Shear viscosity to entropy ratio in A+A collisions at NICA energies

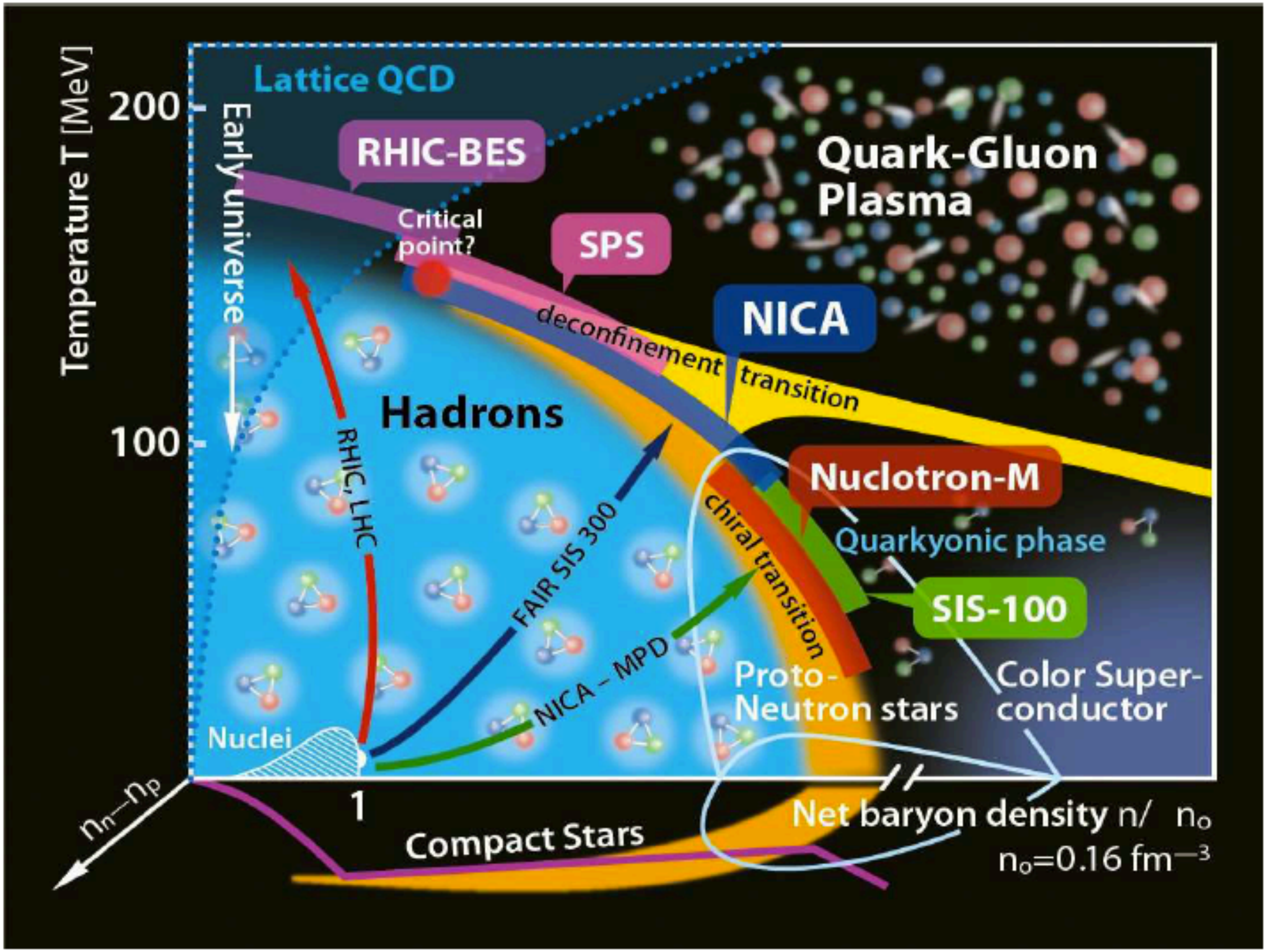
E. Zabrodin

in collaboration with

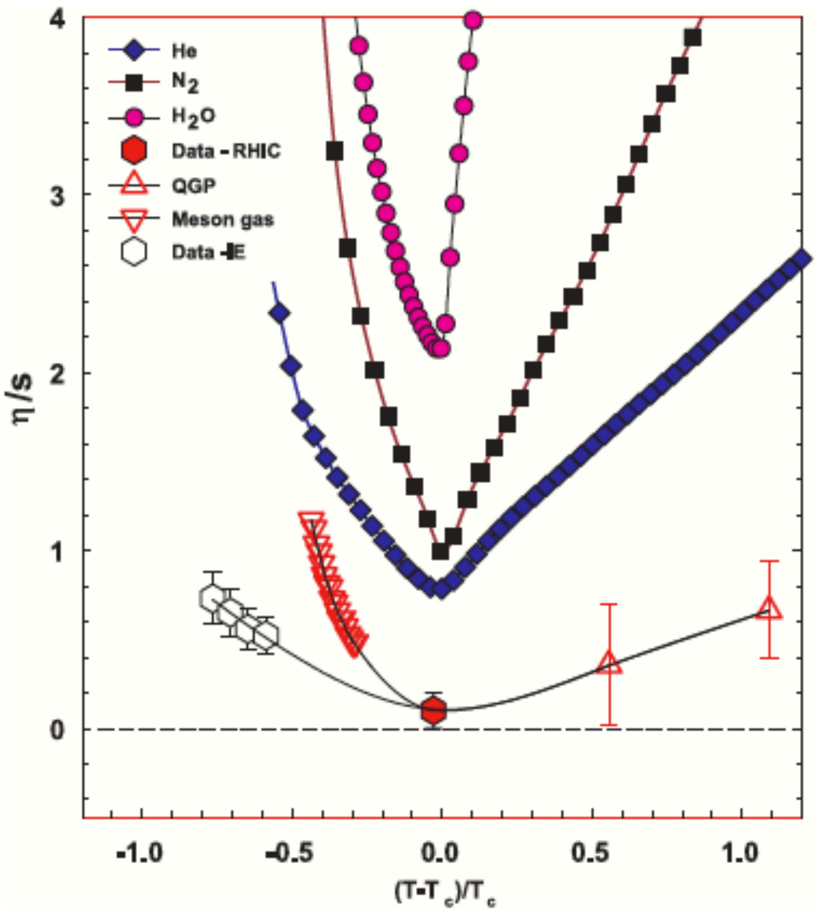
L. Bravina, M. Teslyk, and O. Vitiuk

Simposium “Four decades of hydrodynamics at UiO” ,
Dedicated to Laszlo Csernai 70th birthday. Bergen, 9.09.2019

Motivation


Temperature T [MeV]
200
100
Lattice QCD
Early universe
RHIC-BES
Critical point?
SPS
NICA
Quark-Gluon Plasma
deconfinement transition
Hadrons
RHIC, LHC
FAIR SIS 300
NICA – MPD
Nuclotron-M
Quarkyonic phase
chiral transition
SIS-100
Proto-Neutron stars
Color Super-conductor
Nuclei
$n_n - n_p$
1
Compact Stars
Net baryon density n/ n_0
n_0=0.16 fm^{-3}

Motivation



taken from

R.Rapp, H.Hees. arXiv:0803.0901[hep-ph]

- A.Muronga. PRC 69, 044901 (2004)
- L.Csernai, J.Kapusta, L.McLerran. PRL 97, 152303 (2006)
- P.Romatschke, U.Romatschke. PRL 99, 172301 (2007)
- S.Plumari et al. PRC 86, 054902 (2012)
- ALICE collaboration, CERNCOURIER (14.10.2016)
- J.Rose et al. PRC 97, 055204 (2018)

Green-Kubo: shear viscosity η may be defined as:

$$\eta(t_0) = \frac{1}{\hbar}\frac{V}{T}\int_{t_0}^{\infty} \mathrm{d}t \langle \pi(t)\,\pi(t_0)\rangle_t = \frac{\tau}{\hbar}\frac{V}{T}\langle \pi(t_0)\,\pi(t_0)\rangle,$$

where

$$\begin{aligned}
\langle \pi(t)\,\pi(t_0)\rangle_t &= \frac{1}{3}\sum_{\substack{i,j=1\\ i\neq j}}^{3} \lim_{t_{\max}\to\infty} \frac{1}{t_{\max}-t_0}\int_{t_0}^{t_{\max}} \mathrm{d}t'\,\pi^{ij}(t+t')\,\pi^{ij}(t') \\
&= \langle \pi(t_0)\,\pi(t_0)\rangle \exp\left(-\frac{t-t_0}{\tau}\right)
\end{aligned}$$

with

$$\pi^{ij}(t) = \frac{1}{V}\sum_{\mathrm{particles}} \frac{p^i(t)\,p^j(t)}{E(t)}$$

t_0: initial cut-off time to start with

Model setup: cell calculations

- UrQMD calculations, central Au+Au collisions at energies $E \in [10, 20, 30, 40]$ AGeV of the projectile, 51200 events per each
- central cell $5 \times 5 \times 5$ fm^3 $\Rightarrow$ $\{\varepsilon, \rho_{\mathrm{B}}, \rho_{\mathrm{S}}\}$ at times $t_{\mathrm{cell}} = 1 \div 20$ fm/c
- statistical model (SM): $\{\varepsilon, \rho_{\mathrm{B}}, \rho_{\mathrm{S}}\}$ $\Rightarrow$ $\{T, s, \mu_{\mathrm{B}}, \mu_{\mathrm{S}}\}$

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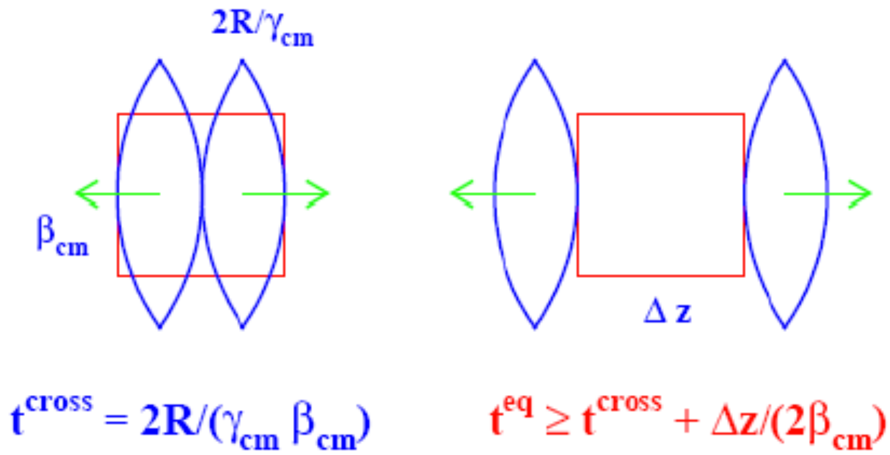
Equilibration in the Central Cell



Kinetic equilibrium:

Isotropy of velocity distributions

Isotropy of pressure

Thermal equilibrium:

Energy spectra of particles are described by Boltzmann distribution

L.Bravina et al., PLB 434 (1998) 379; JPG 25 (1999) 351

$$\frac{dN_i}{4\pi p E dE} = \frac{V g_i}{(2\pi\hbar)^3} \exp\left(\frac{\mu_i}{T}\right) \exp\left(-\frac{E_i}{T}\right)$$

Chemical equlibrium:

Particle yields are reproduced by SM with the same values of $(T,\ \mu_B,\ \mu_S)$:

$$N_i = \frac{V g_i}{2\pi^2\hbar^3} \int_0^\infty p^2 dp \exp\left(\frac{\mu_i}{T}\right) \exp\left(-\frac{E_i}{T}\right)$$

Statistical model of ideal hadron gas

input values — **output values**

$$\varepsilon^{\rm mic} = \frac{1}{V}\sum_i E_i^{\rm SM}(T,\mu_{\rm B},\mu_{\rm S}),$$

$$\rho_{\rm B}^{\rm mic} = \frac{1}{V}\sum_i B_i\cdot N_i^{\rm SM}(T,\mu_{\rm B},\mu_{\rm S}),$$

$$\rho_{\rm S}^{\rm mic} = \frac{1}{V}\sum_i S_i\cdot N_i^{\rm SM}(T,\mu_{\rm B},\mu_{\rm S}).$$

Multiplicity

$$N_i^{\rm SM} = \frac{Vg_i}{2\pi^2\hbar^3}\int_0^\infty p^2 f(p,m_i)dp,$$

Energy

$$E_i^{\rm SM} = \frac{Vg_i}{2\pi^2\hbar^3}\int_0^\infty p^2\sqrt{p^2+m_i^2}\,f(p,m_i)dp$$

Pressure

$$P^{\rm SM} = \sum_i\frac{g_i}{2\pi^2\hbar^3}\int_0^\infty p^2\frac{p^2}{3(p^2+m_i^2)^{1/2}}\,f(p,m_i)dp$$

Entropy density

$$s^{\rm SM} = -\sum_i\frac{g_i}{2\pi^2\hbar^3}\int_0^\infty f(p,m_i)\,\left[\ln f(p,m_i)-1\right]\,p^2dp$$

Model setup: box calculations

- UrQMD box calculations at $\{\varepsilon, \rho_{\mathrm{B}}, \rho_{\mathrm{S}}\}$ for every energy and cell time t_{cell} from cell calculations, 80 points in total, 12800 events per each
  - ρ_{B} is included as $N_p : N_n = 1 : 1$
  - ρ_{S} is included via kaons K^-
  - box size: $10 \times 10 \times 10$ fm^3
  - box boundaries: transparent
- $\pi^{ij}(t)$ data extraction: $t = 1 \div 1000$ fm/c in box time, all types of hadrons are taken into account

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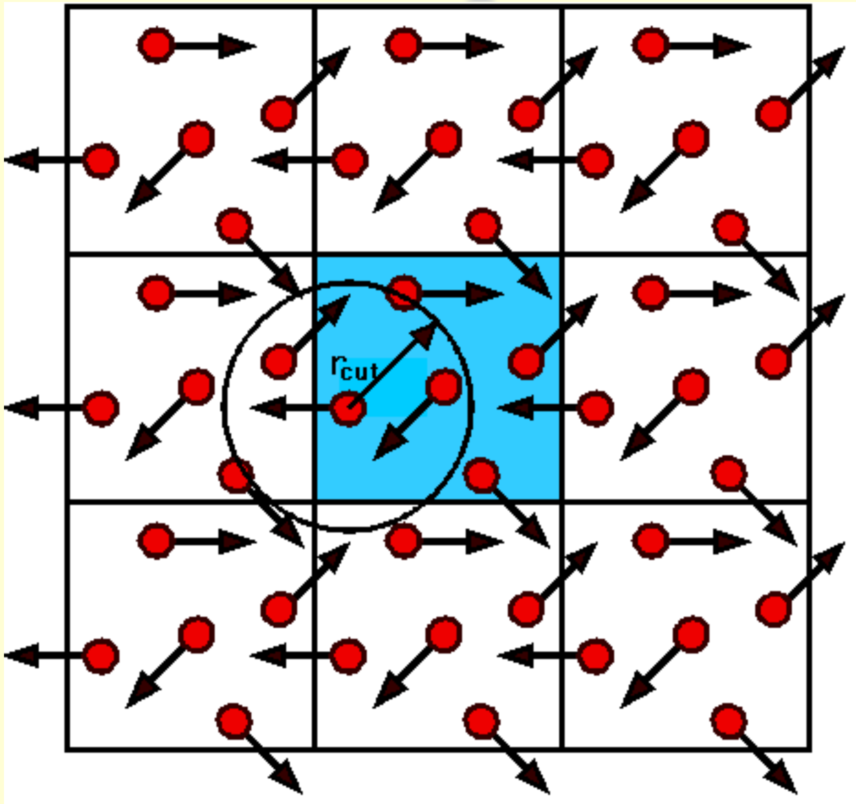
Box with periodic boundary conditions



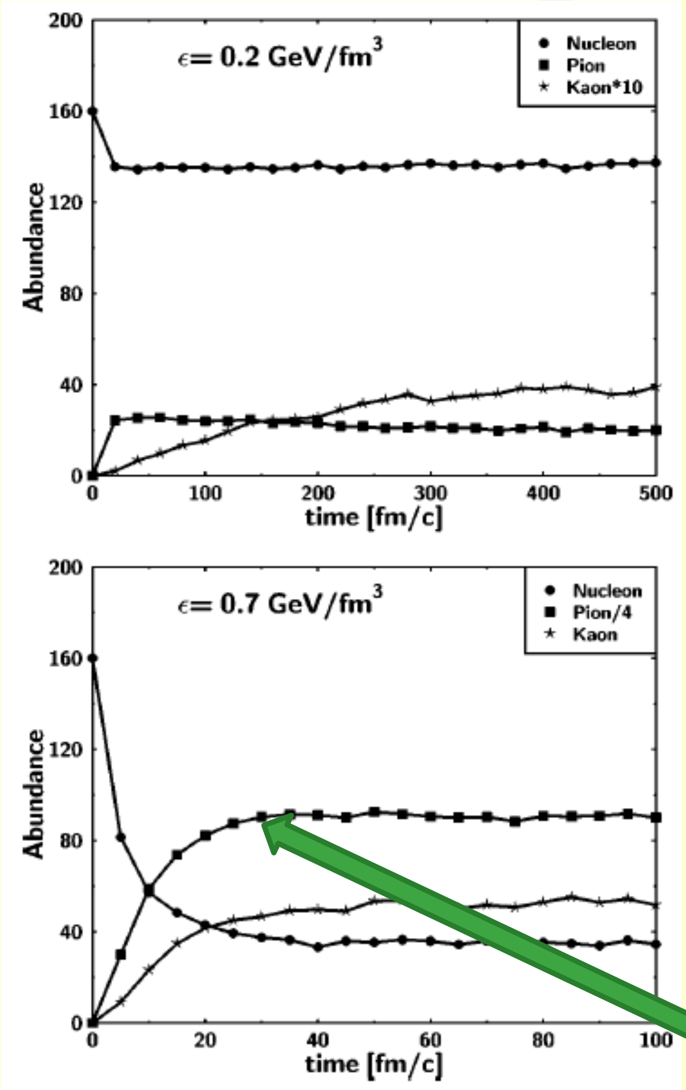
M.Belkacem et al., PRC 58, 1727 (1998)

Model employed: UrQMD
55 different baryon species (N, Δ, hyperons and their resonances with $m \leq$ 2.25 GeV/c^2)
32 different meson species (including resonances with $m \leq$ 2 GeV/c^2) and their respective antistates.
For higher mass excitations a string mechanism is invoked.

Initialization: (i) nucleons are uniformly distributed in a configuration space; (ii) Their momenta are uniformly distributed in a sphere with random radius and then rescaled to the desired energy density.

Test for equilibrium: particle yields and energy spectra

Box: particle abundances



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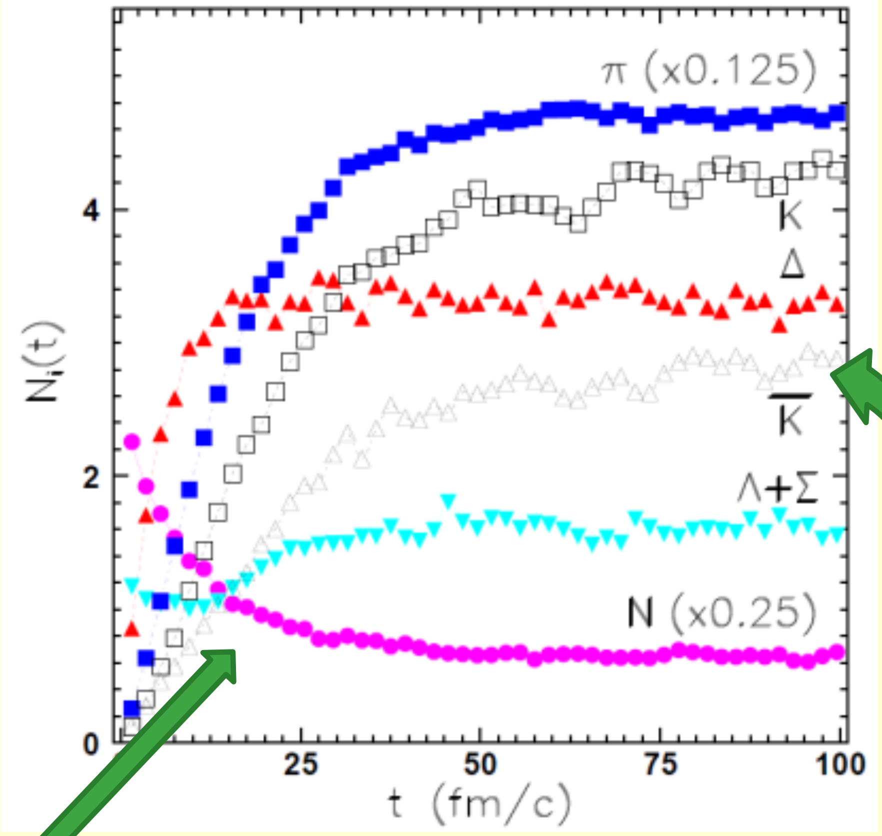
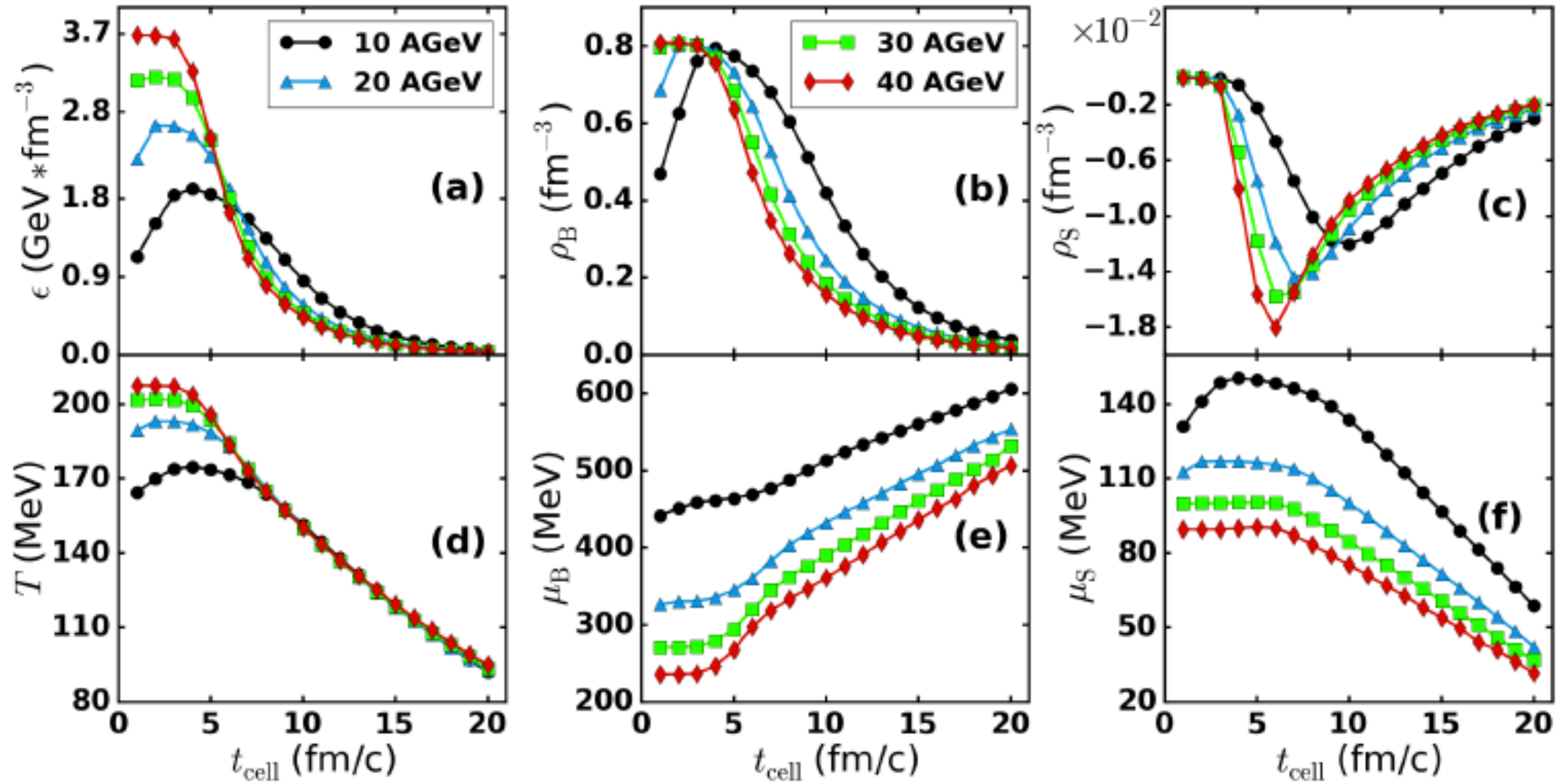


L.Bravina et al., PRC 62, 064906 (2000)

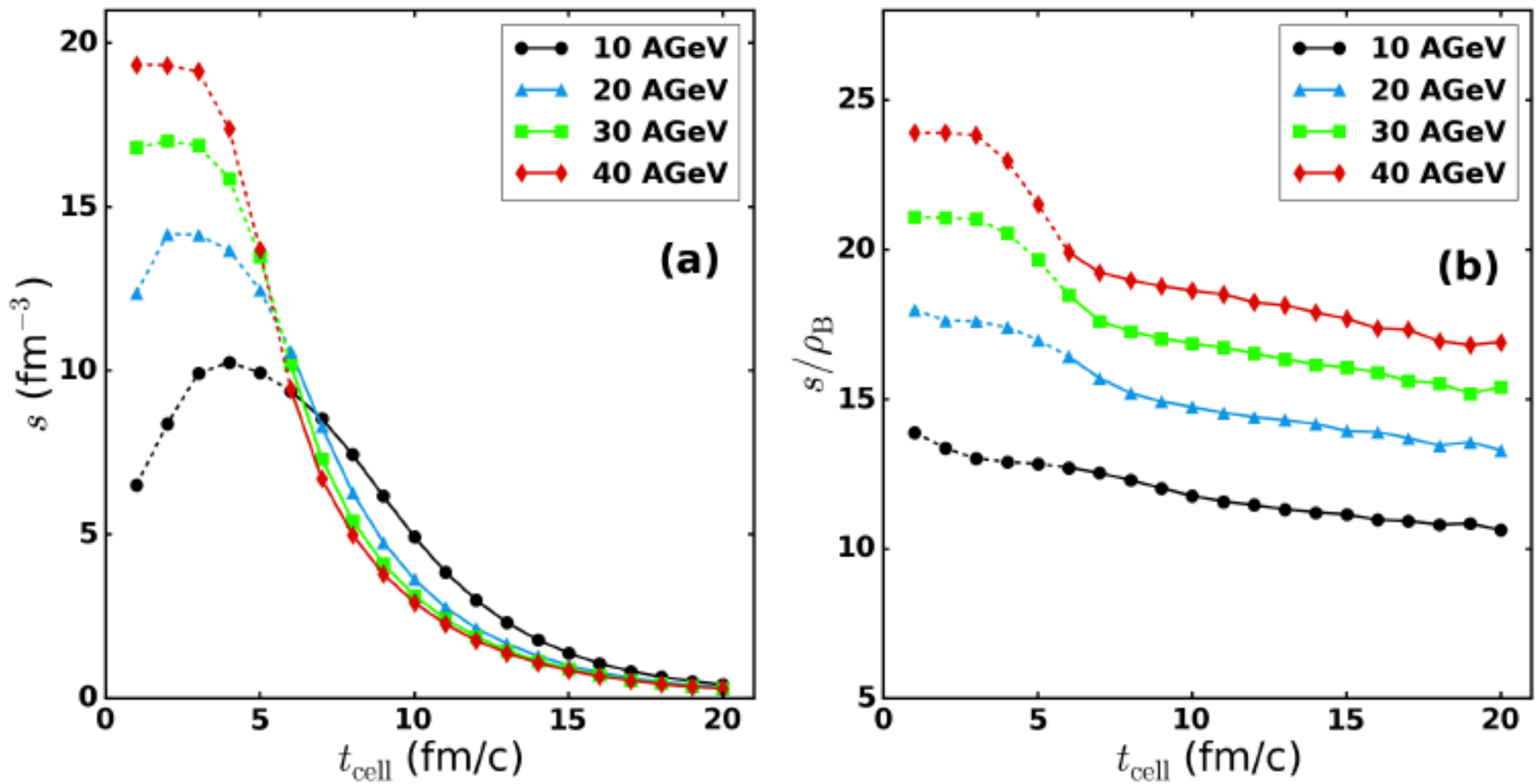
Saturation of yields after a certain time. Strange hadrons are saturated longer than others (at not very high energy densities)

Cell + SM



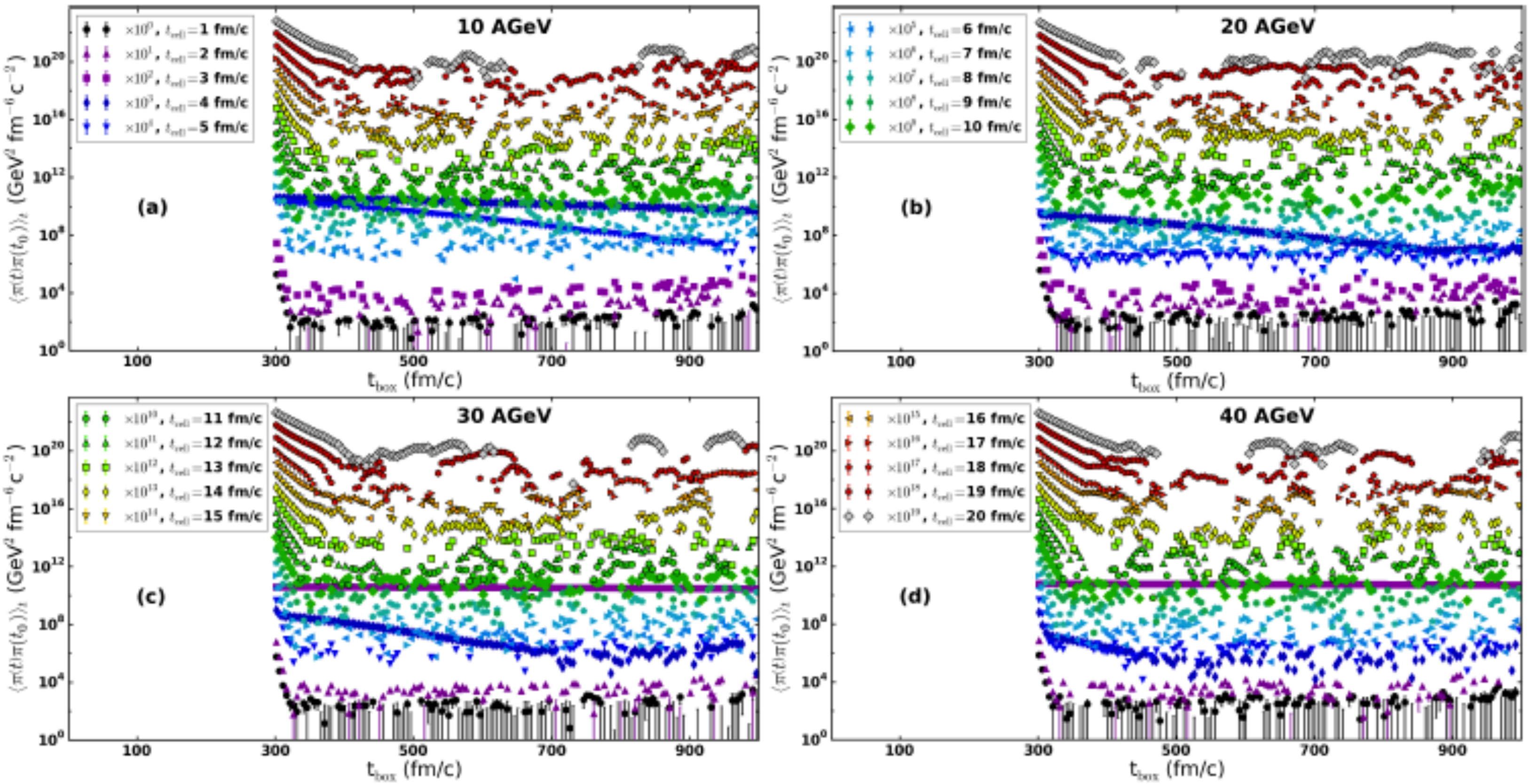
Dependence of $\varepsilon, \rho_{\rm B}, \rho_{\rm S}$ (from cell) and of $T, \mu_{\rm B}, \mu_{\rm S}$ (from SM) on $t_{\rm cell}$

SM, Boltzmann entropy s



Dynamics of Boltzmann entropy density s and of $s/\rho_{\rm B}$ in cell

Results: $\langle \pi(t)\pi(t_0) \rangle_t$ at $E \in [10, 20, 30, 40]$ AGeV

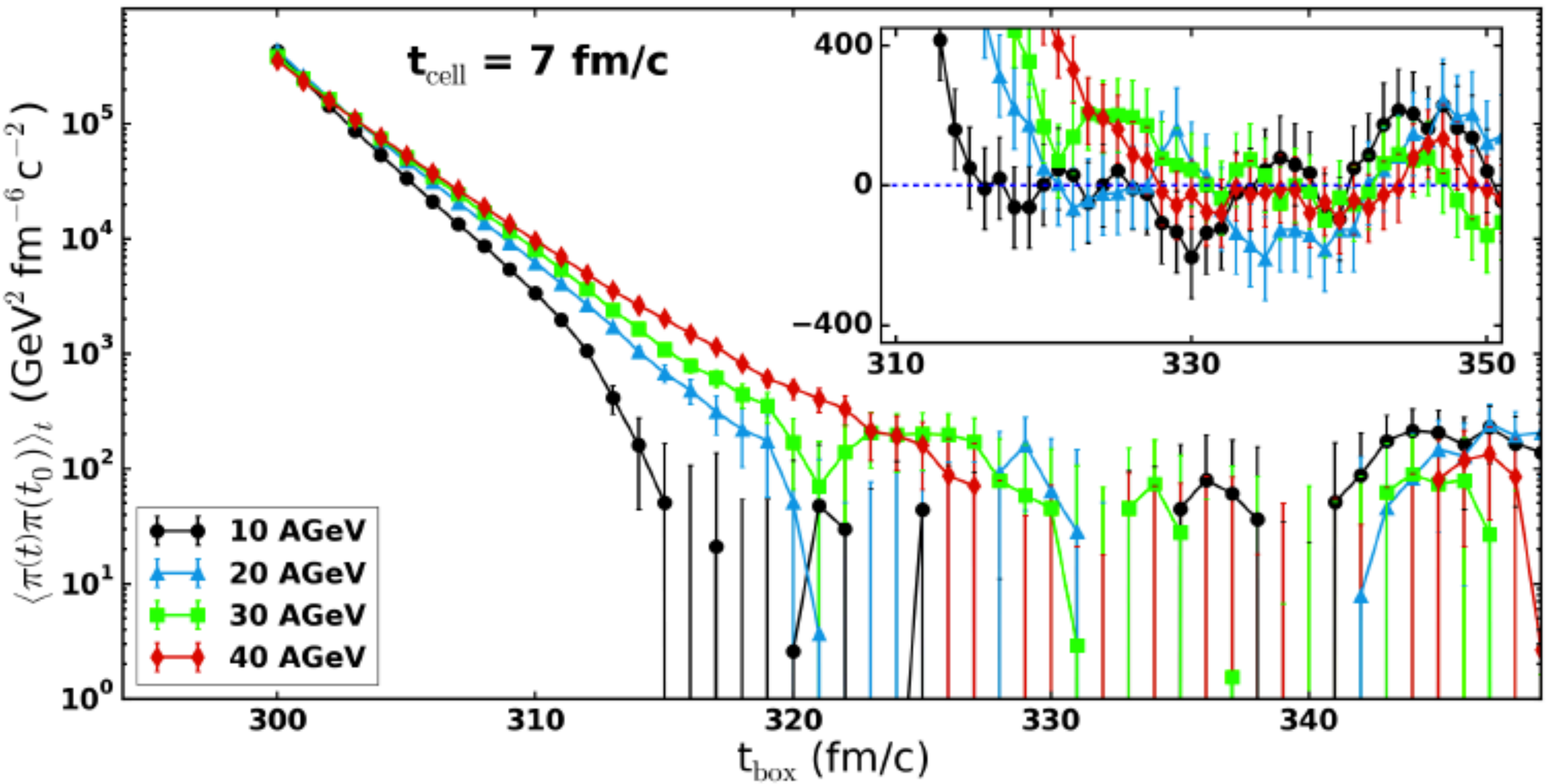


Time dependence of correlators $\langle \pi(t)\pi(t_0) \rangle_t$

$t_0 = 300$ fm/c

$t_{\rm cell} \in \{1 \div 20\}$ fm/c

Results: $\langle \pi(t) \pi(t_0) \rangle_t$ at fixed t_{cell}



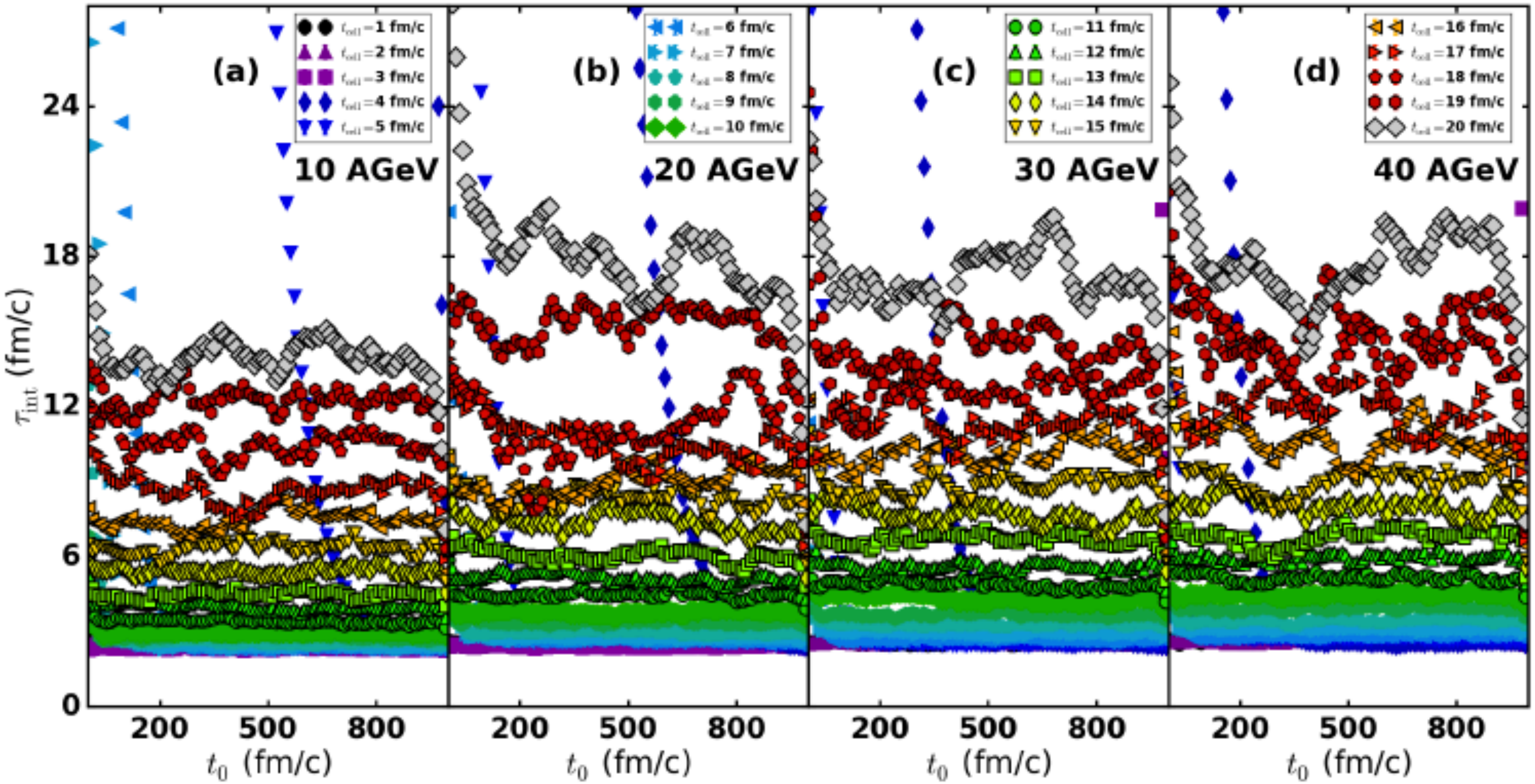
Time dependence of correlators $\langle \pi(t) \pi(t_0) \rangle_t$

Subplot: the same but at linear scale

$t_0 = 300$ fm/c

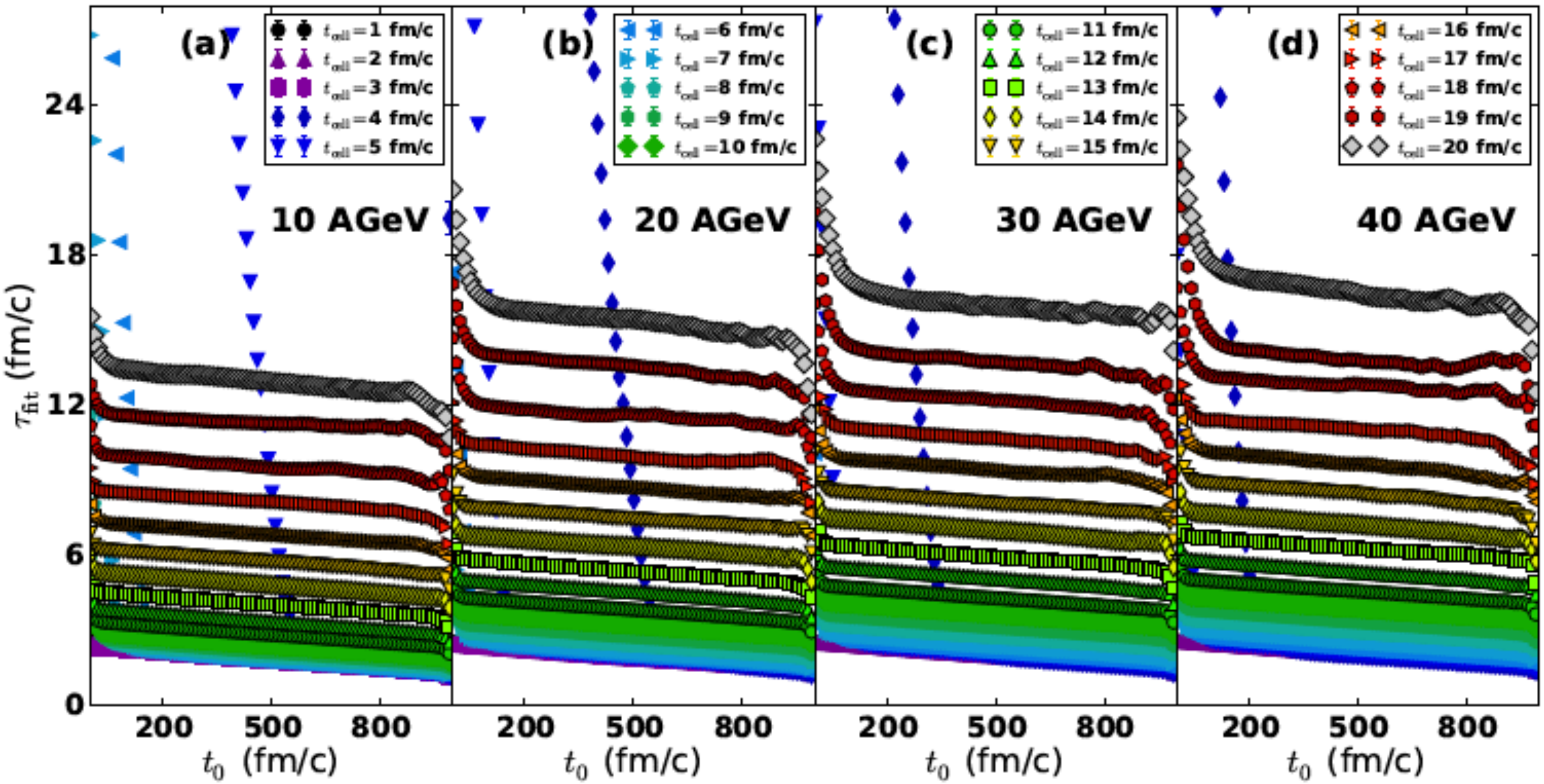
$t_{cell} = 7$ fm/c

Results: $\tau\,(t_0)$



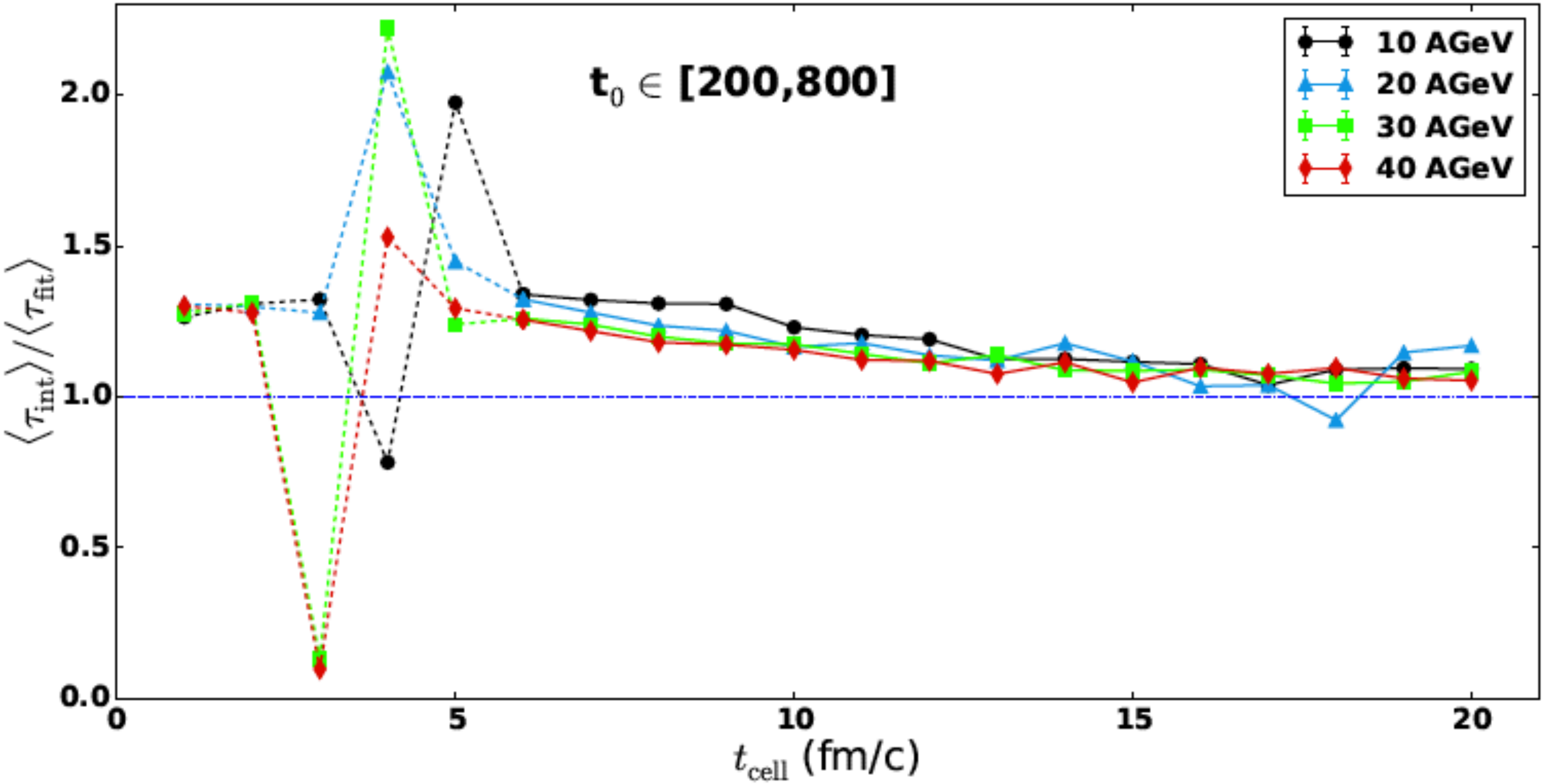
Dependence of τ on t_0

Results: τ from the fit



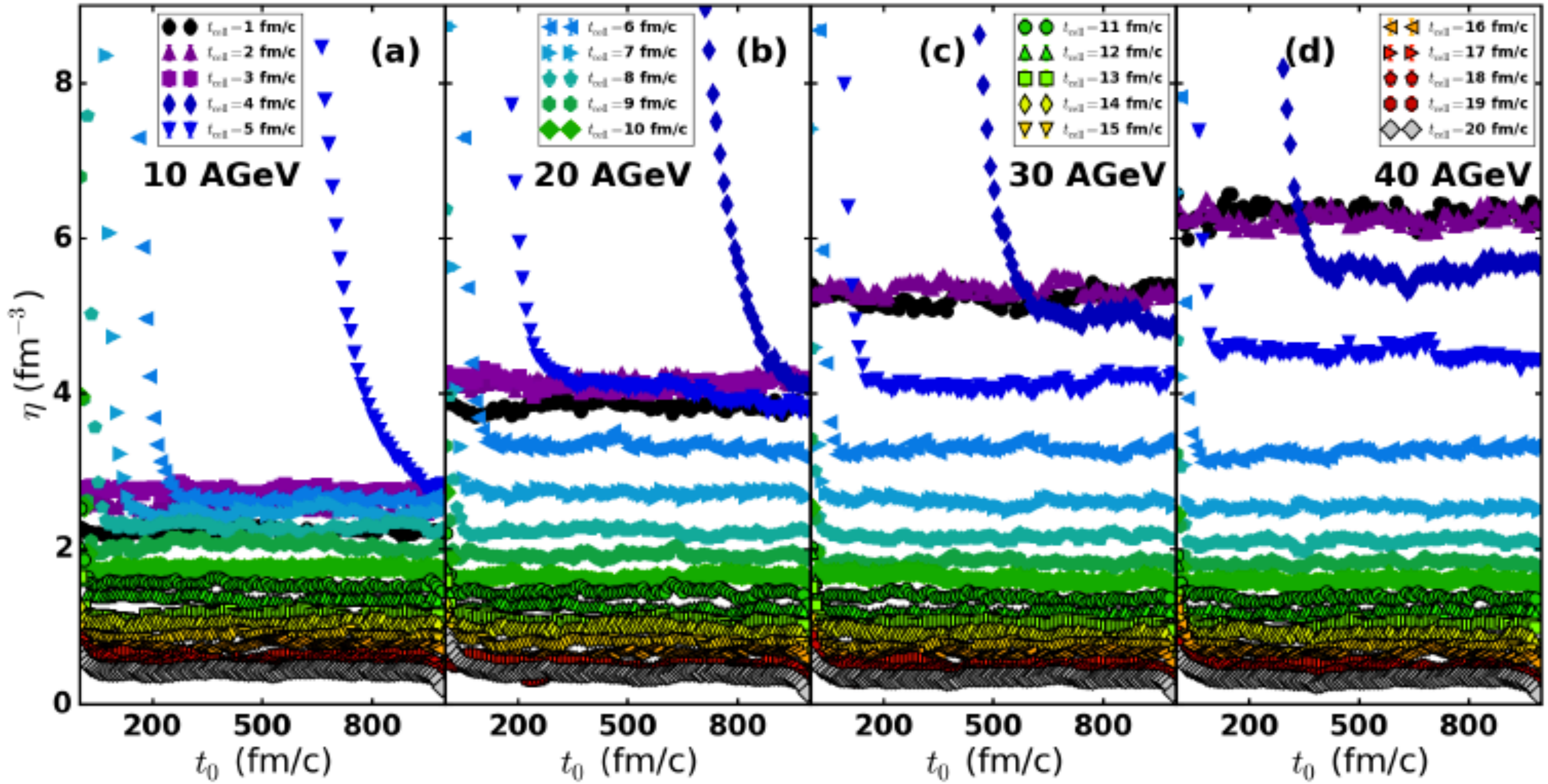
Dependence of τ_{fit} on t_0

Results: Comparison of τ_{int} and τ_{fit}



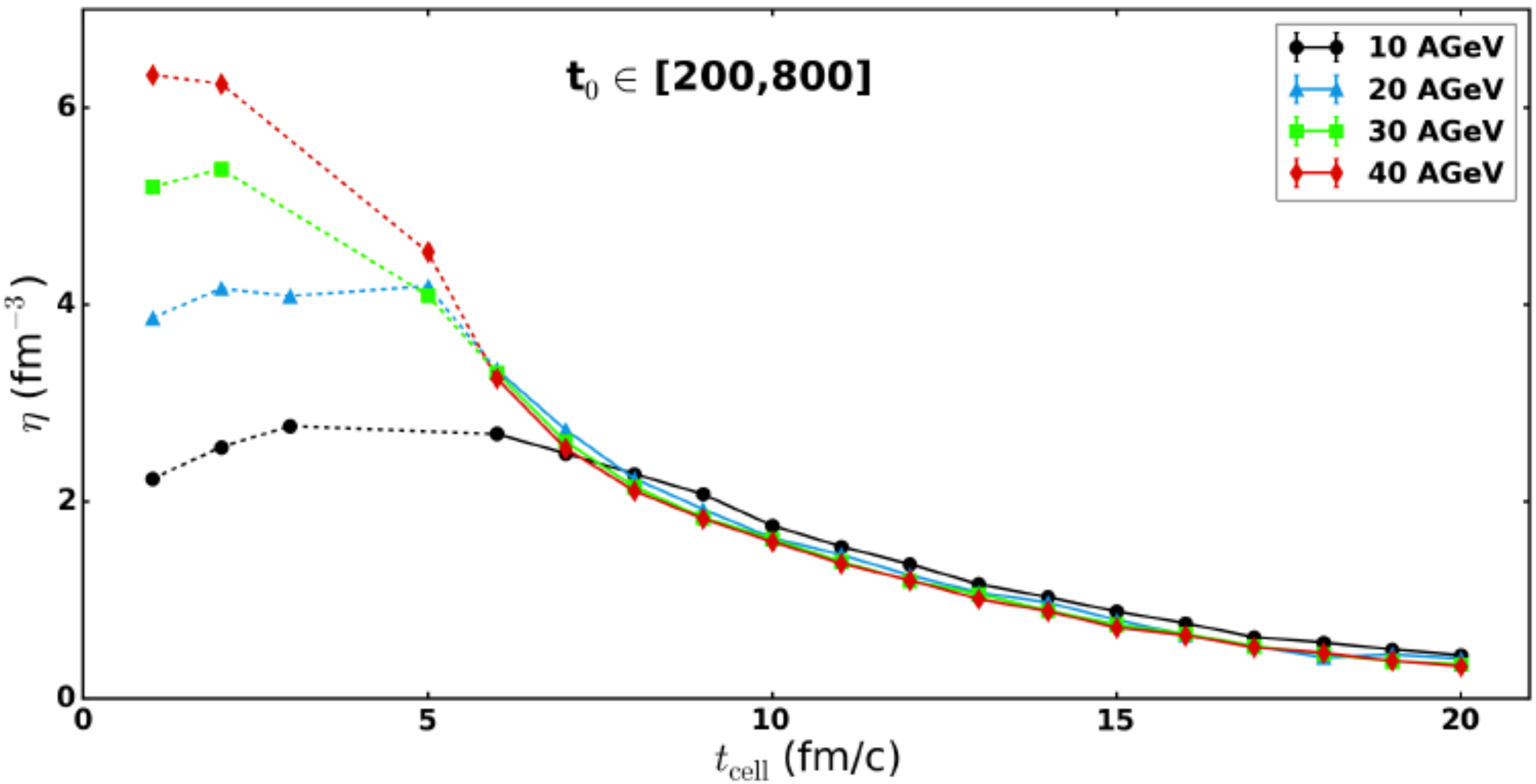
Ratio of τ_{int}/τ_{fit}

Results: viscosity $\eta\,(t_0)$



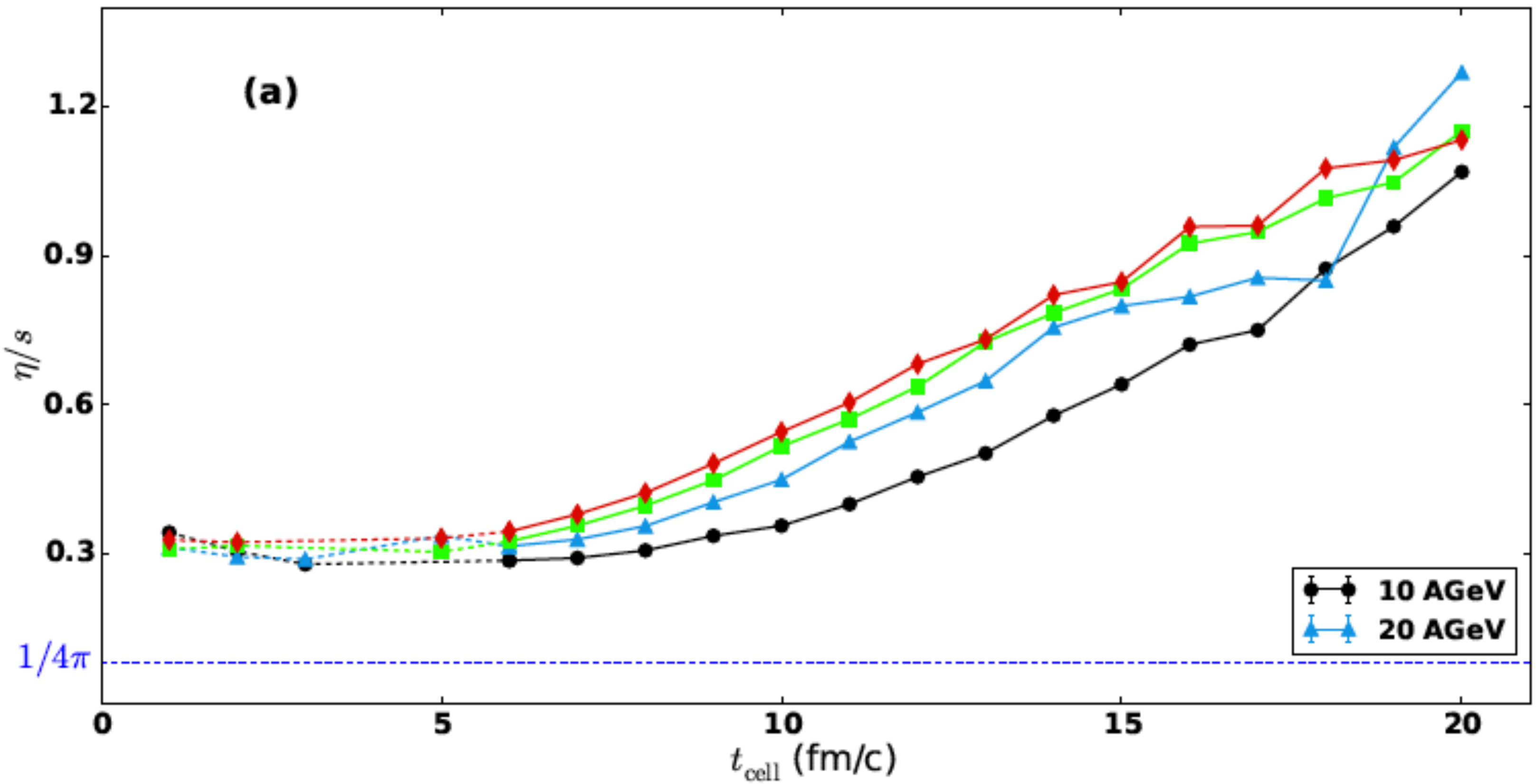
Dependence of η on t_0

Results: viscosity $\eta\,(t_{\rm cell})$



Dynamics of η in cell
All curves sit on the top of each other for $t_{\rm cell} \geq 7$ fm/c

Results: η/s

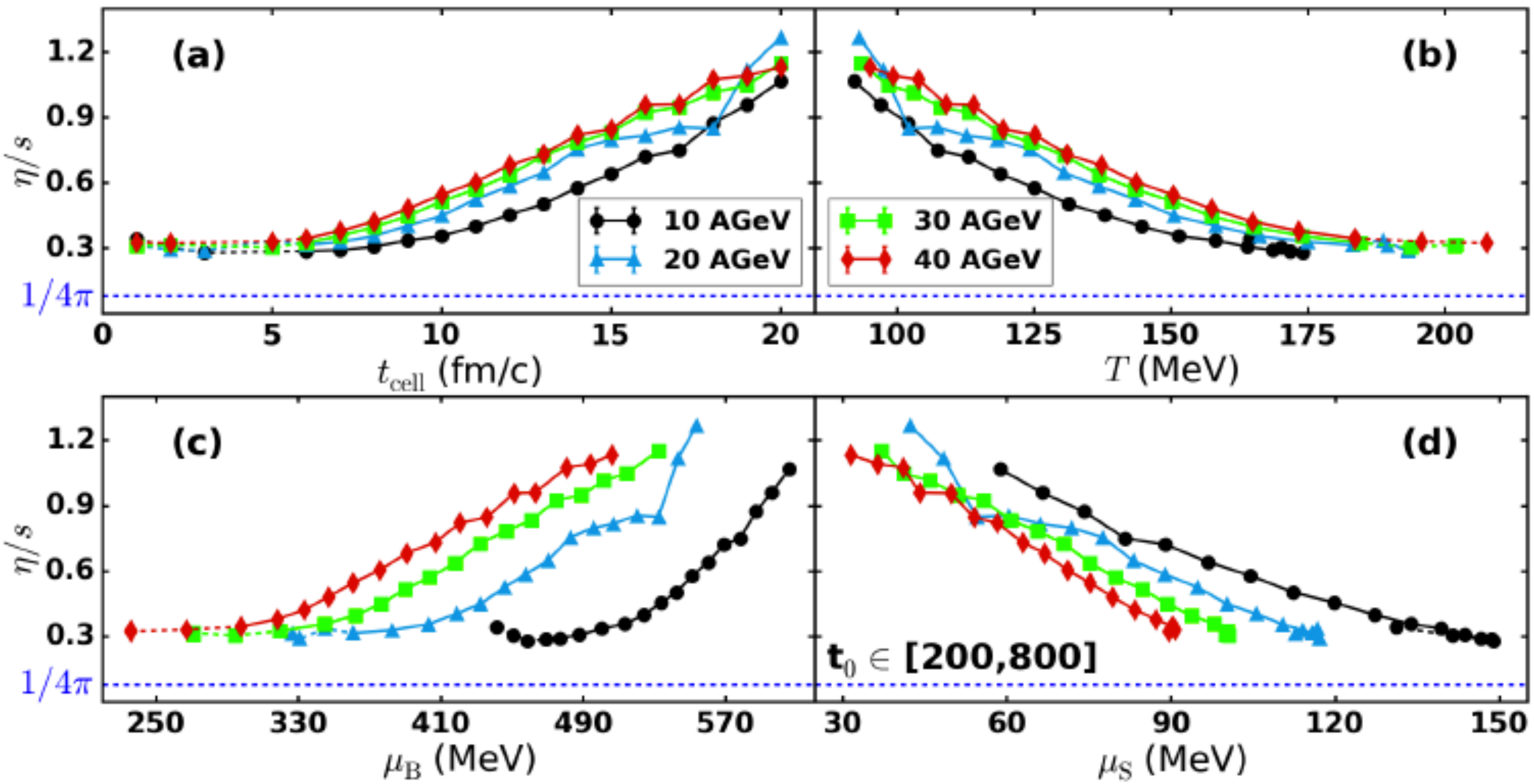


Dynamics of η/s in cell

η/s increases with time for $t_{\rm cell} \geq 6$ fm/c for all four energies

Minimum - for 10 AGeV, corresponding to 4.5 GeV in c.m. frame

Results: η/s



Dynamics of η/s in cell
as function of time, T, μ_B, μ_S

Conclusions

- data from central cell of UrQMD calculations are used as input for SM to calculate temperature T and Boltzmann entropy density s, and for UrQMD box calculations in order to estimate shear viscosity η
- box output data are taken within the range $200 \leq t_0 \leq 800$ fm/c because:
  - values at $t_0 < 200$ fm/c are distorted by the initial fluctuation in the box
  - values at $t_0 > 800$ fm/c may be disturbed by the analog of Brownian motion
- it is shown that for all four tested energies η and s in the cell drop with time
- ratios η/s reach minima about 0.3 at $t \approx 5$ fm/c for all energies. Then, the ratios rise to $1.0 \div 1.2$ at $t = 20$ fm/c
- this increase is accompanied by the simultaneous rise of μ_B and drop of both T and μ_S in the cell