Quantum Open Systems with Matrix Product States

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September 16, 2019

Overview

- Non-equilibrium systems
- Markov approximation and Lindblad equation
- Matrix Product States (MPS)
- MPS methods applied to quantum open systems
- Work done so far and future steps

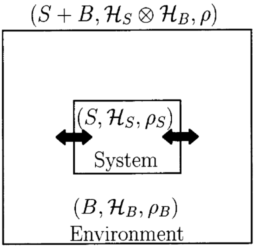
Non-Equilibrium Systems

- The majority of systems in nature are actually found out of equilibrium
- Despite this fact, much less is know about these systems than their equilibrium counterparts
- Recently there has been a lot of interest in the field, motivated by advances in the development of quantum technologies

Quantum Open Systems

- The dynamics of a closed system can be given in terms of a unitary time evolution $i\frac{d}{dt}|\psi(t)\rangle = H(t)|\psi(t)\rangle$
- This is not true in general for open systems, whose dynamics are instead formulated in terms of an equation of motion for the density matrix (quantum master equation)

$$\frac{d}{dt}\rho_S(t) = -iTr_B\left[H(t), \rho(t)\right]$$



Sketch of an open quantum system [Heinz-Peter Breuer, 2002].

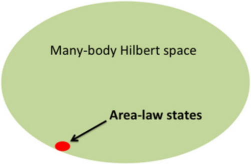
Markov Approximation

- The Markov approximation consists in assuming the time scale over which the state of the system evolves is much larger than the characteristic time scale of the reservoir
- This simplifies the master equation into the Lindblad form
- Note that the non-hermitian terms are responsible for dissipation

$$\frac{d}{dt}\rho_S(t) = \mathcal{L}\rho_S(t)$$
$$= -i[H, \rho_S] + \sum_l \left[W_l \rho_S W_l^\dagger - \frac{1}{2} W_l^\dagger W_l \rho_S - \frac{1}{2} \rho_S W_l^\dagger W_l \right]$$

Hilbert Space Scaling

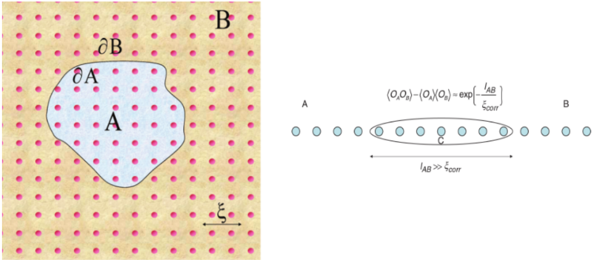
- The dimension of the Hilbert space of a composite system scales exponentially with the number of elements
- This poses the biggest challenge to the study of quantum many body systems
- However, the physical states of interest are typically confined to a small submanifold of the overall Hilbert space
- This motivates variational approaches
- The goal is then to find the best way to parametrize the states of interest



Comparison of area law state's submanifold and full Hilbert space [Orús, 2014].

Area law states

- It can be shown that MPS, and Tensor networks in general, are the good parametrization for states obeying an area law
- This is intimately connected with the fact that the systems they describe have finite correlation lengths
- For 1D gapped systems the density matrix renormalization group (DMRG) method also targets these area law states, this being the reason there is a connection between this method and MPS



Partition of quantum system and behavior of correlation functions [Verstraete et al., 2008].

A general state in the Hilbert space of N elements is given by

$$|\psi\rangle = \sum_{i_1,i_2,\ldots,i_N=1}^{p} C_{i_1 i_2 \ldots i_N} |i_1\rangle \otimes |i_2\rangle \otimes \ldots \otimes |i_N\rangle \,,$$

where p is the dimension of the space of a single particle.
A MPS state is given by

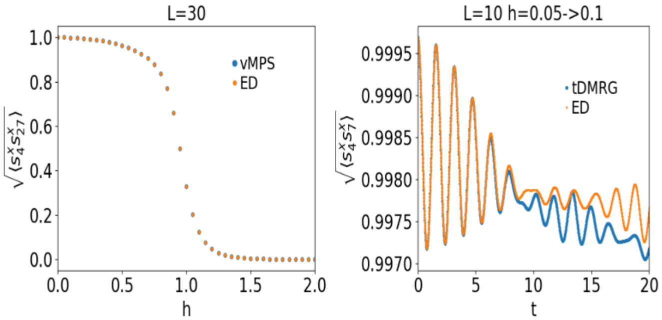
$$|\psi\rangle = \sum_{i_1,i_2,\ldots,i_N}^{p} A_1^{i_1} A_2^{i_2} \ldots A_N^{i_N} |i_1\rangle \otimes |i_2\rangle \otimes \ldots \otimes |i_N\rangle \,,$$

where A_i are $D \times D$ matrices, with D being the so-called bond dimension. This controls the precision of the approximation and is deeply connected with the amount of entanglement of the state

MPS Methods

The MPS formalism allows for the

- computation of expectation values of physical observables
- determination of groundstates using a variational algorithm
- time evolution of pure quantum states



Spin spacial correlation function for a) the groundstate of the Ising model with transverse field and b) during time evolution of a state after a quench in the magnetic field. Both images compare the results obtained with the MPS methods with those from exact diagonalization .

MPS Methods for the study of Quantum Open Systems

The MPS methods are, strictly speaking, applied to pure states. How can they then be used to study open systems?

- One way is to consider the mapping $\rho_S = \sum_{ij} \rho_{ij} |i\rangle \langle j| \rightarrow ||\rho_S\rangle\rangle = \sum_{ij} \rho_{ij} |i\rangle |j\rangle$
- Another way is to consider a purification of the physical system

In both these ways the MPS machinery can be used to study open quantum systems, in particular to obtain their steady states and compute the observables that characterize them.

Work done so far and future steps

- We have developed a computer program to compute groundstates and perform time evolution using the MPS formalism
- We will use it to study the XYZ model in a 1D open chain, which shall serve as a benchmark
- Then we will choose a suitable problem to tackle using the tools mentioned above

Heinz-Peter Breuer, F. P. (2002).
The theory of open quantum systems.
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Annals of Physics, 349:117 – 158.

Verstraete, F., Murg, V., and Cirac, J. (2008).
Matrix product states, projected entangled pair states, and variational renormalization group methods for quantum spin systems.
Advances in Physics, 57(2):143–224.

Thank You