TÉCNICO LISBOA

Disordered topological systems

Jornadas CeFEMA

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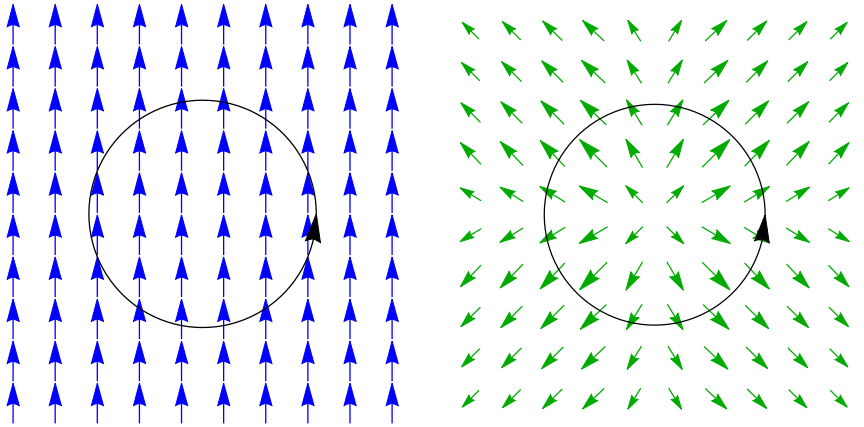
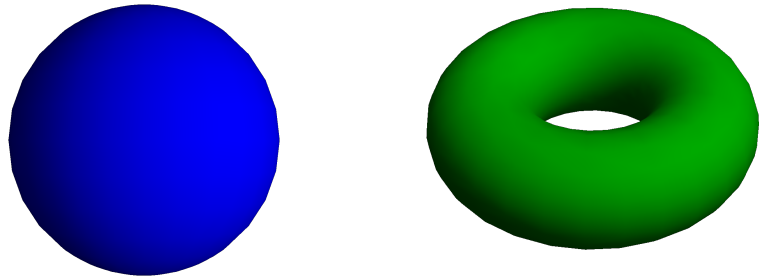
Topological systems

- Phases of matter without an order parameter:
  → No spontaneous symmetry breaking.

- Characterized by topological invariants
  → Robust under continuous deformations;
    → Robust properties.

- Example:
  → Chern number, proportional to the Hall conductivity at null temperature.





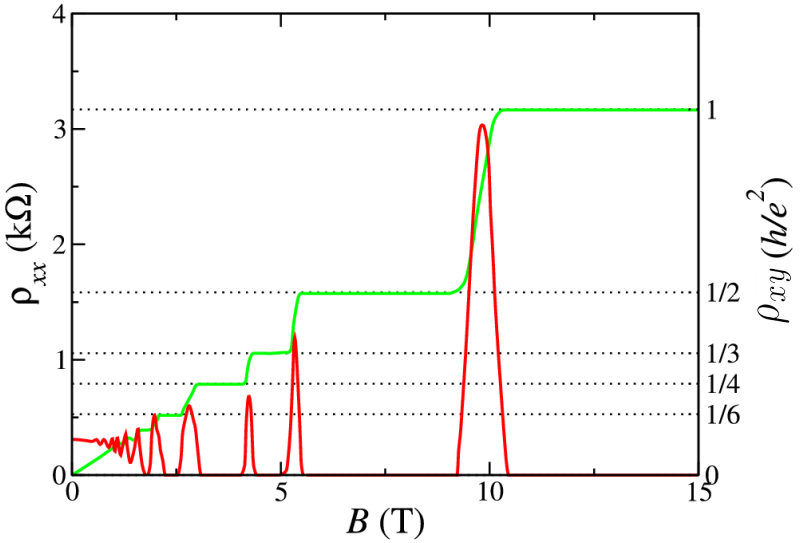
n=0 n=1

Topological systems

Quantum Hall effect

K.V.Klitzing et al., *Physical Review Letters* **1980**, *45*, 494-497

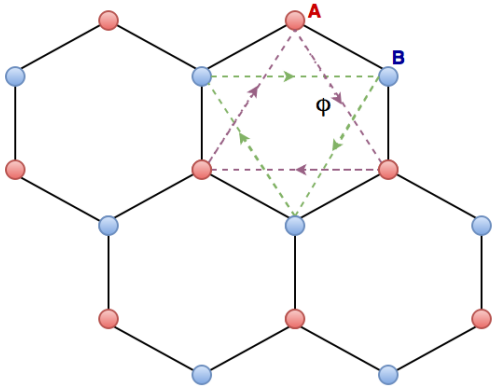
$$\rho_{x,y} = \frac{h}{e^2}\frac{1}{n}, n = 1, 2, \cdots$$

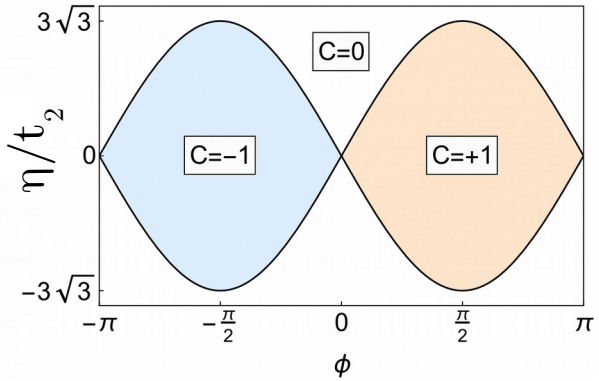


→ Thouless et al., *Physical Review Letters*, **1982**, *49*, 405-408 : Quantization of Hall conductivity explained.

Haldane model

F.D.M. Haldane, *Physical Review Letters*, **1988**, *61*, 2015-2018





$$H = -t\sum_{\langle i,j\rangle} c_i^\dagger c_j + t_2 \sum_{\langle\langle i,j\rangle\rangle} e^{-i\phi_{i,j}} c_i^\dagger c_j + h.c. + \eta \sum_i \zeta_i c_i^\dagger c_i$$

$$\phi_{i,j} = \pm\phi \qquad \zeta_i = \begin{cases} +1 & i \in A \\ -1 & i \in B \end{cases}$$

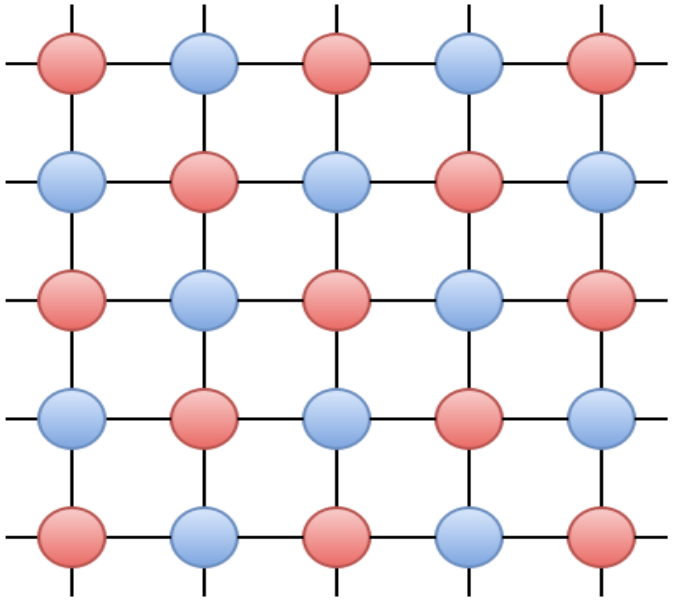
Disordered systems and localization

- Disorder can appear in many different ways:

→ Impurities;

→ Lattice distortions;

→ ...



Large enough degree of disorder

↓

P. W. Anderson, *Physical Review Letters* **1958**, *109*, 1492-1505

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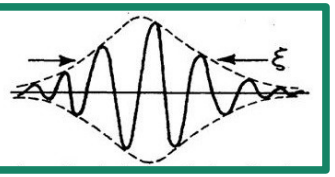
Localization of eigenstates

$$\psi(\mathbf{r}) = f(\mathbf{r})e^{-|\mathbf{r}-\mathbf{r}'|/\xi}$$



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Metal-insulator transition

Anderson insulators (gapless)

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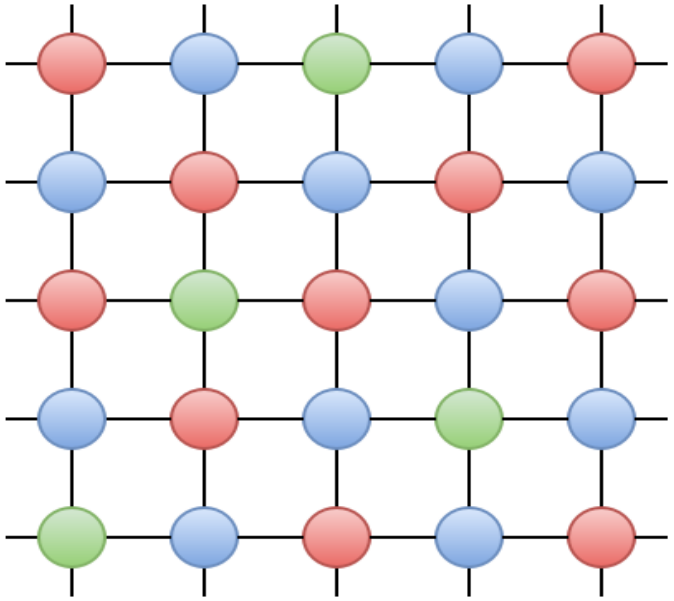

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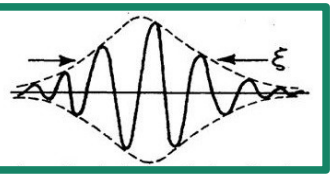

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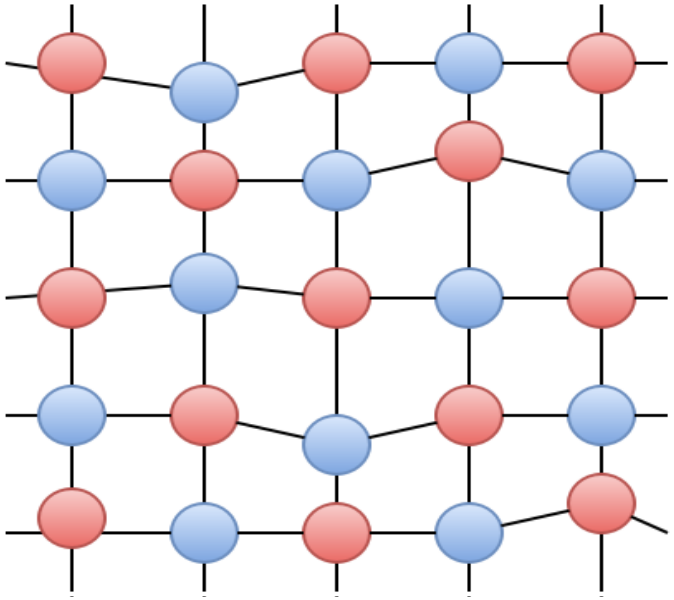

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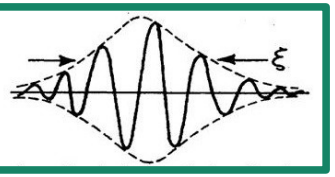

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Metal-insulator transition

Anderson insulators (gapless)

Topology + disorder

E. V. Castro et al., *Physical Review B* **2015**, *92*, 085410
E. V. Castro et al., *Physical Review B* ***2016***, *93*, 245414
M. Gonçalves, P. Ribeiro, E. V. Castro,*Arxiv eprints*, **2018**, *1807.11247*

Disordered Haldane model

Topology + quenched disorder

M. Gonçalves, P. Ribeiro, E. V. Castro, *Arxiv eprints*, **2018**, *1807.11247*

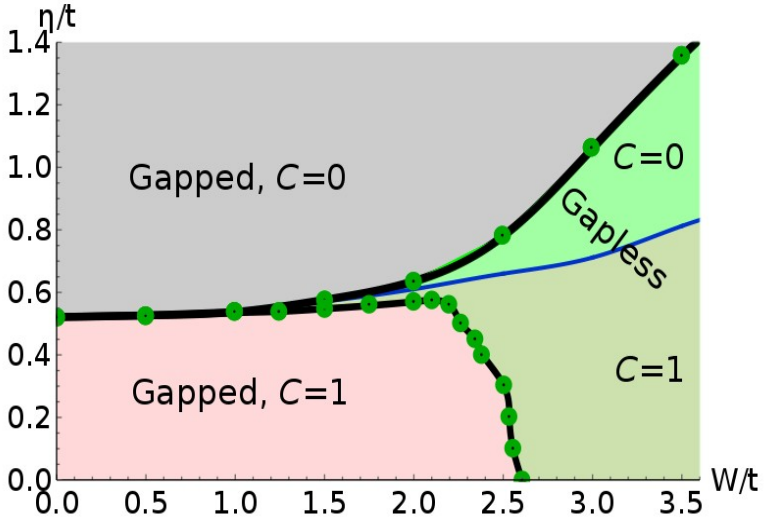
- Topological phase suppressed for large enough disorder strength W/V;

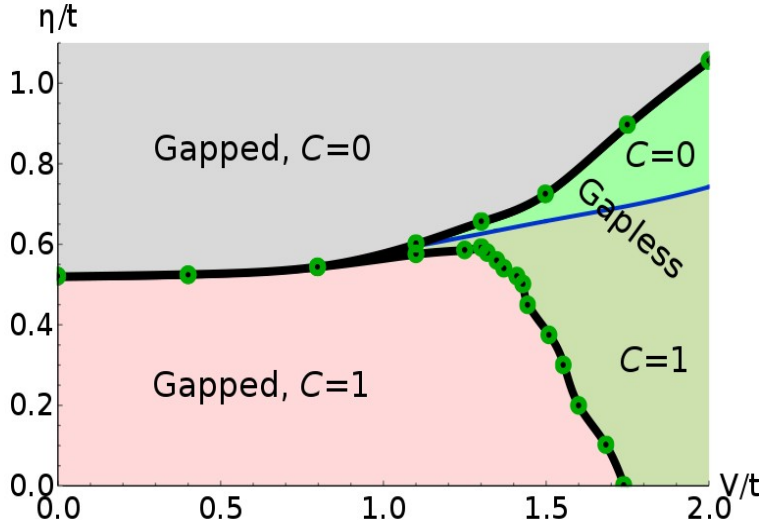
- Gapped (gapless) topological insulating phases (C=1) for low (intermediate) disorder strength;

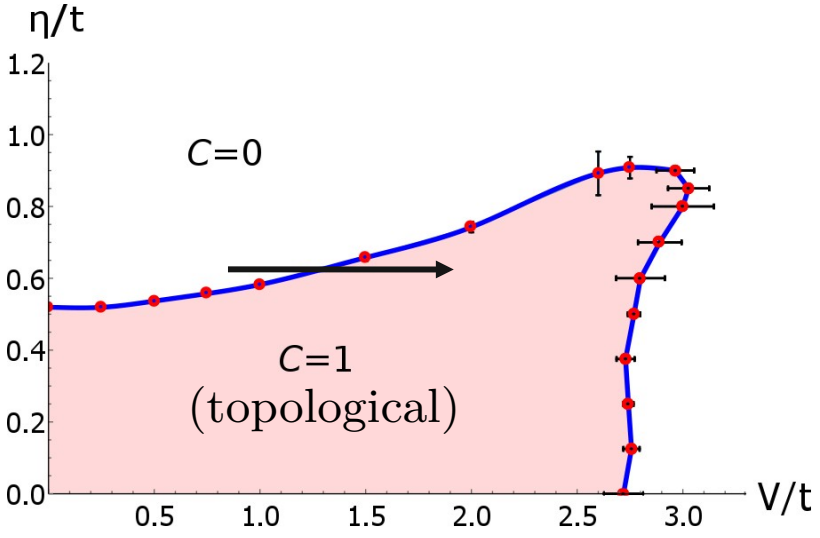
- Possible disorder-driven topological phase.

$$\mathrm{H}=\mathrm{H}_H+\sum_i \xi(i) c_i^\dagger c_i$$

$$\begin{cases} P_W(\xi_i)=\frac{1}{W}\Theta(W/2-|\xi_i|) & \text{, Anderson disorder} \\ P_V(\xi_i)=\frac{1}{2}(\delta(V-\xi_i)+\delta(\xi_i)) & \text{, binary disorder} \end{cases}$$



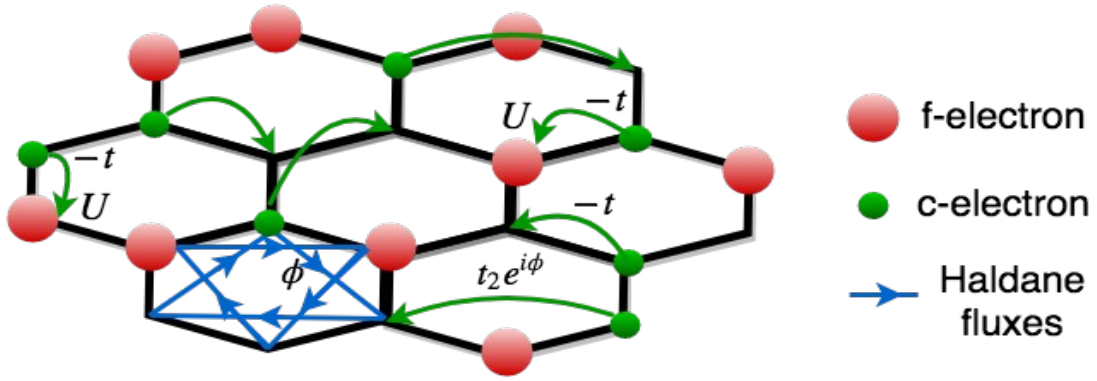




Haldane-Falicov-Kimball model

Topology + interactions (annealed disorder)

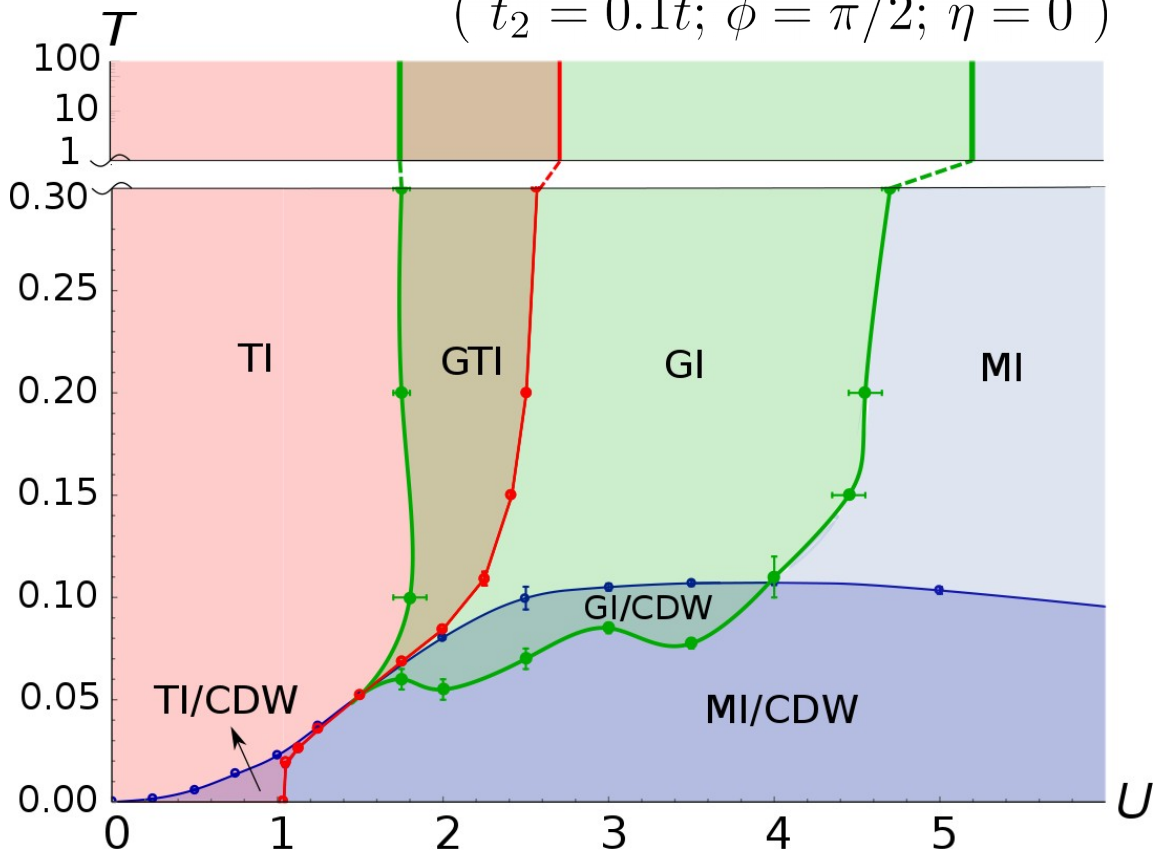
M. Gonçalves, P. Ribeiro, R. Mondaini, E. V. Castro,
Physical Review Letters, **2019**, *122*, 126601



$$H = -t\sum_{\langle i,j\rangle} c_i^\dagger c_j + t_2 \sum_{\langle\langle i,j\rangle\rangle} e^{i\phi_{ij}} c_i^\dagger c_j + h.c. + \eta \sum_i \zeta_i c_i^\dagger c_i$$
$$+ U\sum_i c_i^\dagger c_i n_{f,i} - \sum_i (\mu_c c_i^\dagger c_i + \mu_f n_{f,i})$$

Half-filling: $\mu_c = \mu_f = \frac{U}{2}$

$(t_2 = 0.1t;\ \phi = \pi/2;\ \eta = 0)$

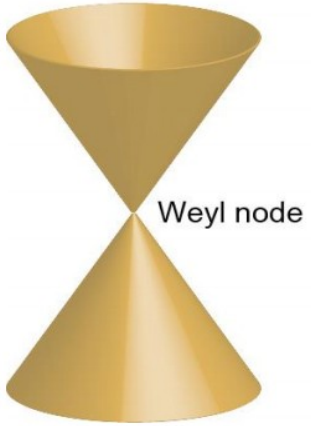


TI: Gapped topological insulator
GI: Gapped insulator
GTI: Gapless topological insulator
MI: Mott insulator

Disordered Weyl nodal loop

M. Gonçalves, P. Ribeiro, E. V. Castro, M. A. N. Araújo,
Arxiv eprints, **2019**, *1908.06910*

Weyl semimetal

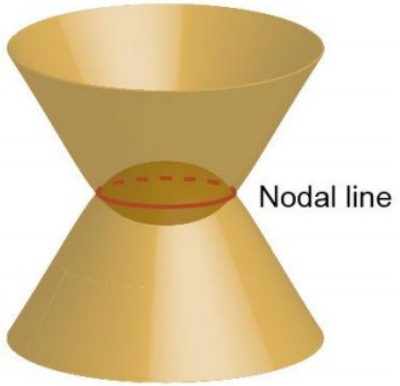


No Kramers degeneracy

Dirac semimetal

Kramers degeneracy

Nodal-line semimetal



- Besides topological insulators, **3D topological semimetals** are also a current hot topic of research. These include:

→ Dirac/Weyl semimetals;

→ Nodal-loop semimetals.

For Dirac/Weyl semimetals:

→ conduction and vallence bands linearly touch at topologically protected points in momentum-space (Dirac/Weyl nodes);

→ Novel disorder-driven quantum phase transition between **semimetallic** and **diffusive metallic** phases (intense debate!);

→ Anderson insulator for large disorder.

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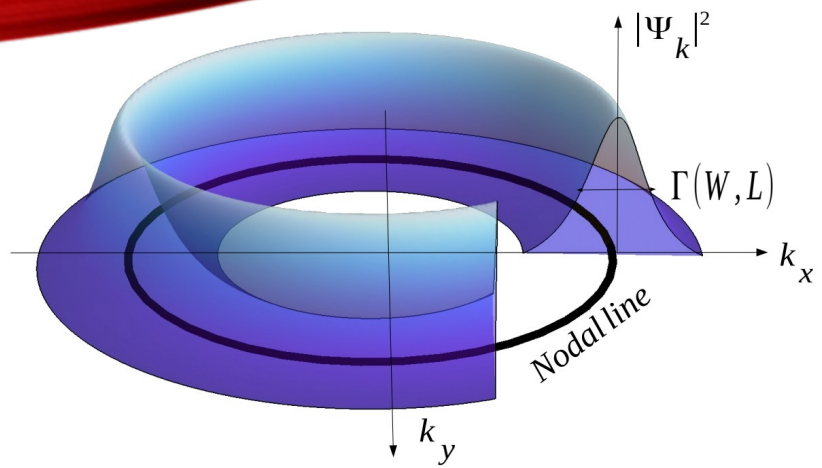

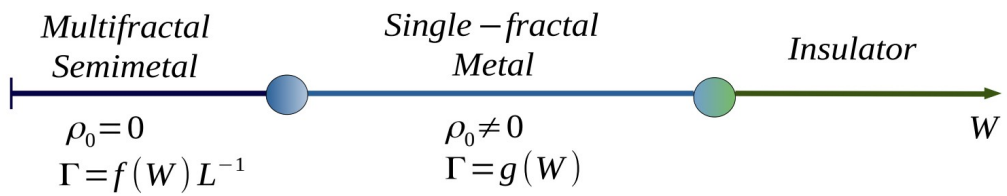

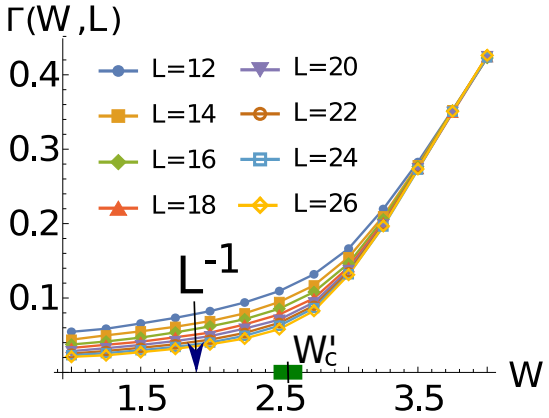

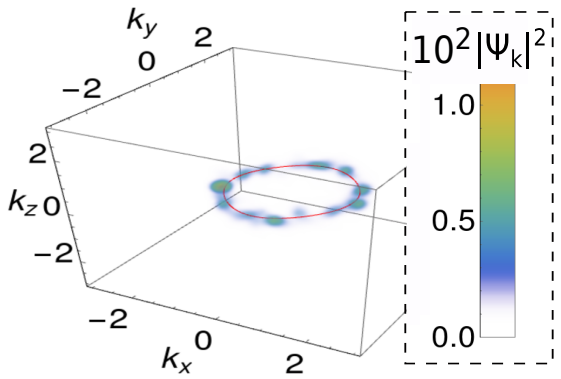

- What about **Weyl nodal-loop semimetals**?

→ Bands touch linearly at a topologically protected nodal line in momentum space;

→ Novel **multifractal semimetallic phase** for finite disorder strength below a critical value;

→ Single-fractal diffusive metallic phase for intermediate disorder;

→ Anderson insulator for large disorder.

Thank you!