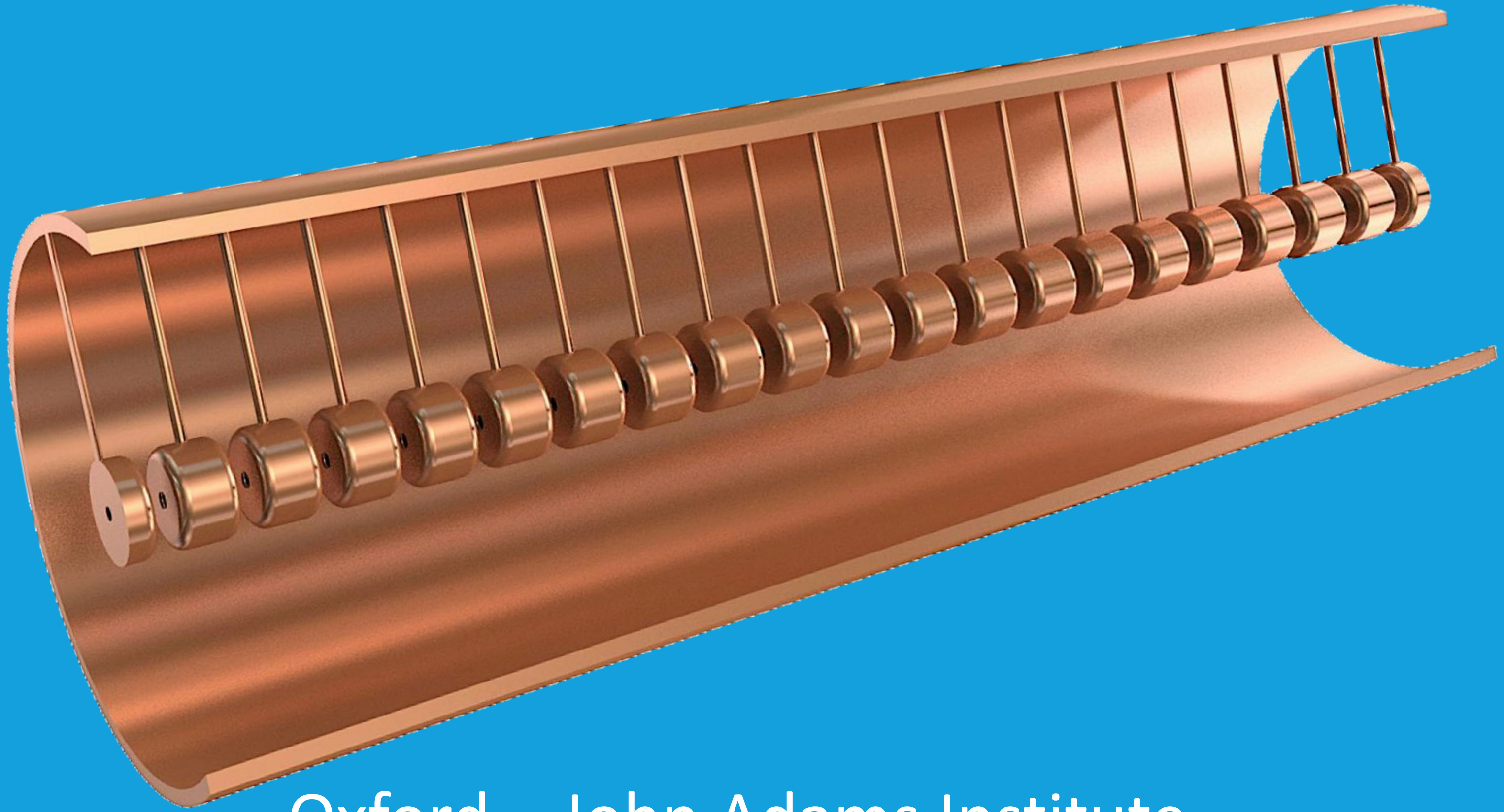
RF Cavity Design
- an introduction -



Oxford – John Adams Institute

21 November 2019

Ciprian Plostinar

Lecture Structure

- RF Cavity Design
  - Design Criteria
  - Figures of Merit
- Introduction to Superfish (2D)
- Examples:
  - Pill-box type cavity
  - DTL type cavity
  - Elliptical cavity
  - A ferrite loaded cavity
- Study of a simple cavity model
- A small RF measurement
  - (If equipment is available)

- Later in Hilary Term:

- CST MicroWave Studio Demo (3D)
  - Open to all
  - Project work

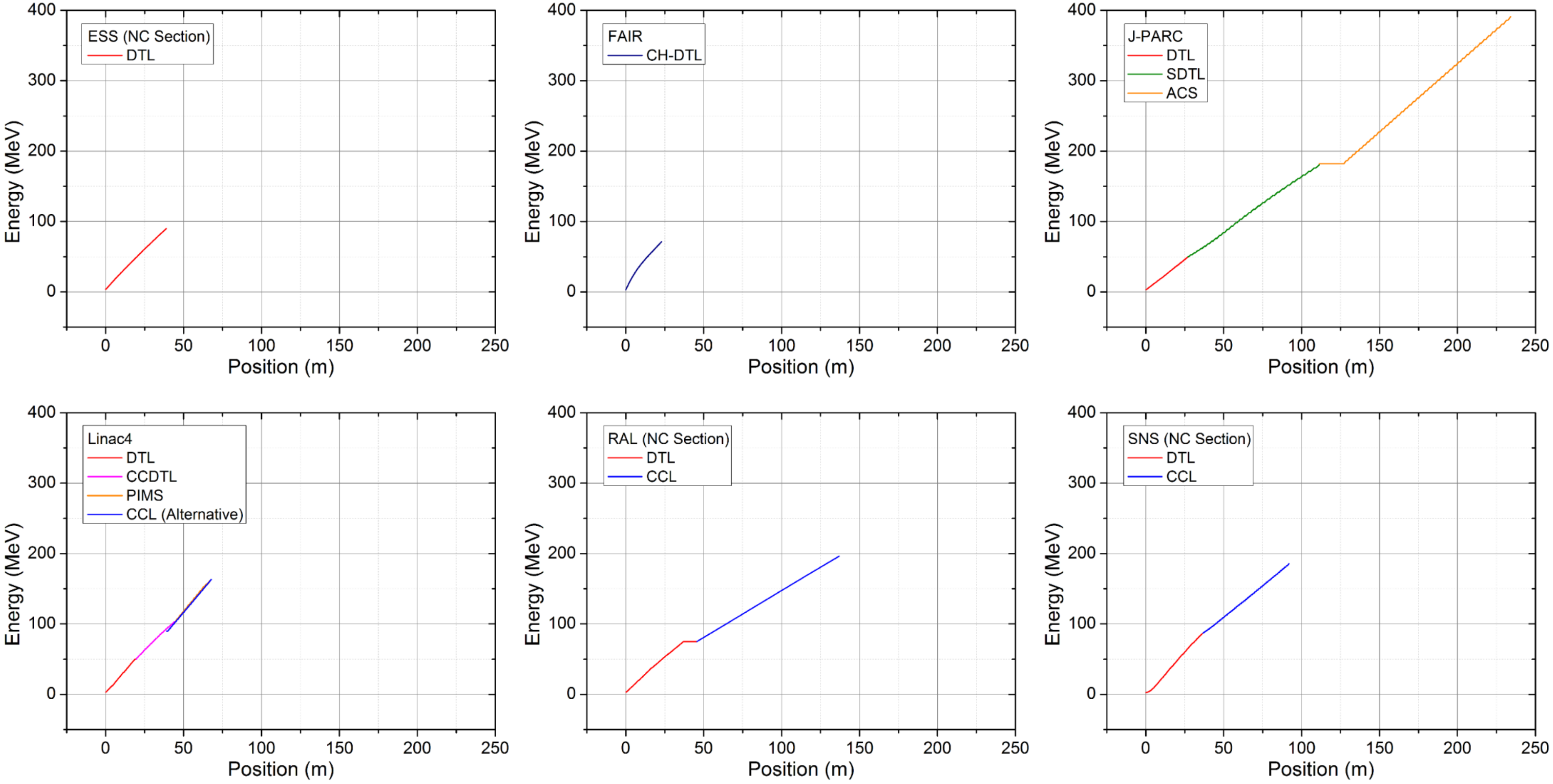
RF Cavity Design

- the basics -

- In most particle accelerators, the energy is delivered to the particle by means of a large variety of devices, normally know as cavity resonators.
- The ideal cavity: volume of perfect dielectric limited by infinitely conducting walls.
- Hollow cylindrical resonator excited by a radio transmitter -> standing wave -> accelerating fields (the pillbox cavity).

Why Cavity Design Is Important?

Acceleration Profile in Several Linacs



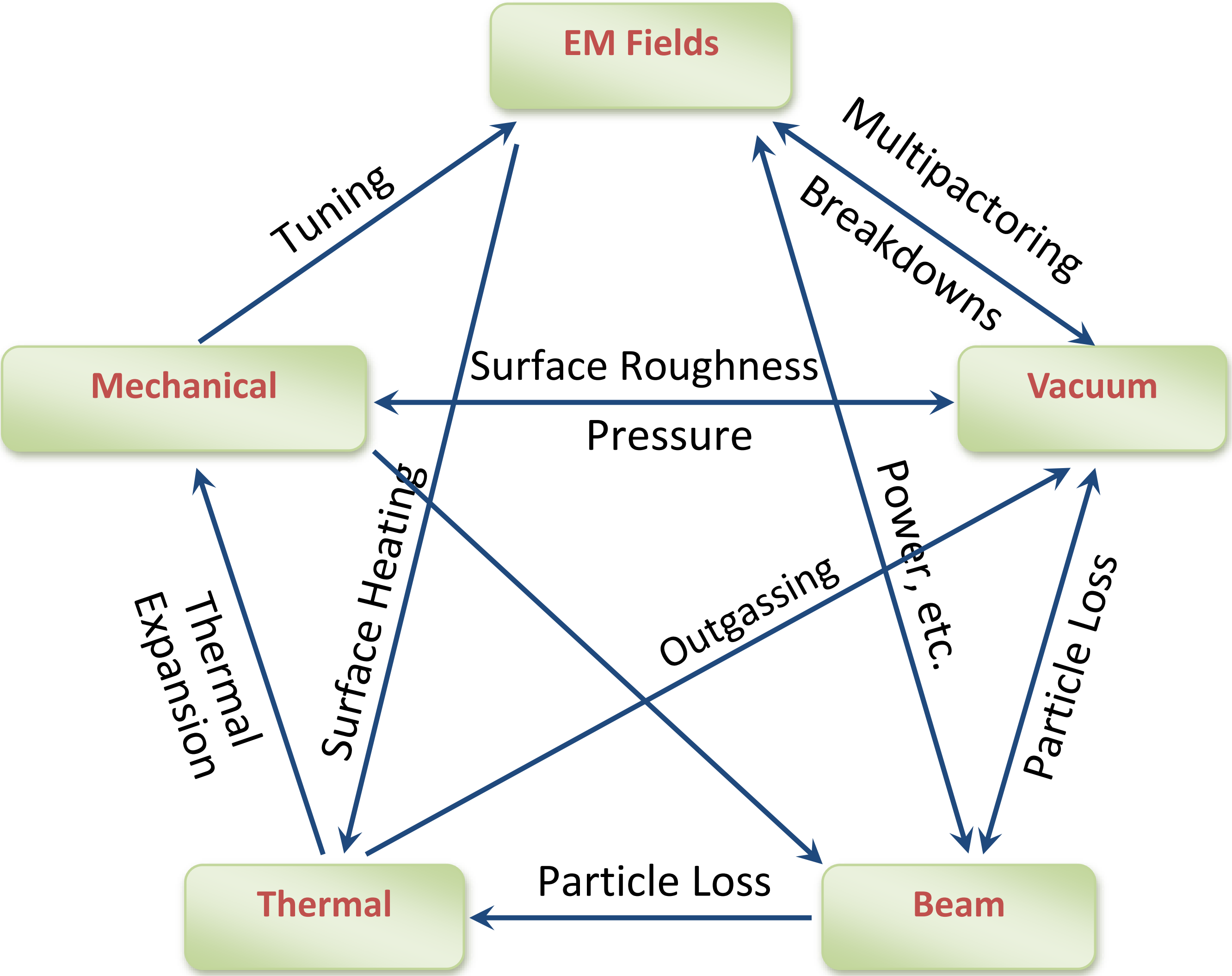
Design Process and Criteria

- Define the requirements:
  - Intended application
  - RF frequency
  - NC/SC
  - Voltage
  - Tuning
  - Etc.

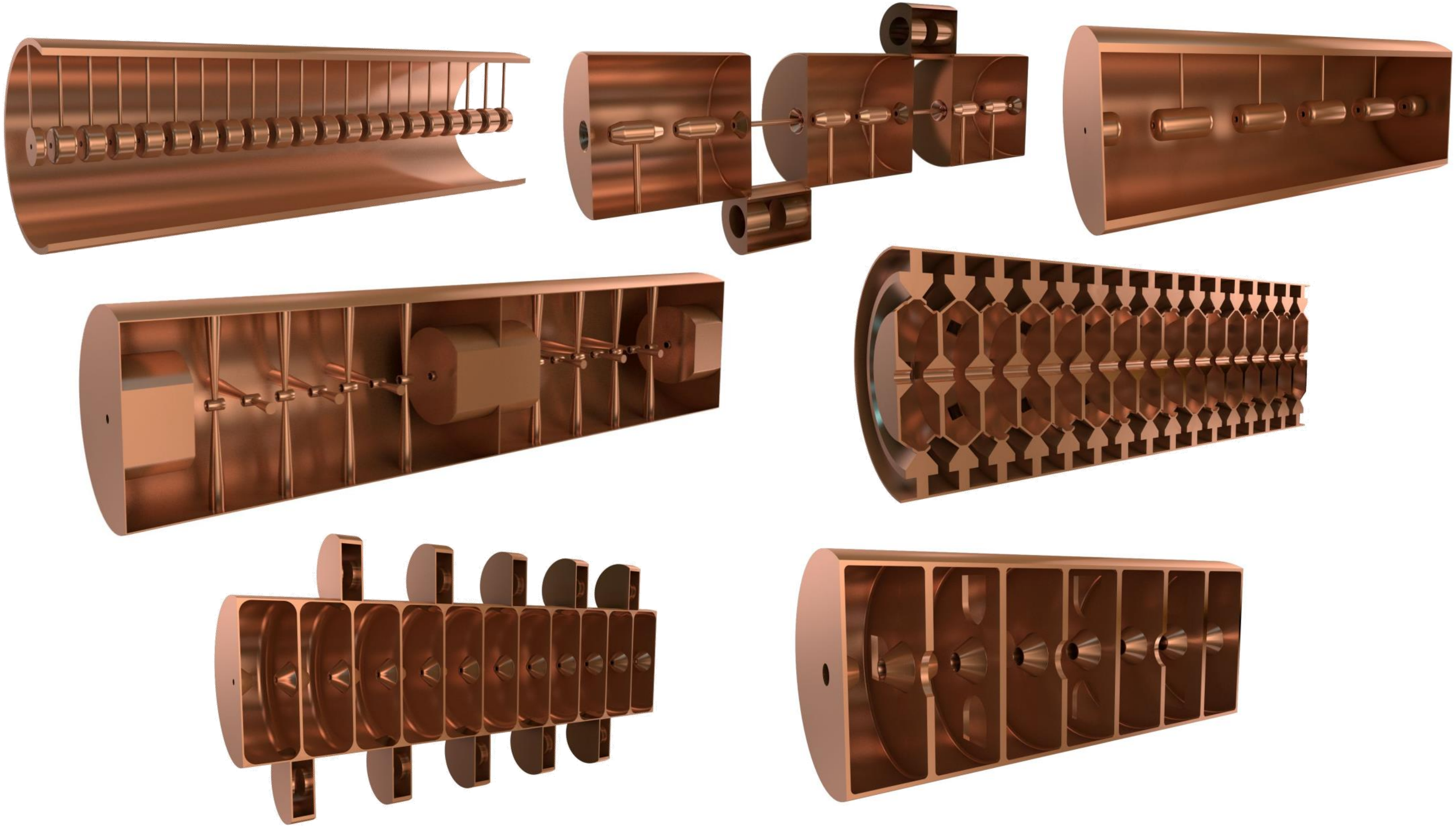
- General design criteria:
  - Power Efficiency & RF Properties
  - Beam Dynamics considerations (control of loss and emittance growth, etc.) – especially true for linacs
  - Technologies and precisions involved
  - Tuning procedures (frequency, field profile, stability against perturbations)
  - Sensitivity to RF errors (phase and amplitude)
  - Etc.

The “Magic Pentagon” of Cavity Design

Interdependent Technologies



Cavity “Zoo”

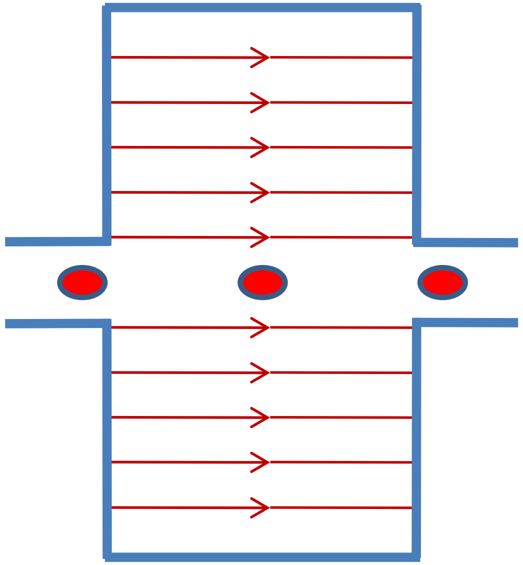


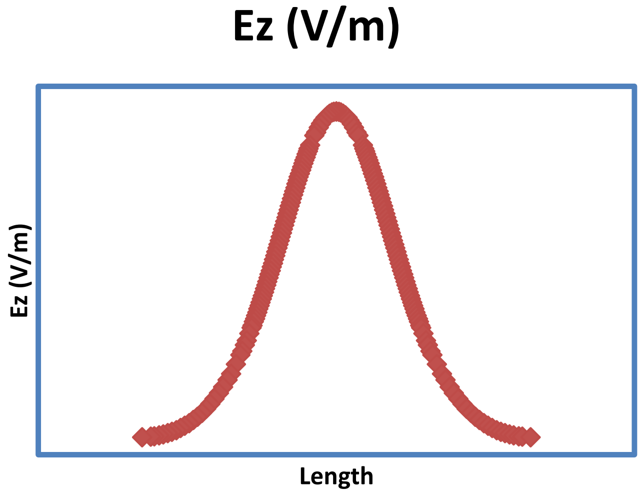
Figures of Merit

The Transit Time Factor, T

- While the particle crosses the cavity, the field is also varying
- The particle sees only a fraction of the peak voltage
- T is a measure of the reduction in energy gain cause by the sinusoidal time variation of the field in the cavity.

$$\boxed{T = \frac{\int_{-L/2}^{L/2} E(0,z)\cdot\cos\frac{2\pi z}{\beta\lambda}\,dz}{\int_{-L/2}^{L/2} E(0,z)\,dz}}$$





Figures of Merit
The Quality Factor, Q

- To first order, the Q-value will depend on the conductivity of the wall material only
- High Q -> narrower bandwidth -> higher amplitudes
- But, more difficult to tune, more sensitive to mechanical tolerances (even a slight temperature variation can shift the resonance)
- Q is dimensionless and gives only the ratios of energies, and not the real amount of power needed to maintain a certain resonant mode
- For resonant frequencies in the range 100 to 1000 MHz, typical values are 10,000 to 50,000 for normal conducting copper cavities; 10^8 to 10^{10} for superconducting cavities.

$$\boxed{Q_0 = \frac{2\pi \cdot stored\ energy}{energy\ consumed\ \ per\ \ period} = \frac{2\pi W}{TP_0} = \omega \frac{U}{P_0}}$$

Figures of Merit

Shunt Impedance

- A measure of the effectiveness of producing an axial voltage V_0 for a given power dissipated
- Typical values of ZT^2 for normal conducting linacs is 30 to 50 MΩ/m. The shunt impedance is less relevant for superconducting cavities.

$$\boxed{r_s = \frac{V_0^{\,2}}{P_0}}$$

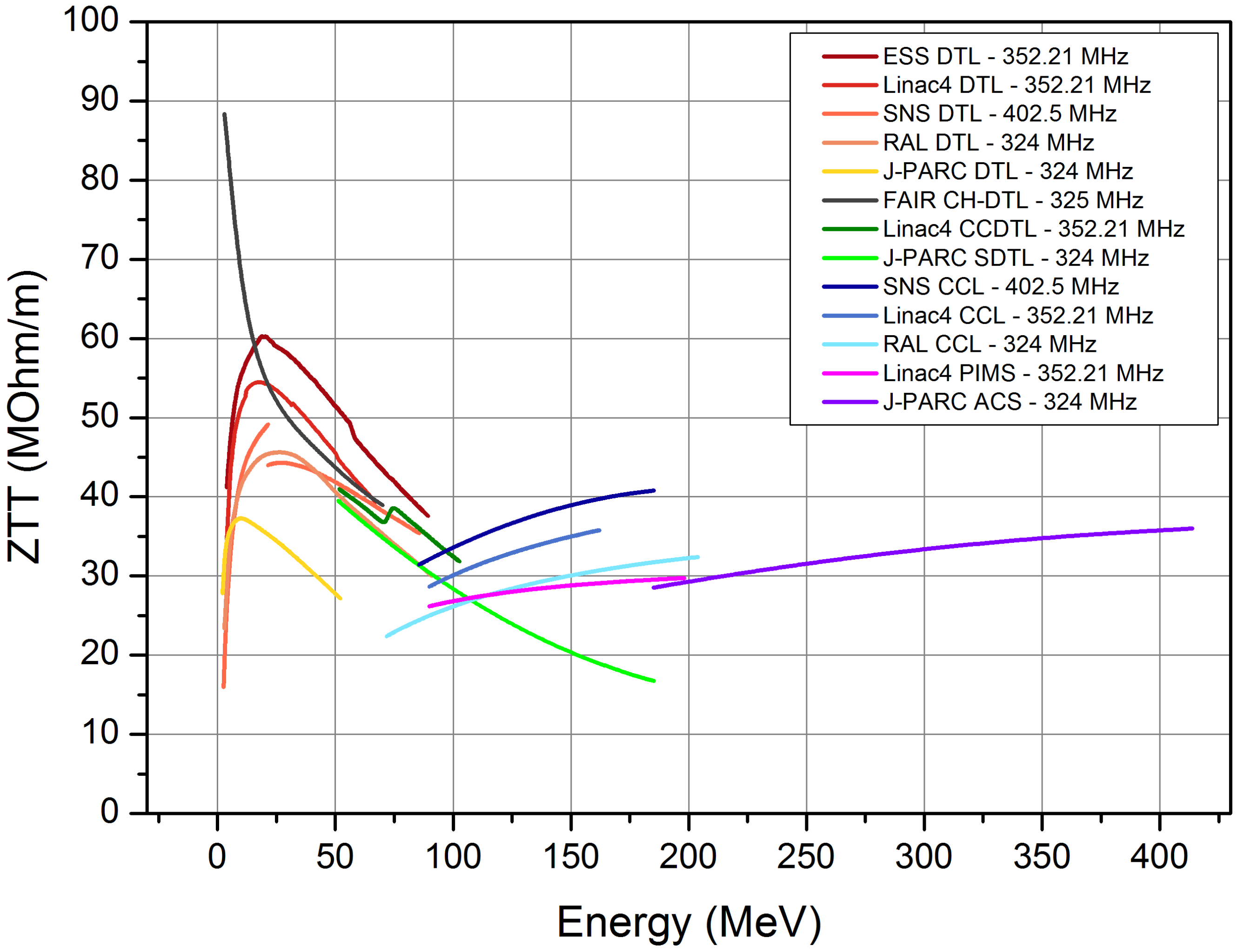
Shunt Impedance

$$\boxed{ZT^2 = \frac{r}{L} = \frac{(E_0 T)^2}{P_0 / L}}$$

Effective Shunt Impedance
per unit length

Figures of Merit

Shunt Impedance



Figures of Merit

r/Q

- Measures the efficiency of acceleration per unit of stored energy at a given frequency

- It is a function only of the cavity geometry and is independent of the surface properties that determine the power losses.

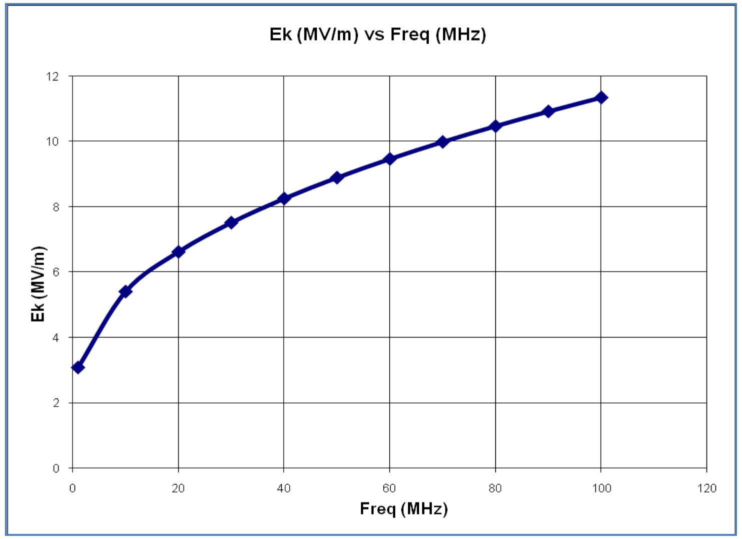
$$\boxed{\frac{r}{Q} = \frac{(V_0 T)^2}{\omega U}}$$

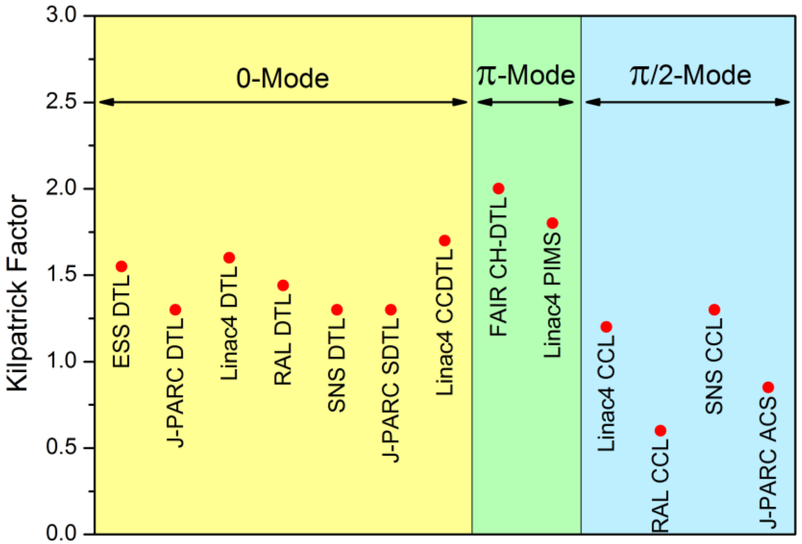
Figures of Merit

The Kilpatrick Factor

- High Field -> Electric breakdown
- Maximum achievable field is limited

$$f = 1.64 E_k^2 e^{-8.5/E_k}$$





Figures of Merit

The Kilpatrick Factor

SC Cavities

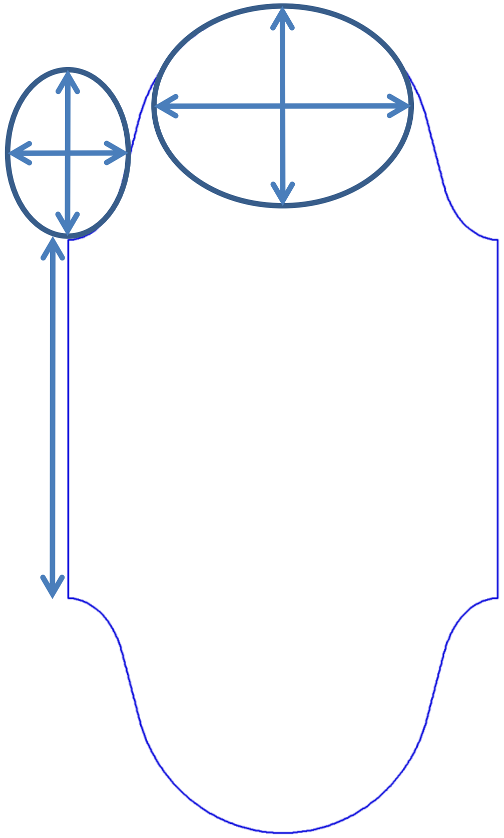
… some other factors to consider

- Epeak/Eacc – field emissions limit (Eacc limit)
- Bpeak/Eacc – quench limit (SC breakdown)
- G (Geometric Factor - the measure of energy loss in the metal wall for a given surface resistance)
- Higher Order Modes – manage and suppress HOM (e.g.: dipole modes can degrade the beam -> suppression scheme using HOM couplers)
- Multicell cavities: Field Flatness
- Kcc – Cell to cell coupling
- Etc.

SC Cavities

Basic design guidelines

- Optimise geometry to increase both r/Q and G resulting in less stored energy and less wall loss at a given gradient (low cryogenic losses)
- Optimise geometry to reduce Epeak/Eacc and Bpeak/Eacc
- Find optimum Kcc. (e.g.: a small aperture increases r/Q and G (!), but reduces Kcc. A small Kcc increases the sensitivity of the field profile to cell frequency errors.)



Introduction to Poisson Superfish

Before you start

- You will need a laptop running Windows. If you have Linux/MacOS install VMWare/Wine.
- Please download and install Poisson Superfish. To do this go to the following address and follow the instructions:
  - http://laacg.lanl.gov/laacg/services/download_sf.phtml
- Please download the example files to your computer from the JAI website.
- An extensive documentation can be found in the Superfish home directory (usually C:/LANL).
  - Have a look at the SFCODES.DOC file. Table VI-4 explains how the object geometry is defined in Superfish (page 157).
  - For a list of Superfish variables, see SFINTRO.doc, Table III-3 (page 76)
- For any questions, email Ciprian (ciprian.plostinar@esss.se) or Emmanuel (emmanuel.tsesmelis@cern.ch). Good luck!

Introduction to Poisson Superfish

The basics

- Poisson and Superfish are the main solver programs in a collection of programs from LANL for calculating static magnetic and electric fields and radio-frequency electromagnetic fields in either 2-D Cartesian coordinates or axially symmetric cylindrical coordinates.

- Finite Element Method

Introduction to Poisson Superfish

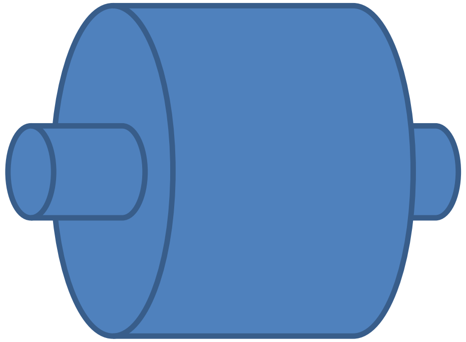
Solvers

- **Automesh** – generates the mesh (always the first program to run)
- **Fish** – RF solver
- **Cfish** – version of Fish that uses complex variables for the rf fields, permittivity, and permeability.
- **Poisson** – magnetostatic and electrostatic field solver
- **Pandira** – another static field solver (can handle permanent magnets)
- **SFO, SF7** – postprocessing
- **Autofish** – combines Automesh, Fish and SFO
- **DTLfish, DTLCells, CCLfish, CCLcells, CDTfish, ELLfish, ELLCAV, MDTfish, RFQfish, SCCfish** – for tuning specific cavity types.
- **Kilpat, Force, WSFPlot**, etc.

A Pillbox Cavity

The simplest RF cavity

- For the accelerating mode (TM_{010}), the resonant wavelength is:
  - independent of the cell length
- Example: a 40 MHz pillbox type cavity would have a diameter of ~ 5.7 m
- In the picture, CERN 88 MHz



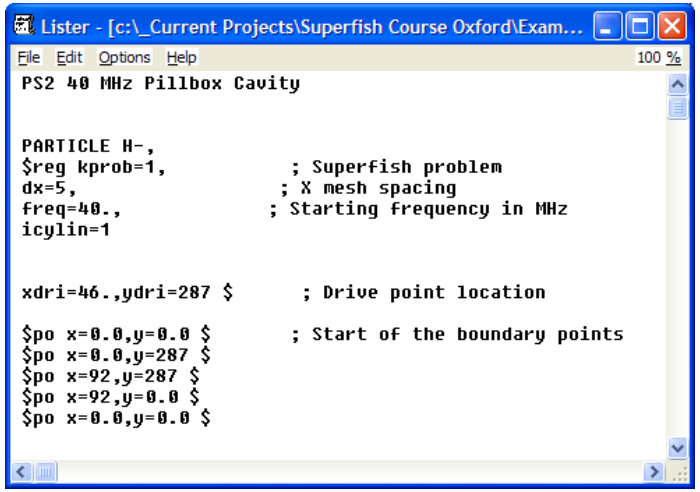
$$\boxed{\begin{array}{l}\lambda = \dfrac{\pi D}{x_1} \\ x_1 = 2.40483\end{array}}$$

x_1 - first root of the zero-th order Bessel function J_0 (x)

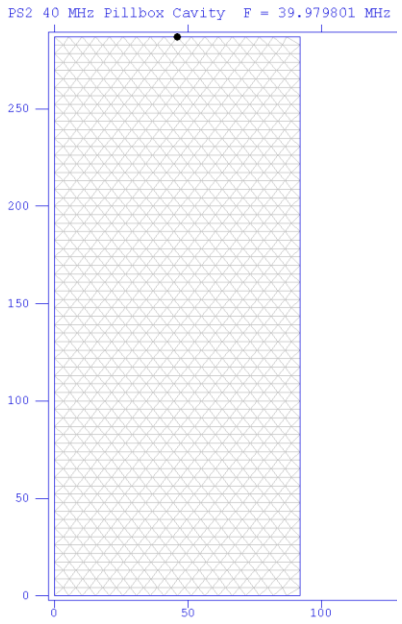
A Pillbox Cavity

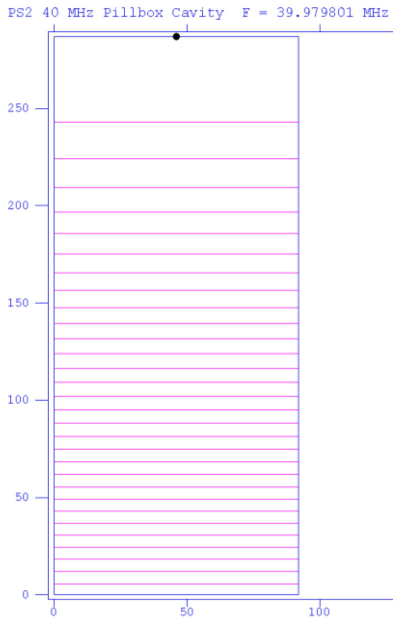
Superfish Implementation

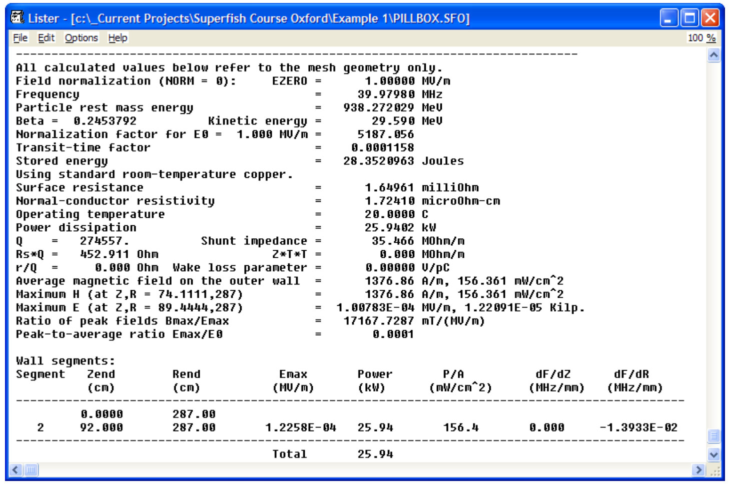
Superfish input file



```
PS2 40 MHz Pillbox Cavity


PARTICLE H-,
$reg kprob=1,             ; Superfish problem
dx=5,                    ; X mesh spacing
freq=40.,               ; Starting frequency in MHz
icylin=1


xdri=46.,ydri=287 $       ; Drive point location

$po x=0.0,y=0.0 $        ; Start of the boundary points
$po x=0.0,y=287 $
$po x=92,y=287 $
$po x=92,y=0.0 $
$po x=0.0,y=0.0 $
```



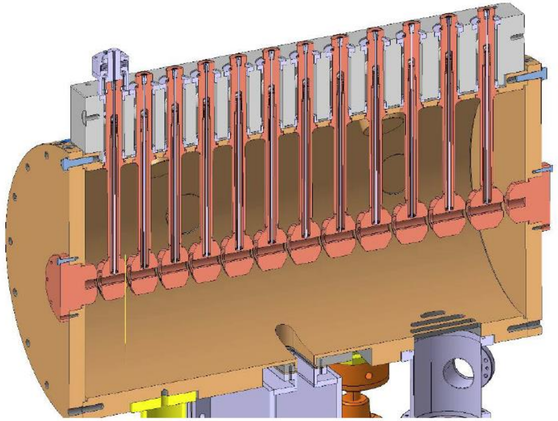




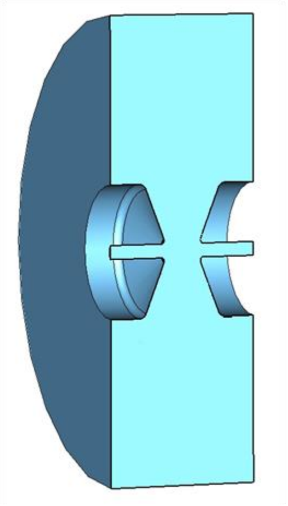
```
-------------------------------------------------------------------------------
All calculated values below refer to the mesh geometry only.
Field normalization (NORM = 0):     EZERO =      1.00000 MV/m
Frequency                                 =     39.97980 MHz
Particle rest mass energy                 =   938.272029 MeV
Beta =  0.2453792          Kinetic energy =       29.590 MeV
Normalization factor for E0 =  1.000 MV/m =     5187.056
Transit-time factor                       =    0.0001158
Stored energy                             =   28.3520963 Joules
Using standard room-temperature copper.
Surface resistance                        =      1.64961 milliOhm
Normal-conductor resistivity              =      1.72410 microOhm-cm
Operating temperature                     =      20.0000 C
Power dissipation                         =      25.9402 kW
Q    =   274557.          Shunt impedance =       35.466 MOhm/m
Rs*Q =   452.911 Ohm                Z*T*T =        0.000 MOhm/m
r/Q  =     0.000 Ohm  Wake loss parameter =      0.00000 V/pC
Average magnetic field on the outer wall  =      1376.86 A/m, 156.361 mW/cm^2
Maximum H (at Z,R = 74.1111,287)          =      1376.86 A/m, 156.361 mW/cm^2
Maximum E (at Z,R = 89.4444,287)          =  1.00783E-04 MV/m, 1.22091E-05 Kilp.
Ratio of peak fields Bmax/Emax            =   17167.7287 mT/(MV/m)
Peak-to-average ratio Emax/E0             =       0.0001

Wall segments:
Segment   Zend        Rend          Emax       Power      P/A        dF/dZ     dF/dR
          (cm)        (cm)          (MV/m)     (kW)     (mW/cm^2)   (MHz/mm)  (MHz/mm)
------------------------------------------------------------------------------------------
         0.0000      287.00
   2     92.000      287.00       1.2258E-04   25.94      156.4       0.000    -1.3933E-02
------------------------------------------------------------------------------------------
                                    Total      25.94
```

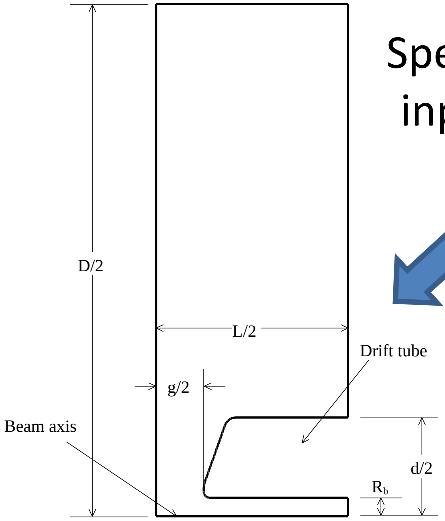
A Drift Tube Linac-type Cavity (DTL)

Basic Geometry

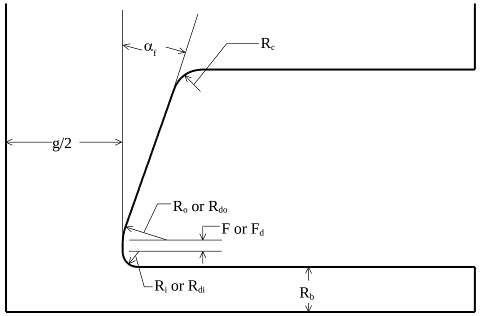


CERN Linac4 DTL prototype





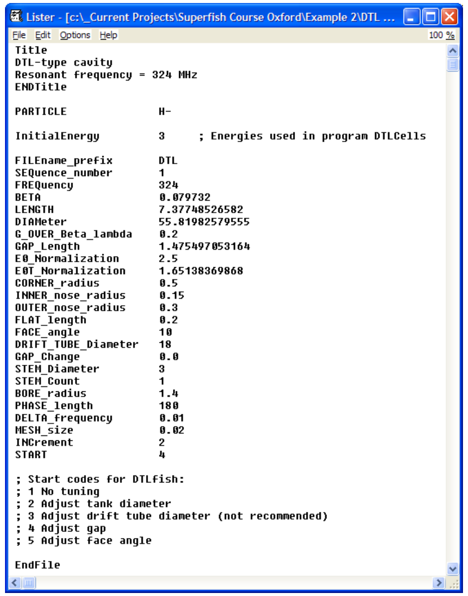
Special Superfish input geometry



A Drift Tube Linac-type Cavity (DTL)

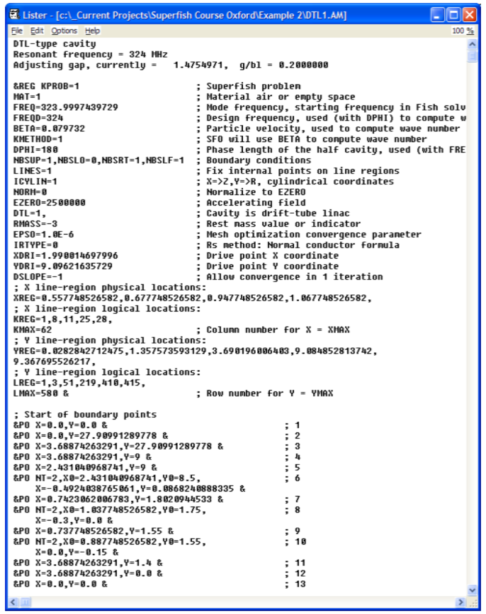
Superfish Implementation

Superfish input file



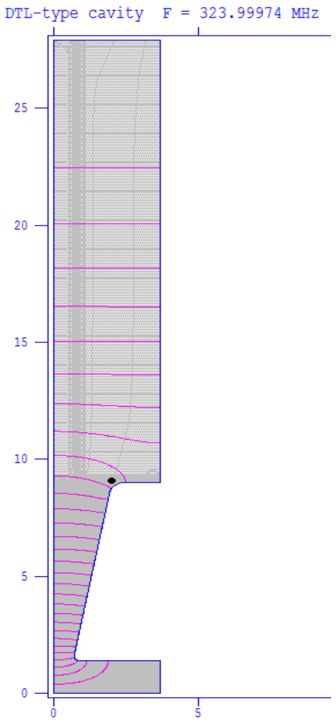
```
Title
DTL-type cavity
Resonant frequency = 324 MHz
ENDTitle

PARTICLE              H-

InitialEnergy         3     ; Energies used in program DTLCells

FILEname_prefix       DTL
SEQuence_number       1
FREQuency             324
BETA                  0.079732
LENGTH                7.37748526582
DIAMeter              55.81982579555
G_OVER_Beta_lambda    0.2
GAP_Length            1.475497053164
E0_Normalization      2.5
E0T_Normalization     1.65138369868
CORNER_radius         0.5
INNER_nose_radius     0.15
OUTER_nose_radius     0.3
FLAT_length           0.2
FACE_angle            10
DRIFT_TUBE_Diameter   18
GAP_Change            0.0
STEM_Diameter         3
STEM_Count            1
BORE_radius           1.4
PHASE_length          180
DELTA_frequency       0.01
MESH_size             0.02
INCrement             2
START                 4

; Start codes for DTLfish:
; 1 No tuning
; 2 Adjust tank diameter
; 3 Adjust drift tube diameter (not recommended)
; 4 Adjust gap
; 5 Adjust face angle

EndFile
```

Geometry file



```
DTL-type cavity
Resonant frequency = 324 MHz
Adjusting gap, currently =   1.4754971,  g/bl = 0.2000000

&REG KPROB=1                         ; Superfish problen
MAT=1                                ; Material air or empty space
FREQ=323.9997439729                  ; Mode frequency, starting frequency in Fish solv
FREQD=324                            ; Design frequency, used (with DPHI) to compute w
BETA=0.079732                        ; Particle velocity, used to compute wave number
KMETHOD=1                            ; SFO will use BETA to compute wave number
DPHI=180                             ; Phase length of the half cavity, used (with FRE
NBSUP=1,NBSLO=0,NBSRT=1,NBSLF=1      ; Boundary conditions
LINES=1                              ; Fix internal points on line regions
ICYLIN=1                             ; X=>Z,Y=>R, cylindrical coordinates
NORM=0                               ; Normalize to EZERO
EZERO=2500000                        ; Accelerating field
DTL=1,                               ; Cavity is drift-tube linac
RMASS=-3                             ; Rest mass value or indicator
EPSO=1.0E-6                          ; Mesh optimization convergence parameter
IRTYPE=0                             ; Rs method: Normal conductor formula
XDRI=1.990014697996                  ; Drive point X coordinate
YDRI=9.09621635729                   ; Drive point Y coordinate
DSLOPE=-1                            ; Allow convergence in 1 iteration
; X line-region physical locations:
XREG=0.557748526582,0.677748526582,0.947748526582,1.067748526582,
; X line-region logical locations:
KREG=1,8,11,25,28,
KMAX=62                              ; Column number for X = XMAX
; Y line-region physical locations:
YREG=0.0282842712475,1.357573593129,3.690196006403,9.084852813742,
9.367695526217,
; Y line-region logical locations:
LREG=1,3,51,219,410,415,
LMAX=580 &                           ; Row number for Y = YMAX

; Start of boundary points
&PO X=0.0,Y=0.0 &                                    ; 1
&PO X=0.0,Y=27.90991289778 &                         ; 2
&PO X=3.68874263291,Y=27.90991289778 &               ; 3
&PO X=3.68874263291,Y=9 &                            ; 4
&PO X=2.431040968741,Y=9 &                           ; 5
&PO NT=2,X0=2.431040968741,Y0=8.5,                   ; 6
    X=-0.4924038765061,Y=0.0868240888335 &
&PO X=0.7423062006783,Y=1.8020944533 &               ; 7
&PO NT=2,X0=1.037748526582,Y0=1.75,                  ; 8
    X=-0.3,Y=0.0 &
&PO X=0.737748526582,Y=1.55 &                        ; 9
&PO NT=2,X0=0.887748526582,Y0=1.55,                  ; 10
    X=0.0,Y=-0.15 &
&PO X=3.68874263291,Y=1.4 &                          ; 11
&PO X=3.68874263291,Y=0.0 &                          ; 12
&PO X=0.0,Y=0.0 &                                    ; 13
```

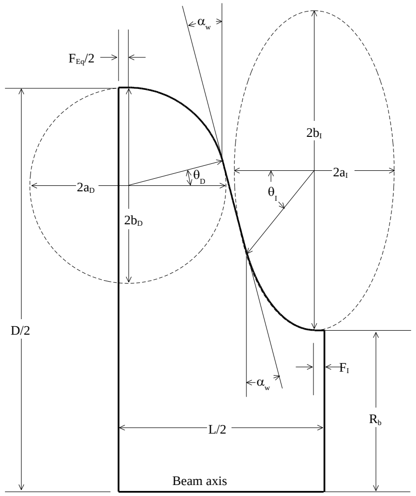
Solution



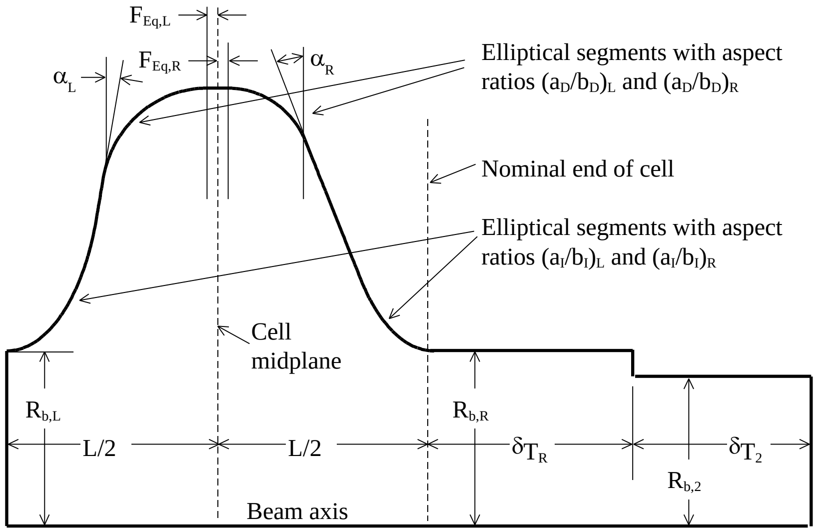
An Elliptical-Type Cavity (think SC)

Basic Geometry

INFN & CEA 704 MHz elliptical SC cavities



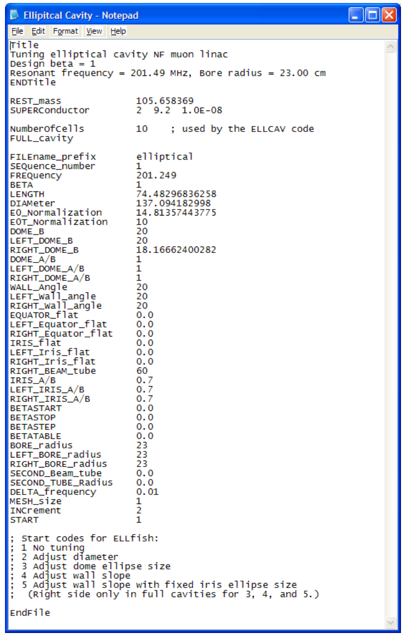
Special Superfish input geometry



An Elliptical-Type Cavity (think SC)

Superfish Implementation

Superfish input file

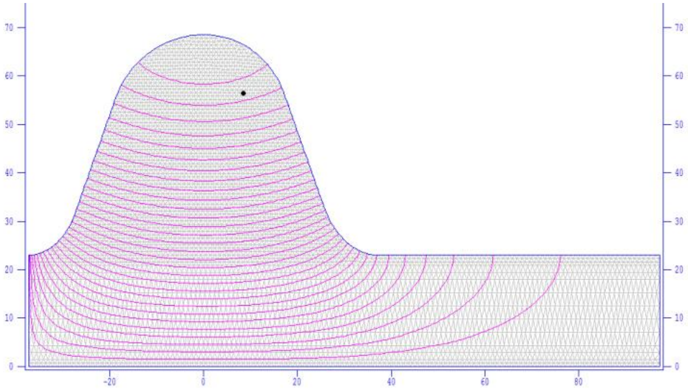


```
Title
Tuning elliptical cavity NF muon linac
Design beta = 1
Resonant frequency = 201.49 MHz, Bore radius = 23.00 cm
ENDTitle

REST_mass             105.658369
SUPERConductor        2  9.2  1.0E-08

NumberOfCells         10    ; used by the ELLCAV code
FULL_cavity

FILEname_prefix       elliptical
SEQuence_number       1
FREQuency             201.249
BETA                  1
LENGTH                74.48296836258
DIAMeter              137.094182998
E0_Normalization      14.81357443775
E0T_Normalization     10
DOME_B                20
LEFT_DOME_B           20
RIGHT_DOME_B          18.16662400282
DOME_A/B              1
LEFT_DOME_A/B         1
RIGHT_DOME_A/B        1
WALL_Angle            20
LEFT_Wall_angle       20
RIGHT_Wall_angle      20
EQUATOR_flat          0.0
LEFT_Equator_flat     0.0
RIGHT_Equator_flat    0.0
IRIS_flat             0.0
LEFT_Iris_flat        0.0
RIGHT_Iris_flat       0.0
RIGHT_BEAM_tube       60
IRIS_A/B              0.7
LEFT_IRIS_A/B         0.7
RIGHT_IRIS_A/B        0.7
BETASTART             0.0
BETASTOP              0.0
BETASTEP              0.0
BETATABLE             0.0
BORE_radius           23
LEFT_BORE_radius      23
RIGHT_BORE_radius     23
SECOND_Beam_tube      0.0
SECOND_TUBE_Radius    0.0
DELTA_frequency       0.01
MESH_size             1
INCrement             2
START                 1

; Start codes for ELLfish:
; 1 No tuning
; 2 Adjust diameter
; 3 Adjust dome ellipse size
; 4 Adjust wall slope
; 5 Adjust wall slope with fixed iris ellipse size
;  (Right side only in full cavities for 3, 4, and 5.)

EndFile
```

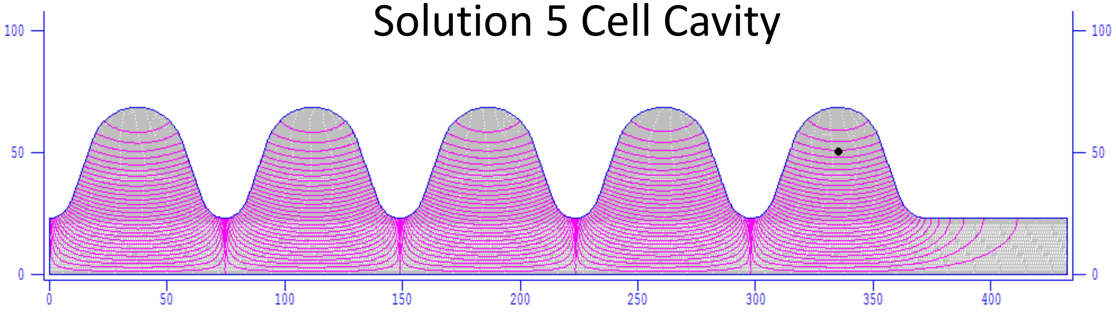
Geometry file

```
Tuning elliptical cavity NF muon linac
Design beta = 1
Resonant frequency = 201.49 MHz, Bore radius = 23.00 cm
No tuning on this cavity.

&REG KPROB=1                          ; Superfish problem
MAT=1                                 ; Material air or empty space
FREQ=201.2471888163                   ; Mode frequency, starting frequency ir
FREQD=201.249                         ; Design frequency, used (with DPHI) to
BETA=1                                ; Particle velocity, used to compute wa
KMETHOD=1                             ; SFO will use BETA to compute wave nur
DPHI=180                              ; Phase length of the cavity, used (wit
NBSUP=1,NBSLO=0,NBSRT=0,NBSLF=0       ; Boundary conditions
LINES=1                               ; Fix internal points on line regions
ICYLIN=1                              ; X=>Z,Y=>R, cylindrical coordinates
NORM=1                                ; Normalize to EZEROT
EZEROT=1.E+07                         ; Accelerating field times T
SCCAV=1,                              ; Superconducting elliptical cavity
RMASS=105.658369                      ; Rest mass value or indicator
EPSO=1.0E-6                           ; Mesh optimization convergence paramet
IRTYPE=1                              ; Rs method: Superconductor formula
TEMPK=2                               ; Superconductor temperature, degrees K
TC=9.2                                ; Critical temperature, degrees K
RESIDR=1.0E-08                        ; Residual resistance
XDRI=8.531253899282                   ; Drive point X coordinate
YDRI=56.50146824014                   ; Drive point Y coordinate
DSLOPE=-1                             ; Allow convergence in 1 iteration
DX=1                                  ; Mesh spacing in X direction
; Y line-region physical locations:
YREG=1.414213562373,4.242640687119,18.05025253169,20.87867965644,
; Y line-region logical locations:
LREG=1,3,5,10,12,
LMAX=80 &                             ; Row number for Y = YMAX

; Start of boundary points
&PO X=0.0,Y=0.0 &                                       ; 1
&PO X=-37.24148418129,Y=0.0 &                           ; 2
&PO X=-37.24148418129,Y=23 &                            ; 3
&PO NT=2,X0=-37.24148418129,Y0=38.66951733241,          ; 4
    A=10.96866213269,AOVRB=0.7,
    X=9.731750709735,Y=-7.228709358454 &
&PO X=-18.79385241572,Y=55.38749436551 &                ; 5
&PO NT=2,X0=0.0,Y0=48.547091499,                        ; 6
    A=20,AOVRB=1,
    X=0.0,Y=20 &
&PO NT=2,X0=0.0,Y0=50.38046749618,                      ; 7
    A=18.16662400282,AOVRB=1,
    X=17.07104252004,Y=6.213351345188 &
&PO X=25.63376923568,Y=33.06792054119 &                 ; 8
&PO NT=2,X0=37.24148418129,Y0=41.69008937394,           ; 9
    A=13.08306256176,AOVRB=0.7,
    X=0.0,Y=-18.69008937394 &
&PO X=97.24148418129,Y=23 &                             ; 10
&PO X=97.24148418129,Y=0.0 &                            ; 11
&PO X=0.0,Y=0.0 &                                       ; 12
```

Solution 1 Cell



Solution 5 Cell Cavity

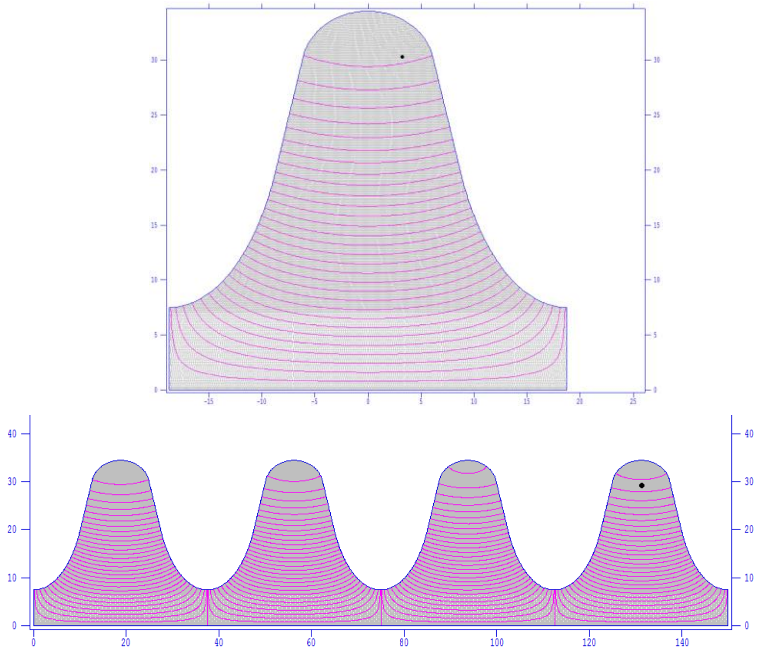


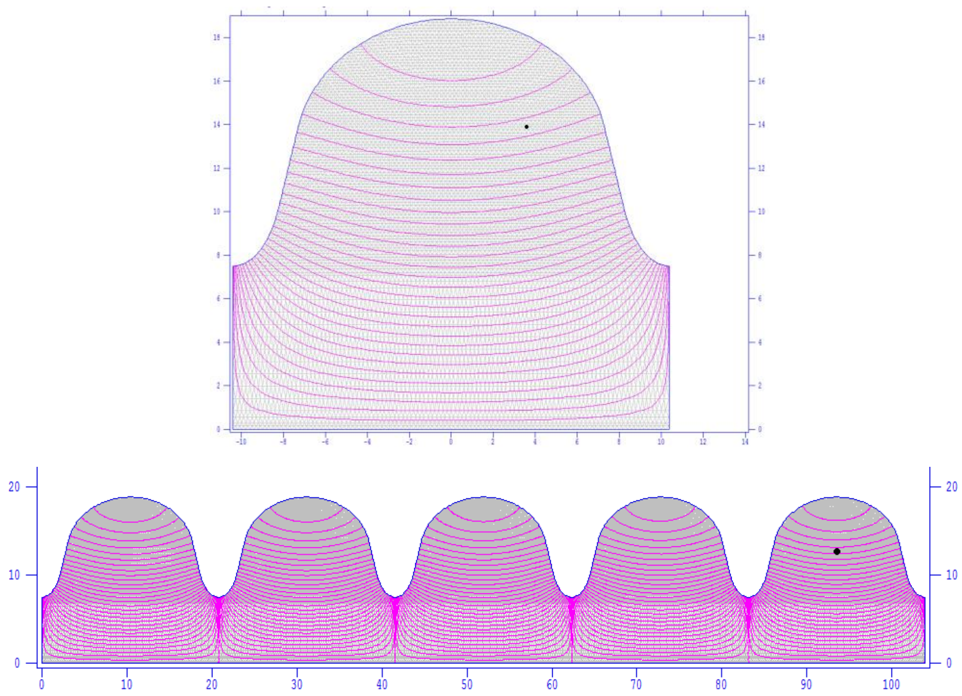
An Elliptical-Type Cavity (think SC)

Two Superfish Examples

- Example 1: 400 MHz
  - Like the LHC 400 MHz RF
  - 4-cell cavity, 4 cavities/Cryomodule

- Example2: 721.4 MHz
  - SPL-like cavities
  - 5-cell cavity





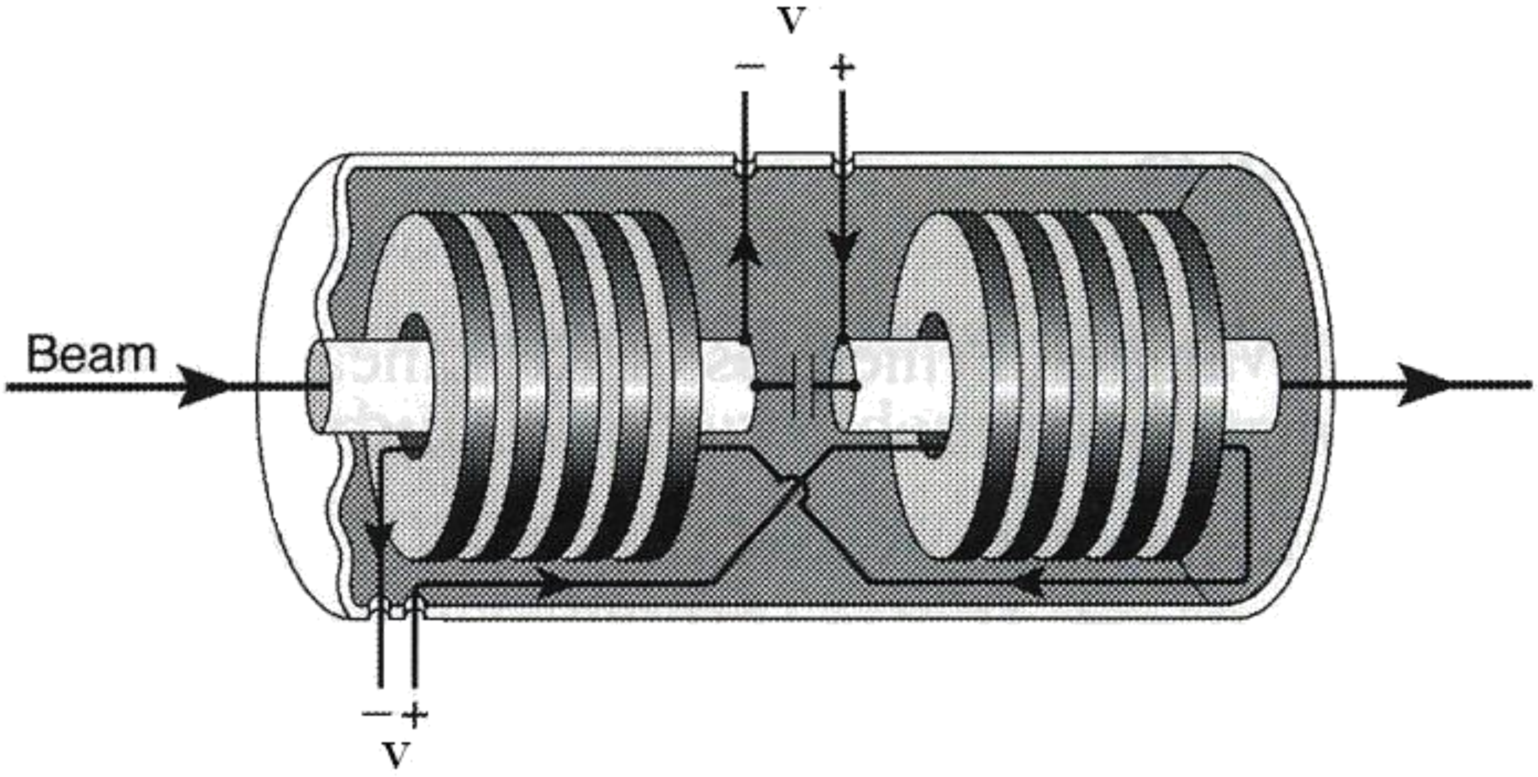
Ferrite Loaded Cavities

The Basics

- Bias current -> Variable magnetic field -> Variable magnetic permeability of the ferrite -> Frequency change
- The structure can be thought of as a resonant transformer in which the beam constitutes a one-turn secondary winding.
- Used when a variable resonant frequency is needed
- The torus of the ferrite encircles the beam path
- Ferrite properties are important (limit the cavity capabilities)
- Frequencies domain: 100 kHz and 60 MHz
- Typical gap voltage of up tens of kV
- Different requirements (large frequency ranges, rapid swings, space, etc) -> various designs.

Ferrite Loaded Cavities

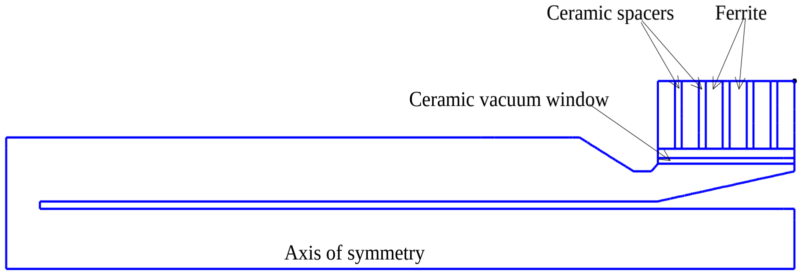
The Basics

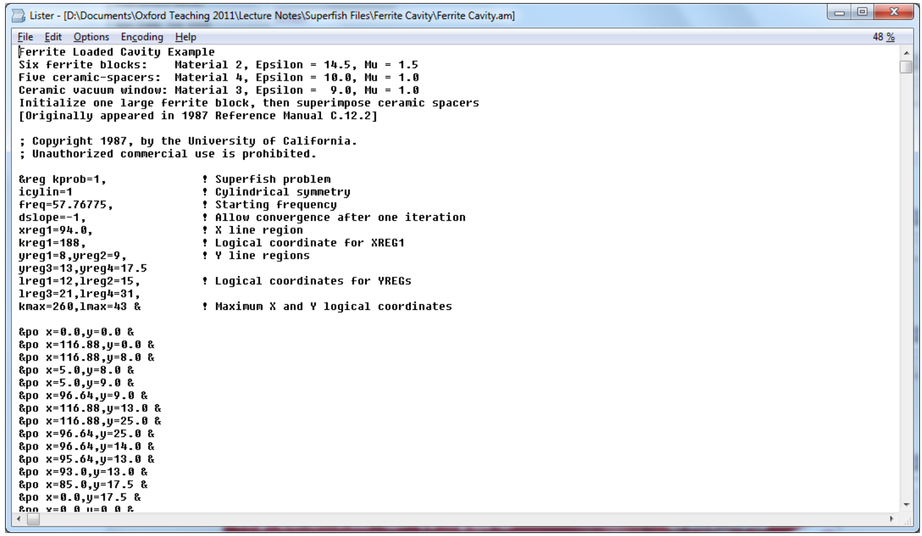


Ferrite Loaded Cavities

Superfish Implementation

Six ferrite blocks: Epsilon = 14.5, Mu = 1.5
Five ceramic-spacers: Epsilon = 10.0, Mu = 1.0
Ceramic vacuum window: Epsilon = 9.0, Mu = 1.0
Cavity length: 116 cm
Number of gaps: 1





```
Ferrite Loaded Cavity Example
Six ferrite blocks:    Material 2, Epsilon = 14.5, Mu = 1.5
Five ceramic-spacers:  Material 4, Epsilon = 10.0, Mu = 1.0
Ceramic vacuum window: Material 3, Epsilon =  9.0, Mu = 1.0
Initialize one large ferrite block, then superimpose ceramic spacers
[Originally appeared in 1987 Reference Manual C.12.2]

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&reg kprob=1,              ! Superfish problem
icylin=1                   ! Cylindrical symmetry
freq=57.76775,             ! Starting frequency
dslope=-1,                 ! Allow convergence after one iteration
xreg1=94.0,                ! X line region
kreg1=188,                 ! Logical coordinate for XREG1
yreg1=8,yreg2=9,           ! Y line regions
yreg3=13,yreg4=17.5
lreg1=12,lreg2=15,         ! Logical coordinates for YREGs
lreg3=21,lreg4=31,
kmax=260,lmax=43 &         ! Maximum X and Y logical coordinates

&po x=0.0,y=0.0 &
&po x=116.88,y=0.0 &
&po x=116.88,y=8.0 &
&po x=5.0,y=8.0 &
&po x=5.0,y=9.0 &
&po x=96.64,y=9.0 &
&po x=116.88,y=13.0 &
&po x=116.88,y=25.0 &
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&po x=95.64,y=13.0 &
&po x=93.0,y=13.0 &
&po x=85.0,y=17.5 &
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&po x=0.0,y=0.0 &
```

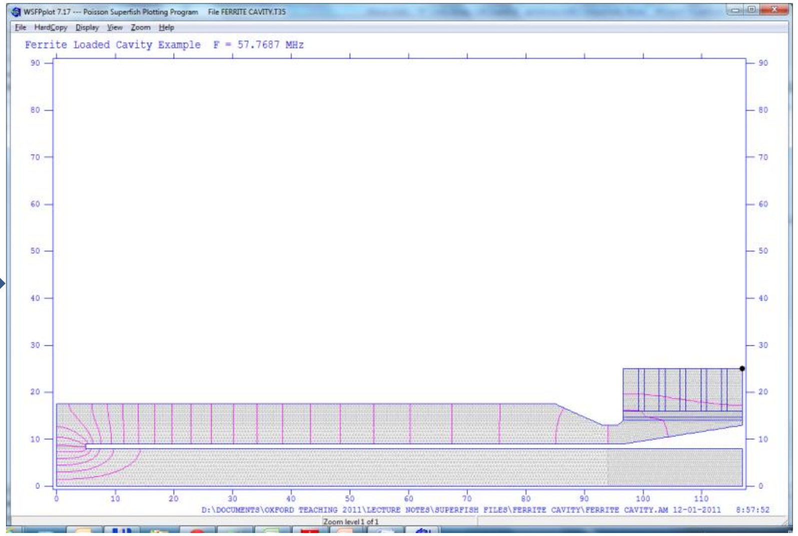

And a lot more…

Now, use your imagination!