Studies of Charged Particle EDM measurement proposal

Malek Haj Tahar and Christian Carli
CERN, Geneva, Switzerland

ABP Group Information Meeting,
September 19th, 2019
Overview

- Original motivation and proposal (all Electric, sensitivity $10^{-29} \text{e.cm}$).
- Proposal: storage ring based EDM search
- Key challenges and systematic errors
- Conclusion
Motivation

- The Electric Dipole Moment (EDM) of a fundamental particle is a measure of the permanent separation of positive and negative electrical charges within the particle volume.

- Fundamental property of particles (like charge, mass, magnetic moment).

- EDM exists only via violations of time reversal and parity symmetry and is aligned with the particle spin.

- Standard model expects EDM on the order of $10^{-38} \text{e.cm}$ which is too weak to explain the matter-antimatter asymmetry.

⇒ EDM is one of the few low energy measurements sensitive to fundamental particle physics at a scale of few TeV and above.
Spin and polarization

- The spin of a particle is inherently quantum mechanical in nature. It is a vector quantity with a component defined in a specific direction. For fermions, its value is half-integer and for bosons it is integer.

- For instance the spin of the proton has only two possible orientations, therefore it is misleading to conjure an image of the particle as a small spinning object.

- In accelerator physics, the observable is the average spin of the beam distribution, also defined as the polarization vector $P$ in the following way:

$$ P = \langle s \rangle = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow} $$

$N_\uparrow$ and $N_\downarrow$ are the number of particles with spins parallel and antiparallel to a specific direction.
The Thomas Bargman-Michel-Telegdi (T-BMT) equation gives the precession rate of the angle between the spin and momentum vectors of a relativistic particle in the presence of electromagnetic fields:

\[ \frac{dP}{dt} = (\Omega_{BMT} + \Omega_{EDM}) \times P \]

where the precession vector due to the particle’s Magnetic Dipole Moment (MDM) is:

\[ \Omega_{BMT} = -\frac{e}{mc} \left[ \left( a + \frac{1}{\gamma} \right) cB - \frac{a\gamma c}{\gamma + 1} (\beta \cdot B)\beta - \left( a + \frac{1}{\gamma + 1} \right) \beta \times E \right] \]

and the precession vector due to the particle’s Electric Dipole Moment (EDM) is:

\[ \Omega_{EDM} = -\frac{e}{mc} \frac{\eta}{2} \left[ E - \frac{\gamma}{\gamma + 1} (\beta \cdot E)\beta + c\beta \times B \right] \]

The spin is defined in the inertial rest frame of the particle while the electromagnetic field vectors are expressed in the laboratory frame. The MDM is usually very well known.
Thomas-BMT equation in non inertial frame

- The accelerator frame is, by definition, a non-inertial frame. In a storage ring, it can be shown that the T-BMT equation transforms into:

\[
\frac{dP}{dt} = (\Omega_{BMT} - \Omega_{cyc} + \Omega_{EDM}) \times P
\]

where \( \Omega_{cyc} \) is the cyclotron angular frequency describing the rotation of the coordinate system and \( P \) is the polarization vector projected in such a frame.

- The idea is to maximize the EDM signal by minimizing the MDM contribution to the spin buildup. Such a condition is set when:

\[
\Omega_{BMT} - \Omega_{cyc} = 0
\]

- For protons, such a condition is fulfilled for an all-electric storage ring for a specific energy, the so called magic energy corresponding to \( E_{\text{kin}} = 232.8 \text{ MeV} \) and the lattice is referred to as the frozen spin lattice.

There is an on-going discussion among the members of the CPEDM collaboration regarding the choice of the coordinate system to properly describe the spin precession. \( \Omega_{BMT} - \Omega_{cyc} \) shall be interpreted with care.
Proposal: storage ring based EDM search

- The stored beam must be spin polarized. The best sensitivity involves rotations as small as micro-radians ⇒ begin with spins aligned along the particle momentum.

- For zero EDM, the beam energy fixes the ratio between magnetic and electric fields to keep the spin in the longitudinal direction ⇒ frozen spin lattice.

- An EDM generates a vertical spin component by coupling with radial E field.

- Difference in the scattering rate between the left and right directions is sensitive to the vertical polarization.

Simplified schematic of the ring: the polarization which is initially longitudinal precesses slowly in the vertical direction due to the radial electric field acting on the EMD.
The full scale ring lattice is the strong focusing lattice proposed by V. Lebedev and used for our studies.

The designed ring lattice requires electric gradient of 8 MV/m hence its circumference.

Weak vertical focusing to maximize the separation between the two counter-rotating beams which is a probe for radial magnetic field imperfections.

Operation below transition reduces IBS growth rates at the thermal equilibrium.

<table>
<thead>
<tr>
<th>parameter</th>
<th>symbol</th>
<th>unit</th>
<th>full scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>bending radius</td>
<td>( r_0 )</td>
<td>m</td>
<td>52.3</td>
</tr>
<tr>
<td>electrode spacing</td>
<td>( g )</td>
<td>cm</td>
<td>3</td>
</tr>
<tr>
<td>electrode height</td>
<td>( d )</td>
<td>cm</td>
<td>20</td>
</tr>
<tr>
<td>deflector shape</td>
<td>( m )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>electrode index</td>
<td>( E_0 )</td>
<td>Mv/m</td>
<td></td>
</tr>
<tr>
<td>radial electric field</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>number long straights</td>
<td>( l_{ss} )</td>
<td>m</td>
<td>4</td>
</tr>
<tr>
<td>long straights sec. leng.</td>
<td></td>
<td></td>
<td>20.8</td>
</tr>
<tr>
<td>polarimeter sections</td>
<td></td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>injection sections</td>
<td></td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>total circumference</td>
<td>( C )</td>
<td></td>
<td>500.0</td>
</tr>
<tr>
<td>harmonic number</td>
<td>( h )</td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>RF frequency</td>
<td></td>
<td></td>
<td>35.878</td>
</tr>
<tr>
<td>number of bunches</td>
<td></td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>particles per bunch</td>
<td></td>
<td></td>
<td>2.5e8</td>
</tr>
<tr>
<td>mom. spread(not/cooled)</td>
<td></td>
<td></td>
<td>±5e-4/1e-4</td>
</tr>
<tr>
<td>max. horz. beta func.</td>
<td>( \beta_{x,\text{max}} )</td>
<td>m</td>
<td>47</td>
</tr>
<tr>
<td>max. vert. beta func.</td>
<td>( \beta_{y,\text{max}} )</td>
<td>m</td>
<td>216</td>
</tr>
<tr>
<td>dispersion</td>
<td>( D )</td>
<td>m</td>
<td>29.5</td>
</tr>
<tr>
<td>horizontal tune</td>
<td>( Q_x )</td>
<td></td>
<td>2.42</td>
</tr>
<tr>
<td>vertical tune</td>
<td>( Q_y )</td>
<td></td>
<td>0.44</td>
</tr>
<tr>
<td>horiz. emit.(not/cool)</td>
<td>( \epsilon_x )</td>
<td>mm-mr</td>
<td>3.2/3</td>
</tr>
<tr>
<td>vert. emit.(not/cool)</td>
<td>( \epsilon_y )</td>
<td>mm-mr</td>
<td>17/3</td>
</tr>
<tr>
<td>slip-factor</td>
<td>( \eta )</td>
<td></td>
<td>-0.192</td>
</tr>
</tbody>
</table>
Spin simulations

- Limited number of codes that can perform beam tracking simulations in electrostatic elements.
  - Even so, not so many codes perform spin tracking. Our numerical simulations so far are based on BMAD code developed by D. Sagan.

- Benchmarking is very delicate: the aimed sensitivity of the experiment is equivalent to detecting a spin buildup of $1.6 \text{ nrad/s}$ or $4.4 \times 10^{-15} \text{ rad/turn}$.
  - Objective: understand the systematic imperfections that can mimic an EDM signal to be measured.

- The spin precession equation can be written in the matrix form $\frac{dP}{dt} = M(t)P(t)$
  - This is a first order linear differential equation with periodic coefficients for which in general a closed form solution is not known. $M(t)$ is a skew-symmetric matrix and the problem is formulated as follows:

\[
\begin{pmatrix}
\frac{dP_r}{dt} \\
\frac{dP_y}{dt} \\
\frac{dP_l}{dt}
\end{pmatrix} =
\begin{pmatrix}
0 & -\Omega_l(t) & \Omega_y(t) \\
\Omega_l(t) & 0 & -\Omega_r(t) \\
-\Omega_y(t) & \Omega_r(t) & 0
\end{pmatrix}
\begin{pmatrix}
P_r \\
P_y \\
P_l
\end{pmatrix}
\]

- Can we solve explicitly for $P_y(t)$? (another popular approach relies on the transfer matrices formalism but non trivial in particular to account for fringe fields).
It can be shown that the second order approximation of the vertical polarization buildup is given by:

\[ P_{y,2}(t) = \xi_{y,2}(t) + \phi_{y,2}(t) \]

where

\[ \xi_{y,2}(t) = -\langle \Omega_r \rangle t + \langle \Omega_l \tilde{\Omega}_y \rangle t - \langle \Omega_y \rangle \langle \tilde{\Omega}_l \rangle t + \frac{\langle \Omega_y \rangle \langle \Omega_l \rangle}{2} t^2 \]

and

\[ \phi_{y,2}(t) = -\tilde{\Omega}_r(t) + \Omega_l(t)\tilde{\Omega}_y(t) + \langle \Omega_y \rangle \left[ t\tilde{\Omega}_l(t) - \tilde{\Omega}_l(t) \right] \]

\[ \tilde{\Omega}_i(t) = \int_0^t [\Omega_i(\tau) - \langle \Omega_i \rangle] d\tau ; \quad i = r, y, l \]

\( \xi_{y,2}(t) \) is the frozen spin solution, i.e. the vertical polarization signal measured at the location of the polarimeter.

Radial magnetic field imperfection:

\[ \frac{dP_y}{dt} = -\langle \Omega_r \rangle = -1.71 \times 10^8 \langle B_x \rangle \]

so that 10 aT of average radial magnetic field yields the same aimed EDM signal of ~ 1.6 nrad/s. CCW beams needed to remediate such an imperfection. Sensitivity needed: pm level.

Achieving \( \langle \Omega_r \rangle = \langle \Omega_y \rangle = \langle \Omega_l \rangle = 0 \) (while neglecting the EDM) is not sufficient to eliminate the imperfections since then \( \xi_{y,2}(t) = \langle \Omega_l \tilde{\Omega}_y \rangle t \) which accounts for the geometric phases (to the second order).

For more details see https://cds.cern.ch/record/2673218
Excellent agreement between the analytical approximation and the tracking simulations using 4th order Runge Kutta integrator.
Example of systematic errors: Geometric phases

- Due to the non-commutativity of spin rotations around different axes, spin buildup mimicking the EDM signal can occur even if $\langle \Omega_x \rangle = \langle \Omega_y \rangle = \langle \Omega_z \rangle = 0$. Such a contribution is generally referred to as geometric phase or Berry phase:

A vertical magnetic field yields a horizontal spin component which is rotated into the vertical plane by means of a longitudinal field component.

This is a second order systematic error which (in some cases) can be eliminated by employing counter-rotating beams.
Conclusion

- Studies of systematic imperfections are still on-going. The main question that remains to be answered is: what is the achievable EDM signal that can be measured?

- The convergence of the numerical simulations is proved for the cases considered so far and are explained by the second order BKM method of averages.

- A CERN yellow report has been drafted by the CPEDM collaboration and will be released soon.
Thank you

“I think I can safely say that nobody understands quantum mechanics... Nobody knows how it can be like that.”

Richard Feynman