Is Dark Energy simulated by structure formation in the Universe ?

THOMAS BUCHERT

CRAL
European Research Council
Established by the European Commission

General Thoughts

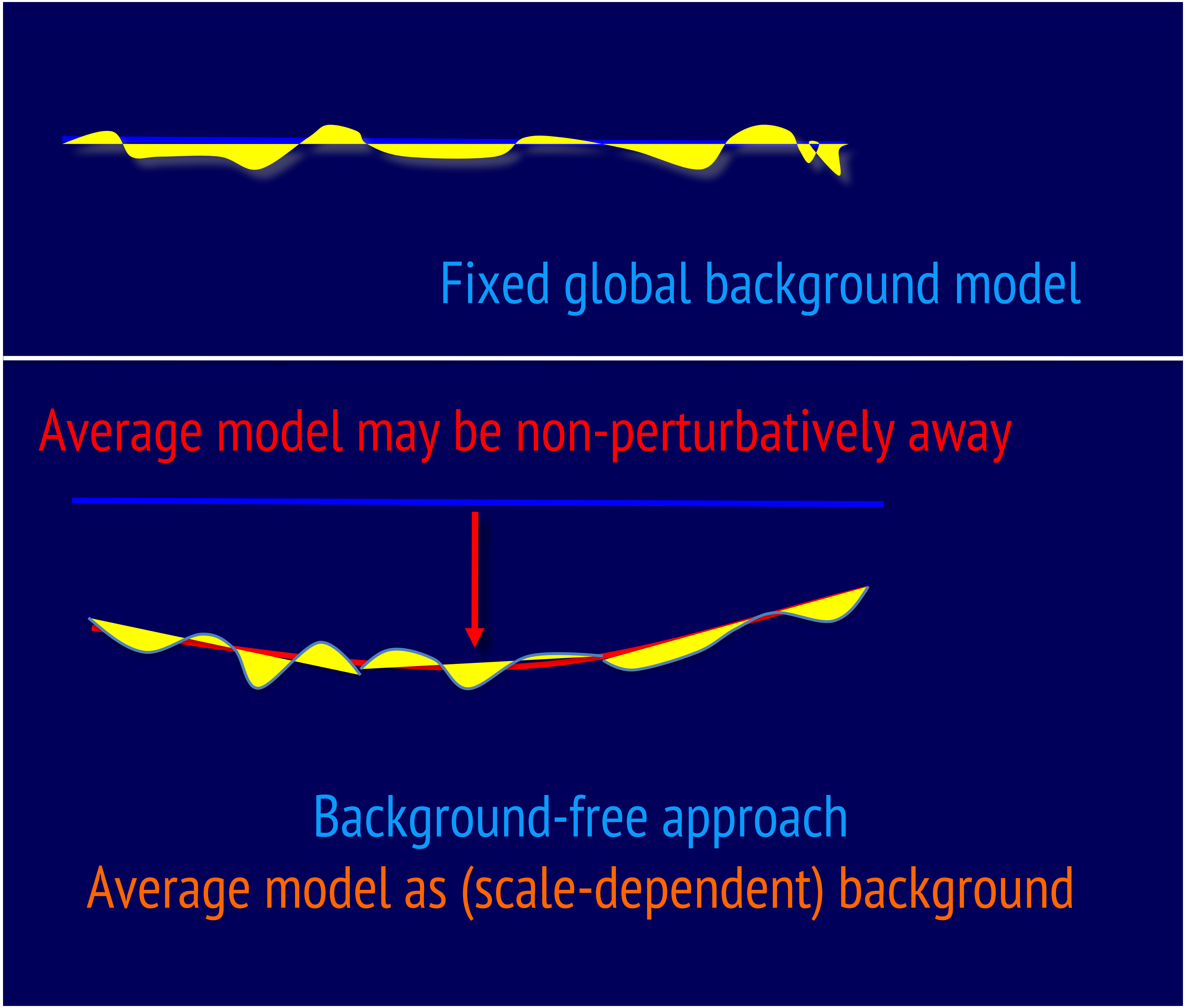
Why averaging ?

We see structures and conceive them as fluctuations
with respect to an assumed background geometry

The description of fluctuations makes only sense
with respect to their average

The average distribution is homogeneous and
large-scale isotropic ...

... but can be dynamically very different
from a homogeneous-isotropic solution


Fixed global background model
Average model may be non-perturbatively away
Background-free approach
Average model as (scale-dependent) background

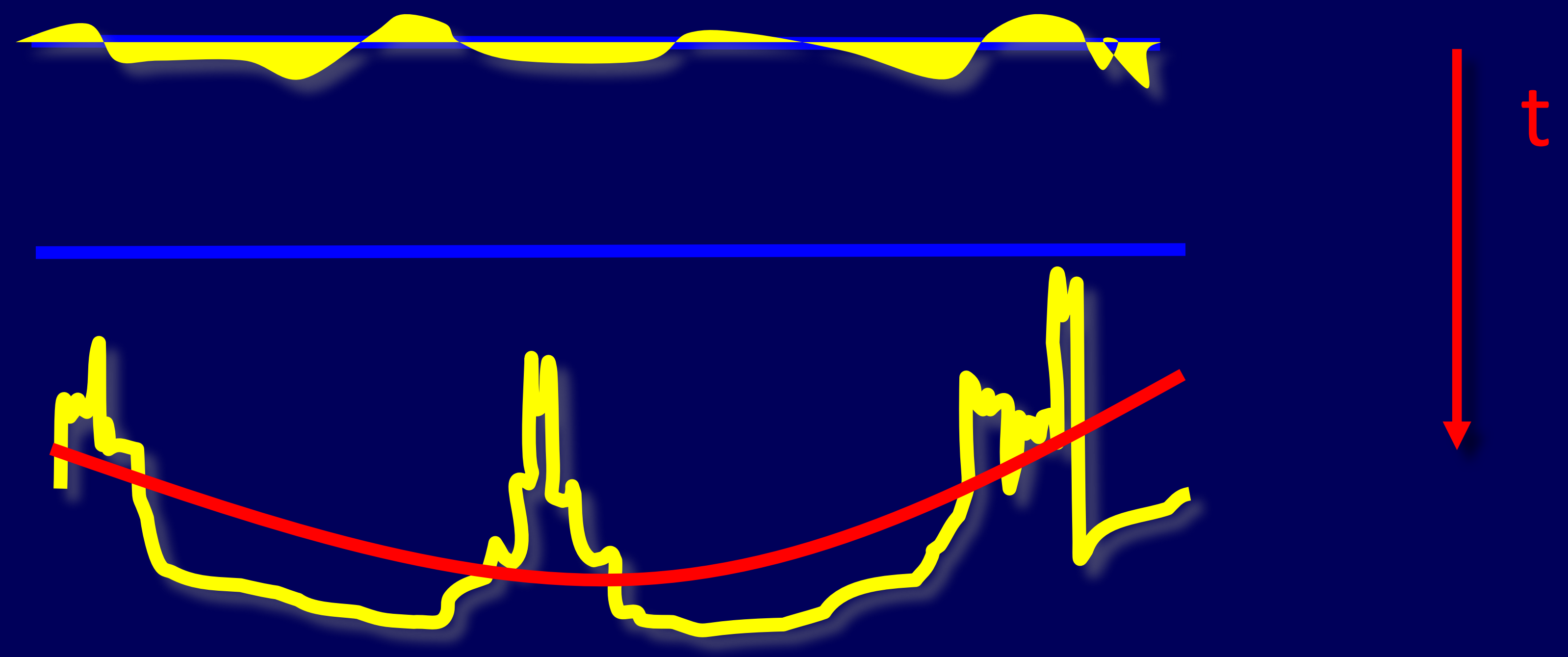
Curvature is Key

The standard ΛCDM model assumes zero curvature

But : there is a geometrical side
to structure formation !

The FLRW geometries allow for constant curvature

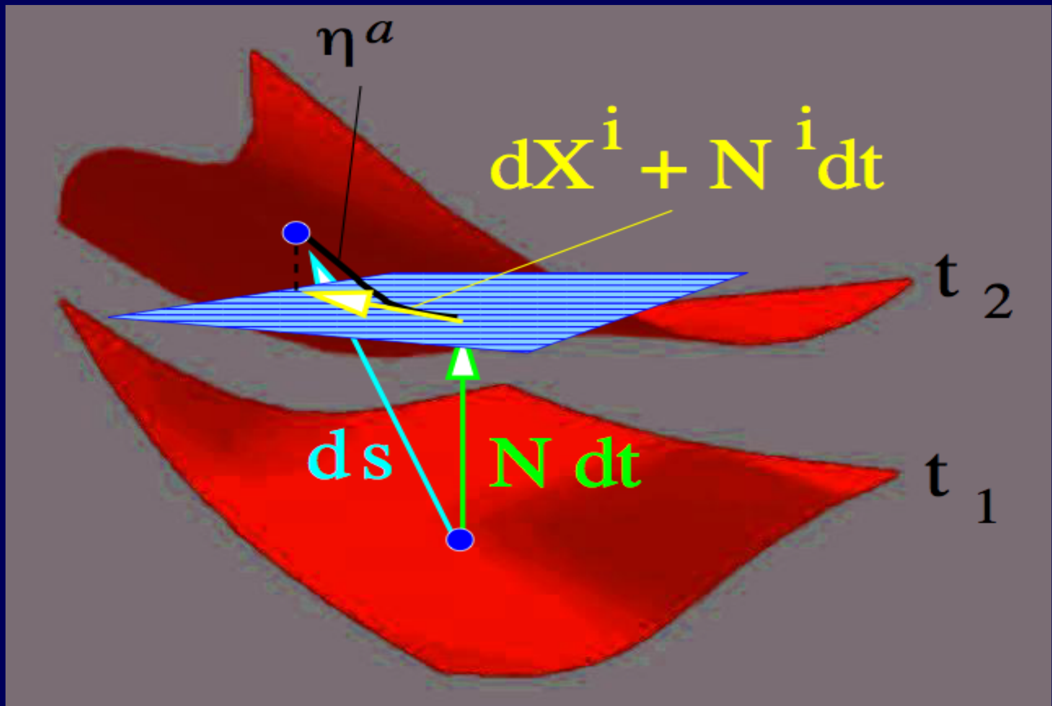
But : The average curvature
can evolve differently !



Curvature is not conserved
while Restmass is conserved

Why spatial averaging ?

Cosmology is conceived as an
evolving space / hypersurface (3+1)
with a synchronous time (vs. local proper time)



Buchert, Mourier, Roy
arXiv: 1805.10455

Take home summary I

Backreaction describes the deviations of the average from an assumed homogeneous-isotropic FLRW solution

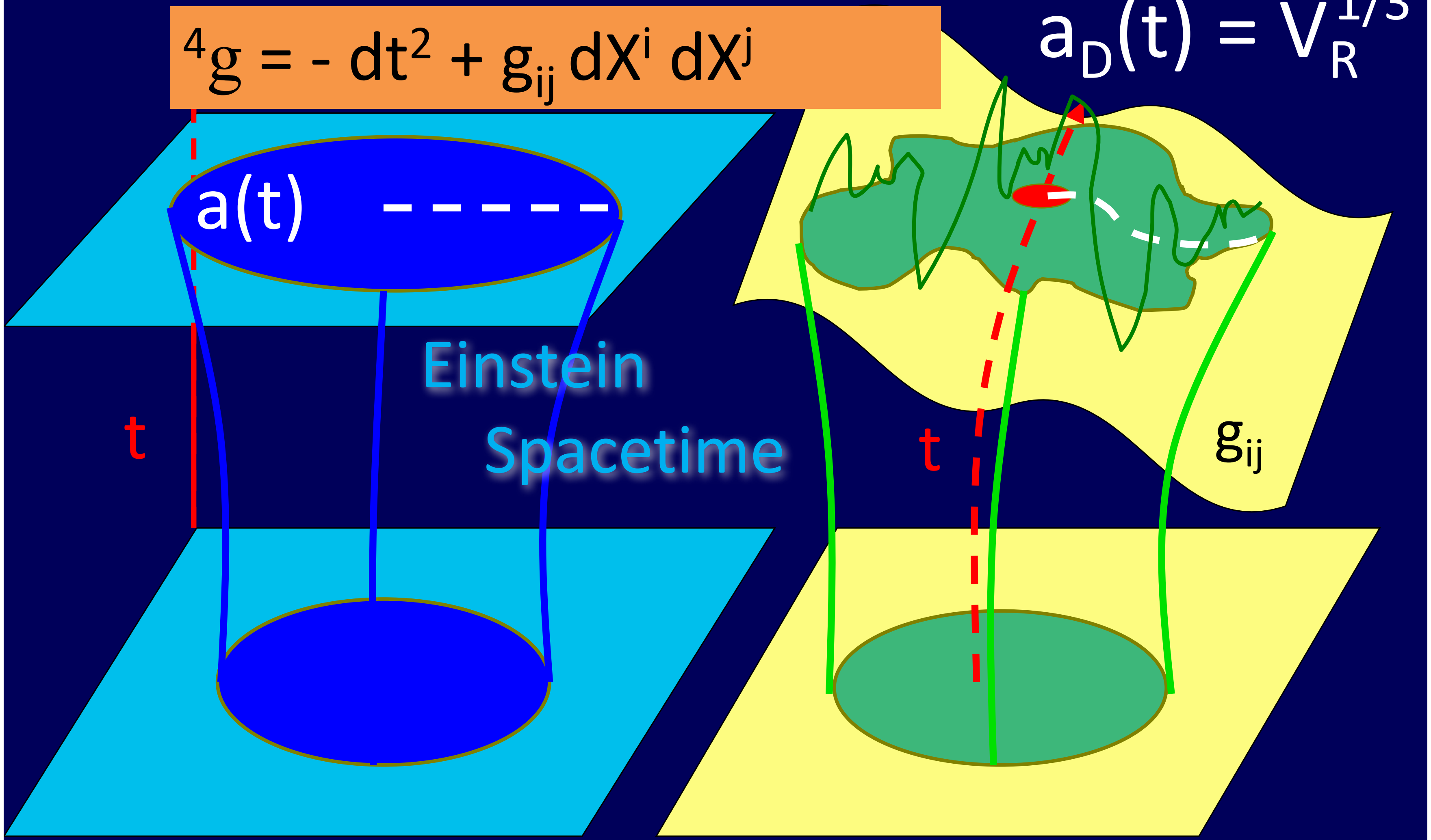
Backreaction arises when the fluctuations are allowed to determine the dynamics of the average model
Structures ‘talk‘ to the ‘background‘

Backreaction arises from inhomogeneities in geometry

Backreaction depends on the choice of foliation of space-time (weakly on cosmological scales)

Averaging in a flow-orthogonal foliation
Irrotational Dust

Averaging dust fluids in free fall



Averaging Operator

Spatial average of scalars on a compact domain :

$$\langle \mathcal{A} \rangle_{\mathcal{D}} := \frac{1}{V_{\mathcal{D}}} \int_{\mathcal{D}} \mathcal{A} \, d\mu_g$$

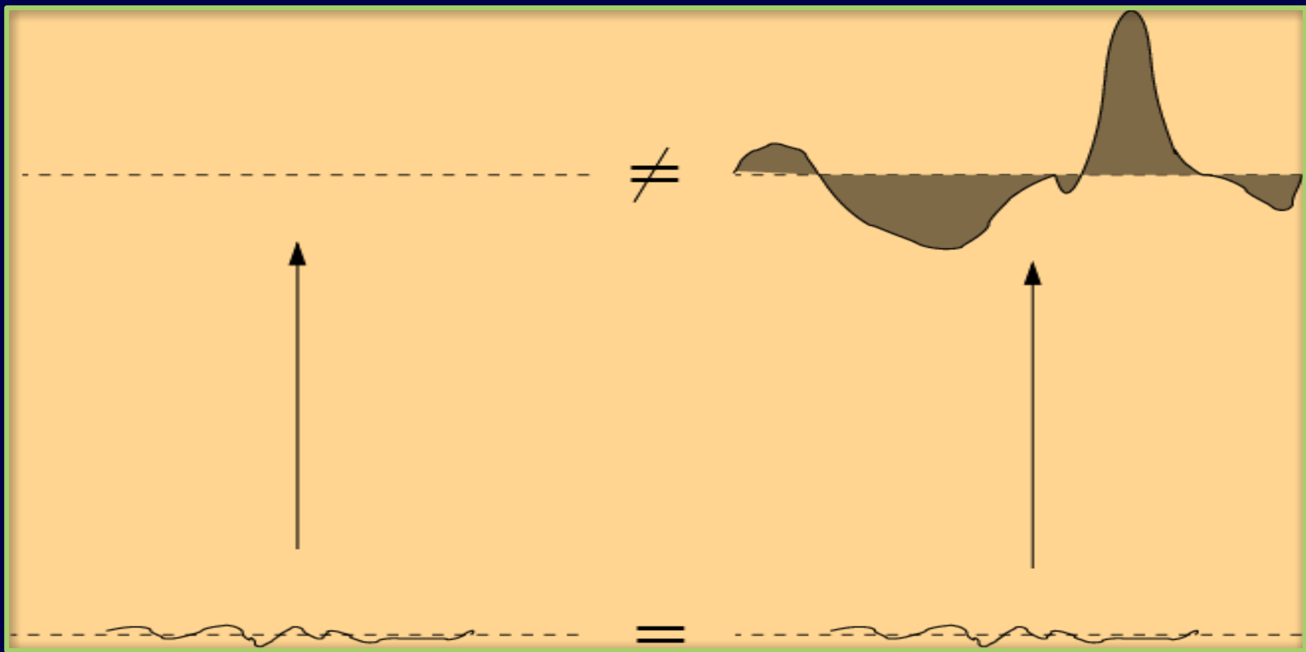
↑

Restmass conservation on the domain

important to compare averages at different times

Non-Commutativity

$$\frac{\partial}{\partial t}\langle \mathcal{A} \rangle \;-\; \langle \frac{\partial}{\partial t}\mathcal{A} \rangle \;=\; \langle \theta \mathcal{A} \rangle \;-\; \langle \theta \rangle \langle \mathcal{A} \rangle$$



Non-Commutativity

$$\frac{\partial}{\partial t}\langle \mathcal{A} \rangle - \langle \frac{\partial}{\partial t}\mathcal{A} \rangle = \langle \theta \mathcal{A} \rangle - \langle \theta \rangle \langle \mathcal{A} \rangle$$

$$\begin{aligned} \frac{\partial}{\partial t}\langle \theta \rangle - \langle \frac{\partial}{\partial t}\theta \rangle &= \langle \theta^2 \rangle - \langle \theta \rangle^2 \\ &= \langle (\theta - \langle \theta \rangle)^2 \rangle \end{aligned}$$

Relative Information Entropy increases

Kullback-Leibler distance : arXiv: gr-qc/0402076

$$\mathcal{S} = \int_{\mathcal{D}} \varrho \ln \frac{\varrho}{\langle \varrho \rangle} \, d\mu_g$$

- Deviations from the standard model increase !

$$\langle \partial_t \varrho \rangle - \partial_t \langle \varrho \rangle = \frac{1}{V} \partial_t \mathcal{S}$$

$$\frac{\partial_t \, \mathcal{S}\{\varrho || \langle \varrho \rangle_{\mathcal{D}}\}}{V_{\mathcal{D}}} = - \langle \delta \varrho \Theta \rangle_{\mathcal{D}} = - \langle \varrho \delta \Theta \rangle_{\mathcal{D}} = - \langle \delta \varrho \delta \Theta \rangle_{\mathcal{D}}$$

Volume acceleration despite local deceleration

$$\partial_t \theta = \Lambda - 4\pi G \varrho + 2II - I^2$$

$$\partial_t \langle\theta\rangle = \Lambda - 4\pi G \langle\varrho\rangle + 2\langle II\rangle - \langle I\rangle^2$$

$$\underline{2II - I^2} = -\frac{1}{3}\theta^2 - 2\sigma^2$$

$$\underline{2\langle II\rangle - \langle I\rangle^2} = \frac{2}{3}\langle(\theta - \langle\theta\rangle)^2\rangle - 2\langle(\sigma - \langle\sigma\rangle)^2\rangle$$

$$-\frac{1}{3}\langle\theta\rangle^2 - 2\langle\sigma\rangle^2$$

Kinematical Backreaction

- Acceleration Law :

$$3\frac{\ddot{a}}{a} + 4\pi G \varrho_H - \Lambda = 0$$

- Expansion Law :

$$3\left(\frac{\dot{a}}{a}\right)^2 - 8\pi G \varrho_H - \Lambda = -\frac{3k}{a^2}$$

- Conservation Law :

$$\dot{\varrho}_H + 3\left(\frac{\dot{a}}{a}\right)\varrho_H = 0$$

- Integrability :

Kinematical Backreaction

- Acceleration Law :

$$3\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} + 4\pi G \langle \varrho \rangle_{\mathcal{D}} - \Lambda = \mathcal{Q}_{\mathcal{D}}$$

- Expansion Law :

$$3\left(\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\right)^2 - 8\pi G \langle \varrho \rangle_{\mathcal{D}} - \Lambda = -\frac{\langle \mathcal{R} \rangle_{\mathcal{D}} + \mathcal{Q}_{\mathcal{D}}}{2}$$

- Conservation Law :

$$\langle \varrho \rangle\dot{}_{\mathcal{D}} + 3\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} \langle \varrho \rangle_{\mathcal{D}} = 0$$

- Integrability :

$$\frac{1}{a_{\mathcal{D}}^6}\partial_t \left(\mathcal{Q}_{\mathcal{D}}\, a_{\mathcal{D}}^6 \right) + \frac{1}{a_{\mathcal{D}}^2}\, \partial_t \left(\langle \mathcal{R} \rangle_{\mathcal{D}}\, a_{\mathcal{D}}^2 \right) = 0$$

Effect of Kinematical Backreaction

$$3\frac{\ddot{a}_{\mathcal{D}_t}}{a_{\mathcal{D}_t}} + 4\pi G\langle\varrho\rangle_{\mathcal{D}_t} - \Lambda = \mathcal{Q}_{\mathcal{D}_t}$$

$$3\frac{\dot{a}^2_{\mathcal{D}_t}}{a^2_{\mathcal{D}_t}} + 3\frac{k_{\mathcal{D}_t}}{a^2_{\mathcal{D}_t}} - 8\pi G\langle\varrho\rangle_{\mathcal{D}_t} - \Lambda = \frac{1}{a^2_{\mathcal{D}_t}}\int_{t_0}^{t} dt'\ \mathcal{Q}_{\mathcal{D}_{t'}}\frac{d}{dt'}a^2_{\mathcal{D}_{t'}}(t') \qquad H_{\mathcal{D}_t} := \frac{\dot{a}_{\mathcal{D}_t}}{a_{\mathcal{D}_t}}$$

Kinematical Dark Energy / Kinematical Dark Matter :

$$\mathcal{Q}_{\mathcal{D}_t} := 2\langle II\rangle_{\mathcal{D}_t} - \frac{2}{3}\langle I\rangle^2_{\mathcal{D}_t}$$

$$\mathcal{Q}_{\mathcal{D}_t} = \frac{2}{3}\left(\langle\theta^2\rangle_{\mathcal{D}_t} - \langle\theta\rangle^2_{\mathcal{D}_t}\right) + 2\langle\omega^2\rangle_{\mathcal{D}_t} - 2\langle\sigma^2\rangle_{\mathcal{D}_t} \ .$$

Effective Form

Recall : Standard Models for Dark Sources

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho_{\rm h}}{3} + \frac{\Lambda}{3} - \frac{k}{a^2};$$

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{4\pi G(\rho_{\rm h} + 3p_{\rm h})}{3} + \frac{\Lambda}{3};$$

$$\dot{\rho}_{\rm h} + 3\left(\frac{\dot{a}}{a}\right)(\rho_{\rm h} + p_{\rm h}) = 0.$$

$$\rho(t) = \frac{1}{2}\dot{\phi}^2 + V(\phi),$$

$$P(t) = \frac{1}{2}\dot{\phi}^2 - V(\phi).$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0.$$

Quintessence

Scalar Dark Matter

Inflation

Effective Equations – Friedmannian Form

$$\varrho^{\mathcal{D}}_{\rm eff} = \langle\varrho\rangle_{\mathcal{D}} - \frac{1}{16\pi G}\mathcal{Q}_{\mathcal{D}} - \frac{1}{16\pi G}\langle\mathcal{R}\rangle_{\mathcal{D}} \quad ; \quad p^{\mathcal{D}}_{\rm eff} = -\frac{1}{16\pi G}\mathcal{Q}_{\mathcal{D}} + \frac{1}{48\pi G}\langle\mathcal{R}\rangle_{\mathcal{D}} \; .$$

$$3\left(\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\right)^2 - 8\pi G\varrho^{\mathcal{D}}_{\rm eff} - \Lambda = 0 \; ;$$

$$3\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} + 4\pi G(\varrho^{\mathcal{D}}_{\rm eff} + 3p^{\mathcal{D}}_{\rm eff}) - \Lambda = 0 \; ;$$

$$\dot{\varrho}^{\mathcal{D}}_{\rm eff} + 3\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\left(\varrho^{\mathcal{D}}_{\rm eff} + p^{\mathcal{D}}_{\rm eff}\right) = 0 \; .$$

$$-\frac{1}{8\pi G}\langle\mathcal{R}\rangle_{\mathcal{D}} = 3U_{\mathcal{D}}$$

$$-\frac{1}{8\pi G}\mathcal{Q}_{\mathcal{D}} = \epsilon\dot{\Phi}^2_{\mathcal{D}} - U_{\mathcal{D}}$$

Effective Scalar Field : ‘Morphon‘

$$\varrho^{\mathcal{D}}_{\Phi} = \epsilon\frac{1}{2}\dot{\Phi}^2_{\mathcal{D}} + U_{\mathcal{D}} \quad ; \quad p^{\mathcal{D}}_{\Phi} = \epsilon\frac{1}{2}\dot{\Phi}^2_{\mathcal{D}} - U_{\mathcal{D}}$$

Buchert, Larena, Alimi
arXiv: gr-qc / 0606020

$$\frac{1}{a^6_{\mathcal{D}}}\partial_t\left(\mathcal{Q}_{\mathcal{D}}\, a^6_{\mathcal{D}}\right) + \frac{1}{a^2_{\mathcal{D}}}\partial_t\left(\langle\mathcal{R}\rangle_{\mathcal{D}}\, a^2_{\mathcal{D}}\right) = 0$$

Effective Equations – Friedmannian Form

$$\varrho_{\rm eff}^{\mathcal{D}} = \langle\varrho\rangle_{\mathcal{D}} - \frac{1}{16\pi G}\mathcal{Q}_{\mathcal{D}} - \frac{1}{16\pi G}\langle\mathcal{R}\rangle_{\mathcal{D}} \quad ; \quad p_{\rm eff}^{\mathcal{D}} = -\frac{1}{16\pi G}\mathcal{Q}_{\mathcal{D}} + \frac{1}{48\pi G}\langle\mathcal{R}\rangle_{\mathcal{D}} \ .$$

$$3\left(\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\right)^2 - 8\pi G\varrho_{\rm eff}^{\mathcal{D}} - \Lambda = 0 \ ;$$

$$3\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} + 4\pi G(\varrho_{\rm eff}^{\mathcal{D}} + 3p_{\rm eff}^{\mathcal{D}}) - \Lambda = 0 \ ;$$

$$\dot{\varrho}_{\rm eff}^{\mathcal{D}} + 3\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\left(\varrho_{\rm eff}^{\mathcal{D}} + p_{\rm eff}^{\mathcal{D}}\right) = 0 \ .$$

$$-\frac{1}{8\pi G}\langle\mathcal{R}\rangle_{\mathcal{D}} = 3U_{\mathcal{D}}$$

$$-\frac{1}{8\pi G}\mathcal{Q}_{\mathcal{D}} = \epsilon\dot{\Phi}_{\mathcal{D}}^2 - U_{\mathcal{D}}$$

Effective Scalar Field : ‘Morphon‘

$$\varrho_{\Phi}^{\mathcal{D}} = \epsilon\frac{1}{2}\dot{\Phi}_{\mathcal{D}}^2 + U_{\mathcal{D}} \quad ; \quad p_{\Phi}^{\mathcal{D}} = \epsilon\frac{1}{2}\dot{\Phi}_{\mathcal{D}}^2 - U_{\mathcal{D}}$$

$$\ddot{\Phi}_{\mathcal{D}} + 3H_{\mathcal{D}}\dot{\Phi}_{\mathcal{D}} + \epsilon\frac{\partial}{\partial\Phi_{\mathcal{D}}}U(\Phi_{\mathcal{D}}, \langle\varrho\rangle_{\mathcal{D}}) = 0$$

Buchert, Larena, Alimi
arXiv: gr-qc / 060602

Closure Assumptions

Closure Assumptions - general

$$\varrho_{\text{eff}}^{\mathcal{D}} = \langle\varrho\rangle_{\mathcal{D}} - \frac{1}{16\pi G}\mathcal{Q}_{\mathcal{D}} - \frac{1}{16\pi G}\langle\mathcal{R}\rangle_{\mathcal{D}} \quad ; \quad p_{\text{eff}}^{\mathcal{D}} = -\frac{1}{16\pi G}\mathcal{Q}_{\mathcal{D}} + \frac{1}{48\pi G}\langle\mathcal{R}\rangle_{\mathcal{D}} \quad .$$

$$3\left(\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\right)^2 - 8\pi G\varrho_{\text{eff}}^{\mathcal{D}} - \Lambda \;=\; 0 \;;$$

$$3\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} + 4\pi G(\varrho_{\text{eff}}^{\mathcal{D}} + 3p_{\text{eff}}^{\mathcal{D}}) - \Lambda \;=\; 0 \;;$$

$$\dot{\varrho}_{\text{eff}}^{\mathcal{D}} + 3\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\left(\varrho_{\text{eff}}^{\mathcal{D}} + p_{\text{eff}}^{\mathcal{D}}\right) \;=\; 0 \;.$$

→ Dynamical Equation of State

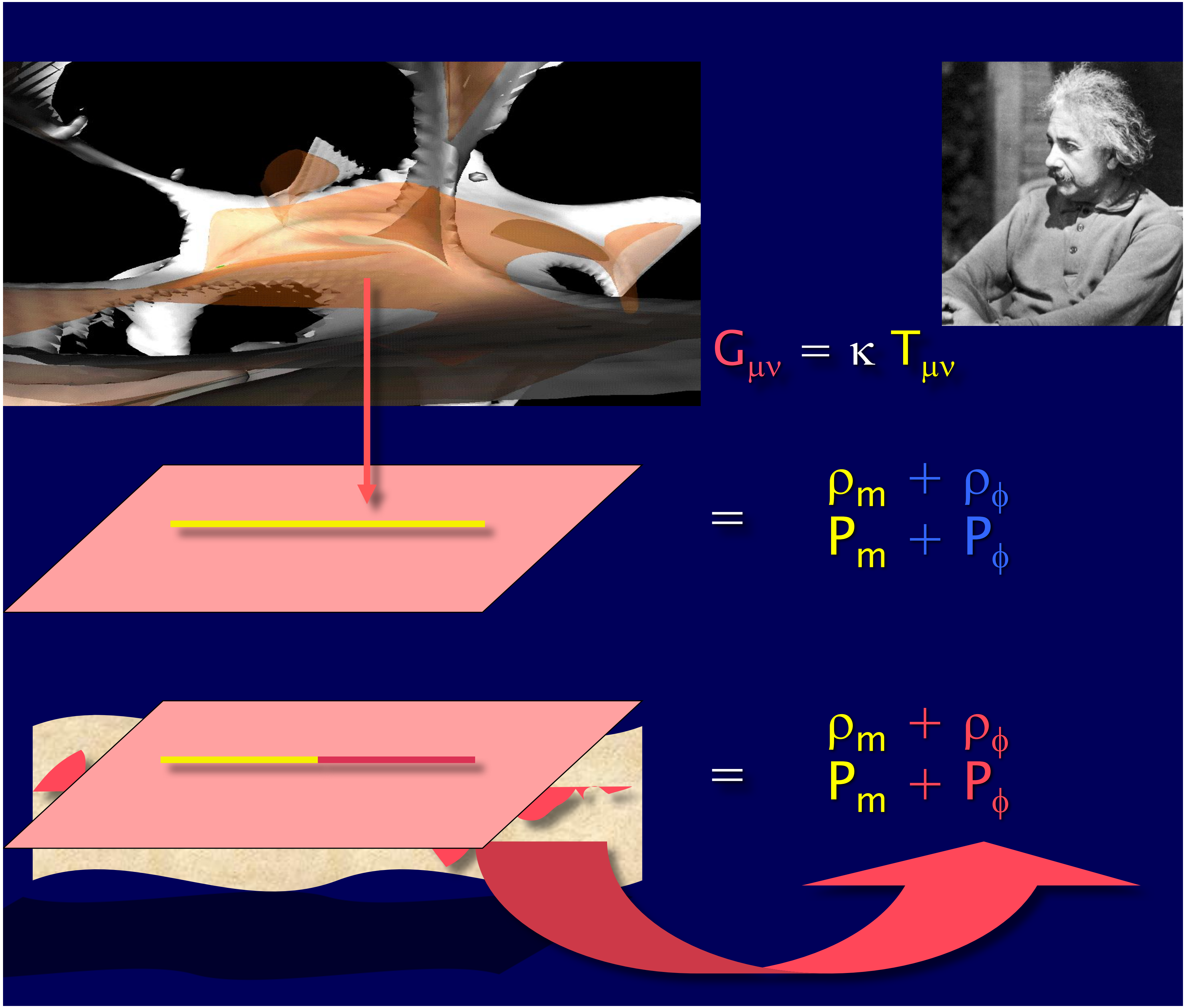
Exact Scaling and Effective Quintessence

$$\mathcal{Q}_{\mathcal{D}} = r\ \langle R \rangle_{\mathcal{D}} = r\ R_{\mathcal{D}_{\mathrm{i}}}\, a_{\mathcal{D}}^{n} \quad ; \ n = -2\frac{(1+3r)}{(1+r)} \ ; \ r = -\frac{(n+2)}{(n+6)}$$

$$U(\Phi_{\mathcal{D}}, \langle \varrho \rangle_{\mathcal{D}_{\mathrm{i}}}) = \frac{2(1+r)}{3}\left((1+r)\frac{\Omega_R^{\mathcal{D}_{\mathrm{i}}}}{\Omega_m^{\mathcal{D}_{\mathrm{i}}}}\right)^{\frac{3}{n+3}} \langle \varrho \rangle_{\mathcal{D}_{\mathrm{i}}} \sinh^{\frac{2n}{n+3}}\left(\frac{(n+3)}{\sqrt{-\epsilon n}}\sqrt{2\pi G}\Phi_{\mathcal{D}}\right)$$

$$\Phi_{\mathcal{D}}(a_{\mathcal{D}}) = \frac{2\sqrt{\epsilon(1+3r)(1+r)}}{(1-3r)\sqrt{\pi G}} \operatorname{arsinh}\left(\sqrt{\frac{-(1+r)\mathcal{R}_{\mathcal{D}_{\mathrm{i}}}}{16\pi G\langle \varrho \rangle_{\mathcal{D}_{\mathrm{i}}}} a_{\mathcal{D}}^{\frac{(1-3r)}{(1+r)}}}\right)$$
$$= \frac{\sqrt{-2\epsilon n}}{(n+3)\sqrt{\pi G}} \operatorname{arsinh}\left(\sqrt{(1+r)\gamma_{\mathcal{R}m}^{\mathcal{D}}}\right) ,$$

$$E_{\mathrm{kin}}^{\mathcal{D}} + \frac{(1+3r)}{2\epsilon} E_{\mathrm{pot}}^{\mathcal{D}} = 0 \ .$$


$G_{\mu\nu} = \kappa T_{\mu\nu}$
$= \rho_m + \rho_\phi$
$P_m + P_\phi$
$= \rho_m + \rho_\phi$
$P_m + P_\phi$

Take home summary II

Backreaction can act accelerating or decelarating
as a function of scale

Backreaction is due to non-local fluctuation terms

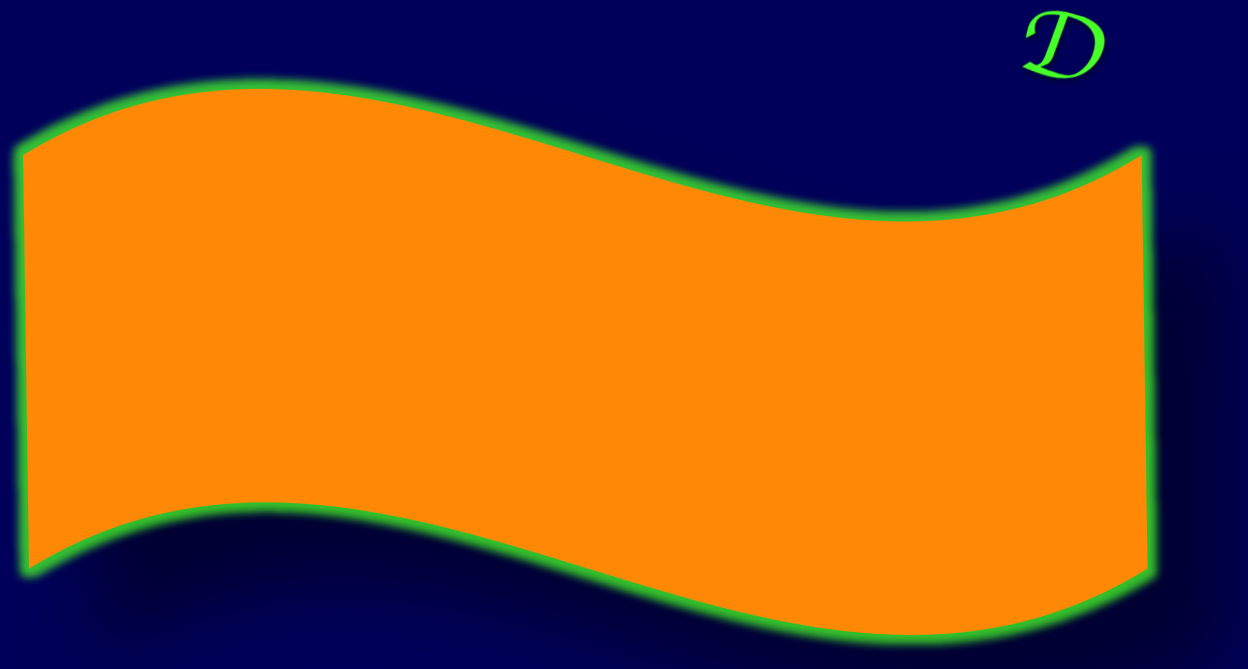
Backreaction couples to the average scalar curvature
Structures ‘talk‘ to the ‘background‘

Backreaction can be described as an effective scalar field

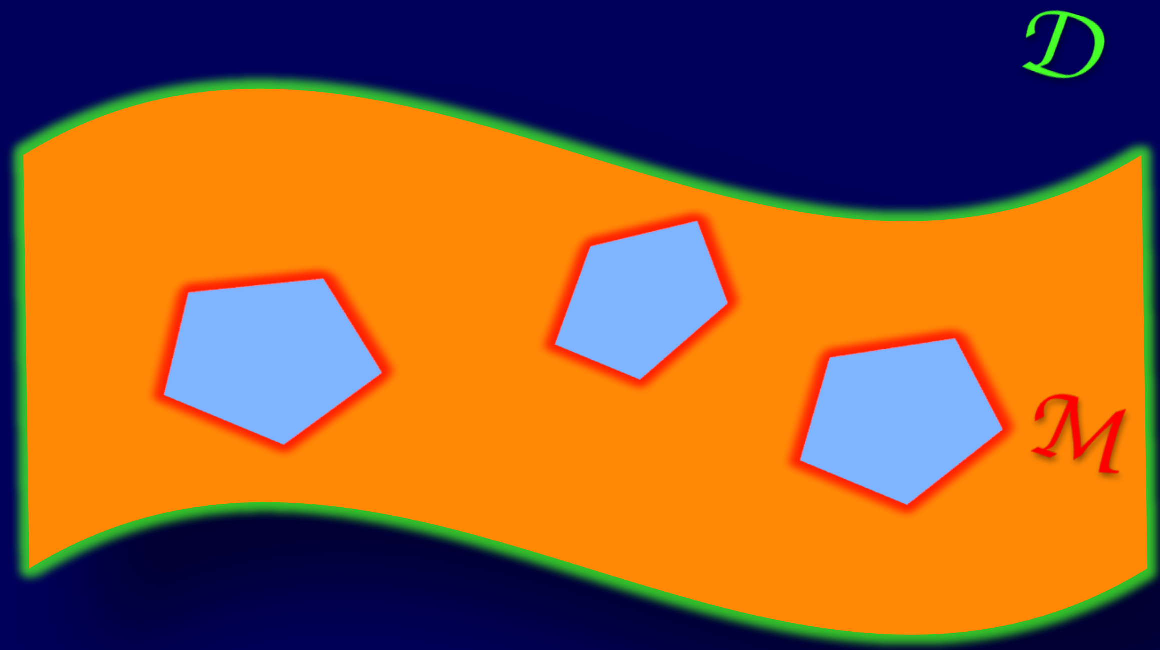
Dark Energy-free models

Background-free Modeling

Example : exact average dynamics
for a volume partitioning of spatial slices



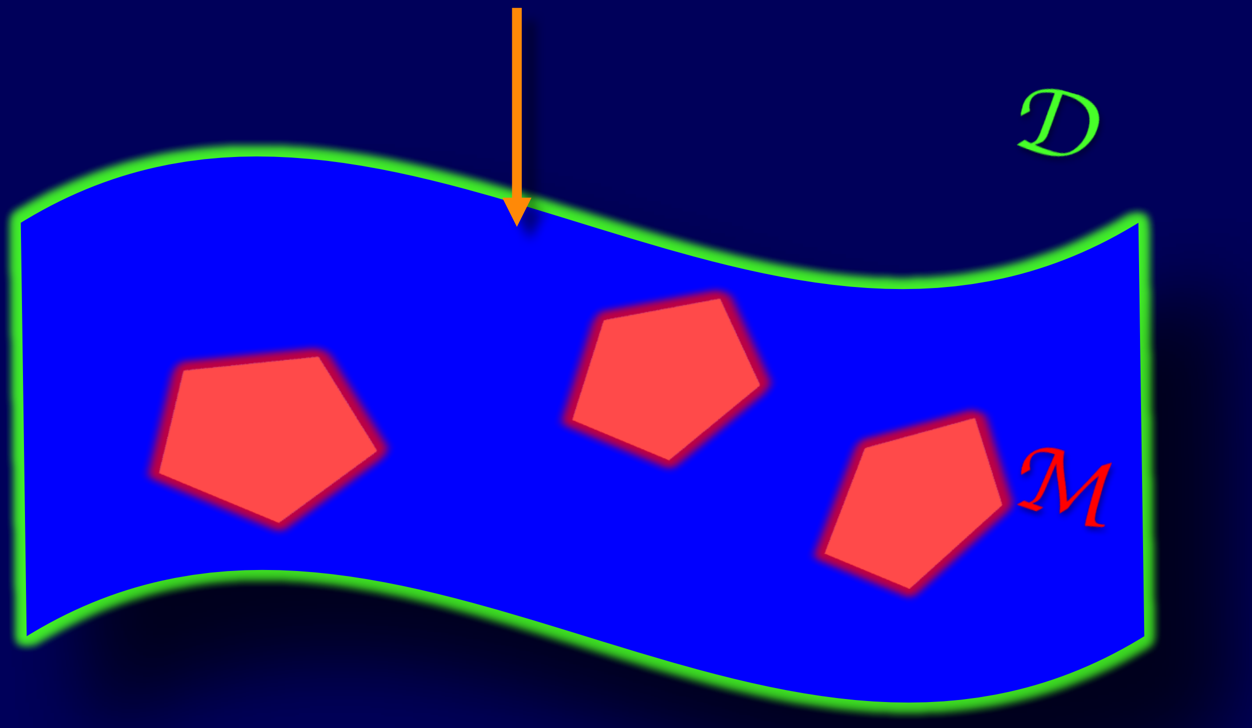
Background-free Two-scale Model



Background-free Two-scale Model

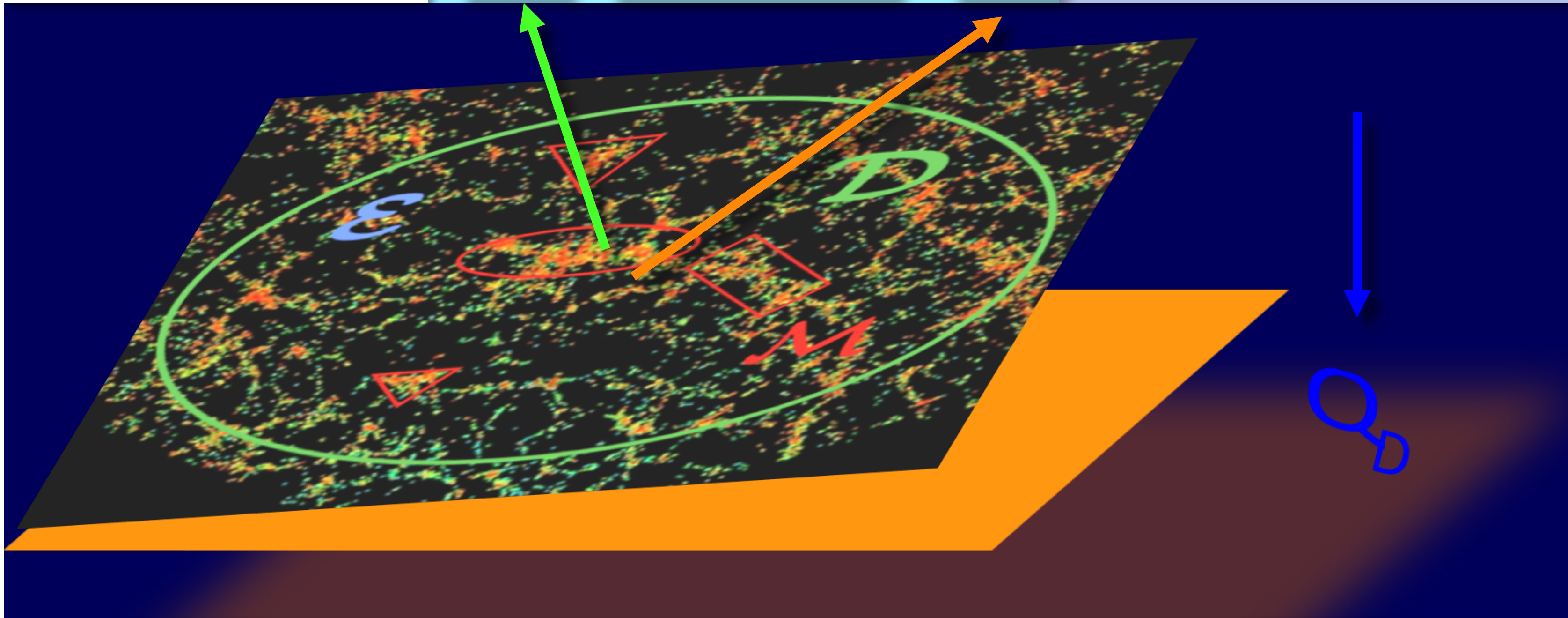
$\lambda_{\mathcal{M}} := |\mathcal{M}| \,/\, |\mathcal{D}|$

$\mathcal{D} = \mathcal{M} \cup \mathcal{E}$



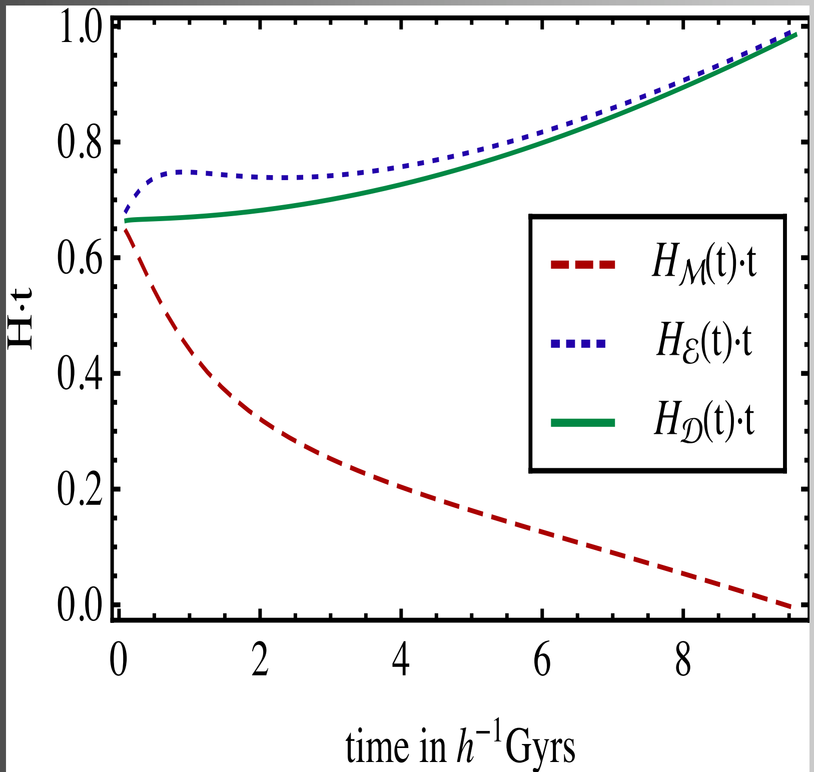
Acceleration in the Two-scale Model

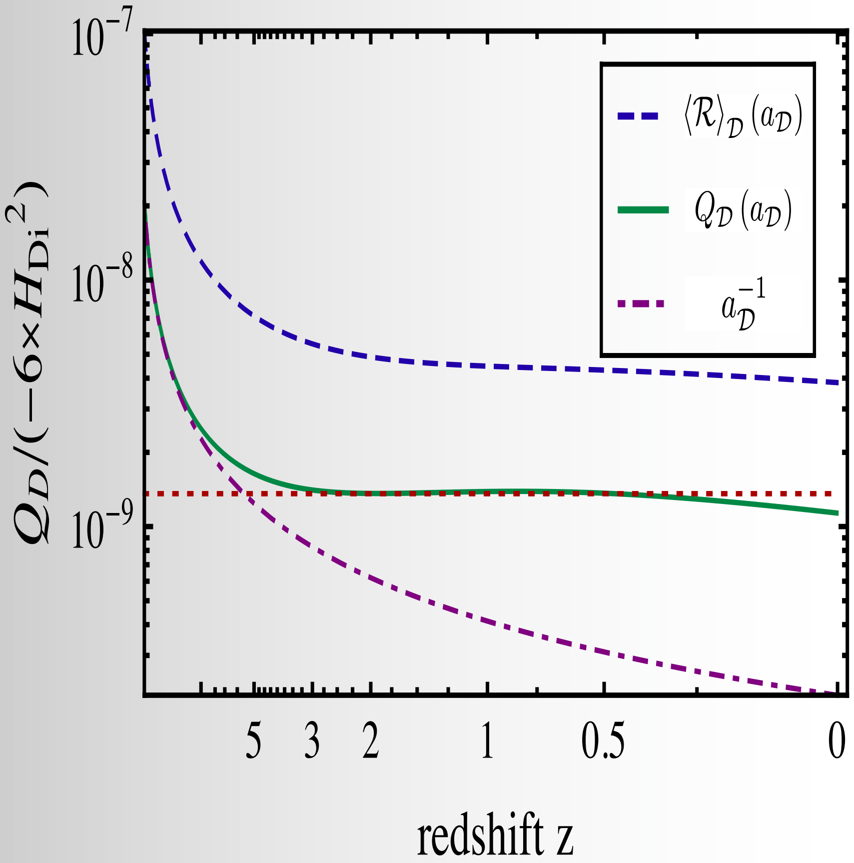
$$a_{\mathcal{D}}(t) := \left(\frac{V_{\mathcal{D}}(t)}{V_{\mathcal{D}_i}}\right)^{1/3} \qquad 3\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} + 4\pi G \langle \varrho \rangle_{\mathcal{D}} - \Lambda \; = \; \mathcal{Q}_{\mathcal{D}} = \frac{2}{3}\left\langle \left(\theta - \langle\theta\rangle_{\mathcal{D}}\right)^2 \right\rangle_{\mathcal{D}} - 2\left\langle \sigma^2 \right\rangle_{\mathcal{D}}$$



$$\mathcal{Q}_{\mathcal{D}} \; = \; \lambda_{\mathcal{M}} \mathcal{Q}_{\mathcal{M}} + (1 - \lambda_{\mathcal{M}})\, \mathcal{Q}_{\mathcal{E}} + 6\lambda_{\mathcal{M}}\, (1 - \lambda_{\mathcal{M}})\, (H_{\mathcal{M}} - H_{\mathcal{E}})^2$$

Acceleration in the Two-scale Model





Wiegand, Buchert
arXiv: 1002.3912

Take home summary III

Backreaction arises due to differential expansion

Structures 'talk' to the 'background'

Backreaction arises from the non-conservation of curvature

Models that feature conserved curvature

describe cosmic variance, not backreaction

Backreaction leads to emerging negative curvature

in a void-dominated Universe

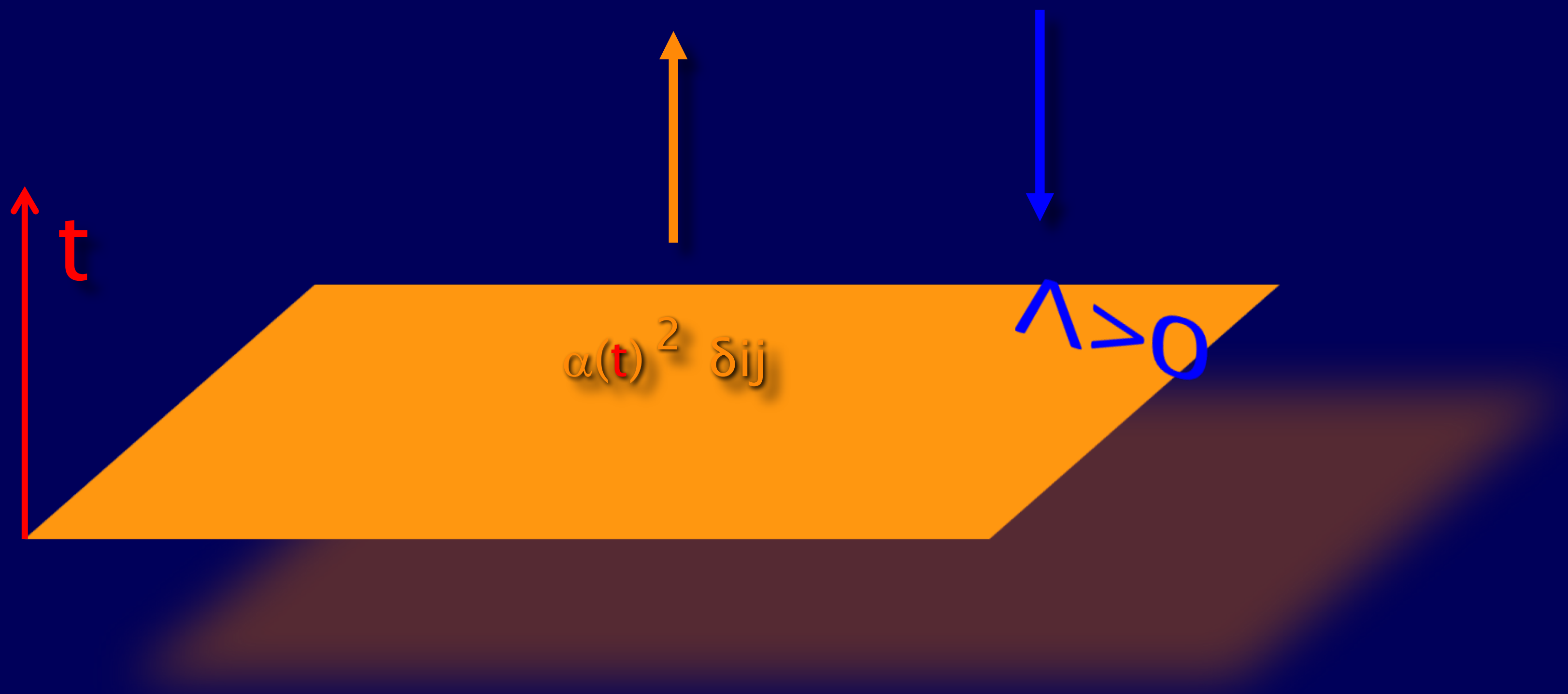
Some words on the link to observations

Acceleration in the Standard Model

local acceleration

global acceleration

$$3\frac{\ddot{a}}{a} + 4\pi G \varrho_H - \Lambda = 0$$



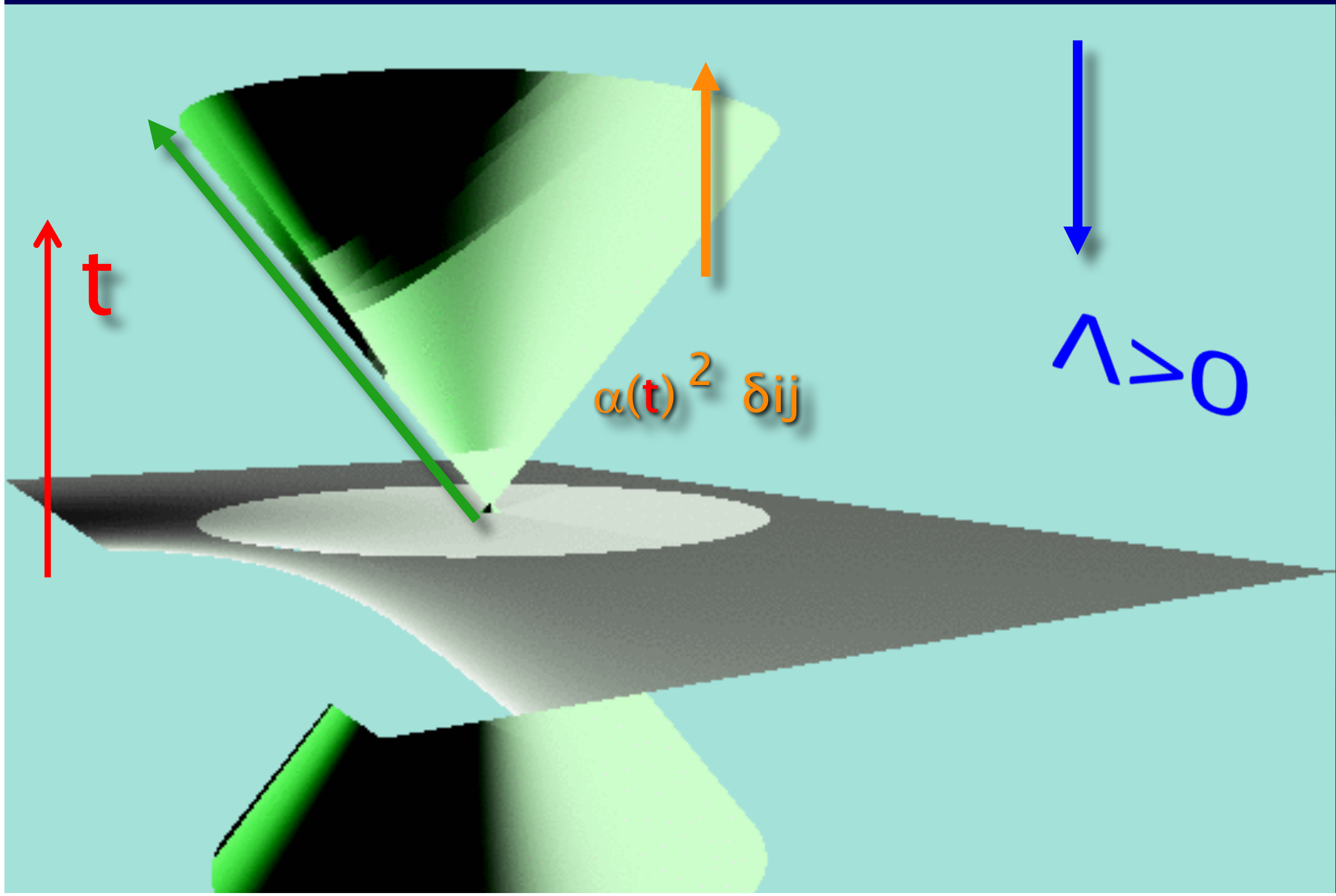
Acceleration in the Standard Model

local acceleration

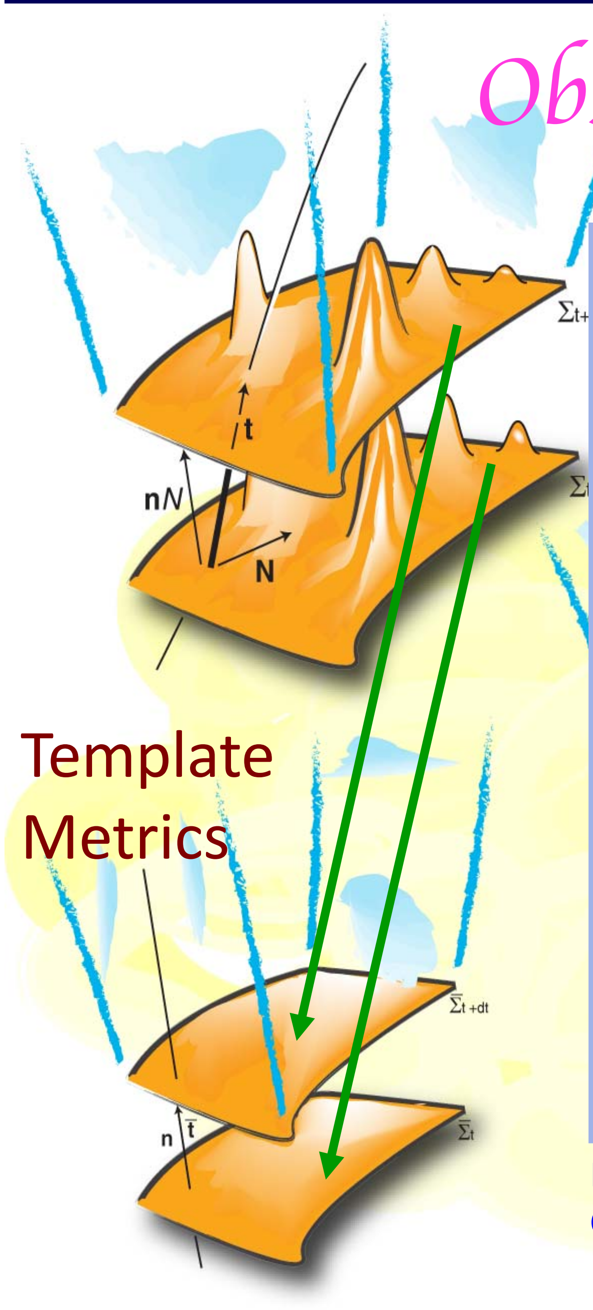
global acceleration

apparent acceleration

$$3\frac{\ddot{a}}{a} + 4\pi G \varrho_H - \Lambda = 0$$



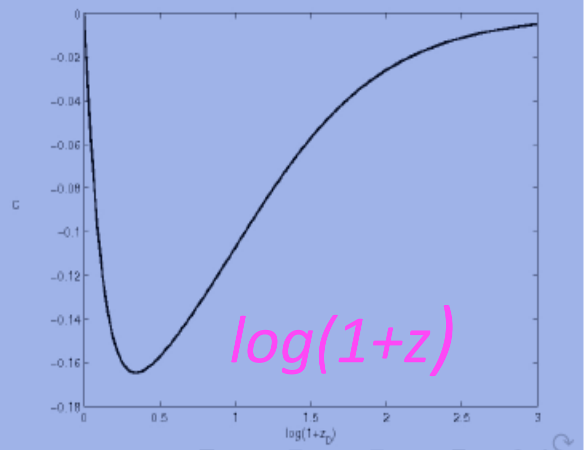
Observational Strategies



- $C(z) = 1 + H^2(DD'' - D'^2) + HH'DD'$ with $D \equiv (1+z)d_A$ is identically zero for FRW, different from 0 otherwise [C. Clarkson & al, arXiv:0712.3457]
- In our models:

$$C(z_{\mathcal{D}}) = -\frac{H_{\mathcal{D}}(z_{\mathcal{D}})r(z_{\mathcal{D}})\kappa'_{\mathcal{D}}(z_{\mathcal{D}})}{2H_{\mathcal{D}_0}\sqrt{1-\kappa_{\mathcal{D}}(z_{\mathcal{D}})r^2(z_{\mathcal{D}})}} .$$

- Testable prediction of the model.
- Can allow to make the difference with a quintessence field with the same n.



Template Metrics

Larena, Alimi, Buchert, Kunz, Corasaniti arXiv: 0808.1161

Euclid

Further Reading :

arXiv:

- gr-qc/0001056
- 0707.2153
- 1103.2016
- 1112.5335