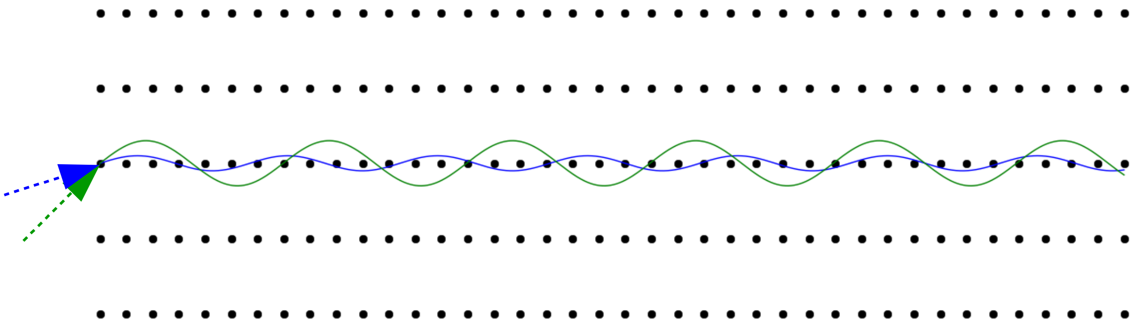
Potential of a muon source based on planar channeling radiation

X. Buffat

- Modeling of channeling radiation
- Muon pair production model
- Electron beam requirement
- Potential source performance
  - Towards collider-compatible performance

More details in arXiv:1910.01541

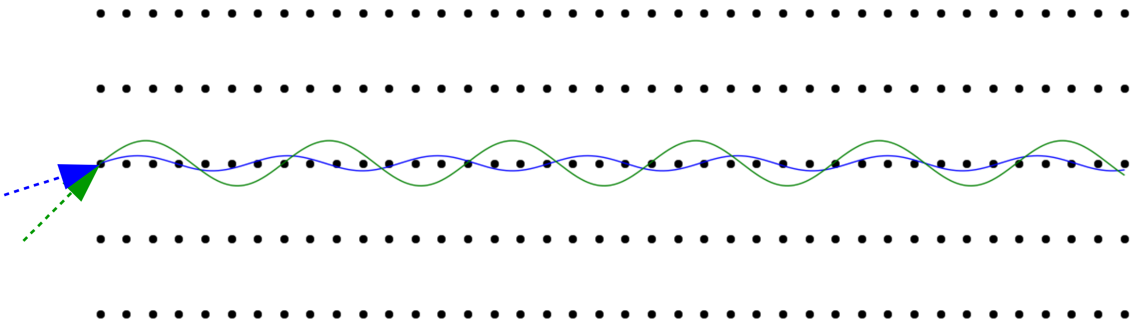
Channeling radiation model

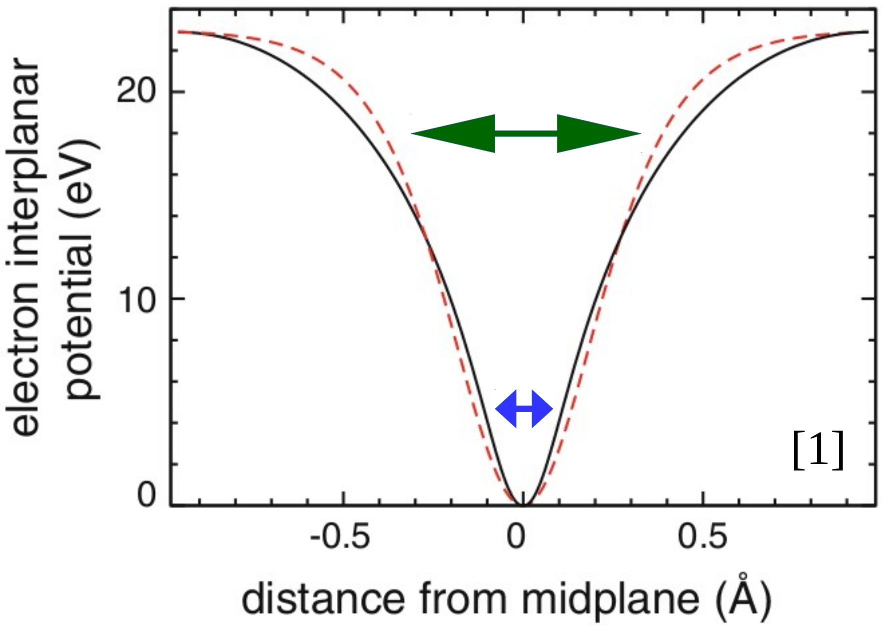


- We consider the motion of high energy electrons trapped transversely in the potential of a crystalline lattice

Channeling radiation model

$$U_{PT}(x) = a_{PT} \tanh^2\left(\frac{x}{b_{PT}}\right)$$





- We consider the motion of high energy electrons trapped transversely in the potential of a crystalline lattice
- Following [1], we derive the emitted photon spectrum by adding the contribution of individual particles based on their trajectory in a Pöschl-Teller potential

$$A_e = b_{PT} \tanh^{-1} \sqrt{\frac{E_e x'^2 + U_{PT}(x)}{a_{PT}}},$$

[1] A. Korol, A. S. yov, and W. Greiner, Channeling and radiation in periodically bent crystals; 2nd ed., Springer Series on Atomic Optical and Plasma Physics (Springer, Berlin, 2014)
[2] P. Schmuser, M. Dohlus, J. Rossbach, and C. Behrens, Free-electron lasers in the ultraviolet and X-ray regime: physical principles, experimental results, technical realization; 2nd ed., Springer Tracts in Modern Physics (Springer, Cham, 2014).

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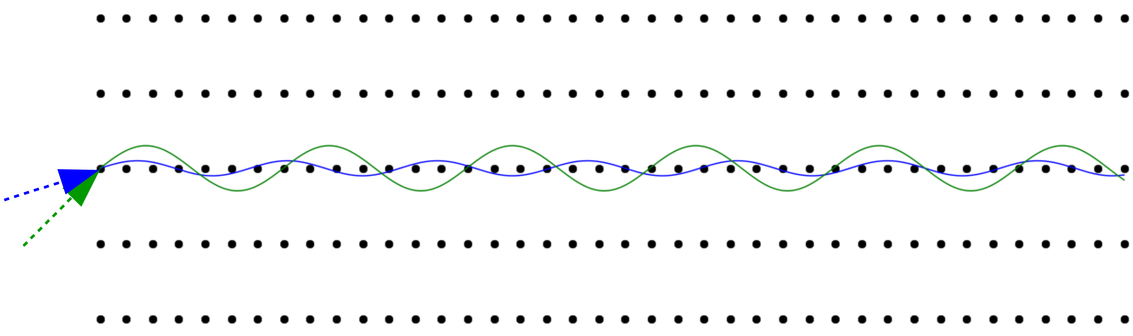

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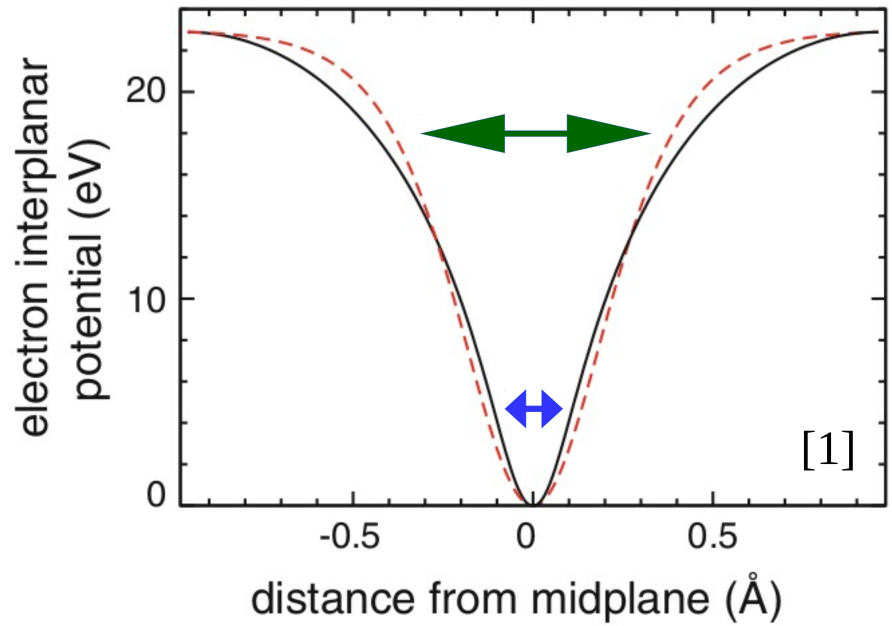

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- Undulator theory [2] can be applied to each particles, but particles oscillating at different amplitudes generate photons at different energies

$$K^2 = 4\gamma_e \frac{a_{PT}}{m_e c^2} \frac{\cosh\left(\frac{A_e}{b_{PT}}\right) - 1}{\cosh^2\left(\frac{A_e}{b_{PT}}\right)} \qquad k_u = \sqrt{\frac{2a_{PT}}{E_e}} \frac{1}{b_{PT} \cosh\left(\frac{A_e}{b_{PT}}\right)}$$

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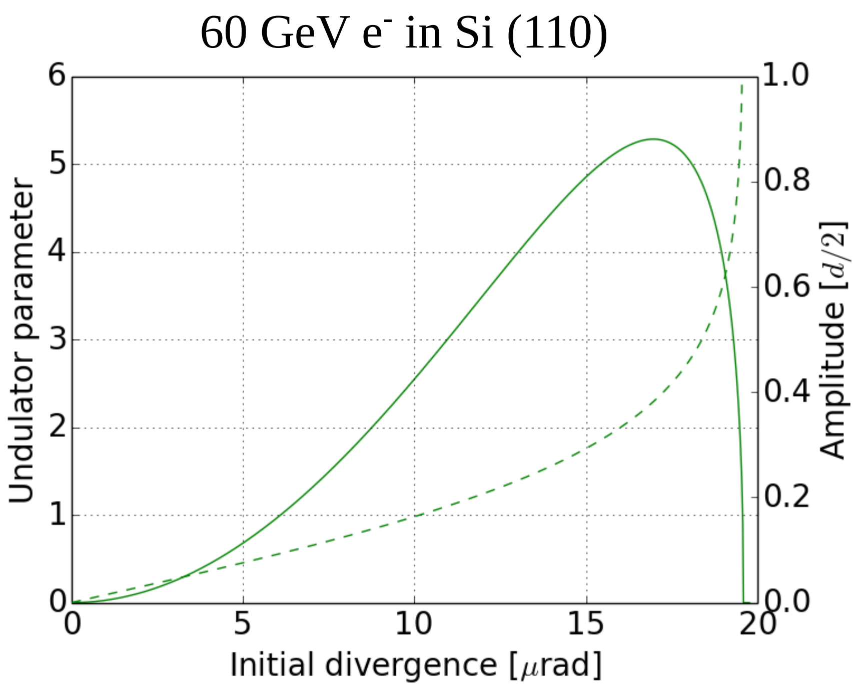
Channeling radiation model

- We consider particles entering the crystal at the center of the channel with a given initial divergence



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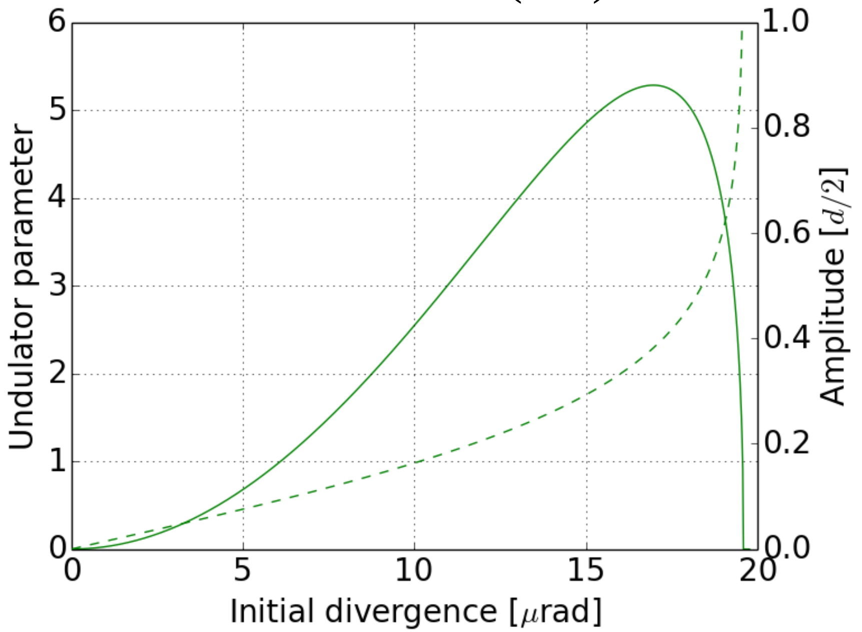
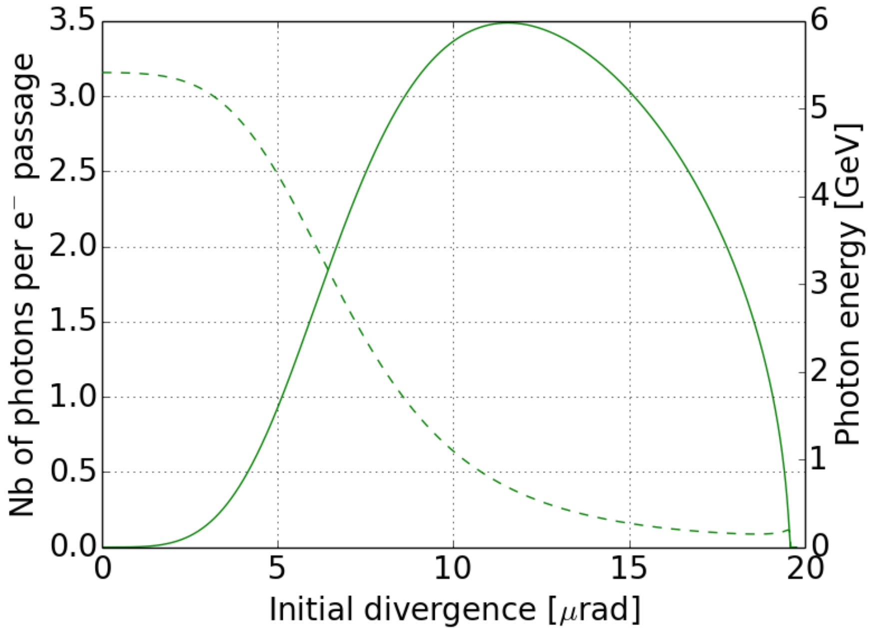
$$E_\gamma = \frac{2\hbar c \gamma_e^2 k_u}{1+\frac{K^2}{2}} \qquad n_\gamma = \frac{e^2 k_u}{24\pi\epsilon_0 \hbar} \frac{K^2}{(1+K^2/2)^2} \frac{L_c}{c}$$

- Particles oscillating at different amplitudes emit at different energies
  - Electron beams size are usually much larger than the interplanar distance the full spectrum should be obtained by integration over the beam phase space at the crystal
  - We need to determine the optimal electron beam divergence

60 GeV e⁻ in Si (110)





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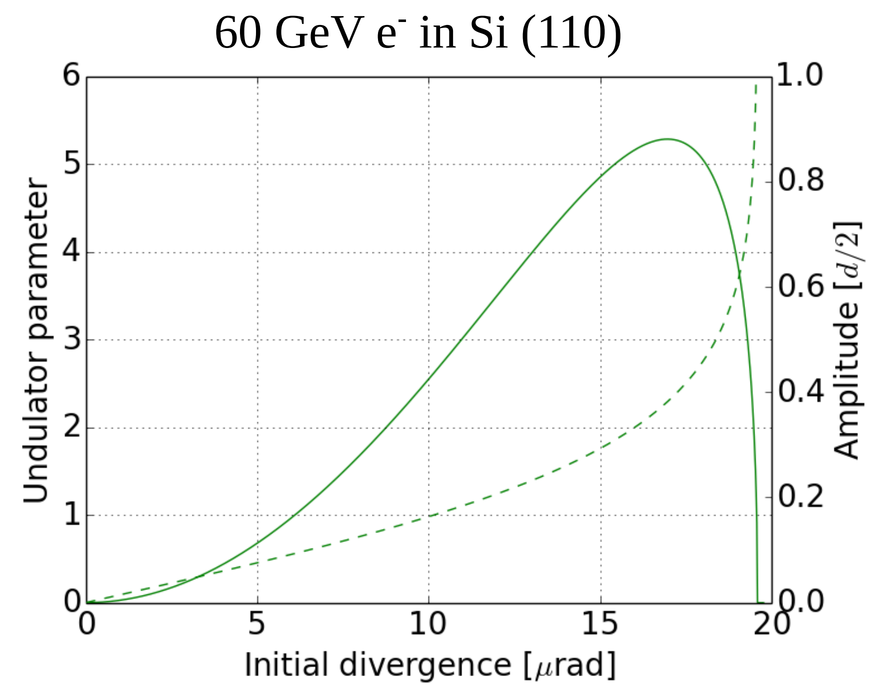
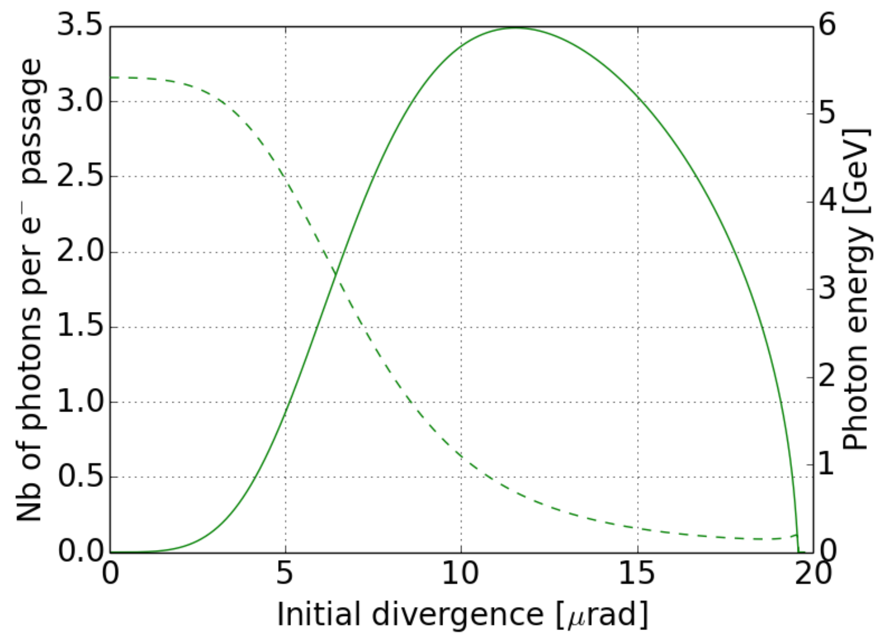
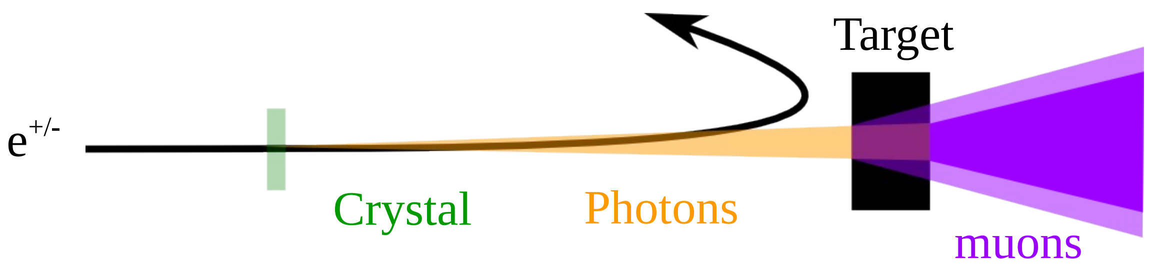
- The crystal length is limited by the energy loss of individual particles with the optimal amplitude
  - In the following we chose L_c such that :

$$\frac{n_\gamma E_\gamma}{E_e} < 0.1 \;\forall\; x, x' \in \mathbb{R}.$$





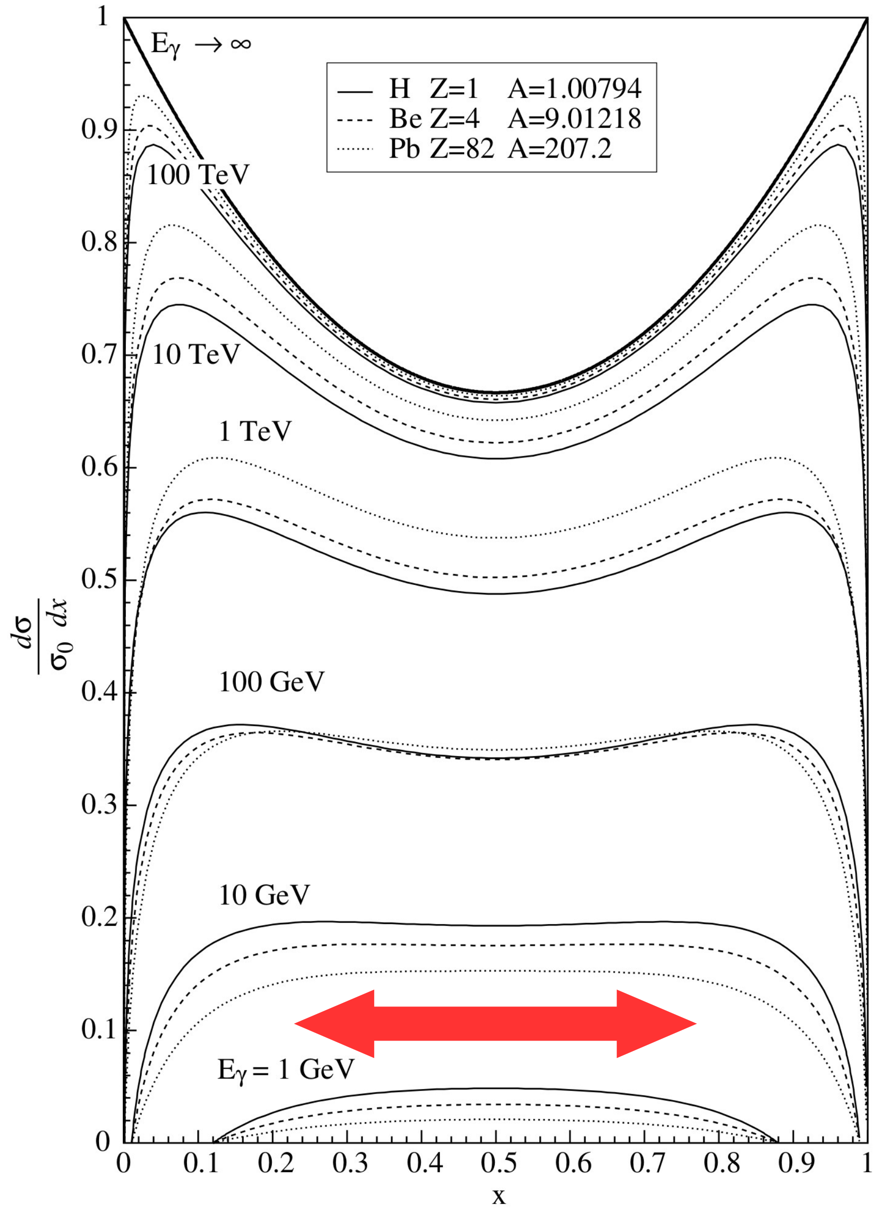
Muon pair production model



- We compute the energy spectrum of the muons generated by the interaction of a photon with a 5 cm tungsten target using [3]
  - At high energy, the energy spectrum is rather flat around the photon energy

$$\frac{d\sigma}{dE_\mu} = 4\frac{\alpha Z^2 r_\mu^2}{E_\gamma}\left(1 - \frac{4}{3}\frac{E_\mu}{E_\gamma}\left(1 - \frac{E_\mu}{E_\gamma}\right)\right)\log W$$

- We assume that the angular is uniform in on cone of opening $1/\gamma_\mu$ (pessimistic)



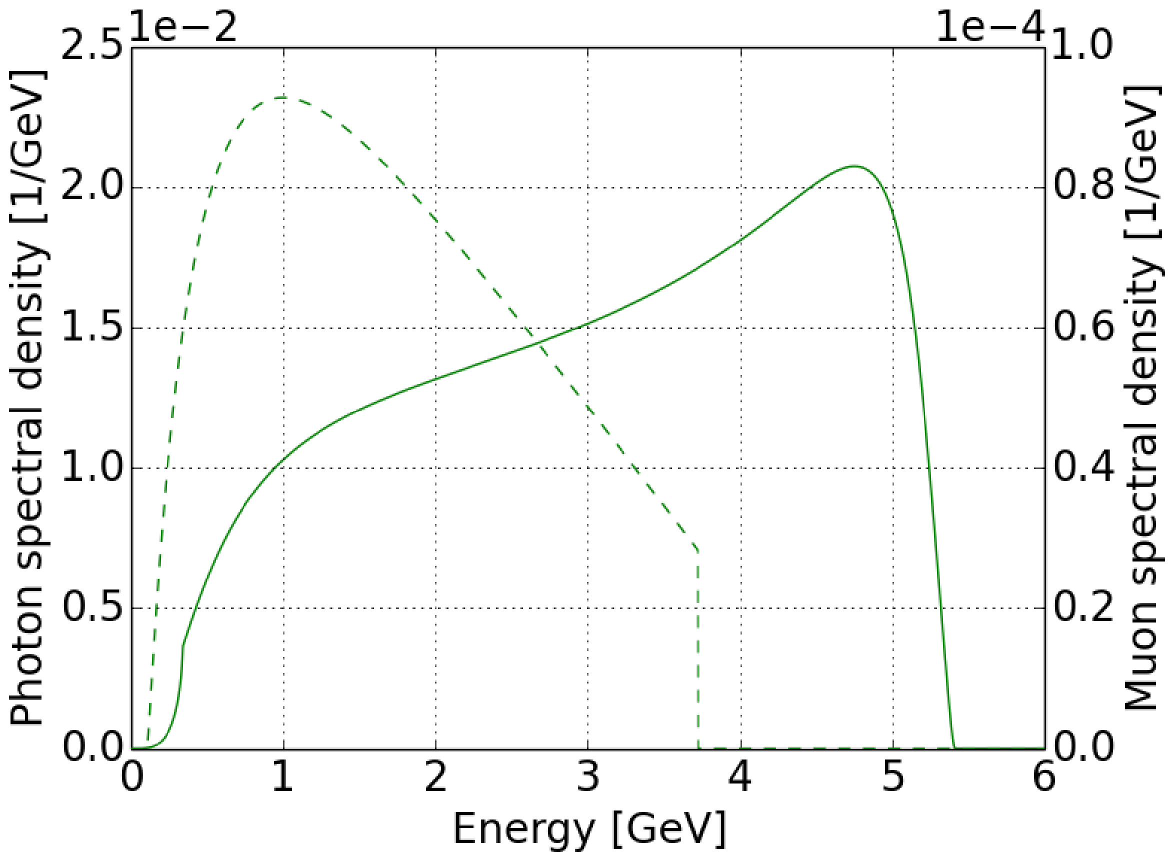
[3] H. Burkhardt, et al. CERN-SL-2002-016-AP

Muon spectrum

- The emission spectrum of each electron is modeled with a delta at the undulator frequency :

$$N_\mu(E) = \int_0^\infty N_\gamma(E') \frac{d\sigma_\mu}{dE}(E, E') dE' \qquad N_\gamma(E') = \frac{1}{d} \int_{-d/2}^{d/2} dx \int_{-\infty}^{\infty} dx' \frac{n_\gamma}{\sqrt{2\pi\sigma_e'^2}} e^{-\frac{x'^2}{2\sigma_e'^2}} \delta(E' - E_\gamma)$$

- The muon spectrum is peaked at low energy, due to the widening of the differential energy cross section at high energy



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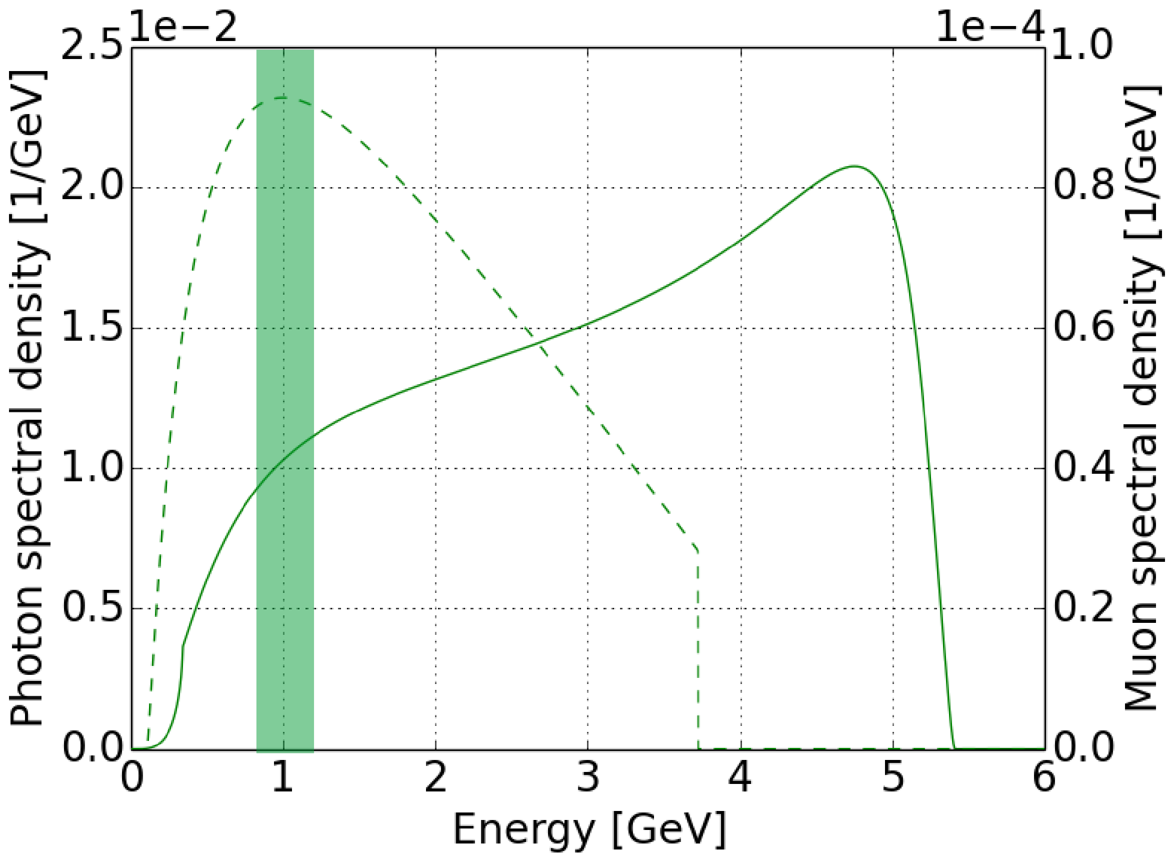
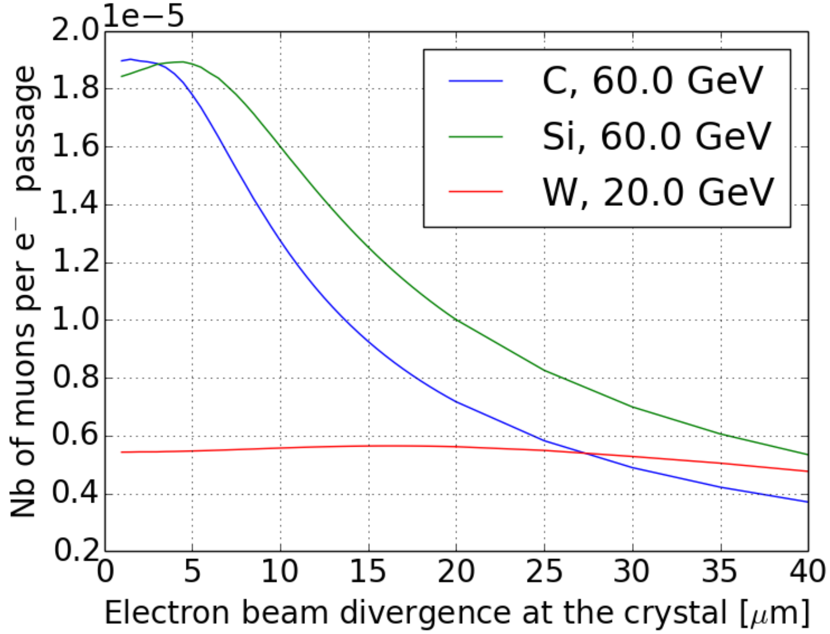
- The muon spectrum is peaked at low energy, due to the widening of the differential energy cross section at high energy
  - We consider the amount of muons in an energy acceptance of ±10%

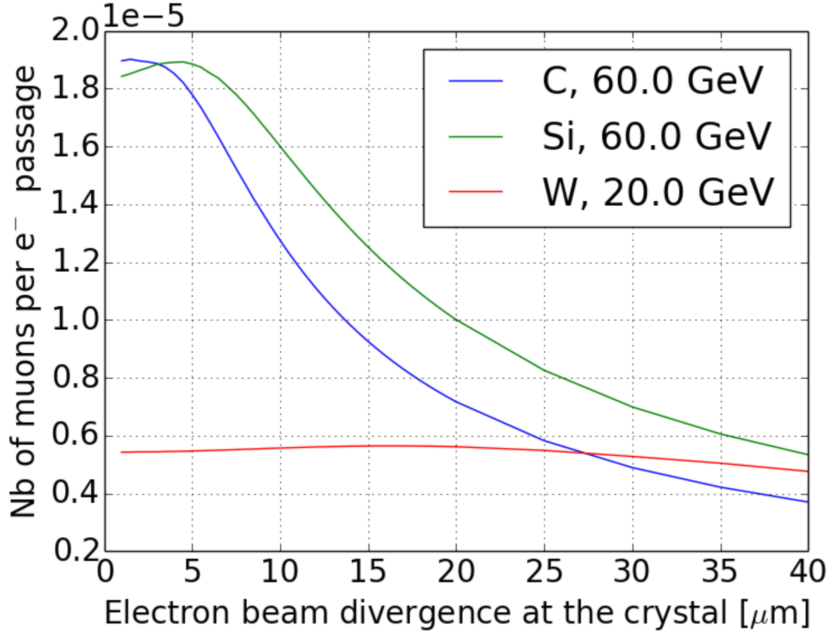


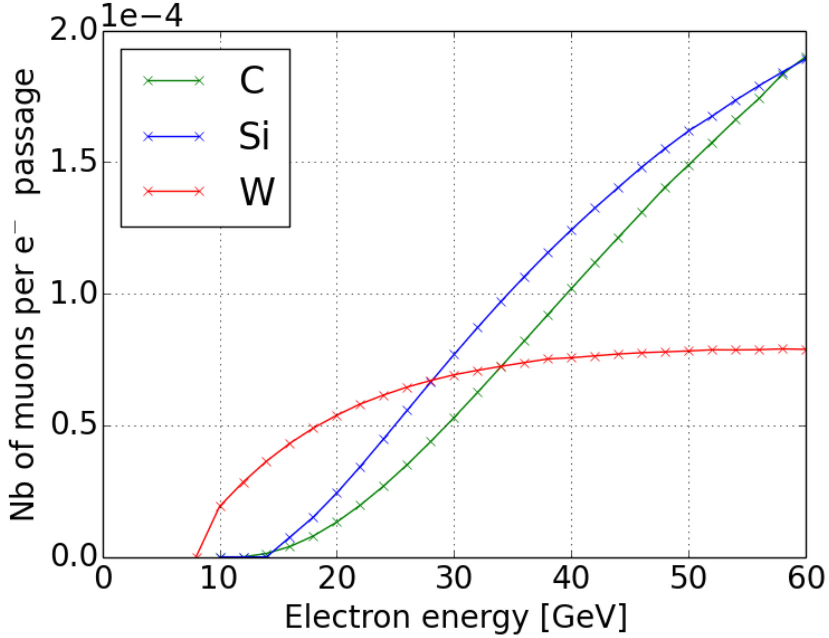
Optimal electron beam parameters



- The optimal electron beam divergence is within reach

Optimal electron beam parameters





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- The usage of light crystals (Si, C) is favourable in terms of conversion efficiency ($e^- \rightarrow \mu^\pm$)
  - Requires high energy electrons (~60 GeV)

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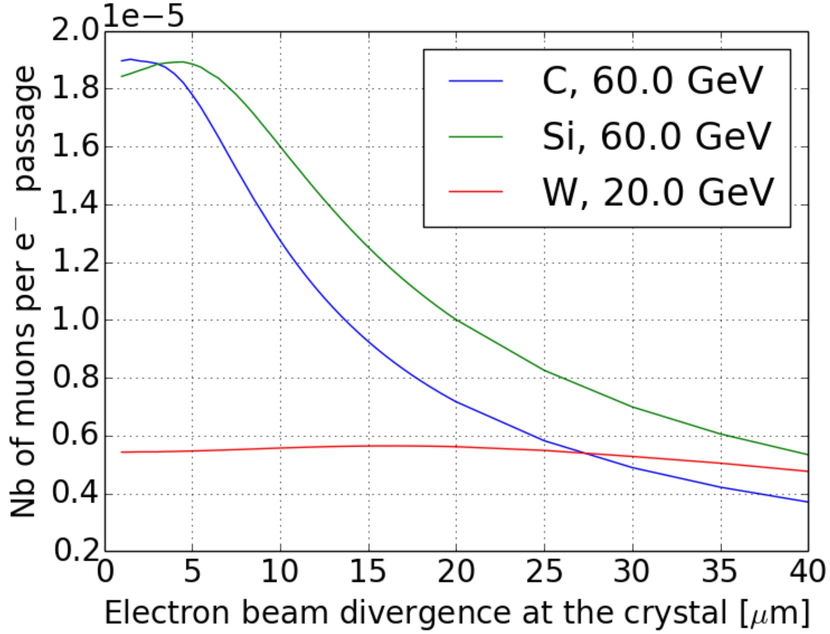
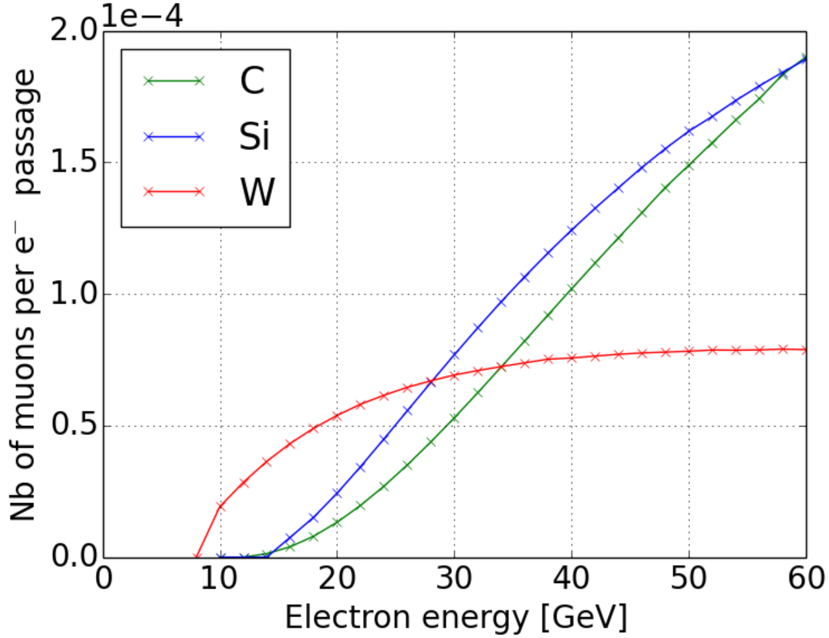
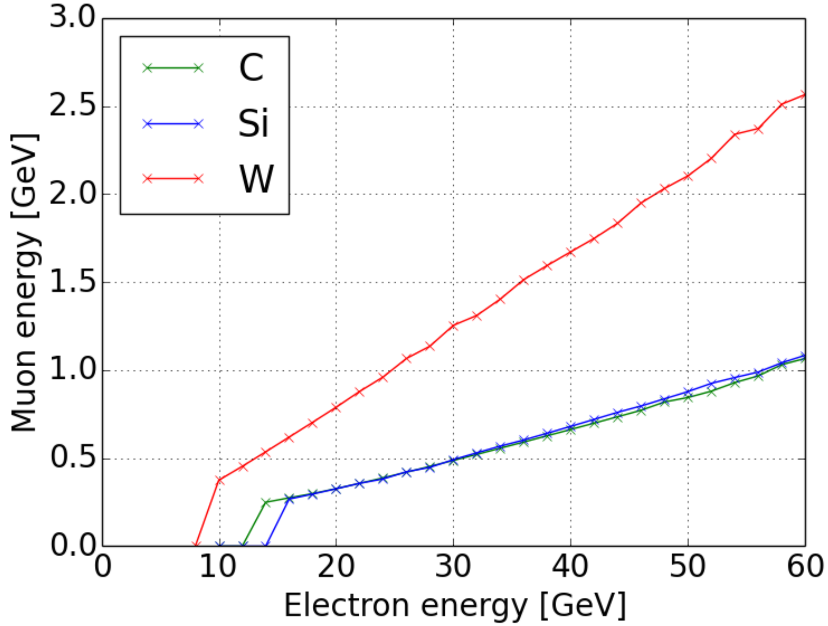
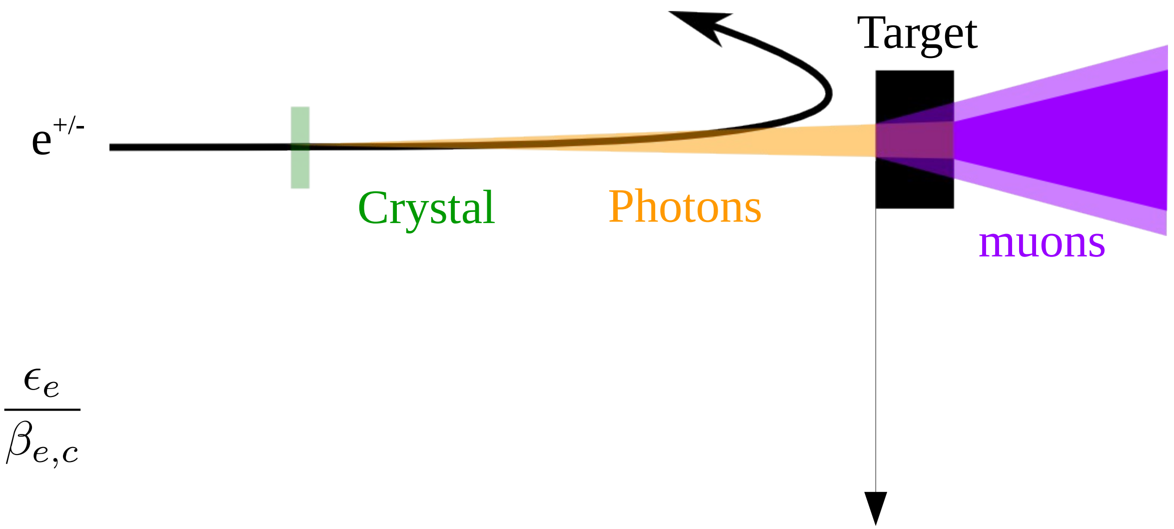




- The optimal electron beam divergence is within reach
- The usage of light crystals (Si, C) is favourable in terms of conversion efficiency ($e^- \rightarrow \mu^\pm$)
  - Requires high energy electrons (~60 GeV)
- The usage of dense crystal (W) is favourable in terms average muon energy and electron beam energy requirement

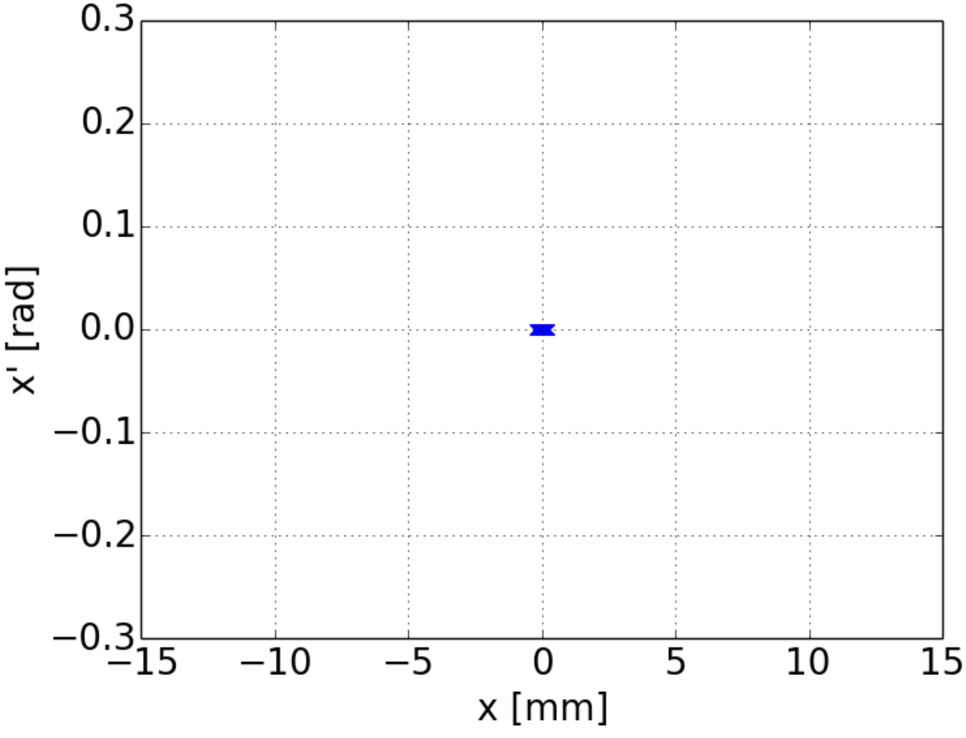


Transverse phase space



$$\sigma^2_{\gamma,\perp} = \epsilon_e \beta_{e,c} + L_{c-t} \frac{\epsilon_e}{\beta_{e,c}}$$

$$\sigma'^2_{\gamma,\perp} = \frac{\epsilon_e}{\beta_{e,c}}$$



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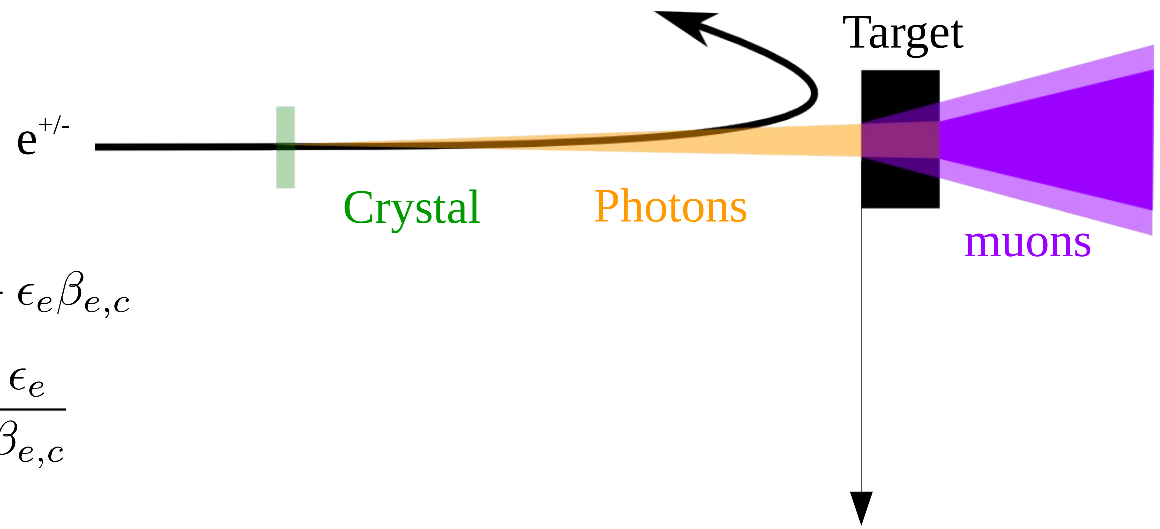

$$\sigma^2_{\gamma,\parallel} = \frac{1}{(4\pi)^2}\frac{\hbar c}{E_\gamma}L_c + \epsilon_e\beta_{e,c}$$

$$\sigma^2_{\gamma,\perp} = \epsilon_e\beta_{e,c} + L_{c-t}\frac{\epsilon_e}{\beta_{e,c}}$$

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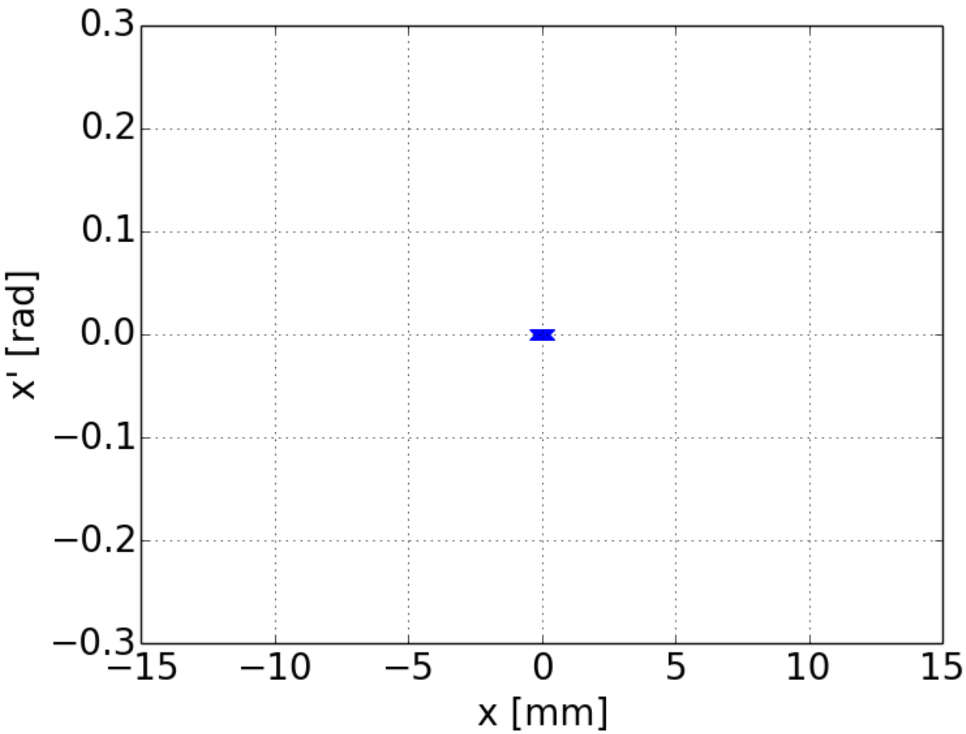

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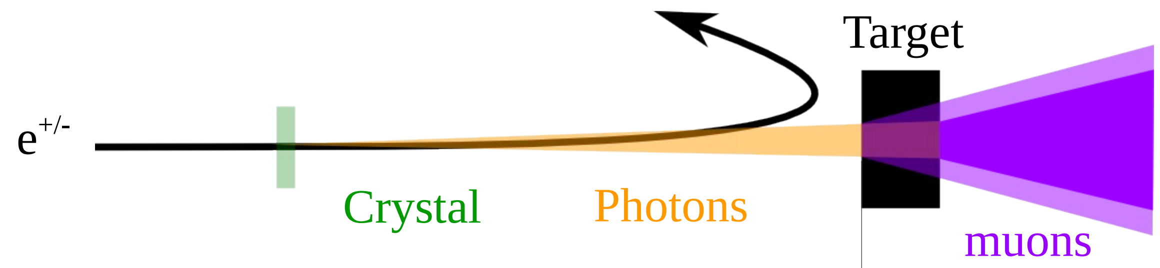


$$\sigma^2_{\gamma,\parallel} = \frac{1}{(4\pi)^2}\frac{\hbar c}{E_\gamma}L_c + \epsilon_e\beta_{e,c}$$

~ 0.1 mm

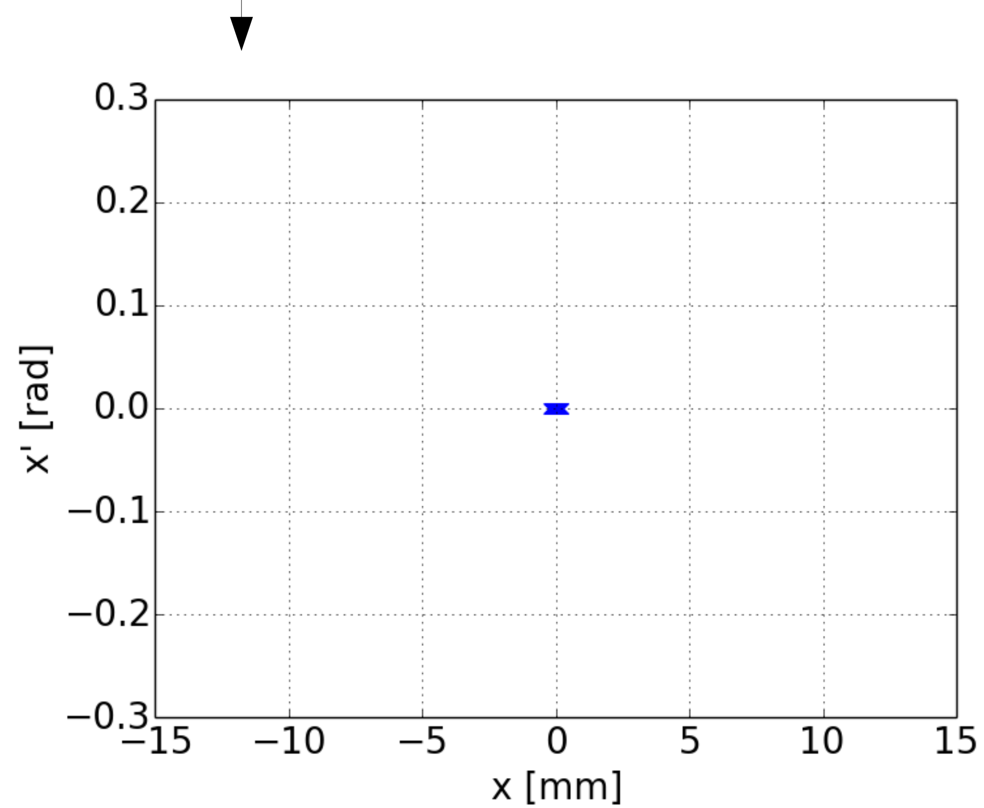
$$\sigma^2_{\gamma,\perp} = \epsilon_e\beta_{e,c} + L_{c-t}\frac{\epsilon_e}{\beta_{e,c}}$$

$$\sigma'^2_{\gamma,\parallel} = \frac{\hbar c}{E_\gamma L_c}$$

<< 1 nm

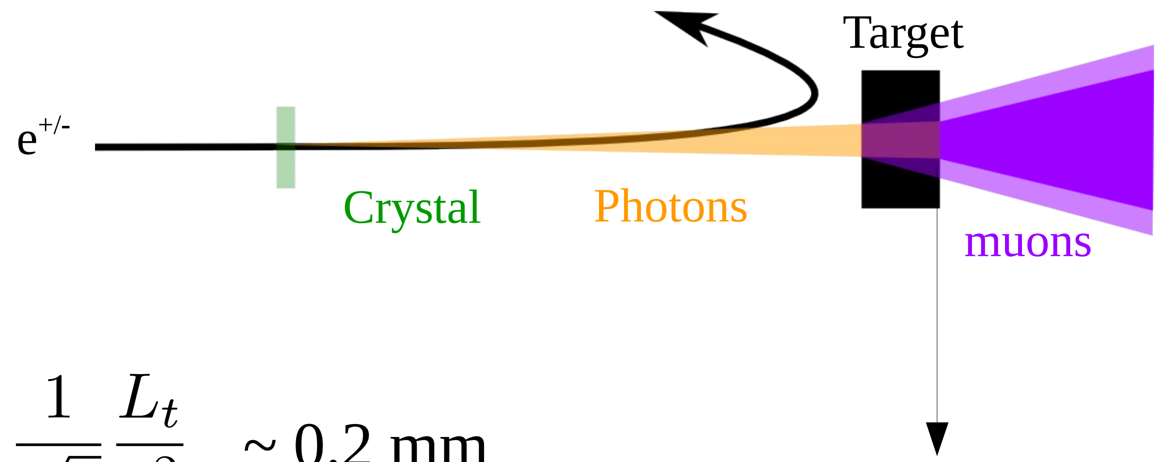
$$\sigma'^2_{\gamma,\perp} = \frac{\epsilon_e}{\beta_{e,c}}$$

~ 6 µm



- GeV electron beams with physical emittances in the nm range are within reach
  - The photon emission process results in an even smaller divergence (in the channeling plane only)

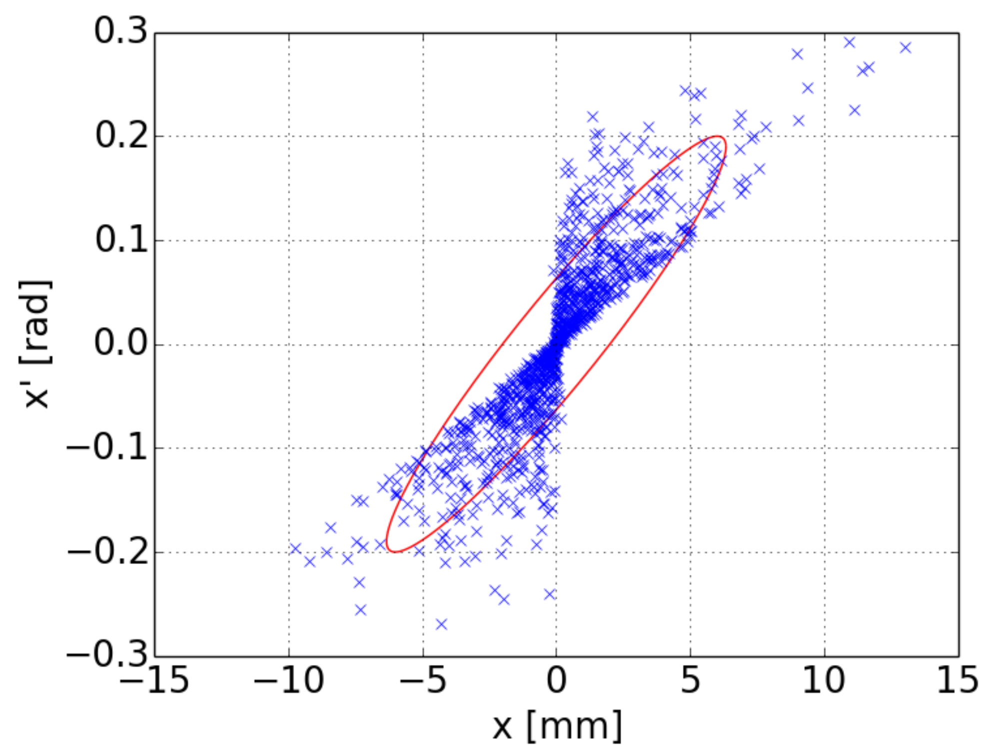
Transverse phase space



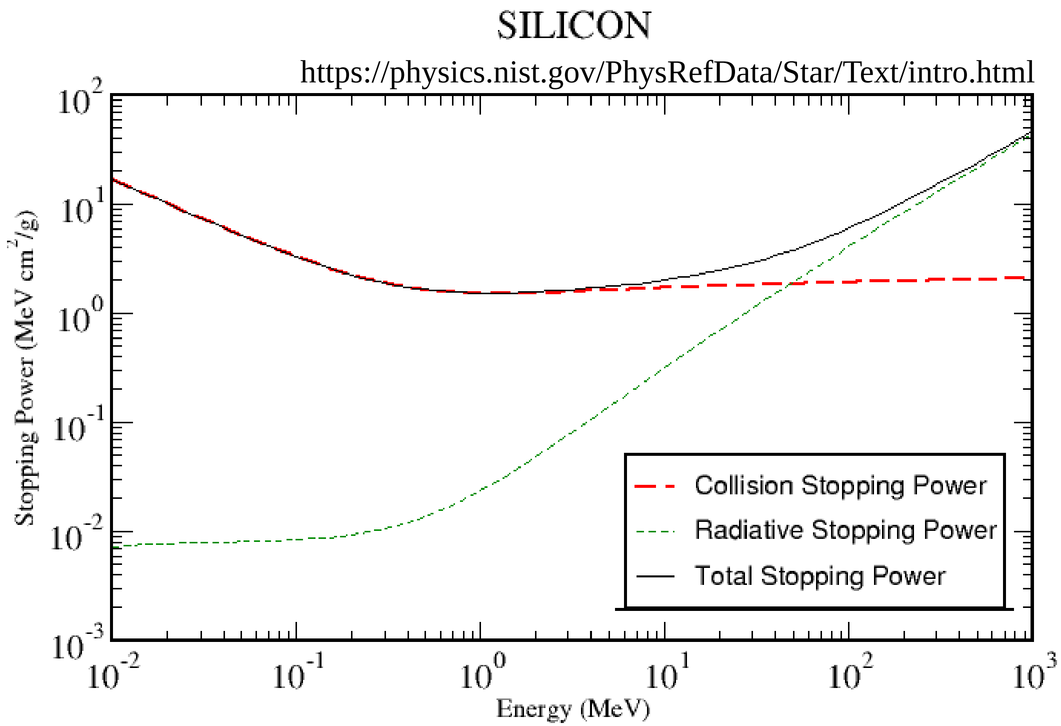
$$\epsilon_\mu \approx \frac{1}{\sqrt{3}} \frac{L_t}{\gamma_\mu^2} \quad \sim 0.2 \text{ mm}$$

- To achieve high conversion efficiency, we chose a dense target with length comparable to its radiation length (→ 5 cm tungsten)
  - The transverse emittance is dominated by the finite length of the target and the emission angle
- The assumption of fixed emission angle is rather pessimistic

  → Monte-Carlo would be needed



Electron beam intensity limit



- With a crystal length much lower than the radiation length, radiative losses are not deposited in the crystal
- The electron beam current is limited by the maximum energy deposition through collisional process in the crysal
  - In the following we assume that **50W** can be dissipated → the electron beam current is limited to a **few mA**

    → Need further understanding of the limit as it affects directly the muon rate

Compatible electron beam design

- A circular electron machine is excluded
  - The damping rate required for the electron beam to compensate multiple scattering is not achievable (less than one turn)

$$\tau_{e,opt} = 2\frac{\sigma'^2_{e,opt}}{\theta_c^2} \qquad \theta_c = \frac{13.6 \cdot 10^6}{eE_e}\sqrt{\frac{L_c}{L_r}}$$

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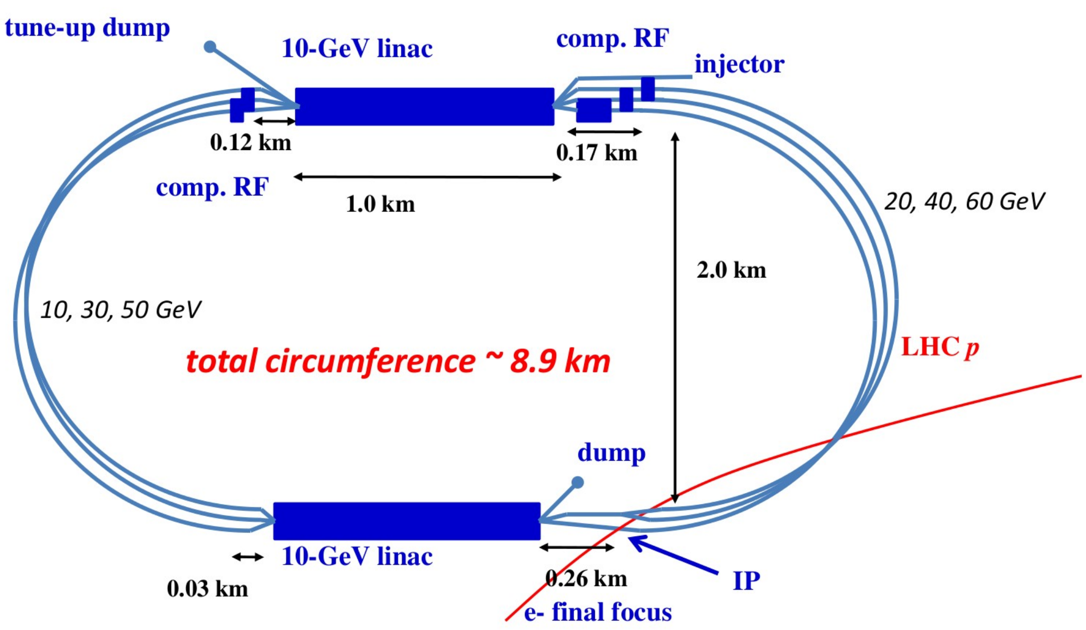
- The electron beam parameters from the ERL design from the LHeC linac-ring option [4] match the requirements for a crystal based muon source



	electrons
beam energy [GeV]	60
Lorentz factor γ	117400
normalised emittance $\gamma\epsilon_{x,y}$ [μm]	50
geometric emittance $\epsilon_{x,y}$ [nm]	0.43
a IP beta function $\beta^*_{x,y}$ [m]	0.12
rms IP beam size $\sigma^*_{x,y}$ [μm]	7
initial rms IP beam divergence $\sigma^*_{x',y'}$ [μrad]	58
beam current [mA]	6.4
bunch spacing [ns]	(25 or) 50
bunch population [ns]	(1 or) 2×10^9
…	
Bunch length	600μm

[4] CERN-OPEN-2012-015

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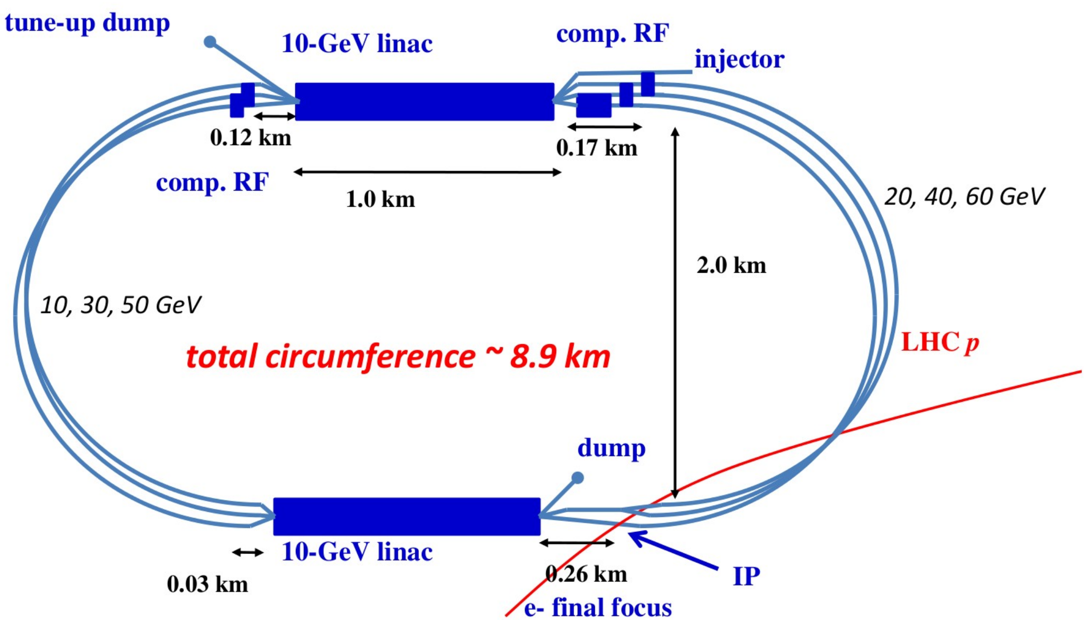
  - To generate beams compatible with a collider, the longitudinal structure of the electron beam needs to be adjusted to an accumulation scheme



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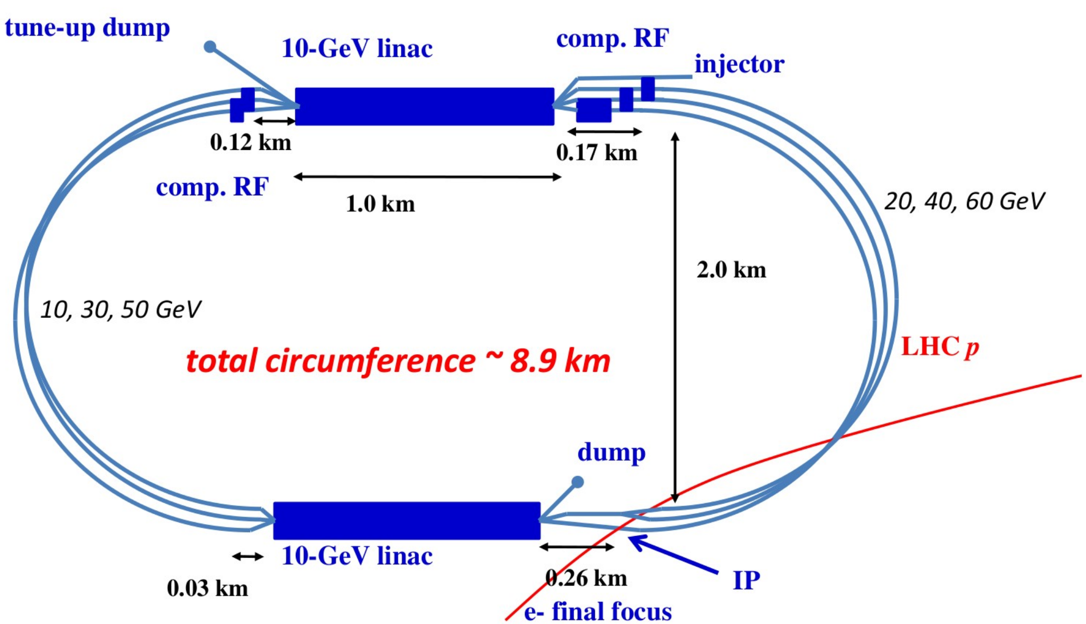
  → LINAC / ERL offer a high flexibility and in particular short bunch lengths



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Source performance

Electron beam		
Energy [GeV]	20	60
Current [mA]	0.6	5
Optimal divergence [μrad]	16.5	4.5
Norm. transverse emit. [μm]	5	50
$\beta_{\|\|}$ at the crystal [m]	0.5	21.0
$\beta_{\perp}$ at the crystal [m]	10.0	
Maximum relative energy loss	0.1	

Crystal		
Crystal Type	W(110)	Si (110)
Radiation Length [m]	0.09	1.56
Dechannelling length [mm]	2.2	13
Crystal length [mm]	0.2	2.1

Target	
Material	W
Length [cm]	5
Distance to the crystal [m]	10

Muon beams		
Energy acceptance	$\pm 10\%$	
Energy [GeV]	2.0	1.1
Efficiency [$10^{-5} \mu$/electron]	0.6	4.5
Rate [$10^{12} \mu/s$]	0.02	1.4
Phys. transverse emit. [mm]	0.08	0.3
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- Interesting muon rates and transverse emittances can be obtained with a crystal based muon source

  → Requires further understanding of the crystal damage / power / power density limits

  → Transverse cooling is still required

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  → Thinner targets would reduced the transverse emittance, at the cost of e-μ conversion efficient

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Current [mA]	0.6	5
Optimal divergence [μrad]	16.5	4.5
Norm. transverse emit. [μm]	5	50
$\beta_{\parallel}$ at the crystal [m]	0.5	21.0
$\beta_{\perp}$ at the crystal [m]	10.0	
Maximum relative energy loss	0.1	

Crystal		
Crystal Type	W(110)	Si (110)
Radiation Length [m]	0.09	1.56
Dechannelling length [mm]	2.2	13
Crystal length [mm]	0.2	2.1

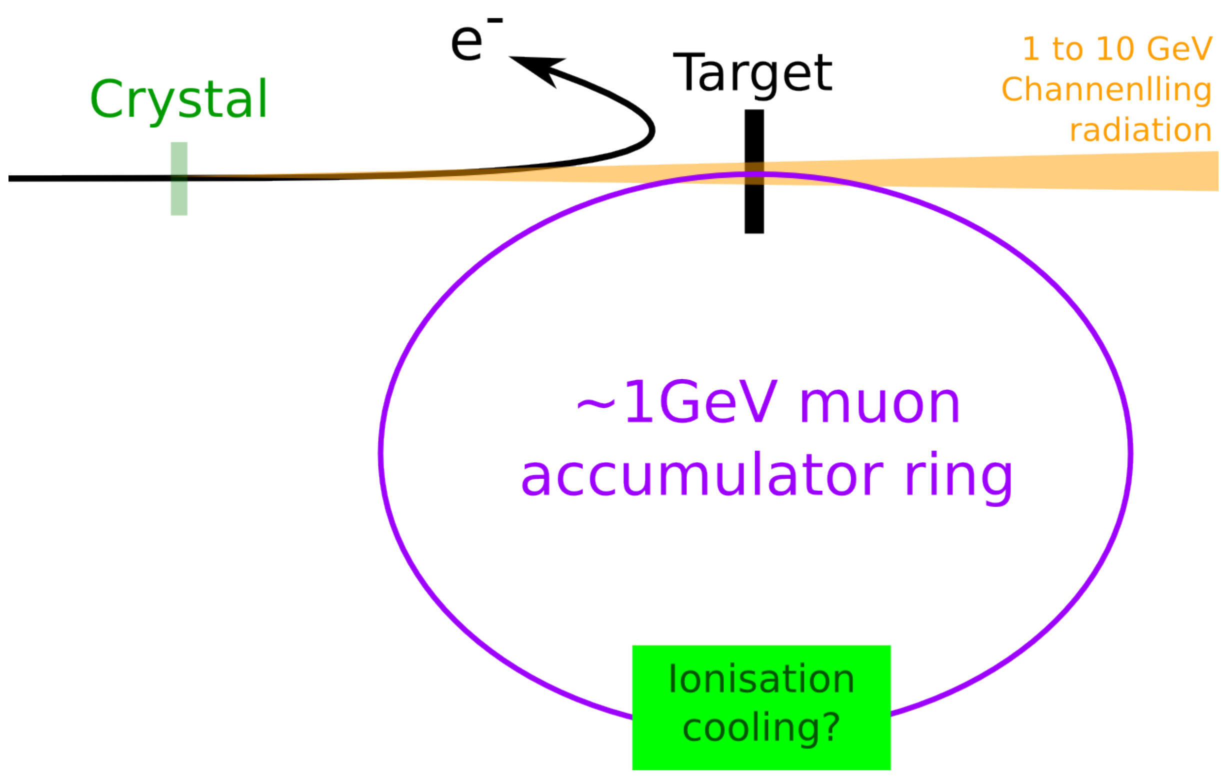
Target	
Material	W
Length [cm]	5
Distance to the crystal [m]	10

Muon beams		
Energy acceptance	$\pm 10\%$	
Energy [GeV]	2.0	1.1
Efficiency [$10^{-5}\mu$/electron]	0.6	4.5
Rate [$10^{12}\mu/s$]	0.02	1.4
Phys. transverse emit. [mm]	0.08	0.3
Norm. transverse emit. [mm]	1.5	2.8

- Interesting muon rates and transverse emittances can be obtained with a crystal based muon source
  - → Requires further understanding of the crystal damage / power / power density limits
  - → Transverse cooling is still required
  - → Thinner targets would reduced the transverse emittance, at the cost of e-μ conversion efficient
- There is quite some margin between the crystal length chosen (limited to 10% energy loss per electron per passage to allow for analytical derivation) and the dechanneling length
  - → Requires further study of the photon spectrum as a function of the crystal type and length

A potential way towards collider-compatible performance

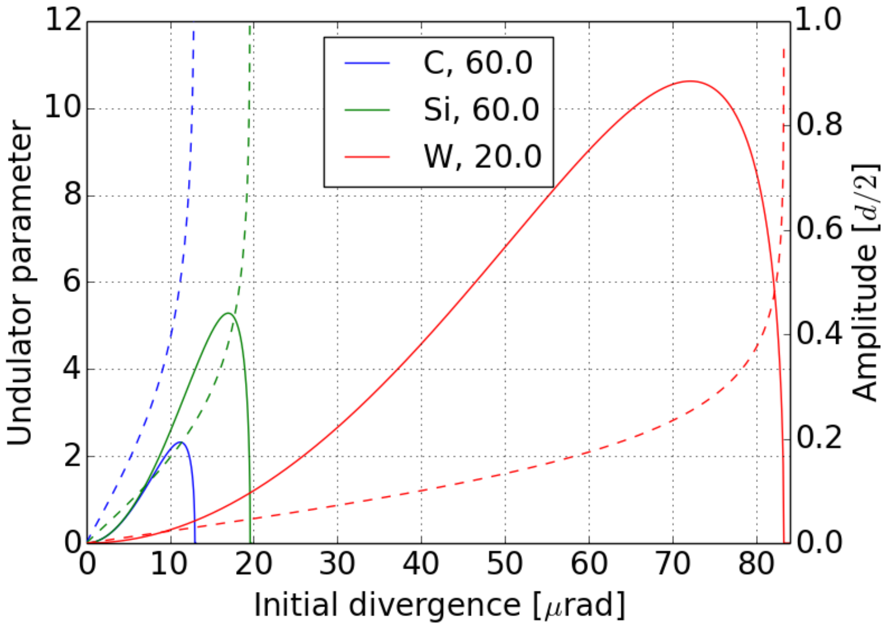
- The muon beam properties may be sufficient to be injected in a high acceptance accumulator ring (i.e. LEMMA-type but at a lower energy and with a higher transverse emittances)
- The usage of a secondary beam (photons) on target prevents the issue with multiple scattering of the primary beam (e.g. positrons)
  - ERL technology offers the hope to remain energy efficient

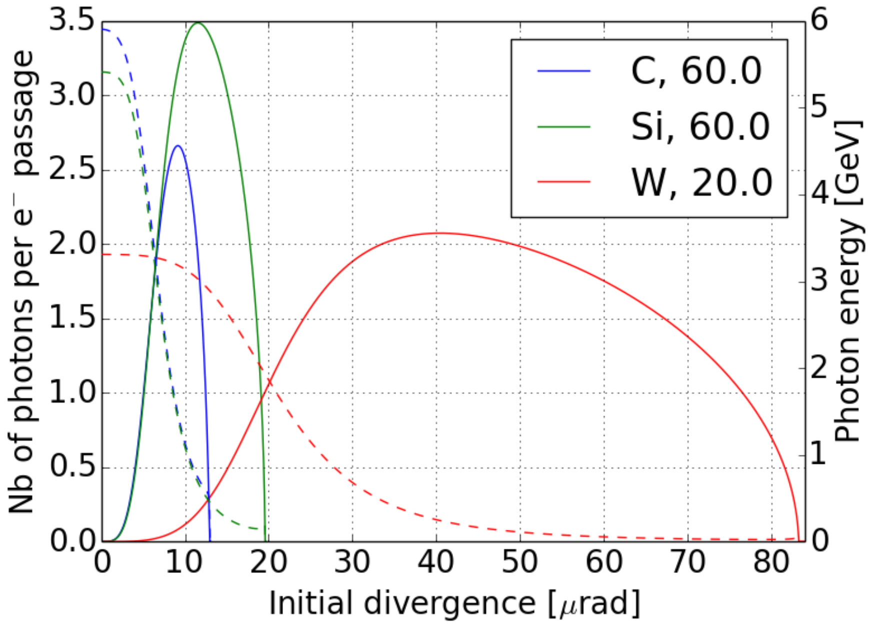


- Transverse cooling during accumulation may offer collider-compatible bunches
  → To be investigated

BACKUP

Channeling radiation from other crystals type





- Materials with a deep channeling potential (W) can channel electron with a higher divergence and higher energy photons can be generated with a lower electron energy