Using Woodcock technique as a way to provide a weight calculation for occurrence biasing of charged particles

16/09/2019
Marc Verderi, LLR
Introduction

• Biasing of interaction occurrence of charged particles is a long pending development item
  – Because of the main difficulty being:
    • the variation of the cross-section over a step, because of energy loss
    • with the need to integrate this cross-section over the step for weight calculations
      – Integration which can not be “brute force” to not penalize the speed-up we try to get by biasing

• During our last meeting, Laurent presented calculations he made in 2012:
  – That demonstrate the correctness of the so-called rejection technique
    • These require the track to do interactions
  – That propose a way to compute the weight in case of free-flights
    • To complement the case of tracks requested not to do interaction

• “Floating in the air” since several years is also the so-called “Woodcock tracking” technique, which is based on the same principle than the rejection technique, but which has an elegant way to rephrase the problem
  – This at the end triggered some neuronal signal in my brain during my post-lunch nap on 30/07/2019 ;)

• Explain the idea after, and perform a series of toy MC tests.
Overview

• Rejection technique
• Woodcock tracking
• Woodcock and biasing
• Discussion / Conclusion
Rejection technique

- **Algorithm:**
  - Determine $\sigma_M$ on $[0, L]$
  - Sample $\ell$ using $\sigma_M$
  - Move track to $\ell$ and compute $\sigma(\ell)$
  - if $\text{rand}(0,1) > \sigma(\ell)/\sigma_M$ reject interaction; else do it

- **Laurent demonstrated the rejection provides the exact amount of interactions $\forall \ell$**
  - Demonstration not straightforward
  - Based of the sum of number of interactions of tracks having suffered $0, 1, 2, \ldots, \infty$ rejected interactions before interacting at $\ell$

- Rejection technique is used in the EM package, to account for variation of cross-section between the start and the end points of the step
  - Not used in the hadronics: not sure why
The Woodcock tracking (invented for medical) rephrases the problem saying that:
- We have a total (and constant) cross-section $\sigma_M$
- This one is the sum of two physics process cross-sections:
  - The physical one $\sigma(\ell)$
  - And $\sigma_{\text{fictitious}}(\ell)$ which generates "fictitious interactions"
    - This is a process that leaves the track unchanged (or that kills it and produces an exact copy, if you prefer a more active process)

**Algorithm:**
- We sample $\ell$ according to the total cross-section $\sigma_M$
- We move the track to $\ell$
- We chose randomly between the two "processes"
  - depending on their relative cross-sections at that point.

This makes *de facto* the exact same sampling than the rejection!
- But easier to understand physically, and with some further advantages for biasing...
“Physical” test cross-section

Related non-interaction probability over a path $0 \rightarrow \ell$

$$P_{NI}(0 \rightarrow \ell) = \exp \left( - \int_{0}^{\ell} \sigma_{phys}(s) \, ds \right)$$

Related probability density function of interactions (product of the two above functions)
Distribution of interactions obtained by Woodcock sampling technique (100 k events)
Same events than previous page, but separated in samples with 0, 1, 2, 3, 4 and > 4 fictitious interactions before the physical one happens.
The Woodcock viewpoint makes it easy the move to biasing:

- In the analog world we have total physical and fictitious cross-sections:
  $$\sigma^a_M = \sigma^a_{phys}(\ell) + \sigma^a_{fictitious}(\ell)$$

- That we replace by their biased version in the biased world:
  $$\sigma^b_M = \sigma^b_{phys}(\ell) + \sigma^b_{fictitious}(\ell)$$

From there, we apply the formalism we already know:

- For a step ending with no interaction (eg: geometry), we multiply the track weight by the non-interaction weight, ratio of the non-interaction probabilities $$P_{NI}^{a(b)}(0 \rightarrow \ell)$$:
  $$w_{NI}(0 \rightarrow \ell) = \frac{P_{NI}^a(0 \rightarrow \ell)}{P_{NI}^b(0 \rightarrow \ell)}$$

  $$P_{NI}^{a(b)}(0 \rightarrow \ell) = \exp \left( - \int_0^\ell \sigma^a_{M(b)} \cdot s \cdot ds \right) = \exp \left( -\sigma^a_{M(b)} \cdot \ell \right)$$

- For a step ending with an interaction by process $$i$$, $$i = \text{“physical” of “fictitious”}$$, we multiply the track weight by the interaction weight:
  $$w_I(\ell) = w_{NI}(0 \rightarrow \ell) \cdot \frac{\sigma_i^a(\ell)}{\sigma_i^b(\ell)}$$

And we’re done!
Biasing sampling:
- $\sigma_{\text{M}} = 0.5$
- $\sigma_{\text{phys}} = \sigma_{\text{M}} \cdot 0.75$
- $\sigma_{\text{fictitious}} = \sigma_{\text{M}} \cdot 0.25$

Reconstructed distribution of physical interactions using biasing

100 k biased events
Reconstructed distribution of fictitious interactions using biasing
Weighted (reconstructed) distributions

Unweighted (generated) distributions

0 fictitious

1 fictitious

2 fictitious

3 fictitious

4 fictitious

> 4 fictitious
Redo with more adapted biased sampling to avoid depletions observed in previous page:

- \( \sigma_M = 1.0 \)
- \( \sigma_{phys} = \sigma_M \cdot 0.5 \)
- \( \sigma_{fictitious} = \sigma_M \cdot 0.5 \)
Reconstructed distribution of physical interactions using biasing

**Biasing sampling:**
- $\sigma_M = 1.0$
- $\sigma_{phys} = \sigma_M \cdot 0.5$
- $\sigma_{fictitious} = \sigma_M \cdot 0.5$

100 k biased events

Reconstructed distribution of physical interactions using biasing
Reconstructed distribution of fictitious interactions using biasing
Discussion / Conclusion

• The good points:
  – The smart Woodcock viewpoint with “fictitious interaction” allows to take into account the variation of cross-sections and allows an easy way move to biasing!
  – The technique looks to have interesting observables to help diagnosing poor biased sampling:
    • Eg: comparison of weighted and unweighted distributions as function of the number of fictitious interaction

• To be tested:
  – The test biasing cross-sections were chosen constant in these first tests
    • Distance dependent biased cross-sections must be tested as well!
  – A free-light scheme might be possible
    • by setting the biased physical cross-section to zero, the track will not interact (by physical post step processes) but will do only fictitious interactions, updating its weight.

• Expected difficulties:
  – Maximum cross-section:
    • The maximum cross-section might be far from the average one in some cases
      – And might a cause of performance penalty
    • The maximum cross-section may even not exist! Specially with biased distributions:
      – For example, the popular forced interaction scheme over a segment $[0, L]$ has a cross-section that diverges at $L$!
    • But using the maximum —and constant— cross-section is for convenience
      – We may extend the Woodcock scheme by choosing a non-constant majorant of physical cross-section:
        $\sigma^b_M(\ell) = \sigma^b_{\text{phys}}(\ell) + \sigma^b_{\text{fictitious}}(\ell)$
        – But still such that $\sigma^b_M(\ell)$ leads to convenient sampling
  – Performance:
    • Their might be a balance to find between relative number of fictitious versus physical interactions
  – And of course as usual: manpower...