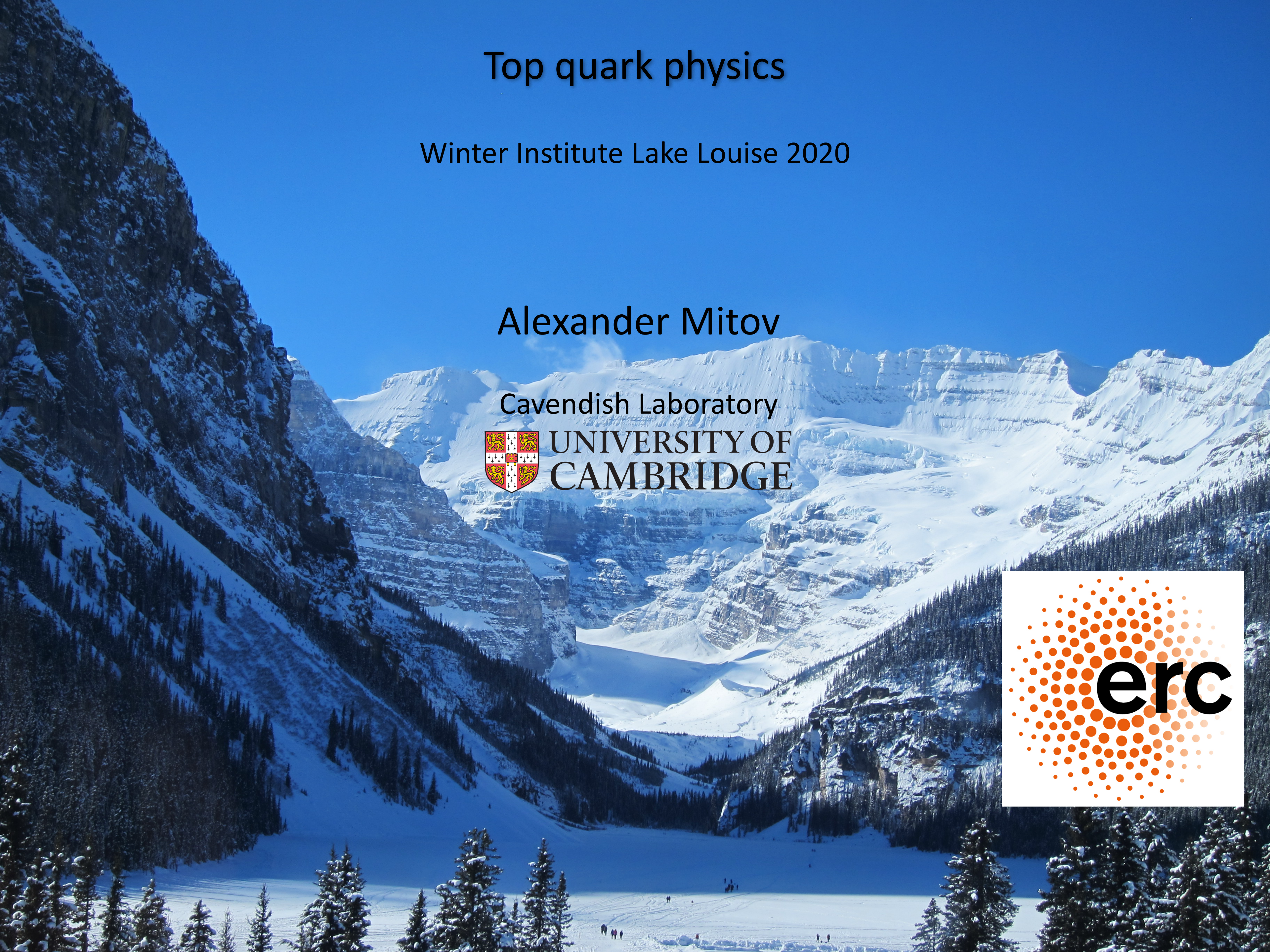
Top quark physics

Winter Institute Lake Louise 2020

Alexander Mitov

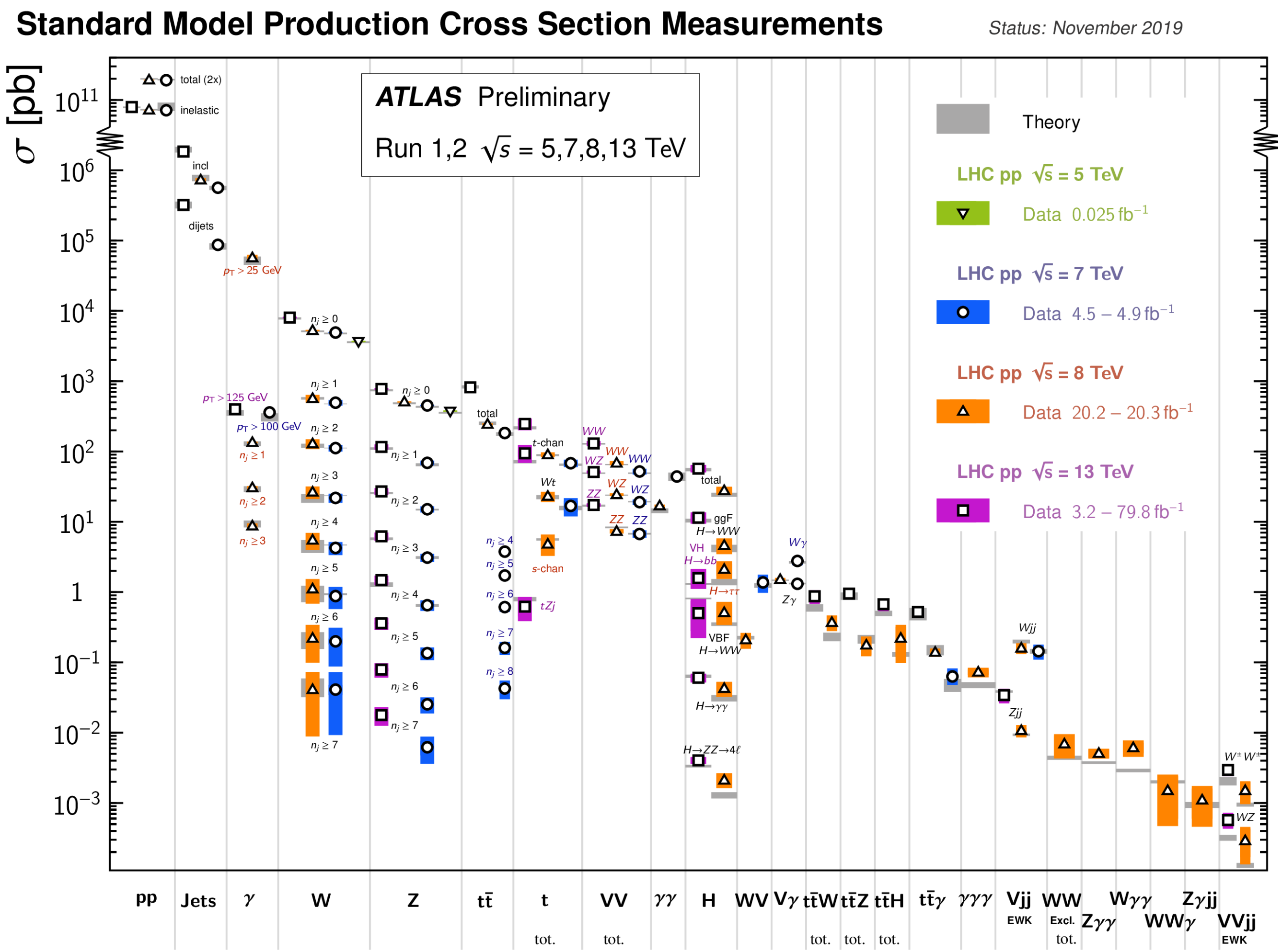
Cavendish Laboratory
UNIVERSITY OF CAMBRIDGE



Introduction: the spirit of these lectures

- Audience with diverse interests (QCD/SM, bSM): discussion should be useful for all
- Will give an overview of important results and state of the art developments;
- Not too many details, but sufficient depth to start thinking about/discussing the subject

- Context: precision in LHC physics is becoming more and more relevant



Introduction: why bother with top physics,
or
why the people who do not work on top physics should nevertheless follow these lectures

✓ The SM prospective

- Top production is the most complex SM process: tame tt, tame the SM (needed for bSM)
  - massive: addition of a mass in a problem adds a dimension to its complexity
  - colored
  - large QCD corrections
  - important EW interactions (strongly interacts with all SM particles)
  - results in very complex final states
- Top <u>can</u> be studied perturbatively: very high accuracy expected (both TH and EXP)
- The only *bare* quark: gives direct access to the SM Lagrangian (with caveats, of course)

✓ The bSM prospective

- Top is a major background for many (most?) bSM processes: search for bSM
- Many prominent past/current discrepancies: spin-correlations, Tevatron top A_{FB} , …
- BSM decays to tops; top loop effects
- Very large coupling to Higgs: if anything in the SM matters for Higgs, this is the top
- In summary, top matters in 2 ways:
  - through its *parametric* values (e.g. M_{top} and EW vacuum stability)
  - directly (through its production rates)

Top quark: the basics

Top quark: the basics

- Is the top special (as we hear all the time?): it depends!
  - From the viewpoint of QCD: NO
  - From the viewpoint EW : YES

- Top gets most of its corrections – and production rates – from QCD effects. But it gets its properties from EW interactions ==> both are very important.

- Top's main attribute: its very large mass: $M_{top} \approx 173$ GeV . Compare:

  * $M_H \approx 125$ GeV
  * $M_W \approx 80$ GeV
  * $M_b \approx 5$ GeV
  * $M_c \approx 1.5$ GeV

Understanding the origin of its mass is a major open problem

- CKM elements relevant for top: $V_{tb} \approx 1$.
  - Top coupling to non-b down-type quarks must be very small (CKM suppression)
  - Top couplings to other up-type quarks is non-zero at loop-level but tiny.

Any significant top coupling to non-b quarks might be a sign of bSM physics

Top quark: the basics

Top's very large mass* dictates its properties (both intrinsic and production ones)

- $M_{top} >> M_W$
  Implication: top readily decays; not true for the other quarks.
- $\Gamma_{top} \approx 1.5$ GeV $>> \Lambda_{QCD} \approx 0.3$ GeV
  Implication: top's lifetime ($\sim 1/\Gamma_{top}$) is much smaller than the typical hadronization time ($\sim 1/\Lambda_{QCD}$).
  Profound consequence: top decays before forming strongly interacting bound states (i.e. mesons).

Top is the only quark that decays as a *bare* particle.

✓ This is of major importance. For the other quarks we have to make conclusions based on modeling of non-perturbative physics. This can be done but can be extremely tricky. In certain cases even beyond our ability to model QCD (not even speaking of solving it).

✓ The fact that top decays (largely*) free of non-perturbative effects gives us added confidence that we know what we are doing regarding SM physics (it really matters in the grand scheme of things…).

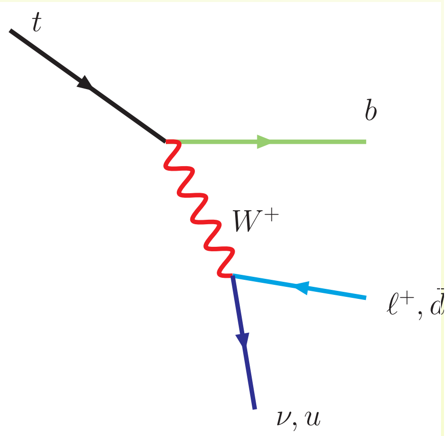
* To be elaborated upon later.

Top quark: the basics

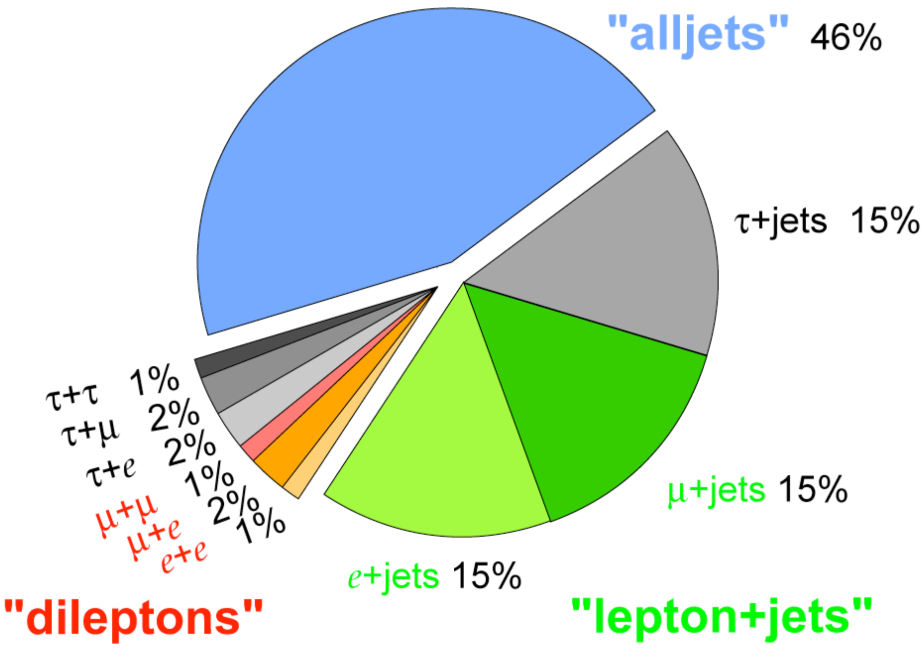
- We refer to the top mode based on the measured final state. Here are the SM options:

$$t \rightarrow W^{+} + b \\ \quad \hookrightarrow l^{+} + \nu$$

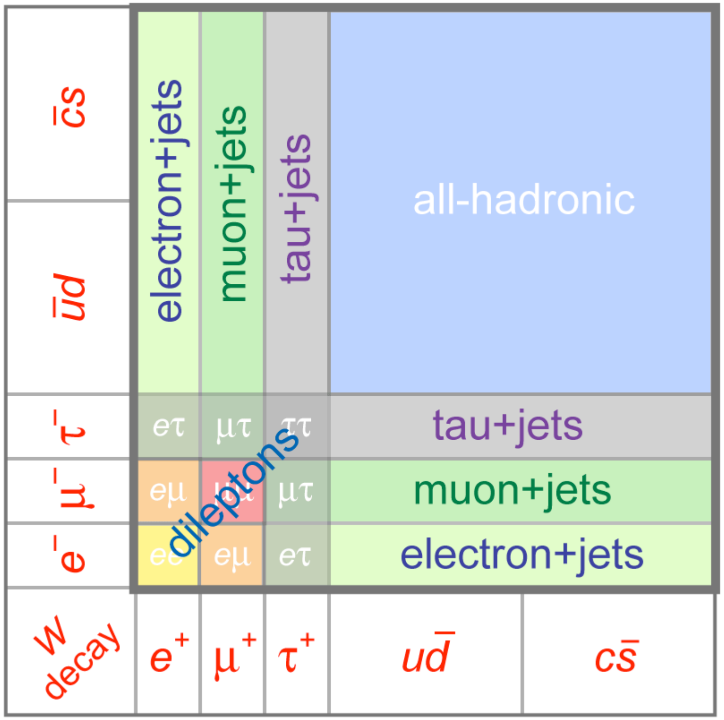
$$t \rightarrow W^{+} + b \\ \quad \hookrightarrow q + \bar{q}$$



Top Pair Branching Fractions



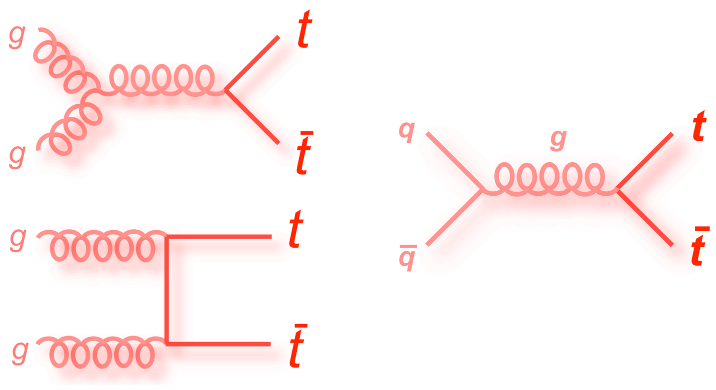
Top Pair Decay Channels

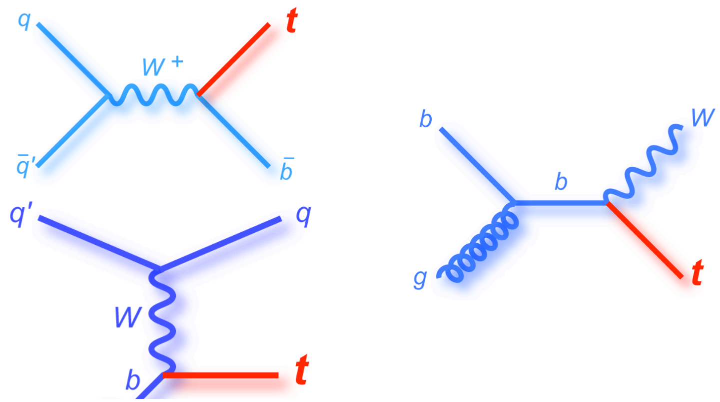


W decay	e^+	μ^+	τ^+	$u\bar{d}$	$c\bar{s}$
$\bar{c}s$	electron+jets	muon+jets	tau+jets	all-hadronic	
$\bar{u}d$					
τ^-	eτ	μτ	ττ	tau+jets	
μ^-	eμ	μμ	μτ	muon+jets	
e^-	ee	eμ	eτ	electron+jets	

Top quark: the basics

- At hadron colliders top quarks are produced in pairs (dominant) or singly.





- Top quark production rates, for various initial states and colliders:

Top pairs only

	TeVatron	LHC 7 TeV	LHC 8 TeV	LHC 14 TeV
gg	15.4%	84.8%	86.2%	90.2%
$qg + \bar{q}g$	-1.7%	-1.6%	-1.1%	0.5%
qq	86.3%	16.8%	14.9%	9.3%

From W. Bernreuther '08

$t\bar{t}$ pairs	dominant reaction	$N_{t\bar{t}}$
Tevatron: $p\bar{p}$ (1.96 TeV)	$q\bar{q} \to t\bar{t}$	$\sim 7 \cdot 10^4 \times L$
LHC: pp (14 TeV)	$gg \to t\bar{t}$	$\sim 9 \cdot 10^5 \times L$
ILC: e^+e^- (400 GeV)	$e^+e^- \to t\bar{t}$	$\sim 800 \times L$
single top	dominant reaction	$(N_t + N_{\bar{t}})$
Tevatron:	$u + b \xrightarrow{W} d + t$	$\sim 3 \cdot 10^3 \times L$
LHC:	$u + b \xrightarrow{W} d + t$	$\sim 3.3 \cdot 10^5 \times L$

Question: any guesses why the rate for the qg reaction (starts at NLO) is negative? Is this OK?

Top quark: the basics

Top quark quantum numbers

- Electric charge = +2/3|e|.
- Because tops are mostly pair produced, it was only recently shown that the exotic charge -4/3 (i.e. decay to bW^-) is unlikely.

- CKM: from weak decays it follows that:

$$B(t \to bW^+) = 0.998, \qquad B(t \to sW^+) \simeq 1.9 \times 10^{-3}, \qquad B(t \to dW^+) \simeq 10^{-4}.$$

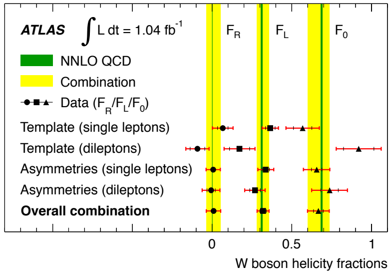
- Limits from measurements of top decays are much weaker.

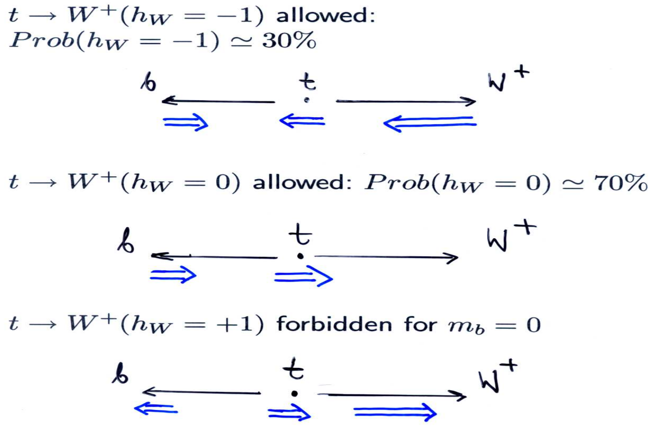
- Top spin: strongly correlated with the helicity of the W

SM predictions for the W helicity fractions:

$$F_0 = 0.99 \times F_0^B\,, \qquad F_- = 1.02 \times F_-^B\,, \qquad F_+ = 0.001$$

$$F_0^B \approx 0.7 \qquad F_-^B \approx 0.3$$




$t \to W^+(h_W = -1)$ allowed: $Prob(h_W = -1) \simeq 30\%$

$t \to W^+(h_W = 0)$ allowed: $Prob(h_W = 0) \simeq 70\%$

$t \to W^+(h_W = +1)$ forbidden for $m_b = 0$

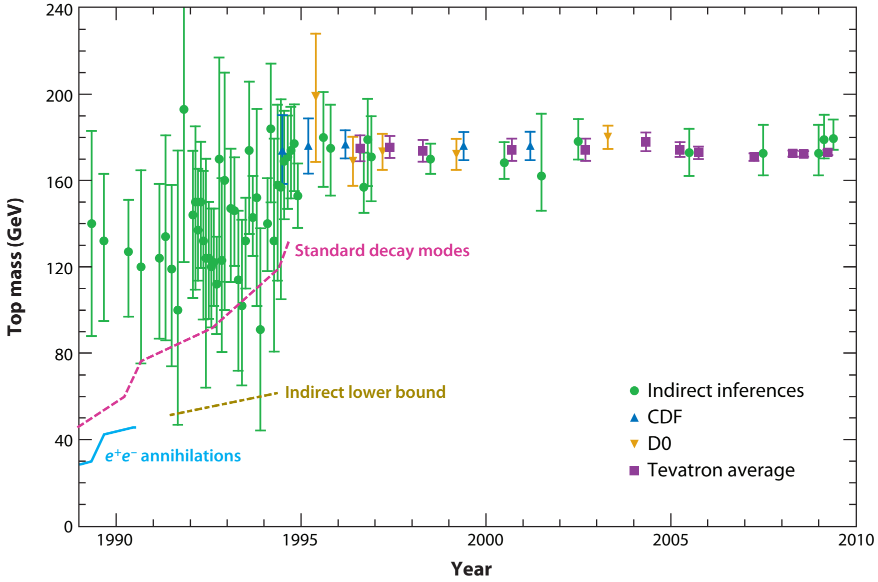
Very sensitive to V-A structure of the tbW vertex

Courtesy of W. Bernreuther

Top quark and EW precision fits

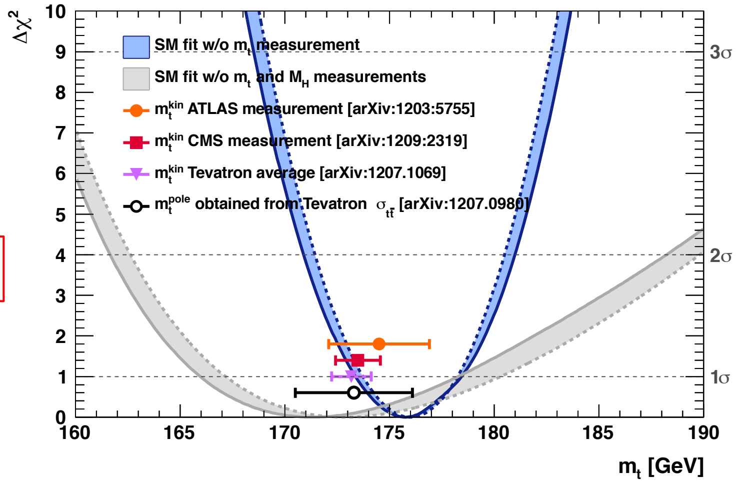
The run-up to the discovery of the top quark is an important lesson in today searches.

- We had an idea about M_{top} before top quarks were first seen:

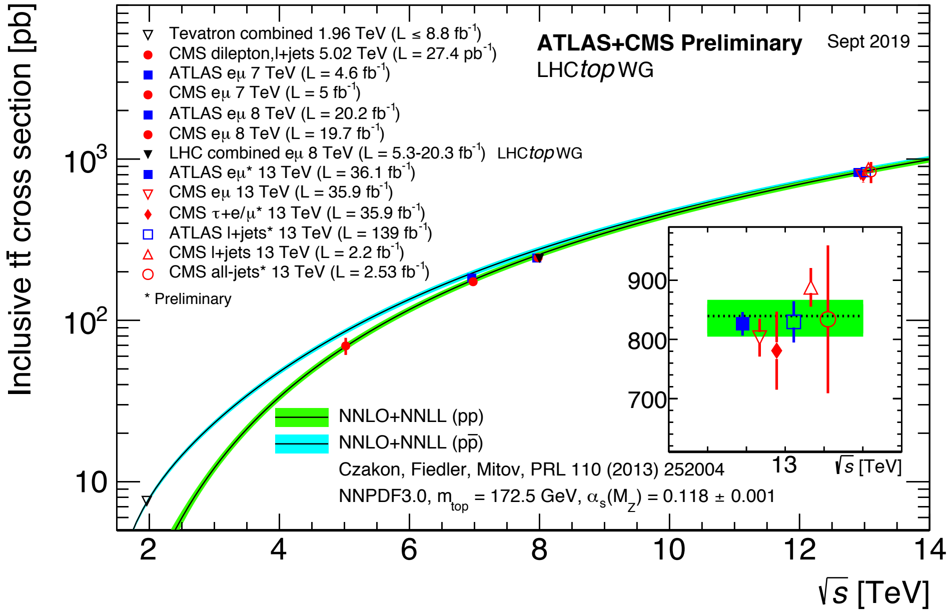


- Using the known Higgs and W masses one can again indirectly "rediscover" the top. The returned mass is $m_t = 175.8^{+2.7}_{-2.4}\ \mathrm{GeV}$ in impressive agreement with direct determinations.

arXiv:1209.2716



Top pair production at hadron colliders



Impressive agreement between EXP and SM!

- The total inclusive cross-sections is known in NNLO + NNLL QCD

Czakon, Mitov et al 2012-13
Catani et al 2018

- EW corrections are also know at NLO but are negligible (below 1%)

- This results in theoretical prediction with O(5%) accuracy:

  - scales (i.e. missing yet-higher order corrections) ~ 3%
  - pdf (at 68% cl) ~ 2-3%
  - alpha$_S$ (parametric) ~ 1.5%
  - m$_{top}$ (parametric) ~ 3%

- Soft gluon resummation makes a difference: scale uncertainty 5% → 3%
- Clearly, most sources of TH error are comparable so further progress will be hard

- Stable top quark pair production is aiming at as high precision as possible

- Results are becoming “mature” and well established. Computed by two groups with different methods. Impressive agreement!

Czakon, Mitov, Poncelet at al 2013 –
Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Sargsyan 2019

- At present this means NNLO QCD + EW + resummation (soft and collinear in the high-energy limit)

Czakon et al;
Pagani, Tsinikos, Zaro
Ferrogglia, Pecjak, Scott, Wang, Yang

- Calculations are fully differential and can handle any safe observable. Up to two dimensional distributions computed

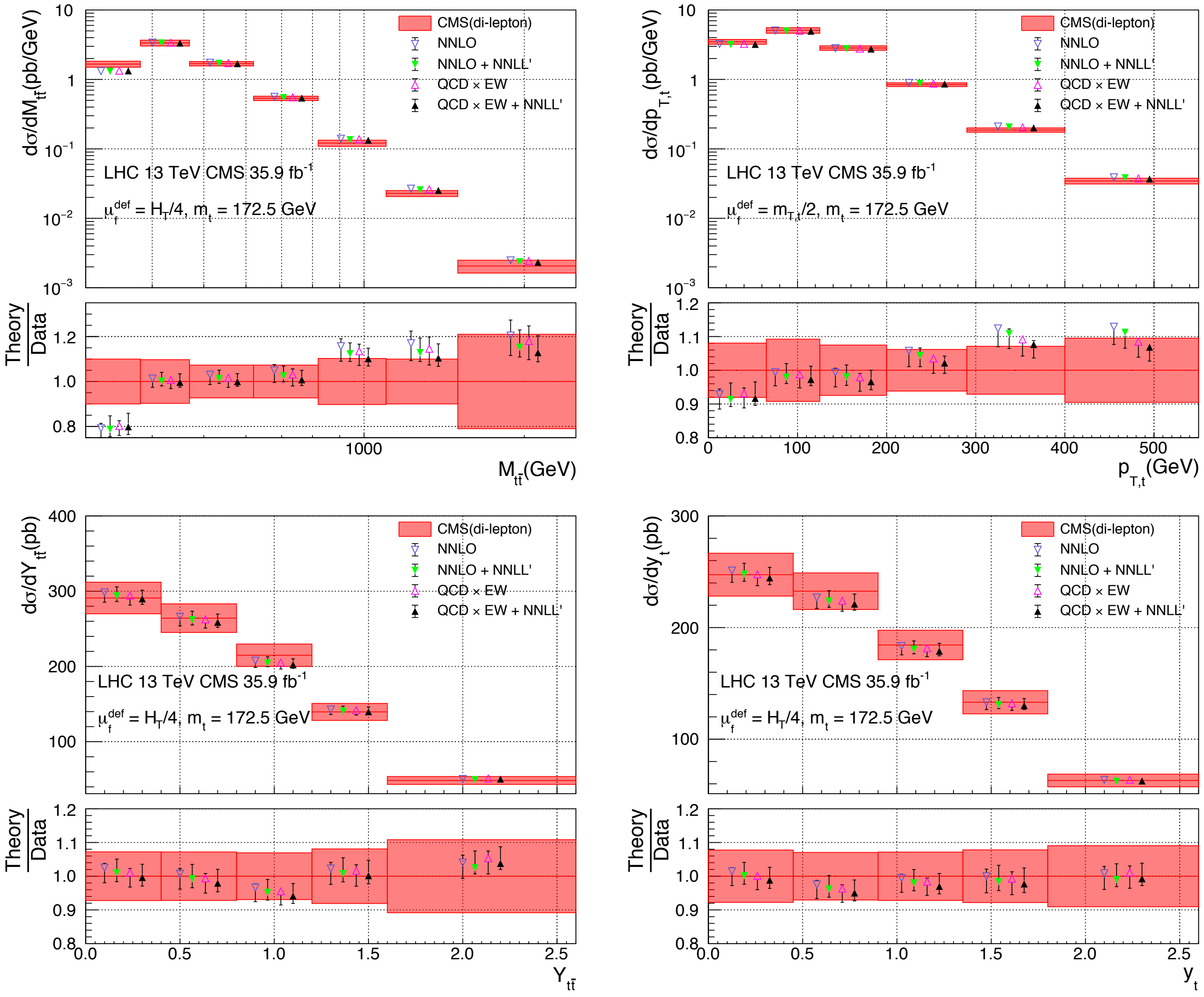
- Many interesting applications:

- PDFs (studied by all groups: conclusions vary from group to group)

- Top parametric impact on Higgs and BSM

- Direct searches with tops

- Results ready for used by SMEFT fits (theoretical predictions available as fastNLO tables; even more convenient and flexible formats in development)



Stable top-pair production in NNLO QCD + NLO EW + NNLL soft/collinear resummation

- Stable top production in fixer order perturbation theory vs. Monte Carlos
    - “*The top PT problem*”
- It was noticed long ago that LHC data at large PT is not described well by MC’s (even at NLO)
- It turns out, there are important higher order effects due to hard radiation.
- Once included, much better agreement
- Nowadays, top-pair MC predictions are often rescaled to repoduse the NNLO top PT

Threshold approximations and resummations in top pair production

- This has been an extremely fertile and useful field.
- Helps in our understanding of QCD at higher orders and non-perturbative phenomena.
- Limited kinematical applicability: certain phase-space regions need it, most do not.

What is threshold?

- Kinematical configuration where all the partonic energy is taken by the top pair and very little, if any, energy is left for radiation.

- Distinguish "absolute threshold" and "threshold":

- Absolute threshold is a particular case of a threshold, where almost all the partonic energy is used to produce the tops at rest.

- Replacing the (unknown) exact NNLO result with its soft approximation (prev. page) became known as $\text{NNLO}_{\text{approx}}$ approaches.

- Warning: the reliability of such approaches in approximating the full result is not guaranteed. Comparisons with exact results show that subleading terms could be numerically large

Threshold approximations and resummations in top pair production

- Another subtlety: in top production, there is another effect that lives close to threshold (i.e. same kinematics, different physics): bound state formation.

- Resummation of bound state effects

From Li Lin Yang et al. 1908.02179

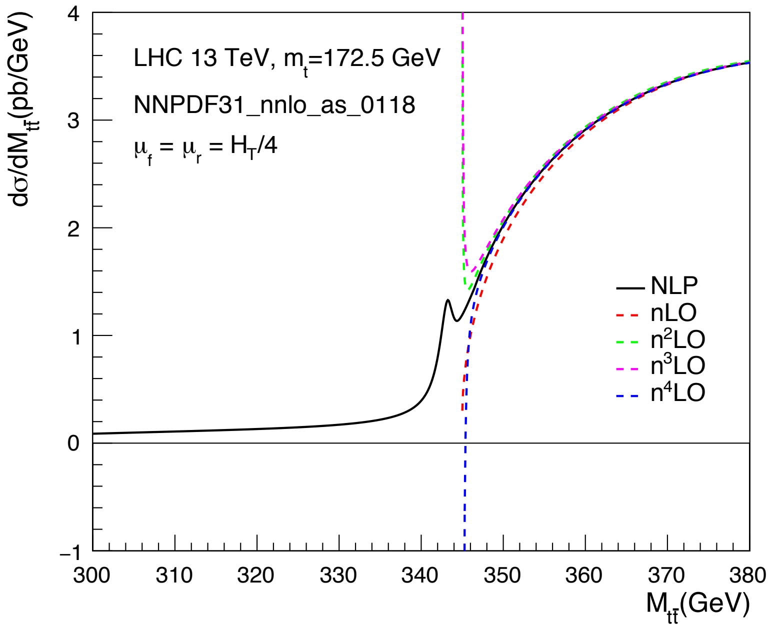


FIG. 3. The NLP resummed result and its fixed-order expansion.

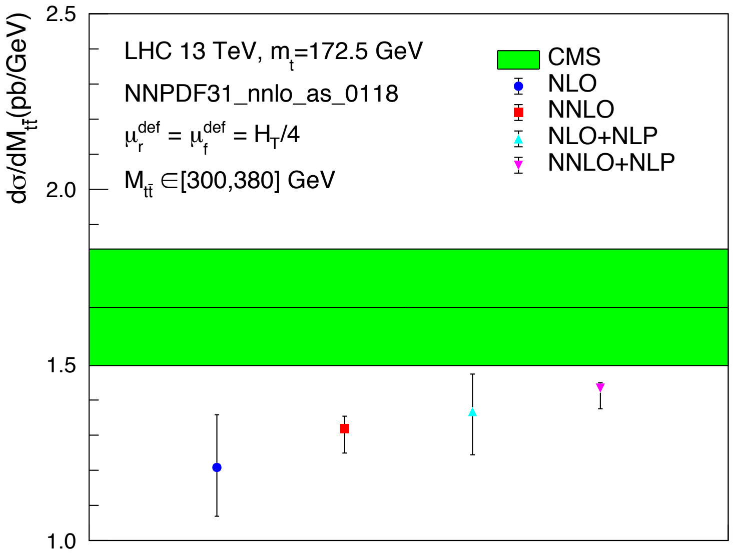


FIG. 4. The averaged $t\bar{t}$ invariant mass distribution in the [300-380] GeV range. The CMS result [2] is shown as the green band. The various theoretical predictions are shown in comparison, with NNLO+NLP being our best prediction.

- Above are updates or new state of the art predictions for the top-pair invariant mass close to threshold.

Effect of running scales:

From Arxiv:1207.5018

- Nowadays all NNLO calculations employ them.
- They are particularly relevant for observables at high P_T

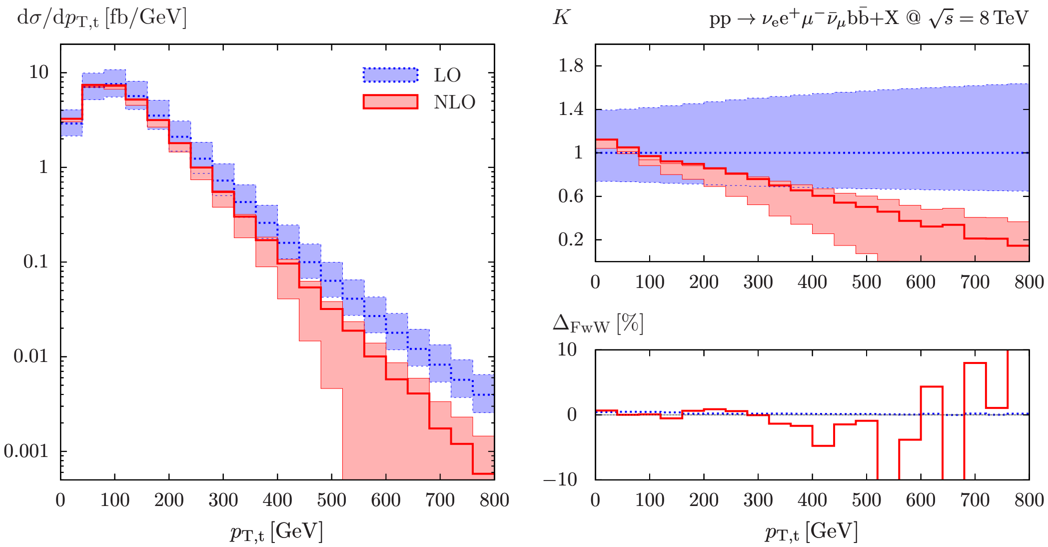


Figure 9: Transverse-momentum distribution of the top quark with standard cuts for the LHC at $\sqrt{s} = 8\,\text{TeV}$ for fixed scale $\mu_0 = m_t/2$.

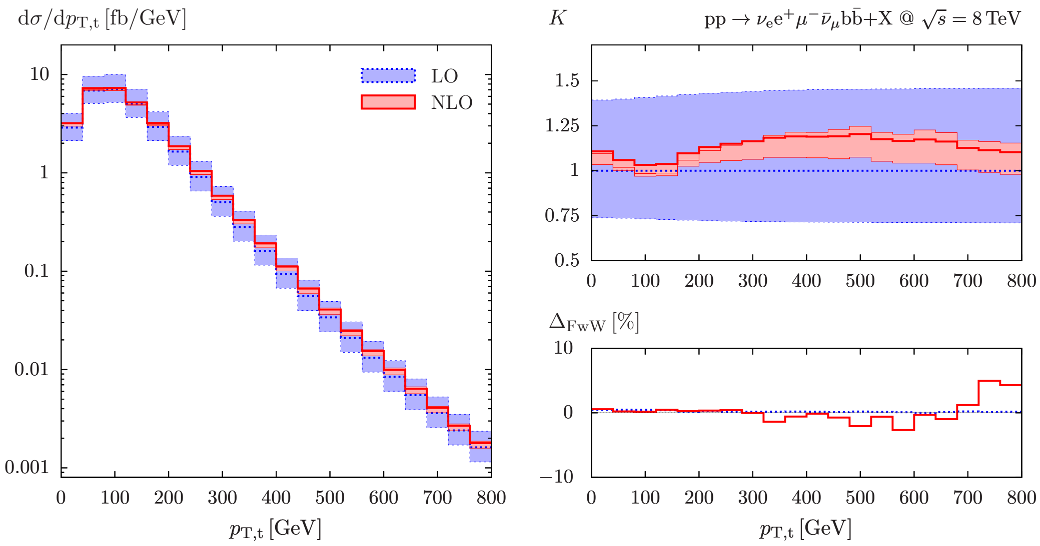
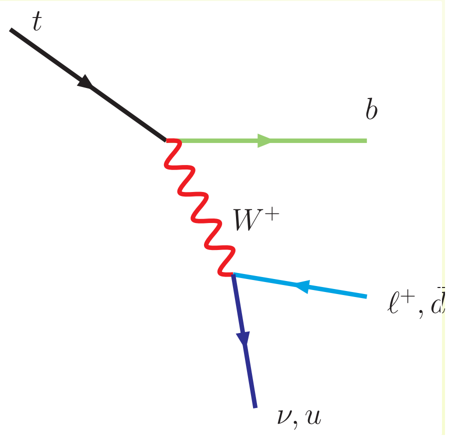


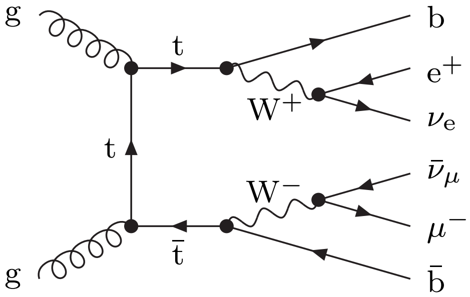
Figure 11: Transverse-momentum distribution of the top quark with standard cuts for the LHC at $\sqrt{s} = 8\,\text{TeV}$ for dynamical scale $\mu_0 = E_T/2$.

Top quark decay



$$t \to W^+ + b$$
$$\quad \hookrightarrow l^+ + \nu$$

$$t \to W^+ + b$$
$$\quad \hookrightarrow q + \bar{q}$$



- The top decays very fast, so it is unrealistic to treat it as a stable particle.
- But how to include the top decay?

- Use narrow width approximation

$$\lim_{\Gamma_t \to 0} \frac{1}{(p_t^2 - m_t^2)^2 + m_t^2 \Gamma_t^2} = \frac{\pi}{m_t \Gamma_t}\, \delta(p_t^2 - m_t^2) \qquad \int d\sigma_{\mathrm{NtWA}} = \sigma_{t\bar{t}}\, \mathrm{BR}_{t \to i}\, \mathrm{BR}_{\bar{t} \to j},$$

- Treat the top as a resonance with a complex mass

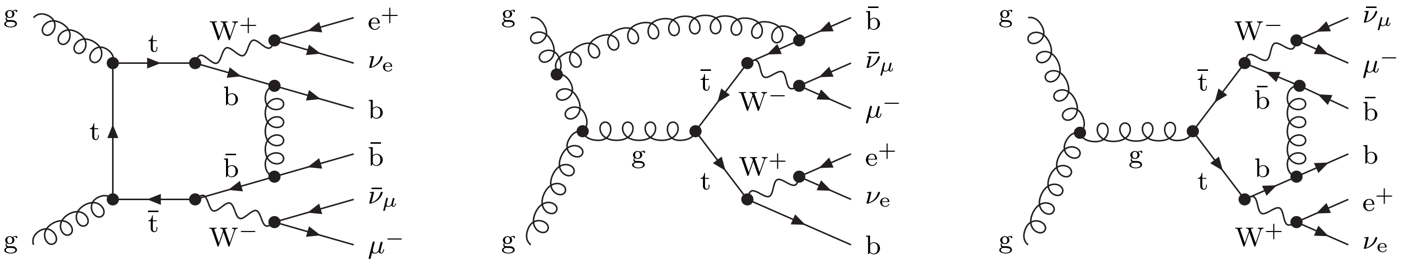
$$m_t^2 - i m_t \Gamma_t$$

- This way we completely separate top production from top decay; a tremendous simplification!

- Some factorizable corrections

Plots from 1207.5018

- … and some non-factorizable ones



Computing the full non-factorizable contributions is at the edge of current capabilities
The real question is if they matter?

- The Narrow Width approximation is correct up to correction of $\sim \Gamma_{top}/M_{top} \approx 1\%$.
- When is this the case?

- In general, we expect that inclusive observables are not very sensitive to NWA breaking effects
- Until few years ago no complete calculation existed and thus we didn't know for sure.
- Complete NLO calculations of tt production showed that indeed, this is the case
- In addition, large corrections are found in certain kinematic regions.

Bevilacqua, Czakon, van Hameren, Papadopoulos, Worek '10
Denner, Dittmaier, Kallweit, Pozzorini '11

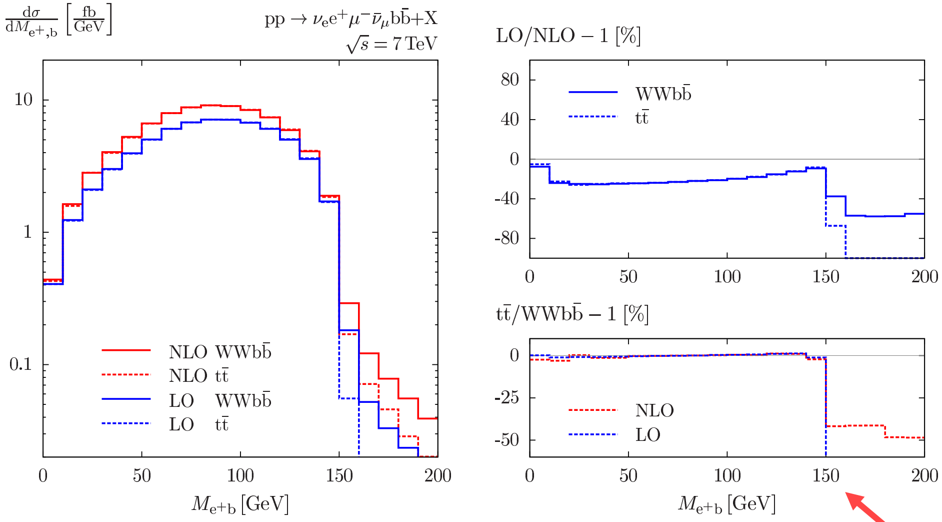


Fig. 27: Distribution in the invariant mass of the positron–b-jet system (as defined in the text) at the 7 TeV LHC: LO (blue) and NLO (red) predictions in narrow-width approximation ($t\bar{t}$, dashed) and including finite-top-width effects (WW$b\bar{b}$, solid). Plotted are absolute predictions (left) and relative deviations of LO (upper-right) and narrow-width (lower-right) approximations w.r.t. NLO and WW$b\bar{b}$ predictions, respectively.

From 1203.6803, page 62

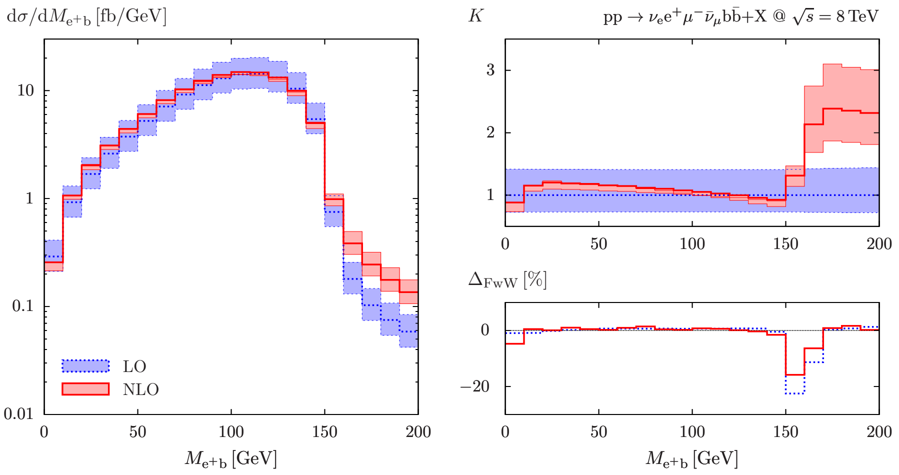


Figure 17: Invariant-mass distribution of positron–b-jet system with standard cuts for the LHC at $\sqrt{s} = 8\,TeV$ for dynamical scale $\mu_0 = E_T/2$.

Dramatic off-shell effects

- This tail corrections might be relevant, for example, in top mass measurements (more later)

History of calculations for top production with decay

- ✓ Top production and decay was first computed at NLO 10-15 years ago

Bernreuther, Brandenbourg, Si, Uwer 2004
Melnikov,Schulze 2008

- ✓ Later expanded to include off-shell/non-resonant effects

Denner, Dittmaier, Kallweit, Pozzorini 2010-
Bevilacqua, Czakon, van Hameren, Papadopoulos, Worek 2010
Frederix 2013
Cascioli, Kallweit, Maierhöfer, Pozzorini 2013

- ✓ Extension for NLO+PS:

Campbell, Ellis, Nason, Re 2014
Jezo, Lindert, Nason, Oleari, Pozzorini 2016

  - ✓ NLO is still the state of the art for off-shell calculations

- ✓ Progress to higher orders was made in the Narrow Width Approximation:
  - ✓ approx NNLO (prod) x NNLO (decay)

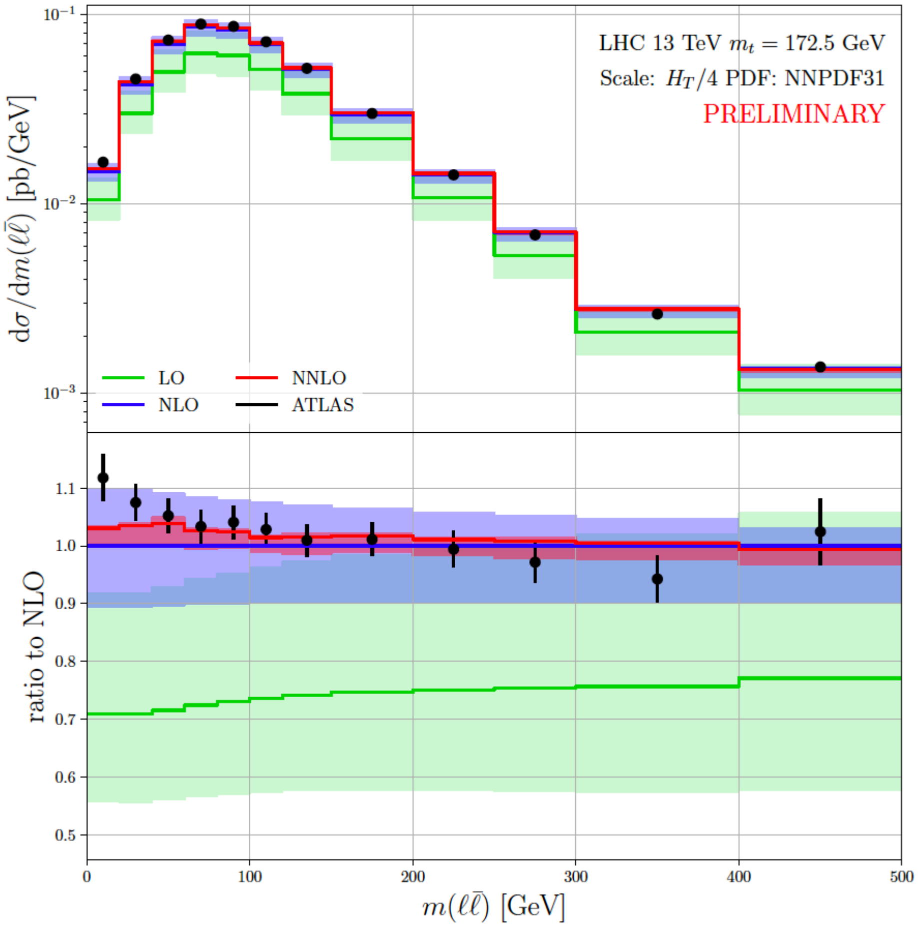
Gao, Papanastasiou 2017

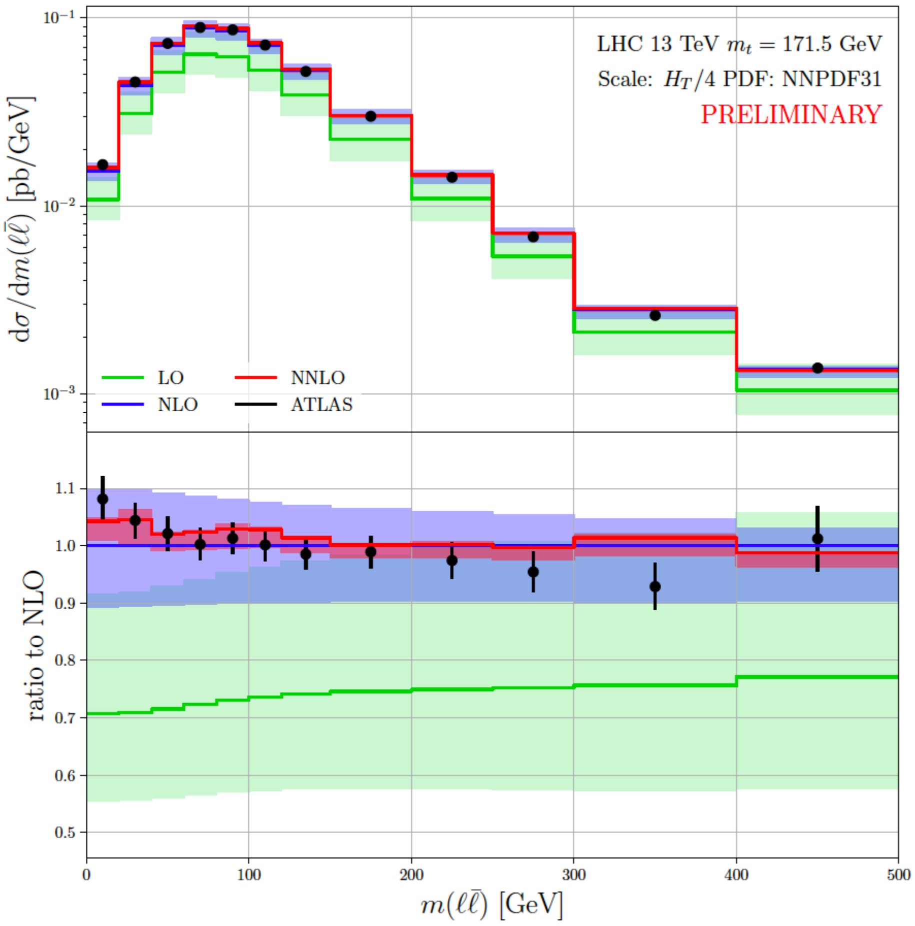
  - ✓ Full NNLO (prod) x NNLO (decay)

Behring, Czakon, Mitov, Papanastasiou, Poncelet 2019

NNLO QCD vs ATLAS data

- ✓ m(lepton pair)
  - ✓ Great reduction of scale error at NNLO (vs NLO). Tiny K-factor.
  - ✓ m_t=171.5GeV better than m_t=172.5GeV.
  - ✓ Improved MC error required to draw quantitative conclusion (especially for m_t determ.)

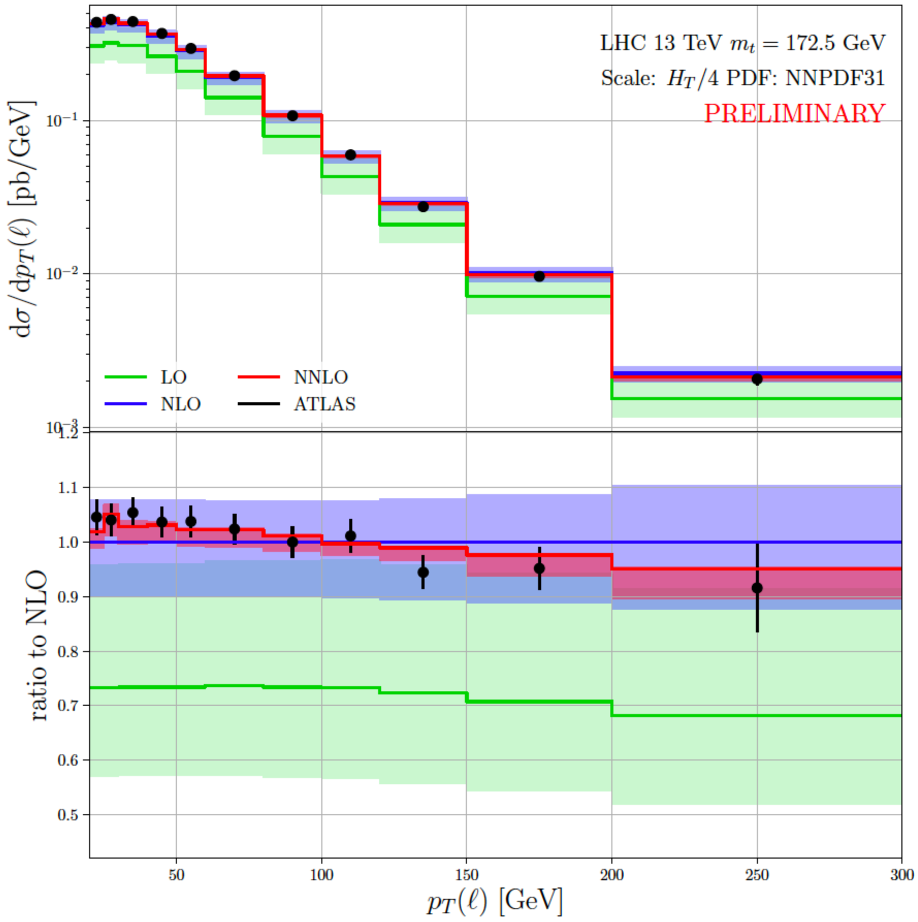


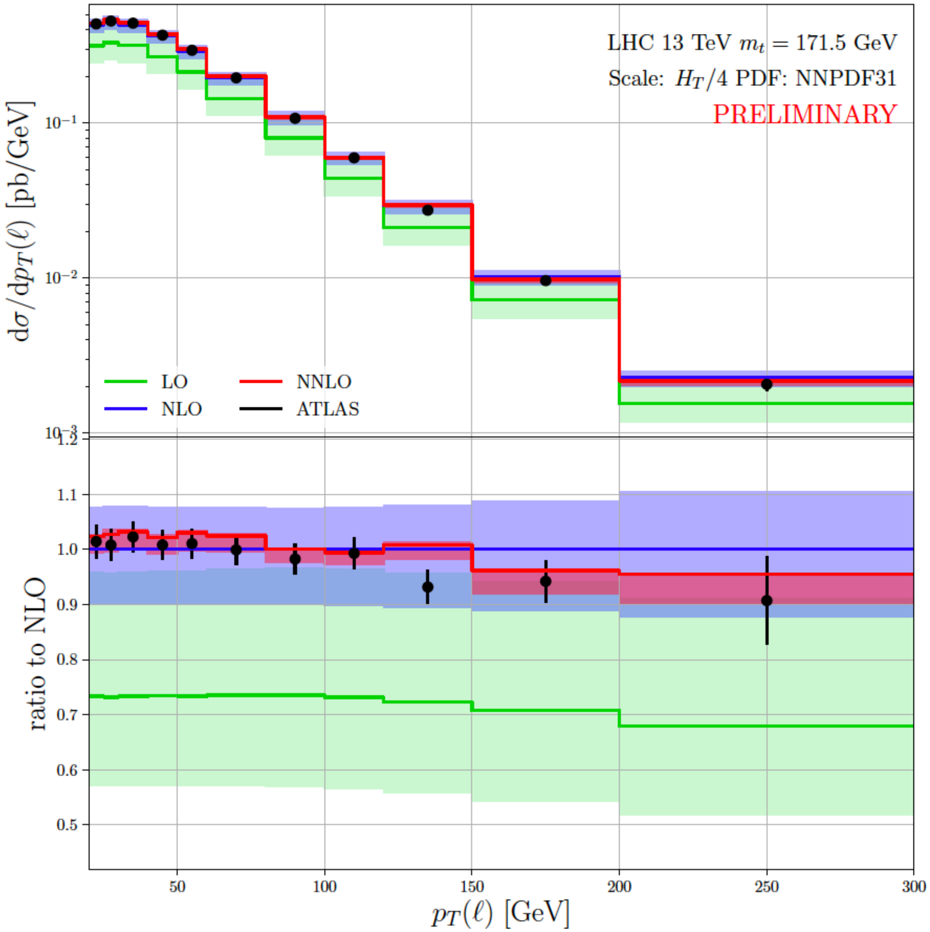


Preliminary: Czakon, Mitov, Poncelet

NNLO QCD vs ATLAS data

- ✓ P_T(lepton)
  - ✓ MC error of NNLO visible albeit small (work in progress)
  - ✓ Great reduction of scale error at NNLO (vs NLO)
  - ✓ m_t=171.5GeV seems better than m_t=172.5GeV





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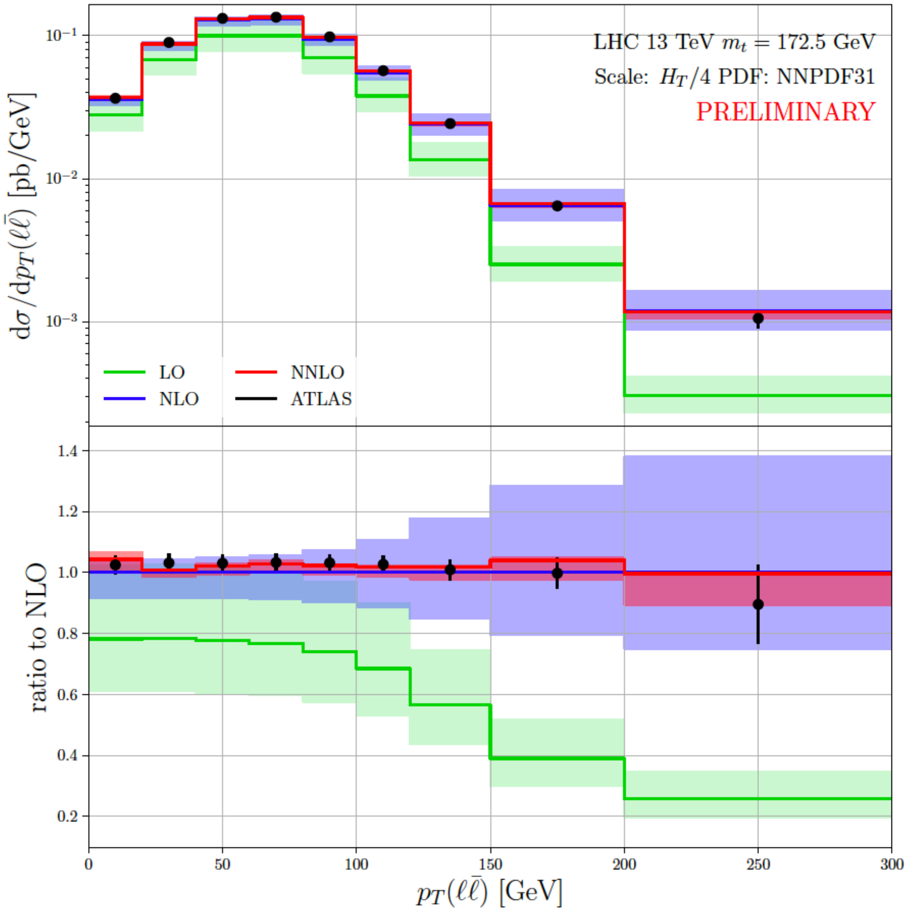
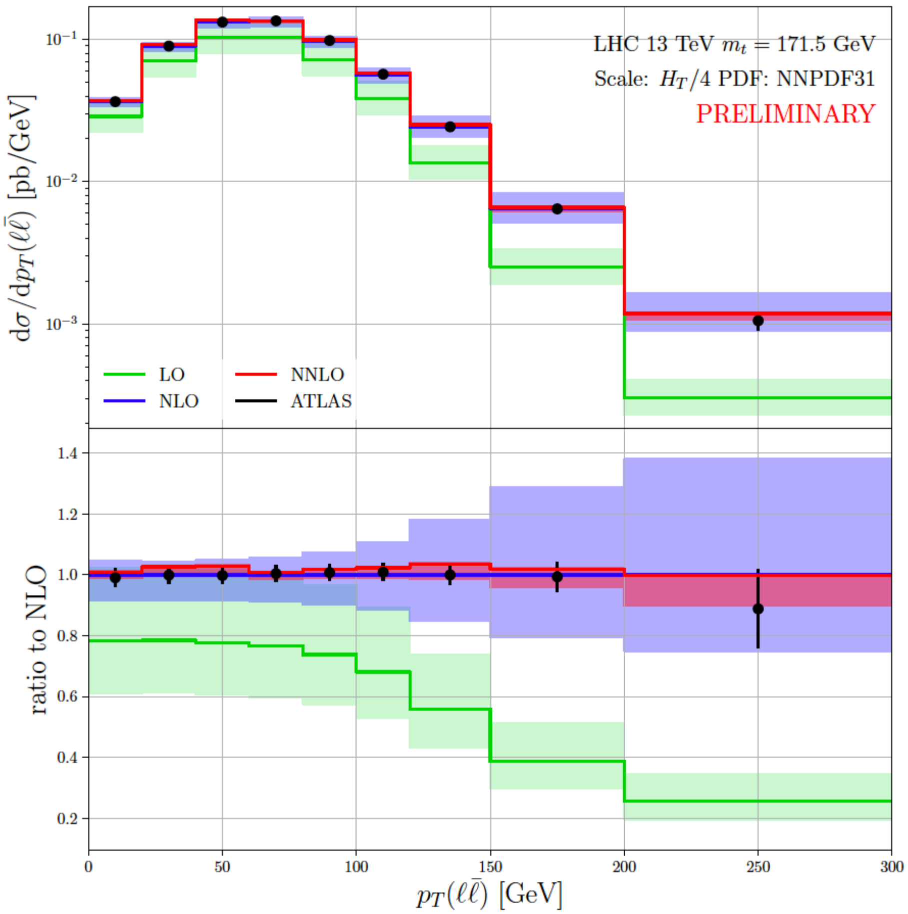
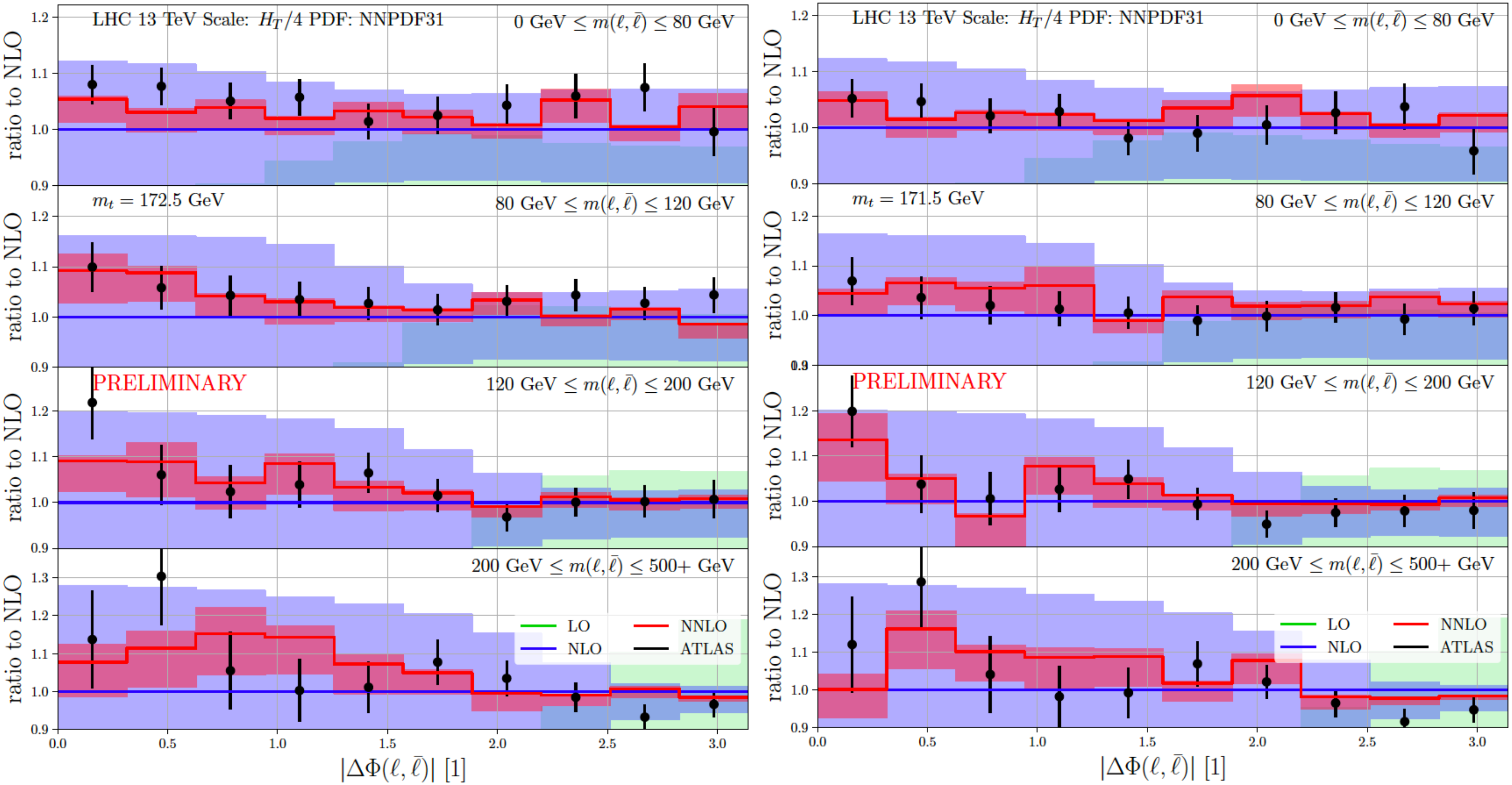




Preliminary: Czakon, Mitov, Poncelet

NNLO QCD vs ATLAS data: 2-dim

- ✓ **Δφ** vs. m(tt) (others are computed, too, not shown)
  - ✓ Great reduction of scale error at NNLO (vs NLO). Mostly small K-factors
  - ✓ Both m_t=171.5GeV and m_t=172.5GeV seem to work
  - ✓ Improved MC error required to draw quantitative conclusion (m_t sensitivity is apparent)



Preliminary: Czakon, Mitov, Poncelet

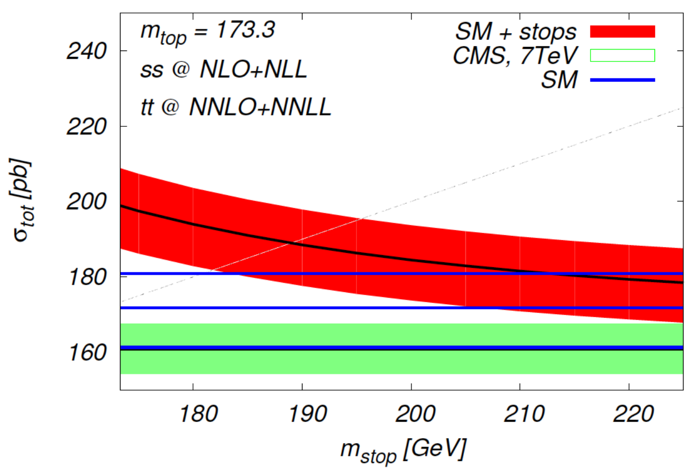
Top quark and BSM

How can a high-precision result be useful?
(i.e. what can be done with it, that could not be achieved with other commonly available tools)

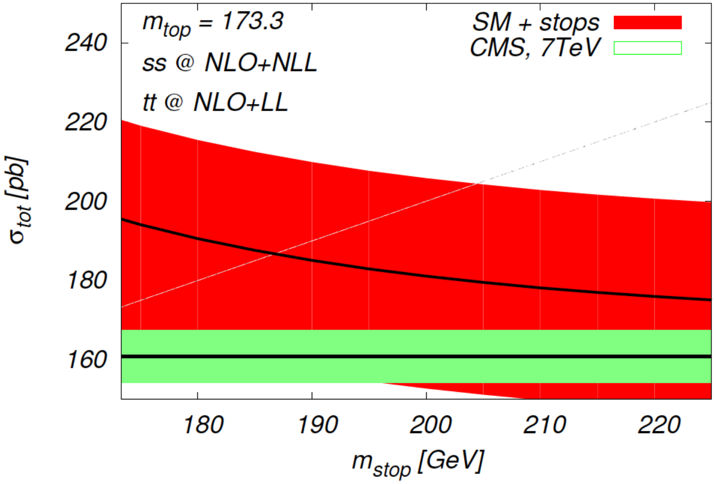
Closing the stop gap (i.e. excluding light “stealthy” top squarks)

See arXiv:1407.1043 for more

SM @ NNLO+NNLL does it…



… SM @ NLO+LL doesn’t do it.



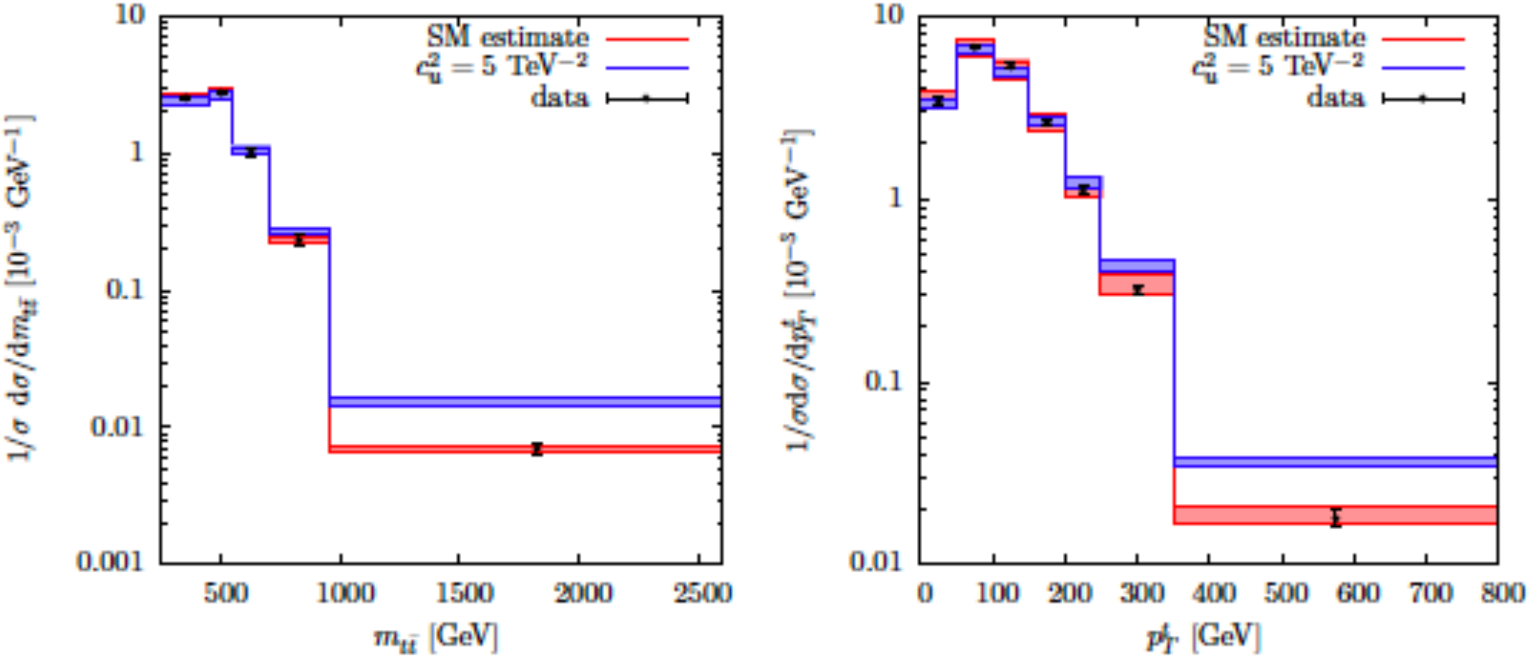
Light stop can be excluded based on rates:

- 5% uncertainty
- For $M_{stop} \sim M_{top}$ we have: $\sigma_{stop} \approx 0.15\, \sigma_{top}$
- Thus 3σ exclusion can be expected.

High-precision is a powerful tool!

- SMEFT fits in top physics can also be strong discriminant on BSM physics

From M. Russell arXiv:1709.10508

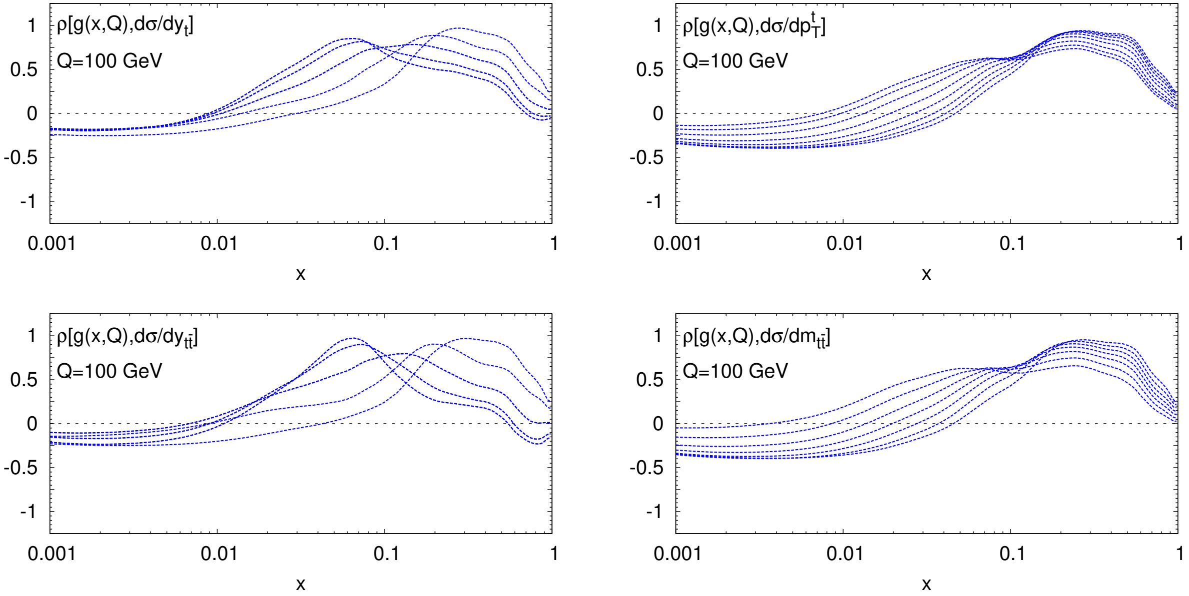


For up to date status see, for example:

Durieux, Gu, Vryonidou, Zhang arXiv:1809.03520
Durieux, Irles, Miralles, Peñuelas, Pöschl, Perelló, Vos arXiv:1907.10619

Top quark and PDFs

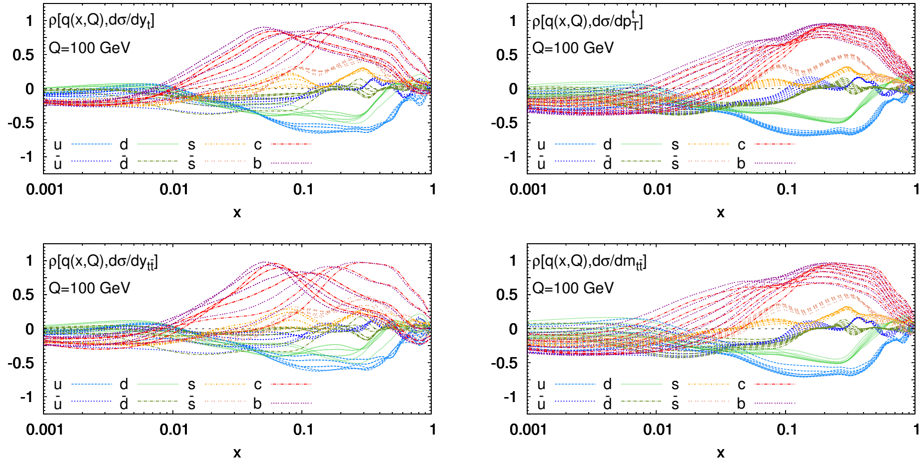
- Top quark data can be used for fitting PDFs
- It is easy to see why: there is strong correlation between top data and gluon pdf at large x



From arXiv:1611.08609

Figure 3: The correlation coefficient ρ between the gluon $g(x, Q^2)$, evaluated at $Q = 100$ GeV, and each of the bins of the y_t, p_T^t, $y_{t\bar{t}}$ and $m_{t\bar{t}}$ top-quark differential distributions at the LHC 8 TeV.

Large x is where heavy BSM production at the LHC hides



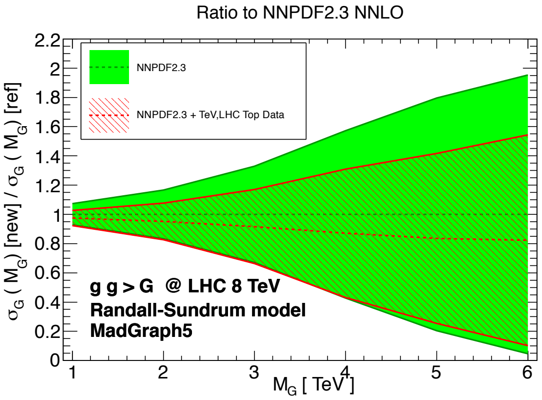


Figure 4: Same as Fig. 3, but for quarks and antiquarks, $q(x, Q^2)$, $q = u, \bar{u}, d, \bar{d}, s, \bar{s}, c, b$.

- The power of top data to fit PDF seems to depend a lot on the PDF fitting methodology

- NNPDF generally finds strong impact of top data on the gluon

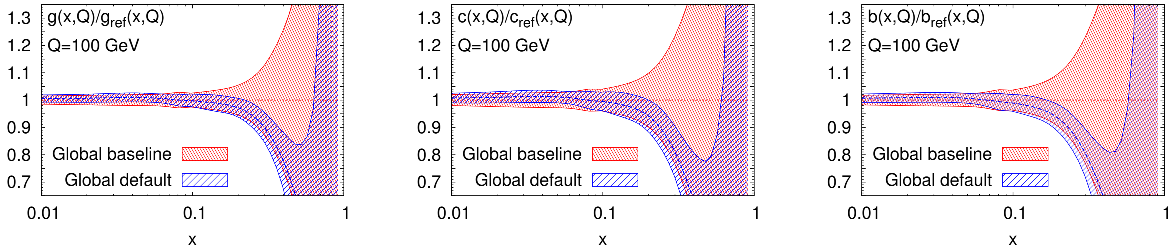
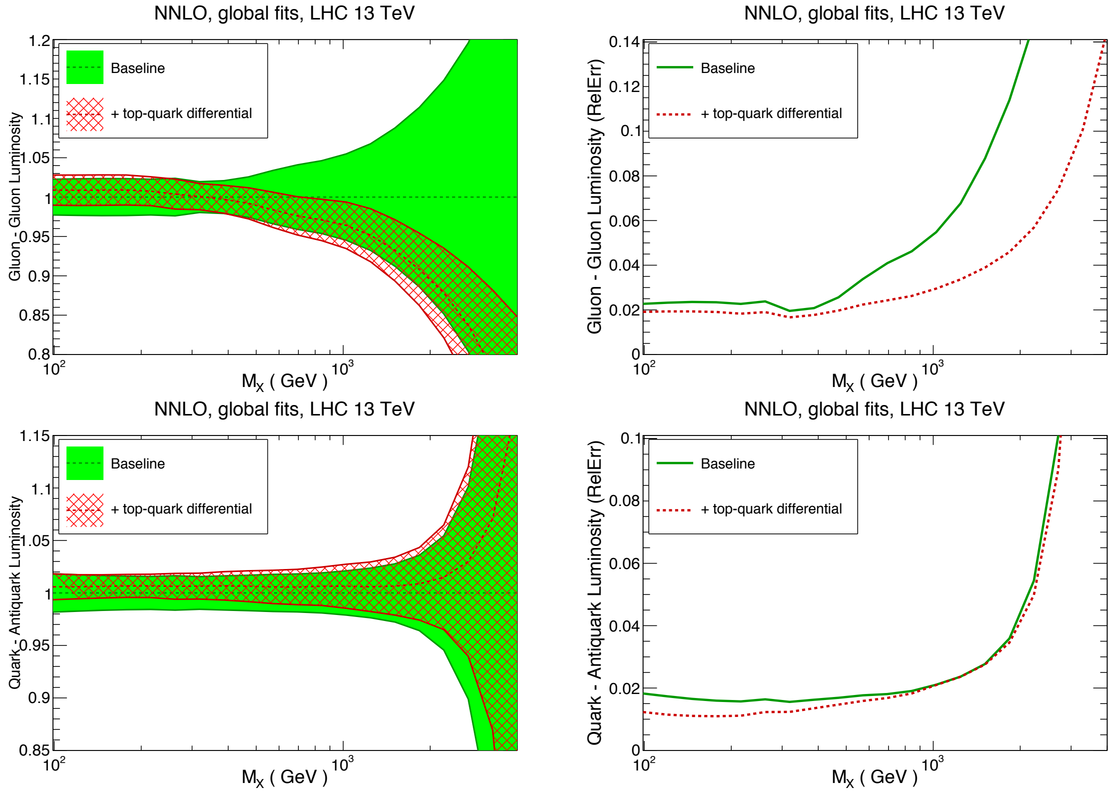


Figure 17: The gluon, charm and bottom PDFs from the global baseline fit compared to the optimal fit including our optimal combination of LHC top-quark data.

From arXiv:1611.08609


NNLO, global fits, LHC 13 TeV

- The power of top data to fit PDF seems to depend a lot on the PDF fitting methodology

- CTEQ finds marginal impact of top data on the gluon; arXiv:1912.08801

  - the reason is the amount of top data is much less than jet data

- MMHT conclusions are also not very clear. They quote the available data (the one at 8 TeV) as not being able to fit it well. In particular, the correlations provided with these measurements

  arXiv:1912.08801

  - This has also been pointed out in the NNPDF study above

  - Note that CTEQ fit different 8 TeV data using doubly differential observables

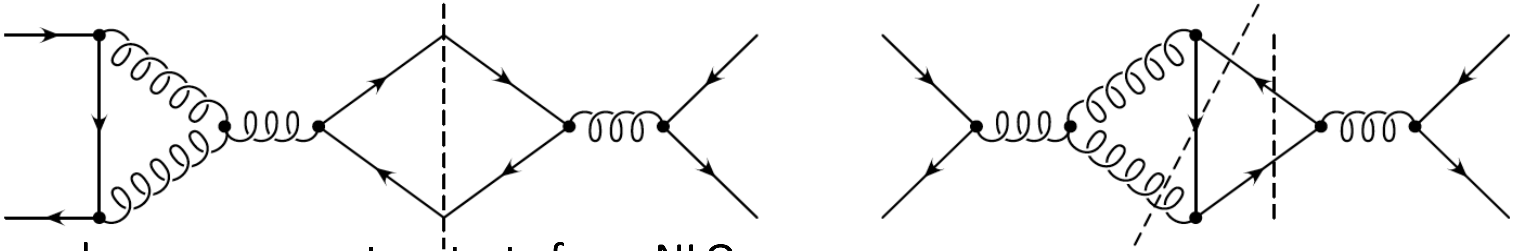
Some interesting observables showing discrepancy w/r to SM:

The charge asymmetry

QCD diagrams that generate asymmetry:

Kuhn, Rodrigo ‘98

… and some QCD diagrams that do not:



✓ For ttbar: charge asymmetry starts from NLO

✓ For ttbar + jet: starts already from LO

✓ Asymmetry appears when sufficiently large number of fermions (real or virtual) are present.

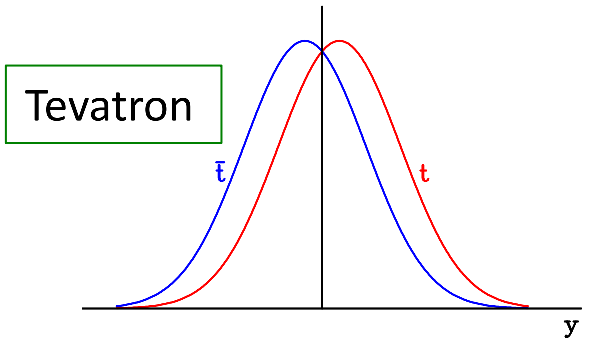
✓ The asymmetry is QED like.

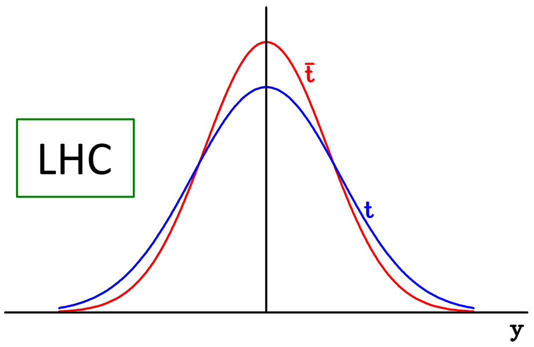
✓ It does not need massive fermions.

✓ It is the twin effect of the perturbative strange (or c- or b-) asymmetry in the proton!

Definition of the asymmetry:

$$A_{\mathrm{FB}} = \frac{N(\Delta y > 0) - N(\Delta y < 0)}{N(\Delta y > 0) + N(\Delta y < 0)}$$



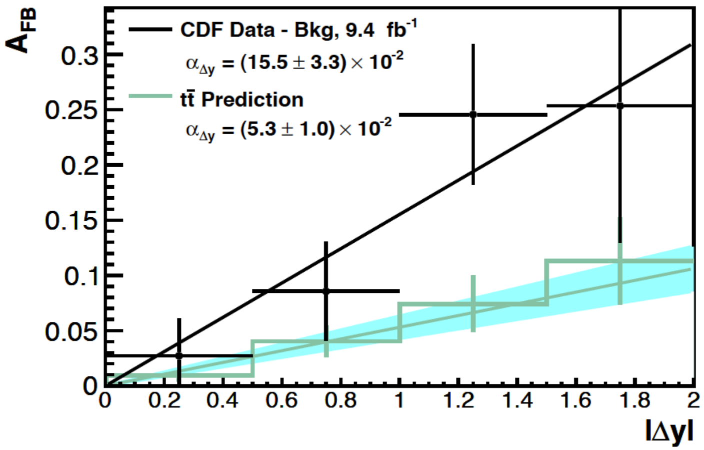


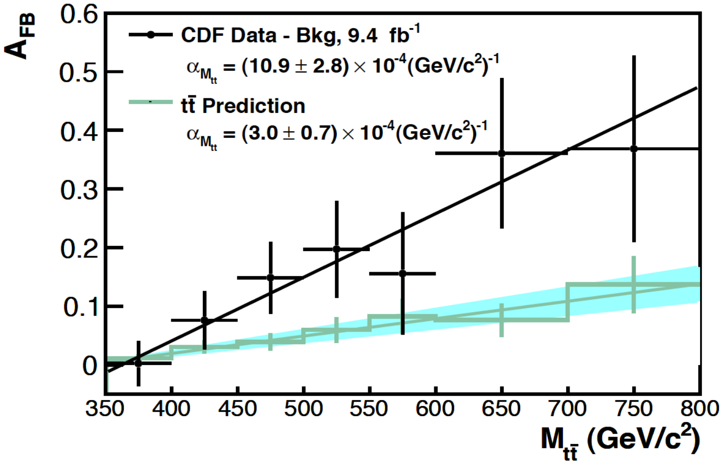
- Expect net asymmetry at the Tevatron, but not LHC
- At the LHC one has to look for the difference in the rapidity shapes

(2014): D0 finds near agreement with SM…

The CDF measurement versus (known) SM:

Discrepancy w/r to SM ≤ 3σ

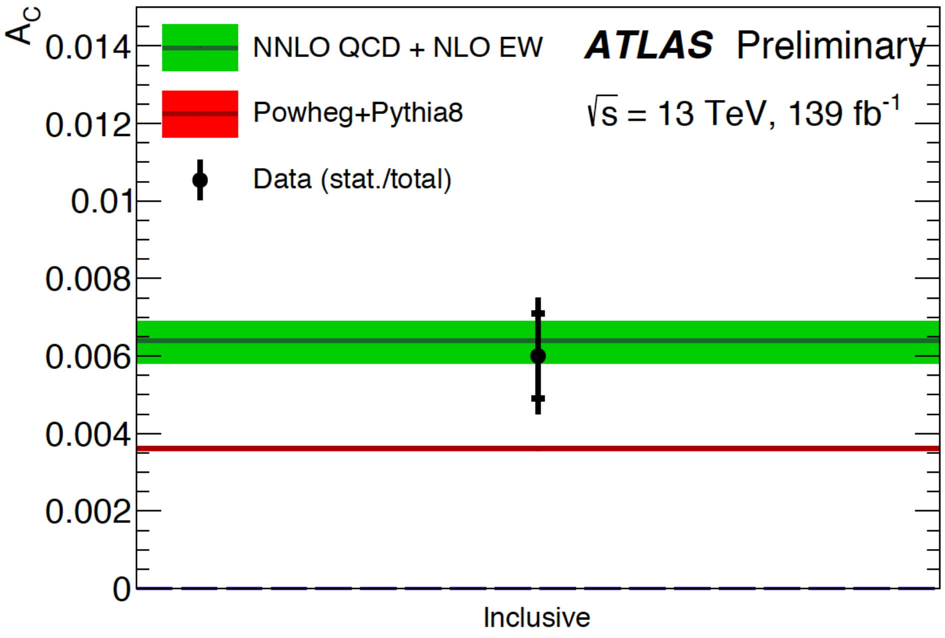


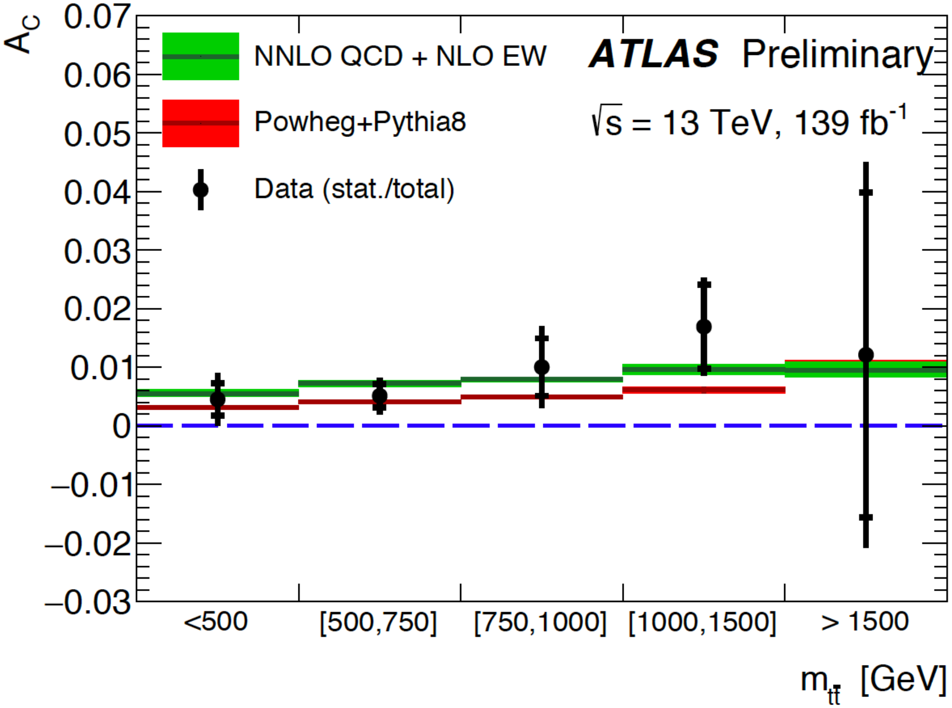


- Impressive measurements from the LHC (note that the asymmetry is diluted at the LHC relative to Tevatron by a factor of 10)

- Impressive improvement of the precision of the measurement

- It starts to differentiate between different SM predictions.
  - Recall: NNLO QCD corrections played important role at the Tevatron, MC did not model this observable satisfactorily





- One can even plot the two observables together:

From M. Russell arXiv:1709.10508

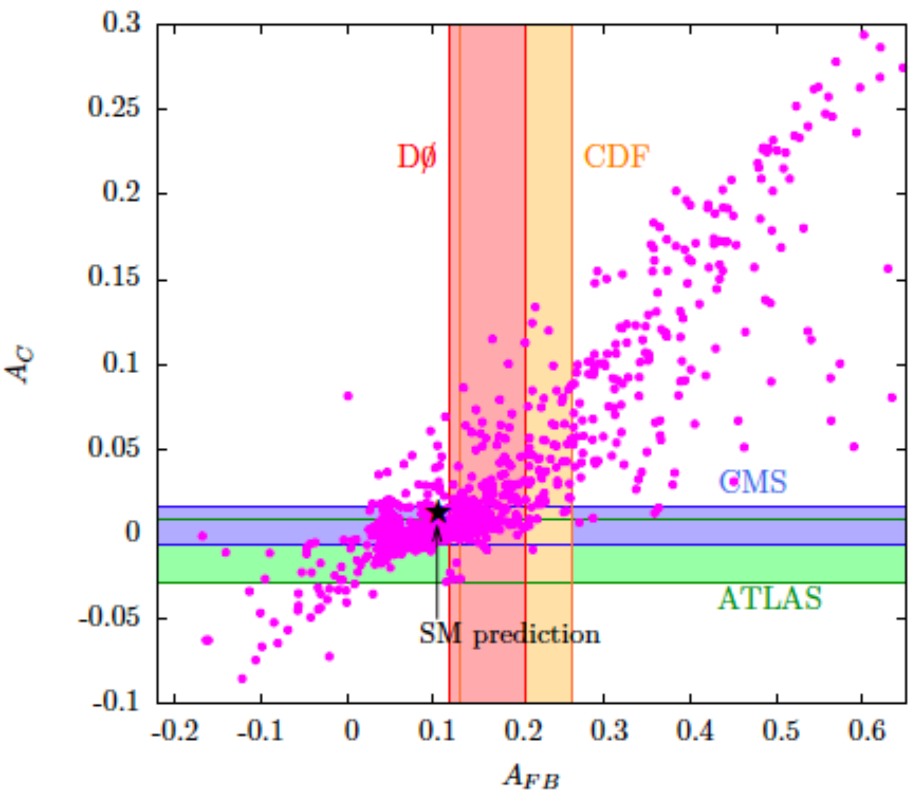


Figure 3.13: *Results of a 1000 point parameter space scan over -10 TeV* $^{-2} < c^{1,2}_{u,d}/\Lambda^2 <$ *10 TeV*$^{-2}$ *overlaid with the most up to date measurements of* A_{FB} *and* A_{C}*, showing clearly the correlation between them.*

Some interesting observables showing discrepancy w/r to SM:

Top quark spin correlations

- Some background:
  - Individual top quarks are produced unpolarized
  - However the spins of the two top quarks in the pair are strongly correlated
  - Since the top decays very fast (the only quark we could observe as a bare quark) its spin information is passed to its decay products
  - Measuring distributions of decay products one can see the imprint of these spin correlations

$$|\mathcal{M}(pp \to t\bar{t} \to (\ell^+\ell - \nu\bar{\nu}b\bar{b})|^2 \sim \mathrm{Tr}[\rho R \bar{\rho}]$$

$$R \sim \underbrace{\bar{A}\mathbb{1}\otimes\mathbb{1}}_{\text{spin-averaged}} + \underbrace{\bar{B}_i^+\sigma^i\otimes\mathbb{1} + \bar{B}_i^-\mathbb{1}\otimes\sigma^i}_{\text{top-quark polarization}} + \underbrace{\bar{C}_{ij}\sigma^i\otimes\sigma^j}_{\text{spin-correlation}}$$

- Why is this observable interesting? It can help differentiate non-SM contributions to top pair production:

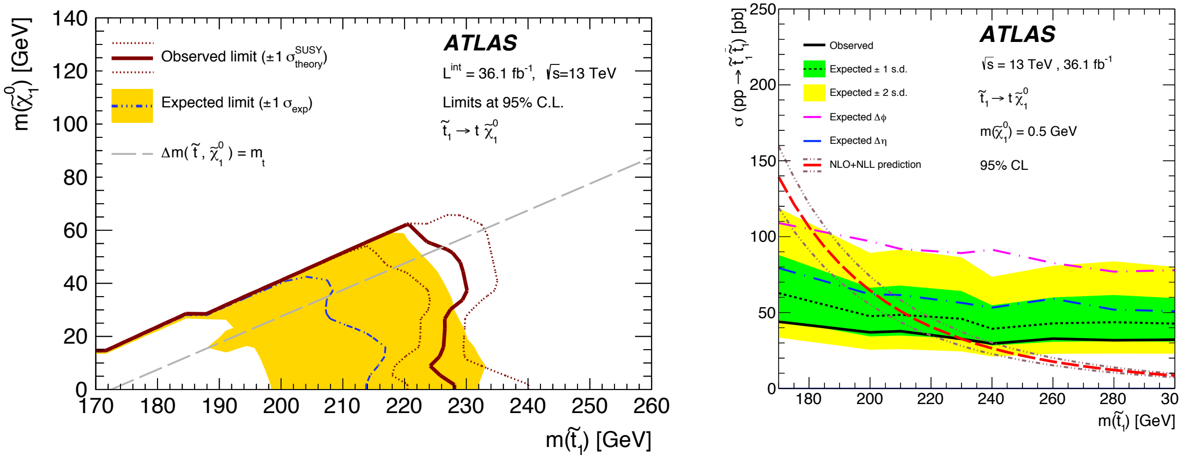
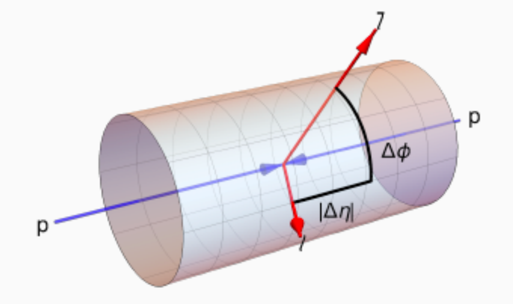


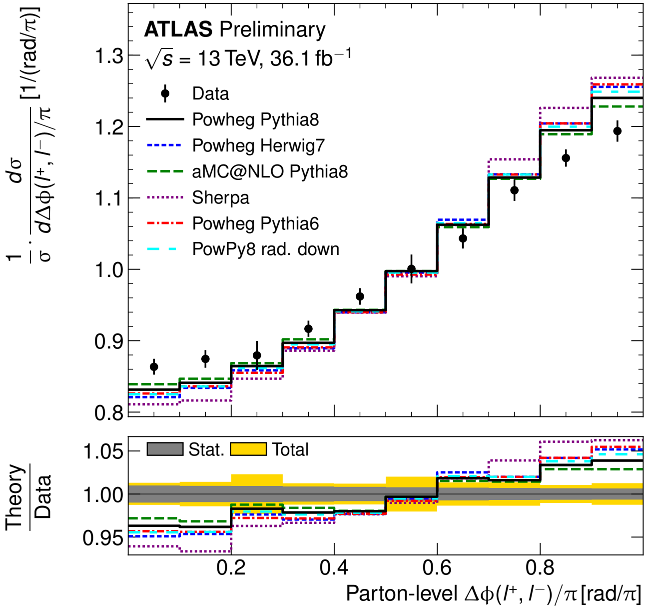
Figure 7 – Left: observed and expected 95% CL exclusion in the plane of $m_{\tilde{t}}$ and m_{LSP} [7]. Right: limits on the $\tilde{t}\tilde{t}^*$ cross section at 95% CL as a function of $m_{\tilde{t}}$, assuming $m_{\text{LSP}} = 0.5$ GeV. The expected limits when using the $|\Delta\phi_{\ell\ell}|$ and $|\Delta\eta|$ distributions alone are shown by the magenta and blue dashed lines, respectively [7].

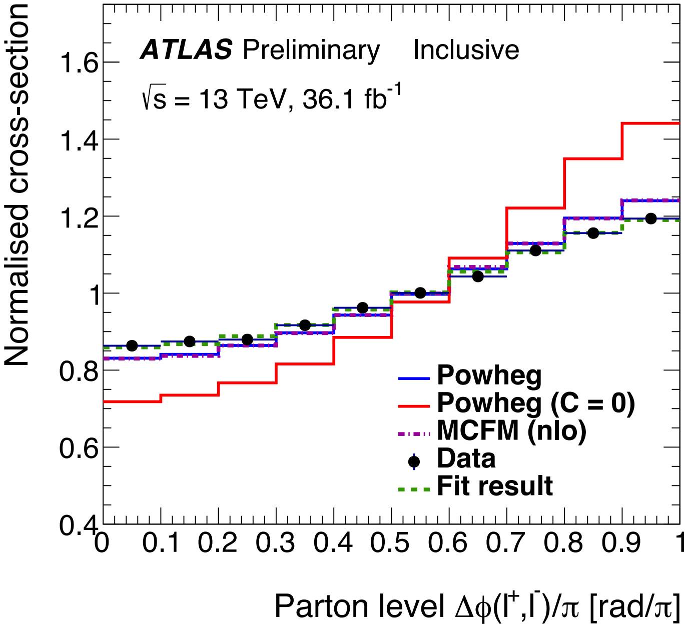
From arXiv:1905.08634

- ✓ In principle the full spin density matrix can be measured
- ✓ However, precision is low (since special frames are needed)

- ✓ To improve precision, use lab-frame distributions (they mix spin-correlation with kinematics)
- ✓ Best candidate: **Δφ** - the angle between the two leptons in the transverse plane







ATLAS-CONF-2018-027

Region	f_{SM}	Significance (incl. theory uncertainties)
$m_{t\bar{t}} < 450$ GeV	1.11 ± 0.04 ± 0.13	0.85 (0.84)
$450 < m_{t\bar{t}} < 550$ GeV	1.17 ± 0.09 ± 0.14	1.00 (0.91)
$550 < m_{t\bar{t}} < 800$ GeV	1.60 ± 0.24 ± 0.35	1.43 (1.37)
$m_{t\bar{t}} > 800$ GeV	2.2 ± 1.8 ± 2.3	0.41 (0.40)
inclusive	1.250 ± 0.026 ± 0.063	3.70 (3.20)

Significant deviation!

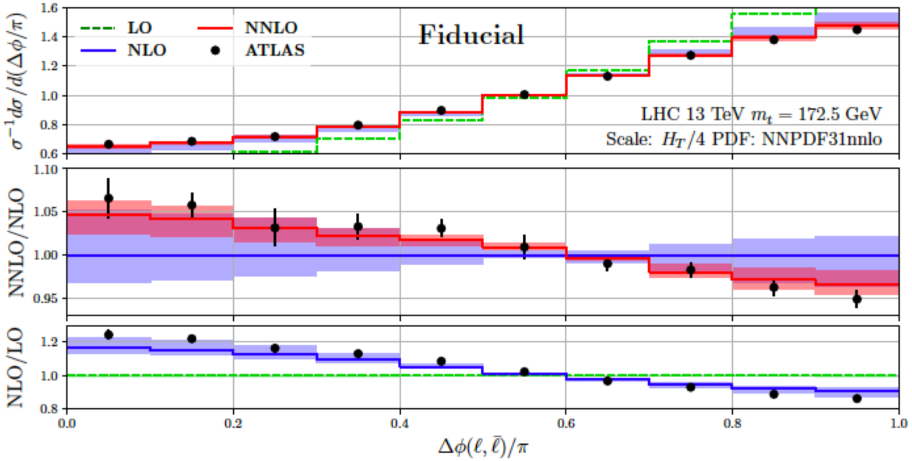
- So, what’s the explanation?

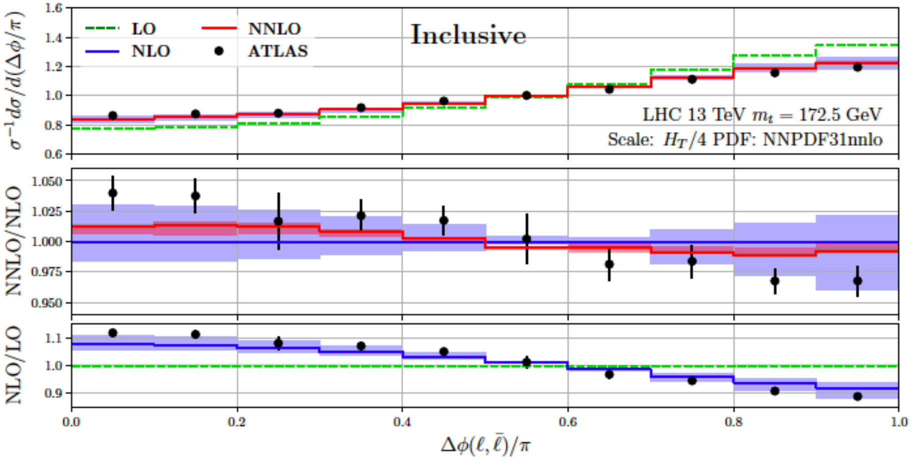
- Months after ATLAS published, the NNLO calculation with top decay also at NNLO appeared

Behring, Czakon, Mitov, Papanastasiou, Poncelet arXiv:1901.05407

- An extensive analysis was made. All but one sources were dismissed:
  - Scale choice
  - m_{top}
  - PDF
  - Finite width and EW corrections

- What was found was very surprising:



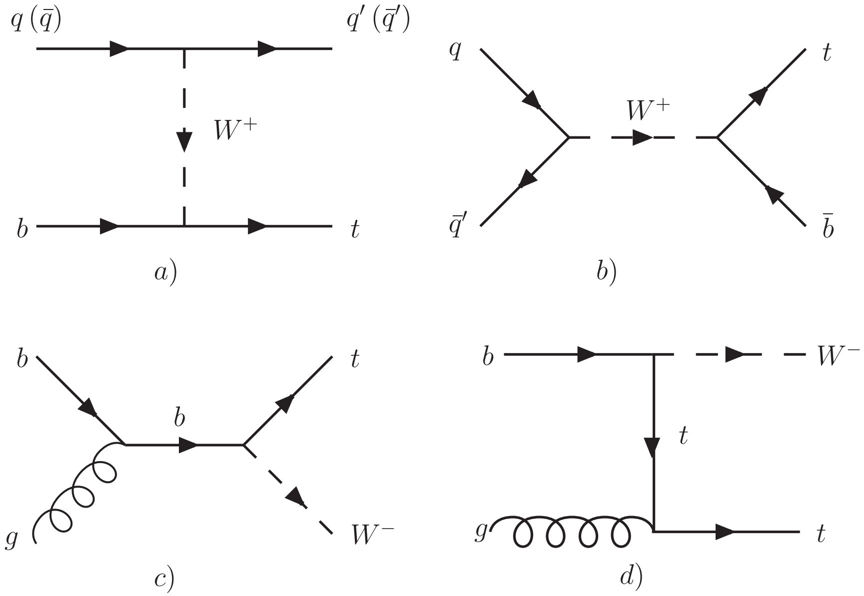


- NNLO describes the data in fiducial volume but not in the inclusive one! How can that be?

Single top production

- The three channels for single top production:

t-channel



s-channel

Wt - production

Wt - production

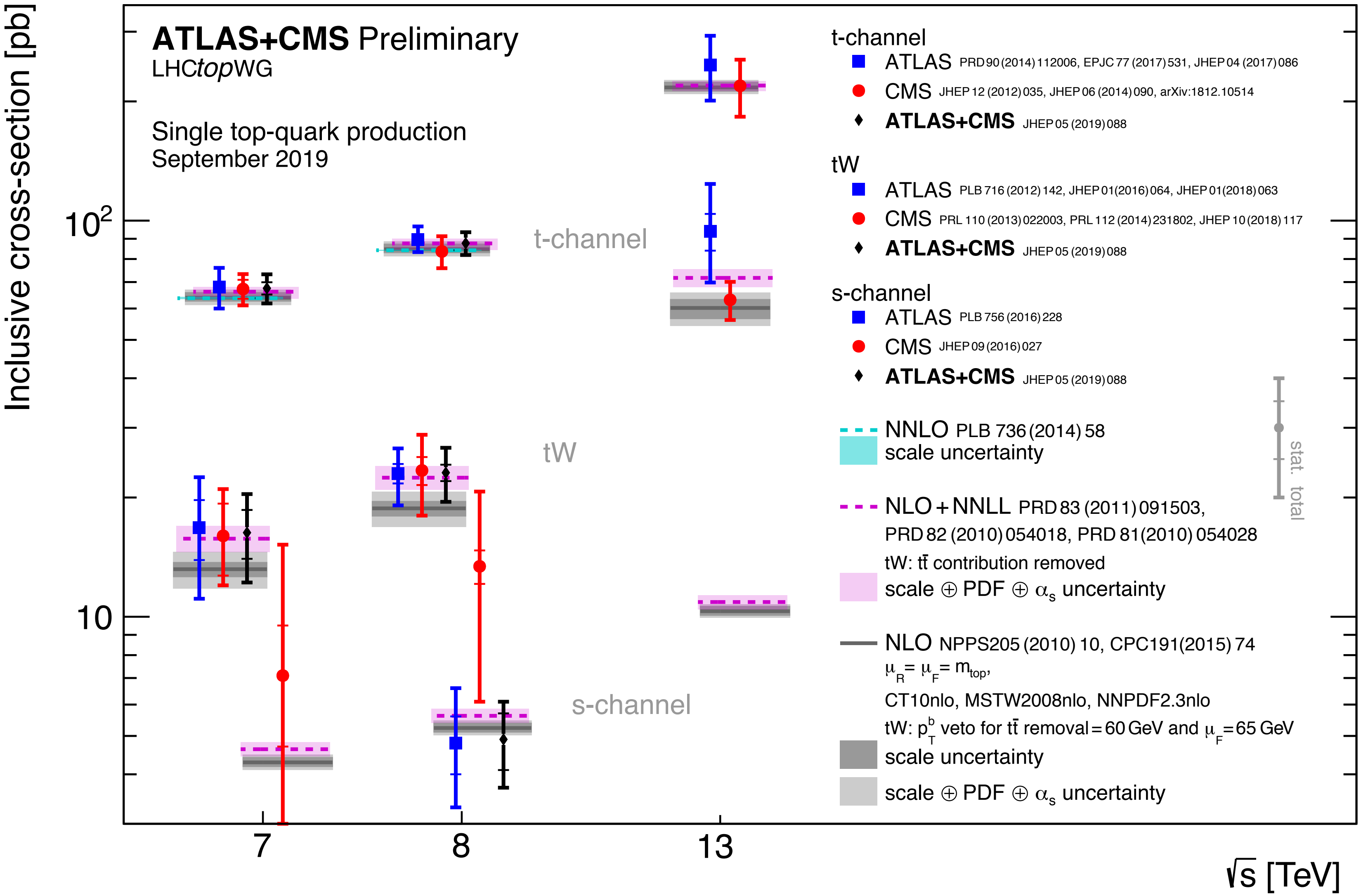
- Typical cross-section values

cross section	t channel	s channel	tW mode
$\sigma^t_{\text{Tevatron}}$	1.15 ± 0.07 pb	0.54 ± 0.04 pb	0.14 ± 0.03 pb
σ^t_{LHC}	150 ± 6 pb	7.8 ± 0.7 pb	44 ± 5 pb
$\sigma^{\bar{t}}_{\text{LHC}}$	92 ± 4 pb	4.3 ± 0.3 pb	44 ± 5 pb

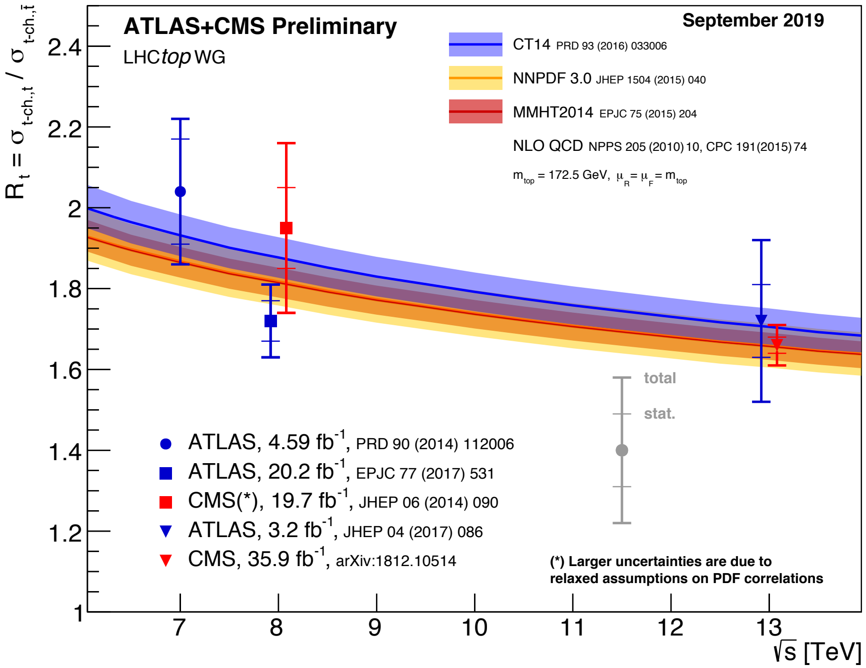
Note that top and anti-top s/t-channel x-sections are different at the LHC (due to pdf's)

- Good agreement between SM theory and measurements

From PDG 2018



- Single top t-channel production is now know through NNLO.
- Theory uncertainties are now tiny: 1% or less
- The production rate for single top at the LHC is large and comparable to top-pair
- It is much harder to measure single top due to not-so-distinct final state
- Single top could be used to measure directly top quark properties, especially Vtb
- Good playground for testing 4- versus 5-flavor number schemes
- Search for FCNC in top production
- Charged light Higgs boson

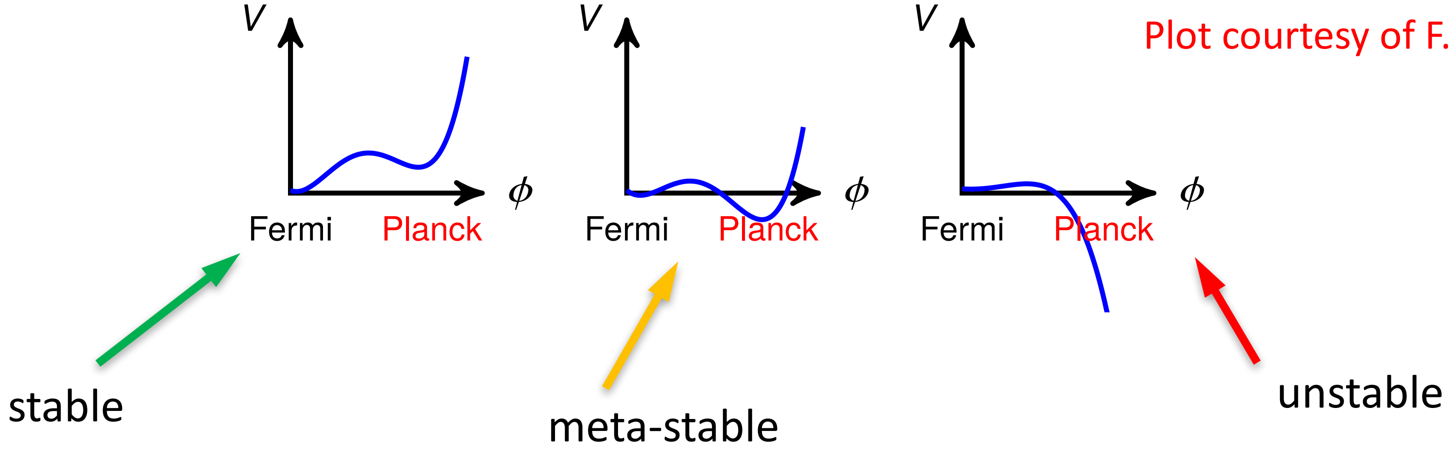


Top quark mass

Why the top mass?

- ✓ It is a fundamental parameter of the SM
- ✓ Its precision affects many precision observables in the SM.
- ✓ Its precision affects the searches for new physics.
- ✓ However, the most relevant case is: extrapolation of the SM to very high energies.
  - ✓ Once the Higgs boson was found (and the mass measured quite precisely) m_{top} is the SM parameter that mostly parametrically affects SM predictions
  - ✓ Prime example: stability of EW vacuum

Higgs potential $V(\phi) \simeq \lambda(\phi)\frac{\phi^4}{4}$



stable

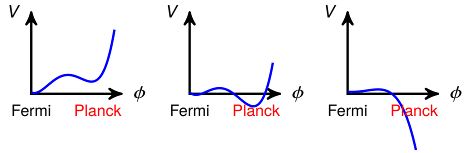
meta-stable

unstable

Plot courtesy of F. Bezrukov

- ✓ Here is how m_{top} enters the game:

Higgs potential $V(\phi) \simeq \lambda(\phi)\frac{\phi^4}{4}$



- ✓ Take the pole-masses m_{top} and m_h as input parameters. Then:

$$\lambda(\mu) = \frac{G_\mu}{\sqrt{2}} m_h^2 + \text{loop corrections}$$

$$y_t(\mu) = \frac{\sqrt{2}}{v} m_t + \text{loop corrections}$$

$\overline{MS}$- running parameters

Defs:

$$\mathcal{L} = \frac{y_t}{\sqrt{2}} h\bar{t}t$$

$$G_\mu = \frac{1}{\sqrt{2}v^2} + \text{loop corrections}$$

Size of loop effects:

$\bar{\mu} = M_t$	λ	y_t
LO	0.12917	0.99561
NLO	0.12774	0.95113
NNLO	0.12604	0.94018

- ✓ In other words in SM both λ and y_t are derived parameters. Their values are:

All numbers on this slide adapted from Buttazzo et al arXiv:1307.3536v4

$$\lambda(\mu = m_t) \approx 0.126 - 0.00004 \left(\frac{\Delta m_t}{1\text{GeV}}\right) + 0.000412 \left(\frac{\Delta m_h}{0.2\text{GeV}}\right) \pm \ldots$$

$$\lambda(\mu = m_{\rm PL}) \approx -0.0143 - 0.0066 \left(\frac{\Delta m_t}{1\text{GeV}}\right) + 0.0026 \left(\frac{\Delta \alpha_s}{0.001}\right) + 0.0006 \left(\frac{\Delta m_h}{0.2\text{GeV}}\right) \pm \ldots$$

$$y_t(\mu = m_t) \approx 0.9369 + 0.0056 \left(\frac{\Delta m_t}{1\text{GeV}}\right) - 0.0006 \left(\frac{\Delta \alpha_s}{0.001}\right) \pm \ldots$$

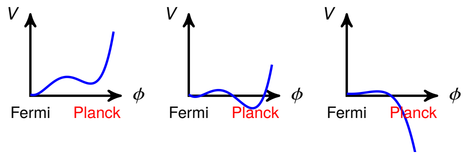
$$y_t(\mu = m_{\rm PL}) \approx 0.3825 + 0.0051 \left(\frac{\Delta m_t}{1\text{GeV}}\right) - 0.003 \left(\frac{\Delta \alpha_s}{0.001}\right) \pm \ldots$$

Where: $\boxed{\Delta x \equiv x - x^{\rm ref}}$

Driven by m_{top}, not m_h!

$$\lambda(\mu = m_{\rm PL}) \approx -0.0143 - 0.0066\left(\frac{\Delta m_t}{1\mathrm{GeV}}\right) + 0.0026\left(\frac{\Delta \alpha_s}{0.001}\right) + 0.0006\left(\frac{\Delta m_h}{0.2\mathrm{GeV}}\right) \pm \ldots$$

Higgs potential $V(\phi) \simeq \lambda(\phi)\frac{\phi^4}{4}$

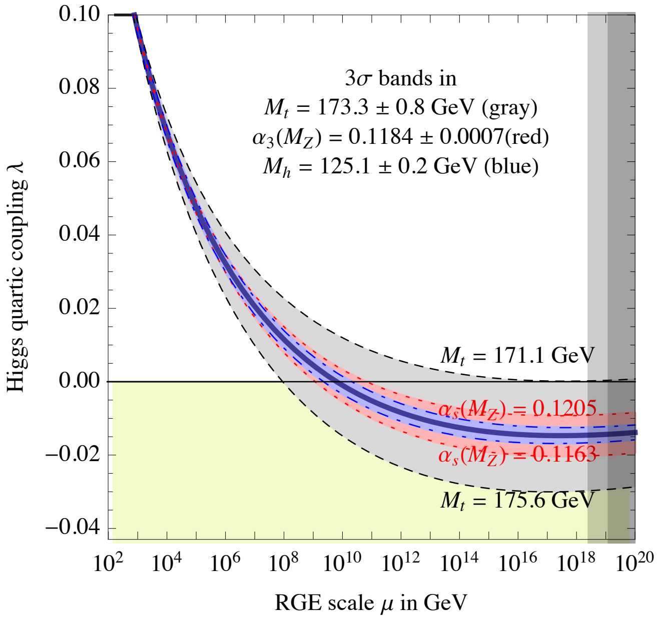


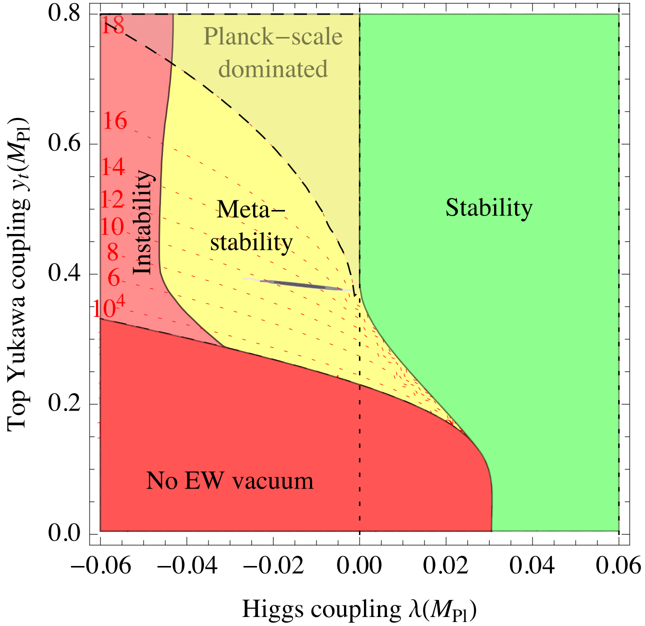
- ✓ The effective potential can be non-negative all the way to m_{PL} if the top mass were **lower** than the current world average by about 2 GeV.

Buttazzo et al arXiv:1307.3536v4

- ✓ Stated differently, stability requires:

$$M_t < (171.53 \pm 0.15 \pm 0.23_{\alpha_3} \pm 0.15_{M_h})\ \mathrm{GeV} = (171.53 \pm 0.42)\ \mathrm{GeV}.$$



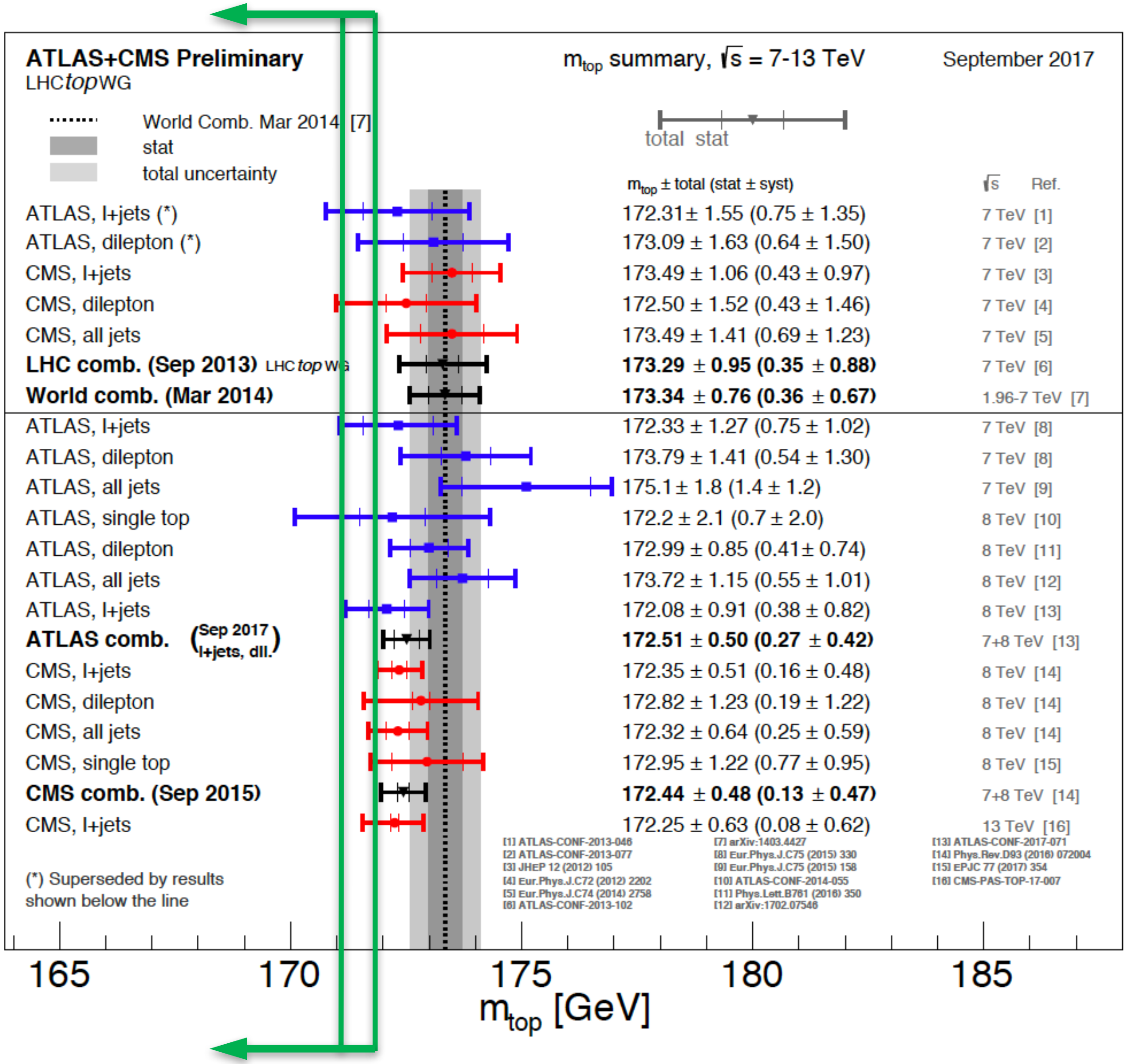


So, what is the value of m_{top} and how well do we know it?

So how well do we (think) we know the top mass?

✓ And the latest LHCtopWG combination:

$$M_t < (171.53 \pm 0.15 \pm 0.23_{\alpha_3} \pm 0.15_{M_h})\,\mathrm{GeV} = (171.53 \pm 0.42)\,\mathrm{GeV}.$$



✓ At face value, the World Average is more than 3σ away from stability.

✓ In practice, the most-precise LHC measurements are almost consistent with stability!

Top mass: precision and scheme dependence

✓ Computing in terms of the pole mass is easy and natural.

✓ However, that particular mass has non-perturbative corrections that restrict its ultimate precision

✓ Recent estimate based on the 4-loop relation: pole mass <--> Msbar mass

Marquard, Smirnov, Smirnov, Steinhauser ’15

$$m_P = 163.643 + 7.557 + 1.617 + 0.501 + (0.195 \pm 0.005)\,\mathrm{GeV}$$

Assuming: $\overline{m}_t = m_t(\overline{m}_t) = 163.643\ \mathrm{GeV}$ and $\alpha_s^{(6)}(m_t) = 0.1088$

✓ Exploring the leading asymptotic behavior of the above relation Beneke ‘94

✓ One can derive an improved relation which predicts (approximately) higher terms in the above expansion.

✓ The ultimate precision is taken for the term where the term-to-term difference is smallest

Beneke, Marquard, Nason, Steinhauser ‘16

  ✓ Error from the terms beyond 4 loops: ~ 250 MeV
  ✓ Ultimate intrinsic error in the above relation: ~ 70 MeV

✓ All this is very important at e^+e^- colliders

However see A. Hoang et al. ‘17

Ongoing developments: POWHEG

✓ Peak position of the “direct” measurement (plus: strong correlation with m_{top})

Ferrario-Ravasio, Jezo, Oleari, Nason, arXiv:1801.03944

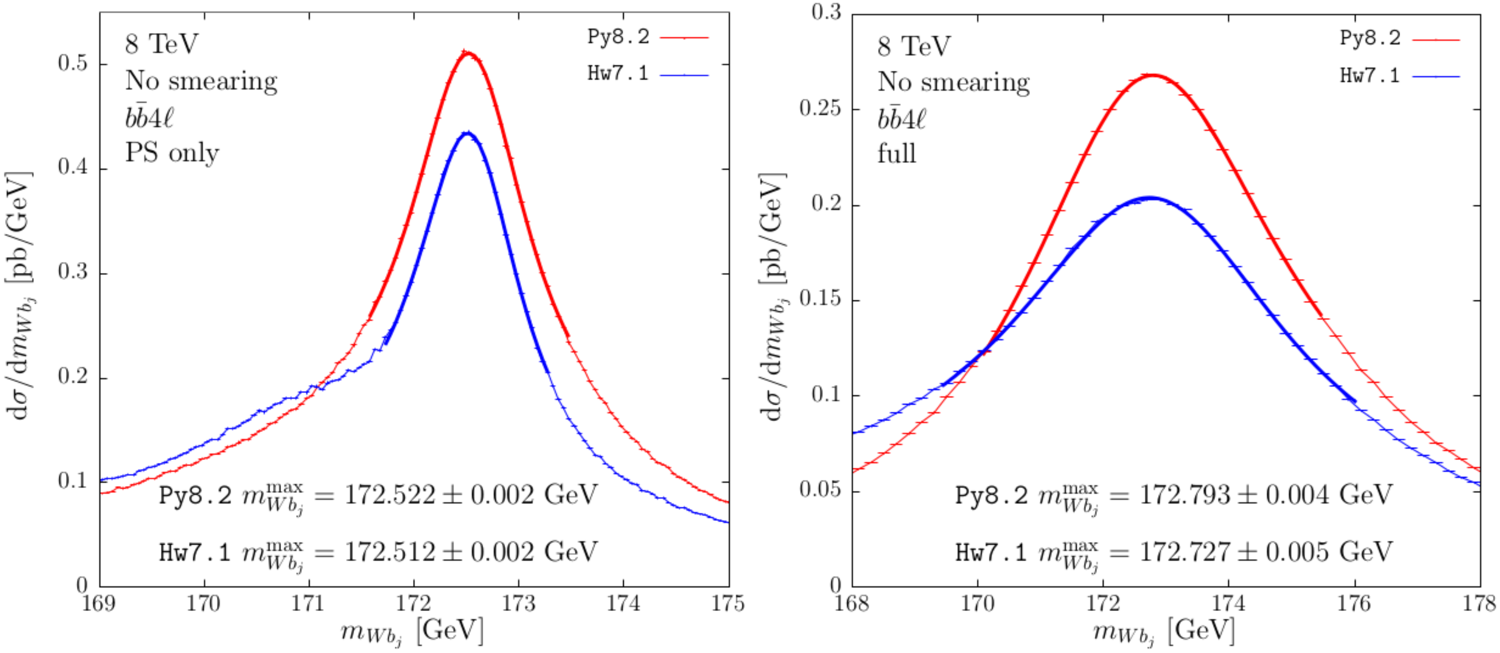


Figure 12. $d\sigma/dm_{Wb_j}$ distribution obtained by showering the $b\bar{b}4\ell$ results with `Pythia8.2` and `Herwig7.1`, at parton-shower level (left) and with hadronization and underlying events (right).

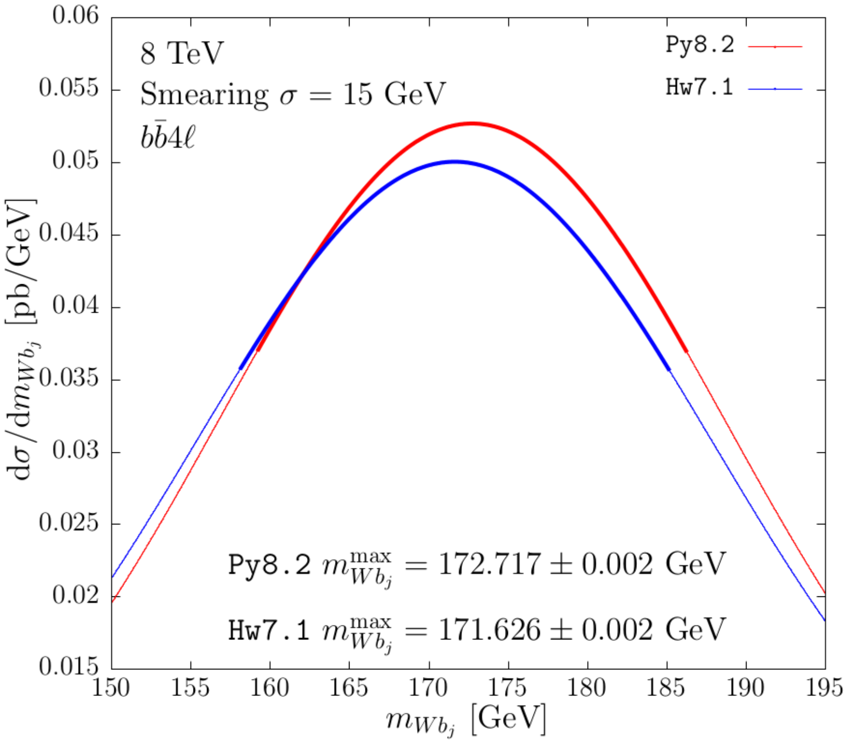
No large difference in the peak position (i.e. no indication here of large NP effects that displace the peak.). However, the marked difference in shape is bound to lead to problems when the experimental resolution is taken into account.

Ongoing/future developments: POWHEG

✓ Peak position of the “direct” measurement (plus: strong correlation with m_{top})

Ferrario-Ravasio, Jezo, Oleari, Nason, arXiv:1801.03944

✓ After smearing (i.e. experimental resolution)



When the resolution is accounted for, we find a 1.1 GeV difference between Herwig7 and Pythia8.

New Physics contributions to m_{top}

✓ One hardly mentioned problem!

✓ There is the possibility that undetected corrections to top production might shift the top mass measurements (measure top+bSM but theory assumes pure SM).

Example: stop -> top+X we discussed earlier

If the stop is light, the event looks top-like!

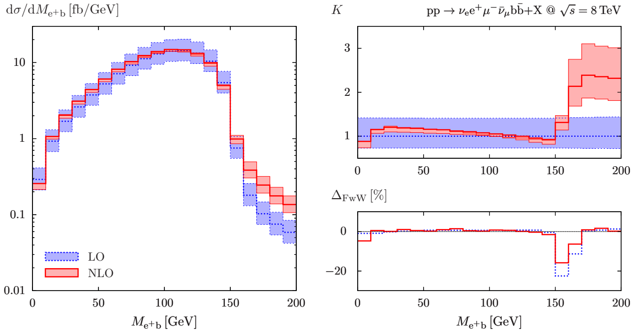


Figure 17: Invariant-mass distribution of positron–b-jet system with standard cuts for the LHC at $\sqrt{s} = 8\,\text{TeV}$ for dynamical scale $\mu_0 = E_T/2$.

✓ The strongest constraint on bSM contributions to m_{top} comes from the CMS end-point method

S. Chatrchyan *et al.* [CMS Collaboration], arXiv:1304.5783

✓ The method is kinematic: it measures the position of the end-point of the spectrum of top decay products. This is independent of the top production mechanism.

✓ The total error from the measurement is just above 2.0 GeV and agrees with the world average.

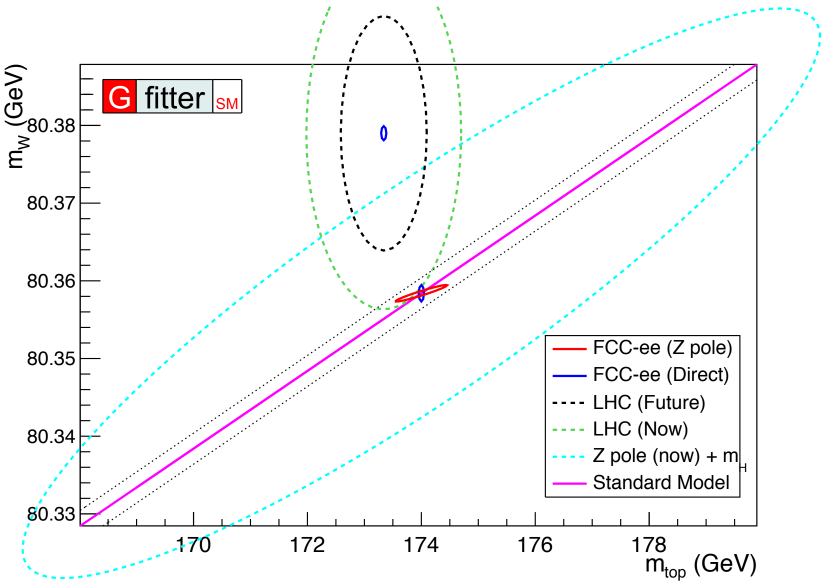
✓ From here we can conclude that bSM contributions to M_{top} are not larger than ~2GeV.

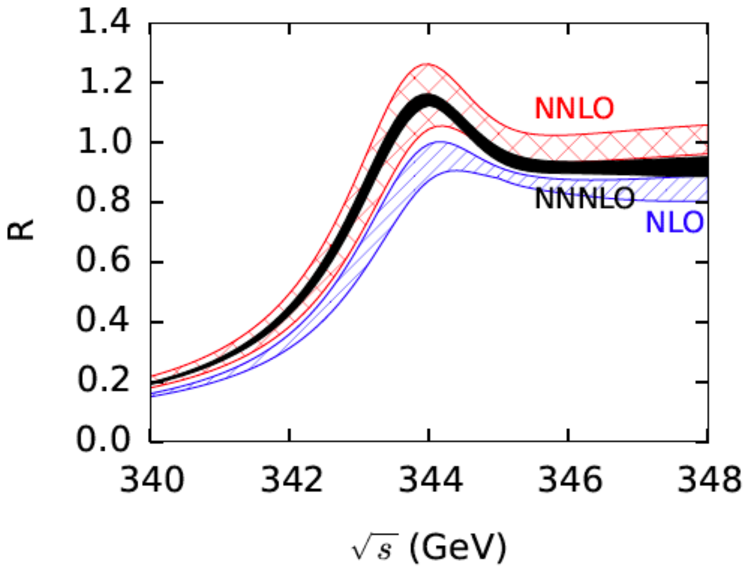
✓ Dedicated studies are welcome. Likely they will be model dependent; any model-independent arguments would be very valuable.

Top quarks at a future e^+e^- collider

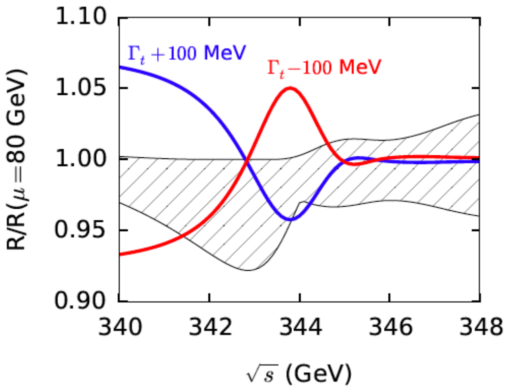
✓ The machine where the ultimate precision of 50 MeV on m_{top} can be achieved!

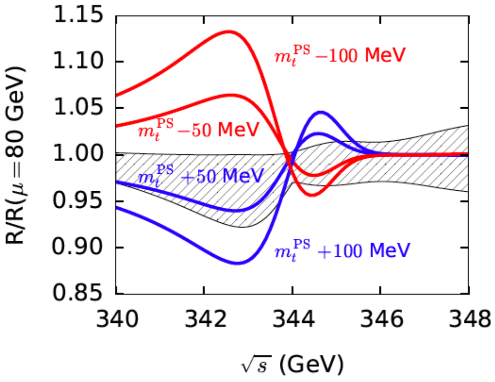
Future Circular Collider Study CERN-ACC-2018-0057





Beneke, Kiyo, Marquard,
Penin, Piclum, Steinhauser
arXiv:1506.06864





✓ Best approach is threshold scan.

✓ Continuum production also possible, depending on the collider

Conclusions and Outlook

- ✓ Top quark physics is a major subject
- ✓ It is actively being developed at the LHC
- ✓ Great prospects at future Colliders
  - ✓ HL-LHC
  - ✓ FCC-pp
  - ✓ e^+e^- machines
- ✓ Some things I did not cover (lack of time, not importance)
  - ✓ Top Yukawa measurements:
    - ✓ Great progress at LHC – direct and indirect
    - ✓ Great prospects at future pp and e^+e^- machines

I would be super happy to discuss any of the above in detail