# Longitudinal Problem Set

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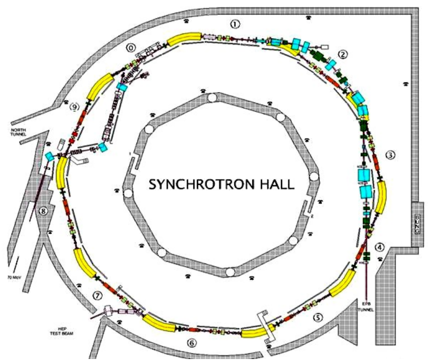
(a) ISIS Synchrotron




(b) ISIS Schematic

**Problem 1.1.** Warm up by deriving the following relativistic relationships from $E = \gamma mc^2$, $p = \gamma mv$, $\beta = v/c$ and $\gamma = 1/\sqrt{1-\beta^2}$

a. $dE = \beta c\, dp$

b. $dE/E = \beta^2\, dp/p$

c. $d(\beta\gamma)/dt = c\, d\gamma/dz$

**Problem 2.1.** ISIS is a proton synchrotron that operates from 70 to 800 MeV on a 50 Hz sinusoidal main magnet field. Given the main dipole field at 70 MeV is 0.17639 T calculate the magnetic rigidity, bending radius $\rho$ and the dipole field at top energy.

**Problem 2.2.** Plot out the dipole field as a function of time over 20 ms.

**Problem 2.3.** Given the mean radius $R$ of the synchrotron is 26 m, calculate and plot the revolution frequency as a function of time through the 10 ms acceleration

**Problem 2.4.** Calculate and plot over 10 ms the

a. Momentum

b. Kinetic Energy

c. Relativistic parameters $\beta$, $\gamma$

What does $\gamma_t$ have to be for ISIS to remain below transition throughout acceleration?

**Problem 2.5.** What is the minimum RF voltage required as a function of time to accelerate a proton at ISIS? Why do we need more?

**Problem 2.6.** Given a mean dispersion of 1 m, what is the $\gamma_t$? What transition kinetic energy does that correspond to? Calculate and plot the slip factor $\eta$ as a function of time over 10 ms.

From the longitudinal equation of motion on slide 29 one can derive the symplectic mapping equations

$$\begin{aligned} \Delta E_{n+1} &= \Delta E_n + V_1(\sin\phi_n - \sin\phi_s) \quad (1) \\ \phi_{n+1} &= \phi_n - \frac{2\pi h\eta}{E_0\beta^2\gamma}\Delta E_{n+1} \quad (2) \end{aligned}$$

from the $n^{\text{th}}$ to the $(n+1)^{\text{th}}$ turn where $V_1$ is the peak gap voltage of the cavity and $E_0$ is the rest energy . These can be used to follow a particle's trajectory in the longitudinal phase space $(\Delta E, \phi)$ on each turn.

**Problem 3.1.** Write a simulation program in the programming language of your choice that assumes several initial particle co-ordinates $(0,\ \phi)$ in the range $0 < \phi < \pi$ and calculates and applies the mapping equations over $n$ turns. Assume a constant energy (70 MeV, $\phi_s = 0$) and use the parameters you calculated in exercise 1. (Other key parameters: $h = 2$, $V_1 = 19$ kV per turn and a sensible number for $n$). Track four more trajectories with start co-ordinates $(0.5\text{ MeV}, \pi)$, $(-0.5\text{ MeV}, -\pi)$, $(1\text{ MeV}, \pi)$, $(-1\text{ MeV}, -\pi)$ and plot out all the longitudinal phase space trajectories on one graph.

**Problem 3.2.** For the above problem pick out the three different types of trajectory and briefly describe the motion in each. Compare to the analogous situation of a simple pendulum.

**Problem 3.3.** Now assume we are half way through acceleration (5 ms). Using the parameters you calculated in exercise 1 and an RF voltage of 150 kV calculate $\phi_s$ and plot some more trajectories, this time with initial co-ordinates $(0,\ \phi)$ between $-\pi < \phi < \pi$. Ensure you track a particle with start co-ordinates $(0, \pi - \phi_s - 0.02)$ and thus trace out the maximum bucket size.

**Problem 3.4.** Calculate the RF bucket height (energy acceptance) with the equation on slide 47 for the two cases (2.1 and 2.3) and compare with the bucket size from your trajectories.

**Problem 3.5.** How does the RF bucket change, as a function of

a. $\phi_s$?

b. $V$?

**Problem 3.6.** What if we now introduce a second harmonic RF system at twice the frequency of the first $(h = 4)$ such that we now have the energy gain per turn

$$\Delta E_{\text{turn}} = e\left(V_1 \sin\phi_s + V_2\sin(2\phi_s + \theta)\right), \quad (3)$$

where $V_2$ is the second harmonic RF voltage and $\theta$ is the phase difference between the two RF systems. How does this change our mapping equations? Try putting this into your tracking code with $V_2 = V_1$ for the case $\phi_s = 0$. How does the RF bucket change with

a. $\theta = \pi$?

b. $\theta = 0$?

Hand in a copy of your program(s) with its (their) output.