HEPHY VIENNA

Institute for High Energy Physics

What is mSUGRA?

Helmut Eberl

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Outline

- Chiral and Vector multiplets
- SUSY Lagrangian
- The MSSM
- Mass Matrices
- R-parity
- Gauge coupling unification
- Renormalisation group equations (RGE)
- mSUGRA

Main references: talk of A. B. Lahanas on SUSY in Corfu workshop 2009
S. Martin, SUSY primer, hep-ph/9709356

Chiral and antichiral multiplets

The general superfield involves many components. Multiplets with fewer components can be constructed !

Covariant derivatives :

$$D_\alpha = i\frac{\partial}{\partial\theta^\alpha} + (\sigma^m\overline{\theta})_\alpha \frac{\partial}{\partial x^m}\,,\quad \overline{D}_{\dot\alpha} = -i\frac{\partial}{\partial\overline{\theta}^{\dot\alpha}} - (\theta\sigma^m)_{\dot\alpha}\frac{\partial}{\partial x^m}$$

Commute with SUSY transformations :

$$[\delta_{SUSY}\,,\,D] = [\delta_{SUSY}\,,\,\overline{D}] = 0$$

Chiral superfield : $\overline{D}\Phi = 0$, **Antichiral superfield :** $D\Phi^\dagger = 0$

These properties are preserved by SUSY transformations !

- A chiral superfield contains a complex scalar A a Left - handed Weyl fermion ψ and an auxiliary complex field F. Its particle content is 2 spin-0 and 2 - spin 1/2 states .
- In terms of the variable $y^m \equiv x^m - i\,\theta\,\sigma^m\,\overline{\theta}$

$$\Phi \;=\; A(y) + \sqrt{2}\theta\,\psi(y) + \theta\,\theta\,F(y)$$

- The $\theta\theta$ component field F, or last component, carries the highiest dimensionality !

Under infinitesimal SUSY transformations by $\xi, \overline{\xi}$:

$$\begin{aligned}
\delta\,A &= \sqrt{2}\,\xi\,\psi \\
\delta\,\psi &= \sqrt{2}\,\xi\,F - i\,\sqrt{2}\,\sigma^m\,\overline{\xi}\,\partial_m A \\
\delta\,F &= -i\sqrt{2}\,\overline{\xi}\,\overline{\sigma}^m\,\partial_m \psi
\end{aligned}$$

The last component transform as a total derivative $\Longrightarrow$

$$\int d^4x\,F \;=\; \textbf{SUSY invariant}$$

- An **antichiral** superfield includes a complex scalar, a Right - handed Weyl fermion and an auxiliary complex field.
- If Φ is a **chiral** superfield with components (A, ψ, F) its Hermitian superfield $\Phi^\dagger$ is **antichiral** with components $(A^*, \overline{\psi}, F^*)$

Products of chiral superfields :

- If $\Phi \sim (A, \psi, F)$ and $\Phi' \sim (A', \psi', F')$ are chiral superfields their product $\Phi'' = \Phi\,\Phi'$ is also a chiral field with components

$$\begin{aligned} A'' &= \; A\,A' \\ \psi'' &= \; A\,\psi' + A'\,\psi \\ F'' &= \; A'\,F + A\,F' - \psi\,\psi' \end{aligned}$$

- The product $\Phi^\dagger\,\Phi$ is a real superfield which includes kinetic terms in its last $\theta^2\,\overline{\theta}^2$ component, producing SUSY invariant kinetic terms upon $\int d^4x$!

$$\Phi^\dagger\,\Phi|_{\theta^2\,\overline{\theta}^2} \; = \;$$
$$-\tfrac{1}{4}\,A\Box A^* - \tfrac{1}{4}\,A^*\,\Box A + \tfrac{1}{2}\,|\partial_m A|^2 + \tfrac{i}{2}\,(\psi\sigma^m\partial_m\overline{\psi} - h.c.) + FF^*$$

Vector multiplets

A Hermitian superfield V defines a vector multiplet

$$\textbf{Vector superfield:} \quad V = V^{\dagger}$$

including, among other components, a vector field A_μ a Weyl fermion λ and an auxiliary field D as its last $\theta^2 \bar{\theta}^2$ component.

- For any chiral field $\Phi \sim (A, \psi, F)$ the transformation

  $$V \Longrightarrow V + \Phi + \Phi^{\dagger}$$

  defines a gauge transformation with gauge parameter $\Lambda = -2\, Im\, A$
- Under the gauge transformation

  $$A_\mu \Longrightarrow A_\mu + \partial_\mu \Lambda \quad , \quad \lambda, D \Longrightarrow \textbf{themselves}$$

- The remaining components are not gauged invariant and can be gauged way ! This defines the **Wess - Zumino** gauge

- In the Wess - Zumino gauge

$$V = -(\theta\,\sigma^\mu\,\bar\theta)\,A_\mu + i\,\theta\theta\bar\theta\,\bar\lambda - i\,\bar\theta\bar\theta\,\theta\,\lambda + \frac{1}{2}\,\theta\theta\,\bar\theta\bar\theta\,D$$

and $V^n = 0$ for $n \geq 3$

- The vector field A_μ and its partner, **gaugino** , λ are the physical d.o.f. describing 2 spin-1 and 2 spin-1/2 states.

Under infinitesimal SUSY transformations λ, D and the field strength $F_{\mu\nu} = \partial_\mu\,A_\nu - \partial_\nu\,A_\mu$ transform to each other :

$$\begin{aligned}
\delta\,F_{\mu\nu} &= -i\,(\,\xi\sigma_\nu\partial_\mu\bar\lambda + \bar\xi\,\bar\sigma_\nu\partial_\mu\lambda\,) - (\nu \leftrightarrow \mu) \\
\delta\,\lambda &= i\,\xi\,D + i\,\sigma^{\mu\nu}\,F_{\mu\nu} \\
\delta\,D &= \xi\sigma^\mu\partial_\mu\bar\lambda - \bar\xi\,\bar\sigma^\mu\partial_\mu\lambda
\end{aligned}$$

- The gauge interactions of the chiral fields can be easily read in the Wess-zumino gauge since $V^n = 0\,,\ n \geq 3$.

$$\Phi^\dagger e^{2gV}\Phi = \Phi^\dagger\Phi + 2g\,\Phi^\dagger V\Phi + g^2\,\Phi^\dagger V^2\Phi$$

- The SUSY Yang-Mills Lagrangian

$$\mathcal{L}_{gauge} = -\frac{1}{4}\,G_{\mu\nu}^{(a)\,2} + \frac{i}{2}\,\overline{\lambda}^{(a)}\overline{\sigma}^{\mu}D_\mu\lambda^{(a)} + \frac{1}{2}\,D^{(a)2}$$

with $D_\mu\lambda^{(a)} = \partial_\mu\lambda^{(a)} + g\,f^{abc}A_\mu^b\lambda^{(c)}$ covariant derivative and $G_{\mu\nu}^{(a)}$ the non-Abelian gauge field strength is both gauge and SUSY invariant !

- A supersymmetric gauge Lagrangian with chiral multiplets Φ_i put in some representation Φ, reducible in general, is

$$\mathcal{L} = \mathcal{L}_{gauge} + \int d^2\theta\, d^2\overline{\theta}\,\Phi^\dagger e^{2gV}\Phi + \left(\int d^2\theta\, W(\Phi) + h.c.\right)$$

F- and D-terms, Superpotential

- Auxiliary F - fields arise from the kinetic and superpotential terms

$$F_i^\dagger F_i + \left(F_i^\dagger \frac{\partial W(A_i)}{\partial A_i} + h.c. \right)$$

- Auxiliary D - fields arise from the YM kinetic and gauge interaction terms and are eliminated by their eqs. of motion.

$$\frac{1}{2} D^{(a)2} + D^{(a)} \sum_i g A^* T^{(a)} A$$

Using also the eqs. of motions for the F's,

- D and F type auxiliary fields are

$$D^{(a)} = - \sum_i g A^* T^{(a)} A \quad , \quad F_i^\dagger = - \frac{\partial W(A)}{\partial A_i}$$

For a renormalizeable theory the **superpotential** is

$$W(\Phi_i) = \frac{\lambda_{ijk}}{3!} \Phi_i \Phi_j \Phi_k + \frac{m_{ij}}{2!} \Phi_i \Phi_j$$

The complete SUSY Lagrangian

$$\mathcal{L} = \mathcal{L}_{inv}$$

$$\begin{aligned}\mathcal{L}_{inv} &= \\ &- \frac{1}{4}G^{(a)\,2}_{\mu\nu} + |D_\mu A|^2 + \left(\frac{i}{2}\bar{\lambda}^{(a)}\bar{\sigma}^\mu D_\mu \lambda^{(a)} + \frac{i}{2}\bar{\psi}\bar{\sigma}^\mu D_\mu \psi + h.c.\right) \\ &- \left(\frac{1}{2}W_{ij}\psi_i\psi_j + i\sqrt{2}gA^*T^{(a)}\psi\lambda^{(a)} + h.c.\right) - |F_i|^2 - \frac{1}{2}D^{(a)\,2}\end{aligned}$$

No fermion - boson mass degeneracy is observed in nature and thus a SUSY breaking sector should be present !

Only "soft" breaking is allowed, therefore only allowed:

Gaugino masses	M_a
Scalar masses	m_{ij} , b_{ij}
Trilinear couplings	A_{ijk}

$$\mathcal{L}_{soft} = -m^2_{ij}\phi^*_i\phi_j - \left(b_{ij}\phi_i\phi_j + A_{ijk}\phi_i\phi_j\phi_k + \frac{M_a}{2}\lambda_a\lambda_a + h.c.\right)$$

$\phi_i \sim A$ before

The Minimal Supersymmetric Standard Model

- To every **SM particle** a **SUSY partner** is introduced, both members of the same multiplet and the d.o.f. are more than doubled.

- The structure of the SM is automatically included.

- New particles are predicted, super partners (sparticles) of the SM particles – SUSY models have a rich phenomenology.

Superpotential – generalisation of the Yukawa interactions of the SM:

$$W_{\mathrm{MSSM}} = \overline{u}\mathbf{y_u}QH_u - \overline{d}\mathbf{y_d}QH_d - \overline{e}\mathbf{y_e}LH_d + \mu H_u H_d$$

all fields are here chiral multiplets

$\mathbf{y_u}, \mathbf{y_d}, \mathbf{y_e}$ are the 3 x 3 Yukawa matrices

μ can be complex

Chiral supermultiplets:

Names		spin 0	spin 1/2	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
squarks, quarks	Q	$(\widetilde{u}_L\ \ \widetilde{d}_L)$	$(u_L\ \ d_L)$	$(\,\mathbf{3},\ \mathbf{2}\,,\ \frac{1}{6})$
($\times 3$ families)	$\overline{u}$	$\widetilde{u}^*_R$	$u^\dagger_R$	$(\,\overline{\mathbf{3}},\ \mathbf{1},\ -\frac{2}{3})$
	$\overline{d}$	$\widetilde{d}^*_R$	$d^\dagger_R$	$(\,\overline{\mathbf{3}},\ \mathbf{1},\ \frac{1}{3})$
sleptons, leptons	L	$(\widetilde{\nu}\ \ \widetilde{e}_L)$	$(\nu\ \ e_L)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ -\frac{1}{2})$
($\times 3$ families)	$\overline{e}$	$\widetilde{e}^*_R$	$e^\dagger_R$	$(\,\mathbf{1},\ \mathbf{1},\ 1)$
Higgs, higgsinos	H_u	$(H^+_u\ \ H^0_u)$	$(\widetilde{H}^+_u\ \ \widetilde{H}^0_u)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ +\frac{1}{2})$
	H_d	$(H^0_d\ \ H^-_d)$	$(\widetilde{H}^0_d\ \ \widetilde{H}^-_d)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ -\frac{1}{2})$

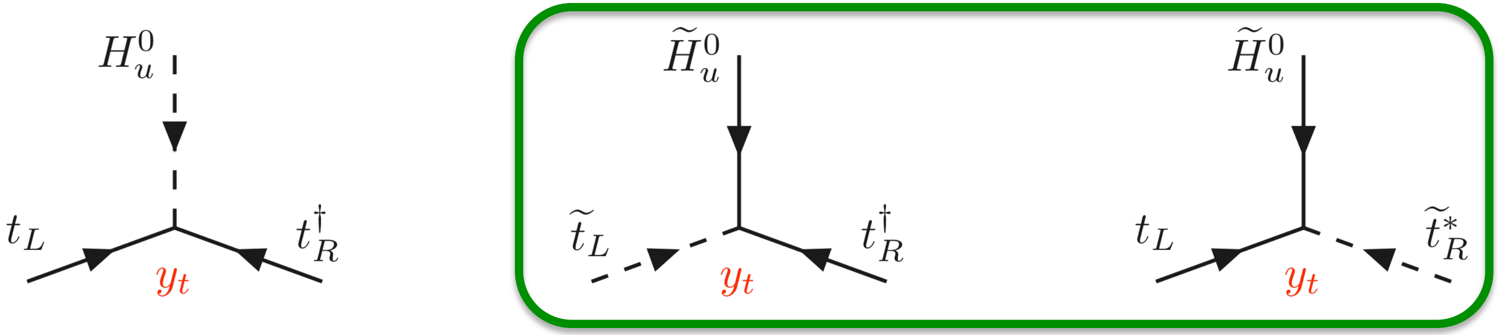
4 neutralinos

2 Charginos

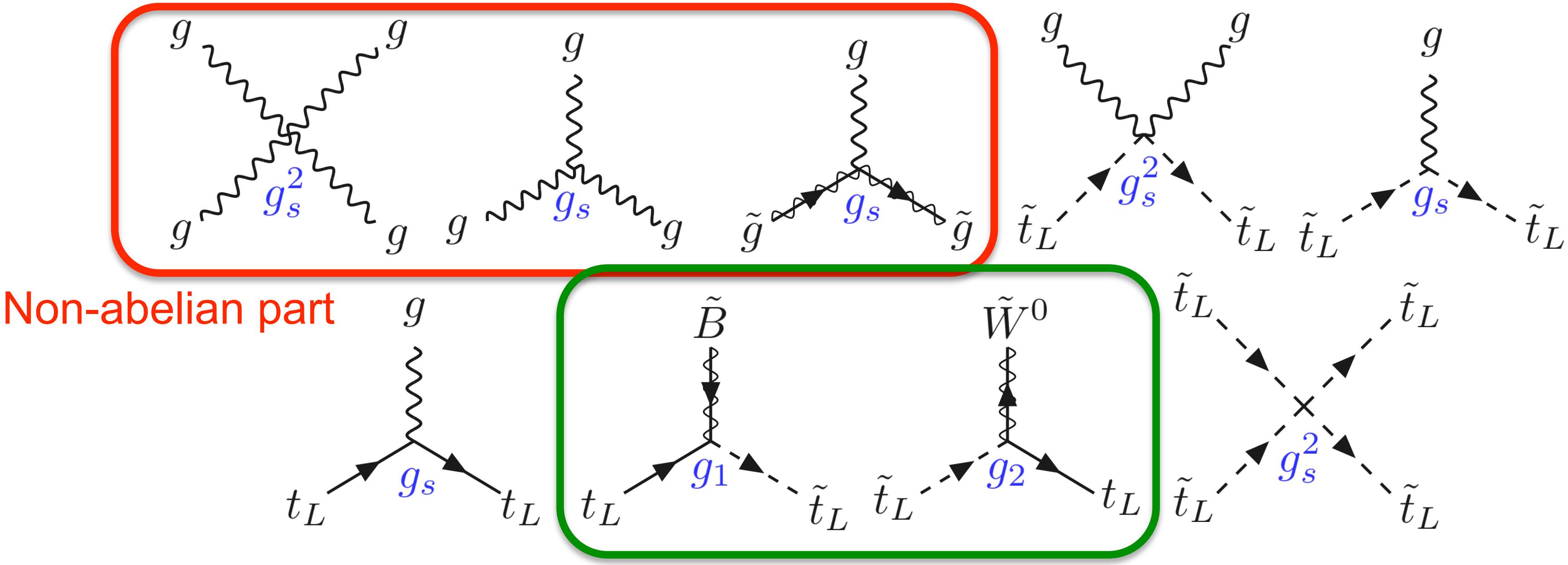
Gauge supermultiplets:

Names	spin 1/2	spin 1	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
gluino, gluon	$\widetilde{g}$	g	$(\,\mathbf{8},\ \mathbf{1}\,,\ 0)$
winos, W bosons	$\widetilde{W}^\pm\ \ \widetilde{W}^0$	$W^\pm\ \ W^0$	$(\,\mathbf{1},\ \mathbf{3}\,,\ 0)$
bino, B boson	$\widetilde{B}^0$	B^0	$(\,\mathbf{1},\ \mathbf{1}\,,\ 0)$

top-neutral Higgs chiral field interaction from superpotential:



Supersymmetric gauge interaction, e.g. for SU(3) or SU(2):



$\tilde{\chi}_i^0 = \alpha\, \tilde{H}_u^0 + \beta\, \tilde{H}_d^0 + \gamma\, \tilde{B} + \delta\, \tilde{W}^0$ mixing states $\tilde{t} = \alpha\, \tilde{t}_L + \beta\, \tilde{t}_R$

Interaction states – physical states

Particle mixing: Higgs bosons, sfermions, charginos, neutralinos

⇓

Mass matrices – Eigenvalue problems

MSSM parameters, most general case

We have the SM parameters + μ

~~SUSY~~ terms, defined by:

$$\begin{aligned}
\mathcal{L}_{\rm soft}^{\rm MSSM} \;=\; & -\frac{1}{2}\left(M_3\widetilde{g}\widetilde{g} + M_2\widetilde{W}\widetilde{W} + M_1\widetilde{B}\widetilde{B} + \text{c.c.}\right) \\
& -\left(\widetilde{\overline{u}}\,\mathbf{a_u}\,\widetilde{Q}H_u - \widetilde{\overline{d}}\,\mathbf{a_d}\,\widetilde{Q}H_d - \widetilde{\overline{e}}\,\mathbf{a_e}\,\widetilde{L}H_d + \text{c.c.}\right) \\
& -\widetilde{Q}^\dagger\,\mathbf{m^2_Q}\,\widetilde{Q} - \widetilde{L}^\dagger\,\mathbf{m^2_L}\,\widetilde{L} - \widetilde{\overline{u}}\,\mathbf{m^2_{\overline{u}}}\,\widetilde{\overline{u}}^\dagger - \widetilde{\overline{d}}\,\mathbf{m^2_{\overline{d}}}\,\widetilde{\overline{d}}^\dagger - \widetilde{\overline{e}}\,\mathbf{m^2_{\overline{e}}}\,\widetilde{\overline{e}}^\dagger \\
& - m^2_{H_u}H_u^*H_u - m^2_{H_d}H_d^*H_d - (bH_uH_d + \text{c.c.})\,.
\end{aligned}$$

$M_1,\ M_2,\ M_3$: complex numbers

$\mathbf{a_u},\ \mathbf{a_d},\ \mathbf{a_d}$: 3×3 matrices, complex

$\mathbf{m^2_Q},\ \mathbf{m^2_L},\ \mathbf{m^2_{\overline{u}}},\ \mathbf{m^2_{\overline{d}}},\ \mathbf{m^2_{\overline{e}}}$: 3×3 hermitian matrices

$m^2_{H_u},\ m^2_{H_u}$: real numbers, b: complex number

105 independent soft breaking parameters, how can this be reduced?

Most of the new parameters imply flavor mixing or ~~CP~~ processes, severe experimental constraints from e.g. rare decays, Kaon physics, B-physics, EDMs of electron, neutron, atoms, DM, …

MSSM parameters: “real” version

All of these potentially dangerous flavor-changing and CP-violating effects in the MSSM can be evaded if one assumes (or can explain!) that supersymmetry breaking is suitably “universal".

$$\mathbf{m^2_Q} = m^2_Q \mathbf{1}, \quad \mathbf{m^2_{\bar{u}}} = m^2_{\bar{u}} \mathbf{1}, \quad \mathbf{m^2_{\bar{d}}} = m^2_{\bar{d}} \mathbf{1}, \quad \mathbf{m^2_L} = m^2_L \mathbf{1}, \quad \mathbf{m^2_{\bar{e}}} = m^2_{\bar{e}} \mathbf{1}.$$

We further assume, that the trilinear couplings are each proportional to the corresponding Yukawa coupling matrix,

$$\mathbf{a_u} = A_{u0}\, \mathbf{y_u}, \qquad \mathbf{a_d} = A_{d0}\, \mathbf{y_d}, \qquad \mathbf{a_e} = A_{e0}\, \mathbf{y_e},$$

and real parameters:

$$\arg(M_1),\ \arg(M_2),\ \arg(M_3),\ \arg(A_{u0}),\ \arg(A_{d0}),\ \arg(A_{e0}) \ = \ 0 \text{ or } \pi,$$
$$+ \text{ real } \mu \text{ and real } b$$

Only CP-violating source left is the CKM phase.

Big step towards mSUGRA!

Mass matrices

Higgs sector in the MSSM

$$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} \qquad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}$$

electroweak SSB

$\Downarrow$

$$\langle H_1^0 \rangle = v_1 = v \cos\beta \text{ and } \langle H_2^0 \rangle = v_2 = v \sin\beta$$

tree-level: 2 free parameters, m_{A^0}, $\tan\beta = \frac{v_2}{v_1}$ chosen

$$m_{h^0}^{\text{tree}} \leq m_{Z^0} \, |\cos 2\beta|$$

one–loop corr. important for m_{h^0}, m_{H^0}, and α, leading terms $\sim \frac{m_t^4}{m_W^2}$

$$m_{h^0}^{\text{corr.}} \lesssim 135 \text{ GeV}$$

Mass matrix

$$M^2(H^0,h^0) = \begin{pmatrix} \sin^2\beta m_{A^0}^2 + \cos^2\beta m_{Z^0}^2 & -\sin\beta\cos\beta(m_{A^0}^2 + m_{Z^0}^2) \\ -\sin\beta\cos\beta(m_{A^0}^2 + m_{Z^0}^2) & \cos^2\beta m_{A^0}^2 + \sin^2\beta m_{Z^0}^2 \end{pmatrix} = (R^{h^0})^T . \begin{pmatrix} m_{H^0}^2 & 0 \\ 0 & m_{h^0}^2 \end{pmatrix} . R^{h^0}$$

$$\text{with} \quad R^{h^0} := \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} .$$

Special cases (at tree level):

Limit $m_{A^0} \gg m_{Z^0}$:

$$M^2(H^0,h^0) \sim \begin{pmatrix} \sin^2\beta & -\sin\beta\cos\beta \\ -\sin\beta\cos\beta & \cos^2\beta \end{pmatrix} m_{A^0}^2 \Rightarrow \begin{array}{l} \cos\alpha \longrightarrow \sin\beta \\ \sin\alpha \longrightarrow -\cos\beta \end{array}$$

Heavy Higgs masses are degenerated:

$$m_{h^0} \ll m_{H^0} \sim m_{A^0} \sim m_{H^\pm}$$

Limit large $\tan\beta$: ($m_{A^0} \sim m_{Z^0}$ – "intense coupling regime")

$$m_{Z^0} < m_{A^0}: \; M^2(H^0,h^0) \sim \begin{pmatrix} m_{A^0}^2 & 0 \\ 0 & m_{Z^0}^2 \end{pmatrix} \quad \alpha \longrightarrow 0$$

$$m_{A^0} < m_{Z^0}: \; M^2(H^0,h^0) \sim \begin{pmatrix} m_{Z^0}^2 & 0 \\ 0 & m_{A^0}^2 \end{pmatrix} \quad \alpha \longrightarrow \pi/2$$

$\sin(\alpha) \longrightarrow 0/1$, $\cos(\alpha) \longrightarrow 1/0$,

Sfermion sector in the MSSM

$$\mathcal{M}^2_{\tilde f} = \begin{pmatrix} m^2_{\tilde f_L} & a_f\, m_f \\ a_f\, m_f & m^2_{\tilde f_R} \end{pmatrix}$$

soft ~~SUSY~~ mass $\Downarrow$ — D–term $\Downarrow$ — F–term $\Downarrow$

$$m^2_{\tilde f_L} = M^2_{\{\tilde Q,\tilde L\}} + (I^{3L}_f - e_f \sin^2\theta_W)\cos 2\beta\, m^2_Z + m^2_f$$

$$m^2_{\tilde f_R} = M^2_{\{\tilde U,\tilde D,\tilde E\}} - e_f \sin^2\theta_W \cos 2\beta\, m^2_Z + m^2_f$$

$$a_f = A_f - \mu \begin{cases} \cot\beta & \ldots \quad \text{up} \\ \tan\beta & \ldots \quad \text{down} \end{cases} \text{–type sfermions}$$

$\Uparrow$ cubic coupling — $\Uparrow$ F–term

$$m^2_{\tilde f_{1,2}} = \frac{1}{2}\left(m^2_{\tilde f_L} + m^2_{\tilde f_R} \mp \sqrt{(m^2_{\tilde f_L} - m^2_{\tilde f_R})^2 + 4a_f^2 m_f^2}\right)$$

Convention: $m_{\tilde f_1} \le m_{\tilde f_2}$

$$\begin{pmatrix} \tilde f_1 \\ \tilde f_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_{\tilde f} & \sin\theta_{\tilde f} \\ -\sin\theta_{\tilde f} & \cos\theta_{\tilde f} \end{pmatrix} \begin{pmatrix} \tilde f_L \\ \tilde f_R \end{pmatrix}$$

$$\cos\theta_{\tilde f} = \frac{-a_f\, m_f}{\sqrt{(m^2_{\tilde f_L} - m^2_{\tilde f_1})^2 + a_f^2 m_f^2}} \qquad (0 \le \theta_{\tilde f} < \pi)$$

$$m_f A_f = \frac{1}{2}\left(m^2_{\tilde f_1} - m^2_{\tilde f_2}\right)\sin 2\theta_{\tilde f} + m_f\, \mu \begin{Bmatrix} \cot\beta \\ \tan\beta \end{Bmatrix}$$

Chargino sector in the MSSM

The tree-level chargino mass matrix

$$X = \begin{pmatrix} M & \sqrt{2} m_W \sin\beta \\ \sqrt{2} m_W \cos\beta & \mu \end{pmatrix}$$

is diagonalized by two matrices U and V leading to

$$m_{\tilde{\chi}^\pm_{1,2}} = \frac{1}{\sqrt{2}} \left(M^2 + \mu^2 + 2m_W^2 \mp \left((M^2 - \mu^2)^2 + 4m_W^2 \cos^2 2\beta + 4m_W^2 (M^2 + \mu^2 + 2M\mu \sin 2\beta)\right)^{1/2} \right)^{1/2}$$

When μ or M is large,

one chargino eigenstate is a pure gaugino and the other one a pure higgsino state

$$|\mu| \gg \quad \rightarrow \quad m_{\tilde{\chi}^\pm_1} \sim M \,, \quad m_{\tilde{\chi}^\pm_2} \sim |\mu|$$

$$|M| \gg \quad \rightarrow \quad m_{\tilde{\chi}^\pm_1} \sim |\mu| \quad m_{\tilde{\chi}^\pm_2} \sim M$$

Neutralino sector in the MSSM

The tree-level neutralino mass matrix

$$Y = \begin{pmatrix} M' & 0 & -m_Z s_W c_\beta & m_Z s_W s_\beta \\ 0 & M & m_Z c_W c_\beta & -m_Z c_W s_\beta \\ -m_Z s_W c_\beta & m_Z c_W c_\beta & 0 & -\mu \\ m_Z s_W s_\beta & -m_Z c_W s_\beta & -\mu & 0 \end{pmatrix}$$

is diagonalized by the unitary matrix Z,

$$Z^* Y Z^{-1} = Y_D = \mathrm{diag}(m_{\chi_1^0}, m_{\chi_2^0}, m_{\chi_3^0}, m_{\chi_4^0})$$

Gauge unification leads to $M' = \frac{5}{3}\tan^2\theta_W M$, valid for the $\overline{\mathrm{DR}}$ parameters.

If μ or M is large,
one neutralino eigenstate is a pure bino, one a pure W^3-ino and the other ones pure higgsino states.

R-parity conservation

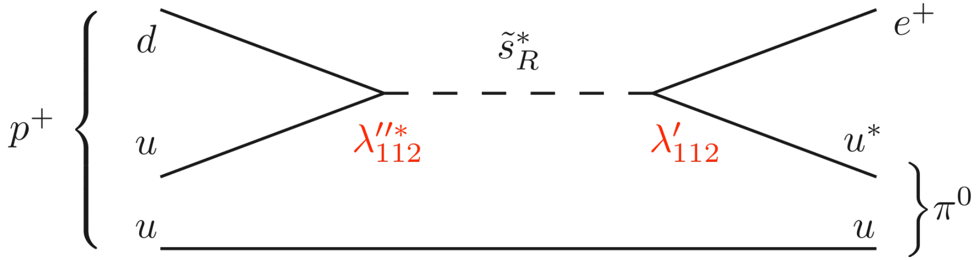
The most general gauge invariant superpotential can have B- and L-violating terms:

$$W_{\Delta \mathrm{L}=1} = \frac{1}{2}\lambda^{ijk} L_i L_j \bar{e}_k + \lambda'^{ijk} L_i Q_j \bar{d}_k + \mu'^i L_i H_u$$

$$W_{\Delta \mathrm{B}=1} = \frac{1}{2}\lambda''^{ijk} \bar{u}_i \bar{d}_j \bar{d}_k$$

Family indices $i = 1, 2, 3$,
Baryon number: $\mathrm{B} = +1/3$ for Q_i; $\mathrm{B} = -1/3$ for $\bar{u}_i, \bar{d}_i$; and $\mathrm{B} = 0$ for all others.
Lepton number: $\mathrm{L} = +1$ for L_i, $\mathrm{L} = -1$ for $\bar{e}_i$, and $\mathrm{L} = 0$ for all others.

Squarks would mediate disastrously rapid proton decay if R-parity were violated by both $\Delta\mathrm{B} = 1$ and $\Delta\mathrm{L} = 1$ interactions. This example shows $p \to e^+\pi^0$ mediated by a strange (or bottom) squark.



The MSSM is defined to conserve R-parity (s = spin):

$$P_R = (-1)^{3(\mathrm{B}-\mathrm{L})+2s} = \begin{cases} +1 \text{ for SM or Higgs particles} \\ -1 \text{ for sparticles} \end{cases}$$

Colliders: pair produced sparticles, stable LSP, sparticle decay chain to LSP

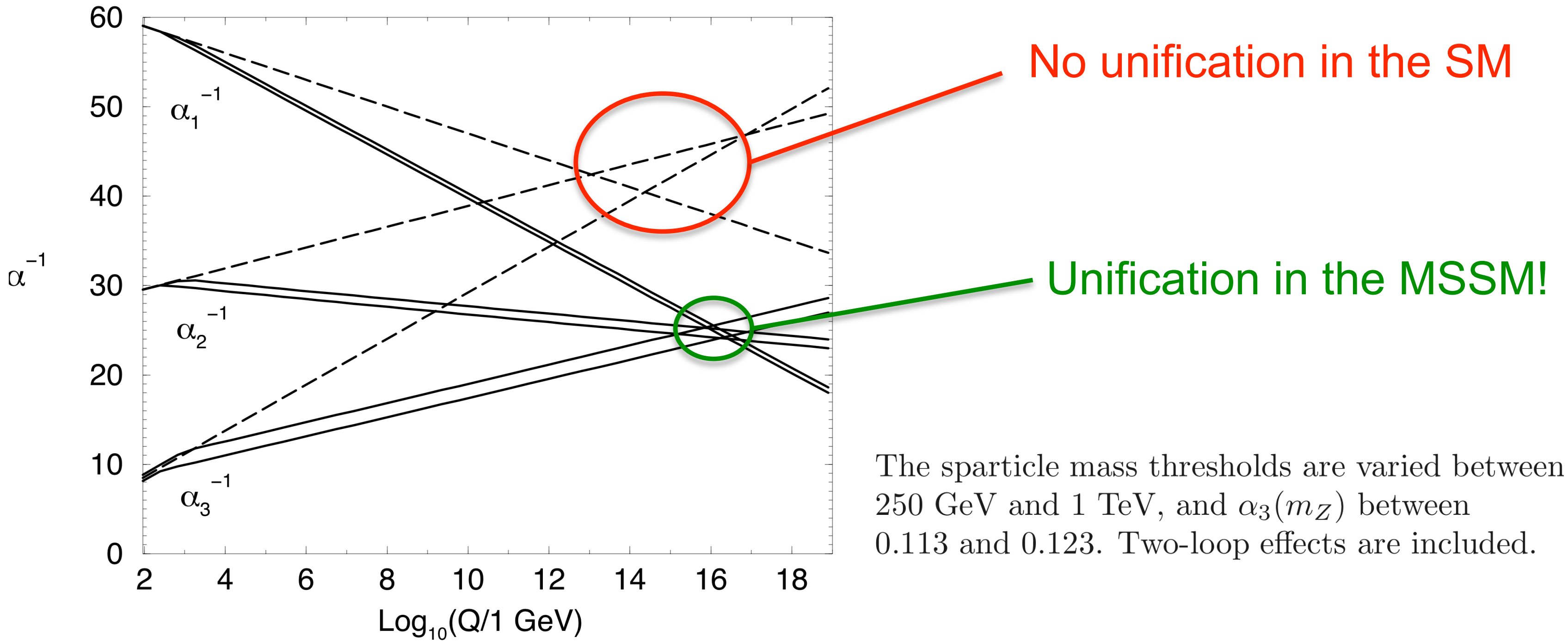
Gauge coupling unification

The 1-loop RG equations for the Standard Model gauge couplings are

$$\beta_{g_a} \equiv \frac{d}{dt} g_a = \frac{1}{16\pi^2} b_a g_a^3, \qquad (b_1, b_2, b_3) = \begin{cases} (41/10,\ -19/6,\ -7) & \text{Standard Model} \\ (33/5,\ 1,\ -3) & \text{MSSM} \end{cases}$$

where $t = \ln(Q/Q_0)$, with Q the RG scale, and for the GUT we use $g_1 = \sqrt{5/3}g'$.
Note, the inverses of $\alpha_a = g_a^2/4\pi$ run linear:

$$\frac{d}{dt}\alpha_a^{-1} = -\frac{b_a}{2\pi} \qquad (a = 1, 2, 3)$$



The sparticle mass thresholds are varied between 250 GeV and 1 TeV, and $\alpha_3(m_Z)$ between 0.113 and 0.123. Two-loop effects are included.

Renormalization Group equations for the MSSM

The soft SUSY breaking parameters run with the energy scale and the complete RGEs are known to 2 - loops
(S. P. Martin & M .T. Vaughn, 1993, Y. Yamada, 1994)

Large top Yukawa coupling drives Higgs masses^2 negative towards small energies, signaling Radiative Electroweak Symmetry Breaking !

$t = \ln(Q/Q_0)$, where Q is the renormalization scale, and Q_0 is a reference scale

Here we use simplified model, "real" version with only 3rd gen. Yuk's and A's

The RGEs for the superpotential parameters are:

$$
\begin{aligned}
\beta_{y_t} \equiv \frac{d}{dt} y_t &= \frac{y_t}{16\pi^2}\left[6y_t^* y_t + y_b^* y_b - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\right], \\
\beta_{y_b} \equiv \frac{d}{dt} y_b &= \frac{y_b}{16\pi^2}\left[6y_b^* y_b + y_t^* y_t + y_\tau^* y_\tau - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2\right], \\
\beta_{y_\tau} \equiv \frac{d}{dt} y_\tau &= \frac{y_\tau}{16\pi^2}\left[4y_\tau^* y_\tau + 3y_b^* y_b - 3g_2^2 - \frac{9}{5}g_1^2\right], \\
\beta_{\mu} \equiv \frac{d}{dt} \mu &= \frac{\mu}{16\pi^2}\left[3y_t^* y_t + 3y_b^* y_b + y_\tau^* y_\tau - 3g_2^2 - \frac{3}{5}g_1^2\right].
\end{aligned}
$$

independent of "soft" terms, prop. to value itself

$$\beta_{M_a} \equiv \frac{d}{dt} M_a = \frac{1}{8\pi^2} b_a g_a^2 M_a \qquad (b_a = 33/5,\ 1,\ -3)$$

b_a^{MSSM} are the same that appear in the gauge coupling RGs. Therefore it holds:

$$\frac{M_1}{g_1^2} = \frac{M_2}{g_2^2} = \frac{M_3}{g_3^2} = \frac{m_{1/2}}{g_U^2} \quad \Rightarrow \quad M_1 : M_2 : M_3 \sim 0.5 : 1 : 3$$

$$16\pi^2 \frac{d}{dt} a_t = a_t \Big[18 y_t^* y_t + y_b^* y_b - \frac{16}{3} g_3^2 - 3 g_2^2 - \frac{13}{15} g_1^2 \Big] + 2 a_b y_b^* y_t + y_t \Big[\frac{32}{3} g_3^2 M_3 + 6 g_2^2 M_2 + \frac{26}{15} g_1^2 M_1 \Big],$$

$$16\pi^2 \frac{d}{dt} a_b = a_b \Big[18 y_b^* y_b + y_t^* y_t + y_\tau^* y_\tau - \frac{16}{3} g_3^2 - 3 g_2^2 - \frac{7}{15} g_1^2 \Big] + 2 a_t y_t^* y_b + 2 a_\tau y_\tau^* y_b + y_b \Big[\frac{32}{3} g_3^2 M_3 + 6 g_2^2 M_2 + \frac{14}{15} g_1^2 M_1 \Big],$$

$$16\pi^2 \frac{d}{dt} a_\tau = a_\tau \Big[12 y_\tau^* y_\tau + 3 y_b^* y_b - 3 g_2^2 - \frac{9}{5} g_1^2 \Big] + 6 a_b y_b^* y_\tau + y_\tau \Big[6 g_2^2 M_2 + \frac{18}{5} g_1^2 M_1 \Big],$$

$$16\pi^2 \frac{d}{dt} b = b \Big[3 y_t^* y_t + 3 y_b^* y_b + y_\tau^* y_\tau - 3 g_2^2 - \frac{3}{5} g_1^2 \Big] + \mu \Big[6 a_t y_t^* + 6 a_b y_b^* + 2 a_\tau y_\tau^* + 6 g_2^2 M_2 + \frac{6}{5} g_1^2 M_1 \Big].$$

Breaking terms can get large corrections from other terms

Using

$$S \equiv \mathrm{Tr}[Y_j m^2_{\phi_j}] = m^2_{H_u} - m^2_{H_d} + \mathrm{Tr}[\mathbf{m^2_Q} - \mathbf{m^2_L} - 2\mathbf{m^2_{\bar{u}}} + \mathbf{m^2_{\bar{d}}} + \mathbf{m^2_{\bar{e}}}].$$

always positive

$$\left\{\begin{aligned} X_t &= 2|y_t|^2(m^2_{H_u} + m^2_{Q_3} + m^2_{\bar{u}_3}) + 2|a_t|^2, \\ X_b &= 2|y_b|^2(m^2_{H_d} + m^2_{Q_3} + m^2_{\bar{d}_3}) + 2|a_b|^2, \\ X_\tau &= 2|y_\tau|^2(m^2_{H_d} + m^2_{L_3} + m^2_{\bar{e}_3}) + 2|a_\tau|^2. \end{aligned}\right.$$

we can write

$$16\pi^2 \frac{d}{dt} m^2_{H_u} = 3X_t - 6g_2^2|M_2|^2 - \frac{6}{5}g_1^2|M_1|^2 + \frac{3}{5}g_1^2 S,$$

$$16\pi^2 \frac{d}{dt} m^2_{H_d} = 3X_b + X_\tau - 6g_2^2|M_2|^2 - \frac{6}{5}g_1^2|M_1|^2 - \frac{3}{5}g_1^2 S.$$

Normally, $X_t >> X_b, X_\tau$, at some scale $m^2_{H_u}$ becomes negative!

dynamical spontaneous $SU(2)_I$ x $U(1)_Y$ symmetry breaking

The 3rd family bilinear soft terms run as:

$$
\begin{aligned}
16\pi^2 \frac{d}{dt} m^2_{Q_3} &= X_t + X_b - \frac{32}{3} g_3^2 |M_3|^2 - 6 g_2^2 |M_2|^2 - \frac{2}{15} g_1^2 |M_1|^2 + \frac{1}{5} g_1^2 S, \\
16\pi^2 \frac{d}{dt} m^2_{\bar{u}_3} &= 2X_t - \frac{32}{3} g_3^2 |M_3|^2 - \frac{32}{15} g_1^2 |M_1|^2 - \frac{4}{5} g_1^2 S, \\
16\pi^2 \frac{d}{dt} m^2_{\bar{d}_3} &= 2X_b - \frac{32}{3} g_3^2 |M_3|^2 - \frac{8}{15} g_1^2 |M_1|^2 + \frac{2}{5} g_1^2 S, \\
16\pi^2 \frac{d}{dt} m^2_{L_3} &= X_\tau - 6 g_2^2 |M_2|^2 - \frac{6}{5} g_1^2 |M_1|^2 - \frac{3}{5} g_1^2 S, \\
16\pi^2 \frac{d}{dt} m^2_{\bar{e}_3} &= 2X_\tau - \frac{24}{5} g_1^2 |M_1|^2 + \frac{6}{5} g_1^2 S.
\end{aligned}
$$

Sfermion masses^2 are $\sim \alpha\, m^2_{1/2} + \beta\, m^2_0$

minimal SUperGRAvity

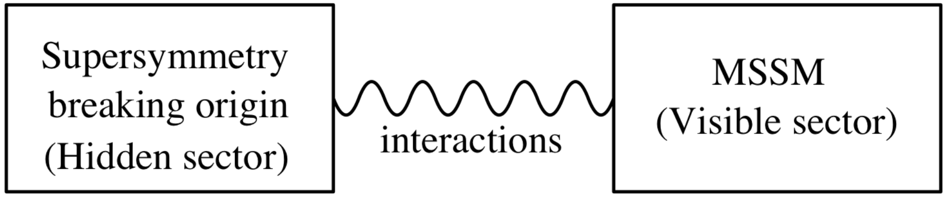
Local SUGRA: Gravity is naturally included

Breaking of SUGRA – at a high scale near to M_{Planck}
goldstino – fermionic comp. of a supermultiplet - auxiliary field F (or D) gets v.e.v.

swallowed into the spin$\pm\frac{1}{2}$ components of the gravitino

Gravity mediated SUSY breaking, dimensional analysis:

$$m_{\text{soft}} \sim \langle F\rangle / M_{\text{P}},$$



$$\mathcal{L}_{\text{NR}} = -\frac{1}{M_{\text{P}}} F \left(\frac{1}{2} f_a \lambda^a \lambda^a + \frac{1}{6} y'^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} \mu'^{ij} \phi_i \phi_j \right) + \text{c.c.} - \frac{1}{M_{\text{P}}^2} F F^* k^i_j \phi_i \phi^{*j}$$

Assume “minimal” form for the normalization of kinetic terms and gauge interactions in the full, non-renormalizable supergravity Lagrangian, then

$$\mathcal{L}_{\text{NR}} = -\frac{1}{M_{\text{P}}} F \left(\frac{1}{2} f \lambda^a \lambda^a + \frac{1}{6} \alpha \phi_i \phi_j \phi_k + \frac{1}{2} \beta \phi_i \phi_j \right) + \text{c.c.} - \frac{1}{M_{\text{P}}^2} F F^* k \phi_i \phi^{*j}$$

After spontaneous SUSY breaking only four parameters left:

$$m_{1/2} = f\frac{\langle F\rangle}{M_{\rm P}}, \qquad m_0^2 = k\frac{|\langle F\rangle|^2}{M_{\rm P}^2}, \qquad A_0 = \alpha\frac{\langle F\rangle}{M_{\rm P}}, \qquad B_0 = \beta\frac{\langle F\rangle}{M_{\rm P}}.$$

Comparing with our "soft" SUSY Lagrangian

$$\mathcal{L}_{soft} = -m_{ij}^2\phi_i^*\phi_j - \left(b_{ij}\phi_i\phi_j + A_{ijk}\phi_i\phi_j\phi_k + \frac{M_a}{2}\lambda_a\lambda_a + h.c.\right)$$

we can write all "soft" parameters as

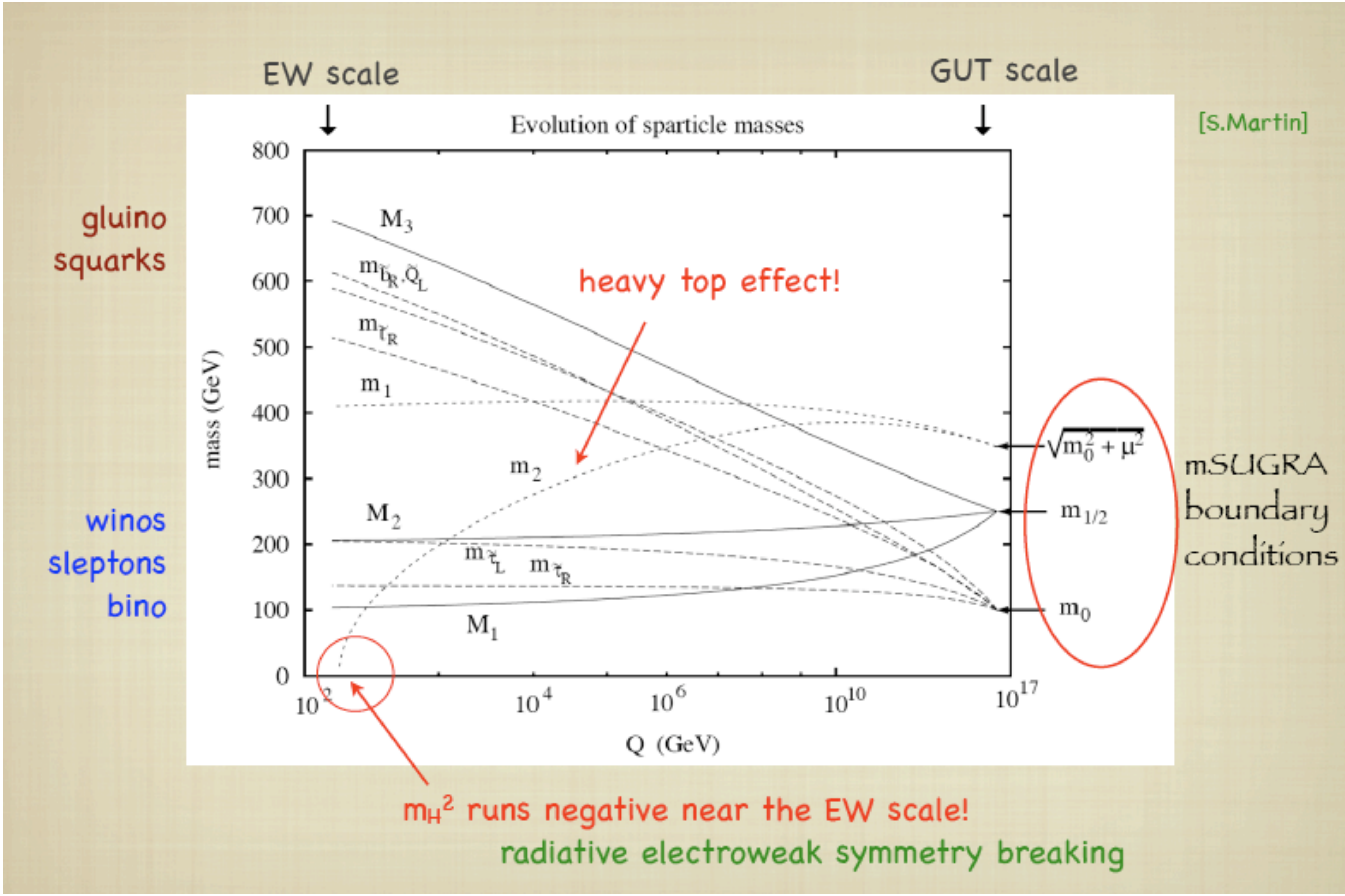
$$M_3 = M_2 = M_1 = m_{1/2},$$
$$\mathbf{m_Q^2} = \mathbf{m_{\bar u}^2} = \mathbf{m_{\bar d}^2} = \mathbf{m_L^2} = \mathbf{m_{\bar e}^2} = m_0^2\,\mathbf{1}, \qquad m_{H_u}^2 = m_{H_d}^2 = m_0^2,$$
$$\mathbf{a_u} = A_0\mathbf{y_u}, \qquad \mathbf{a_d} = A_0\mathbf{y_d}, \qquad \mathbf{a_e} = A_0\mathbf{y_e},$$
$$b = B_0\mu,$$

What's about $\tan\beta$ and μ?

With the two Higgs minimum conditions we get the relations. But only the absolute value of μ is fixed:

$$m_Z^2 = \frac{m_{H_d}^2 + \mu^2 - (m_{H_u}^2 + \mu^2)\tan^2\beta}{\tan^2\beta - 1} \quad \text{and} \quad \sin 2\beta = \frac{B_0\mu}{m_{H_u}^2 + m_{H_d}^2 + \mu^2}$$

From Sabine Kraml's talk at PiP, 3 Dec 09



Reference point SPS1a'

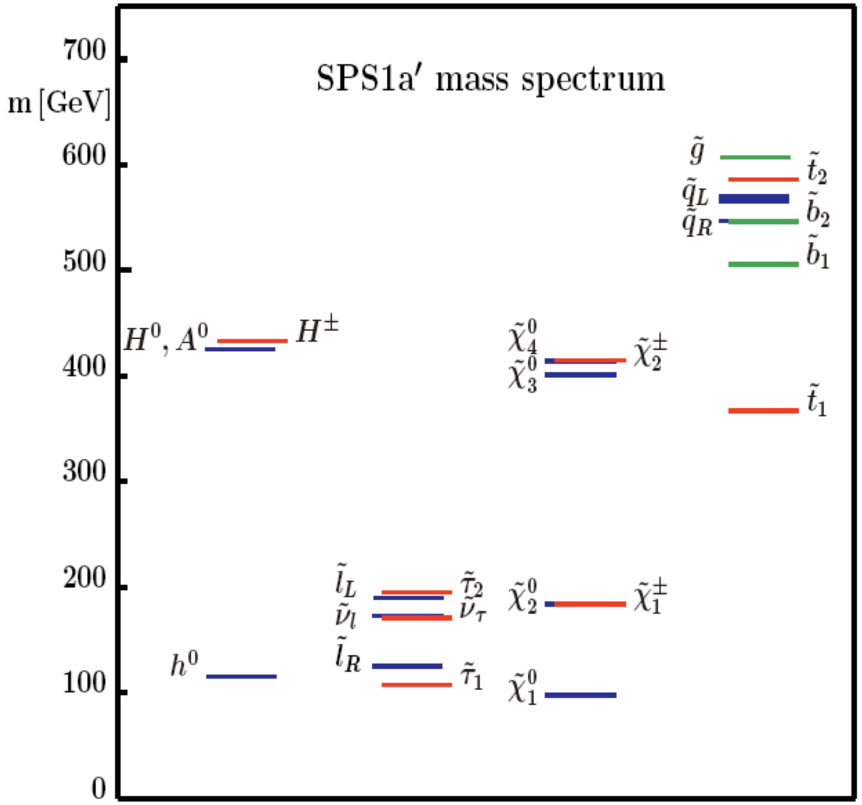
mSUGRA point: $M_{1/2} = 250$ GeV, $M_0 = 70$ GeV, $A_0 = -300$ GeV,

$\tan\beta = 10$, sign$\mu = +1$

Note, the point SPS1a' is close to the original Snowmass point SPS1a (has $M_0 = 100$ GeV, $A_0 = -100$ GeV).

Parameter	SPS1a′ value	Parameter	SPS1a′ value
g'	0.3636	M_1	103.3
g	0.6479	M_2	193.2
g_s	1.0844	M_3	571.7
Y_τ	0.1034	A_τ	−445.2
Y_t	0.8678	A_t	−565.1
Y_b	0.1354	A_b	−943.4
μ	396.0	$\tan\beta$	10.0
M_{H_d}	159.8	$\lvert M_{H_u}\rvert$	378.3
M_{L_1}	181.0	M_{L_3}	179.3
M_{E_1}	115.7	M_{E_3}	110.0
M_{Q_1}	525.8	M_{Q_3}	471.4
M_{U_1}	507.2	M_{U_3}	387.5
M_{D_1}	505.0	M_{D_3}	500.9

DRbar parameter at Q = 1 TeV



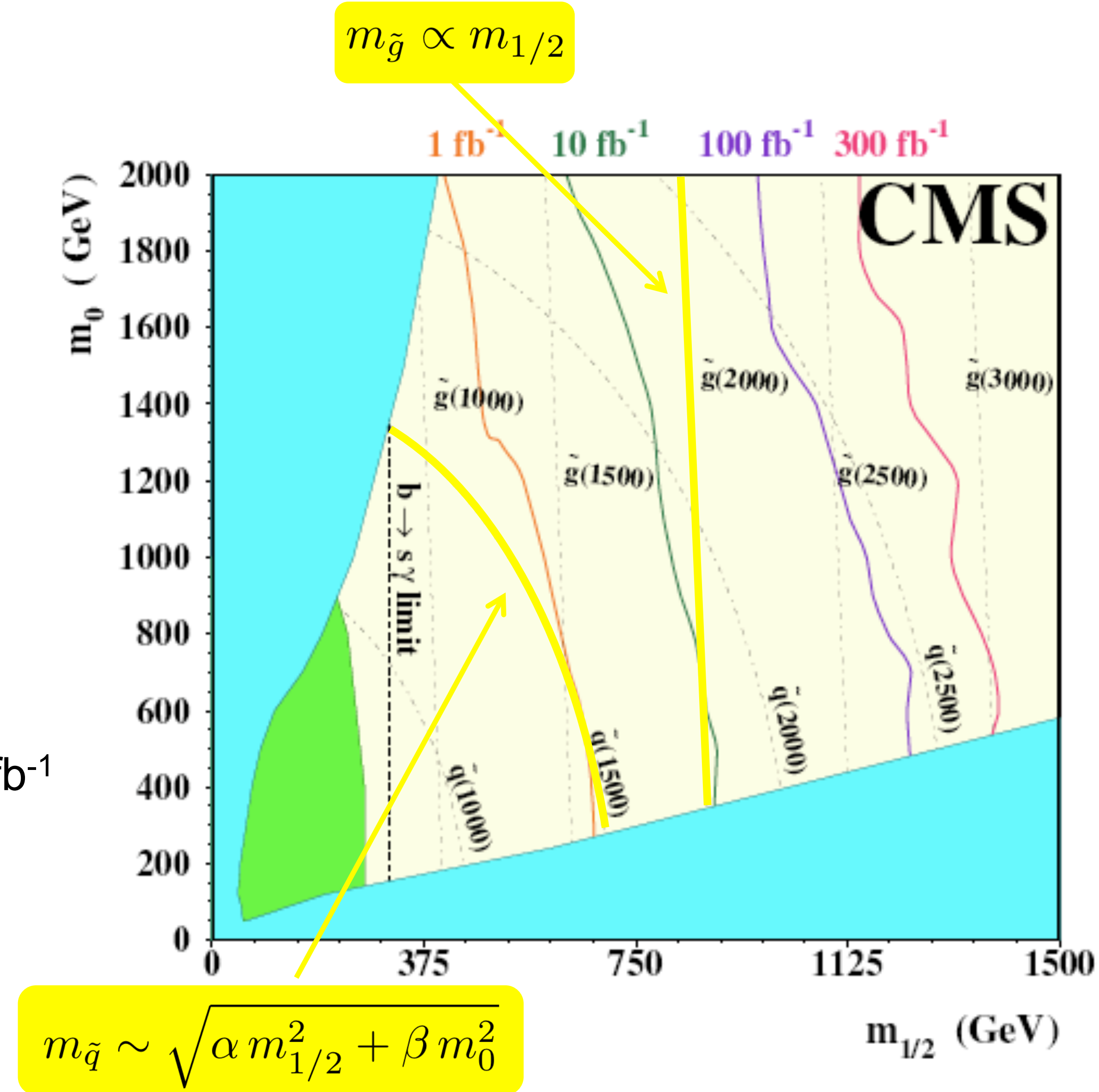
5s discovery reach for gluinos and squarks

[CMS98]:

M < 2 TeV with 1fb^{-1}

M < 2 TeV with 10 fb^{-1}

M < 2.5 – 3 TeV with 300 fb^{-1}



Paige: "Could find SUSY quickly, but must first understand detectors."

talk from Frank E. Paige, Vienna, July 2004

Thank you for your attention!