

Non-perturbative renormalization by decoupling

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OVERVIEW

The challenge

Renormalization in 3M

Exploratory study

Conclusions

THE SCALE OF QCD: Λ -PARAMETER

$$\Lambda_s = \mu \left[b_0 \bar{g}_s^2(\mu) \right]^{\frac{-b_1}{2b_0^2}} e^{-\frac{1}{2b_0 \bar{g}_s^2(\mu)}} \underbrace{\exp \left\{ - \int_0^{\bar{g}_s(\mu)} dx \left[\frac{1}{\beta_s(x)} + \frac{1}{b_0 x^3} - \frac{b_1}{b_0 x} \right] \right\}}_{\mathcal{O}(\bar{g}_s^2(\mu))}$$

The intrinsic scale of QCD

- ▶ Λ_s same units as μ
- ▶ Λ_s is RGI: $d\Lambda_s/d\mu = 0$
- ▶ $\bar{g}_s^2(\mu)$ is a function of Λ_s/μ : Λ_s dictates what are “low” and “high” energies.
- ▶ (i.e. $\alpha_{\overline{\text{MS}}}(M_Z)$ is trivial to compute if one knows $\Lambda_{\overline{\text{MS}}}^{(5)}$)
- ▶ Scheme dependent, but if

$$\bar{g}_{s'}^2(\mu) \xrightarrow{\bar{g}_s \rightarrow 0} \bar{g}_s^2(\mu) + c_{ss'} \bar{g}_s^4(\mu) + \dots$$

then

$$\frac{\Lambda_{s'}}{\Lambda_s} = \exp \left(\frac{-c_{ss'}}{2b_0} \right).$$

- ▶ Defined non-perturbatively (even $\Lambda_{\overline{\text{MS}}}$)

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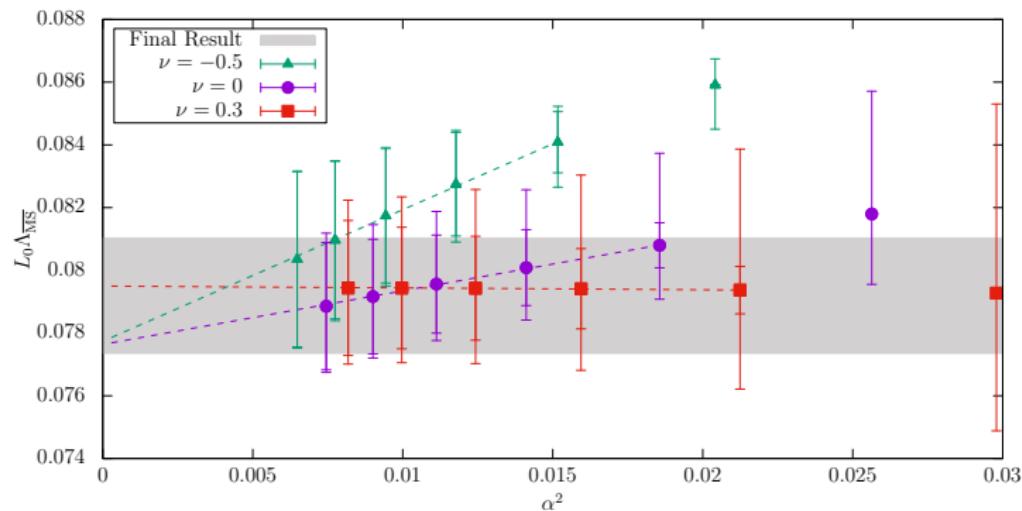
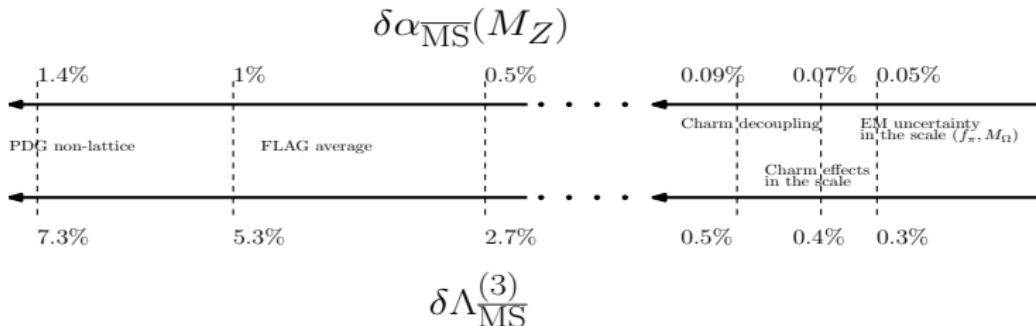
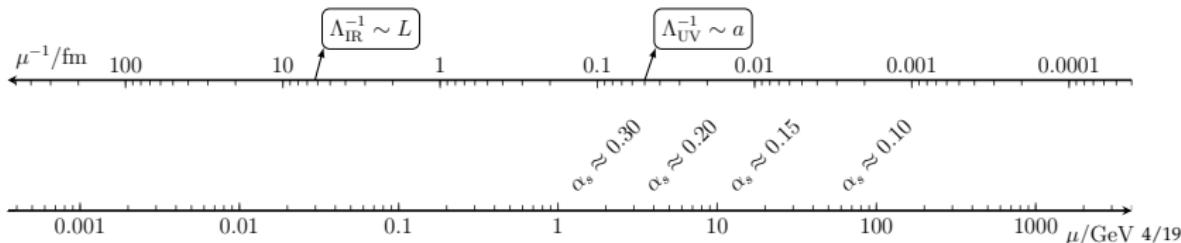


Figure: Source: ALPHA Eur.Phys.J. C78 (2018)

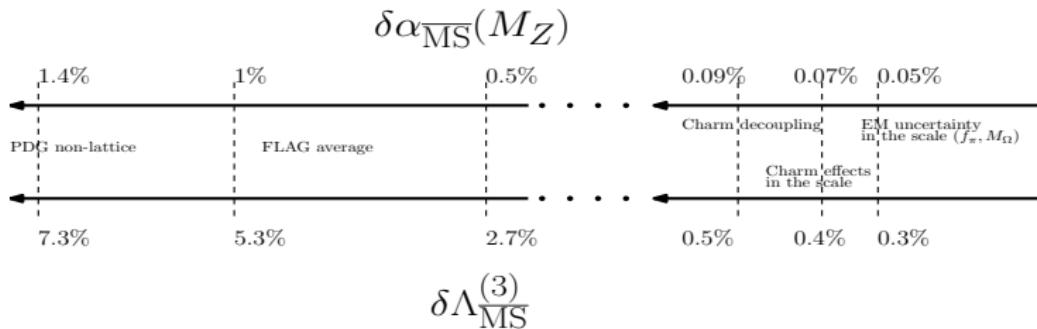
THE CHALLENGE IN DETERMINING Λ : CONNECTING QUARKS WITH HADRONS



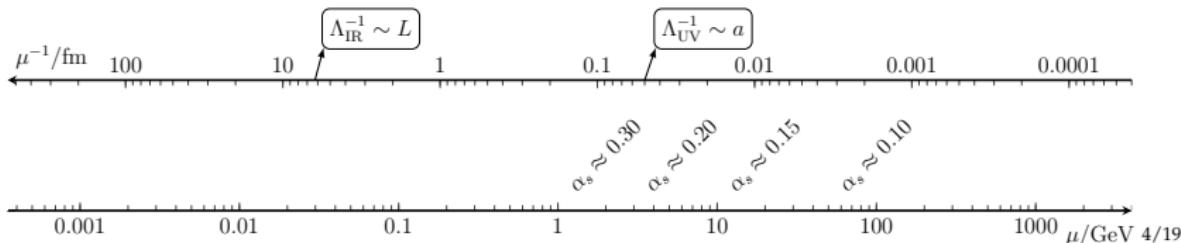
- ▶ 5.3% error in $\Lambda^{(3)} \leftrightarrow$ 1% error in $\alpha_{\overline{\text{MS}}}(M_Z)$:
 - ▶ Electromagnetic corrections
 - ▶ Charm effects (i.e. $N_f = 2 + 1 + 1$ vs. $N_f = 2 + 1$)
 - ▶ Connecting hadronic physics with EW scale **without assumptions** on low scale physics (i.e. perturbation theory)



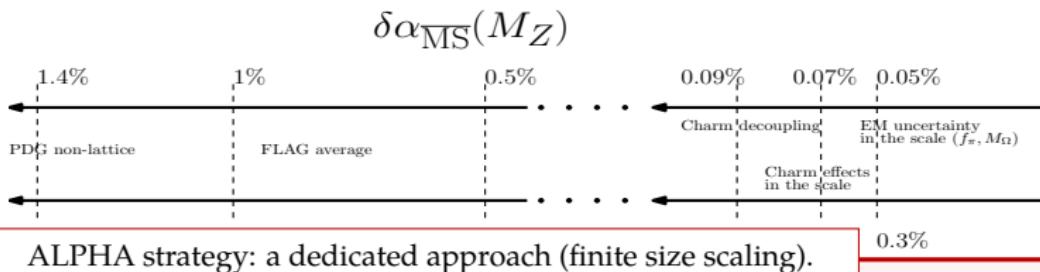
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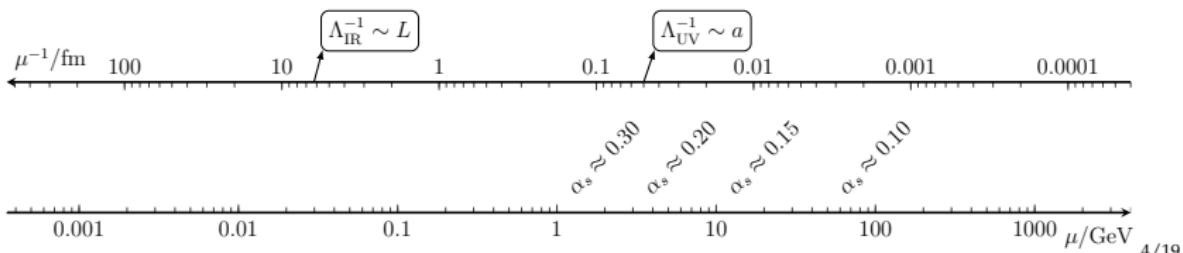


THE CHALLENGE IN DETERMINING Λ : CONNECTING QUARKS WITH HADRONS



We aim at a 3% error in $\Lambda^{(3)}$. We do not care about EM, charm quarks, ...

- ▶ 5.3% error in $\Lambda^{(3)} \leftrightarrow 1\%$ error in $\alpha_{\overline{\text{MS}}}(M_Z)$:
 - ▶ Connecting hadronic physics with EW scale **without assumptions** on low scale physics (i.e. perturbation theory)



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3M: A UNIVERSE WITH THREE HEAVY DEGENERATE QUARKS ($M \gg \Lambda$)

Alice uses fundamental theory

$$S_{\text{fund}}[A_\mu, \psi, \bar{\psi}] = \int d^4x \left\{ -\frac{1}{2g^2} \text{Tr}(F_{\mu\nu}F_{\mu\nu}) + \sum_{i=1}^3 \bar{\psi}_i (\gamma_\mu D_\mu + M) \psi_i \right\}$$

Bob uses effective theory

$$S_{\text{eff}}[A_\mu] = -\frac{1}{2g_{\text{eff}}^2} \int d^4x \{ \text{Tr}(F_{\mu\nu}F_{\mu\nu}) \} + \frac{1}{M^2} \sum_k \omega_k \int d^4x \mathcal{L}_k^{(6)} + \dots$$

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Decoupling

- Dimensionless “low energy quantities” $\sqrt{t_0}/r_0, w_0/\sqrt{8t_0}, r_0/w_0, \dots$ from effective theory

$$\frac{\mu_1^{\text{fund}}(M)}{\mu_2^{\text{fund}}(M)} = \frac{\mu_1^{\text{eff}}}{\mu_2^{\text{eff}}} + \mathcal{O}\left(\frac{\mu^2}{M^2}\right)$$

RENORMALIZATION IN 3M: ALICE DETERMINES THE STRONG COUPLING

$$\frac{\Lambda}{\mu} = \left[b_0 \bar{g}^2(\mu) \right]^{-\frac{b_1}{2b_0^2}} e^{-\frac{1}{2b_0 \bar{g}^2(\mu)}} \exp \left\{ - \int_0^{\bar{g}(\mu)} dx \left[\frac{1}{\beta(x)} + \frac{1}{b_0 x^3} - \frac{b_1}{b_0^2 x} \right] \right\}.$$

- ▶ Determine non-perturbatively the β -function in the fundamental ($N_f = 3$) theory, mass-less scheme.
- ▶ Integral up to $\bar{g}^{(3)}(\mu_{\text{ref}})$ (in a mass-less scheme!) gives:

$$\frac{\Lambda^{(3)}}{\mu_{\text{ref}}}$$

- ▶ Only needs to compute a dimensionless ratio

$$\mu_{\text{ref}} \sqrt{8t_0(M)}$$

- ▶ Result

$$\sqrt{8t_0(M)} \Lambda^{(3)} = \frac{\Lambda^{(3)}}{\mu_{\text{ref}}} \times \mu_{\text{ref}} \sqrt{8t_0(M)}$$

RENORMALIZATION IN 3M: BOB DETERMINES THE STRONG COUPLING

$$\frac{\Lambda}{\mu} = \left[b_0 \bar{g}^2(\mu) \right]^{-\frac{b_1}{2b_0^2}} e^{-\frac{1}{2b_0 \bar{g}^2(\mu)}} \exp \left\{ - \int_0^{\bar{g}(\mu)} dx \left[\frac{1}{\beta(x)} + \frac{1}{b_0 x^3} - \frac{b_1}{b_0^2 x} \right] \right\}.$$

- ▶ Determine non-perturbatively the β -function in the effective ($N_f = 0$) theory.
- ▶ Integral up to $\bar{g}^{(0)}(\mu'_{\text{ref}})$ gives:

$$\frac{\Lambda^{(0)}}{\mu'_{\text{ref}}}$$

- ▶ Determine the dimensionless ratio

$$\mu'_{\text{ref}} \sqrt{8t_0}$$

- ▶ Match across quark threshold to convert to $\Lambda^{(3)}$ (using perturbation theory)

$$\Lambda^{(3)} \sqrt{8t_0} = \Lambda^{(0)} \sqrt{8t_0} \times \frac{1}{P(M/\Lambda)}.$$

- ▶ Matching factor $P(M/\Lambda)$ [ALPHA 1809.03383]:

- ▶ Known in perturbation theory up to three-loops. Power series in $\alpha(m^*)$
- ▶ “Good” PT: corrections very small even at m_c^* .

RELATION BETWEEN ALICE AND BOB COMPUTATION

Relation between Alice and Bob results:

$$\Lambda^{(3)} \sqrt{8t_0(M)} = \Lambda^{(0)} \sqrt{8t_0} \times \frac{1}{P(M/\Lambda)} + \mathcal{O}(\alpha^3(m^*)) + \mathcal{O}\left(\frac{1}{t_0 M^2}\right)$$

Bob is telling us that $\Lambda^{(3)}$ can be computed from $\Lambda^{(0)}$

$$\Lambda^{(3)} = \lim_{M \rightarrow \infty} \frac{1}{\sqrt{8t_0(M)}} \times \Lambda^{(0)} \sqrt{8t_0} \times \frac{1}{P(M/\Lambda)}$$

We need

- ▶ Running in pure gauge: $\Lambda^{(0)} \sqrt{8t_0}$
- ▶ A scale in a world with degenerate massive quarks: $\sqrt{8t_0(M)}$ in fm/MeV.

Lattice QCD can simulate unphysical worlds

$$\sqrt{8t_0(M)} = \sqrt{8t_0^{\text{phys}}} \times \lim_{a \rightarrow 0} \frac{\sqrt{8t_0(M)/a}}{\sqrt{8t_0^{\text{phys}}}/a}$$

with $\sqrt{8t_0^{\text{phys}}} = 0.415(4)(2)$ fm from [Bruno et al. '17]

NON-PERTURBATIVE RENORMALIZATION BY DECOUPLING

Master relation

$$\frac{\Lambda^{(N_f)}}{\mu_{\text{dec}}(M)} = \frac{\Lambda^{(0)}}{\mu_{\text{dec}}} \times \frac{1}{P(M/\Lambda)} + \mathcal{O}(\alpha^3(m^\star)) + \mathcal{O}\left(\frac{\mu_{\text{dec}}^2}{M^2}\right)$$

With a proper limit:

$$\Lambda^{(N_f)} = \lim_{M \rightarrow \infty} \mu_{\text{dec}}(M) \times \frac{\Lambda^{(0)}}{\mu_{\text{dec}}} \times \frac{1}{P(M/\Lambda)}$$

Where

- ▶ Pure gauge running: $\Lambda^{(0)}/\mu_{\text{dec}}$
- ▶ A scale with N_f massive degenerate quarks: $\mu_{\text{dec}}(M)$

NOTE: this is not completely trivial

$$\frac{\mu_{\text{dec}}(M)}{\mu_{\text{dec}}^{\text{phys}}} = \lim_{a \rightarrow 0} \frac{a\mu_{\text{dec}}(M)}{a\mu_{\text{dec}}^{\text{phys}}}$$

is a difficult extrapolation (M wants to be large, aM wants to be small).

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OUR SETUP: CHOICES OPTIMIZED TO BE ABLE TO SIMULATE HEAVY QUARKS

$$\Lambda^{(3)} = \mu_{\text{dec}}(M) \times \frac{\Lambda^{(0)}}{\mu_{\text{dec}}} \times \frac{1}{P(M/\Lambda^{(3)})} + \mathcal{O}(\alpha^3(m^*)) + \mathcal{O}\left(\frac{\mu_{\text{dec}}}{M}\right) + \mathcal{O}\left(\frac{\mu_{\text{dec}}^2}{M^2}\right)$$

- ▶ Work in finite volume schemes with Schrödinger Functional boundary conditions: $T \times L^3$ with Dirichlet bcs. in time. ($\mu \sim 1/L$): "Only" two scales.
- ▶ Use Gradient Flow couplings

$$\bar{g}^2(\mu) = \mathcal{N}^{-1}(c, a/L) t^2 \langle E(t) \rangle \Big|_{\mu^{-1} = \sqrt{8t} = cL}.$$

- ▶ Matching condition ($\{N_f = 3, M\} \leftrightarrow \{N_f = 0\}$) between massive scheme and effective theory

$$\bar{g}^2(\mu_{\text{dec}}(M)) \Big|_{N_f=3, M, T=2L} = \bar{g}^2(\mu_{\text{dec}}) \Big|_{N_f=0, T=2L}.$$

Caveats: Schrödinger Functional boundary conditions break chiral symmetry

- ▶ μ_{dec}/M corrections to decoupling
- ▶ Choose $T = 2L$. Boundary effects very suppressed
- ▶ Most probably completely negligible, but keep in mind...

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$$\frac{\Lambda^{(3)}}{\mu_{\text{dec}}(M)} \times P \left(\frac{M}{\mu_{\text{dec}}} \frac{\mu_{\text{dec}}}{\Lambda^{(3)}} \right) = \frac{\Lambda^{(0)}}{\mu_{\text{dec}}} + \mathcal{O}(\alpha^3(m^*)) + \mathcal{O}\left(\frac{\mu_{\text{dec}}}{M}\right) + \mathcal{O}\left(\frac{\mu_{\text{dec}}^2}{M^2}\right)$$

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$$\rho \times P \left(\frac{M}{\mu_{\text{dec}}} \frac{1}{\rho} \right) = \frac{\Lambda^{(0)}}{\mu_{\text{dec}}} + \mathcal{O}(\alpha^3(m^*)) + \mathcal{O}\left(\frac{\mu_{\text{dec}}}{M}\right) + \mathcal{O}\left(\frac{\mu_{\text{dec}}^2}{M^2}\right)$$

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OUR SETUP: MOST COLUMNS OF THE TABLE ALREADY KNOWN

3-flavor renormalization program by ALPHA

- ▶ $\mu_{\text{dec}}(M)$ [GeV]: Switch to mass-less scheme. Use ALPHA [\[ALPHA 1706.03821\]](#)

$$\bar{g}^2(\mu_{\text{dec}}(M)) \Big|_{N_f=3, M, T=2L} \implies \bar{g}^2(\mu_{\text{dec}}(M)) \Big|_{N_f=3, M=0, T=L} \implies \mu_{\text{dec}}(M) \text{ in [fm].}$$

- ▶ M [GeV]: NP-renormalization ALPHA [\[ALPHA 1802.05243\]](#)

$$LM = \frac{L}{a} Z_m(\mu_{\text{dec}}(M))(1 + b_m am_q) Z_{RGI}(\mu_{\text{dec}}(M)) (am_0 - am_c)$$

- ▶ Z_m determined non-perturbatively (\rightarrow no details here!)
- ▶ Z_{RGI} Known non-perturbatively [\[ALPHA 1802.05243\]](#)
- ▶ 1-loop value for b_m , b_g : Not fully $\mathcal{O}(a)$ -improved.
- ▶ $\Lambda^{(0)}/\mu_{\text{dec}}$: Known very precisely [\[M. Dalla Brida, A. Ramos. 1905.05147\]](#)

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- M [GeV]: NP-renormalization ALPHA [\[ALPHA 1802.05243\]](#)

$$LM = \frac{L}{a} Z_m(\mu_{\text{dec}}(M))(1 + b_m am_q) Z_{RG1}(\mu_{\text{dec}}(M)) (am_0 - am_c)$$

Missing piece: massive \leftrightarrow massless: LCP accurately known at $M = 0$

L/a	β	$\bar{g}^2(\mu_{\text{dec}}(M)) \Big _{N_f=3, M=0, T=L}$	$\mu_{\text{dec}}(M)$ [GeV]
12	4.3020	3.9533(59)	0.789(15)
16	4.4662	3.9496(77)	0.789(15)
20	4.5997	3.9648(97)	0.789(15)
24	4.7141	3.959(50)	0.789(15)
32	4.90	3.949(11)	0.789(15)

DETERMINE MASSIVE COUPLING FOR MATCHING

Example: $L/a = 20$

β	κ	$z = M/\mu_{\text{dec}}(M)$	$M [\text{GeV}]$	$\bar{g}^2(\mu_{\text{dec}}(M)) \Big _{N_f=3, M, T=2L}$
4.5997	0.1352889	0	0	3.9648(97)
4.6083	0.133831710060	1.972(18)	1.6	4.290(16)
4.6172	0.132345249425	4.000(37)	3.2	4.458(15)
4.6266	0.130827894135	6.000(58)	4.7	4.555(15)
4.6364	0.129273827559	8.000(85)	6.3	4.717(16)

Extrapolate results to the continuum: Example $z = 6$

We determine $\bar{g}^2(\mu_{\text{dec}}(M)) \Big|_{N_f=3, M, T=2L}$ change cuts $(aM)^2 < 1/8, 1/4$

L/a	β	aM	$\bar{g}^2(\mu_{\text{dec}}(M)) \Big _{N_f=3, M, T=2L}$
12	4.3499	0.50	4.636(12)
16	4.5008	0.37	4.588(15)
20	4.6266	0.30	4.555(15)
24	4.7359	0.25	4.555(15)
32	4.9159	0.18	4.517(15)
∞	$(aM)^2 < 1/4$		4.504(17)
∞	$(aM)^2 < 1/8$		4.499(26)

CONTINUUM EXTRAPOLATIONS WITH TWO CUTS: $(aM)^2 < 1/4, 1/8$

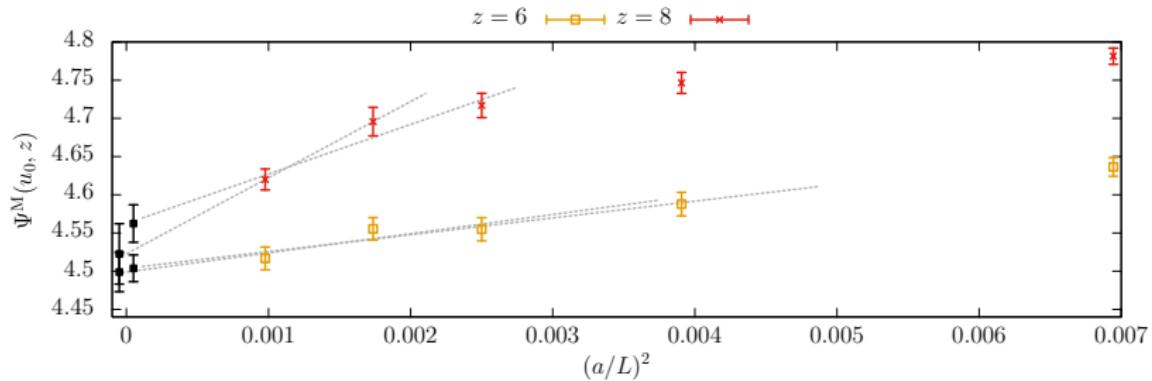
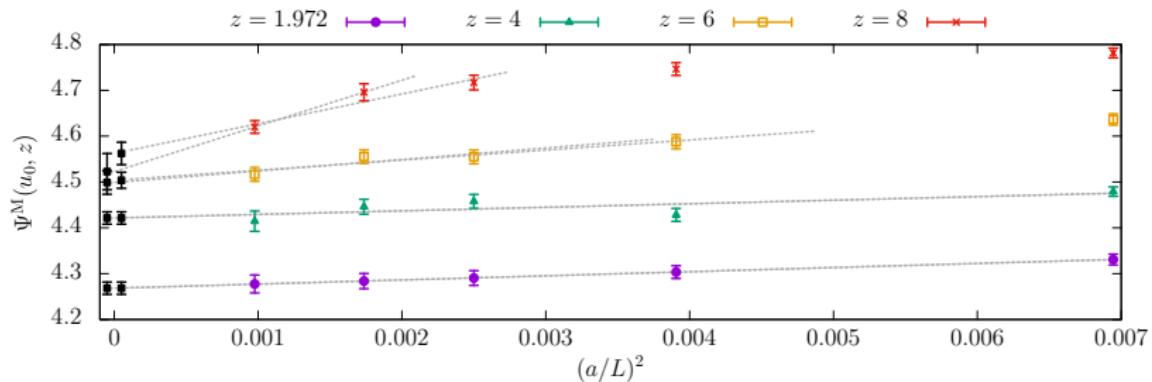
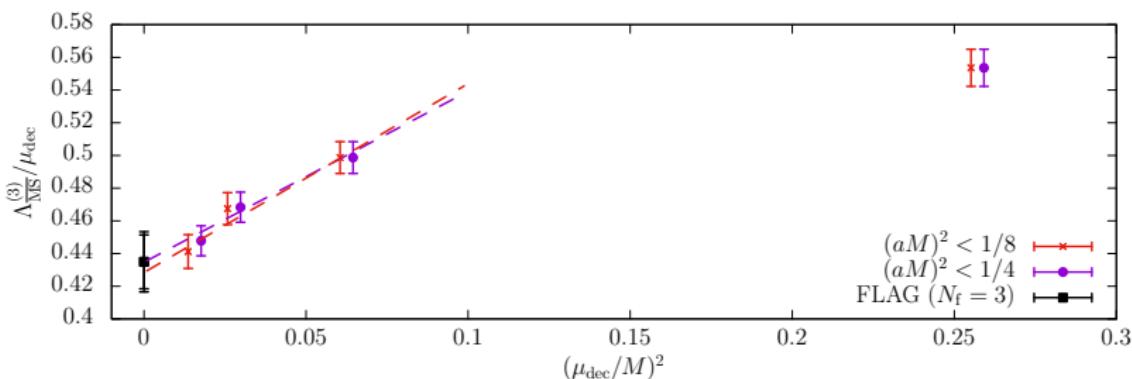


TABLE CAN BE FILLED

M [GeV]	$\bar{g}^2(\mu_{\text{dec}}(M))$	$\Lambda^{(0)}/\mu_{\text{dec}}$	$\Lambda^{(3)}/\mu_{\text{dec}}$	$\frac{1}{P(M/\Lambda)}$	$\Lambda^{(3)}$ [MeV]	Δ_3 [MeV]
1.6	4.268(13)	0.554(11)	0.689(11)	0.8038(63)	437(12)	3.0
3.2	4.421(13)	0.4987(97)	0.725(11)	0.6878(37)	393(11)	1.6
4.7	4.499(26)	0.4675(98)	0.743(12)	0.6291(30)	368(10)	1.0
6.3	4.523(40)	0.441(10)	0.749(14)	0.5895(30)	348(10)	0.8
∞		FLAG19 (lattice)			343(12)	



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CONCLUSIONS: NON-PERTURBATIVE RENORMALIZATION BY DECOUPLING

- Lattice QCD can determine scales at un-physical values of the parameters

$$\Lambda^{(N_f)} = \lim_{M \rightarrow \infty} \mu_{\text{dec}}(M) \times \frac{\Lambda^{(0)}}{\mu_{\text{dec}}} \times \frac{1}{P(M/\Lambda)}$$

with

- $\mu_{\text{dec}}(M)$: Scale with N_f heavy quarks ($M \gg \Lambda$)
- $\Lambda^{(0)}/\mu_{\text{dec}}$: Computed non-perturbatively in pure gauge
- $P(M/\Lambda)$: Perturbative relation between fundamental and effective theories
- Method is generic
 - Similar expressions for other RGI invariants: $M, \hat{B}_K, \hat{B}_B, \dots$
 - Valid in finite or infinite volume renormalization schemes
 - If you can, just compute $\sqrt{8t_0}, w_0$ with 3-4 quarks as heavy as possible!
- Still working (larger L/a) but It works!!
 - Finite volume setup: Small PT corrections $\mathcal{O}(\alpha^3(m^*))$, window problem ameliorated
 - $\mu_{\text{dec}}(M) = 789(15)$ MeV. Applied with $M = 1.6, \dots, 6.3$ GeV
 - Non-perturbative running in pure gauge from $\mu = 789$ MeV to $\mu = \infty$
 - $\Lambda^{(3)}$ in agreement with current knowledge
- Probably best approach to reduce error in α_s substantially
 - Running done in pure gauge!
 - Better precision in pure gauge: Small lattice spacing, efficient algorithms.
 - Switch massive \leftrightarrow massless schemes little effect in total error.
 - $\lim_{M \rightarrow \infty}$ can be controlled

CONCLUSIONS: LATTICE QCD IN THE 2020

- ▶ Electromagnetic corrections
- ▶ Charm effects (i.e. $N_f = 2 + 1 + 1$ vs. $N_f = 2 + 1$)
- ▶ Statistical precision in difficult observables.
- ▶ Connecting hadronic physics with EW scale without assumptions on low scale physics (i.e. perturbation theory).
- ▶ “Non-perturbative extrapolation” is difficult.
 - ▶ Correction decreases very slowly $\alpha''(\mu_{\text{PT}})$
 - ▶ Even in pure gauge, with much more computer power, the situation is far from clear
 - ▶ Is this under control for M, B_K, \dots ?

Lattice in the 2020

- ▶ “custom” tools for each problem.
- ▶ Generating ensembles no longer the crucial/key/challenging element?