Total Cross-sections at high energies.

- Energy dependence of total hadronic cross-sections
- BN Model: Eikonalised minijet model with soft gluon resummation.
- Extension to γp case.
- Cosmic connections.

Some references:

1)A. Corsetti, A. Grau, G. Pancheri,Y.Srivastava, PLB382 (1996)

2) A. Grau, G.Pancheri, Y. Srivasatava, PRD60 1999, hep-ph/9905228

3) R.M. Godbole, A. Grau, G. Pancheri and Y. Srivastava, PRD 72, 2005, 076001, [arXiv:hep-ph/0408355].

4)A. Achilli, R. Hegde, R. M. Godbole, A. Grau, G. Pancheri and Y. Srivastava, arXiv:0708.3626, Phys. Lett. 659 (2007) 137.

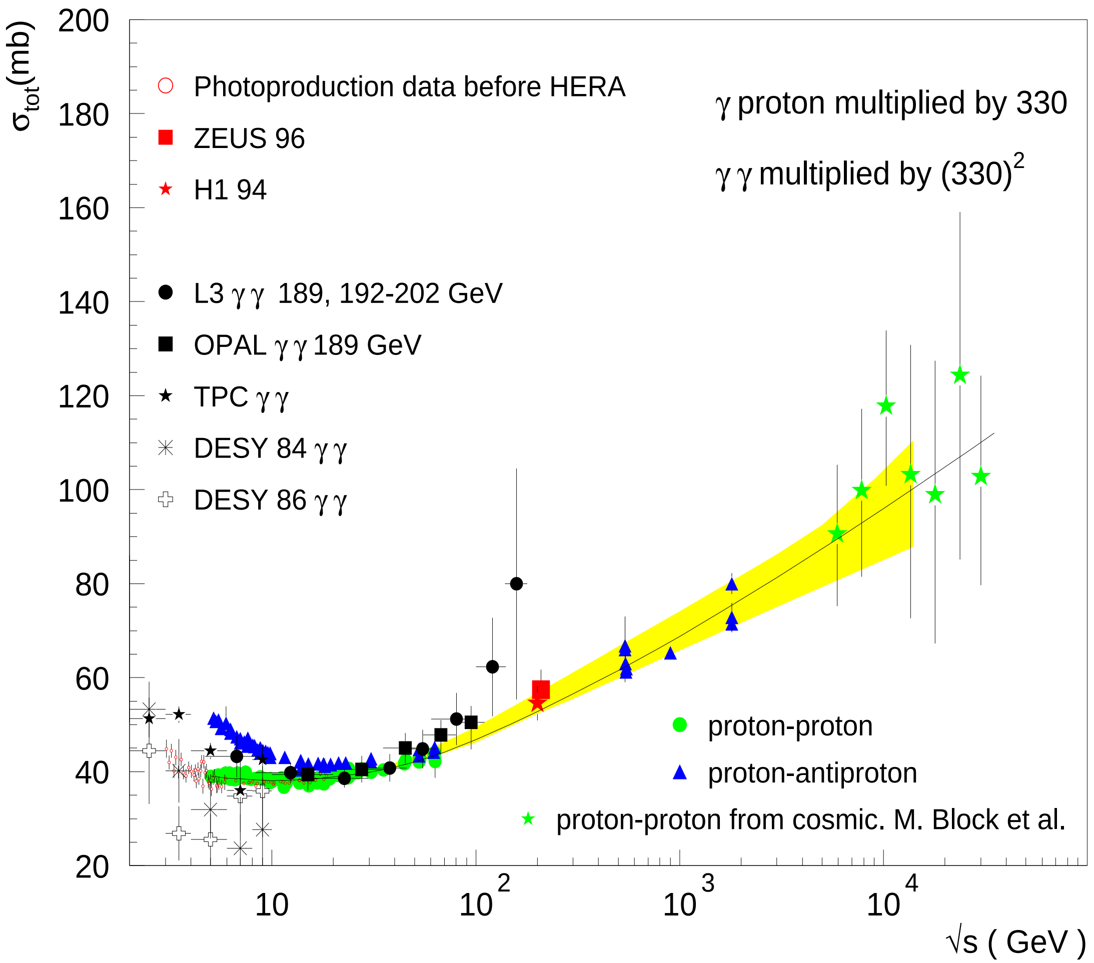
5)R.G., A. Grau, G. Pancheri, Y. Srivastava : Total photoproduction cross-sections at high energy: EJPC 63, (2009) 69.

6)R.G., A. Grau, G. Pancheri, Y. Srivastava: Froissart Bound and soft k_T resummation: Phys. Lett. B 682 (2009) 55.

7)A. Grau, G. Pancheri,O. Shekhovtsova and Y. Srivastava, Modelling pion and proton total cross-sections at LHC, Phys. Lett. B693 (2010) 456

8)F. Cornet, C.A. Garcia Canal, A. Grau, G. Pancheri and G. Sciutto, ICRC 2009, Proceedings, Photoproduction total cross-section and shower development.

All hadronic total cross-sections rise with energy.



Yellow band: range for Eikonal model improved with soft gluon resummation (RG, Grau, Pancehri,Srivastava, RG : G.G.P.S.)

Charge to models: Explain 1) The normalisation, 2) The rise **and** 3) the initial fall with energy.

Need to predict the high energy behaviour.

- Which mechanism drives the rise?
- Do they all rise with same slope?
- Do they satisfy the Froissart bound

  $\sigma_{tot} \leq \log^2(s)$ as $s \to \infty$.

The tools:

- Bounds from Analyticity and Unitarity.
- Regge Pomeron exchange.
- The Eikonal Minijet Model: EMM.
- Bloch-Nordsieck Resummation for the EMM.
- Minijet model and gluon saturation

1]

Regge-Pomeron Exchange:

$$\sigma_{tot} = \beta_R \left(\frac{s}{s_0}\right)^{\alpha_R(0)-1} + \beta_P \left(\frac{s}{s_0}\right)^{\alpha_P(0)-1} \equiv X s^{-\eta} + Y s^{\epsilon}$$

with $\eta \simeq 0.45, \epsilon \simeq 0.08$.

DL Parameterisation: A. Donnachie, P. Landschoff, PLB 296 (1992) 227

The fit had to be extended to include a 'hard' pomeron. DL (PLB 595 (2004) 393)

2] Fits using Unitarity

Block and Halzen: Phys. Rev. D72 (2005) 036006, PRD D73 (2006) 054022

The BH fit for $\sigma^{\pm} = \sigma^{\bar{p}p}/\sigma^{pp}$ as a function of beam energy ν, is given as

$$\sigma^{\pm} = c_0 + c_1 \ln(\nu/m) + c_2 ln^2(\nu/m) + \beta_{P'}(\nu/m)^{\mu-1} \pm \delta(\nu/m)^{\alpha-1},$$

The Froissart bound is saturated. They make predictions for the LHC cross-sections.

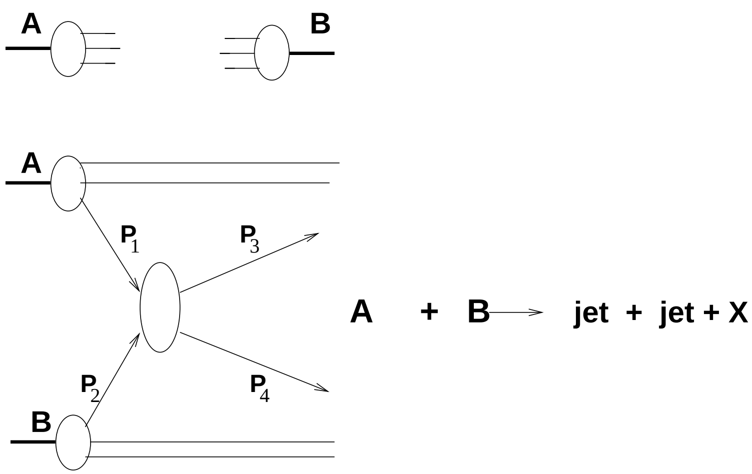
Igi and Ishida: obtain fits using Finite Energy sum rules Igi and M. Ishida, Phys. Lett. B622 (2005) 286 Similar predictions.

COMPETE Program: J. R. Cudell et. al. Phys. Rev. Lett. 89 (2002) 201801 J.-R. Cudell and O.V. Selyugin, hep-ph/061246; J.R. Cudell, E. Martynov, O.V. Selyugin and A. Lengyel, Phys. Lett. B 587 (2004) 78.

Constraints from analyticity, unitarity, factorisation and the fact that cross-section grow at most as $\log^2(s)$, but they also obtain equally good fits assuming the cross-section goes as a constant.

3] Eikonalised Minijet models : there is a whole class of them. (For a recent review : P. Lipari and M. Lusignoli, Phys. Rev. D 80 (2009) 074014)

Basic philosophy:



Try to explain the rise and the initial fall in terms of partons in the colliding hadrons using experimentally determined parton densities and basic QCD interactions among partons.

Increasing beam energy $\Rightarrow$ increase in # and energy of collding partons.

$\sigma_{jet} = \sigma(A + B \rightarrow jet + jet + X)$

calculated in pQCD rises with increasing $\sqrt{s}$.

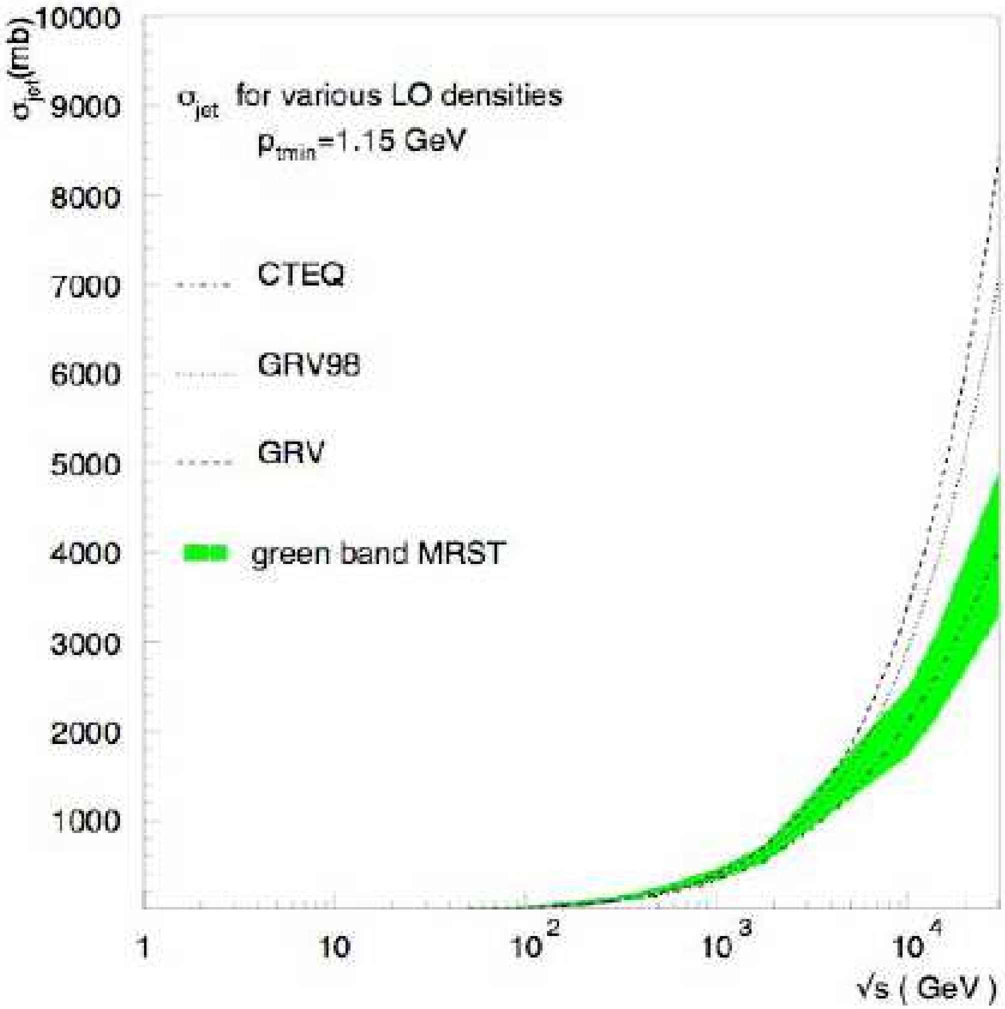
Energy rise in σ_{tot} driven by the rise of σ_{jet}.

Minijet Model Halzen and Cline (1985)

$$\sigma_{jet} = \int_{p_t min} \frac{d^2\sigma_{jet}}{d^2\vec{p}_t} d^2\vec{p}_t = \sum_{partons} \int_{p_t min} d^2\vec{p}_t \int f(x_1) dx_1 \int f(x_2) dx_2 \frac{d^2\sigma^{partons}}{d^2\vec{p}_t}$$

- Dominated by gluons.
- Depends strongly on the transverse momentum cutoff p_{tmin}.
- $\sigma_{jet} \propto \frac{1}{p_{tmin}^2} \left[\frac{s}{4p_{tmin}^2}\right]^{J-1}$ where $f(x) \sim x^{-J}$ and $J \sim 1.3 - 1.4$.

$$\sigma_{jet} \propto \left(\frac{s}{GeV^2}\right)^{J-1}$$



Asymptotic behavior of σ_{jet} for different PDF (LO):

CTEQ: J~1.3

GRV98: J~1.4

GRV: J~1.4

MRST: J~1.3

7

• σ_{jet} rises with s as a power in violation Frossiart Bound too fast towards σ_{tot}.

• Unitarization essential. Provided by finite size of the scatterer. Done using eikonal formalism

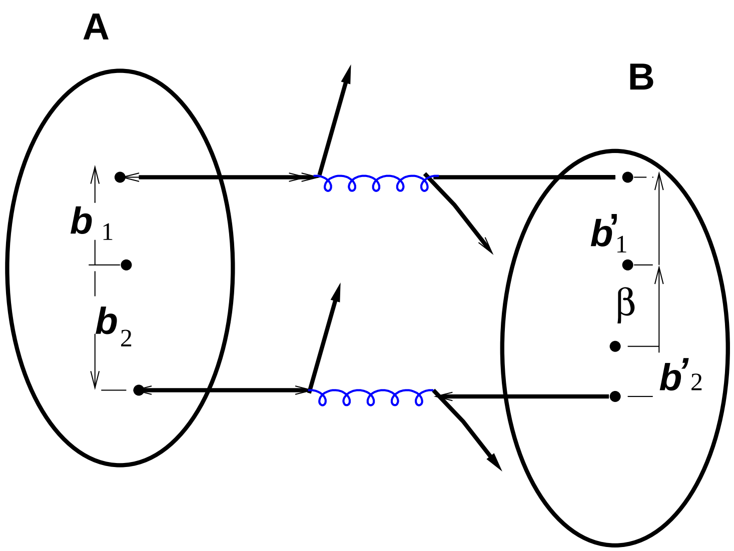
• The steep rise of σ_{jet} with s is NOT reflected in the energy rise of $\sigma_{tot}, \sigma_{inel}$.

With increasing energy the probability of multiple parton scattering (MPS) in a given hard scatter increases

$$\sigma_{AB}^{jet}(s) = < n_{pair}^{jet} > (s)\sigma_{AB}^{inel}(s)$$

Rising MPS $\Rightarrow$ rising jet pair multiplicity

Need to calculate the s dependence of $< n_{pair}^{jet} >$.



Transverse Overlap of the hadrons

s dependence related to that of the MPS probability. This in turn decided by the overlap of the partons in the transverse plane.

$$A_{AB}(\beta) = \int d^2 b_1 \rho_A(\vec{b_1}) \rho_B(\vec{\beta} - \vec{b_1})$$

The different models differ in how one models this overlap.

- $\sigma^{\text{inel}}_{pp(\bar{p})} = 2\int d^2\vec{b}[1 - e^{-n(b,s)}]$

Avergae number of collisions:

$n(b,s) = A_{AB}(b,s)\sigma(s) = 2\chi_I(b,s)$

$\chi(b,s)$ (EIKONAL function.)

Build n(b,s) for σ^{inel} and use it for

$\sigma^{\text{tot}}_{pp(\bar{p})} = 2\int d^2\vec{b}[1 - e^{-n(b,s)/2} cos(\chi_R)],\ \chi_R = 0$ in EMM

Approximations normally used: $n(b,s) = n_{soft}(b,s) + n_{hard}(b,s)$

Further factorisation: $n(b,s) = A(b)[\sigma_{soft} + \sigma_{jet}]$

- Model for A(b).

- σ_{soft} parametrized

- σ_{jet} LO QCD jet x-sections

- Eikonal model not restricted to calculate ONLY c.sections also used to calculate properties of hadronic events. pioneering: T. Sjostrand

At low energies and small σ^{jet}

$$\sigma_{AB}^{\text{inel}} = 2\int d^2\vec{b}[1 - e^{-n(b,s)}] \simeq \sigma_{AB}^{soft} + \sigma_{AB}^{jet}$$

At high energies, the eikonalisation softens the energy rise of σ^{inel} compared to that of σ^{jet}.

• Eikonal $\chi(b,s)$ contains information on the energy and the transverse space distribution of the partons in the hadrons.

• simplest formulation with minijets to drive the rise and eikonalization to ensure unitarity :

$$2\chi_I(b,s) \equiv n(b,s) = A(b)[\sigma_{soft} + \sigma_{jet}]$$

• The normalization depends both on σ_{soft} and on the b-distribution.

How to calculate the transverse overlap function in terms of 'measured' quantities?

- σ^{jet} depends on the parton densities $f_{q/A}(x_1), f_{q/B}(x_2)$ x_i the longitudinal mmtm fraction

- Overlap function on the transverse space (momentum) distribution.

Calculate the overlap functions using Fourier Transforms of form factors.

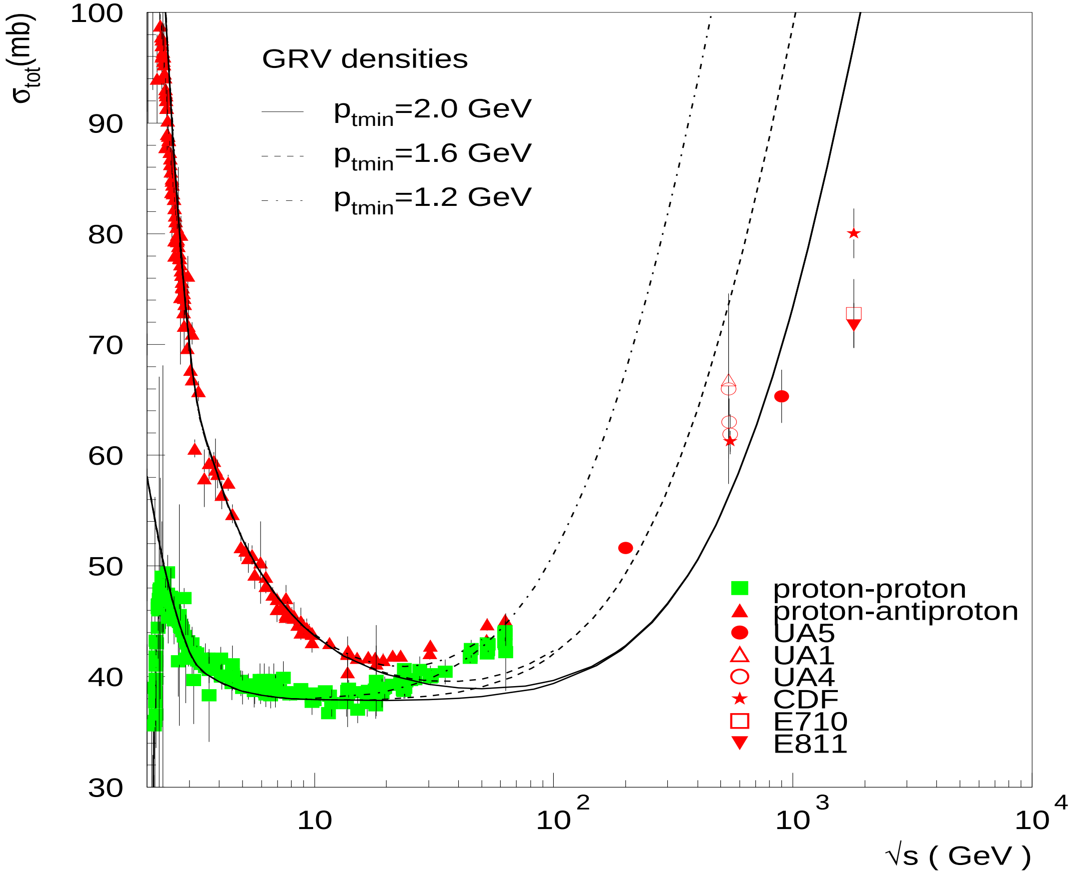
One can also calculate it using Fourier Transform of the transverse momentum distribution of the partons inside the proton.

Even the eikonalised minijet model predicts too strong a growth DIfficult to get the initial fall and the rise both at the right energies.

BN model is a QCD based model to calculate the overlap function $A(b,s)$.

Form Factor Model and factorisation between b,s of A(b,s).

Energy rise still too fast



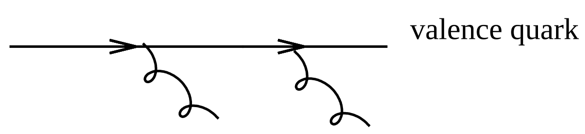
EMM model does O.K. qualitatively but is certainly not the whole story. Improve the model by removing the approximations used.

Recall **assumed** $n(b,s) = A(b)[\sigma_{soft} + \sigma_{jet}]$.

- The separation between s and b dependence only an approximation.

- Writing the overlap function as a $\mathcal{F}.\mathcal{T}$. of measured distributions does not allow for a s dependence of A

Pancheri, Grau, Srivatava had developed a model based on semi-classical method to calculate the impact parameter space distribution of partons in a hadron using resummation of soft gluon emissions.



BN model uses this computation.

• It is like EMM model with σ_{jet}^{QCD} driving the rise

and in addition

Soft Gluon Emission from Initial State Valence Quarks in k_t-space to give impact parameter space distribution of colliding partons

• introduces energy dependence in the **b-distribution** of partons in the hadrons $\Longrightarrow$ which depends on

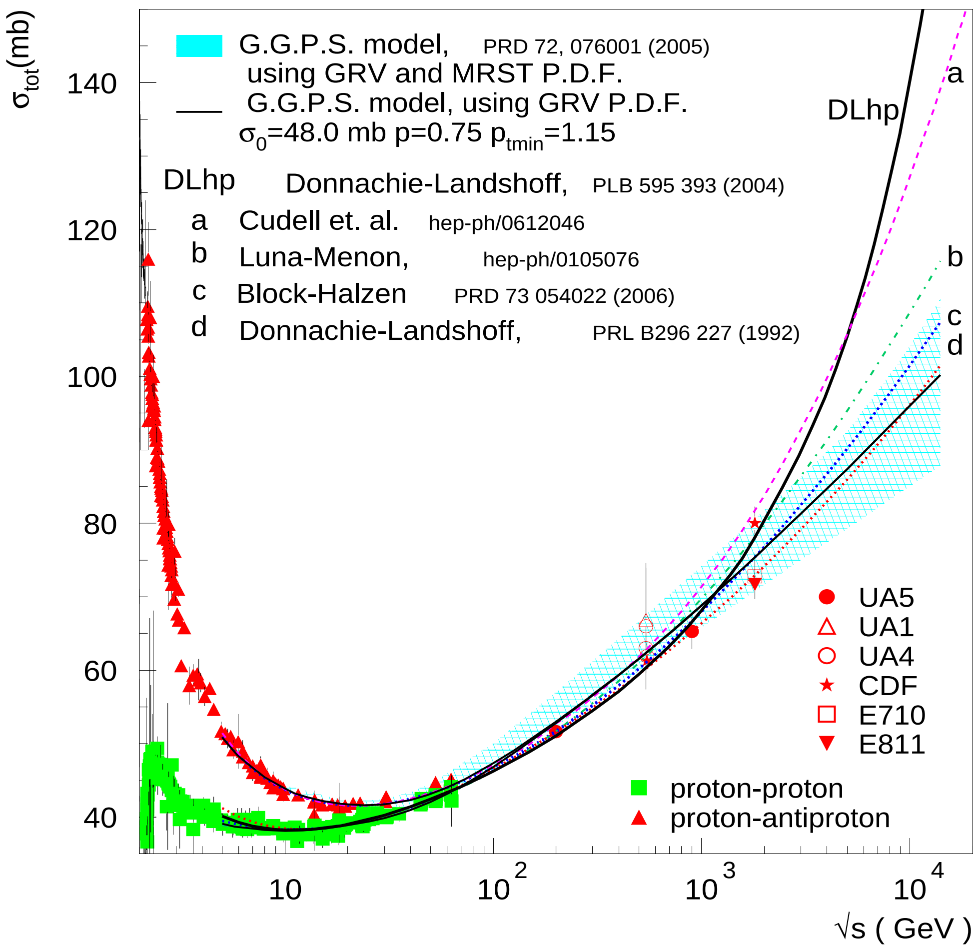
1. p_{tmin}

2. parton densities

Net effect is the energy rise is toned down.

The softening effect happens

- as $\sqrt{s} \Uparrow$ the phase space available for soft gluon emission also $\Uparrow$
- the transverse momentum of the initial colliding pair due to soft gluon emission $\Uparrow$
- more straggling of initial partons $\Rightarrow$ less probability for the collision
- The soft gluons in the eikonal in fact restore the Froissart bound.



$$\sigma_{tot}^{LHC} = 102 + 12 - 13 mb$$

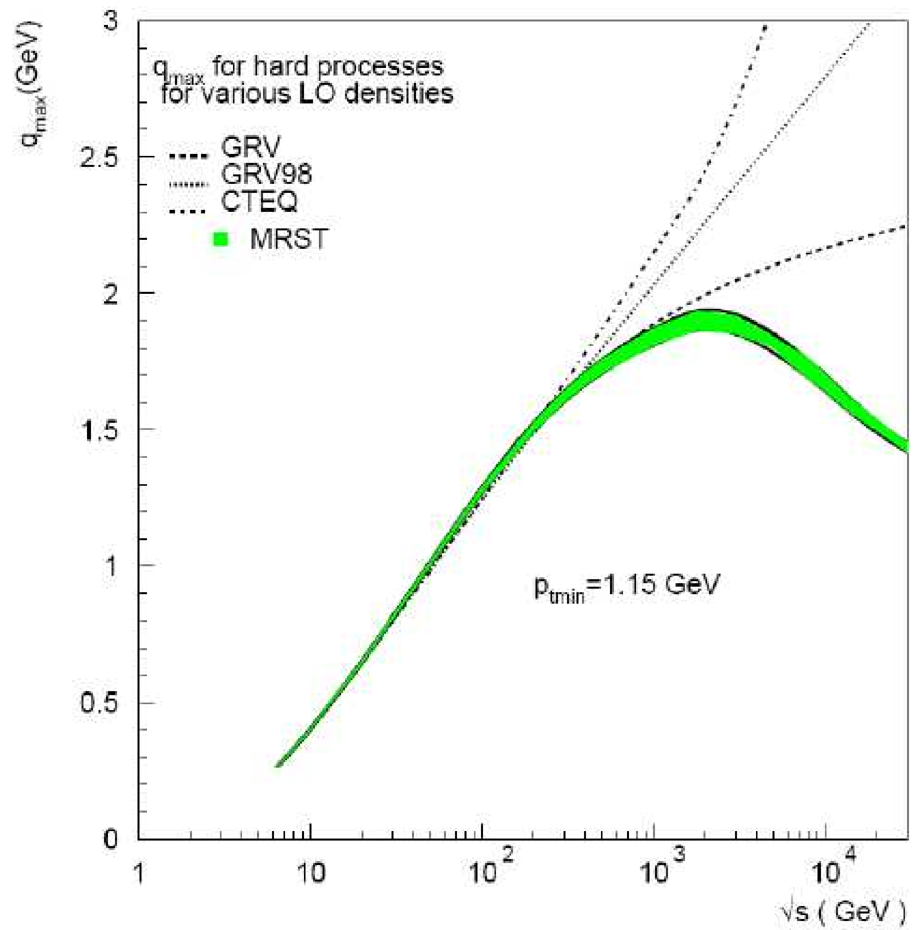
Important issue how to handle α_s for this very soft gluon emissions how to compute $A(b, s)$ for hard and soft contribution and How the energy rise is toned down to restore Froissart bound

In fact our model a very direct direct connection between the behaviour of $\alpha_s(Q^2)$ in the far infra red and high energy behaviour of the cross-section.

$$A_{BN}^{AB}(b,s) = \mathcal{N}\int d^2\mathbf{K}_\perp \frac{d^2P(\mathbf{K}_\perp)}{d^2\mathbf{K}_\perp} e^{-i\mathbf{K}_\perp\cdot\mathbf{b}} = \frac{e^{-h(b,q_{max})}}{\int d^2\mathbf{b}e^{-h(b,q_{max})}} \equiv A_{BN}^{AB}(b,q_{max}(s))$$

$$h(b,q_{max}(s)) = \frac{16}{3}\int_0^{q_{max}(s)} \frac{dk_t}{k_t}\frac{\alpha_s(k_t^2)}{\pi}\left(\log\frac{2q_{\mathrm{max}}(s)}{k_t}\right)[1 - J_0(k_t b)]$$

q_{max} is a the maximum momentum allowed for gluon emission by kinematics.
For hard part, depends on x_1, x_2.
Avergaed over the parton densities.
For soft part it is parmaterised and fitted



Singular but integrable epxressions

For $k_T >> \Lambda_{QCD}$ should reduce to the usual asymptotic freedom expression

Should yield confining potential in the infrared limit.

$$\alpha_s(k_T) = \frac{12\pi}{(33-2N_f)} \frac{p}{\log(1+p(k_T^2/\Lambda_{QCD}^2)^p)}$$

$$1/2 < p < 1.$$

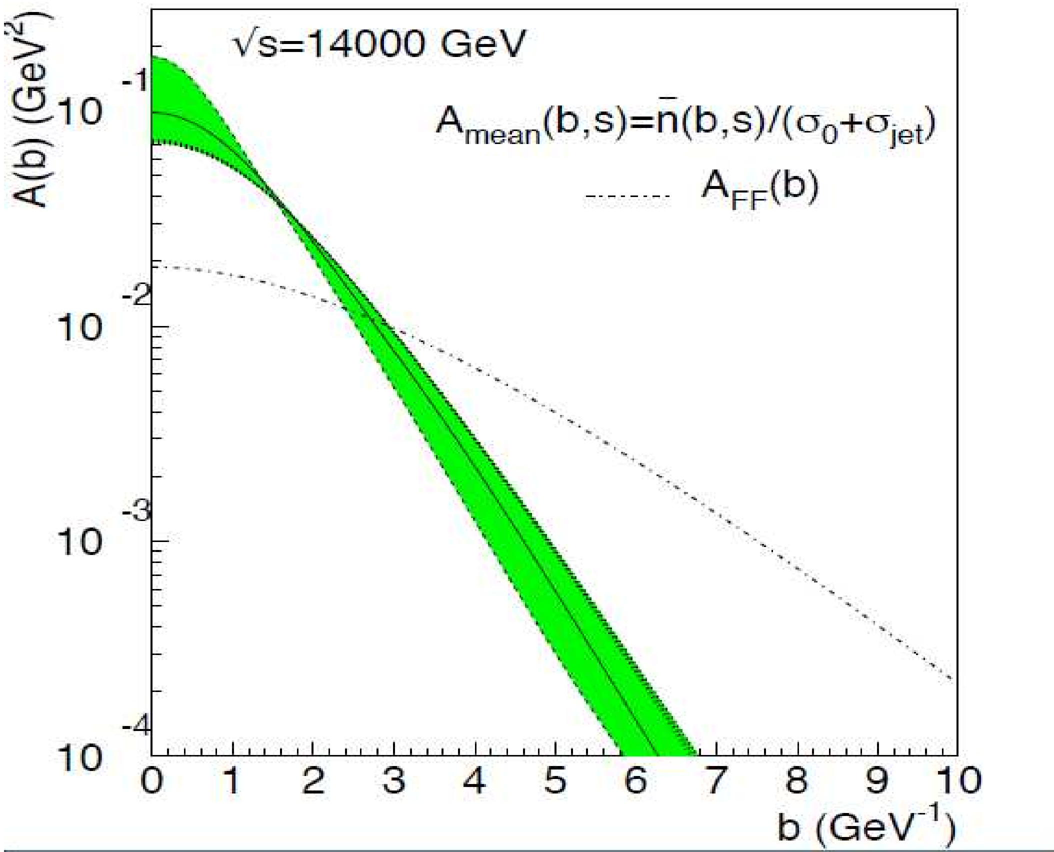
$\alpha_s(Q^2) \to (1/Q^2)^p$ as $Q^2 \to 0$.

$\sigma_{hard} \simeq \left(\frac{s}{s_0}\right)^{\epsilon} \sigma_1 \;\; A(b,s) \sim e^{-(bq)^{2p}}$

ϵ reflects the rise of σ_{jet} and is about $0.3 - 0.4$.

$\sigma_{tot} \simeq (\epsilon \ln s)^{1/p} \;\; 1/2 < p < 1.$

Froissart Bound related to the long distance behaviour of QCD.



A(b) (GeV²)
√s=14000 GeV
A_mean(b,s)=n̄(b,s)/(σ_0+σ_jet)
A_FF(b)
10^-1
10^-2
10^-3
10^-4
0 1 2 3 4 5 6 7 8 9 10
b (GeV⁻¹)

Encouraged to extend to γp.

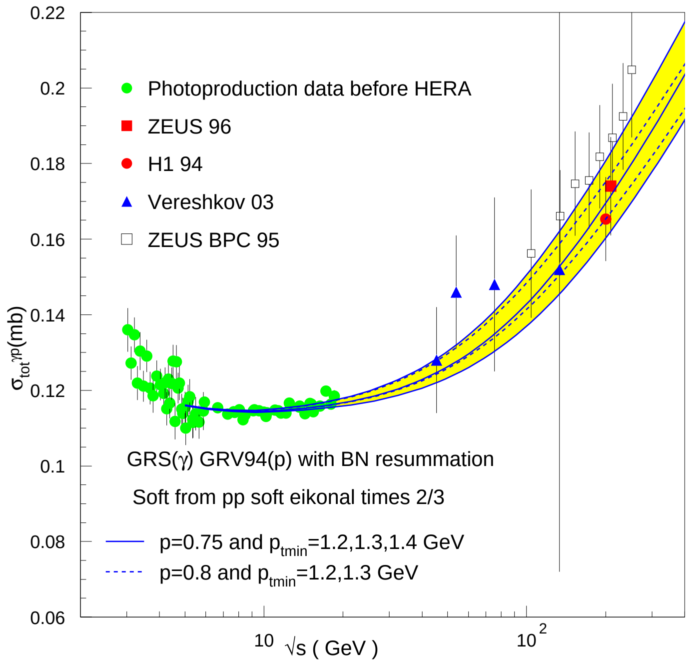
$$\sigma_{tot}^{\gamma p} = 2P_{had} \int d^2\vec{b}[1 - e^{-n^{\gamma p}(b,s)/2}]$$

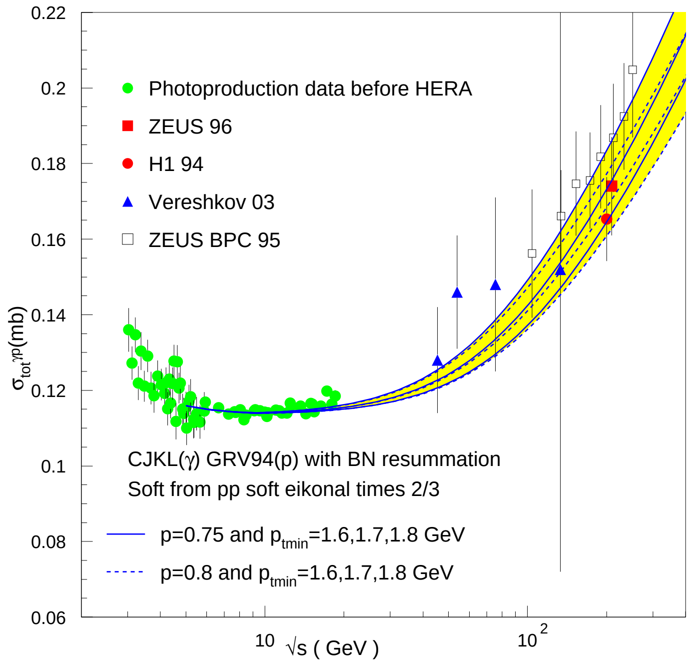
While now calculating $n^{\gamma p}$ one has to be careful. Involves now an additional parameter P_{had}

$$n^{\gamma p}(b,s) = n^{\gamma p}_{soft}(b,s) + n^{\gamma p}_{hard}(b,s) = n^{\gamma p}_{soft}(b,s) + A(b,s)\sigma^{\gamma p}_{jet}(s)/P_{had}$$

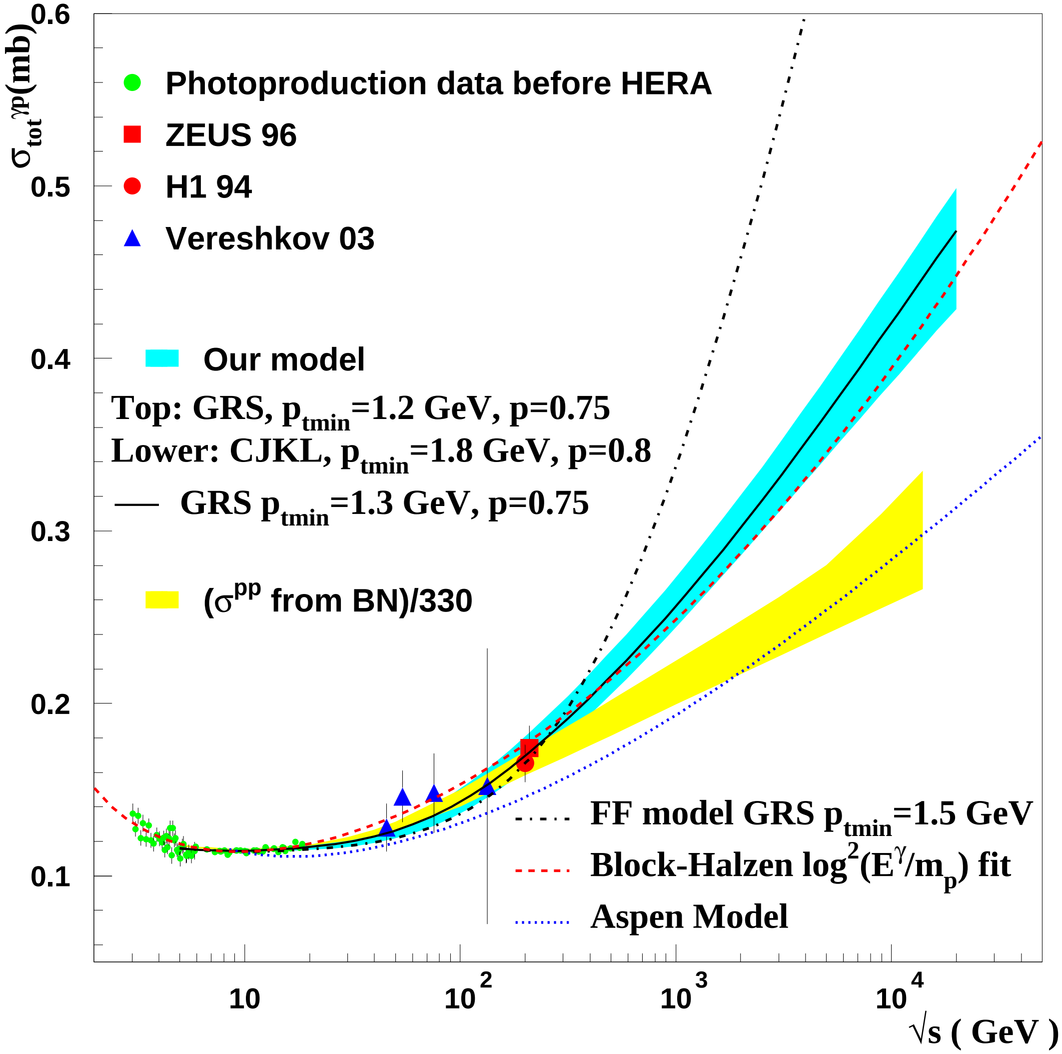
We now use $n_{hard}(b,s) = \frac{A_{BN}^{AB}(b,s)\sigma_{jet}}{P_{had}}$

$$A_{BN}^{AB}(b,s) = \mathcal{N} \int d^2\mathbf{K}_\perp \frac{d^2P(\mathbf{K}_\perp)}{d^2\mathbf{K}_\perp} e^{-i\mathbf{K}_\perp\cdot\mathbf{b}} = \frac{e^{-h(b,q_{max})}}{\int d^2\mathbf{b}e^{-h(b,q_{max})}} \equiv A_{BN}^{AB}(b, q_{max}(s))$$



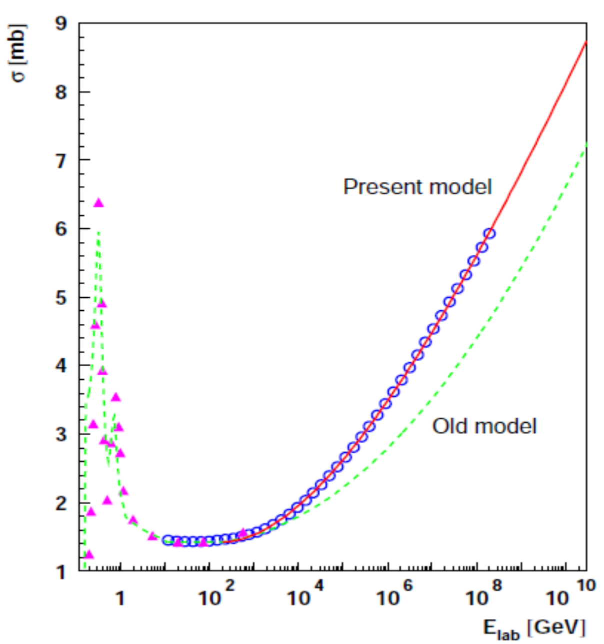


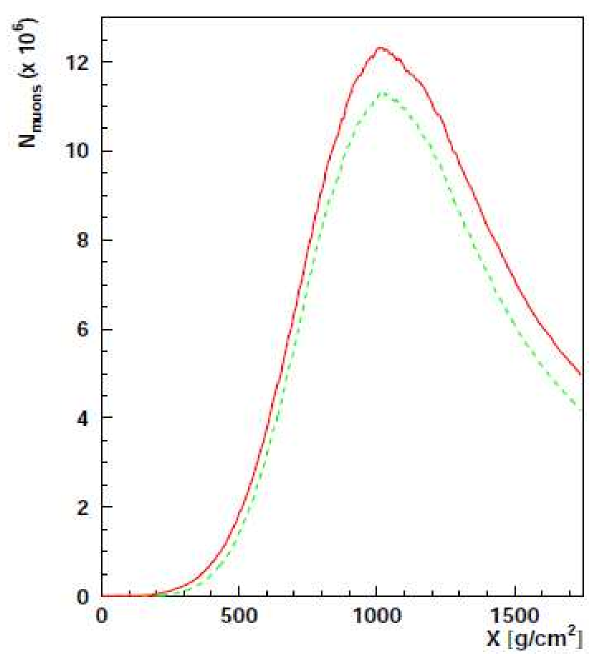
Predictions obtained using the photonic parton densities and parametes p, p_{tmin} and σ_{soft} similar to the proton case.



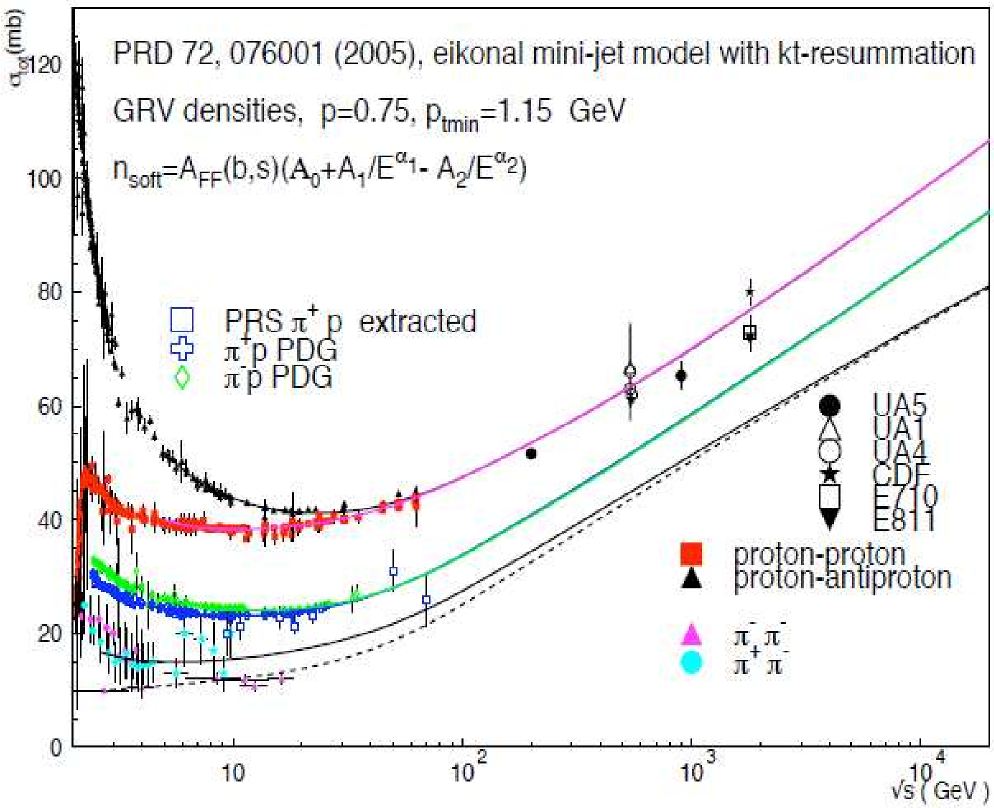
$\sigma_{tot}^{\gamma p}$(mb)
0.6
0.5
0.4
0.3
0.2
0.1
Photoproduction data before HERA
ZEUS 96
H1 94
Vereshkov 03
Our model
Top: GRS, p_{tmin}=1.2 GeV, p=0.75
Lower: CJKL, p_{tmin}=1.8 GeV, p=0.8
GRS p_{tmin}=1.3 GeV, p=0.75
(σ^{pp} from BN)/330
FF model GRS p_{tmin}=1.5 GeV
Block-Halzen $\log^2(E^\gamma/m_p)$ fit
Aspen Model
10
10^2
10^3
10^4
$\sqrt{s}$ (GeV)

F. Cornet et al, 31st ICRC proceedings. Simulation of air showers with AIRES and QGSJSET-II, for 10^{19} eV photon showers.





LHS: Comparison of the predictions of the BN model and the default form in AIRES. RHS: Difference for the longitudinal muon showers due to two models.



A. Grau, G. Pancheri, O. Shekhovtsova, Y. Srivastava: PLB 693 (2010) 456. πp cross-sections available only for lower energies and $\pi\pi$ cross-sections even the rise is not seen. BN models predicts high energy behaviour which may be measured at LHC using ZDC.

Petrov, Ryutin, Sobol: EPJC 65 (2010) 637, hep-ph/1008.4469,hep-ph/1005.2984.

• Soft gluon resummation effects tame the high energy rise of unitarised minijet cross-sections and restore the Froissart bound.

• The BN Eikonalised model predicts LHC cross-sections consistent with other fits which were done requiring analticity and unitarity properties.

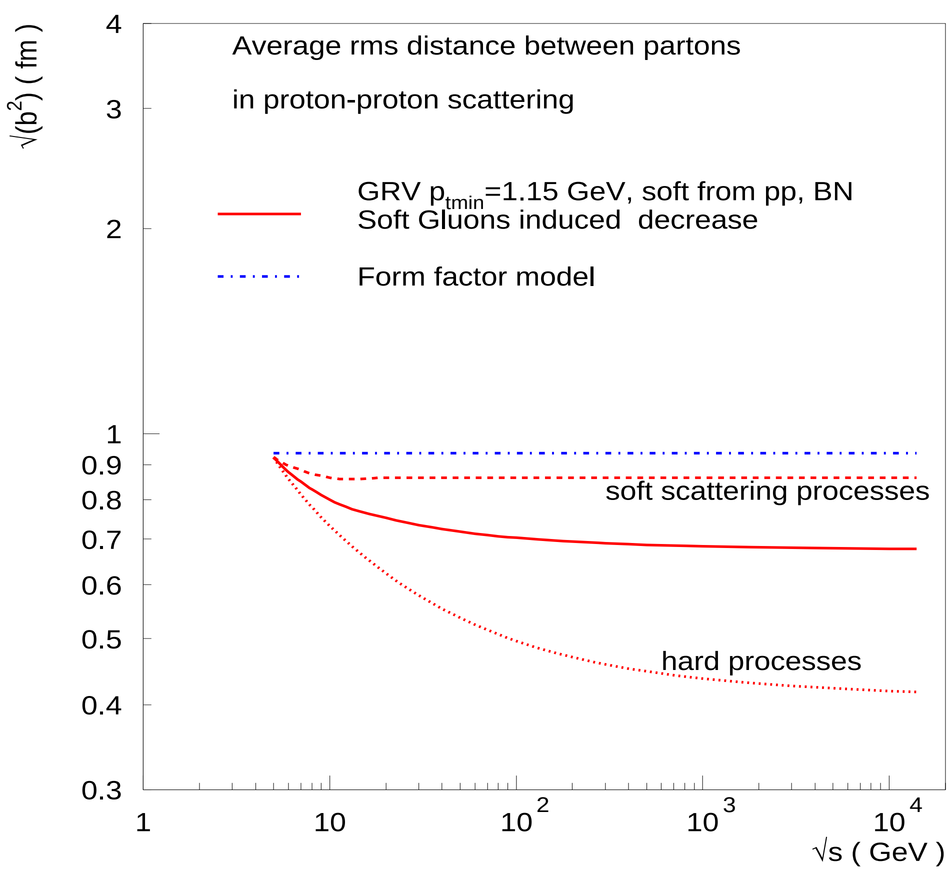
• In particular the predicitons of our model for *both* the pp and γp case agree with those of Block Halzen fits where they argue saturation of Froissart bound.

• This in fact means a violation of simple factorisation in going from p to γp and further $\gamma\gamma$.

• Extension of the pp total cross-section predictions to *very* high energies requires use of parton density parametrisations valid to very small x, using (say) HERA data augmented by LHC(?)

- Some simulations to esitmate effect on air showers induced by high energy photons has been started.

- Model extended to pion proton cross-section. Can this be tested at LHC with expts with ZDC?



- Indeed the rms distance between the centres of two hadrons decreases with energy causing more shadowing and taming the rise
- Similar observation by M. Seymore and collab. from a study of properties of the events in $p\bar{p}$ data from CDF in an eikonal picture.

Can one marry (hence test at LHC) this model of size of proton with simulation of underlying events?

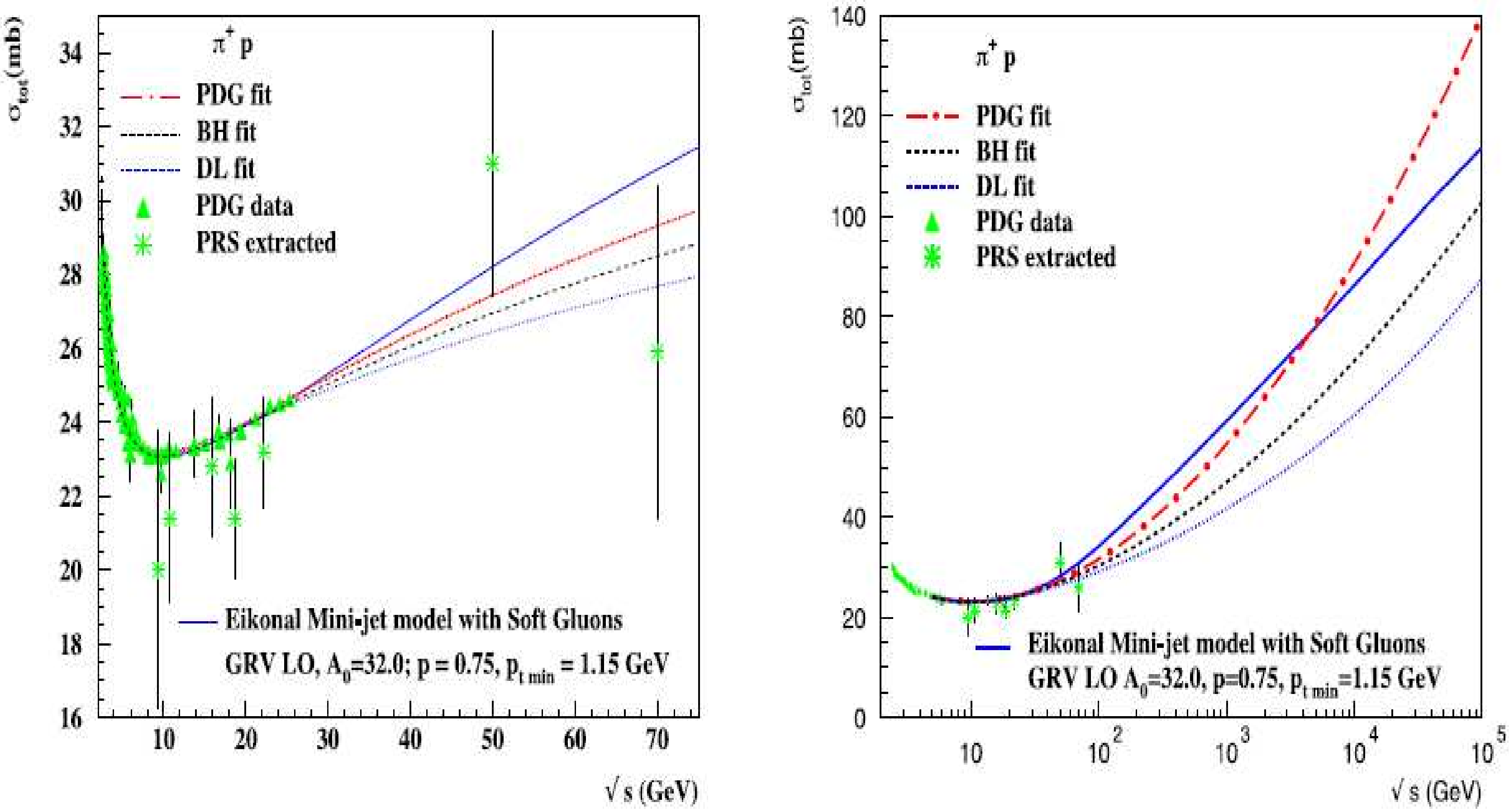
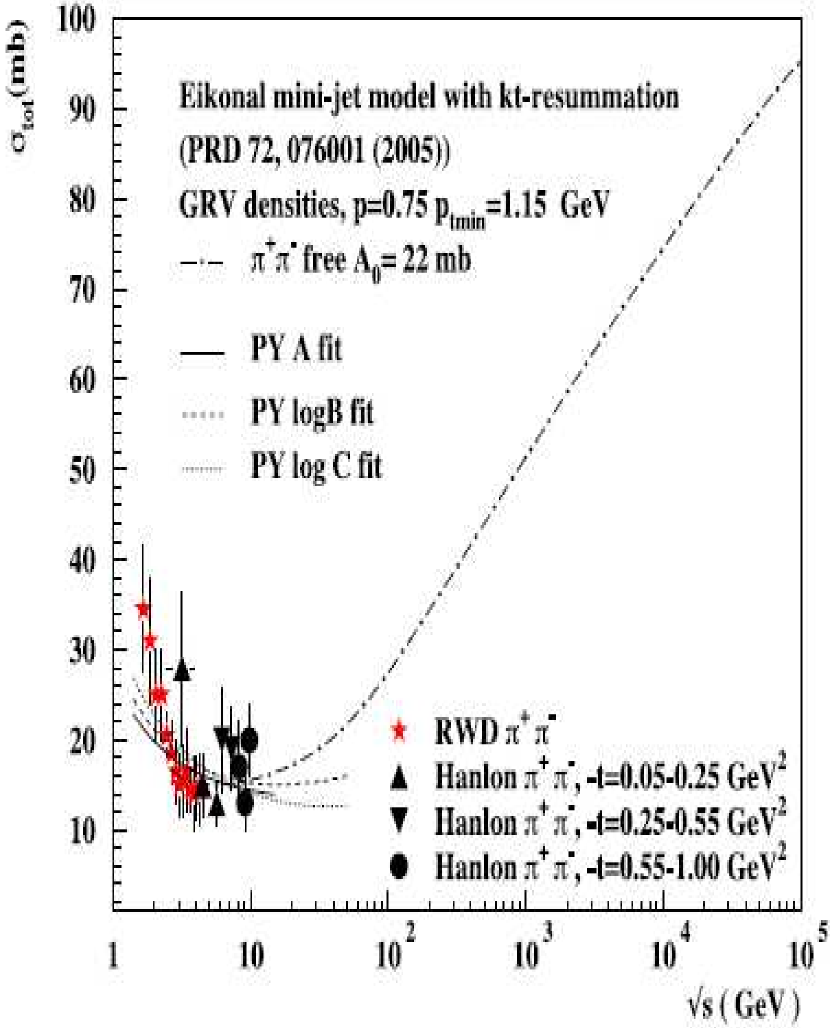
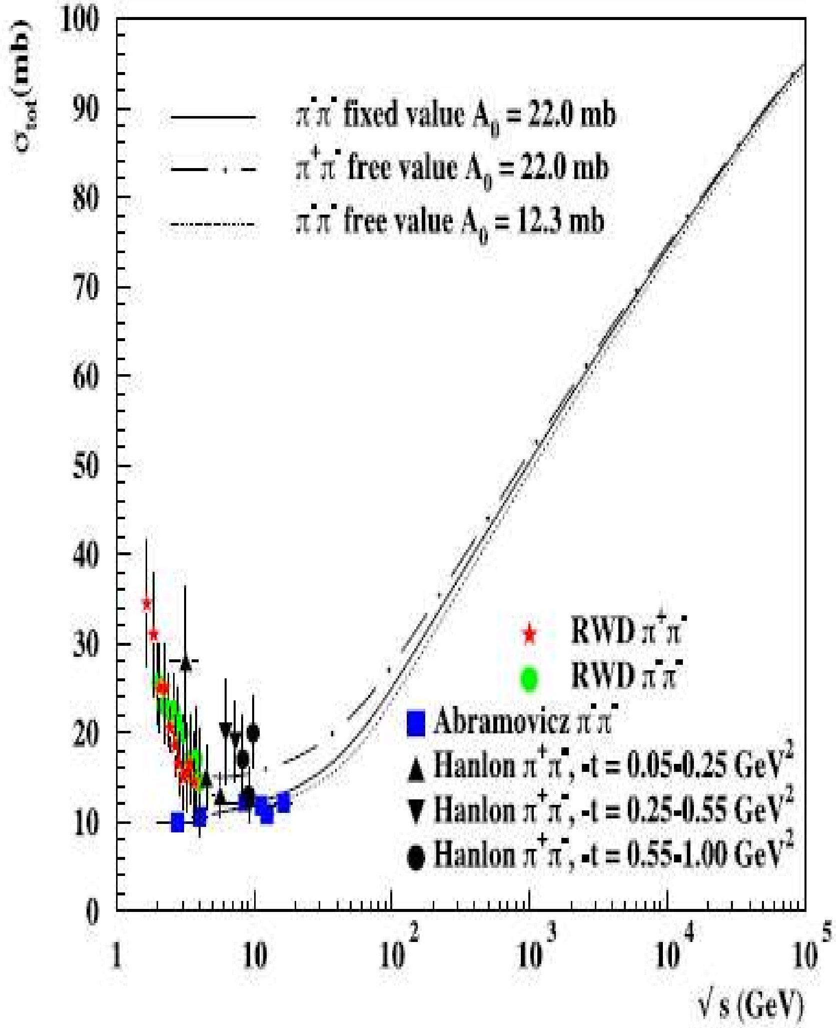


Fig. 2. In the left panel we show the fit for π^+p channel in the low energy region when the parameter A_0 is fixed to the value $2/3\,A_0^{pp}$, and with values of the low energy parameters as in Table 2. At right, we show the full energy range of interest.


σtot(mb)
Eikonal mini-jet model with kt-resummation
(PRD 72, 076001 (2005))
GRV densities, p=0.75 ptmin=1.15 GeV
π+π- free A0= 22 mb
PY A fit
PY logB fit
PY log C fit
RWD π+π-
Hanlon π+π-, -t=0.05-0.25 GeV2
Hanlon π+π-, -t=0.25-0.55 GeV2
Hanlon π+π-, -t=0.55-1.00 GeV2
√s (GeV)


σtot(mb)
π-π- fixed value A0 = 22.0 mb
π+π- free value A0 = 22.0 mb
π-π- free value A0 = 12.3 mb
RWD π+π-
RWD π-π-
Abramovicz π-π-
Hanlon π+π-, -t = 0.05-0.25 GeV2
Hanlon π+π-, -t = 0.25-0.55 GeV2
Hanlon π+π-, -t = 0.55-1.00 GeV2
√ s (GeV)

Aspen Model (BGHP model): QCD inspired model. Use analyticity, unitarity, factorisation etc. Fit the eikonal functions. Overlap function calculated using form factors. M. M. Block, E. M. Gregores, F. Halzen and G. Pancheri, Phys. Rev. D D60 (1999) 054024,

Luna-Menon Model: Dynamical gluon mass used. A further variant of the BGHP model. E. G. S. Luna, A. F. Martini, M. J. Menon, A. Mihara and A. A. Natale, Phys. Rev. D72 (2005) 034019.

Based on Colour Glass condensate Model F. Carvallo et al, 0705.1842 also try to use the gluon saturation phenomena to tone down the fast rise!