Radiation from electrons passing through helical undulator or colliding with circularly polarized laser wave

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Introduction

Longitudinally polarized positrons are obtained by photo-production:

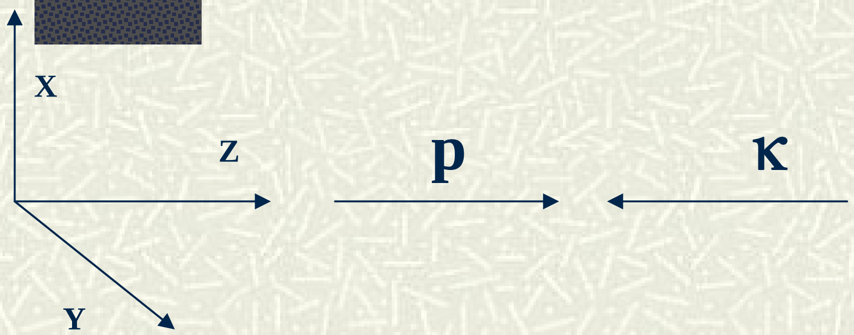
$$\gamma_1 + \gamma_2 \rightarrow e^+ + e^-$$

At least one of two photons should be circularly polarized. When both photons are real, the threshold condition $\omega_1\omega_2 \geq m^2$ implies ω of one photon above 2 TeV (for CO_2 laser)

In a target, the conditions $\omega_1 \geq 2m,\ |\vec{k}_2| \geq 2m^2/\omega_1$ are easily fulfilled

According to existing theory of radiation, the circularly polarized photons are emitted during helical motion of unpolarized electrons, which can be realized in the field of a **circularly polarized laser wave** or in a **helical undulator**

Similarity of radiation in laser wave fields and in undulators



In the laboratory system

$$\vec{H}_L = B[\vec{e}_x \cos\zeta + \mu \vec{e}_y \sin\zeta]$$

$$\zeta = kz \equiv (\kappa_u x), \; k = 2\pi/\lambda_u$$

$$\vec{A}_L = a[\vec{e}_x \cos\chi + \mu \vec{e}_y \sin\chi]$$

$$\chi = \kappa_w^0 (t + z), \; \kappa_w = \kappa_w^0 (1, 0, 0, -1)$$

In the electron rest system

Helical undulator

$$\vec{H}_R = \frac{B\tilde{\kappa}_u^0}{k\mathrm{v}}[\vec{e}_x \cos\zeta + \mu \vec{e} \sin\zeta]$$

$$\vec{E}_R = \frac{B\tilde{\kappa}_u^0}{k}[-\mu \vec{e}_x \sin\zeta + \vec{e}_y \cos\zeta]$$

$$\tilde{\kappa}_u = \gamma k(\mathrm{v},0,0,\text{-}1), \; \gamma = 1/\sqrt{1-\mathrm{v}^2}$$

Laser wave field

$$\vec{H}_R = -\mu a \tilde{\kappa}_w [\vec{e}_x \cos\chi + \mu \vec{e}_y \sin\chi]$$

$$\vec{E}_R = -\mu a \tilde{\kappa}_w [-\mu \vec{e}_x \sin\chi + \vec{e}_y \cos\chi]$$

$$\tilde{\kappa}_w = \gamma \kappa_w^0 (1+\mathrm{v})(1,0,0,-1), \; a \Leftrightarrow B/k$$

Basic parameters, their meaning and magnitude for undulators and lasers

Emission of radiation in the wave is characterized by two parameters:

$$\xi^2 = \frac{e^2\langle \vec{A}^2\rangle}{m^2} = \frac{e^2 a^2}{m^2} = 2\alpha\lambda_w \lambda\!\!\!^{-}{}_c^2 n_w \;,\; \xi^2_{\max} = \frac{\mathrm{E_b(J)}}{\omega_{\mathrm{w}}(\mathrm{eV})}\frac{\lambda_w}{\sigma_z}\frac{(29.4)^2}{\sigma_\perp^2(\mu m)}$$

It is the effective interaction constant, as $\xi^2 \approx \alpha \mathrm{N_{int}}$

At $\xi^2 \ll 1$ perturbation theory (Compt. effect in our case) is applicable

For a helical undulator $\xi_u^2 = (eB/km)^2 \equiv \mathrm{K}^2 = [0.935 B(\mathrm{T})\,\lambda_u(cm)]^2$

The kinematic parameter $s = 2(kp)/m^2$ enters 4-mom. conservation low

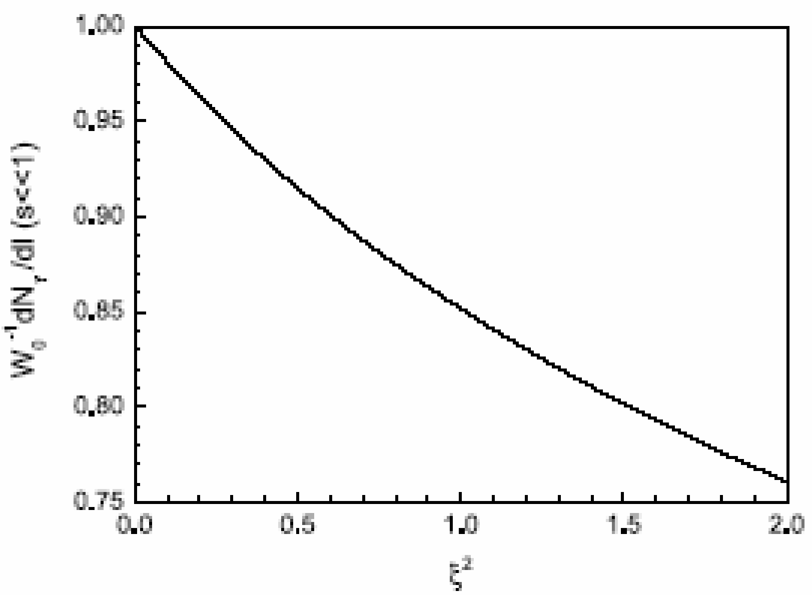
$$x \equiv \frac{\omega}{\varepsilon} = \frac{s}{1+s+(\gamma\vartheta)^2},\quad \gamma = \frac{\varepsilon}{m},\quad x_{\max} = \frac{s}{1+s}$$

$$s_u \approx 0.95\cdot 10^{-6}\,\varepsilon(\mathrm{GeV})/\lambda_{\mathrm{u}}(cm) \quad s_w \approx 1.53\cdot 10^{-2}\,\varepsilon(\mathrm{GeV})\,\omega_w(\mathrm{eV})$$

Photon emission rate at ʔ □]

At $s \ll 1$ the total probability per unit length (rate) reads:

$$\frac{dN_\gamma}{dz} = W_C \cdot F(\xi^2), \; W_C = \frac{2}{3}\alpha\xi^2 \frac{(\kappa p)}{\varepsilon}, \; W_C^u = \frac{4\pi\alpha\xi_u^2}{3\lambda_u}, \; W_C^w = \frac{8\pi\alpha\xi_w^2}{3\lambda_w}$$



W_C is independent of ε , one photon per electron is emitted over

$L \sim W_C^{-1}$ i.e. $L_u \approx \dfrac{32.7\lambda_u}{\xi_u^2}$ and

$L_w \approx \dfrac{16.4\lambda_w}{\xi_w^2}$ so that $L \sim \lambda$

Spectral-angular distribution of radiation and its polarization

Spectral-angular distribution: $\frac{dN_\gamma}{d\Gamma dl} = \frac{1}{2}(A + \vec{B}\vec{\zeta}),\ d\Gamma = d\omega d\Omega$ $\quad \vec{\eta} = \vec{B}/A$

$$(A,\vec{B}) = \frac{\alpha}{\pi}\sum_{n=1}^{\infty}\delta(1+\xi^2+(\gamma\vartheta)^2 - n/\nu)\cdot(a^{(n)},\vec{b}^{(n)}),\ \ \nu = \frac{u}{s},\ u = \frac{x}{1-x}$$

Explicit expressions, discussion etc. in NIM **A 547** (2005) 320

$$(a^{(n)},\vec{b}^{(n)}) \propto \nu^{2(n-1)}\xi^{2n},\ b_2^{(n)} \propto \mu(2\tilde{\nu}-n)\quad \tilde{\nu} = \nu(1+\xi^2)$$

For n=1 (Compton scattering)

$$a^{(1)} = \xi^2\{\frac{f(u)}{4} - \nu(1-\nu)\},\ b_2^{(1)} = \frac{\xi^2}{4}\mu f(u)(2\nu-1),$$

$$b_{lin}^{(1)} = \xi^2\nu(1-\nu),\ f(u) = 1+\nu s+(1+\nu s)^{-1}$$

Conservation law (n-th harmonics)

$$x = \frac{n\cdot s}{1+\xi^2+n\cdot s+(\gamma\vartheta)^2}$$

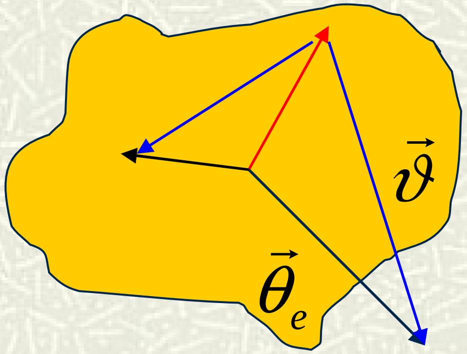
Spectral distribution, collimation and angular divergence in electron beam

Spectral distribution of radiation from the whole electron beam into given collimator reads $\int\limits_{\Omega_{col}} \mathrm{d}^2 n_\perp \int \mathrm{d}^2\theta_e f(\vec{\theta}_e) g(\vec{\theta}_e - \vec{n}_\perp), \int \mathrm{d}^2\theta_e f(\vec{\theta}_e) = 1$

It can be rewritten as

$$\frac{1}{\gamma^2} \int\limits_{\Omega_{col}} \mathrm{d}^2 n_\perp \int \mathrm{d}\varphi\, f(\vec{n}_\perp + \vartheta \vec{e}_\perp) g(\vartheta, \varphi)$$

and



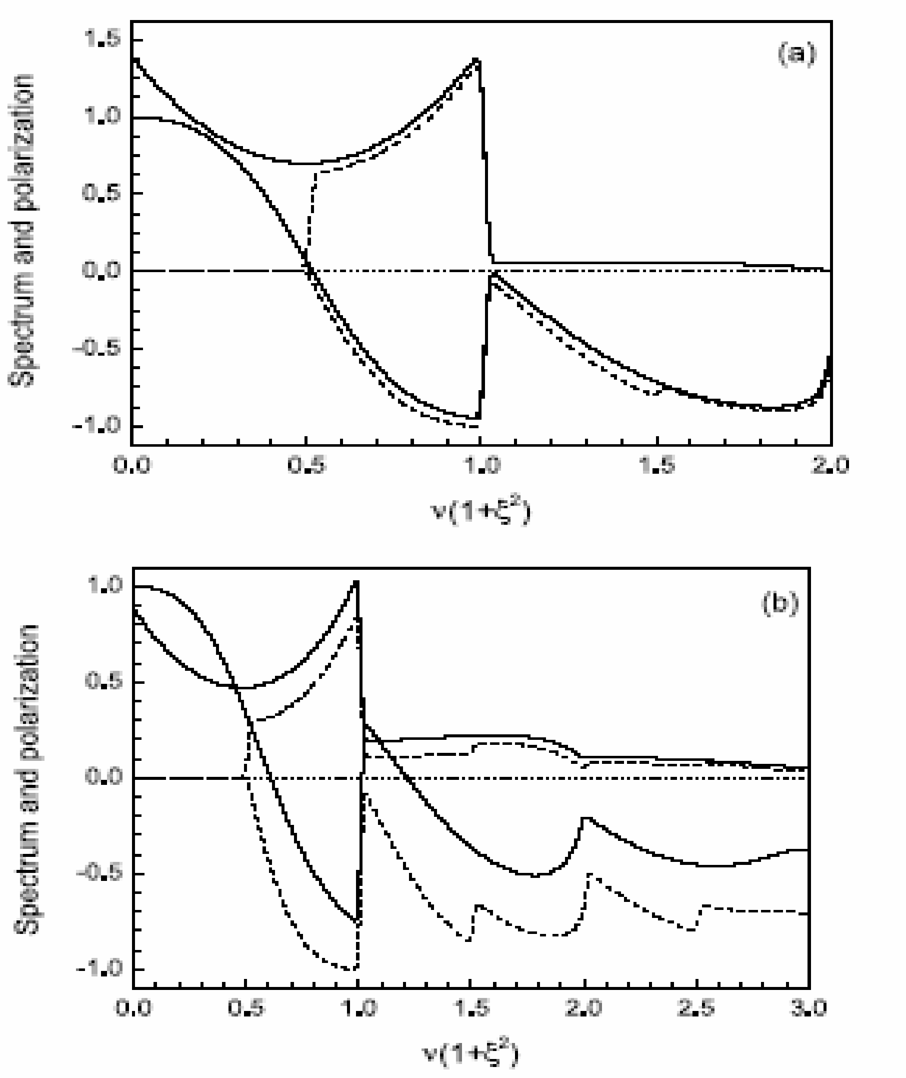
$\frac{\mathrm{dN}_\gamma}{\mathrm{d}\omega \mathrm{dl}}$ as above $(A^\omega, B_2^\omega) = \frac{\alpha}{\gamma^2} \sum_{n=1}^{\infty} \Theta(X_n) \cdot F \cdot (a^{(n)}, b_2^{(n)}),\ X_n = n - \tilde{\nu}$

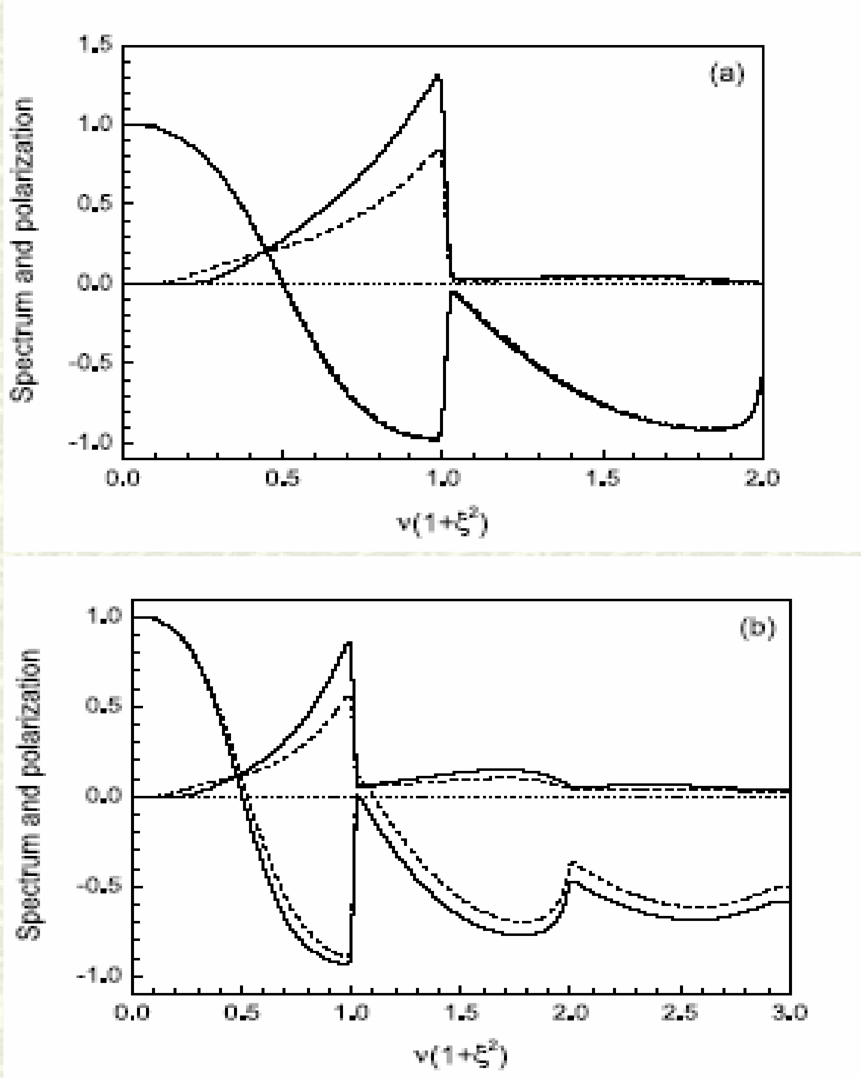
For “round” beam and collimator

$$F(r, \beta),\ r = \frac{\sqrt{X_n}}{\gamma \theta_{col} \sqrt{\nu}} = \frac{\vartheta}{\vartheta_{col}},\ \beta = \frac{\theta_{col}^2}{2\Delta_e^2}$$

$$F \rightarrow \Theta(1 - r^2) \text{ at } \beta □ 1$$

Spectrum and polarization for ξ^2 =0.1 and 1 at s=0.01, $\theta_{col} = \sqrt{1+\xi^2}/\gamma$





Total yield: $\dfrac{dN_\gamma^{tot}}{d\omega} = \int dl \dfrac{dN_\gamma}{d\omega dl}$

The time (l) dependence of ε or θ_{col} if any, can be easily taken into account. The yield per one laser-electron bunch collision is

$$G^{tot} = \int dt \int d^3x\, n_e(\vec{x},t) G(\xi^2(\vec{x},t))$$ but at $\xi^2 \ll 1,\ G(\xi^2) = \xi^2 G_0$

$$G_C^{tot} = 2\alpha\lambda_w \lambda_c^2 G_0 R,\quad R = \int dt \int d^3x\, n_e(\vec{x},t)\, n_w(\vec{x},t)$$ For example, $G_0 = \dfrac{2\alpha(\kappa p)}{3\varepsilon}$ for total number of emitted photons. For Gaussian beams we have

$$R = \frac{N_e N_w Q(\delta_x, \delta_y)}{2\pi\sqrt{\sigma_x^2+\sigma_w^2}\sqrt{\sigma_y^2+\sigma_w^2}},\quad Q = \frac{1}{\sqrt{\pi}}\int_0^\infty dz \frac{\exp(-z^2)}{\sqrt{1+\delta_x z^2}\sqrt{1+\delta_y z^2}}$$ where

$$\delta_{x,y} = \frac{1}{2}\cdot\frac{\sigma_{ze}^2+\sigma_{zw}^2}{\sigma_{x,y}^2+\sigma_w^2}\left[\frac{\sigma_{x,y}^2}{\beta_{x,y}^2}+\frac{\sigma_w^2}{Z_R^2}\right]$$

Conclusion

- Generation of circularly polarized photons using lasers or undulators is due to essentially the same physical mechanism
- The magnitude of basic parameters is very different: $\xi^2_{und} \gg \xi^2_w$ and $s_{und} \ll s_w$ leading to different radiation characteristics
- The laser scheme seems to be more flexible especially for obtaining harder photons and for switching over their helicity