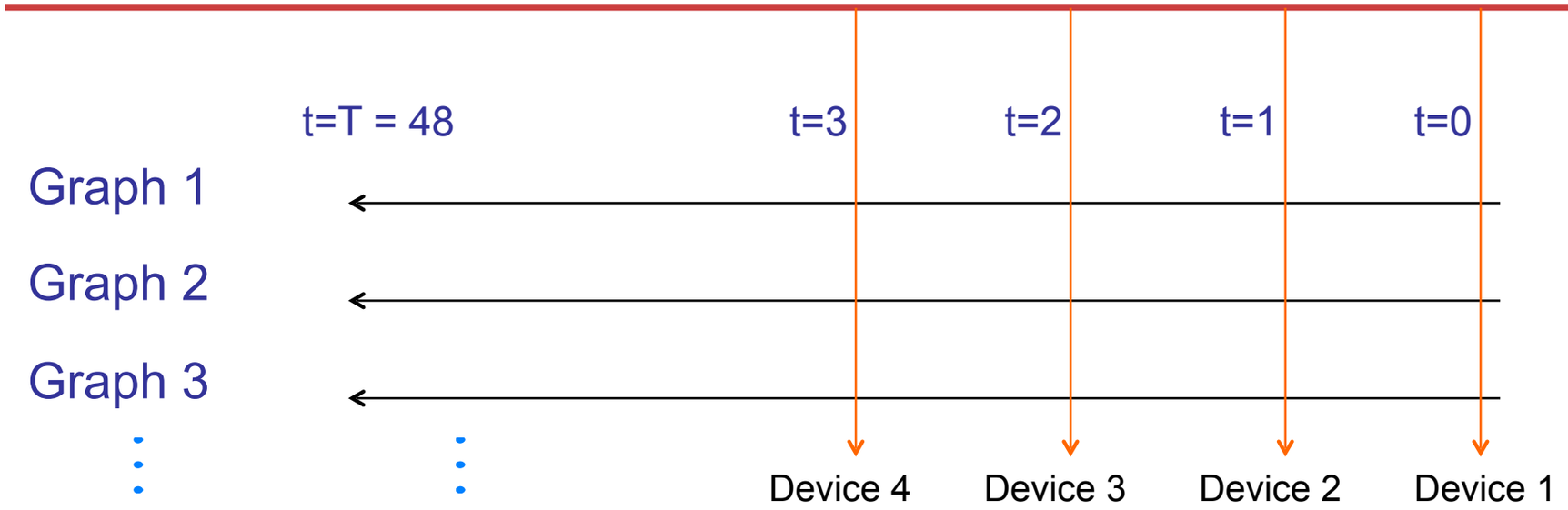

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  - 10,000's of graphs
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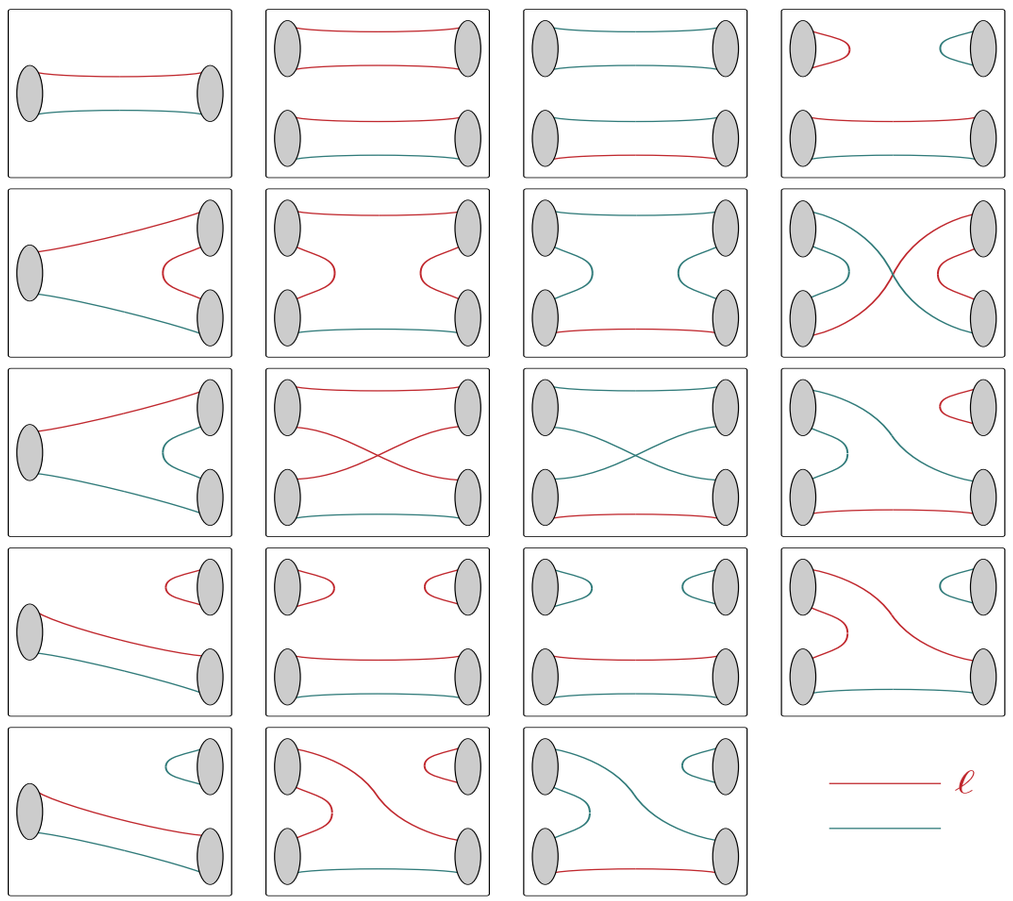
Solutions:

Equivalent graph identification

Dijkstra's method - optimal walk through space of evaluation order



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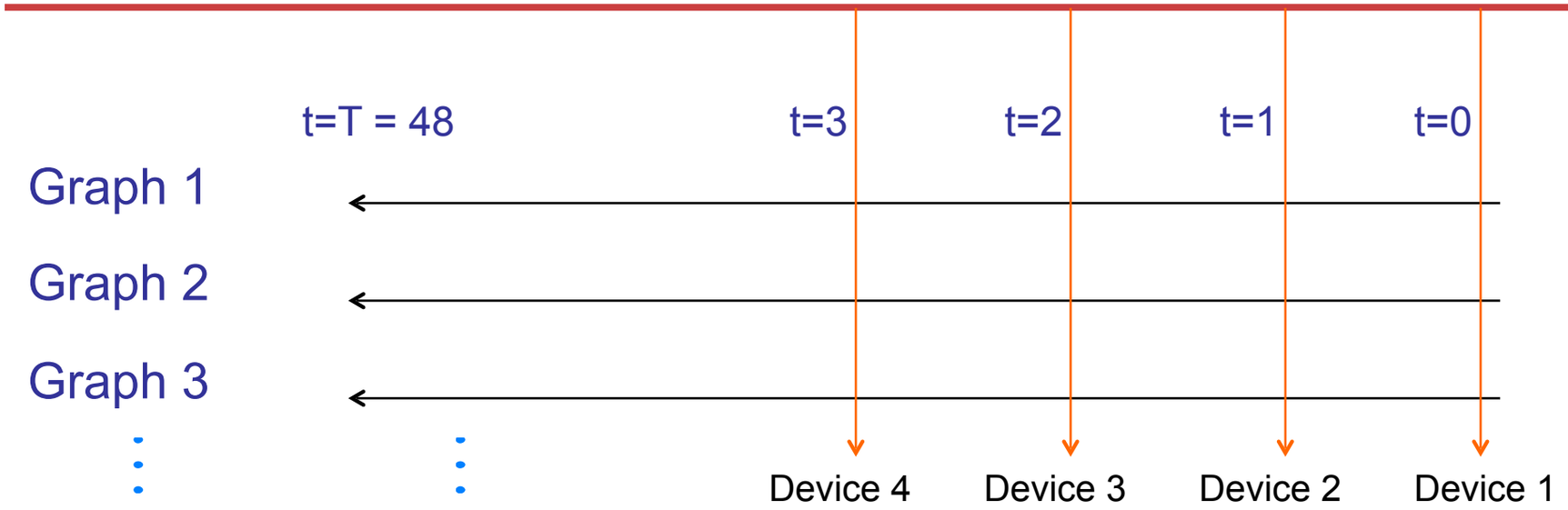
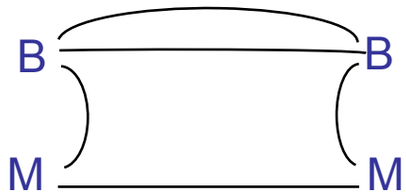
Workflow is choreographed



- Topology
  - Vertices can have different ordinalities



- Order of evaluation is important
  - Baryons ~ $O(N^4)$, Mesons ~ $O(N^3)$

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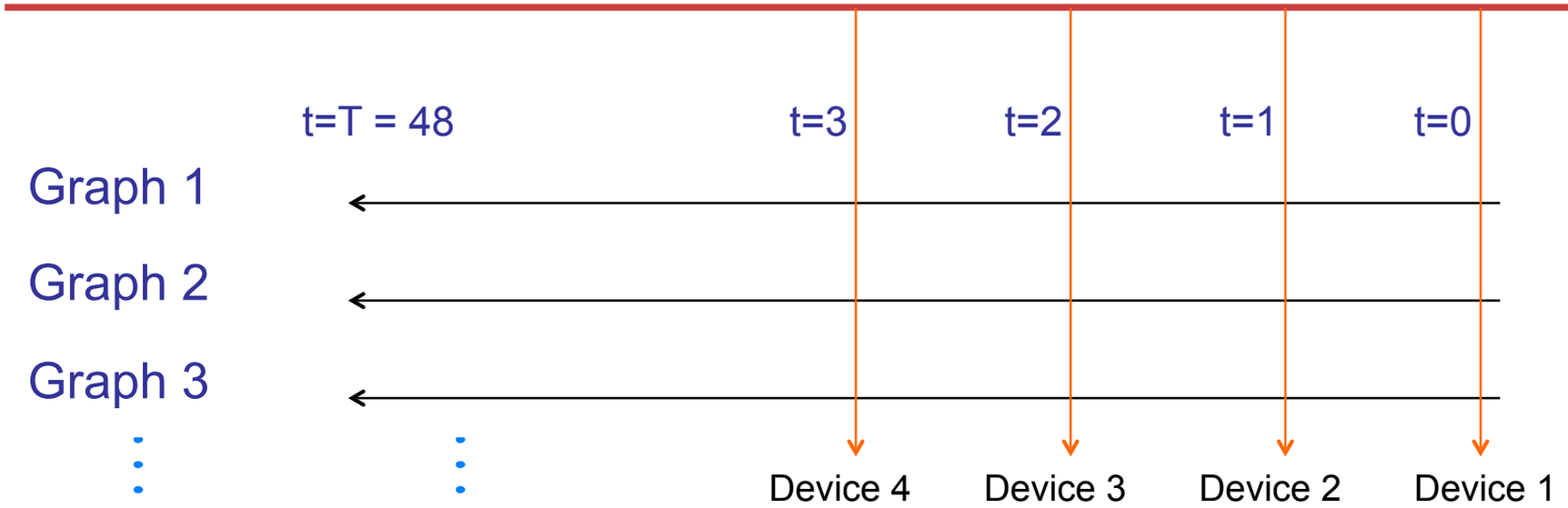

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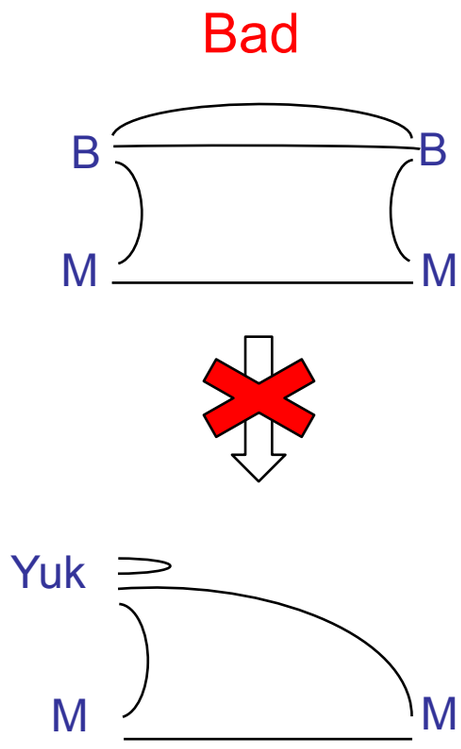

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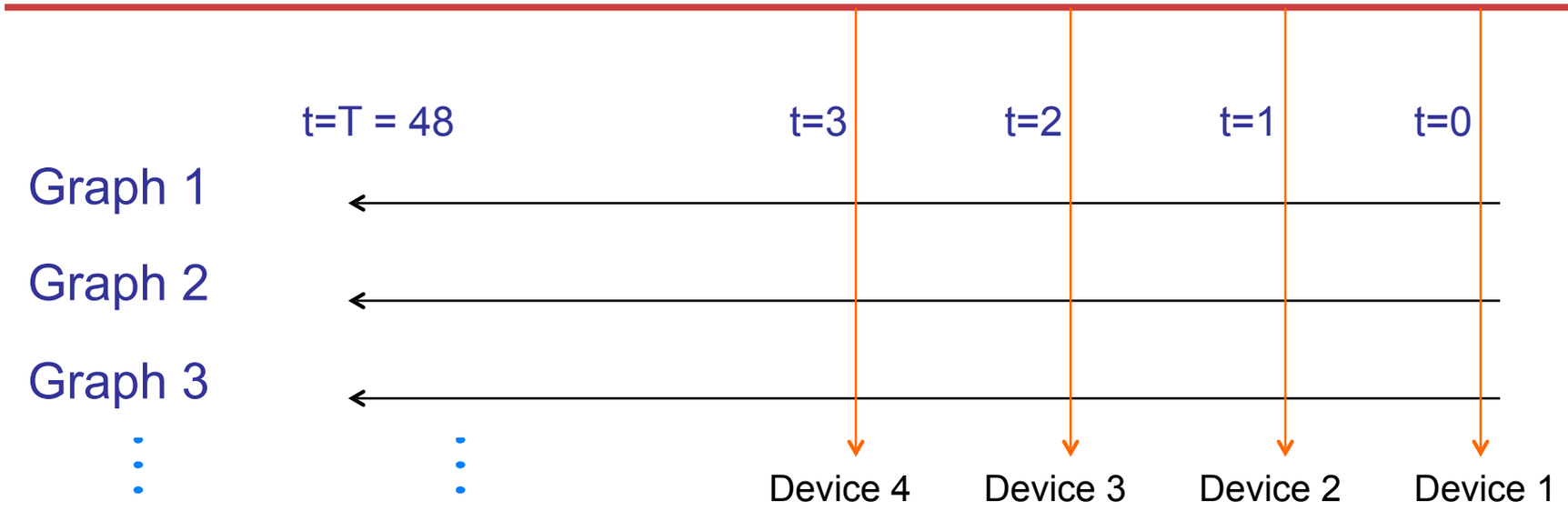

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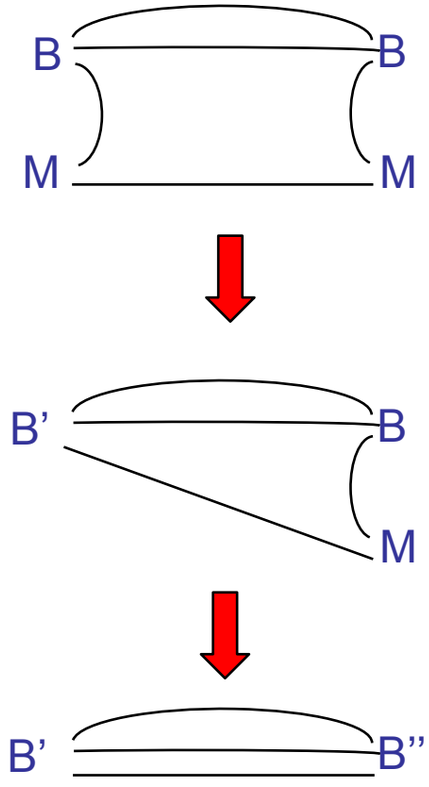
- Lots of mat-muls
- Obvious application for accelerators

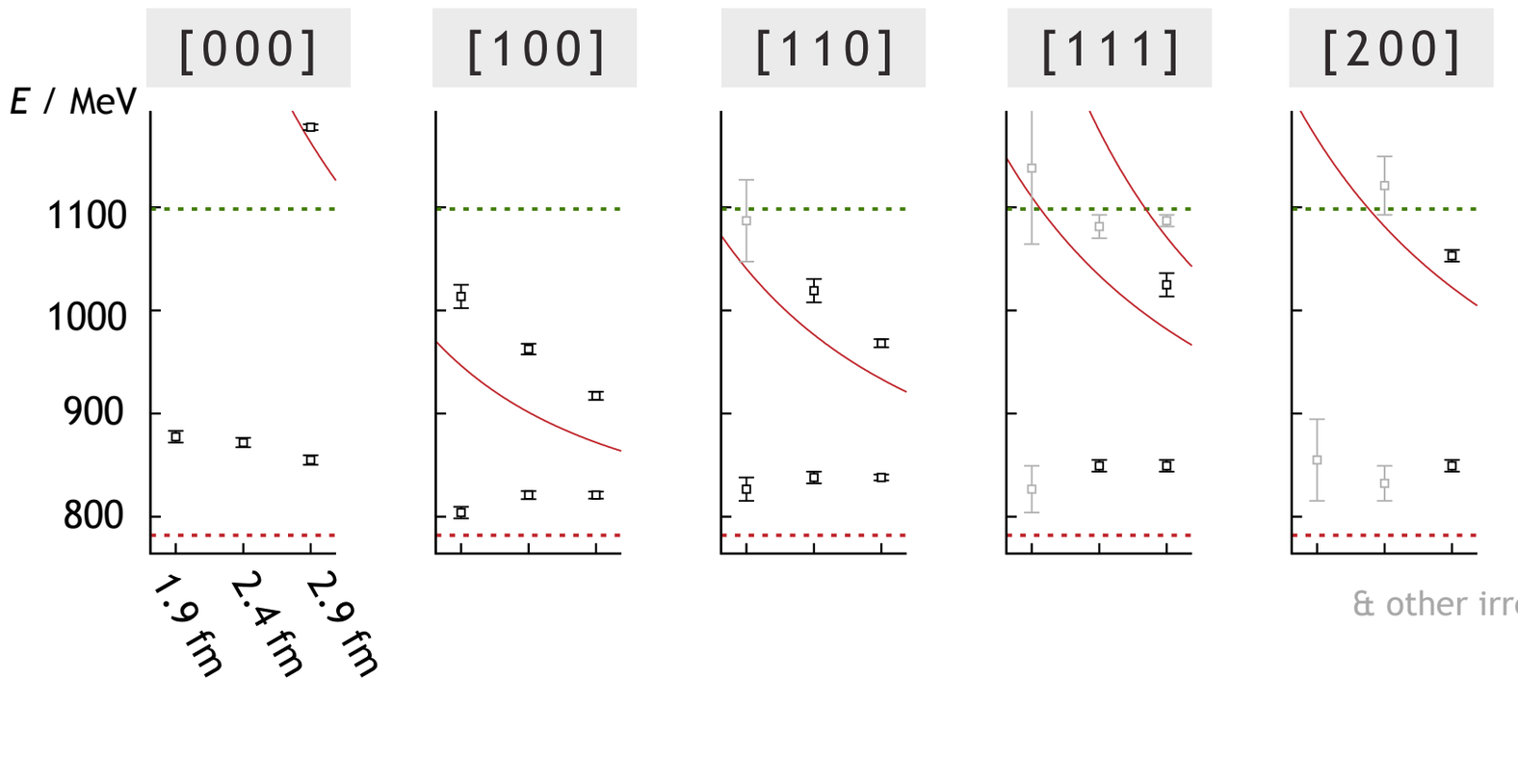
Good



An elastic resonance — the ρ in $\pi\pi$ — lattice QCD

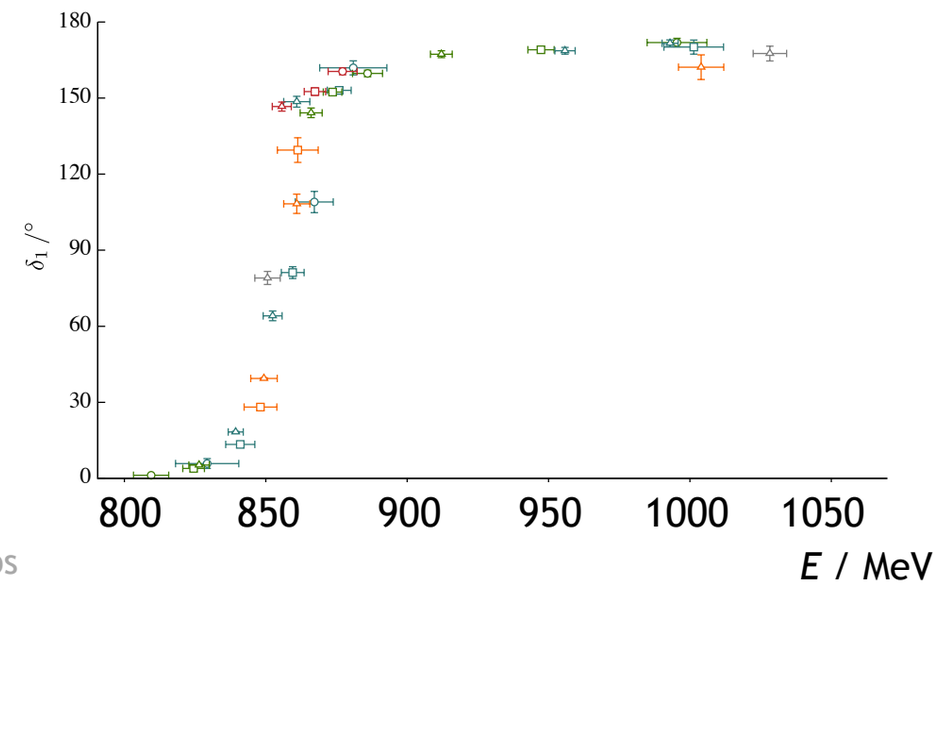
PRD87 034505 (2013)

m_π ~ 391 MeV



& other irreps

scattering phase-shift



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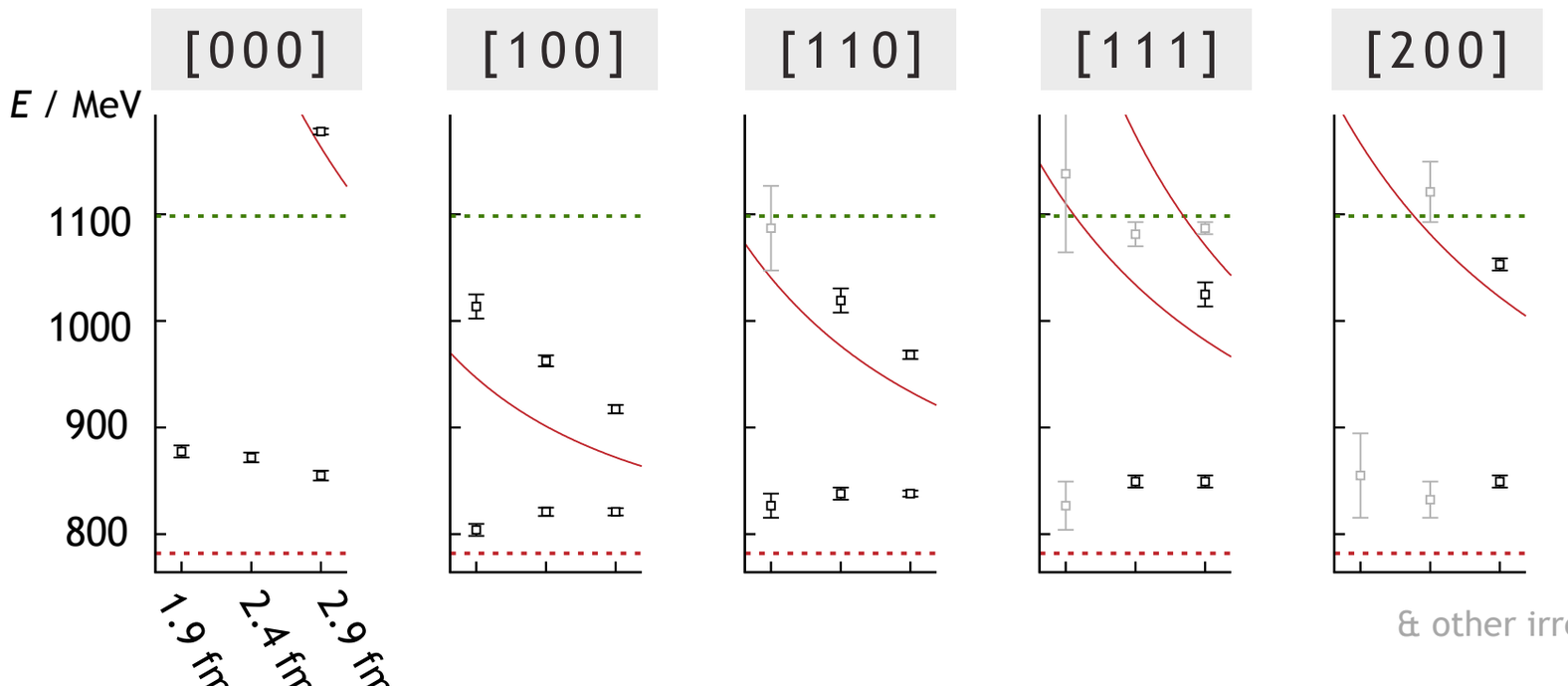

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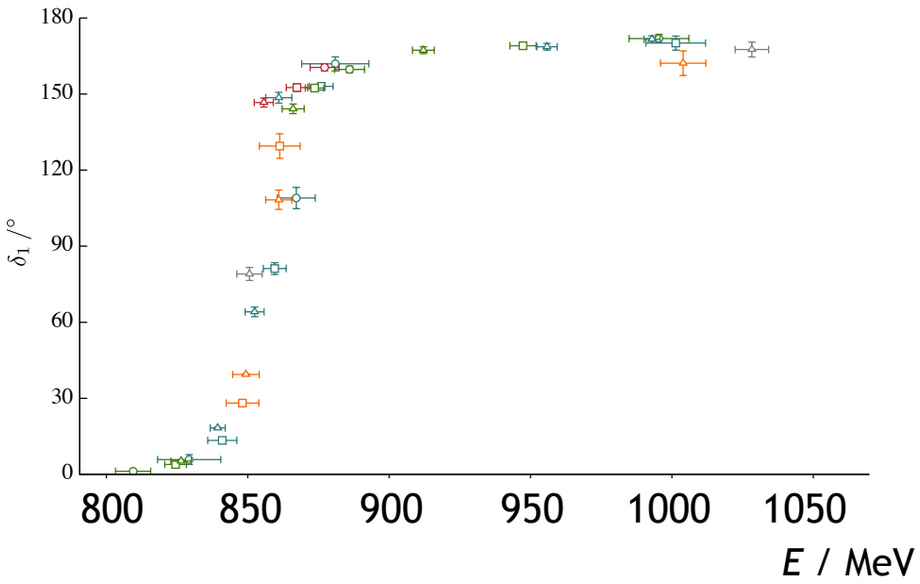
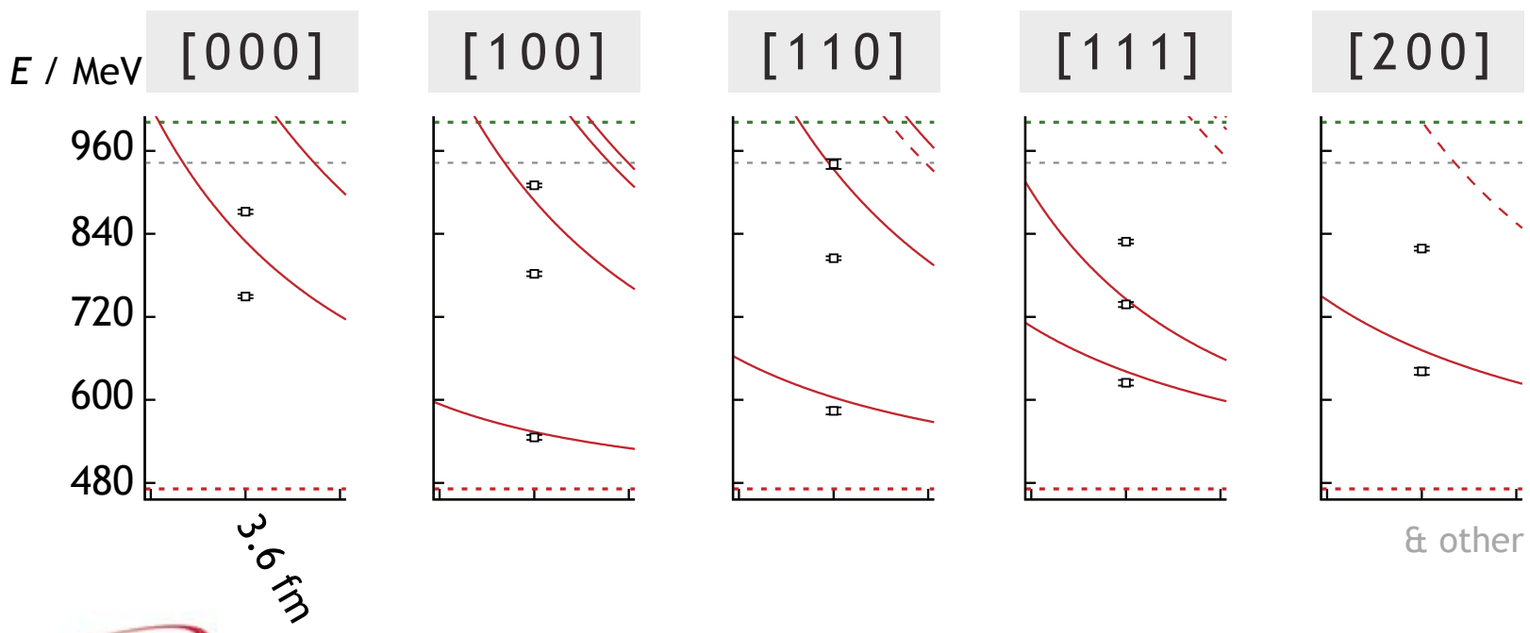


PRD92 094502 (2015) m_π ~ 236 MeV



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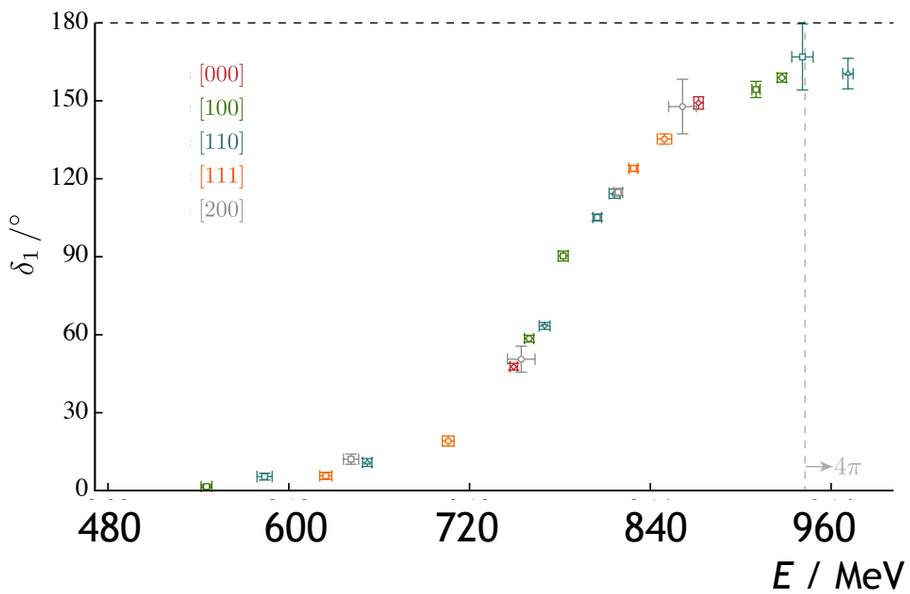

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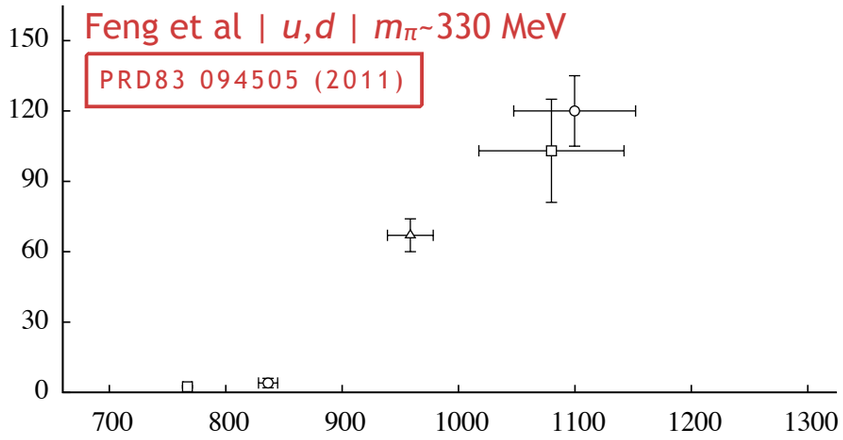
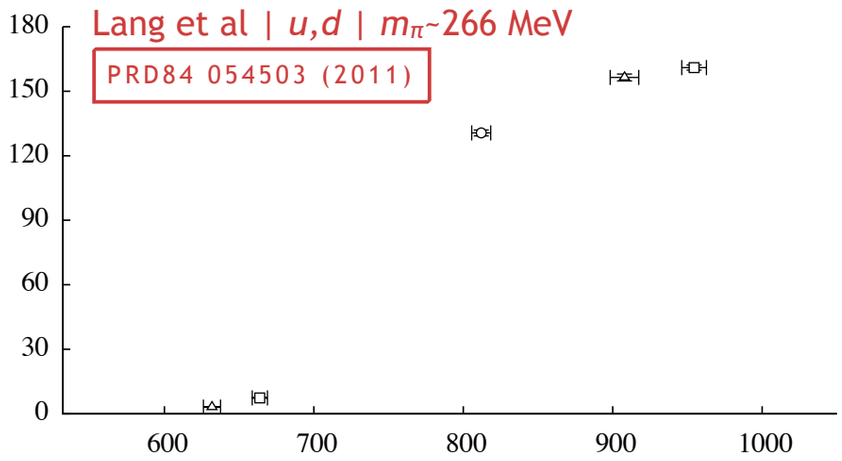
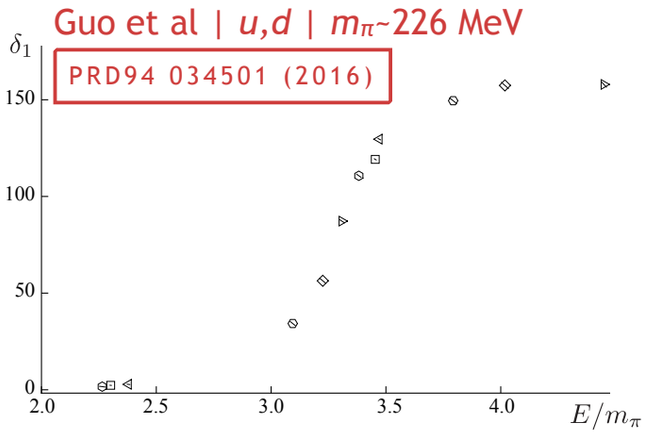
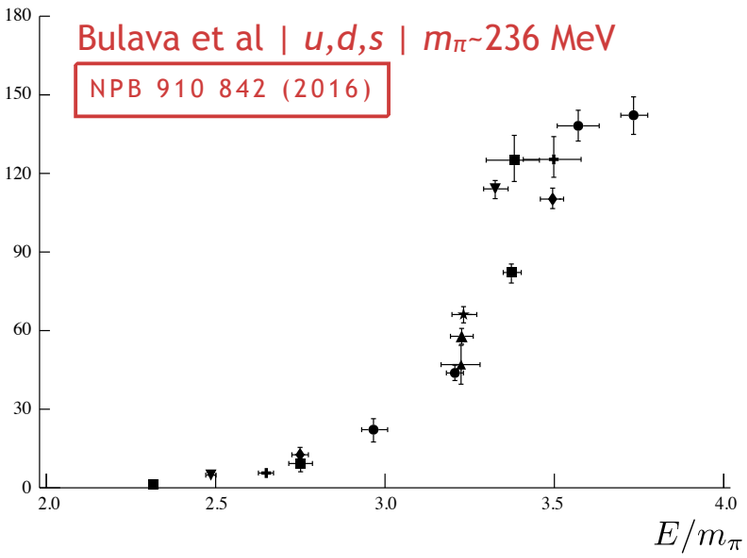
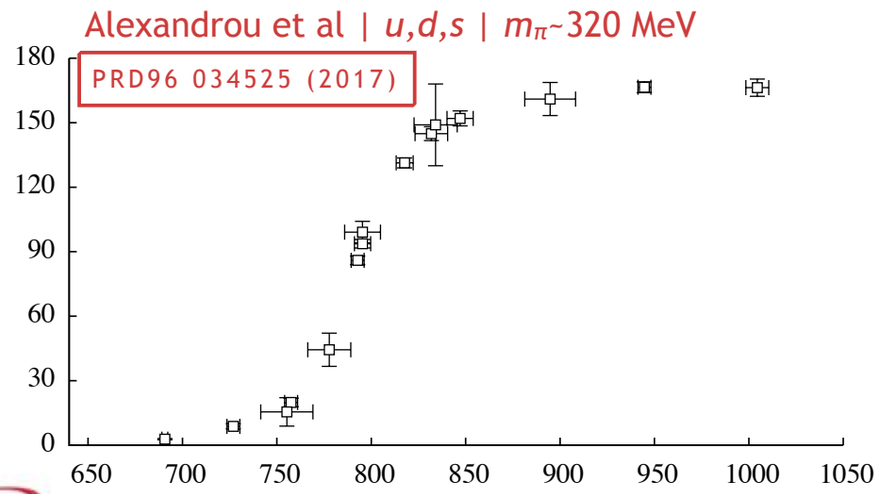
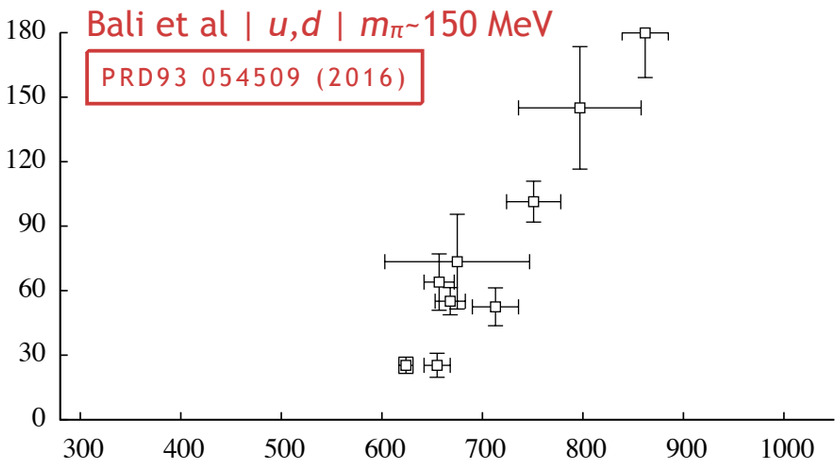












Coupled-channel resonances

Most resonances decay into more than one final state

e.g. two-channel scattering described by a ***t***-matrix $\mathbf{t}(E) = \begin{pmatrix} t_{11}(E) & t_{12}(E) \\ t_{21}(E) & t_{22}(E) \end{pmatrix}$

Finite-volume spectrum as a function of scattering becomes more complicated

coupled-channel spectrum

solutions $E_n(L)$ of

$$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot\boldsymbol{t}(E)\cdot(\mathbf{1} + i\boldsymbol{\mathcal{M}}(E,L))\right] = 0$$

No longer a one-to-one mapping from energy to scattering …

Coupled-channels

solutions, E_n, of

$$\det\left[\mathbf{1} + i\boldsymbol{\rho}\cdot\mathbf{t}\cdot(\mathbf{1}+i\boldsymbol{\mathcal{M}})\right] = 0$$

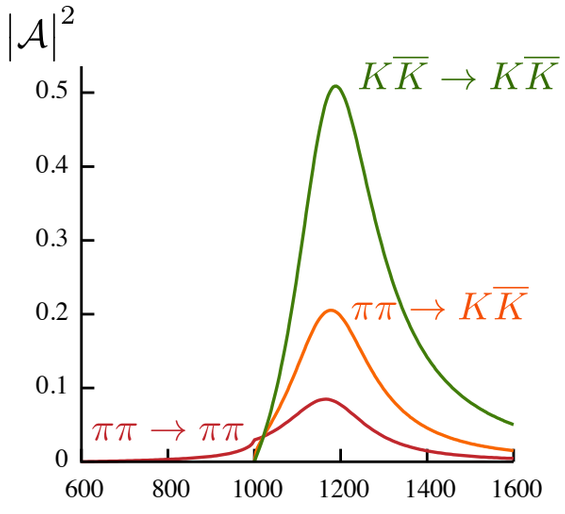
$$\mathbf{t} = \begin{pmatrix} \pi\pi\to\pi\pi & \pi\pi\to K\bar{K} \\ K\bar{K}\to\pi\pi & K\bar{K}\to K\bar{K} \end{pmatrix}$$

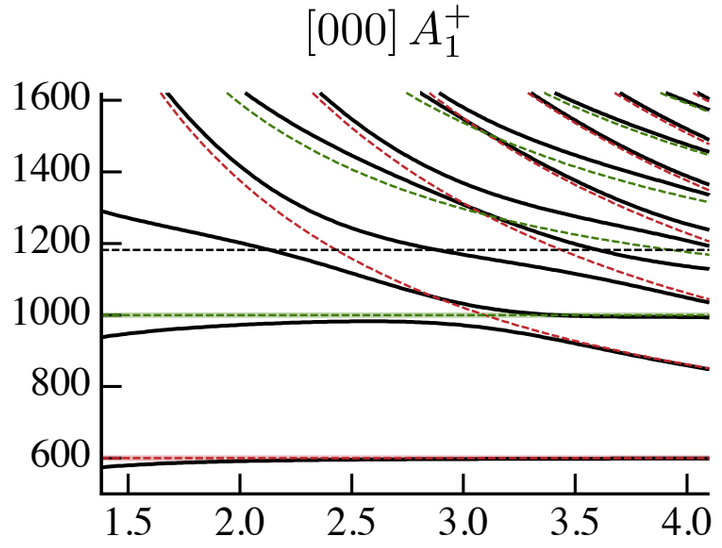
$\boldsymbol{\rho}(E)$ phase-space

$\mathbf{t}(E)$ scattering matrix

$\boldsymbol{\mathcal{M}}(E,L)$ finite-volume function

for a known $\boldsymbol{t}$(E) → can predict E_n





Coupled-channels

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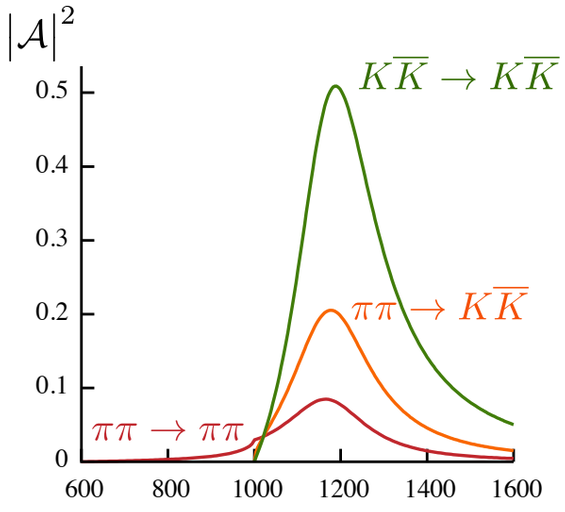
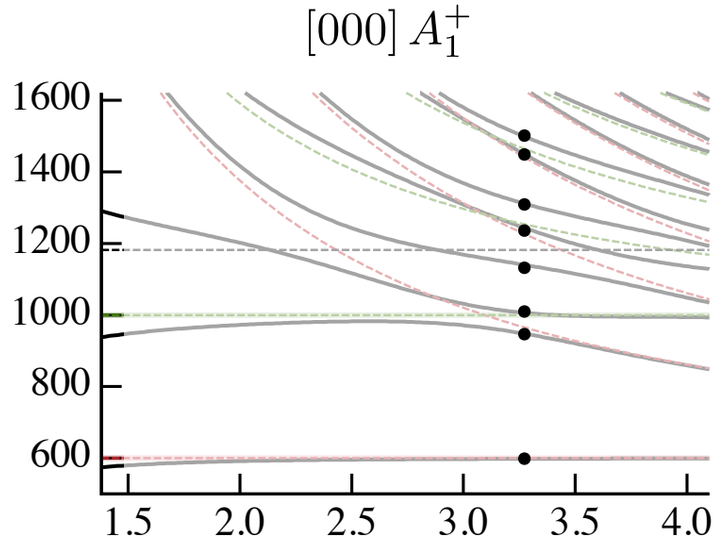


?



but how do we perform the inverse mapping ?

Coupled-channels

solutions, E_n, of

$$\det\Big[\mathbf{1} + i\boldsymbol{\rho}\cdot\mathbf{t}\cdot(\mathbf{1}+i\boldsymbol{\mathcal{M}})\Big] = 0$$

$\boldsymbol{\rho}(E)$ phase-space

$\mathbf{t}(E)$ scattering matrix

$\boldsymbol{\mathcal{M}}(E,L)$ finite-volume function

parameterize the energy dependence of the ***t*-**matrix

***K*-**matrix is a convenient approach (manifest unitarity)

$$\mathrm{Im}\, t_{ij}(s) = -\delta_{ij}\,\rho_i(s)\,\Theta\big(s - s^{(i)}_{\mathrm{thr.}}\big)$$

$\mathbf{t}^{-1}(E) = \mathbf{K}^{-1}(E) - i\boldsymbol{\rho}(E)$ where ***K*-**matrix is “any” real matrix

Coupled-channels

solutions, E_n, of

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$\boldsymbol{\rho}(E)$ phase-space

$\mathbf{t}(E)$ scattering matrix

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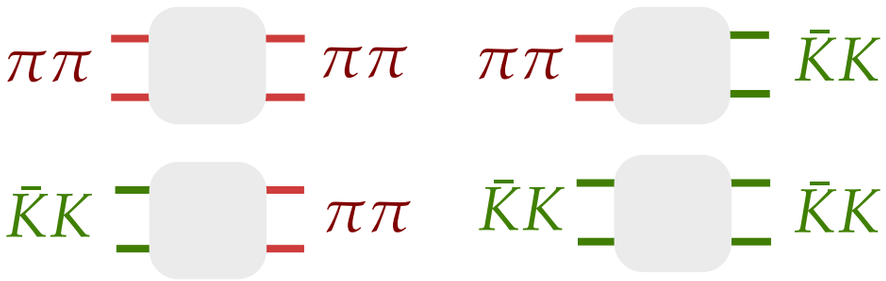
Want pole mass and residues/couplings of t-matrix

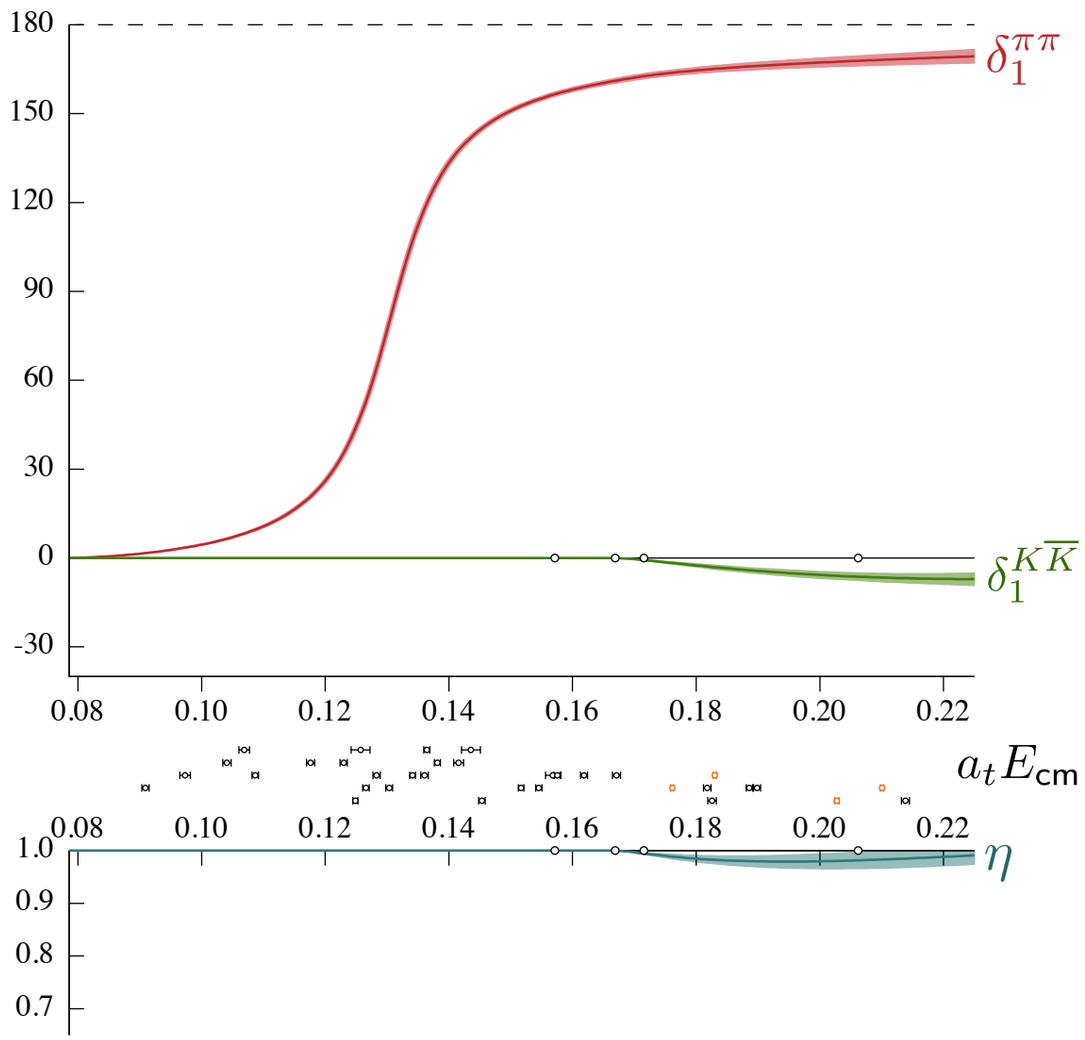
In recent years, progress towards establishing this approach

ρ resonance as a coupled channel system

PRD87 034505 (2013)

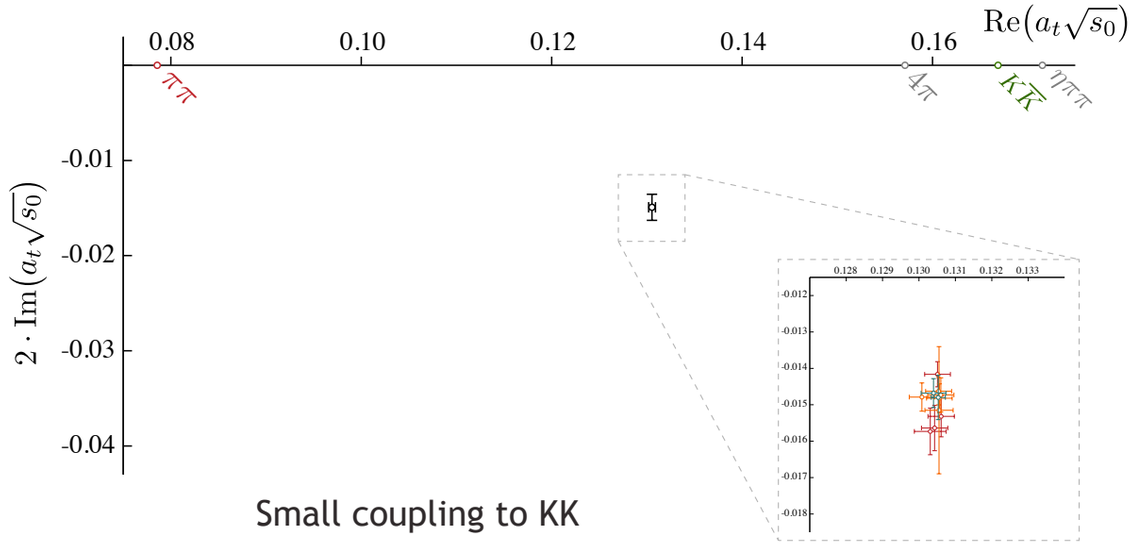


Phase shifts & inelasticity



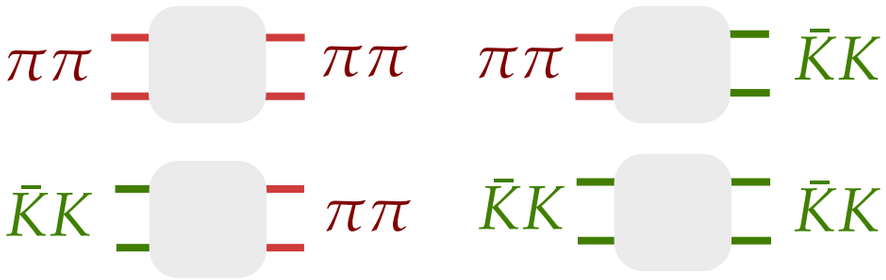
$m_\pi \sim 236$ MeV

t-matrix pole location

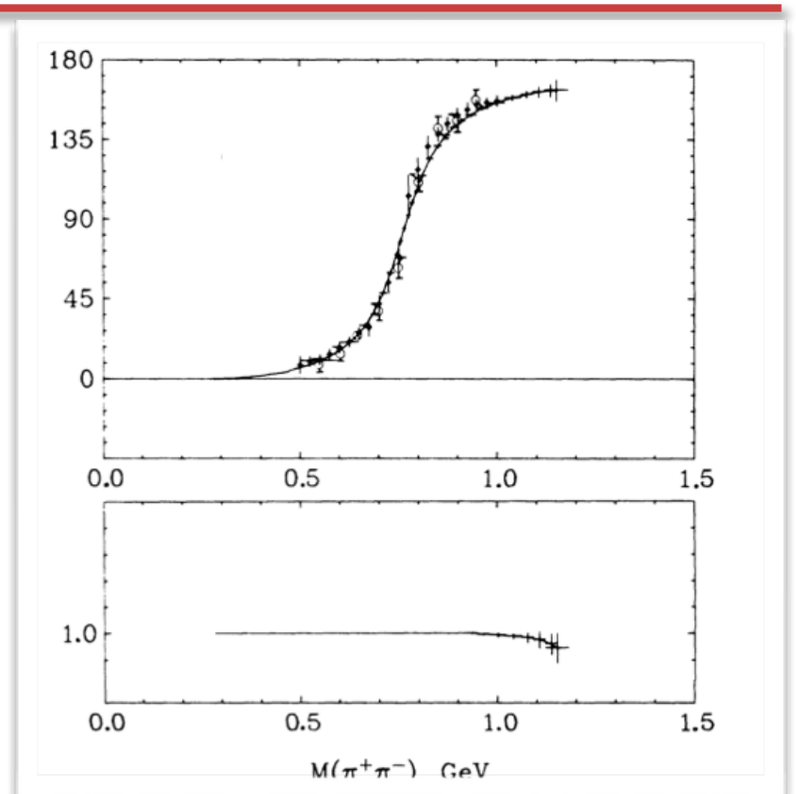


Small coupling to KK

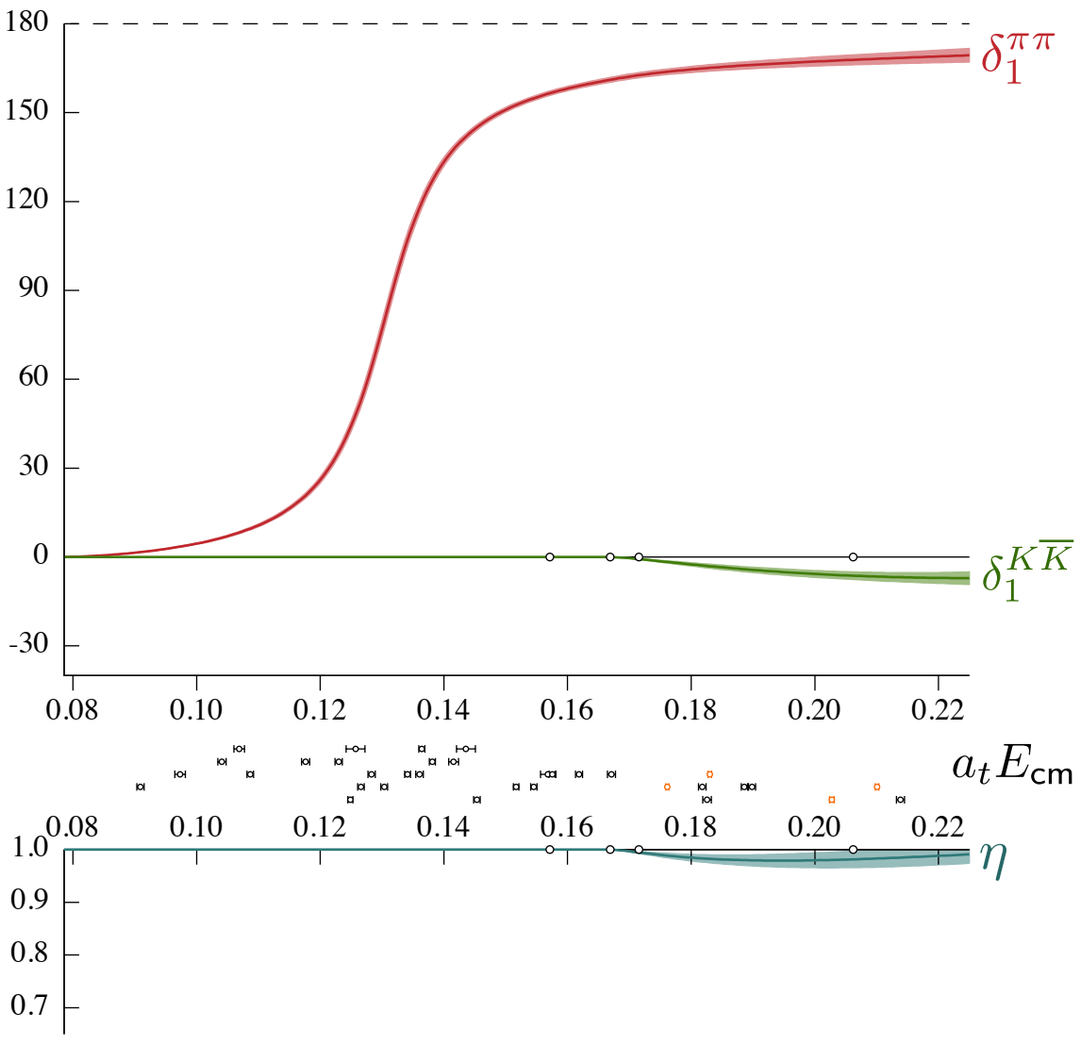
ρ resonance as a coupled channel system



PRD87 034505 (2013)



Phase shifts & inelasticity



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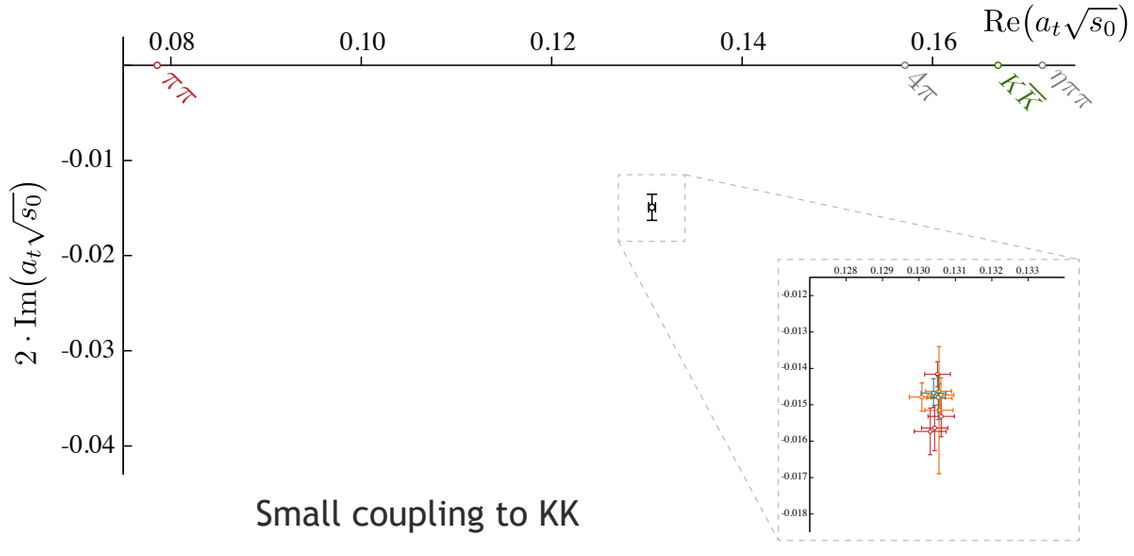


Small coupling to KK

Side comment: four-particle effects

PRD95 074510 (2017)

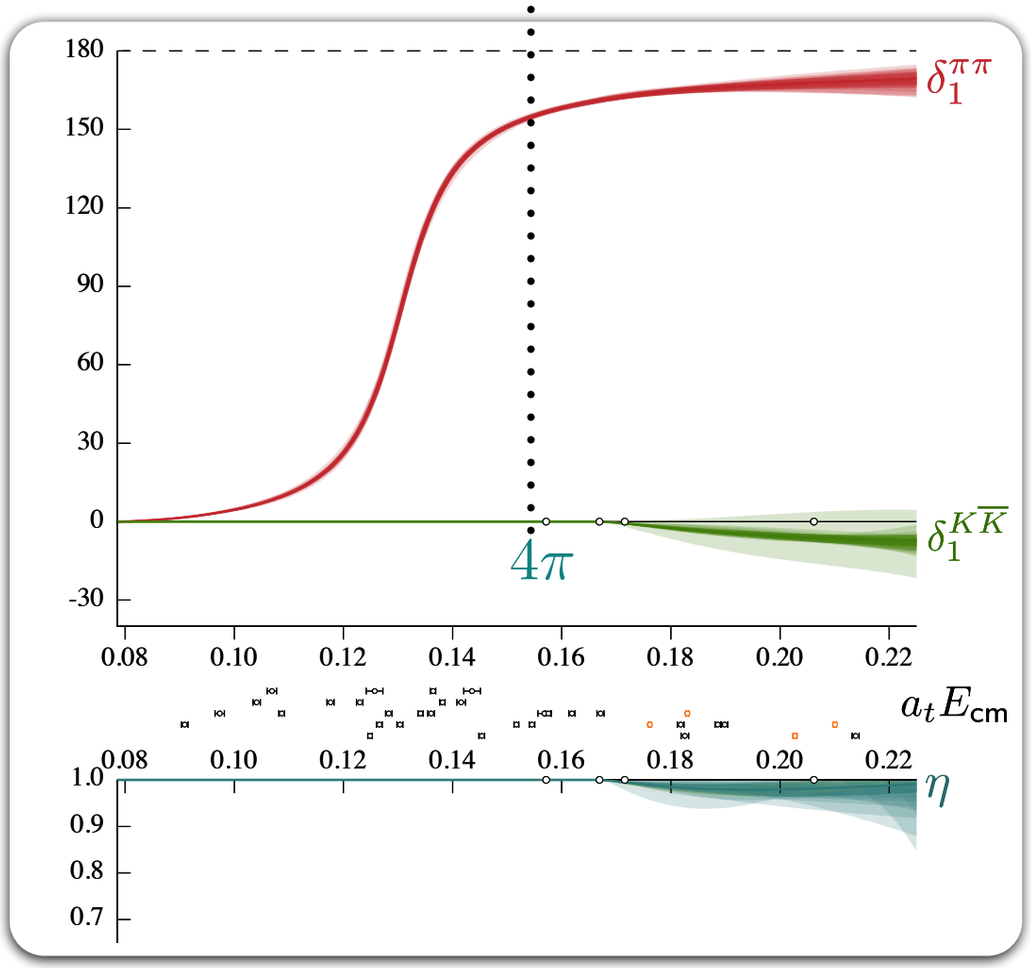
Don't know the equation that describes 2π - 4π, but must have the form:

$$\det\left[\begin{pmatrix}\mathcal{M}_{2\pi} & \\ & \mathcal{M}_{4\pi}\end{pmatrix}^{-1} + \begin{pmatrix}\tilde{K}_{2\pi,2\pi} & \tilde{K}_{2\pi,4\pi} \\ \tilde{K}_{4\pi,2\pi} & \tilde{K}_{4\pi,4\pi}\end{pmatrix}\right] = 0$$

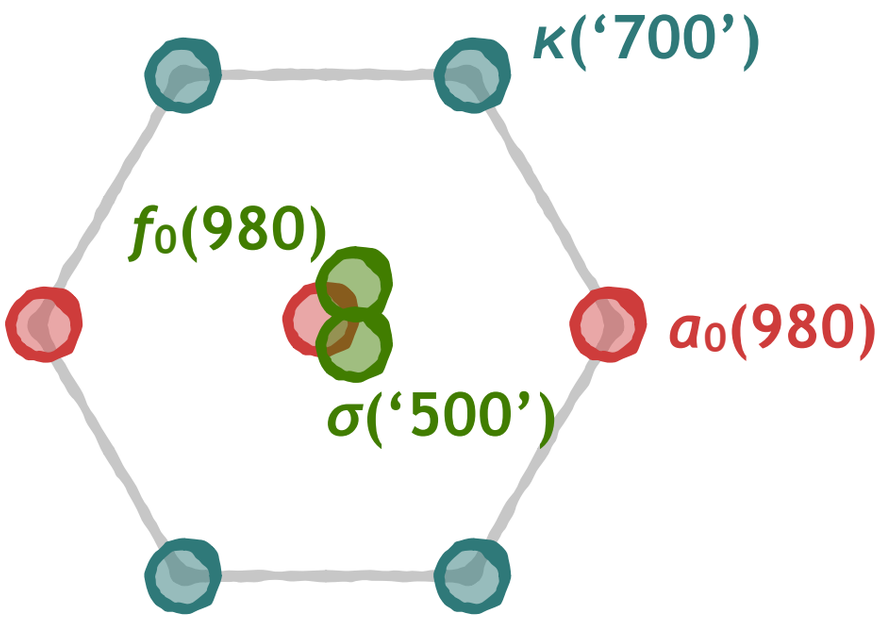
If $\tilde{K}_{2\pi,4\pi} \sim \mathcal{O}(\epsilon)$, then factorizes

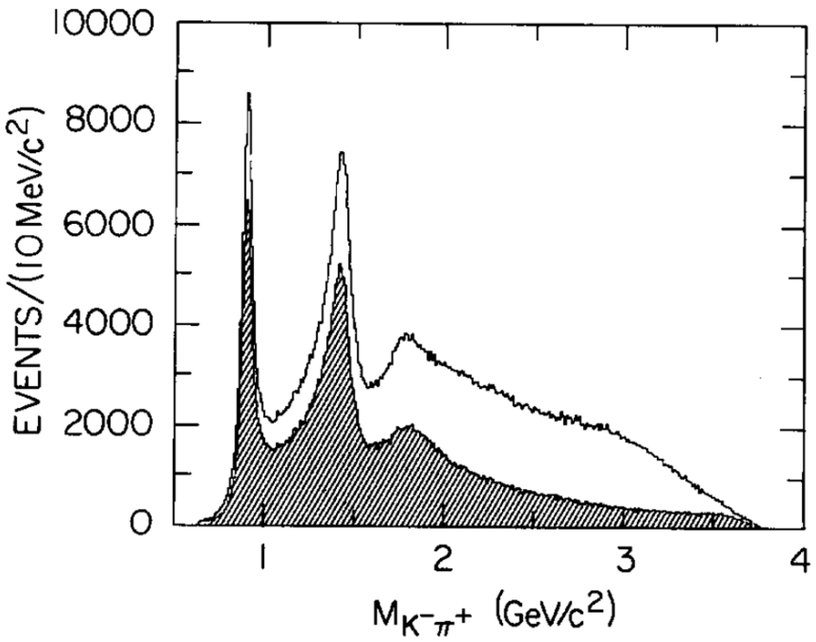
$$\det\left[\mathcal{M}_{2\pi}^{-1} + \tilde{K}_{2\pi,2\pi}\right] \times \det\left[\mathcal{M}_{4\pi}^{-1} + \tilde{K}_{4\pi,4\pi}\right] + \mathcal{O}(\epsilon) = 0$$

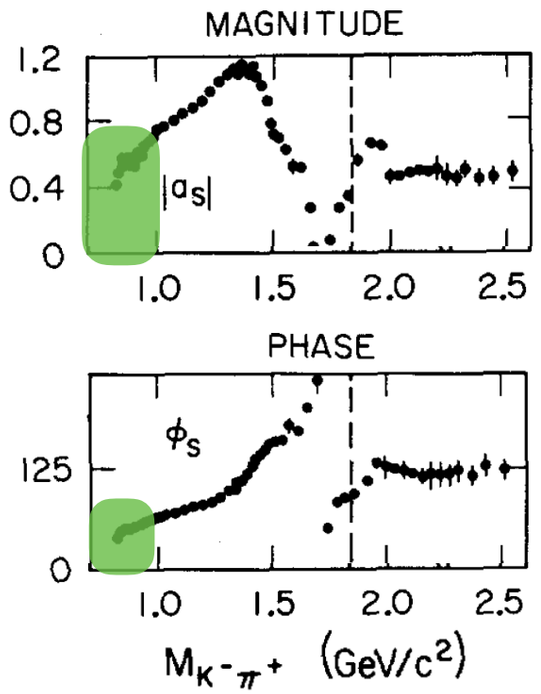
See no clear evidence of 4π

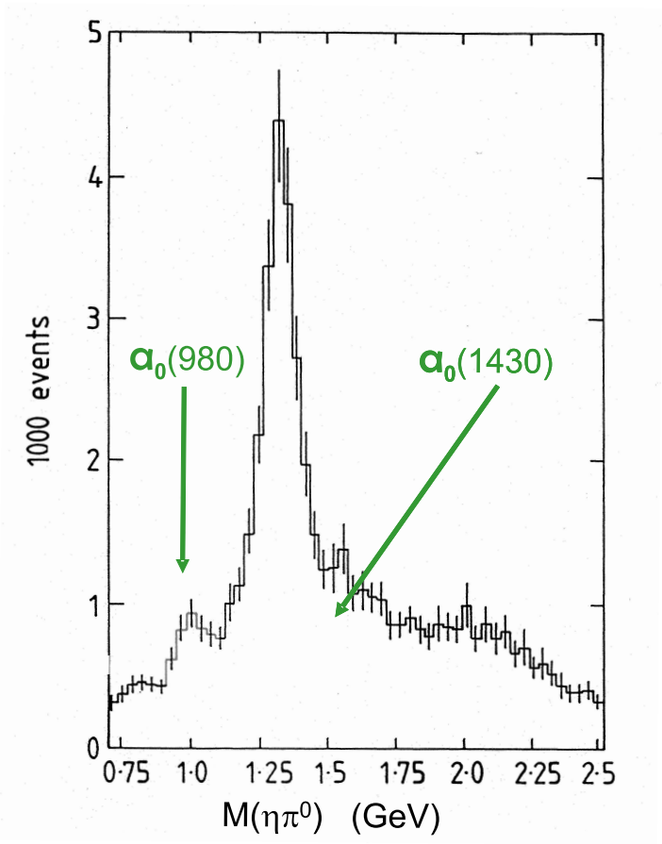


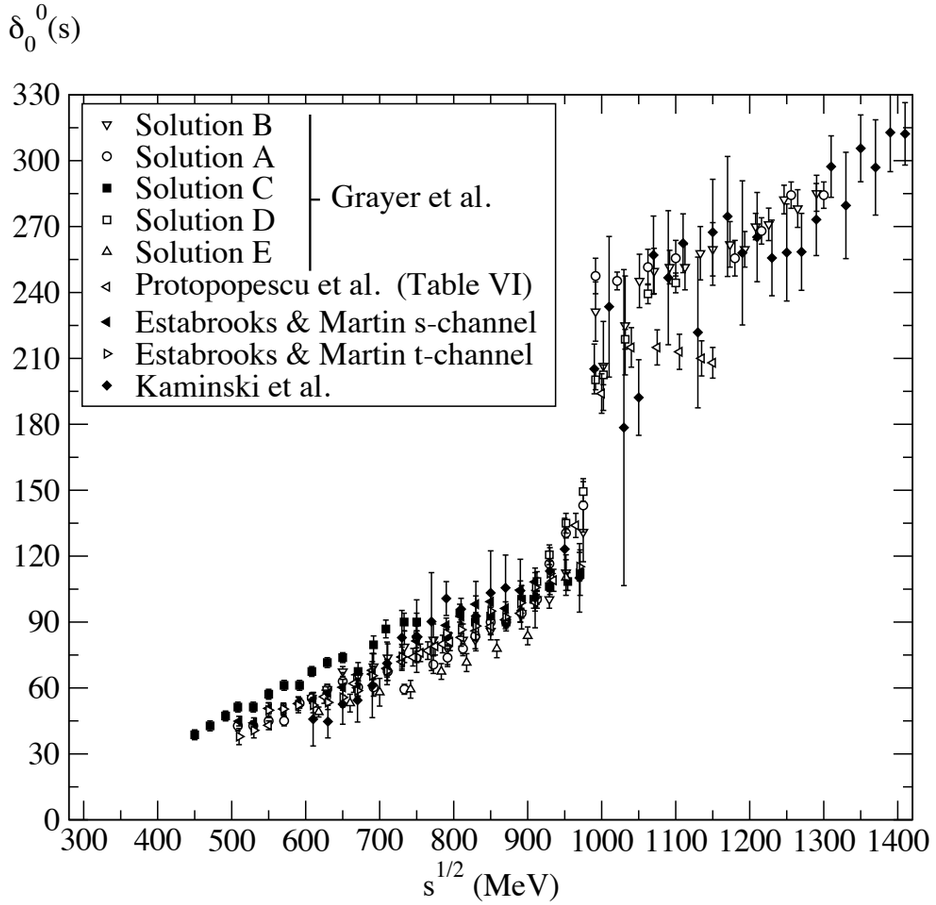
Light scalar mesons - empirically







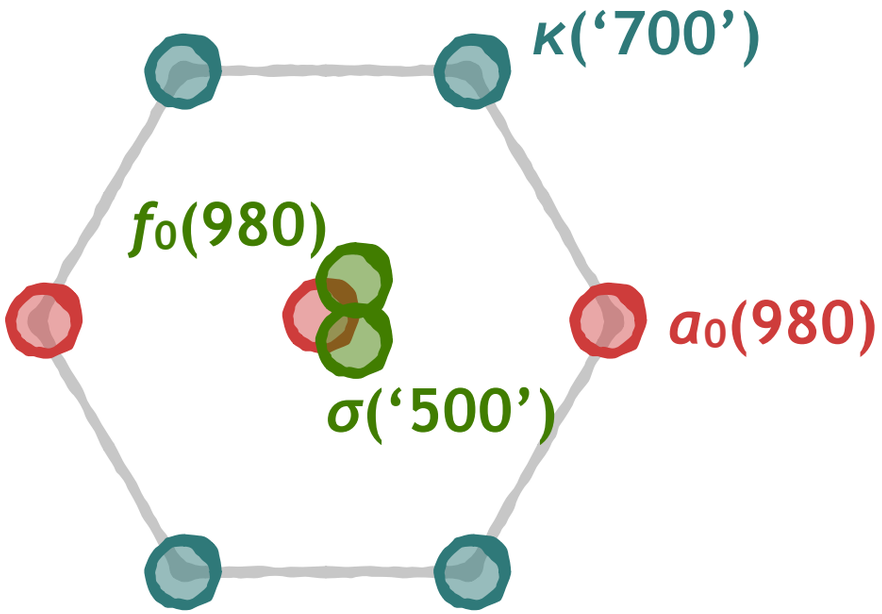




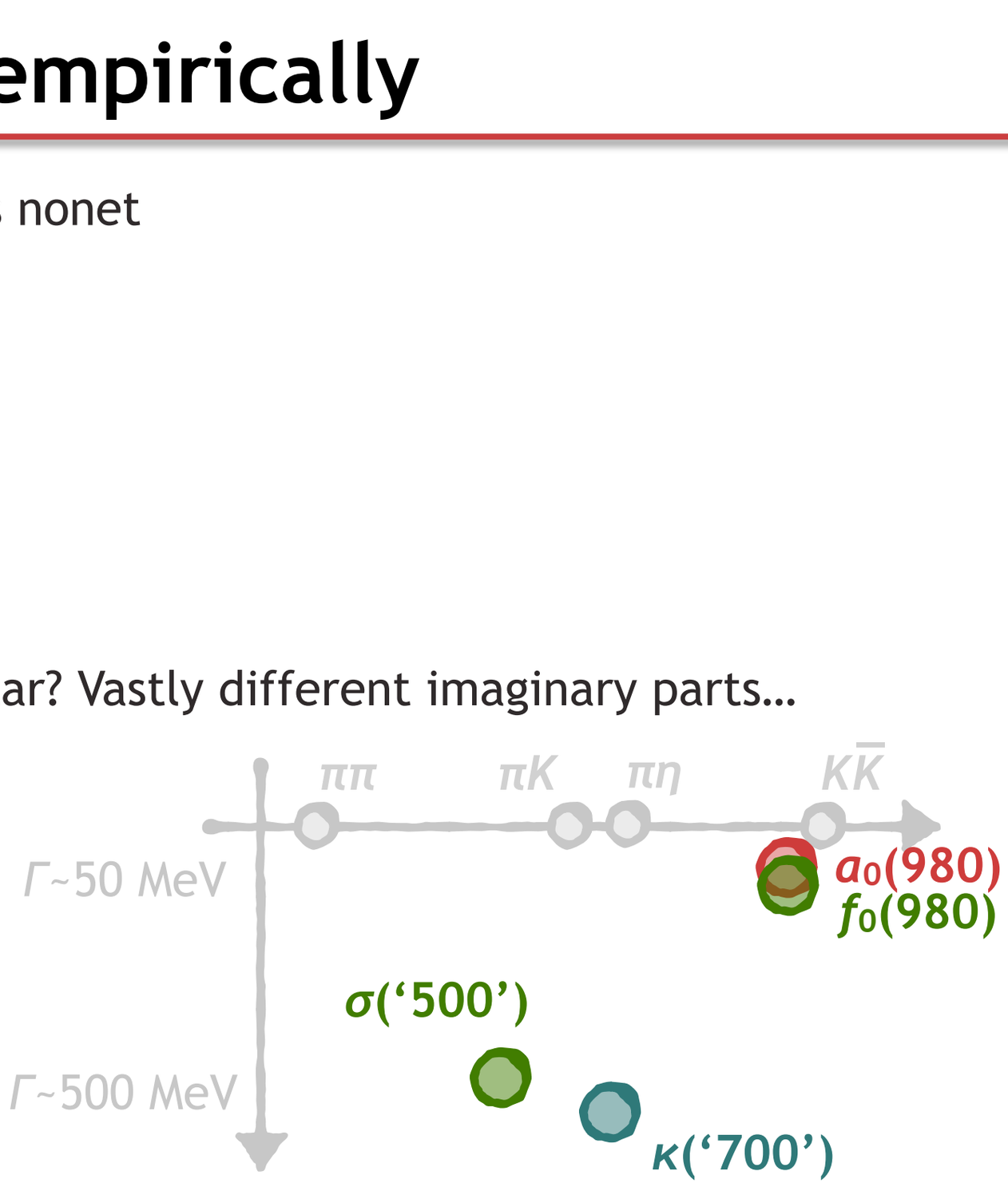
Jefferson Lab

Light scalar mesons - empirically

Conventional wisdom: an ‘inverted’ mass nonet

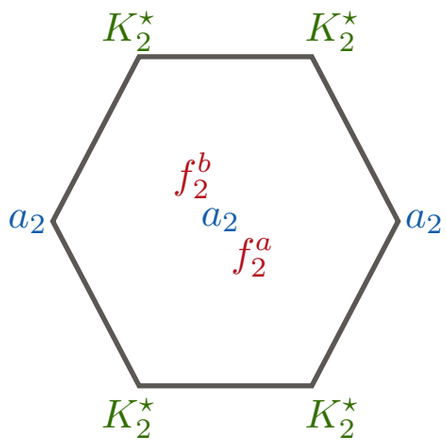


Similar? Vastly different imaginary parts…



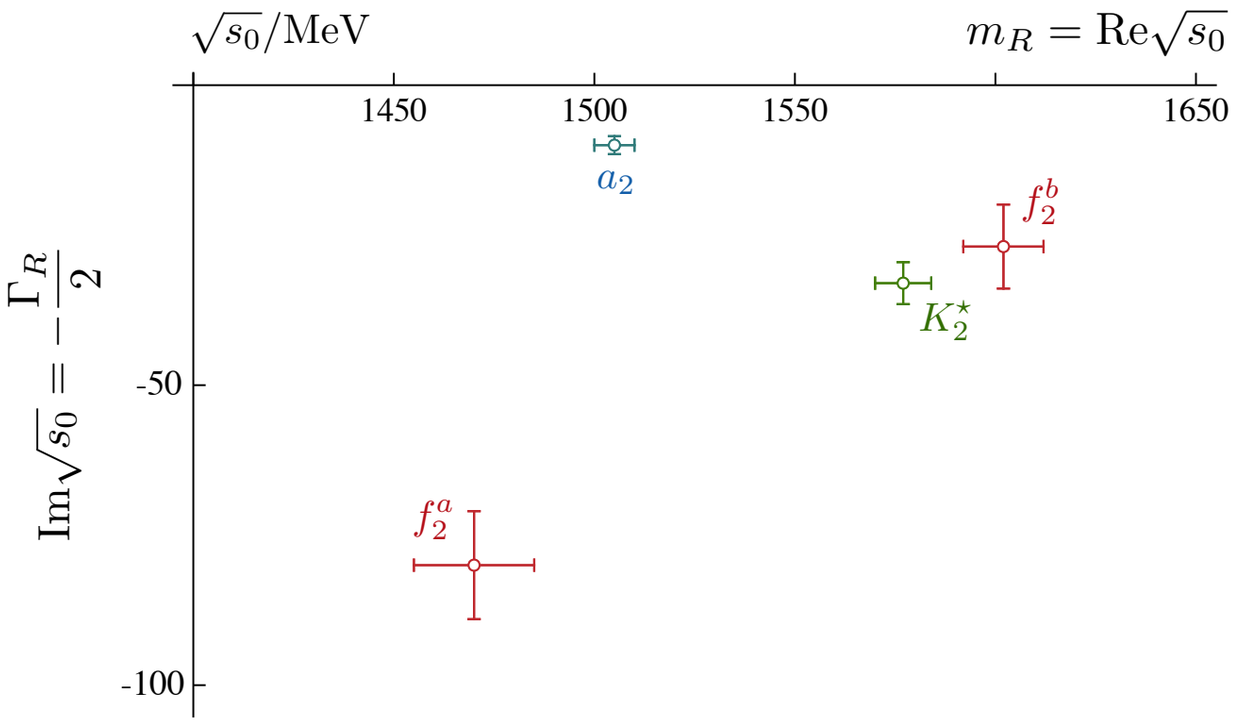
What does QCD have to say?

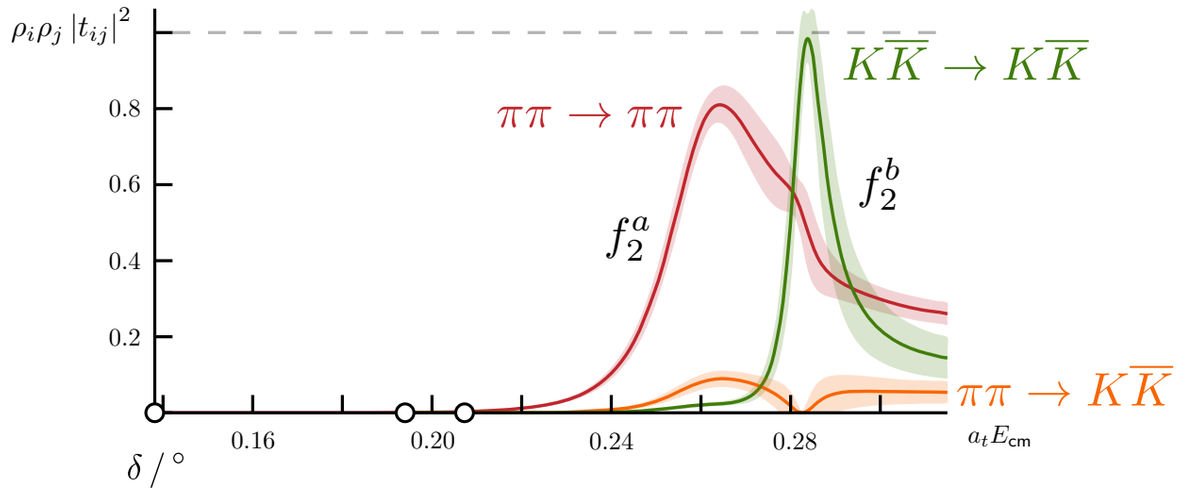
Lightest tensors at m_π=391 MeV

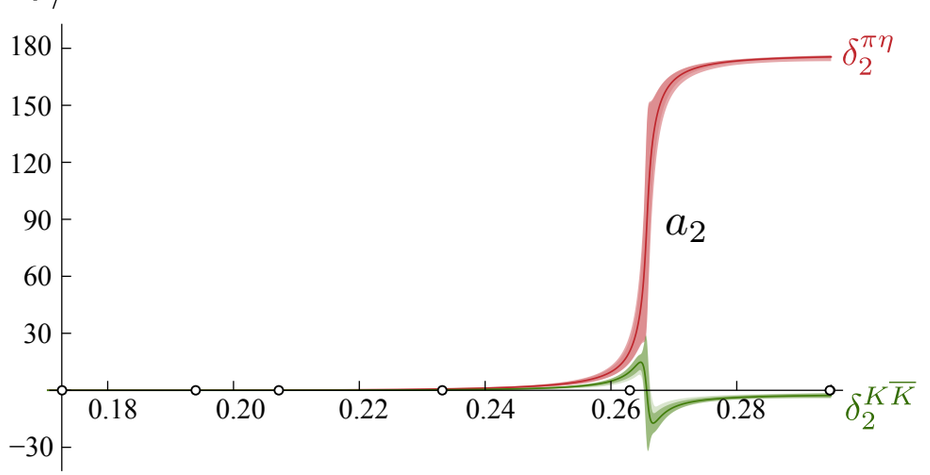


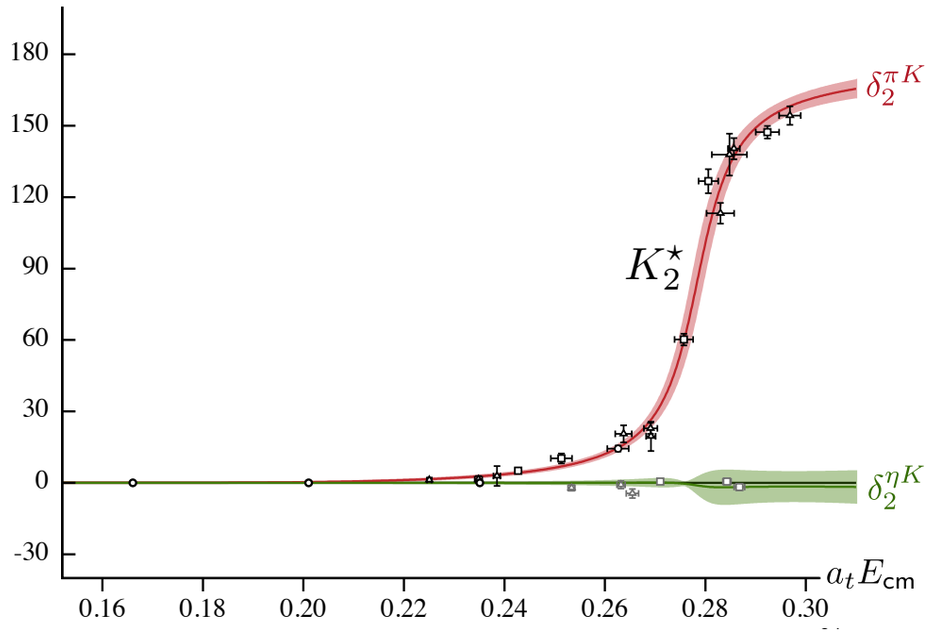
$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$

$$\sqrt{s_0} = m_R - i\frac{\Gamma_R}{2}$$









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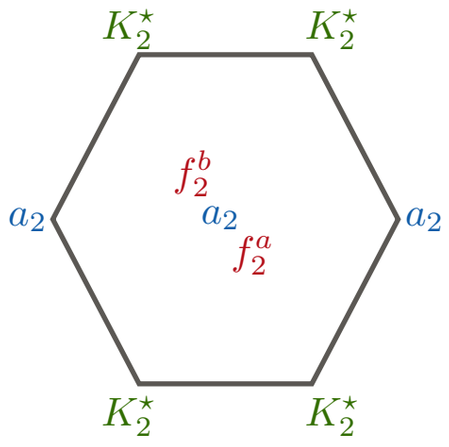
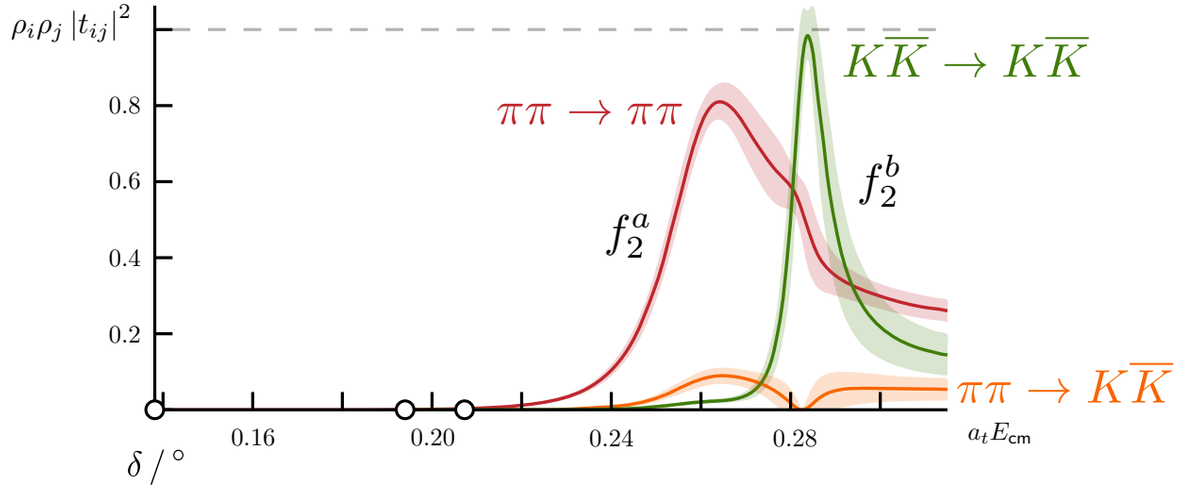
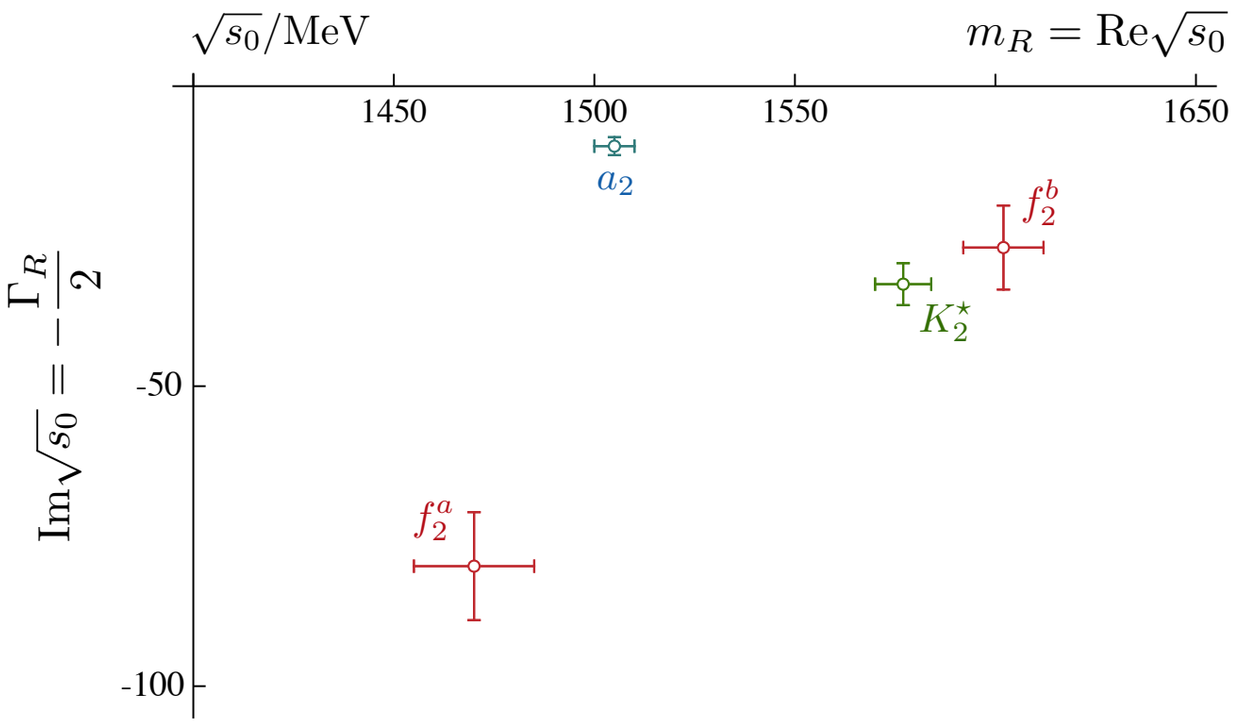




PDG $f_2(1270)$ $f_2(1525)$

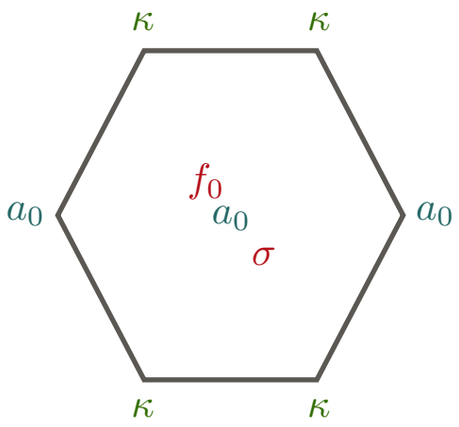
flavor tagging by decays

$\pi\pi$ 84% $K\overline{K}$ 89%



Used as motivation in
GlueX detector upgrade proposal

Lightest scalars at m_π=391 MeV



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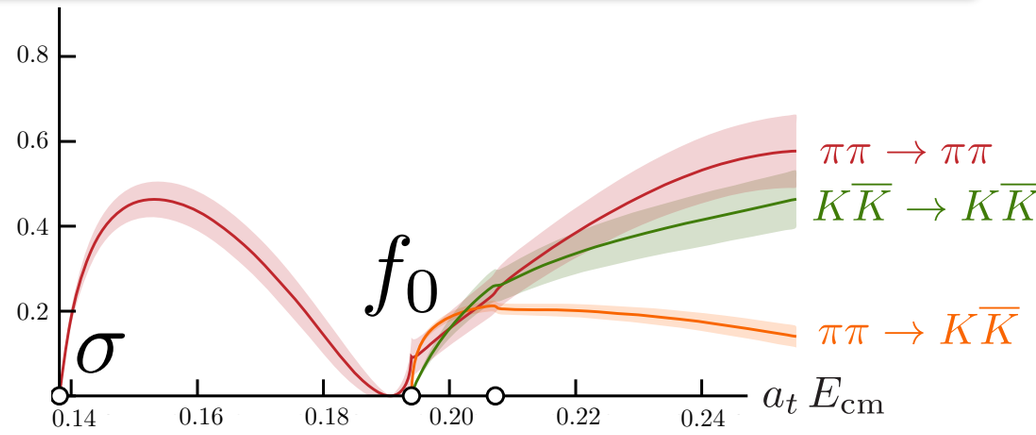
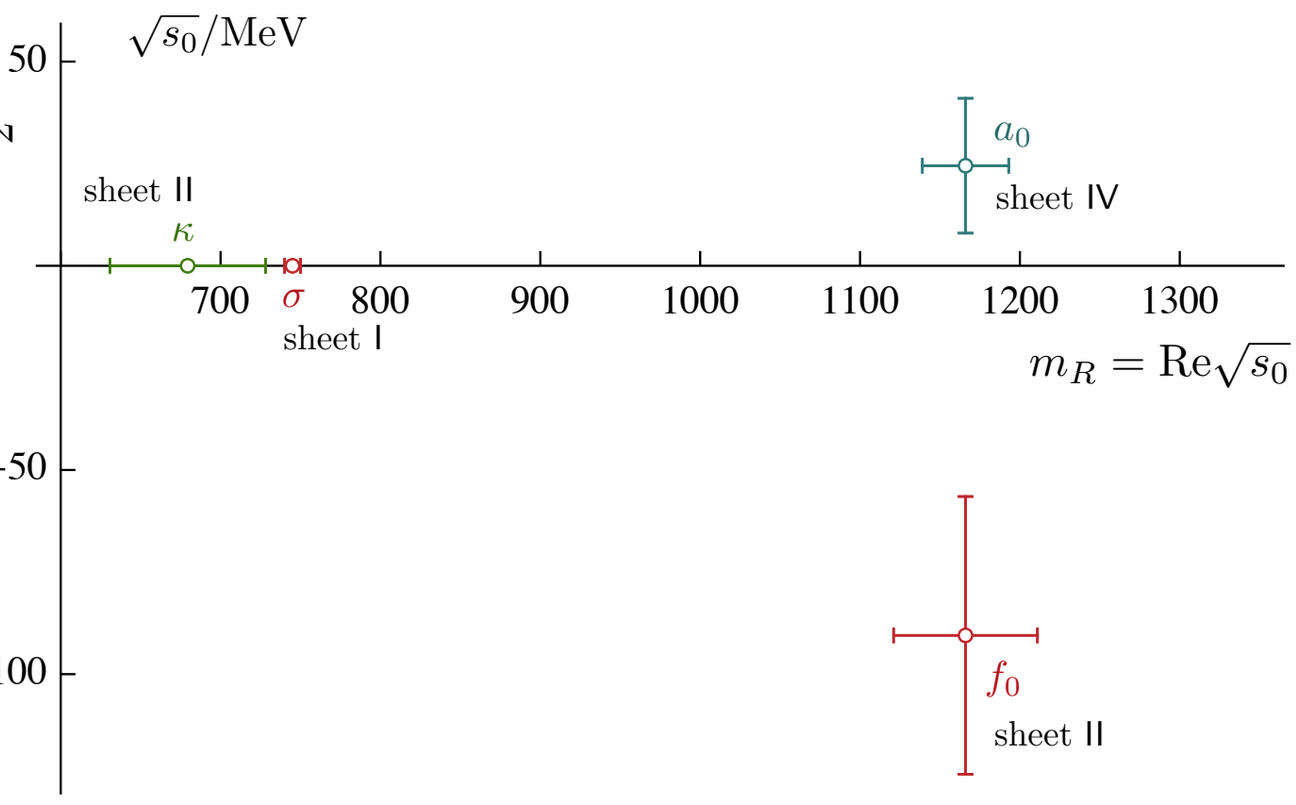
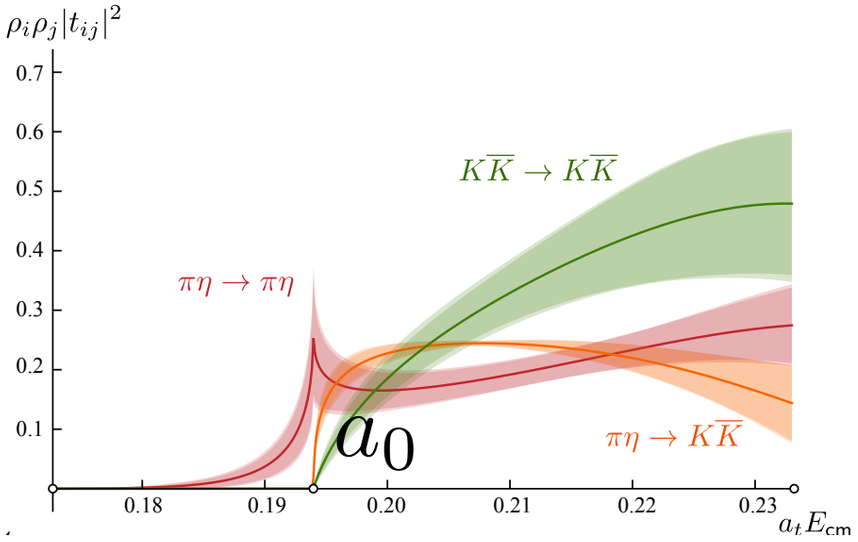
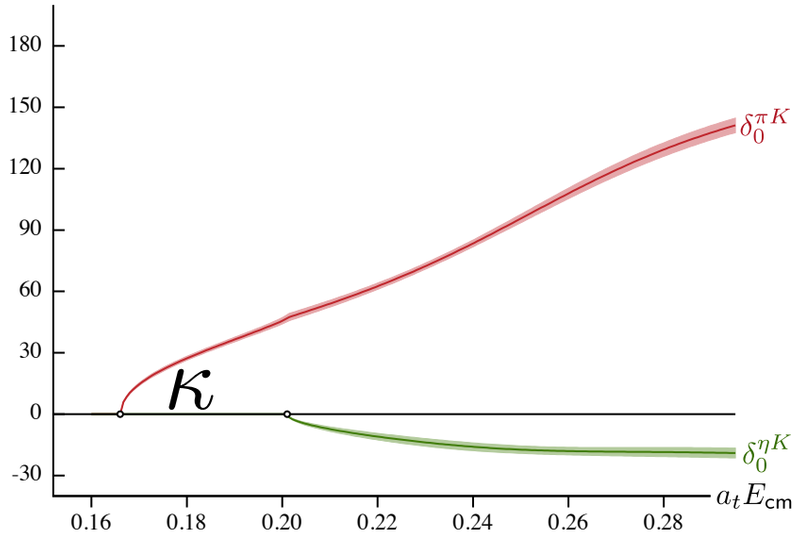
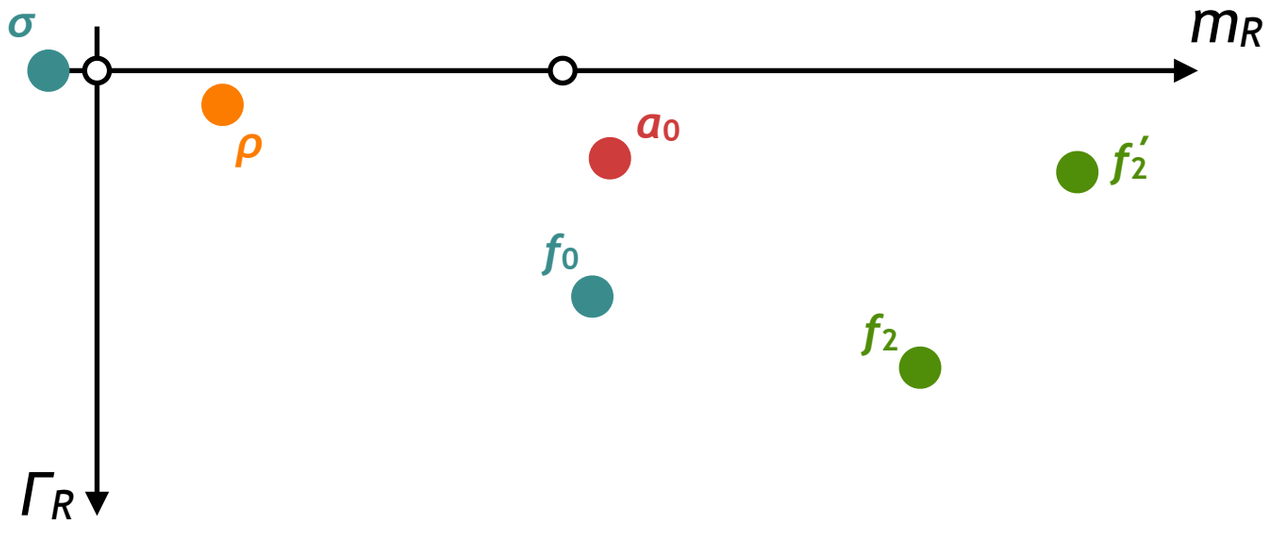




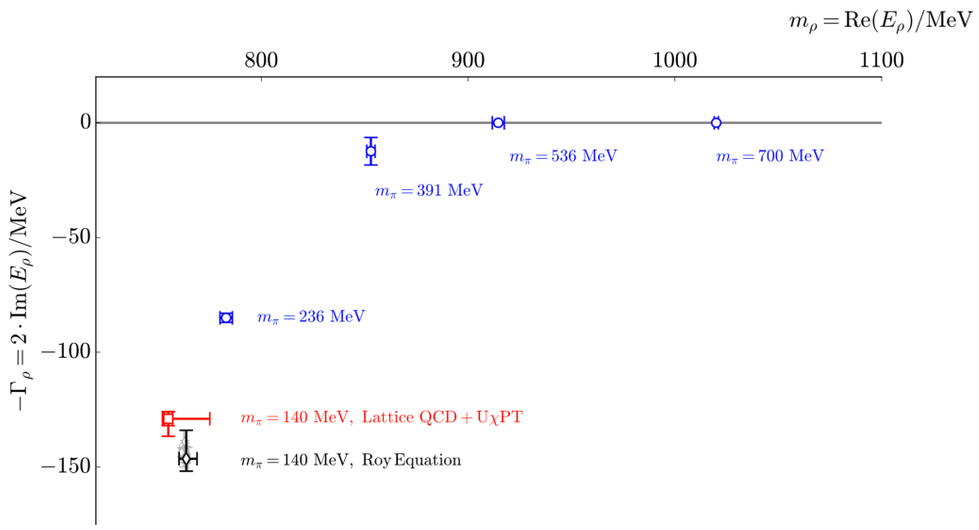


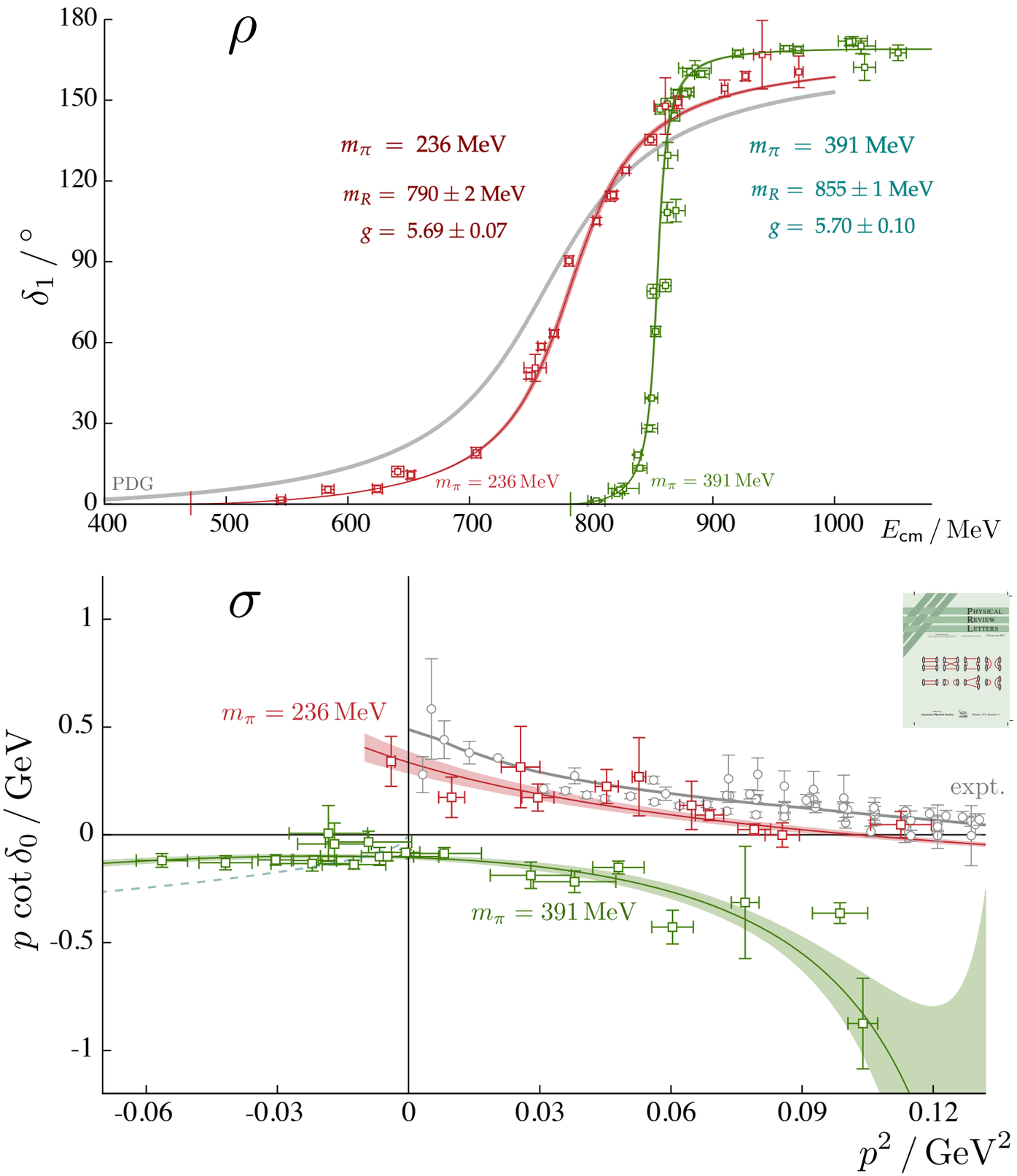


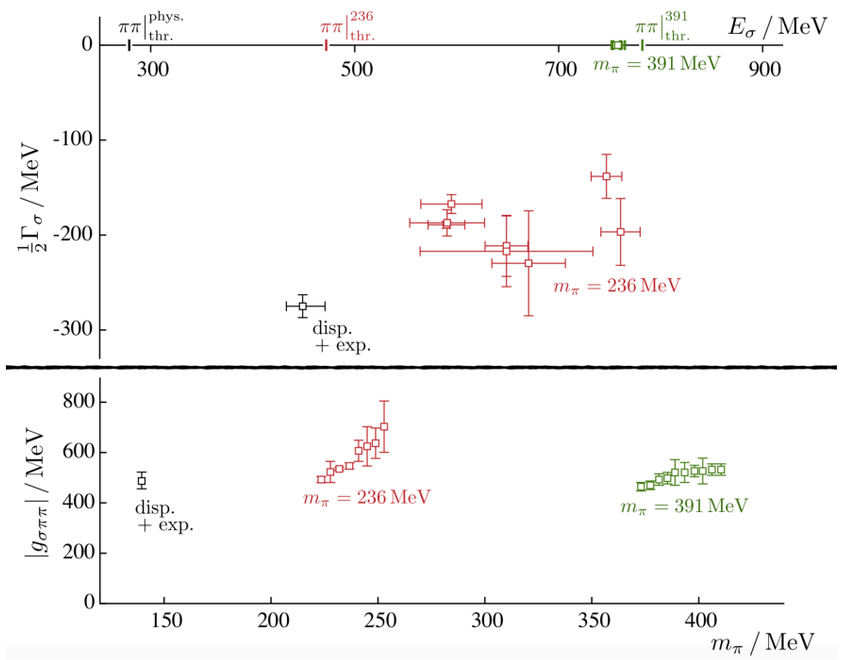
light meson resonances at m_π~391 MeV



Quark mass dependence: I=0 & 1



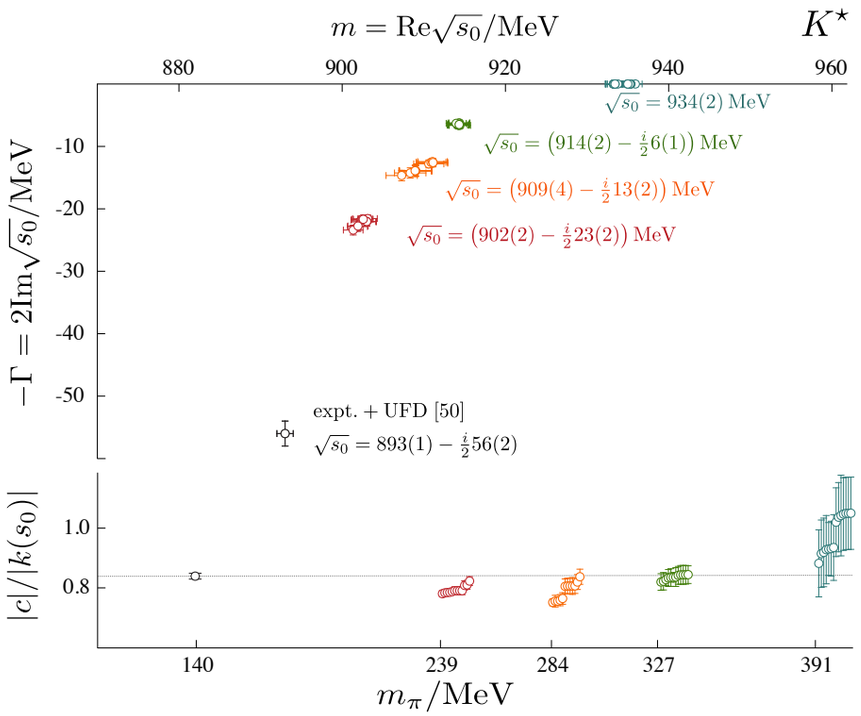


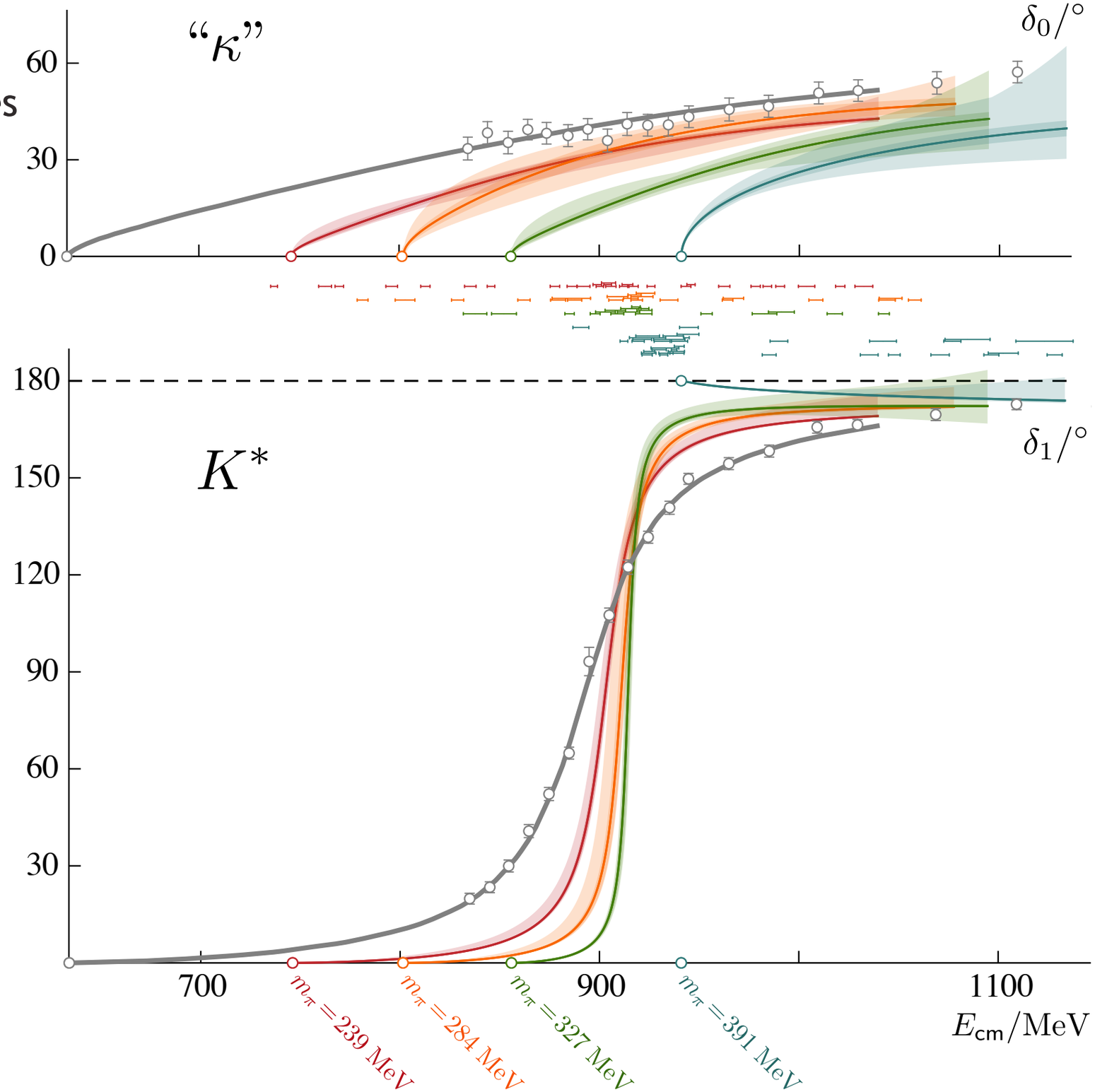


Quark mass dependence: I= 1/2

arXiv:1904:03188

A more complicated story…
need “t” & “u”-channel amplitudes



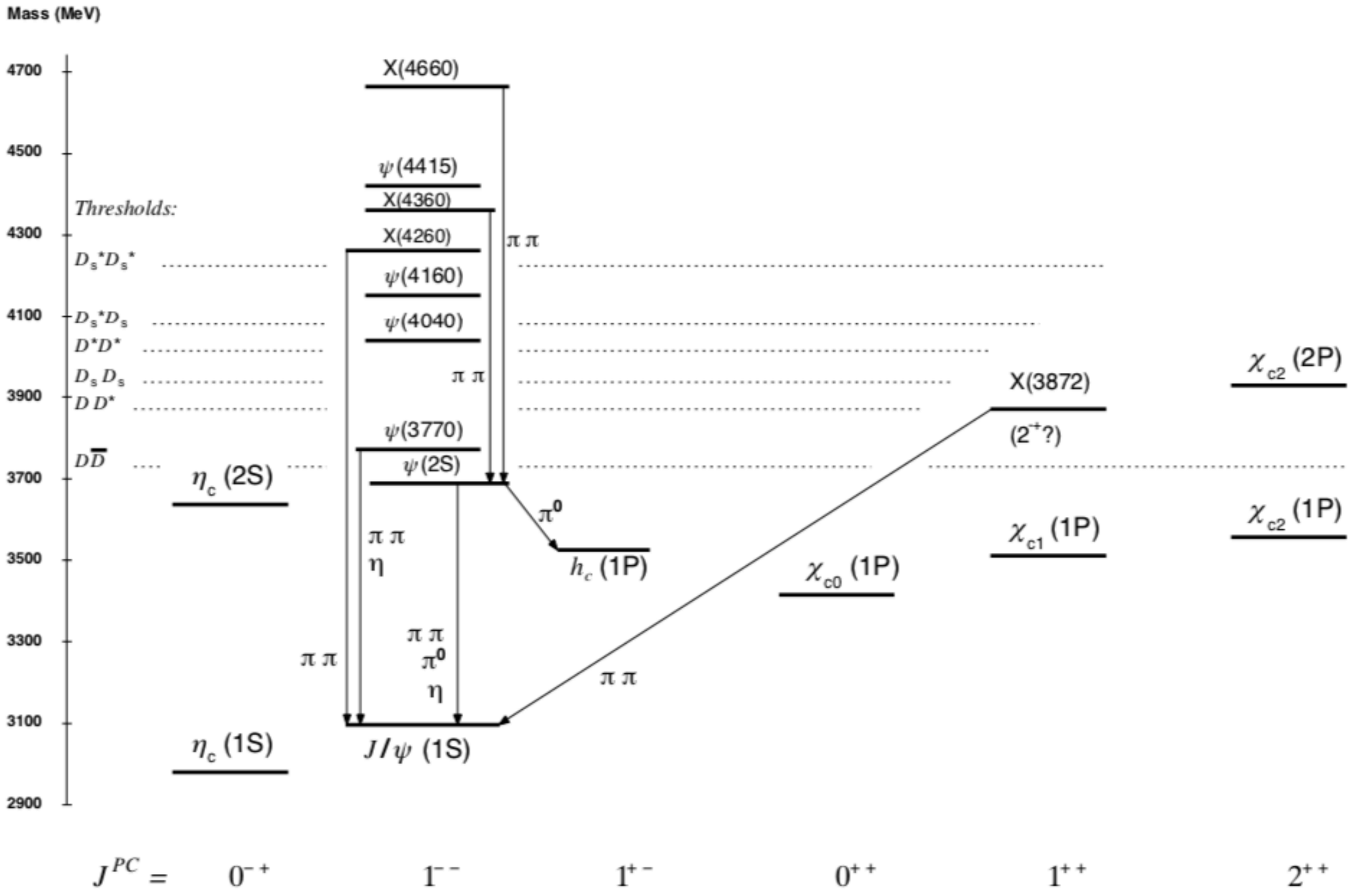


Added new dynamical ensembles

Charmonium resonances

PDG

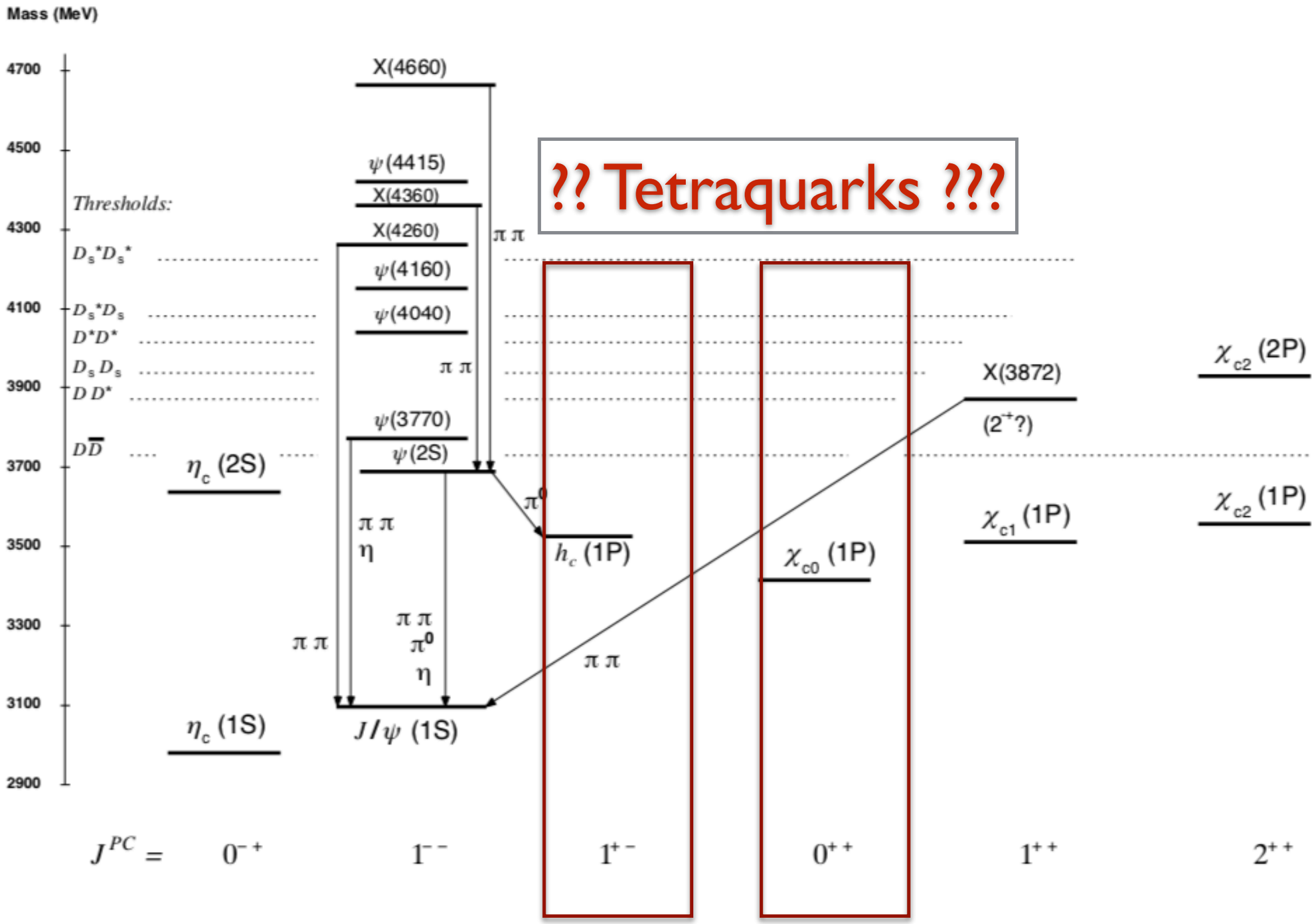
Several resonances reported near $D\bar{D}$ thresholds



Charmonium resonances

PDG

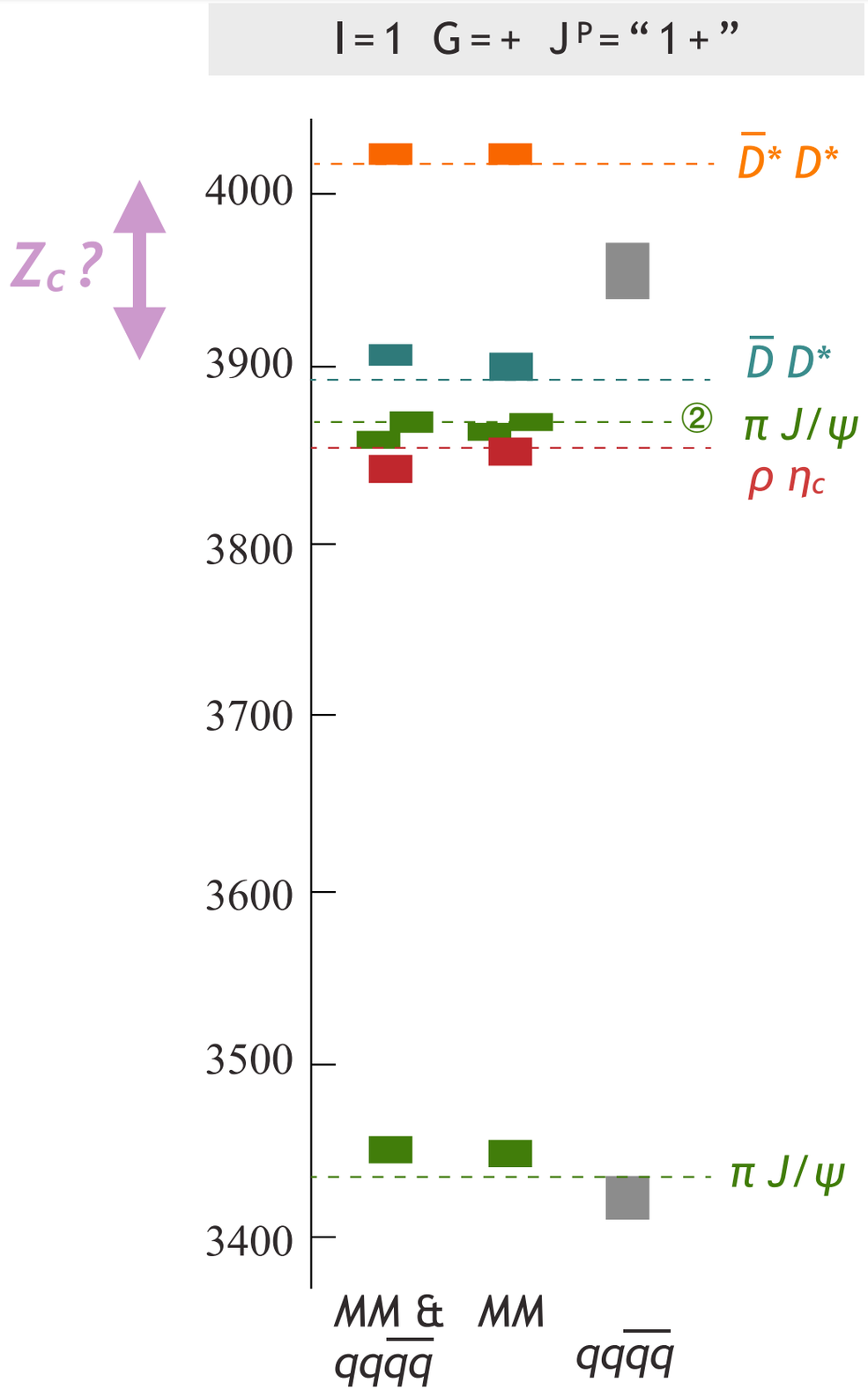
Isospin 1 charmonium?

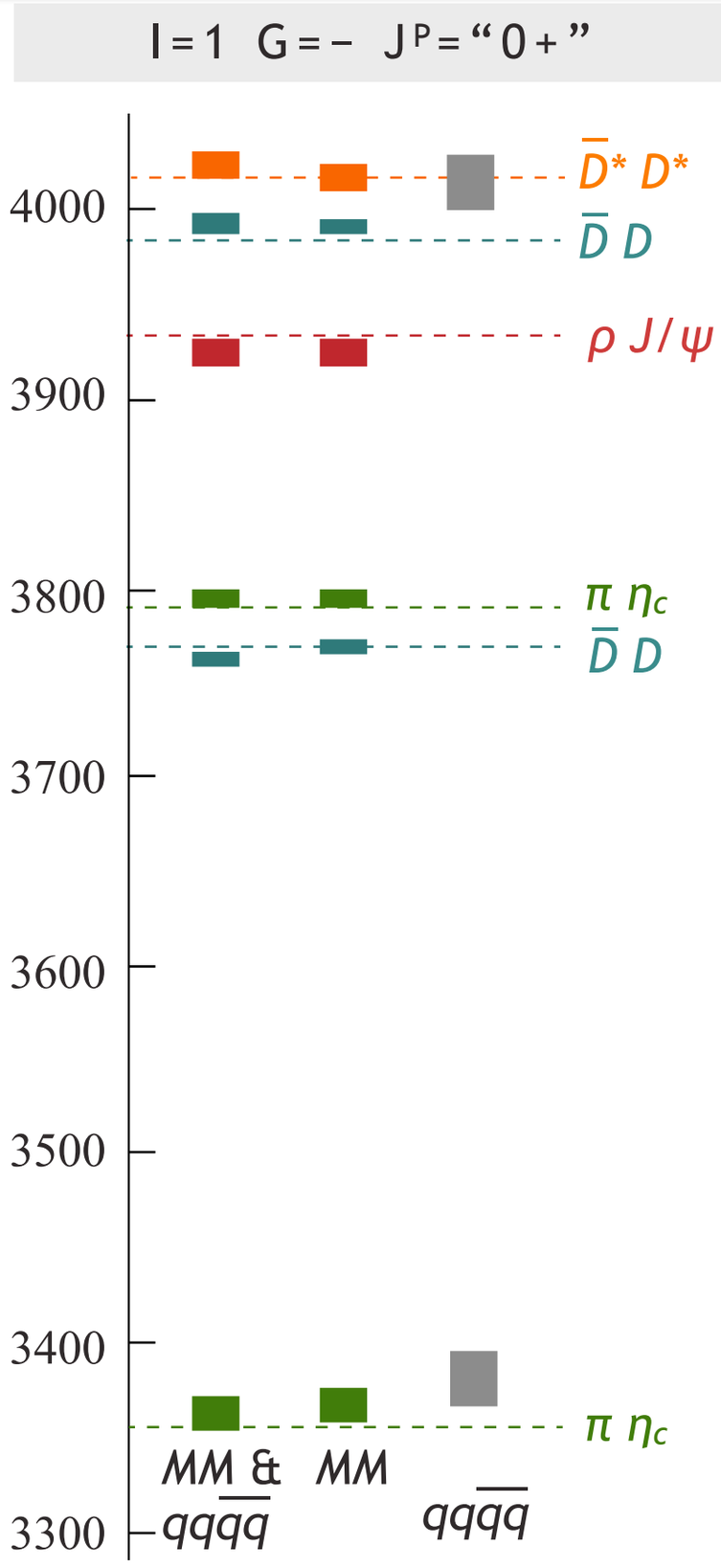


Tetraquarks ?

m_π ~ 391 MeV

JHEP 1711 033 (2017)

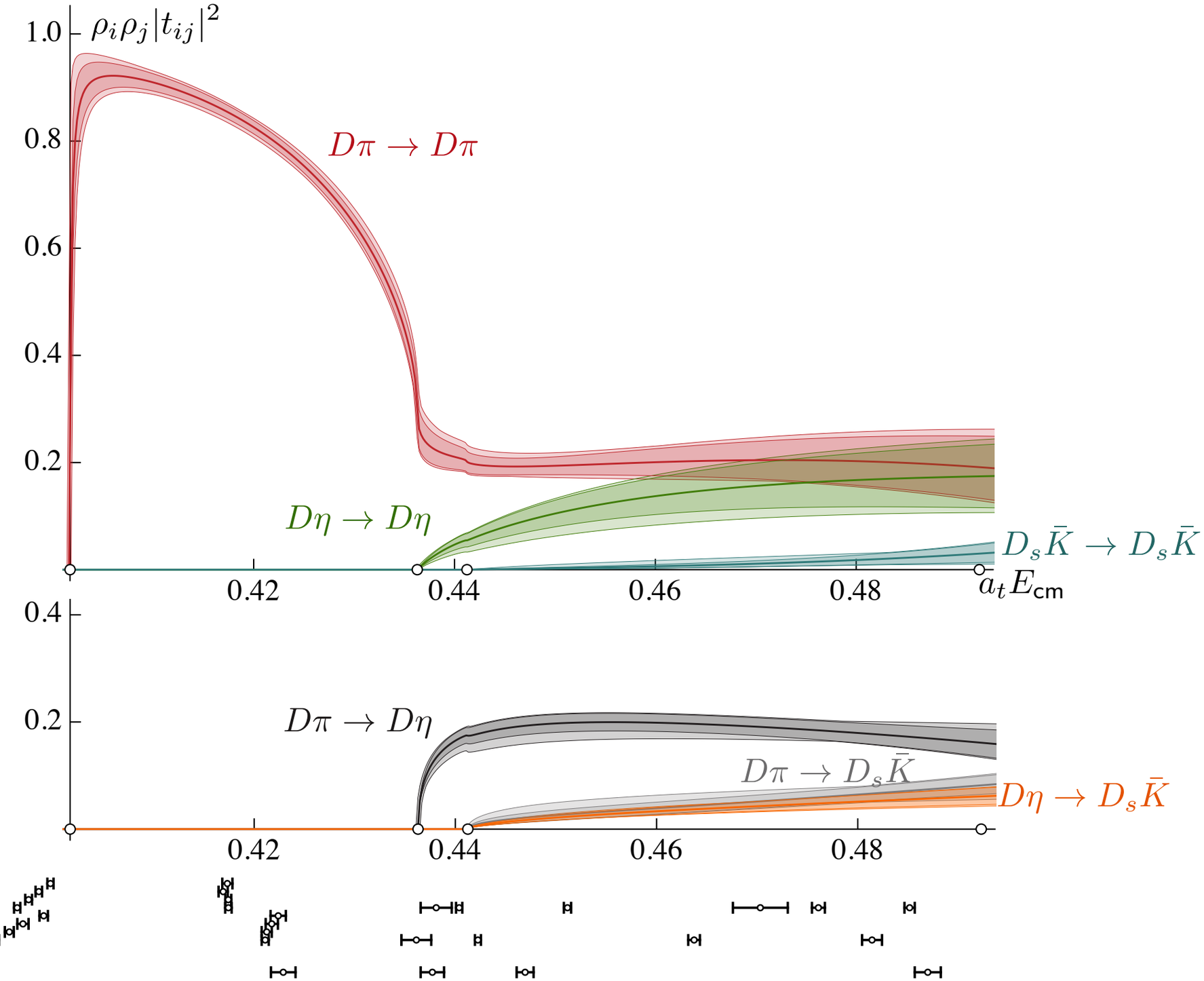




S-wave D decays

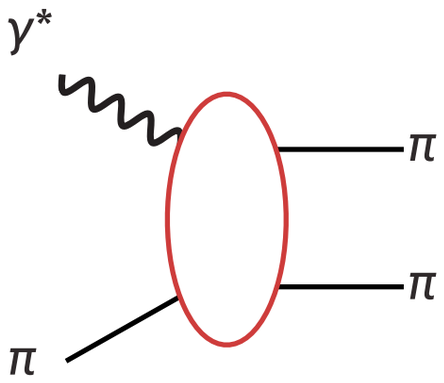
Moir et al, JHEP 1610 (2016) 011

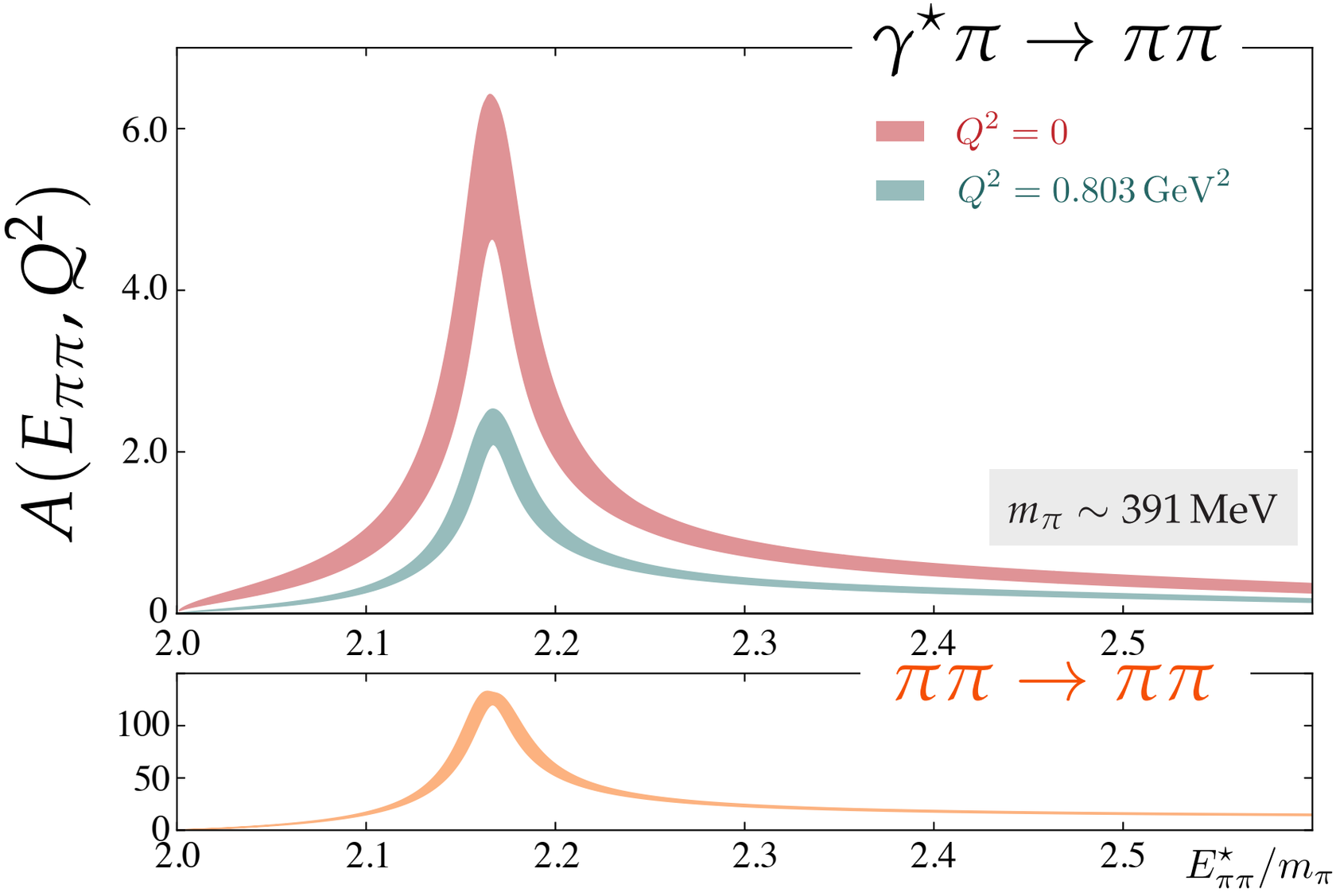
Sharp threshold behavior in D & D_s



Coupling resonances to currents

- Production mechanisms - e.g., photo-production





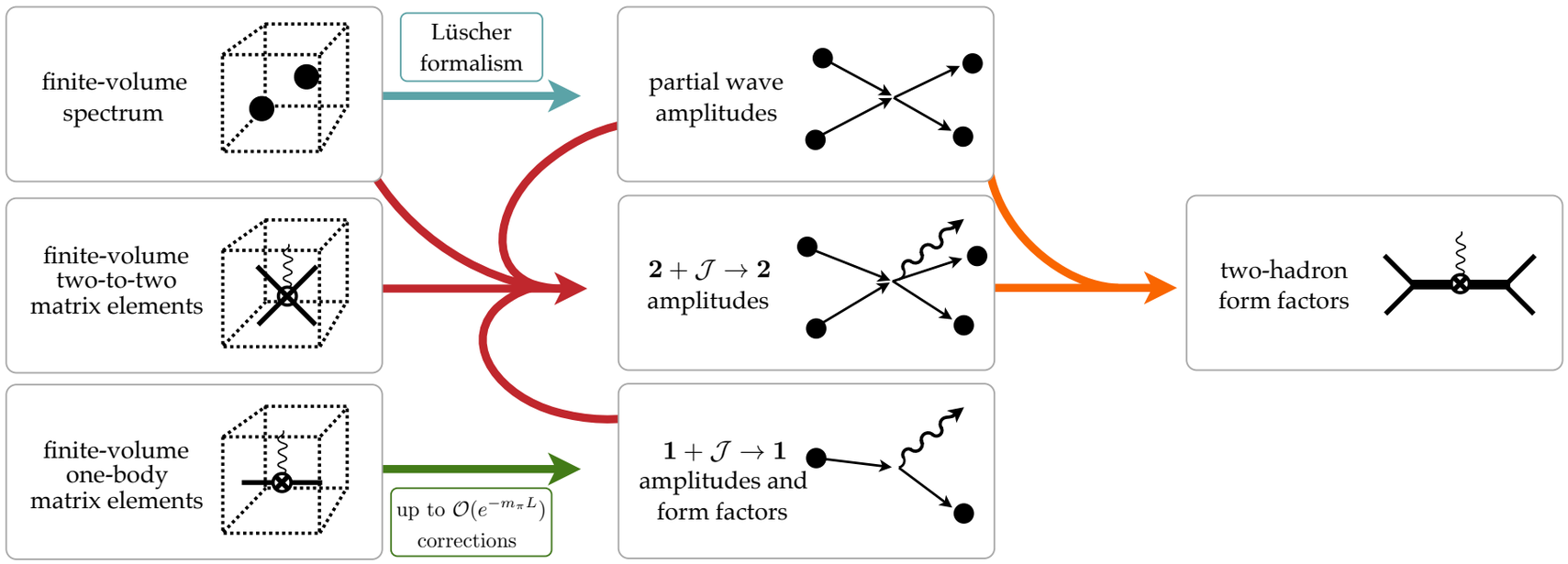
PRL 115 242001 (2015)
arXiv: 1604.03530

2+$\mathcal{J}\to$2 transition amplitudes

arXiv:1812.10504

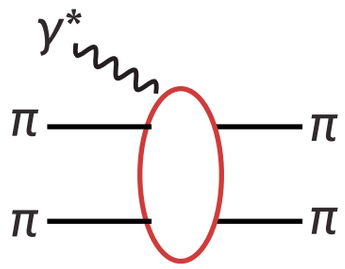
Not directly accessible experimentally

Provides a method to probe internal structure of resonances (e.g., charge radii, form-factors, even PDF-s)



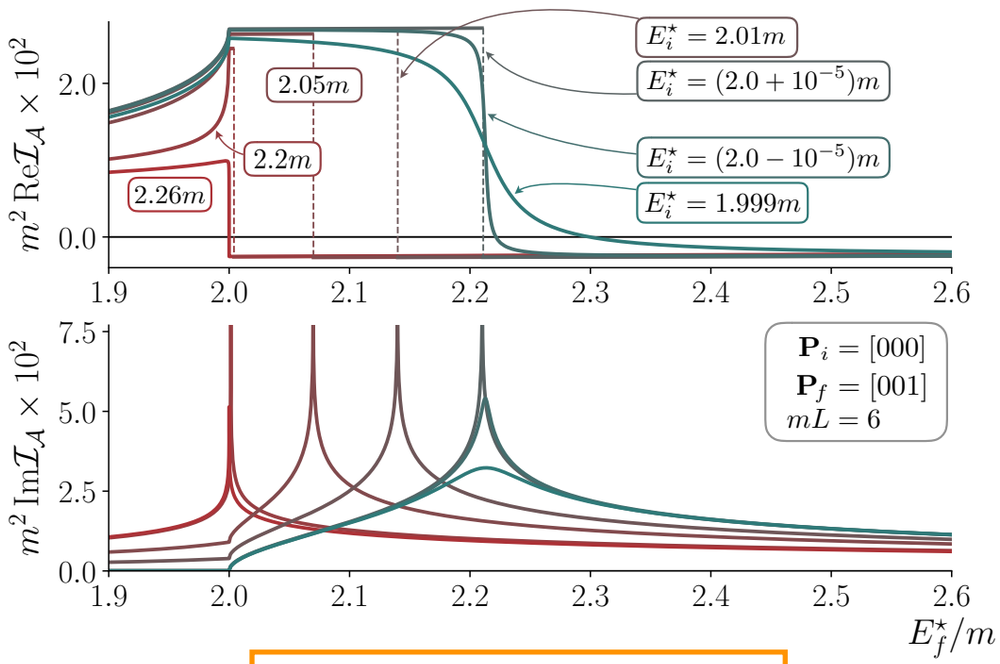
Establishing formalism

E.g., $\gamma^*\ \pi\pi \to \pi\pi$



$$\mathcal{A}\left(E^{\text{in}}_{\pi\pi}, E^{\text{out}}_{\pi\pi}, Q^2\right)$$

Energy regimes probe diagrams contributing



A. Jackura's talk (Fri.)

Status of scattering formalisms

Two-body coupled channel(s)

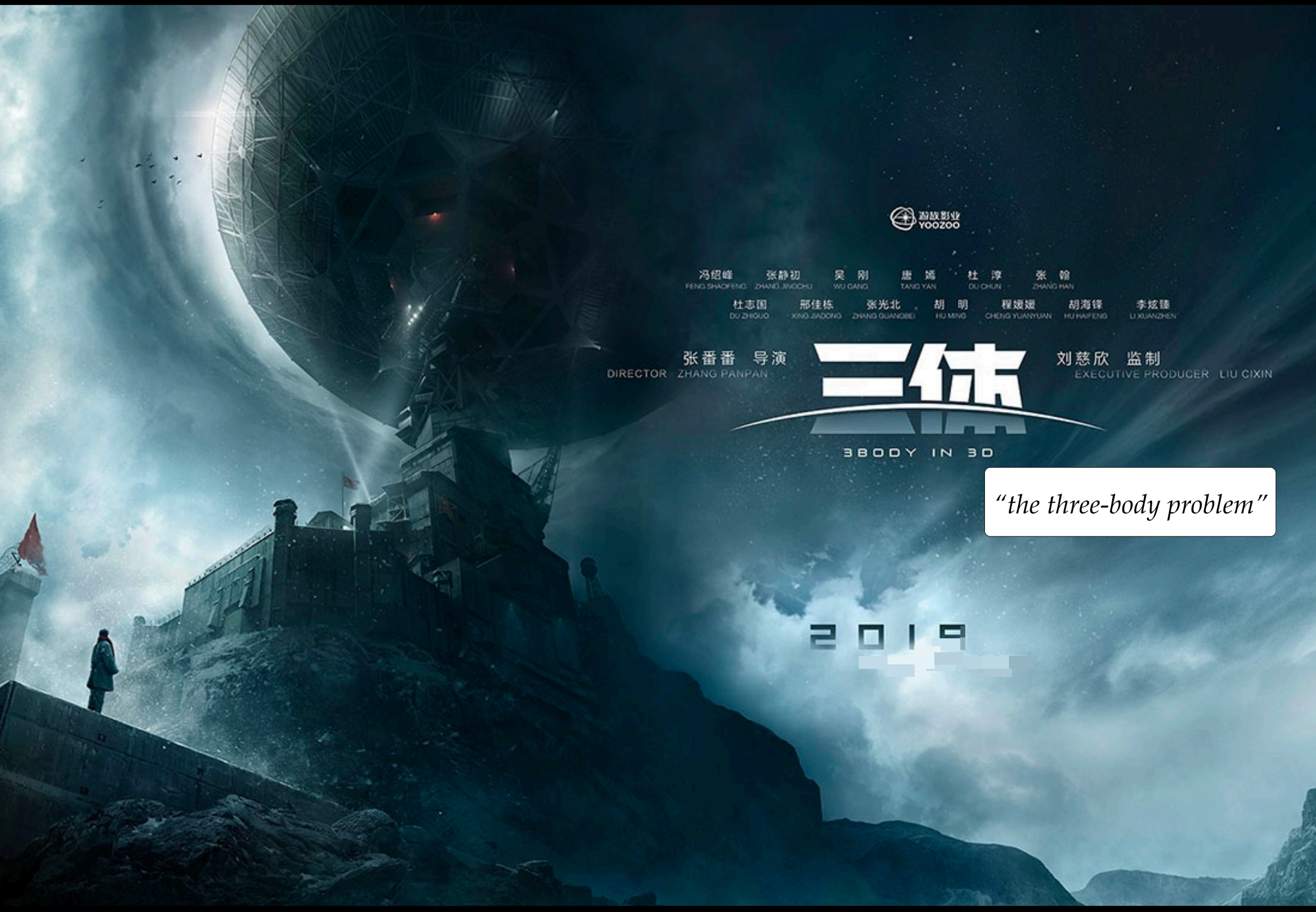
- finite-volume formalism ✔
- amplitude formalism (good) except cross-channels (not-good) 😊 😐
  - can provide branching fractions for phenomenologists ✔

ELEMENTARY PARTICLE THEORY

A. D. MARTIN
Department of Physics, University of Durham

T. D. SPEARMAN
School of Mathematics, Trinity College, Dublin

1970



"the three-body problem"

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The Three-Body Problem (novel)

From Wikipedia, the free encyclopedia

For other uses, see Three-body problem (disambiguation).
"San Ti" redirects here. For the martial arts stance, see Xing Yi Quan.

The Three-Body Problem (Chinese: 三体; literally: 'Three-Body'; pinyin: *sān tǐ*) is a science fiction novel by the Chinese writer Liu Cixin. It is the first novel of the *Remembrance of Earth's Past* (Chinese: 地球往事) trilogy, but Chinese readers generally refer to the whole series by the title of this first novel.[1] The second and third novels in the trilogy are titled *The Dark Forest* and *Death's End*. The title of the first novel refers to the three-body problem in orbital mechanics.

The work was serialized in *Science Fiction World* in 2006, published as a book in 2008 and became one of the most popular science fiction novels in China.[2] It received the Chinese Science

The Three-Body Problem

Thanks to Raul...

Status of scattering formalisms

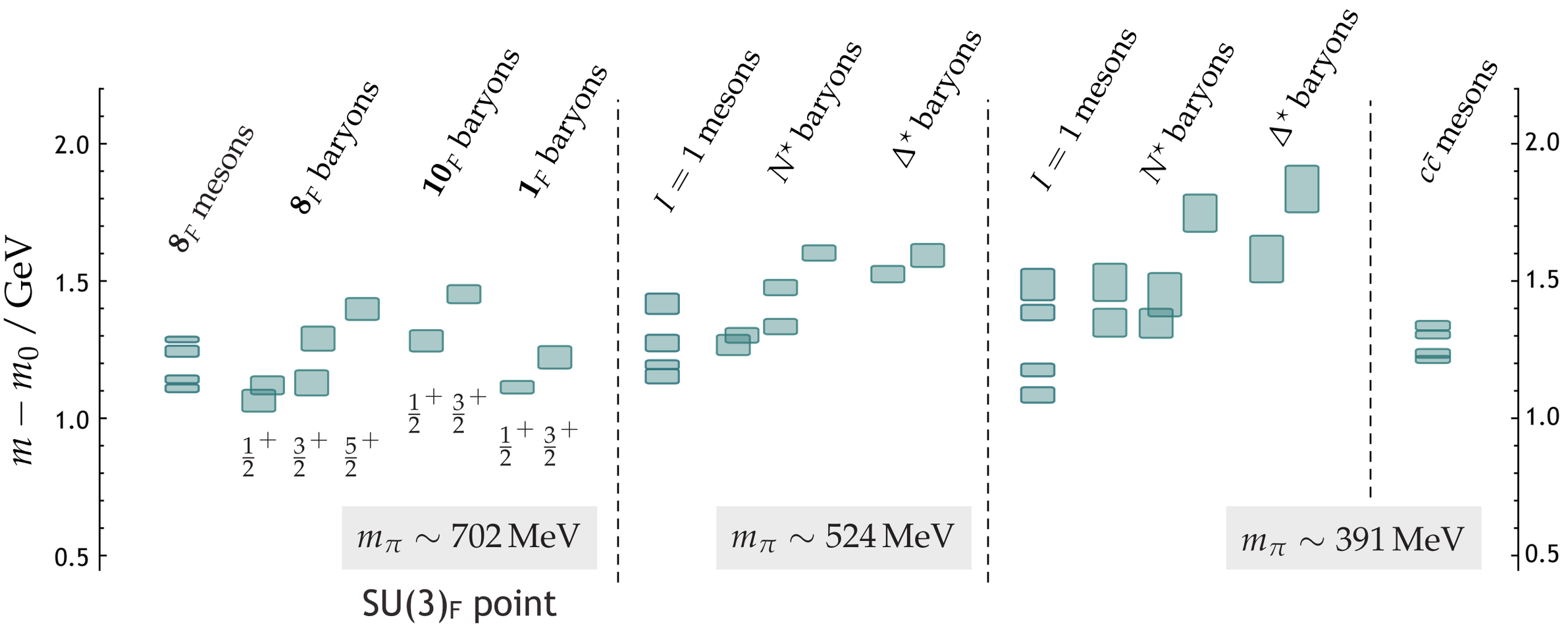
Three-body coupled channel

- unitarity in 3-body — arXiv:1905.11188 ✔
- finite-volume formalism for spinless particles
  - coupled 2-body & 3-body — PRD99 014506 (2018) ✔
  - determinant condition ✔
  - integral equations 😐
- 3-body amplitude formalism - not well understood! 😕
  - comparisons and cross-checks of existing forms — arXiv:1905.12007 ✔
- need 3-body energies. Codes ready and optimized!

NPLQCD | JHEP 1711 033 (2017) - tetraquarks | arXiv:1904.04136 - b1 | arXiv:1905.04277 - 3π

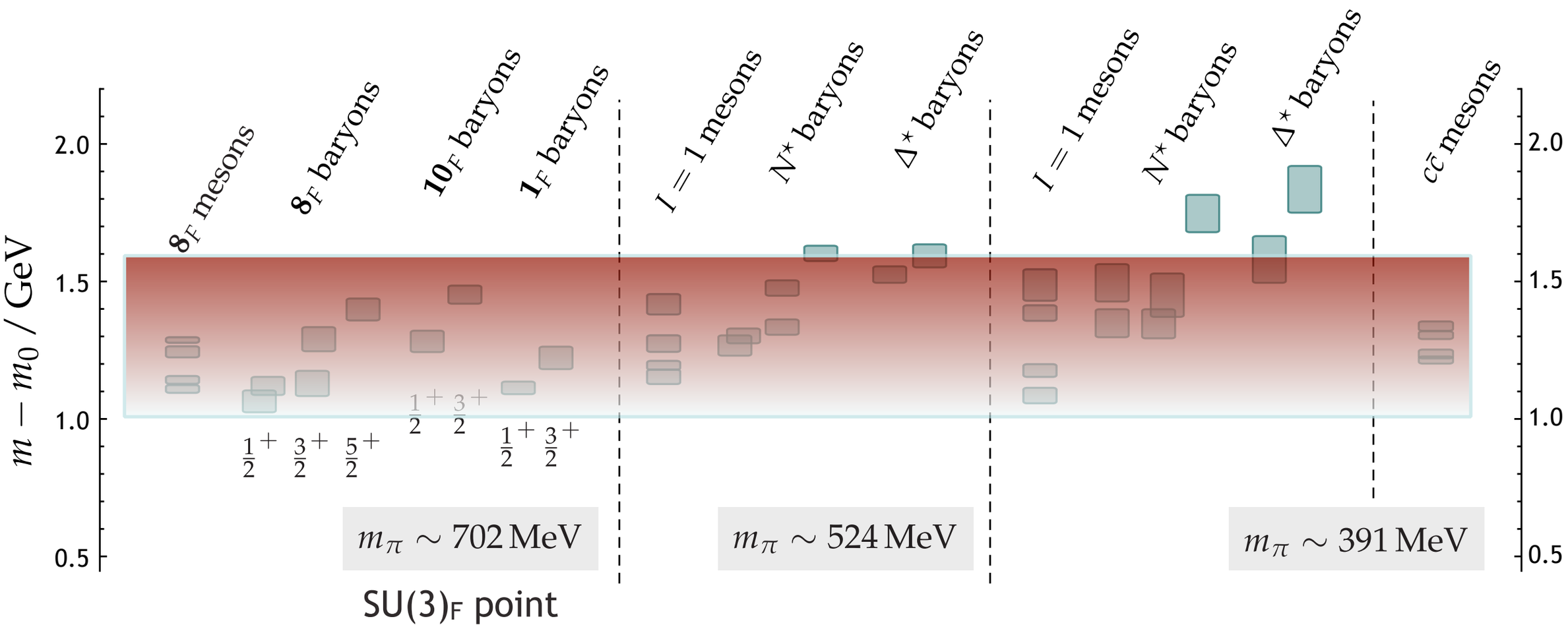
Hadron Spectroscopy - role of the glue

- Subtract the ‘quark mass’ contribution



Hadron Spectroscopy - role of the glue

- Subtract the ‘quark mass’ contribution



- Common energy scale of gluonic excitation ~ 1.3 GeV

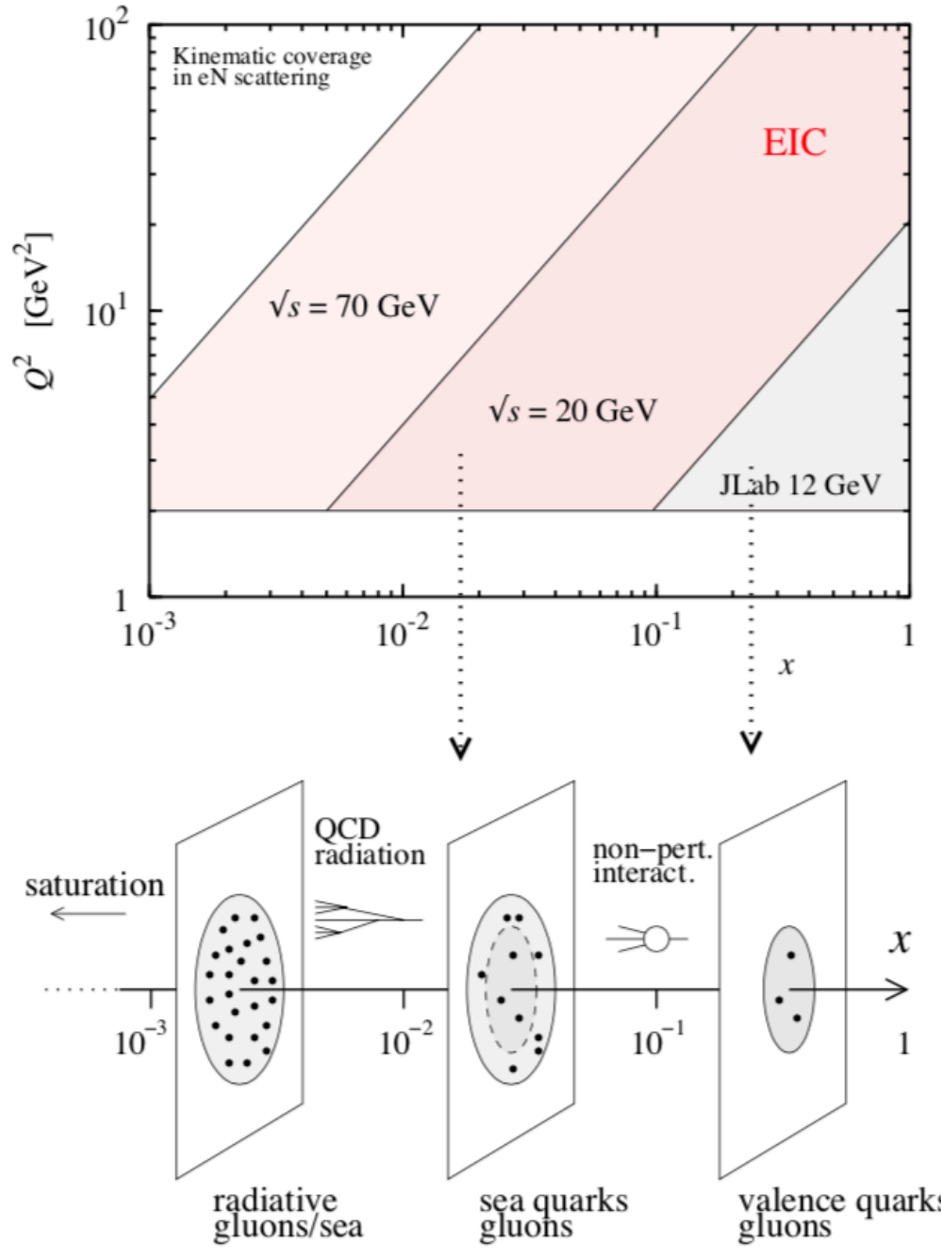
Relevant for EIC?

- Strong overlap of gluonic constructions with ground-states

Electron Ion Collider(s)

Efforts in US (Jefferson Lab & Brookhaven Lab), EicC (IMP, Lanzhou), and eLHC

Particular interest is role of glue in QCD



Partonic structure of gluonic states complementary to such a program

first baby-steps PRD98 014511 (2018)

Distillation is powerful

- Early results suggest rich spectrum of mesons & baryons - including hybrids
  - ➡ @JLab → new expt. proposals
    - ➡ GlueX: kaon PID upgrade, CLAS12: hybrid baryons

Distillation is powerful

- Early results suggest rich spectrum of mesons & baryons - including hybrids
  - ➡ @JLab → new expt. proposals
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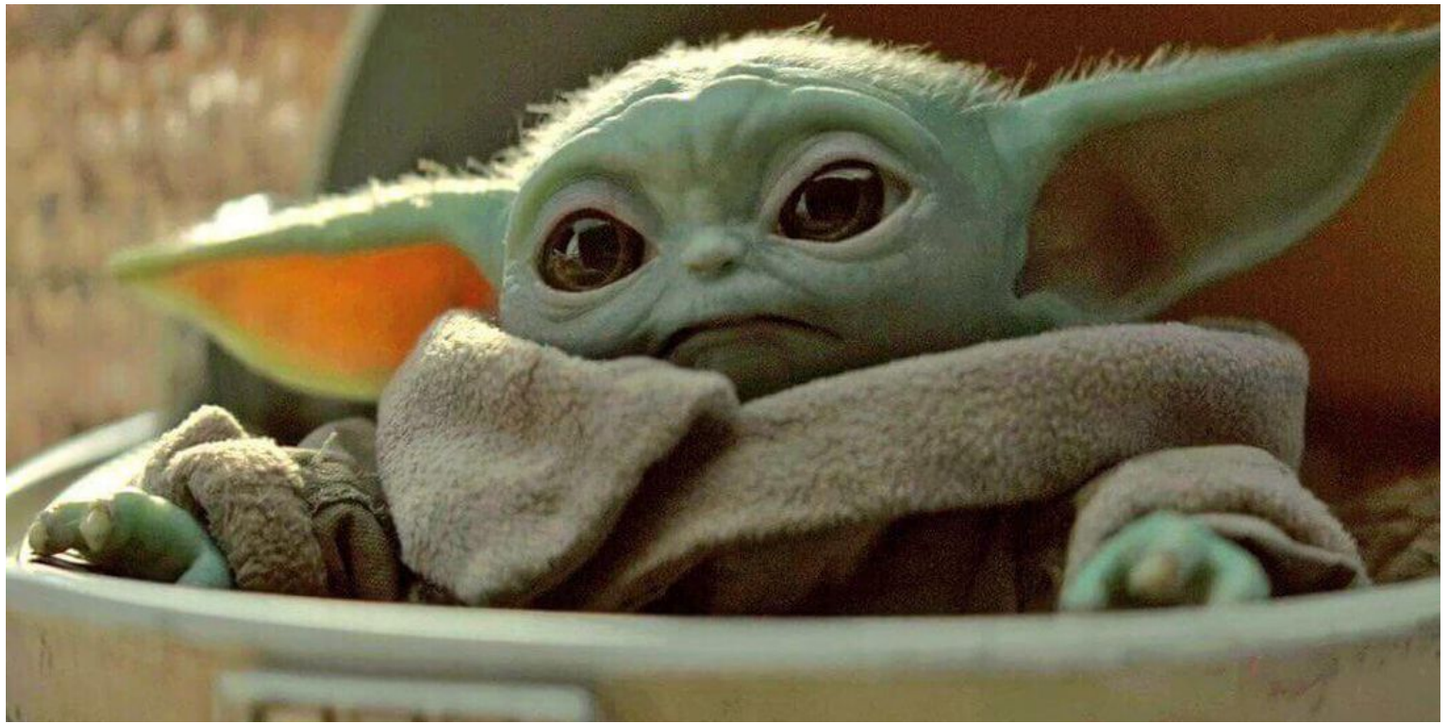
But this is a program in its infancy



Distillation is powerful

- Early results suggest rich spectrum of mesons & baryons - including hybrids
  - ➡ @JLab → new expt. proposals
    - ➡ GlueX: kaon PID upgrade, CLAS12: hybrid baryons

- Near term:
  - See projects use range of light/charm-quark masses → poles & couplings
  - ➡ Estimates of branching fractions useful for expt. analysis
  - 3-body

- Opportunities & challenges
  - XYZ-s & P_c
  - Hybrid hadrons
  - Quantitative understanding of resonant structures

- Long term:
  - Near light-cone formalism
  - Envision merging spectroscopy and structure projects
    - ➡ Understanding role of gluonic structures