

hadron spectroscopy from lattice QCD

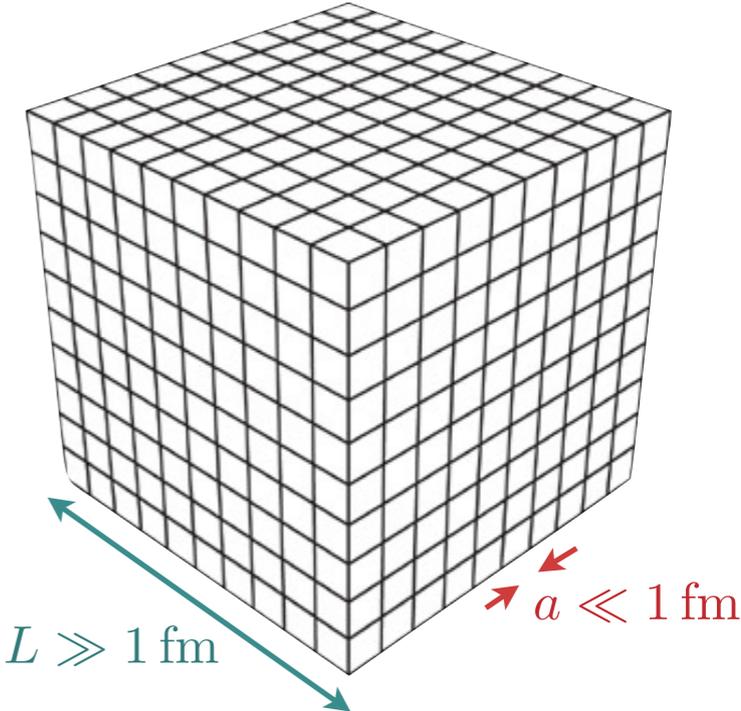
Jozef Dudek

$$\int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A_\mu f(\psi, \bar{\psi}, A_\mu) e^{i \int d^4x \mathcal{L}_{\text{QCD}}(\psi, \bar{\psi}, A_\mu)}$$

sum over quark/gluon field configurations

in Euclidean spacetime, probability for a field configuration

generate field configurations → compute correlation functions



lattice QCD is QCD under controlled approximations

discretisation choice / finite lattice spacing

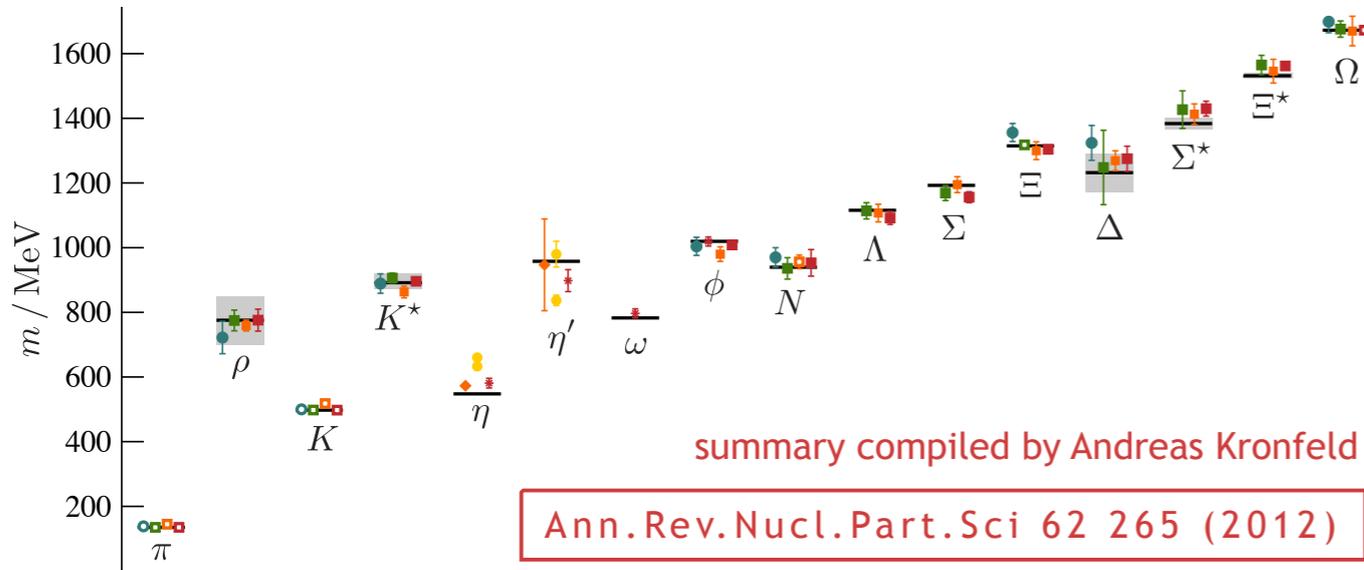
choice of quark mass value

finite spacetime volume

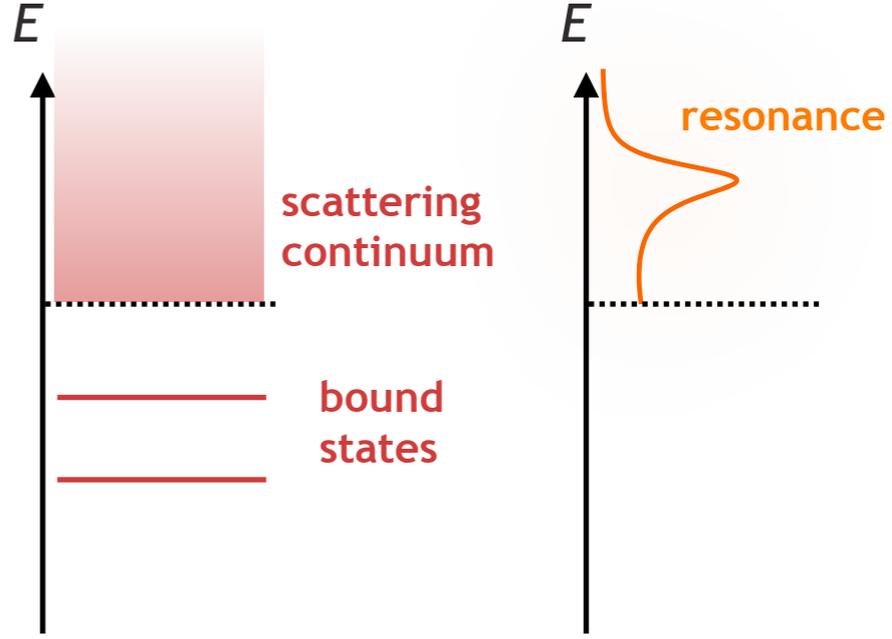
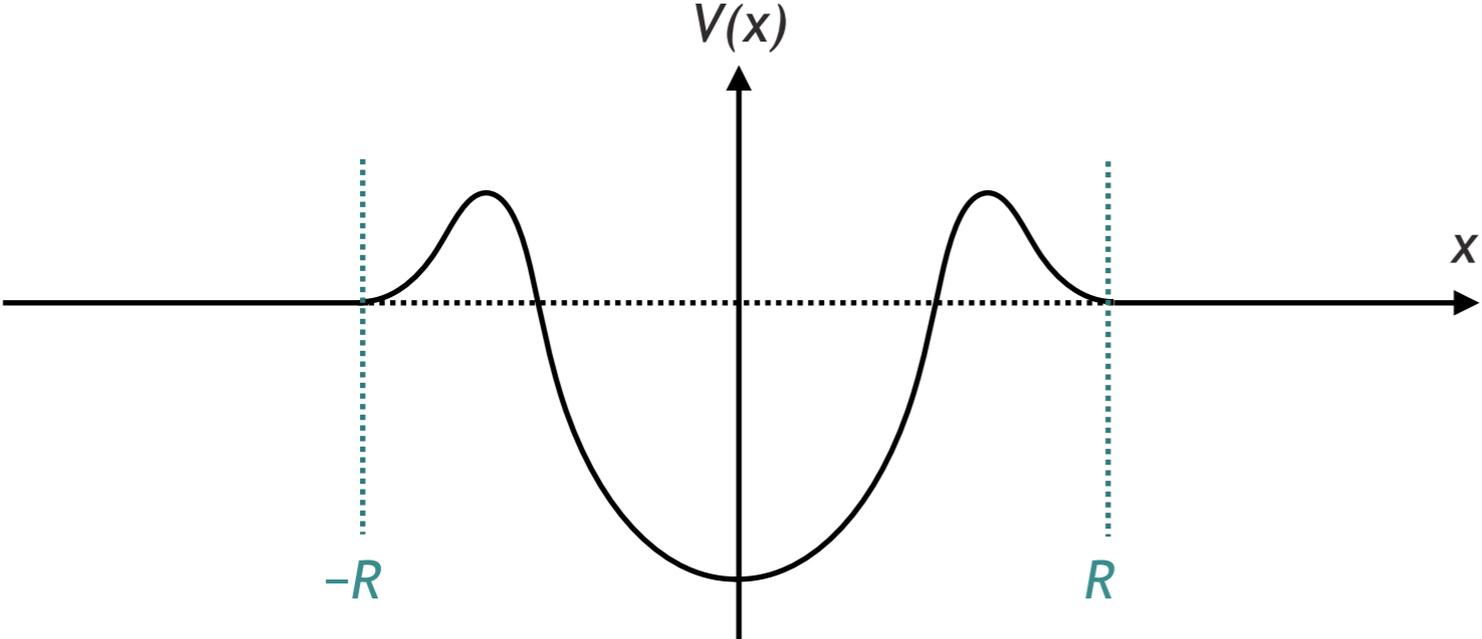
spectrum in two-point correlation functions

$$\langle 0 | \mathcal{O}_i(t) \mathcal{O}_j(0) | 0 \rangle = \sum_n e^{-E_n t} \langle 0 | \mathcal{O}_i | n \rangle \langle n | \mathcal{O}_j | 0 \rangle$$

lattice qcd light hadron spectrum



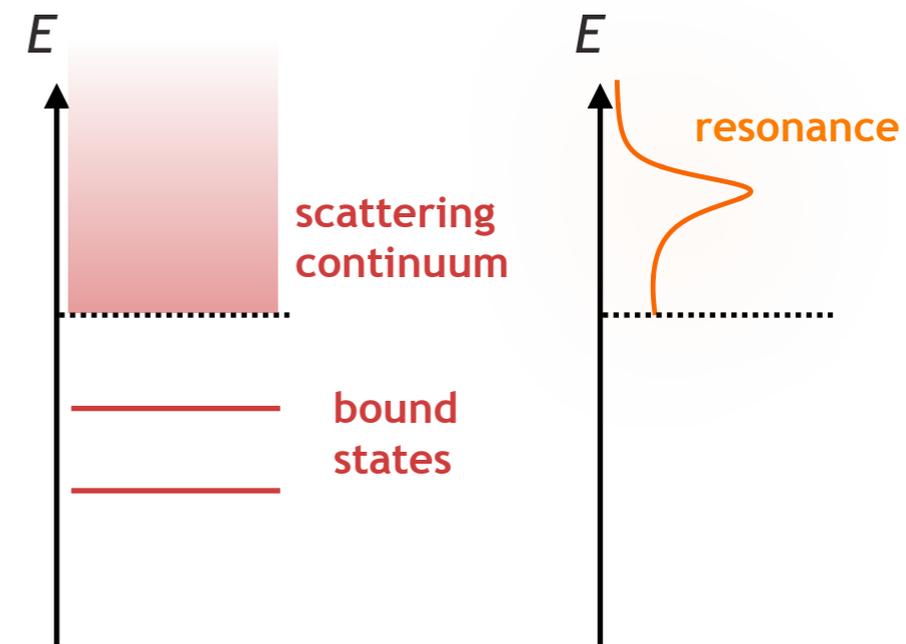
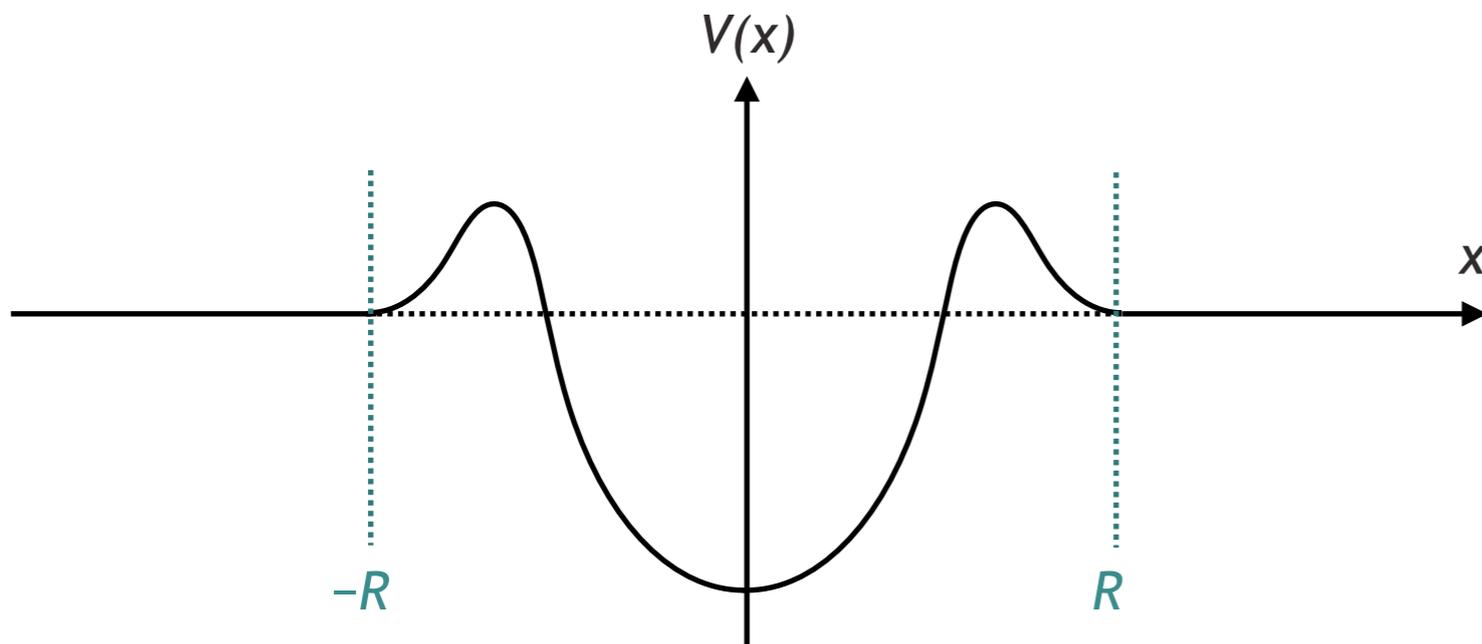
but what about excited states, the resonances of QCD ... ?



$$\psi(|x| > R) \sim \cos(p|x| + \delta(p))$$

phase shift

resonances lie in the continuous spectrum of scattering states



$$\psi(|x| > R) \sim \cos(p|x| + \delta(p))$$

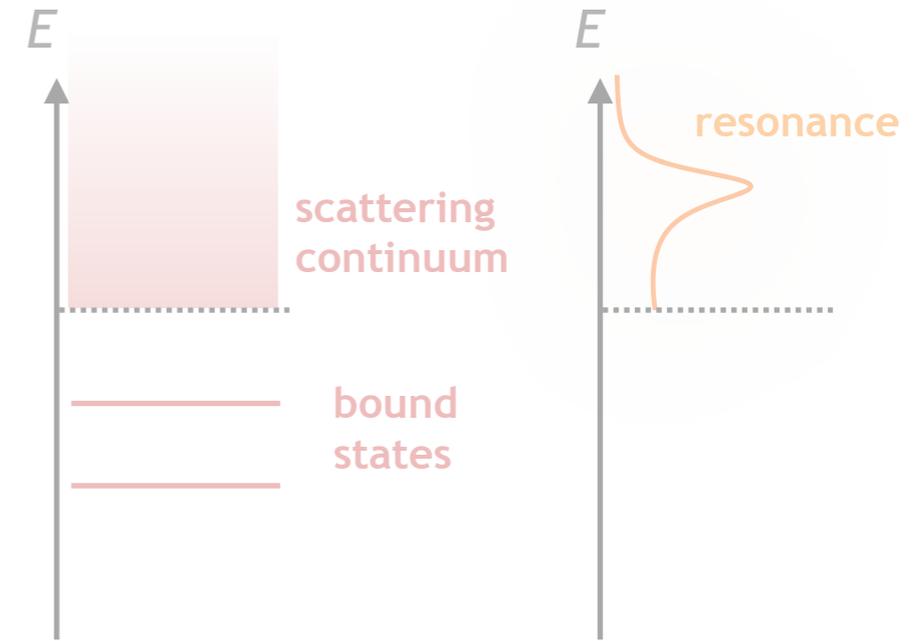
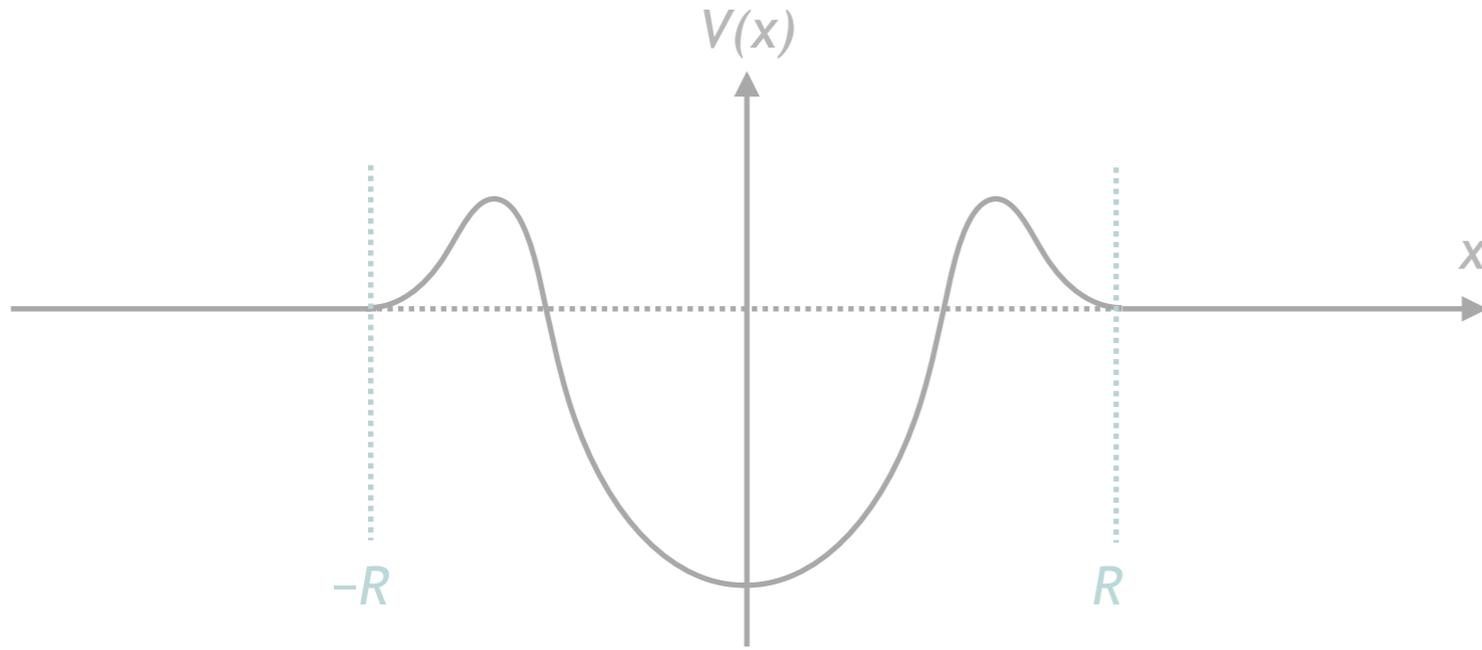
phase shift

resonances lie in the continuous spectrum of scattering states

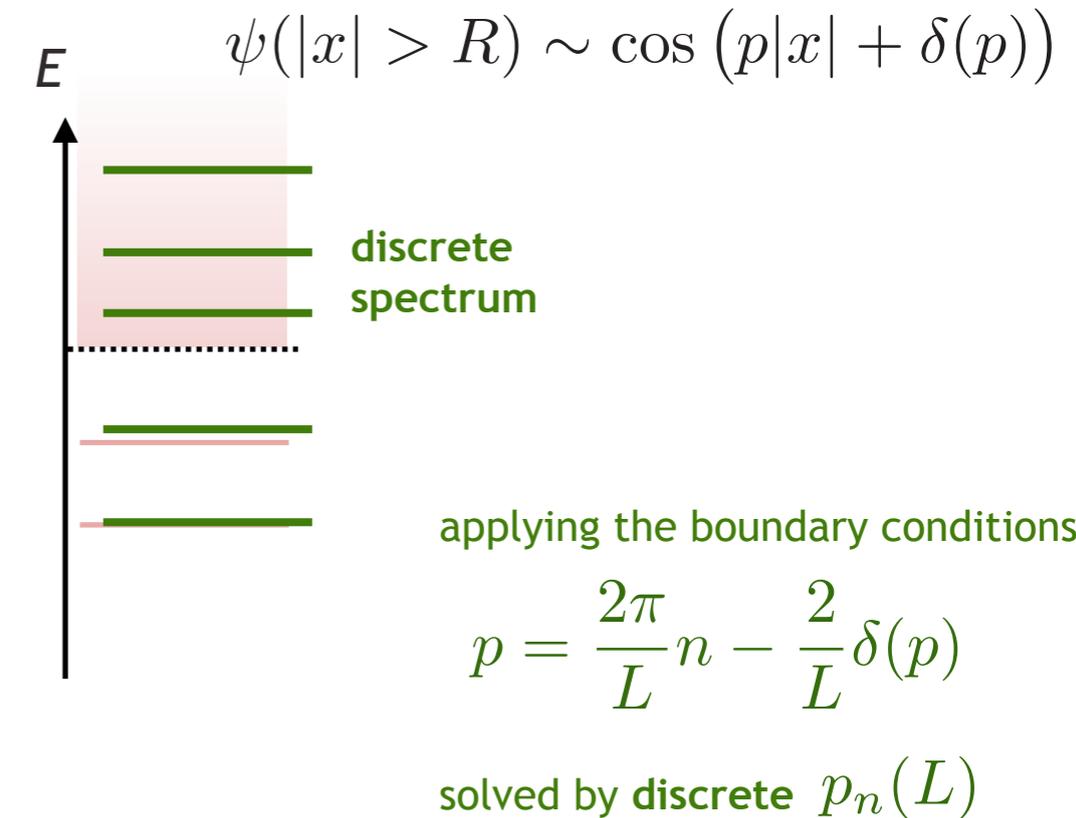
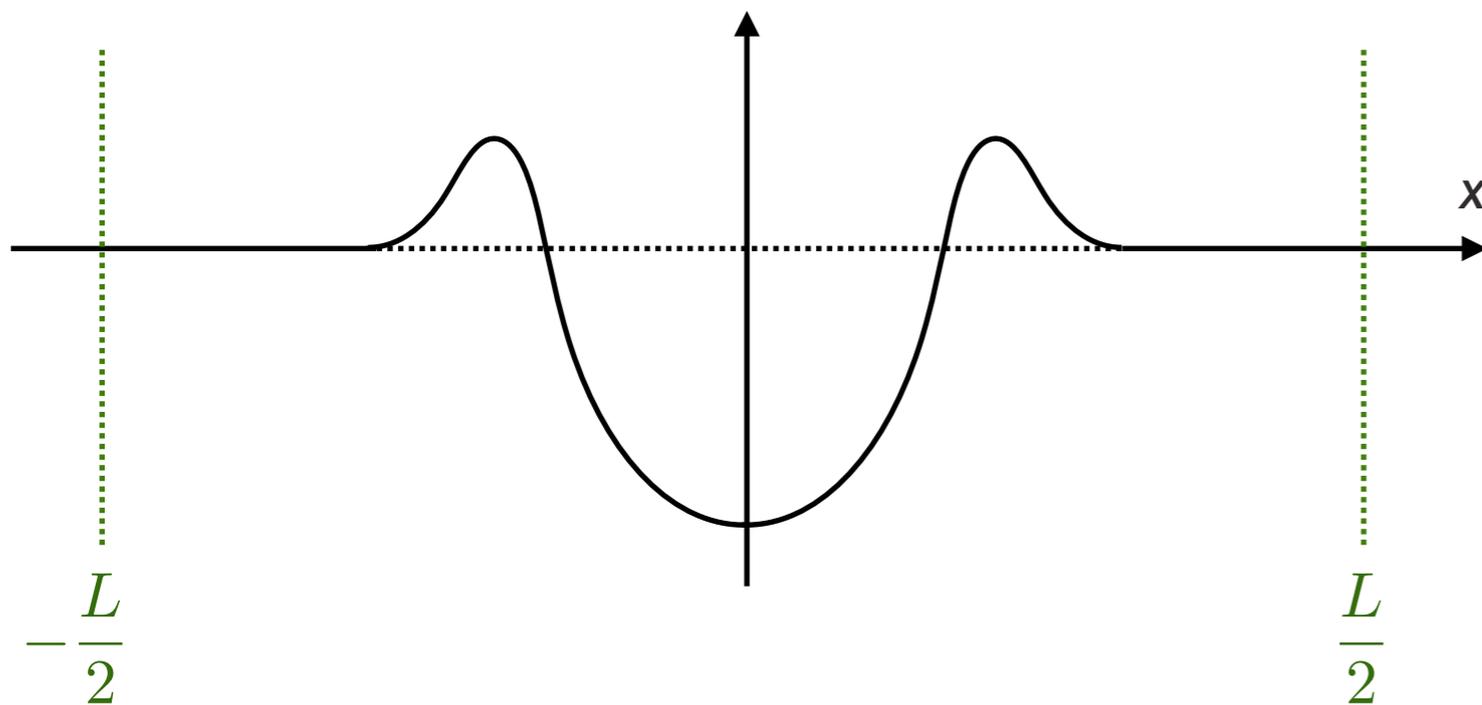
but ...

$$\langle 0 | \mathcal{O}_i(t) \mathcal{O}_j(0) | 0 \rangle = \sum_n e^{-E_n t} \langle 0 | \mathcal{O}_i | n \rangle \langle n | \mathcal{O}_j | 0 \rangle$$

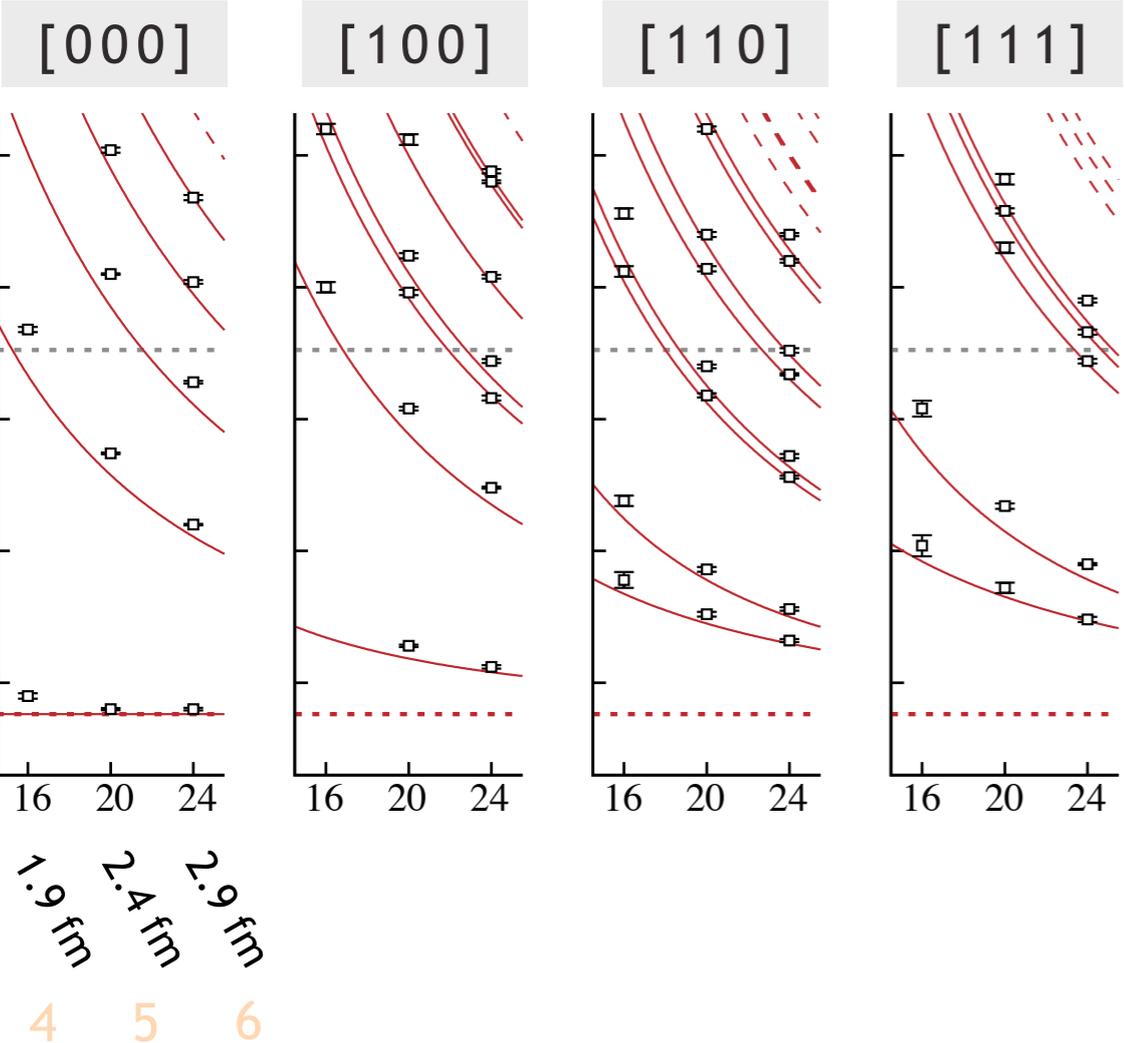
a discrete spectrum in finite-volume



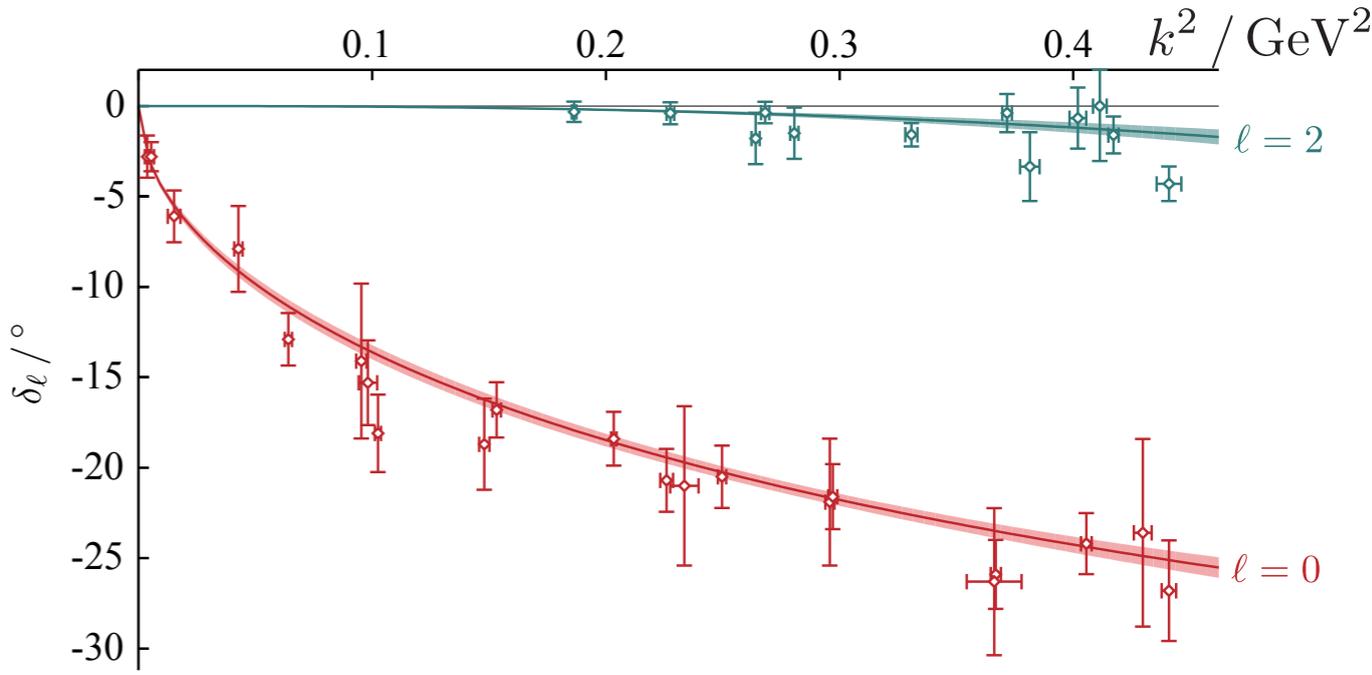
but in a periodic volume ...



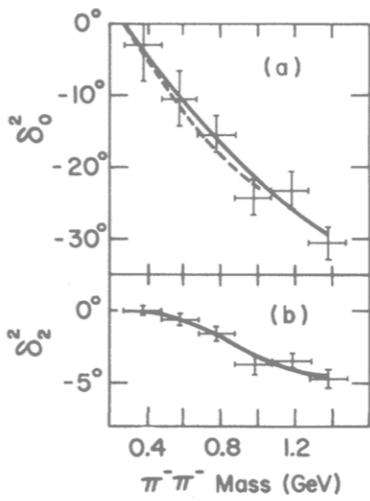
$m_\pi \sim 391$ MeV



scattering phase-shifts



repulsive scattering, no resonances

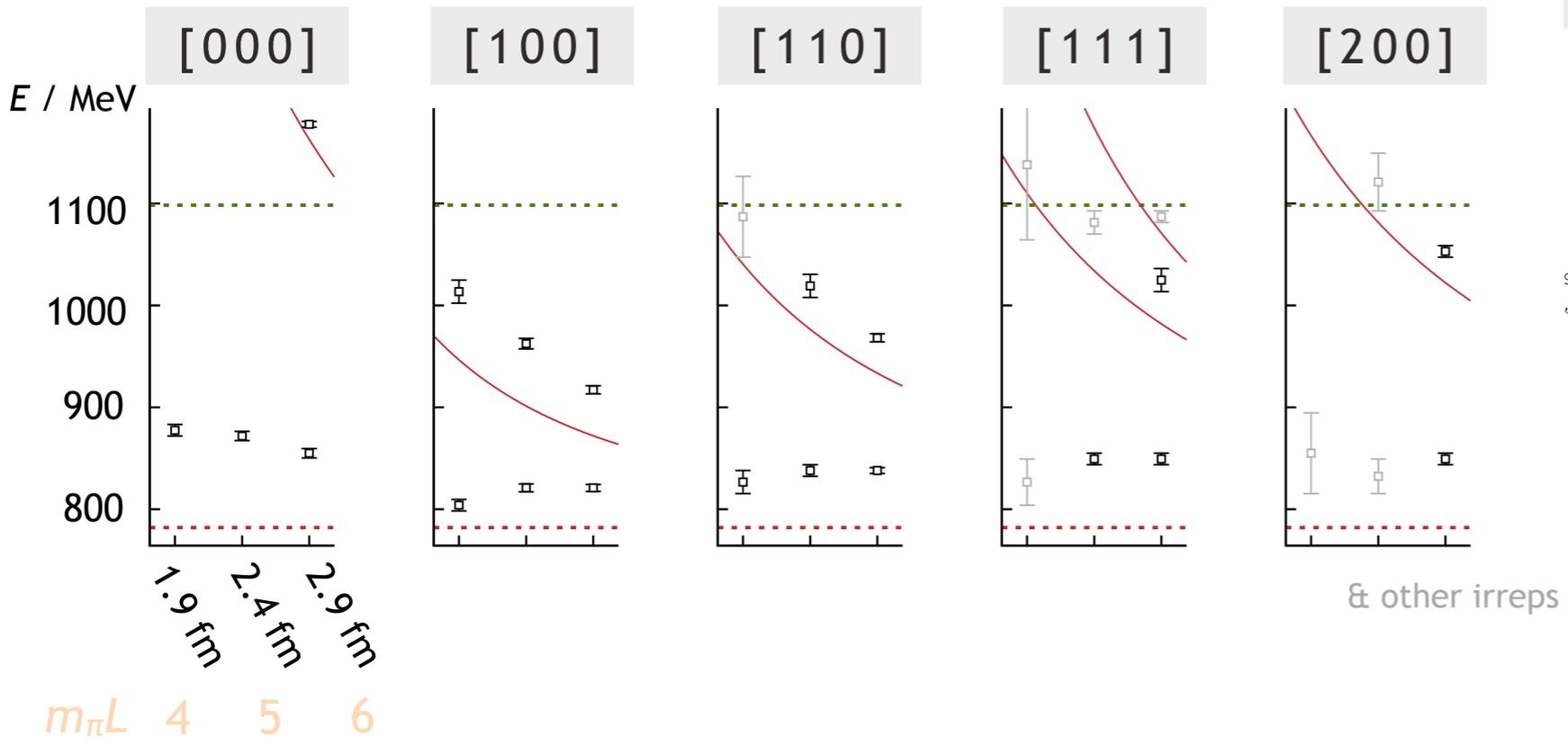


Cohen 1972

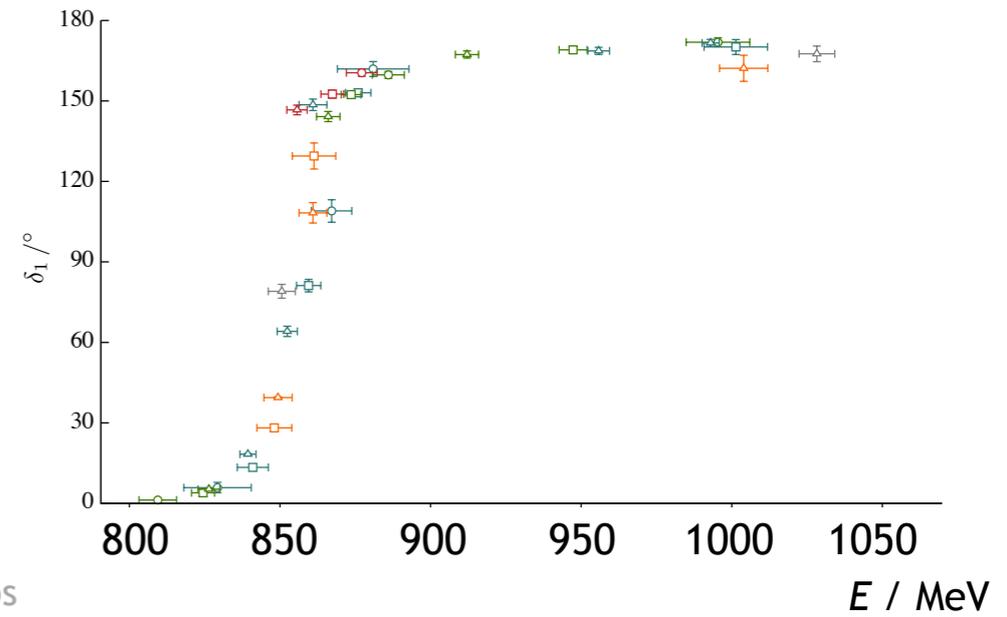
an elastic resonance – the ρ in $\pi\pi$ (isospin=1)

PRD87 034505 (2013)

$m_\pi \sim 391$ MeV

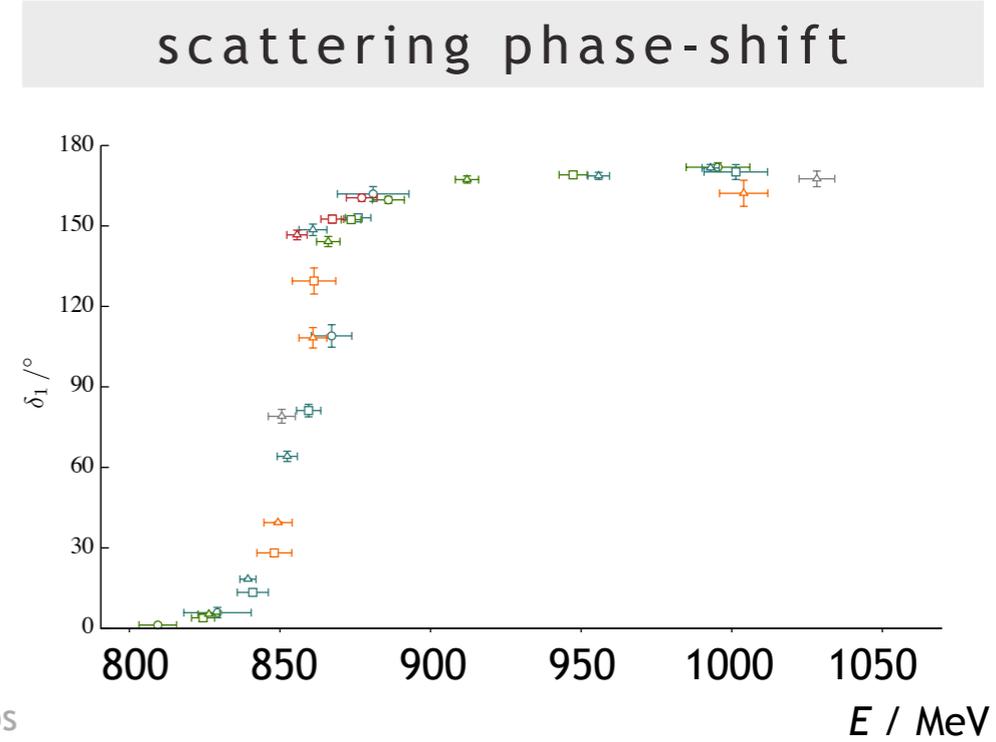
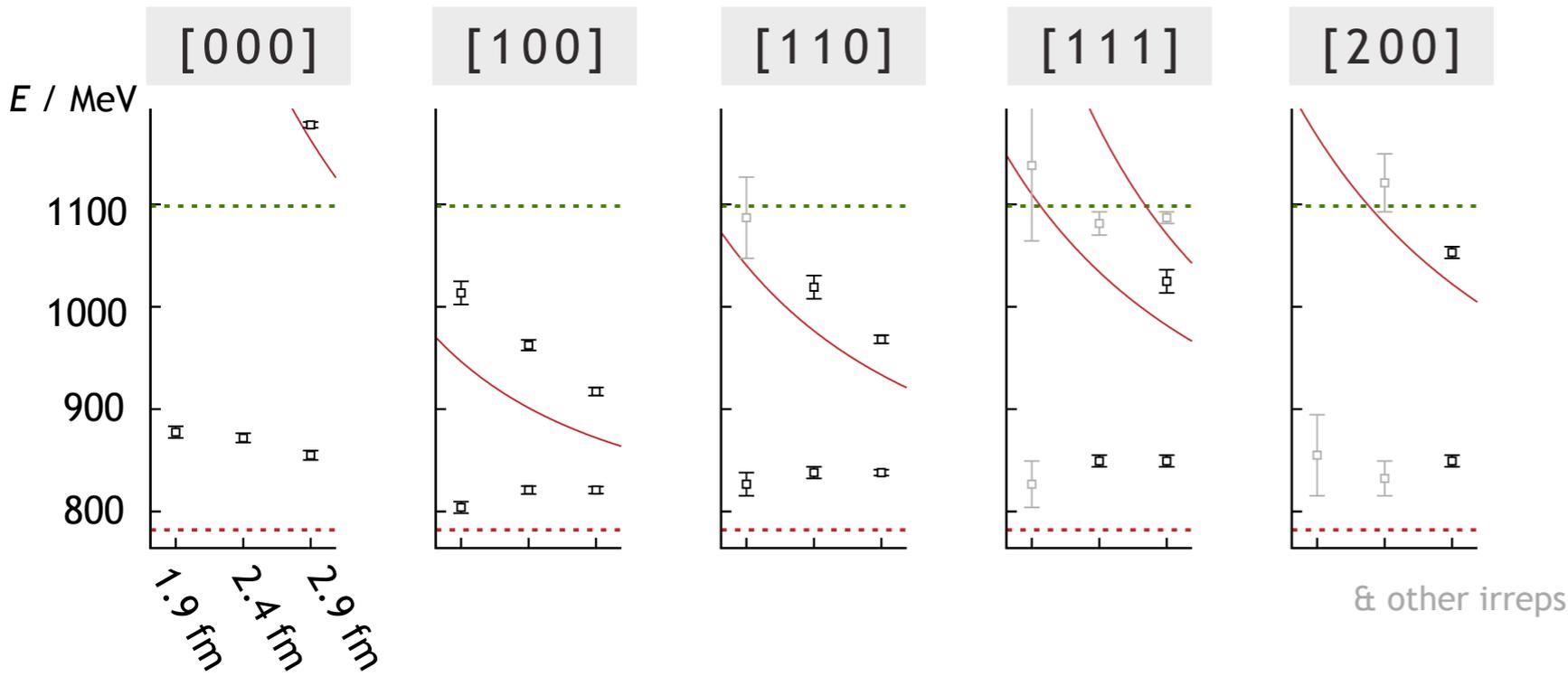


scattering phase-shift

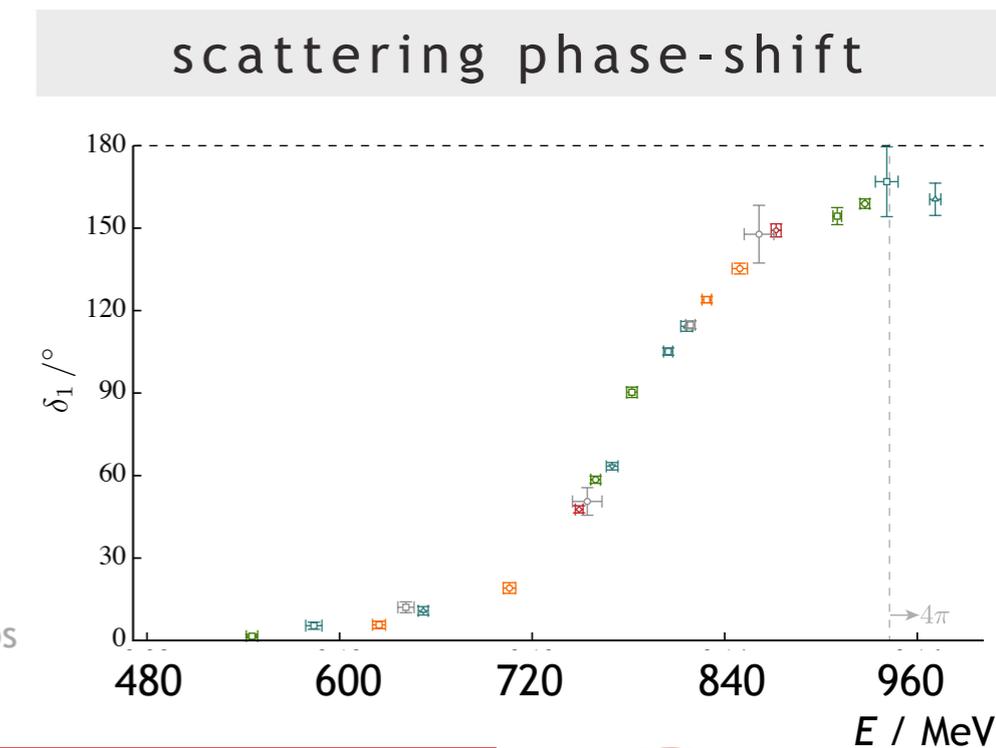
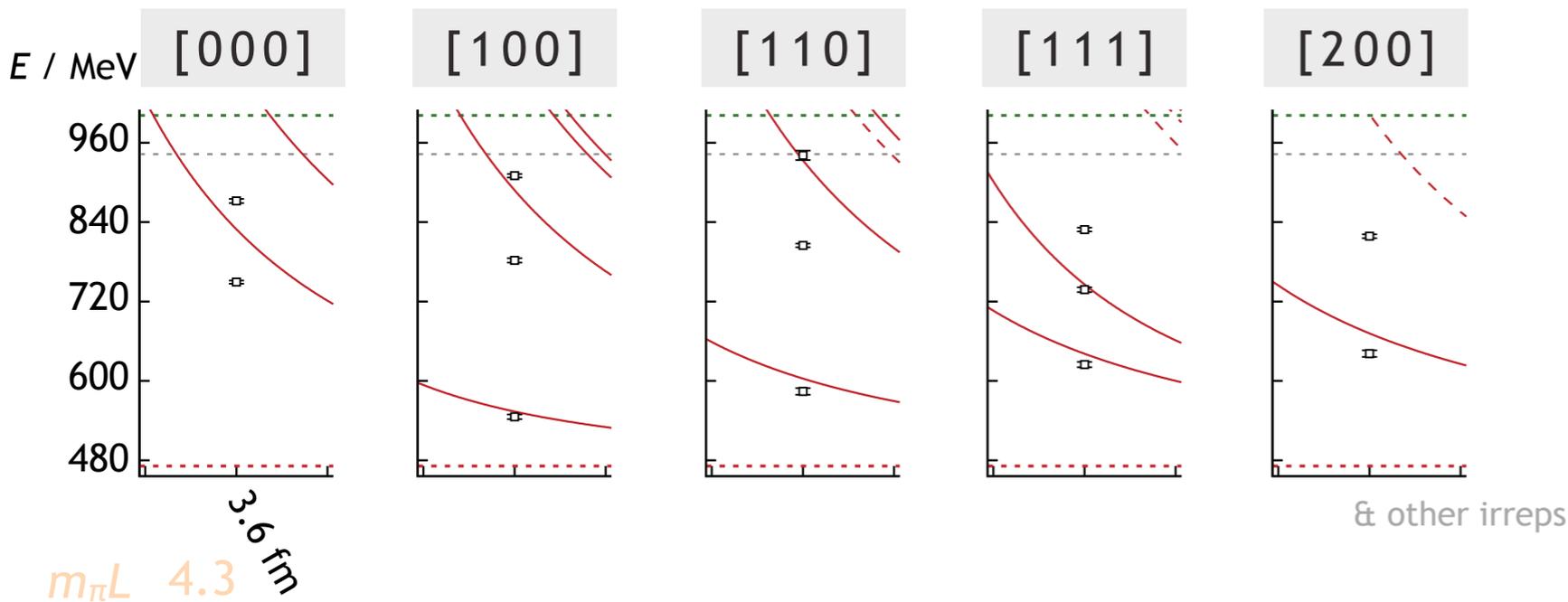


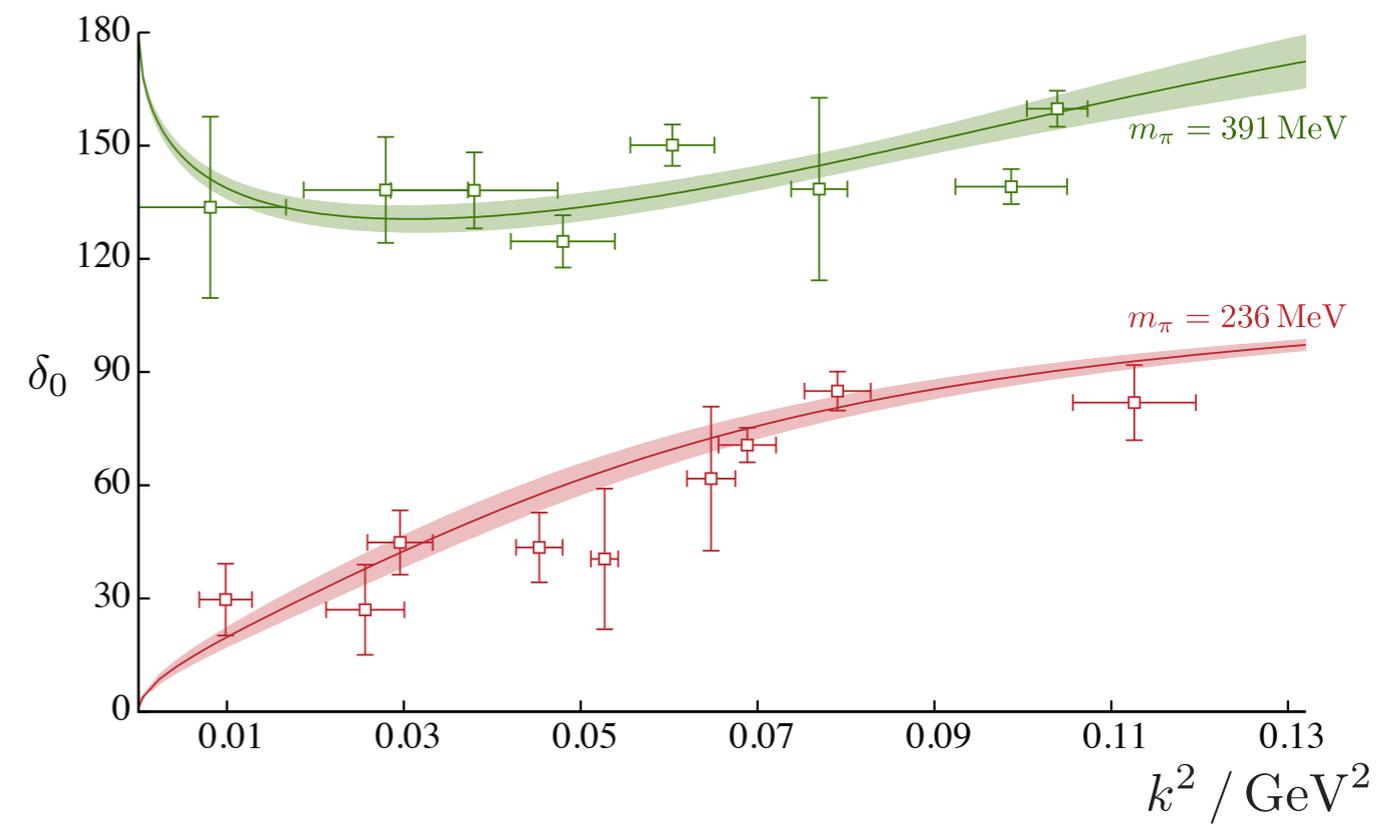
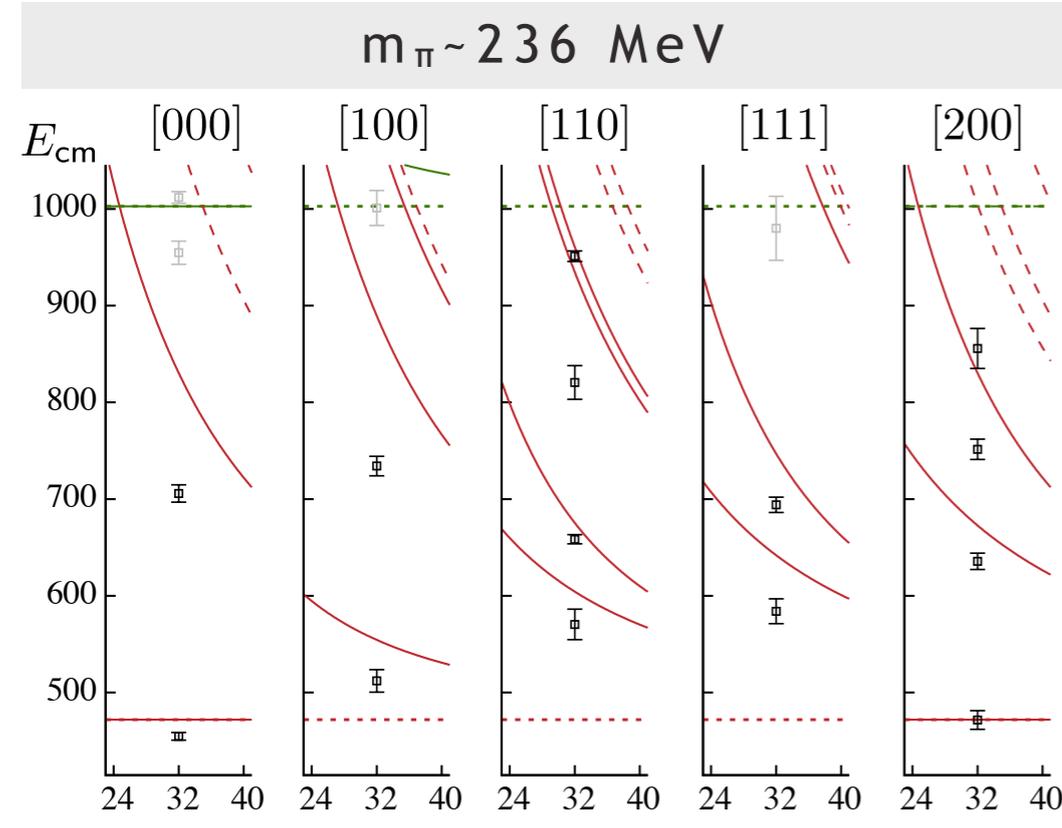
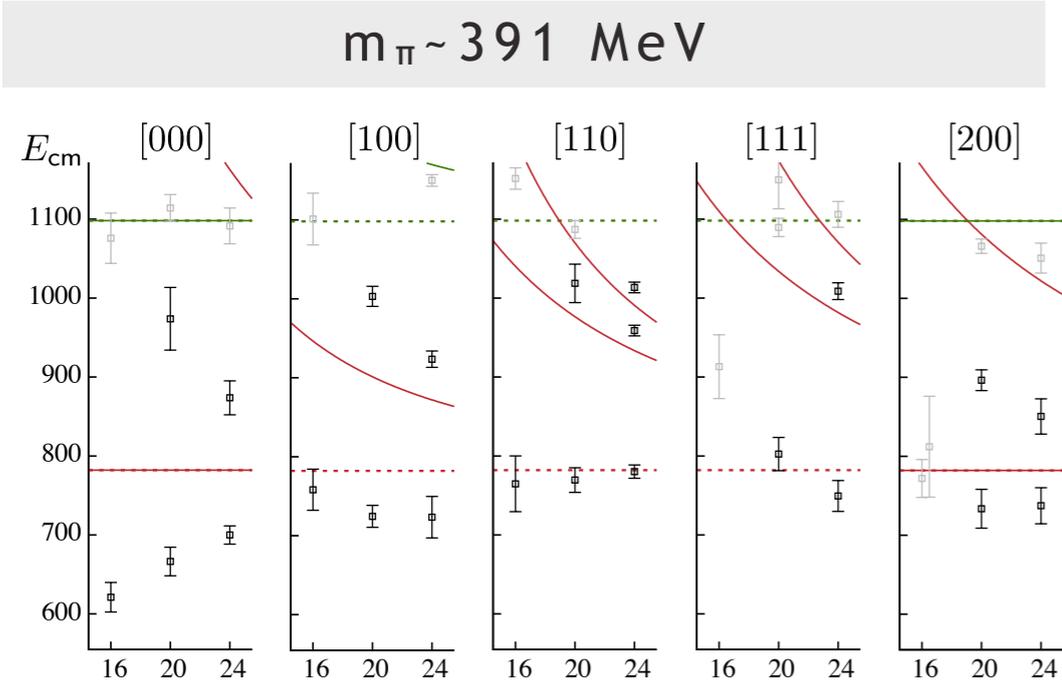
an elastic resonance – the ρ in $\pi\pi$ (isospin=1)

PRD87 034505 (2013) $m_\pi \sim 391$ MeV



PRD92 094502 (2015) $m_\pi \sim 236$ MeV





heavier quark mass — a bound-state

lighter quark mass — attraction, maybe a broad resonance ?

c.f. the experimental σ resonance ...

one possible approach – mimic ‘ideal’ experiment: determine $t_{ab}^{(\ell)}(E)$ for real energies

maybe can then be analytically continued to complex energies to find poles ?

scattering matrix determines the finite-volume spectrum:

$$0 = \det \left[\mathbf{1} + i\rho(E) \cdot \mathbf{t}(E) \cdot (\mathbf{1} + i\mathcal{M}(E, L)) \right]$$

for a given scattering matrix, has a discrete set of solutions $E_n(L)$

the finite-volume spectrum

one approach:

– parameterize the energy dependence of $\mathbf{t}(E)$

– solve $0 = \det \left[\mathbf{1} + i\rho(E) \cdot \mathbf{t}(E) \cdot (\mathbf{1} + i\mathcal{M}(E, L)) \right]$

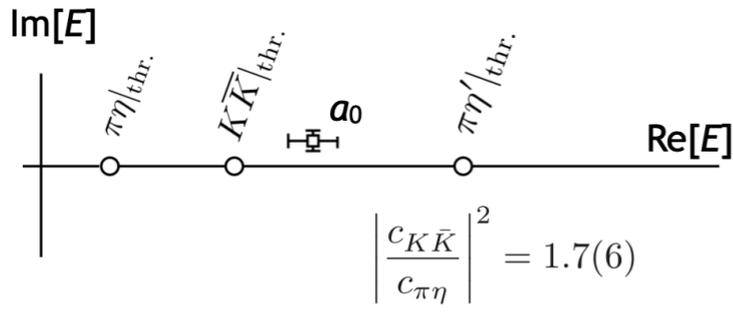
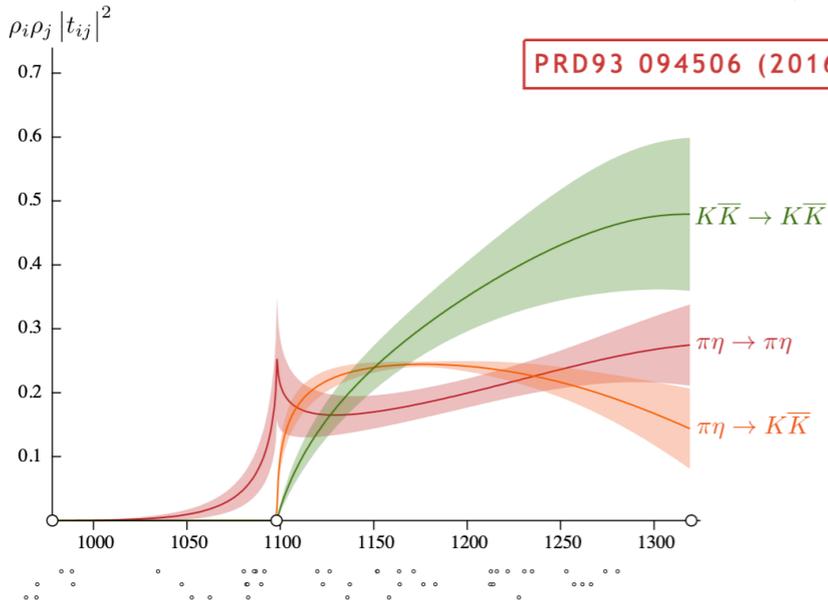
– compare ‘model’ spectrum to lattice spectrum ...

ensure important features are independent of parameterization details by varying parameterization ...

$m_\pi \sim 391 \text{ MeV}$

$a_0 \rightarrow \pi\eta, K\bar{K}$

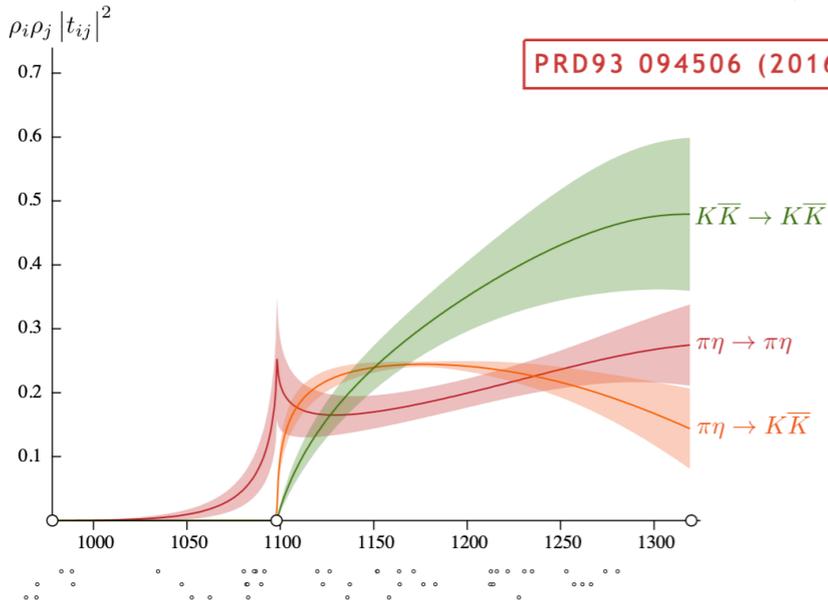
PRD93 094506 (2016)



$m_\pi \sim 391 \text{ MeV}$

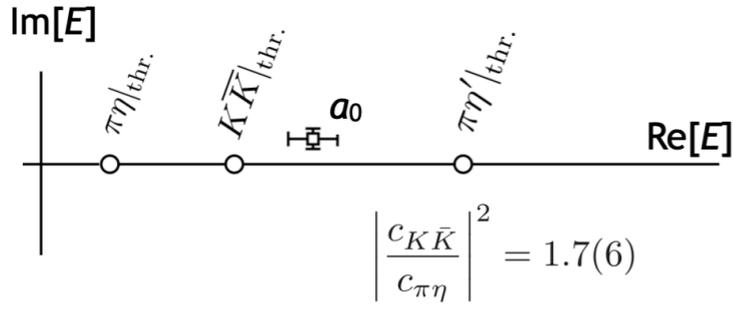
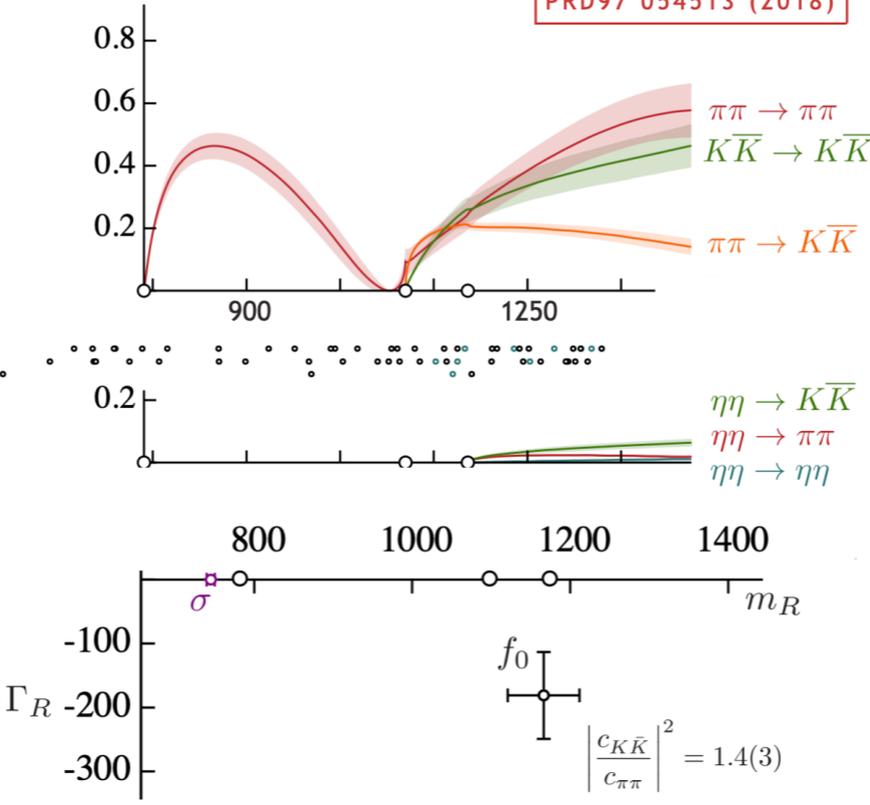
$a_0 \rightarrow \pi\eta, K\bar{K}$

PRD93 094506 (2016)



$f_0 \rightarrow \pi\pi, K\bar{K}, \eta\eta$

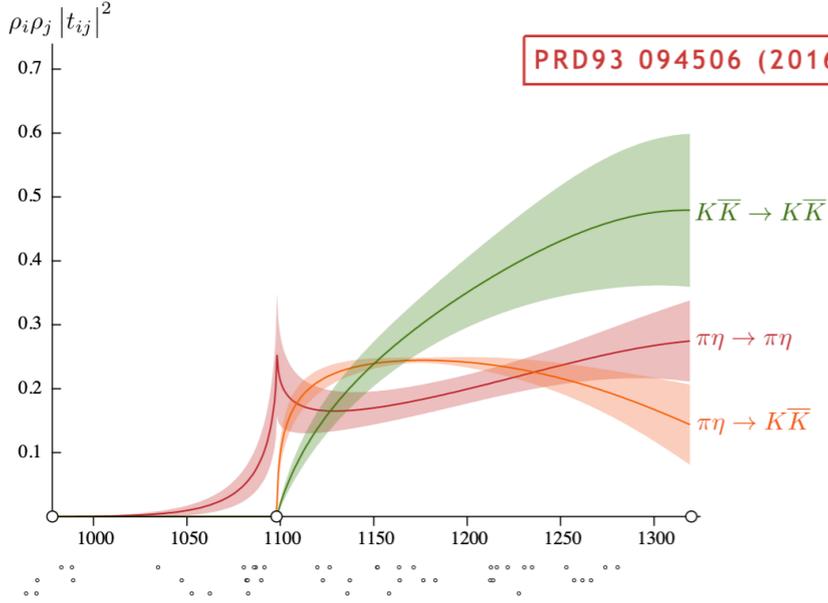
PRD97 054513 (2018)



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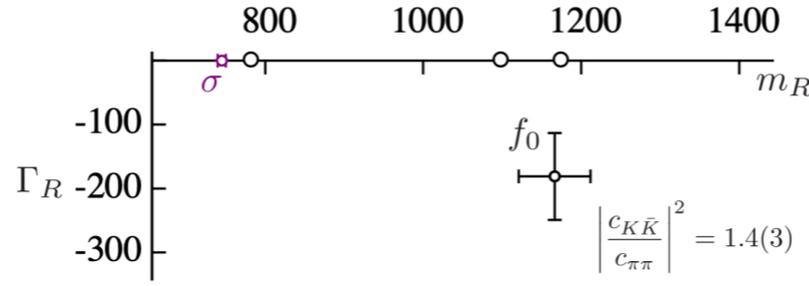
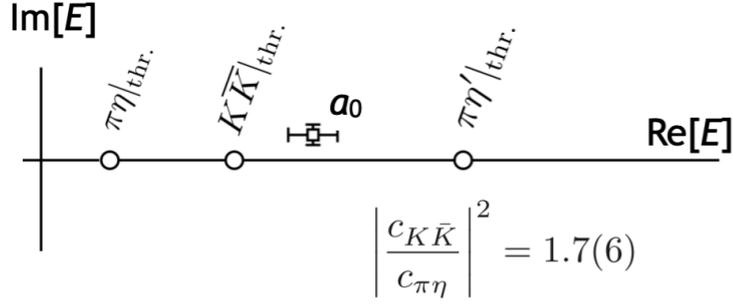
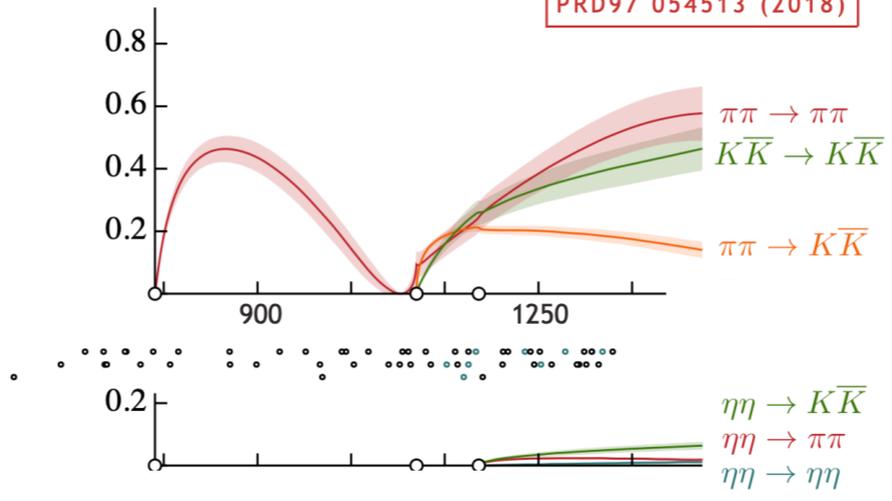
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PRD93 094506 (2016)



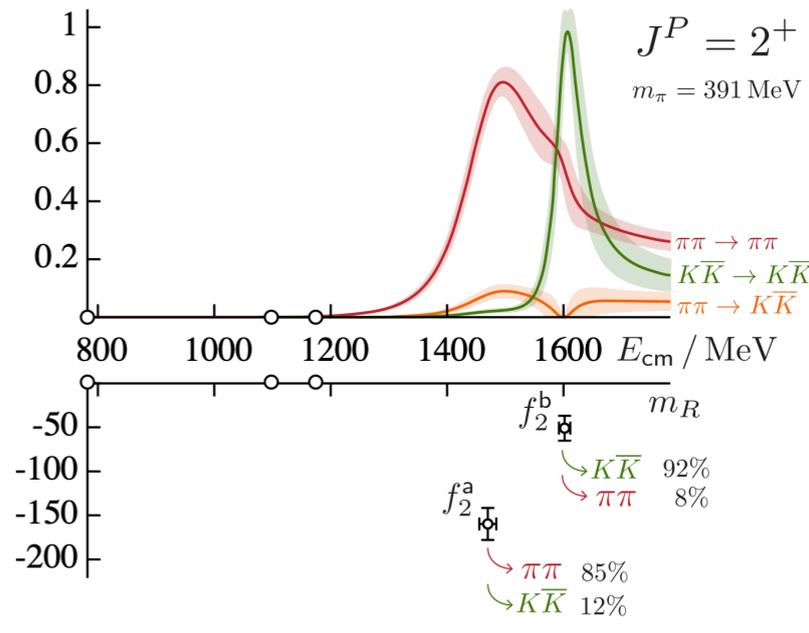
$f_0 \rightarrow \pi\pi, K\bar{K}, \eta\eta$

PRD97 054513 (2018)



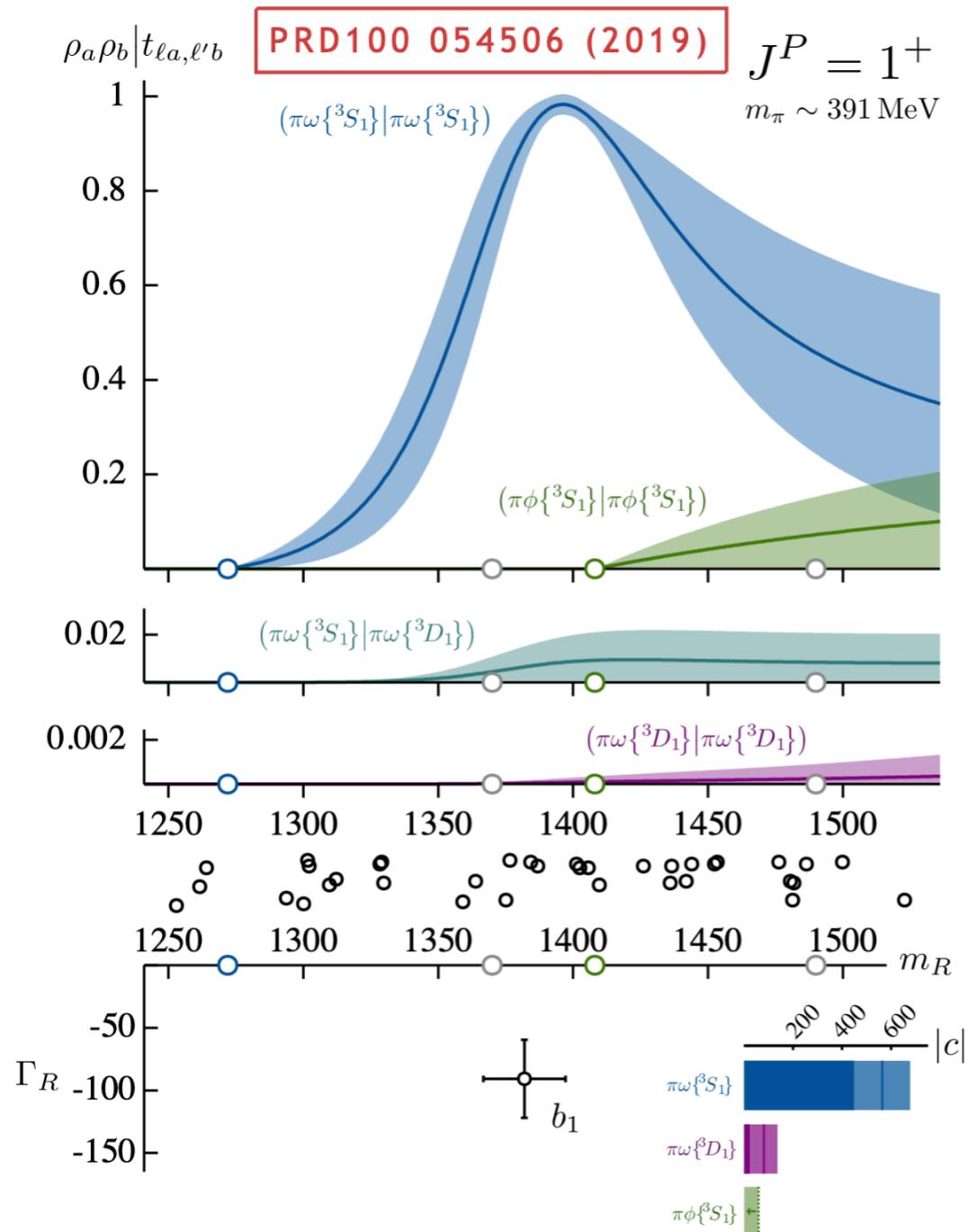
$f_2 \rightarrow \pi\pi, K\bar{K}$

PRD97 054513 (2018)



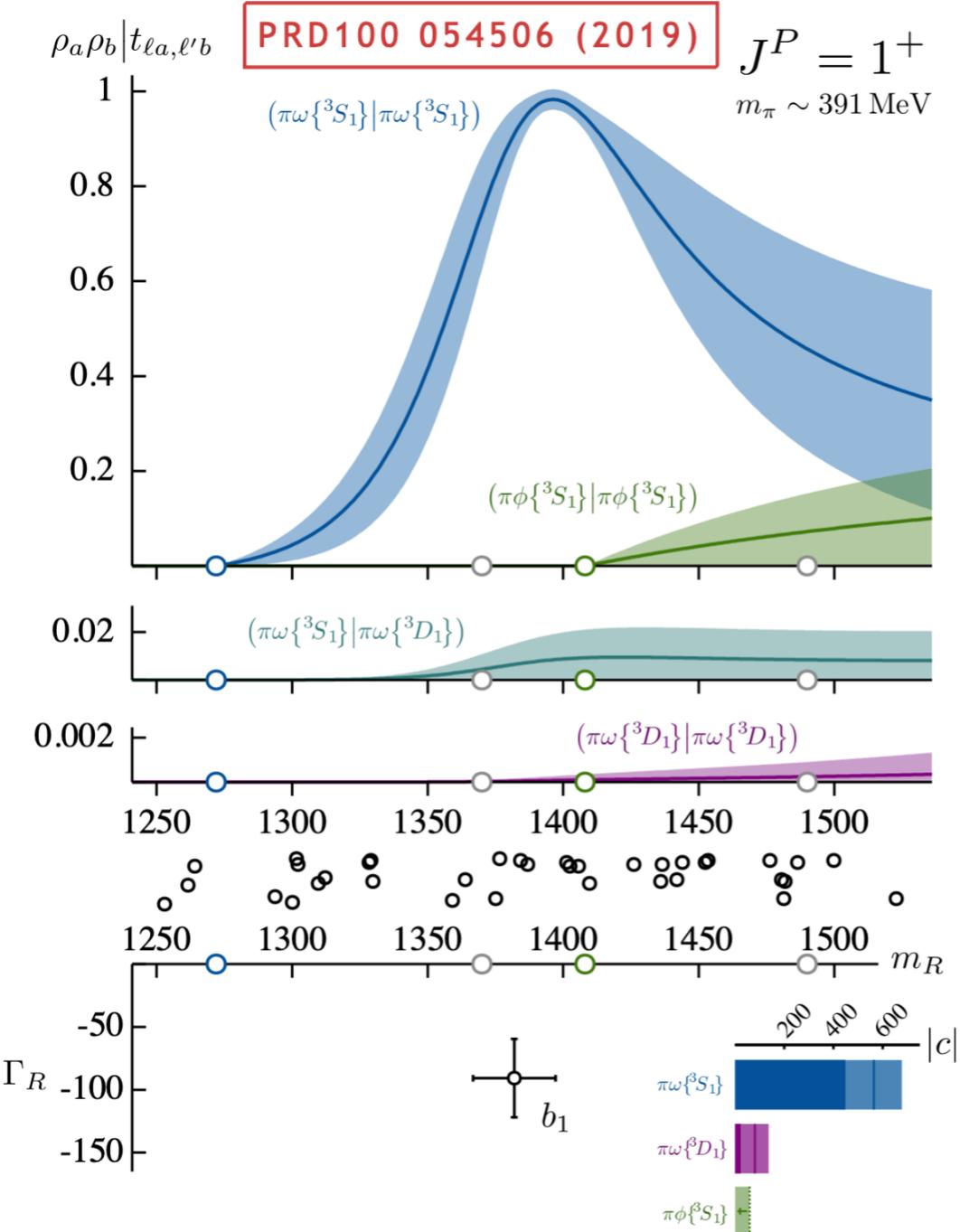
... recently scattering of hadrons with non-zero spin ...

$b_1 \rightarrow \pi\omega, \pi\phi$



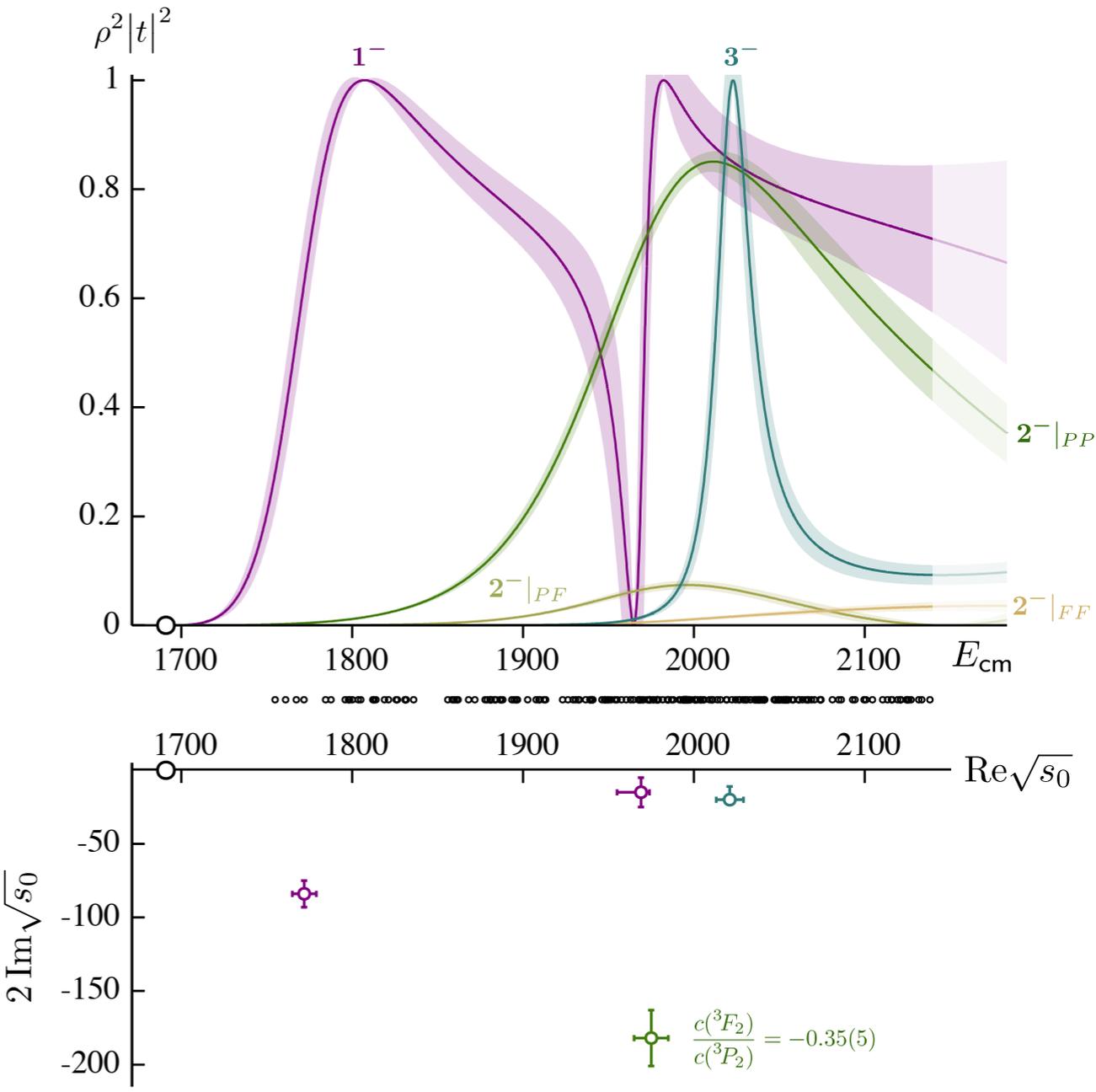
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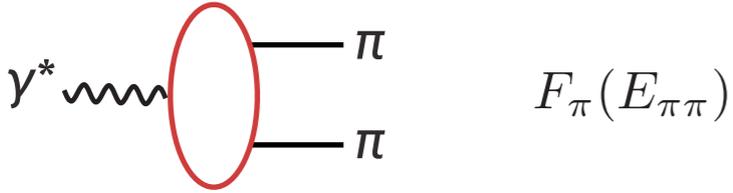


$J^{--} \rightarrow 0^{-+} 1^{--}$
with exact SU(3) flavor

PRD103 074502 (2021)



$\gamma^* \rightarrow \pi\pi$

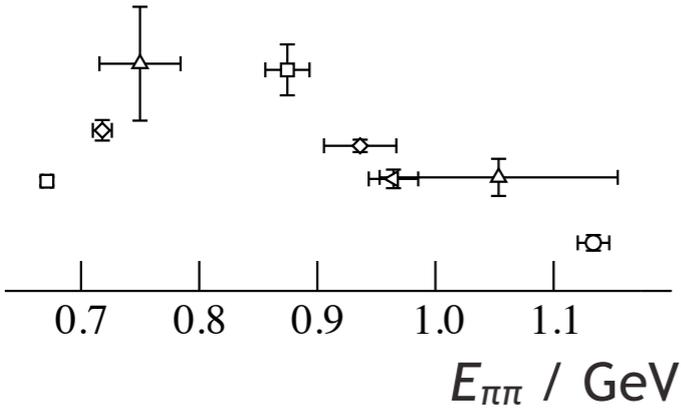


$$F_\pi(E_{\pi\pi})$$



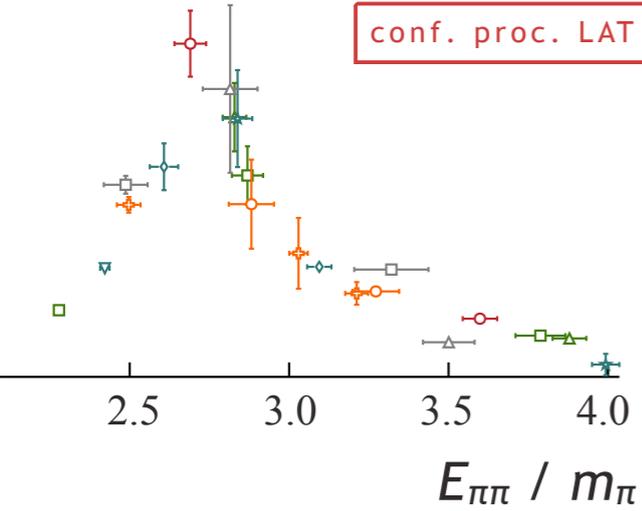
Feng et al | *u,d,s* | $m_\pi \sim 290$ MeV

PRD97 054513 (2018)

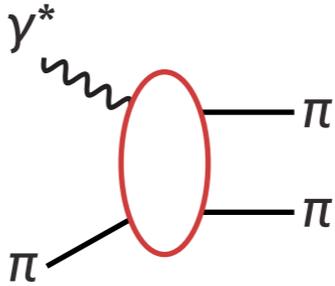


Bulava et al | *u,d,s* | $m_\pi \sim 280$ MeV

conf. proc. LAT '15



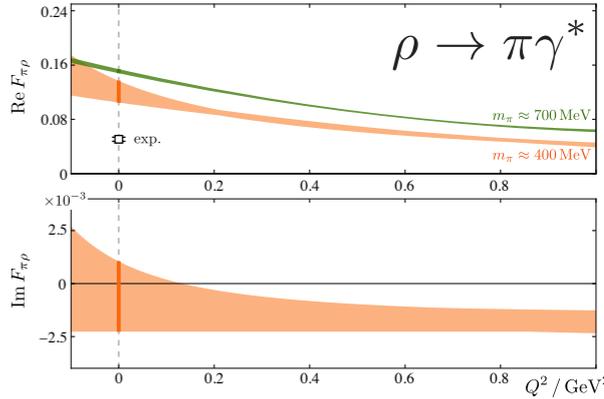
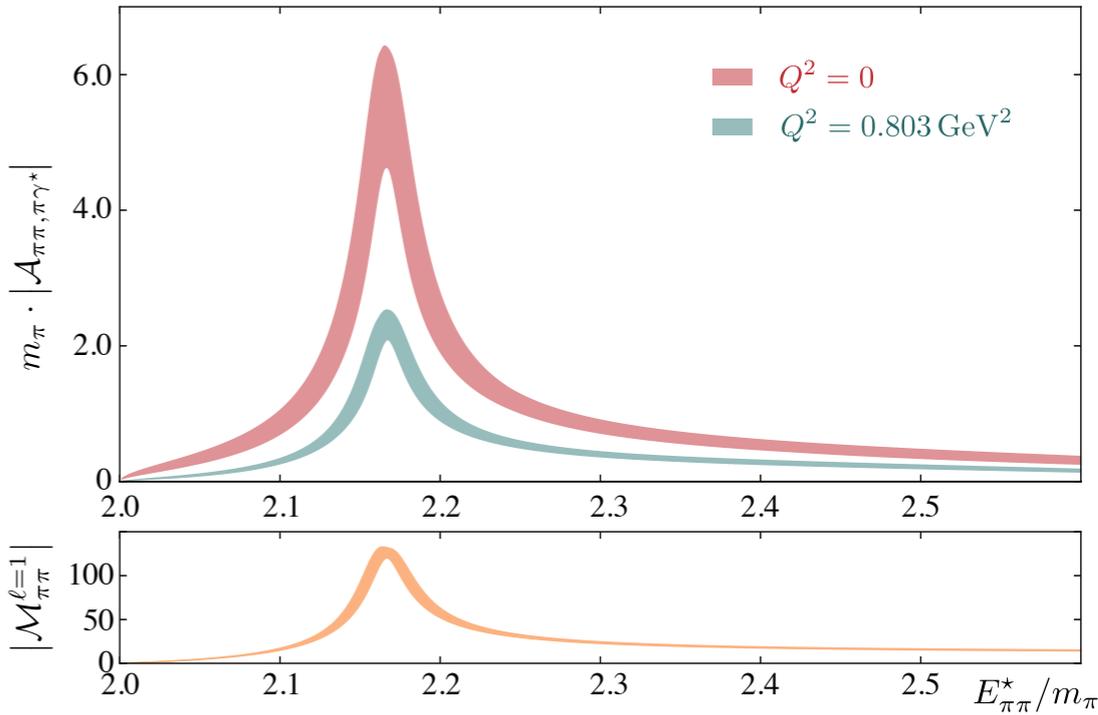
$\gamma^* \pi \rightarrow \pi\pi$



$$\mathcal{A}(E_{\pi\pi}, Q^2)$$

Briceno et al | *u,d,s* | $m_\pi \sim 391$ MeV

PRL115 242001 (2015)



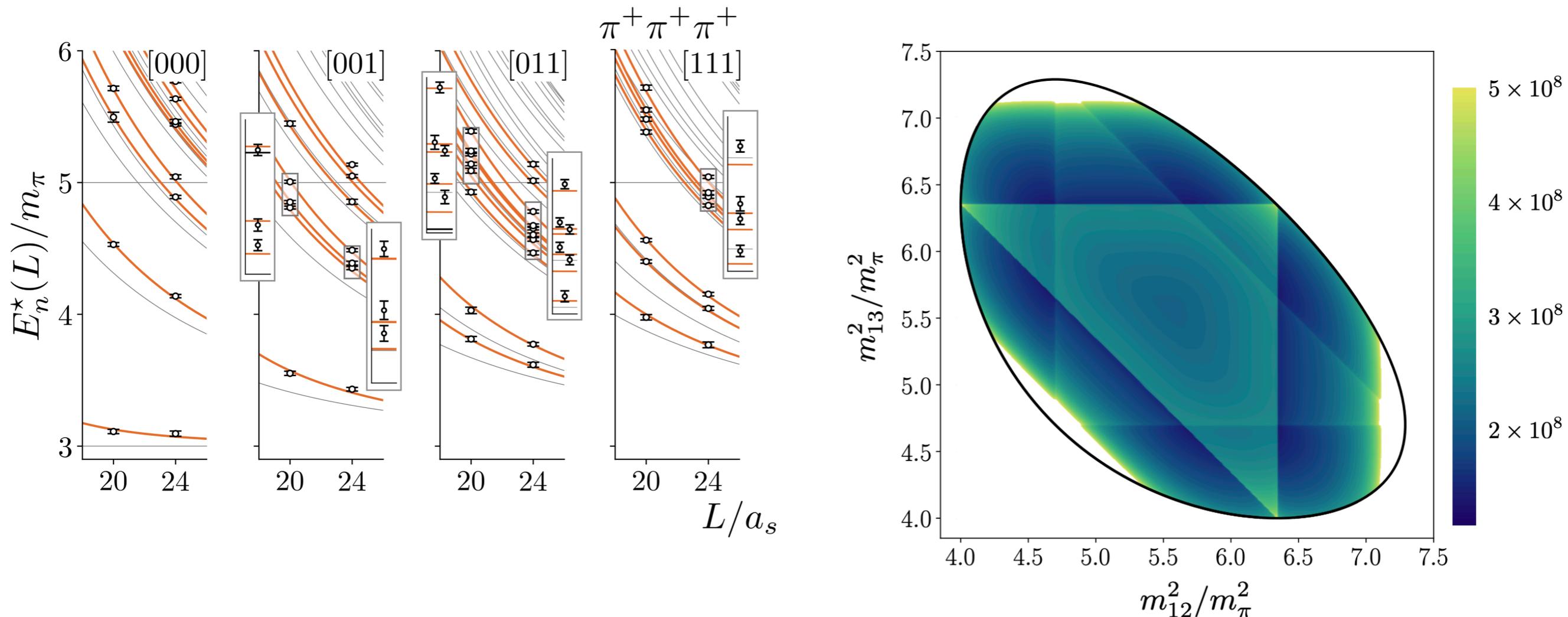
beyond $2 \rightarrow 2$ scattering

formalism used so far only valid for $2 \rightarrow 2$ scattering \Rightarrow required to stay below three-body thresholds ...

three-body formalism now developed and applied in test cases

The energy-dependent $\pi^+\pi^+\pi^+$ scattering amplitude from QCD

Maxwell T. Hansen,^{1,*} Raul A. Briceño,^{2,3,†} Robert G. Edwards,^{2,‡} Christopher E. Thomas,^{4,§} and David J. Wilson^{4,¶}
(for the Hadron Spectrum Collaboration)

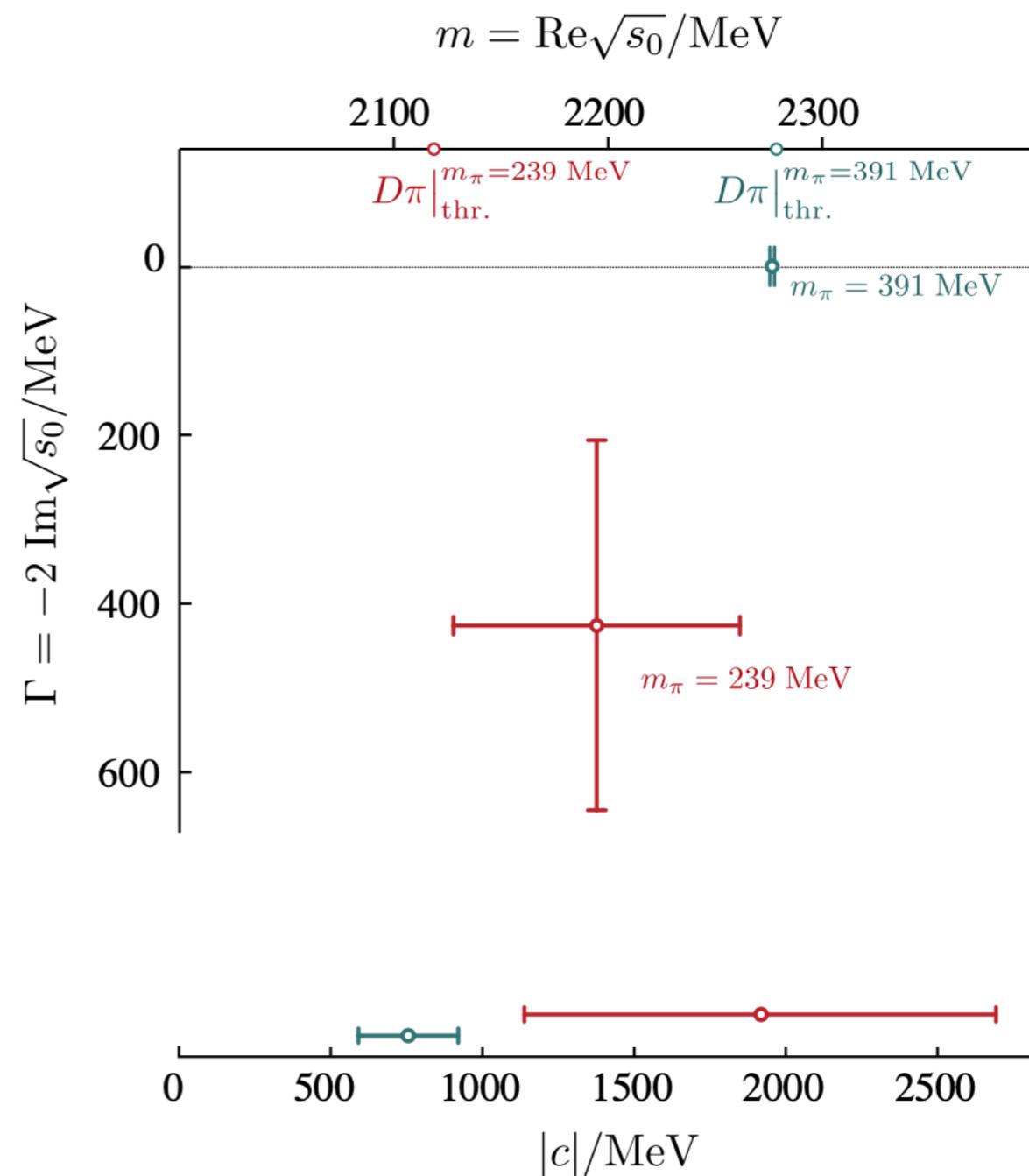
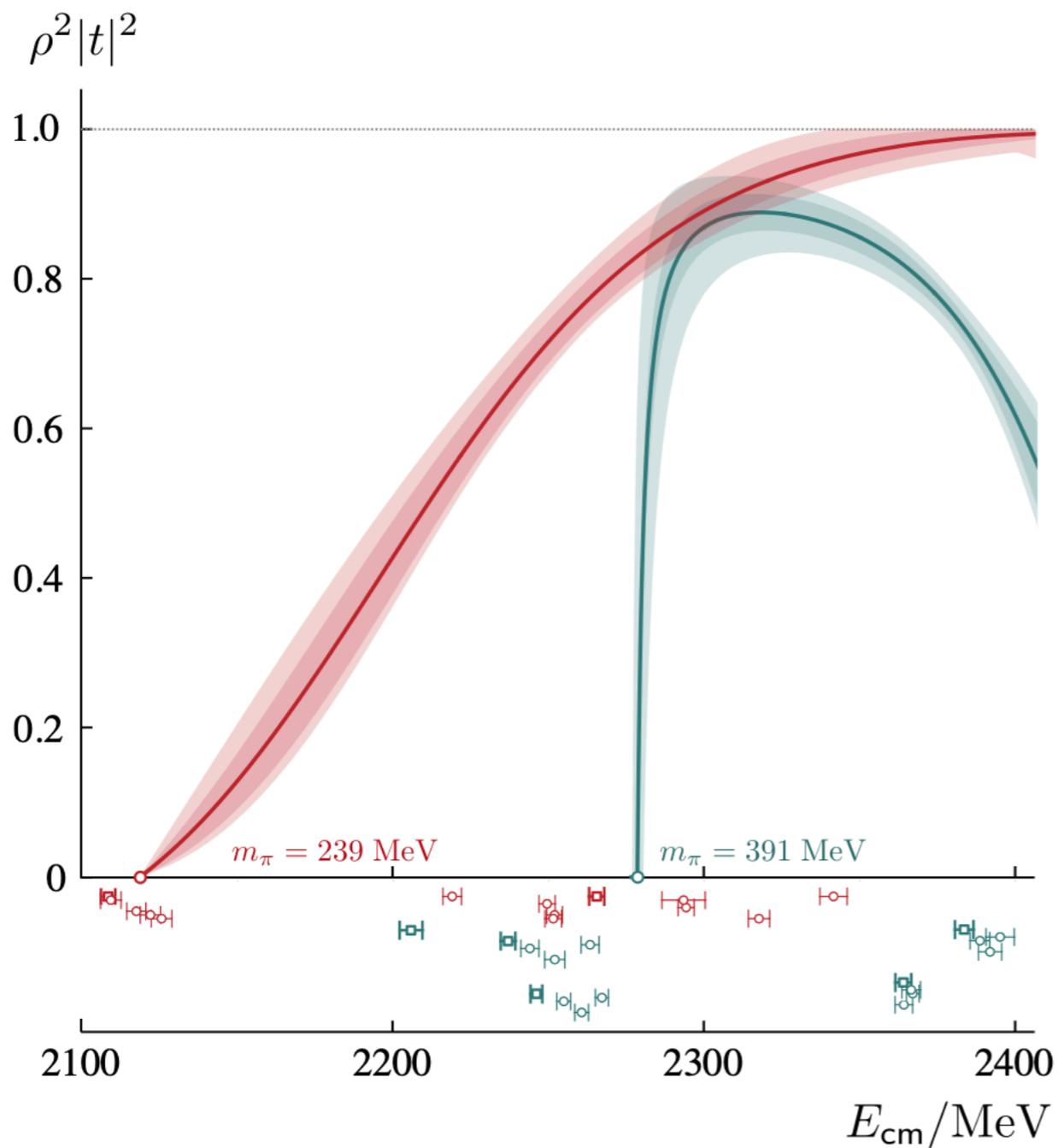


lattice QCD is increasingly becoming a practical tool to study **excited hadron resonances**

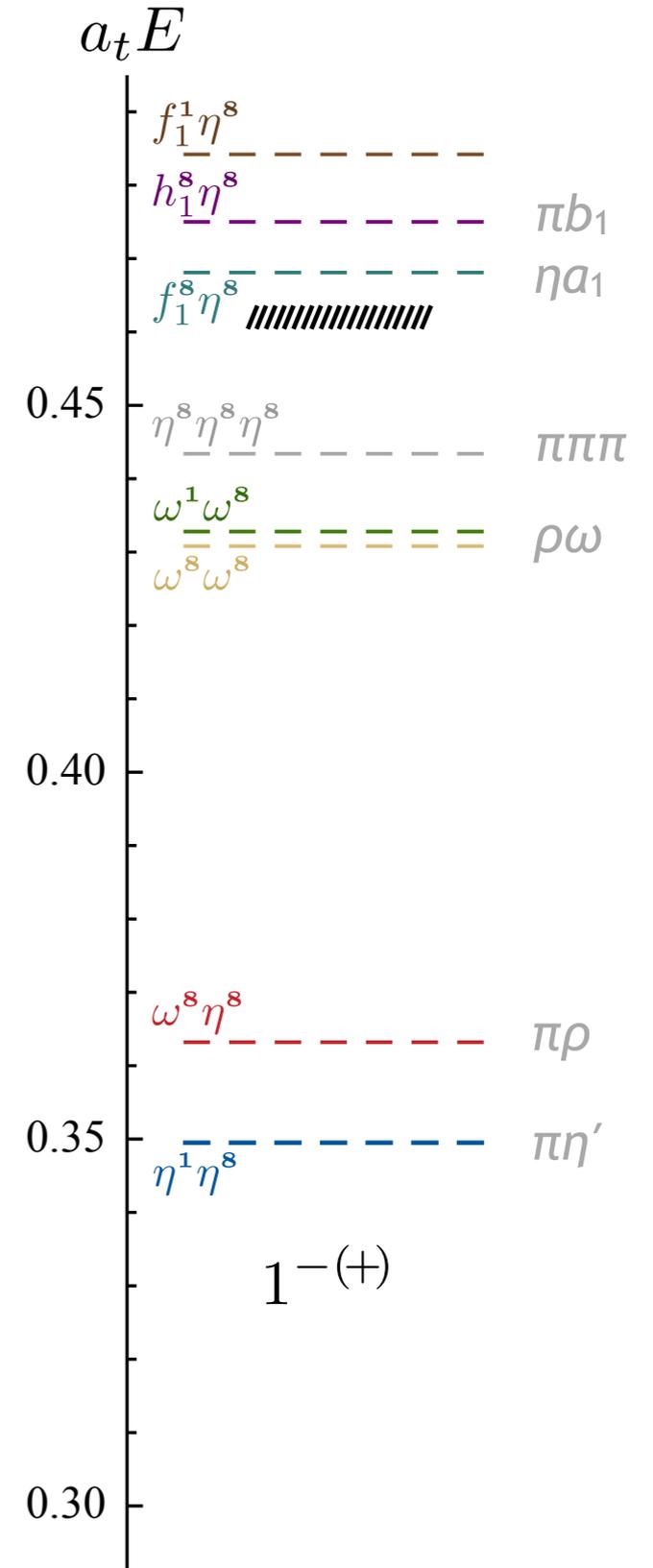
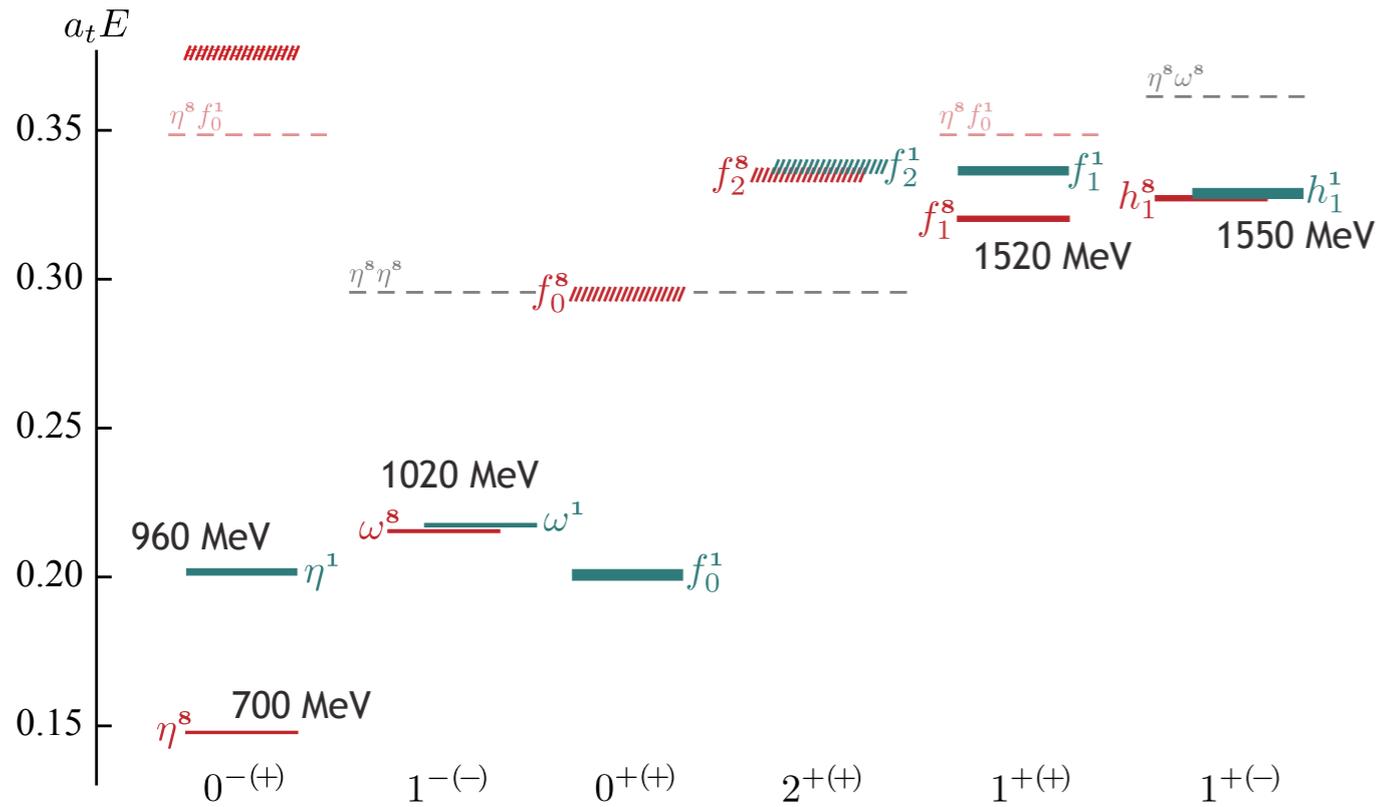
currently somewhat limited to heavier quark masses to avoid many-body decays

but can still learn a lot about strong QCD where previously we relied upon models

a new era of QCD-based hadron spectroscopy



several stable mesons:



please forgive the obscure lattice units,
will convert at the end ...

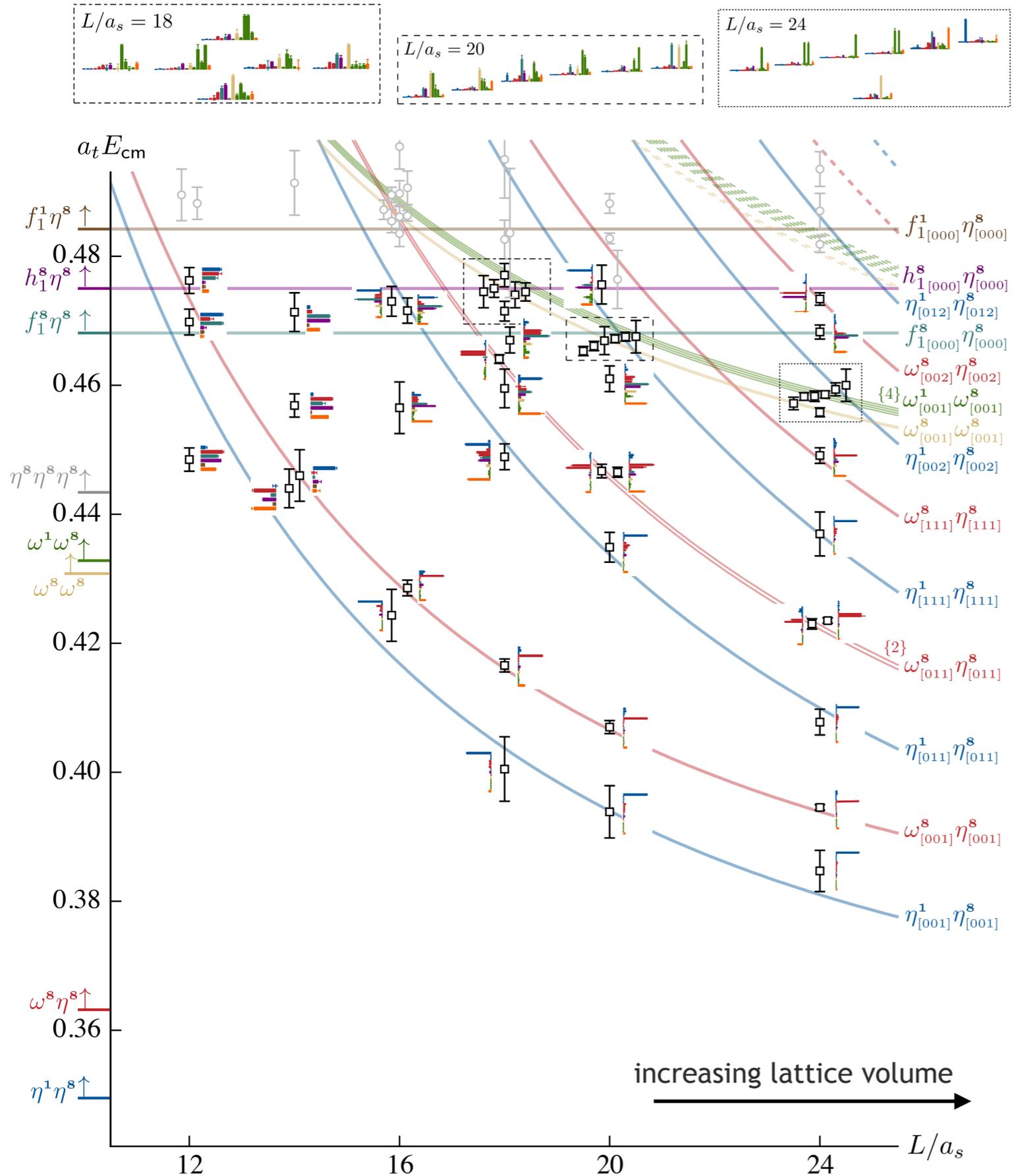
$$\eta^8 \rightsquigarrow \pi, K, \eta$$

$$\eta^1 \rightsquigarrow \eta'$$

$$\omega^8, \omega^1 \rightsquigarrow \rho, K^*, (\omega, \varphi)$$

$$h_1^8, h_1^1 \rightsquigarrow b_1, K_1, (h_1, h_1')$$

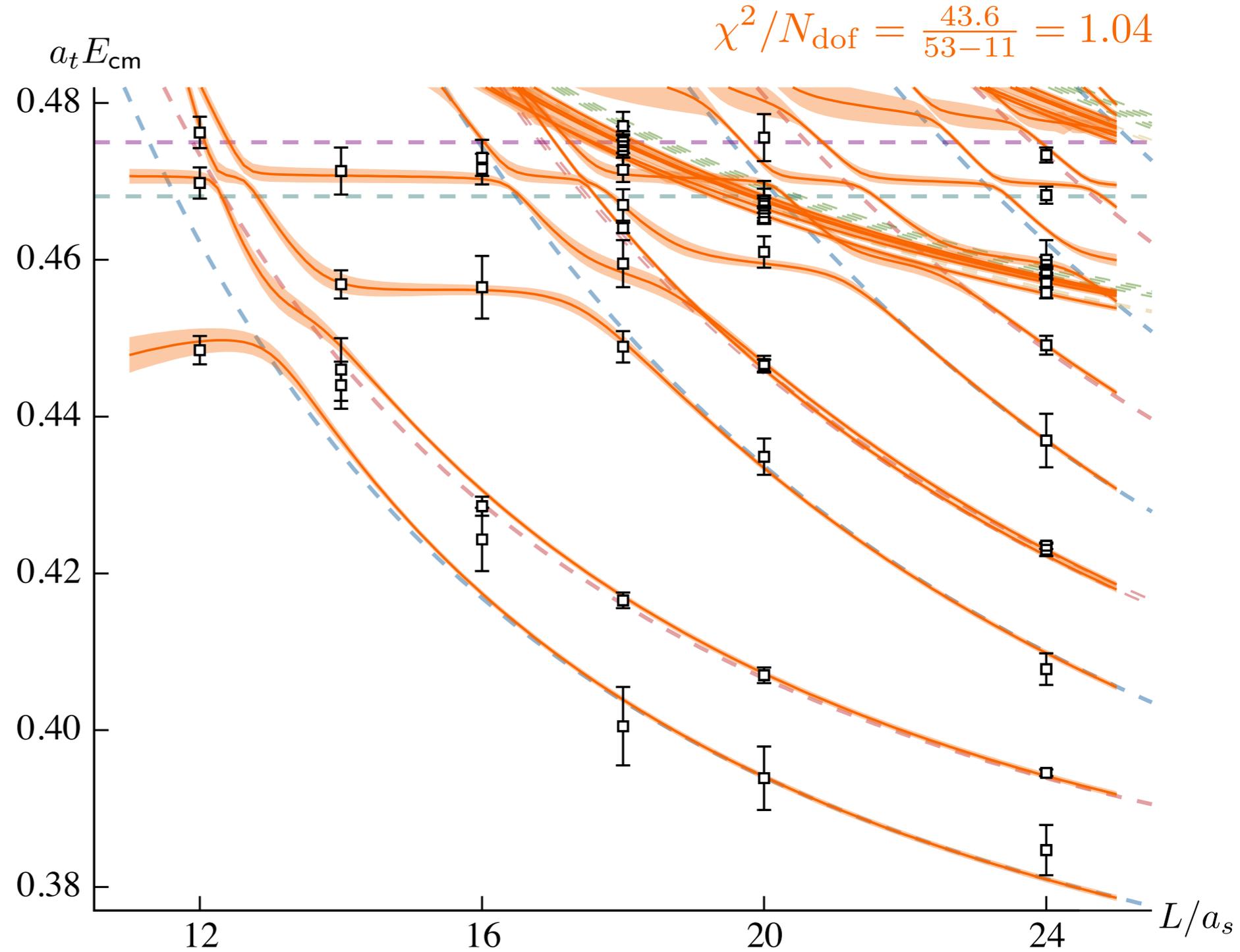
$$f_1^8, f_1^1 \rightsquigarrow a_1, K_1, (f_1, f_1')$$



$\eta^8 \rightsquigarrow \pi, K, \eta$
 $\eta^1 \rightsquigarrow \eta'$
 $\omega^8, \omega^1 \rightsquigarrow \rho, K^*, (\omega, \varphi)$
 $h_1^8, h_1^1 \rightsquigarrow b_1, K_1, (h_1, h_1')$
 $f_1^8, f_1^1 \rightsquigarrow a_1, K_1, (f_1, f_1')$

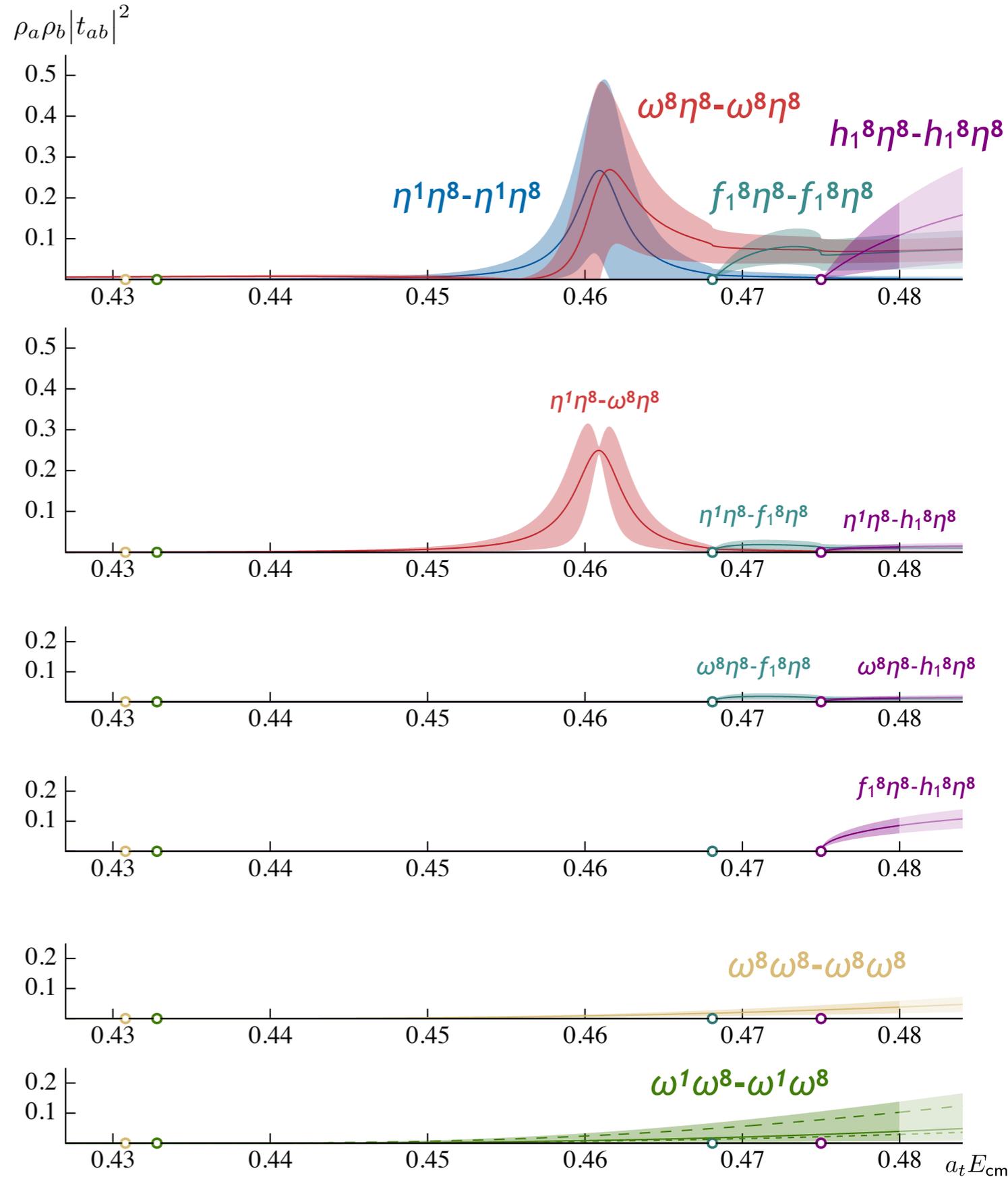
53 energy levels to constrain 'eight' channel scattering

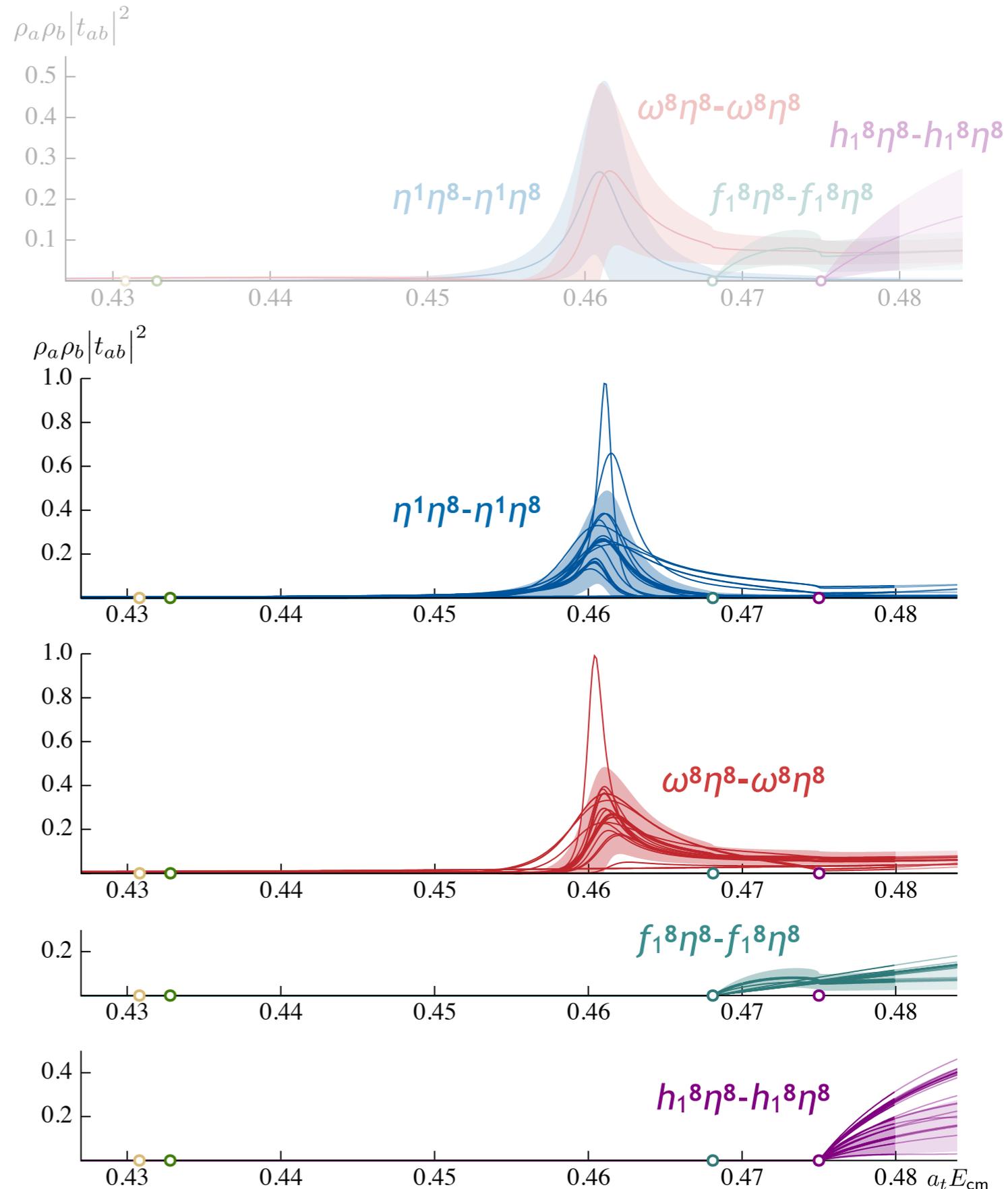
describe scattering by a unitarity-preserving K -matrix featuring a pole
(11 free parameters)

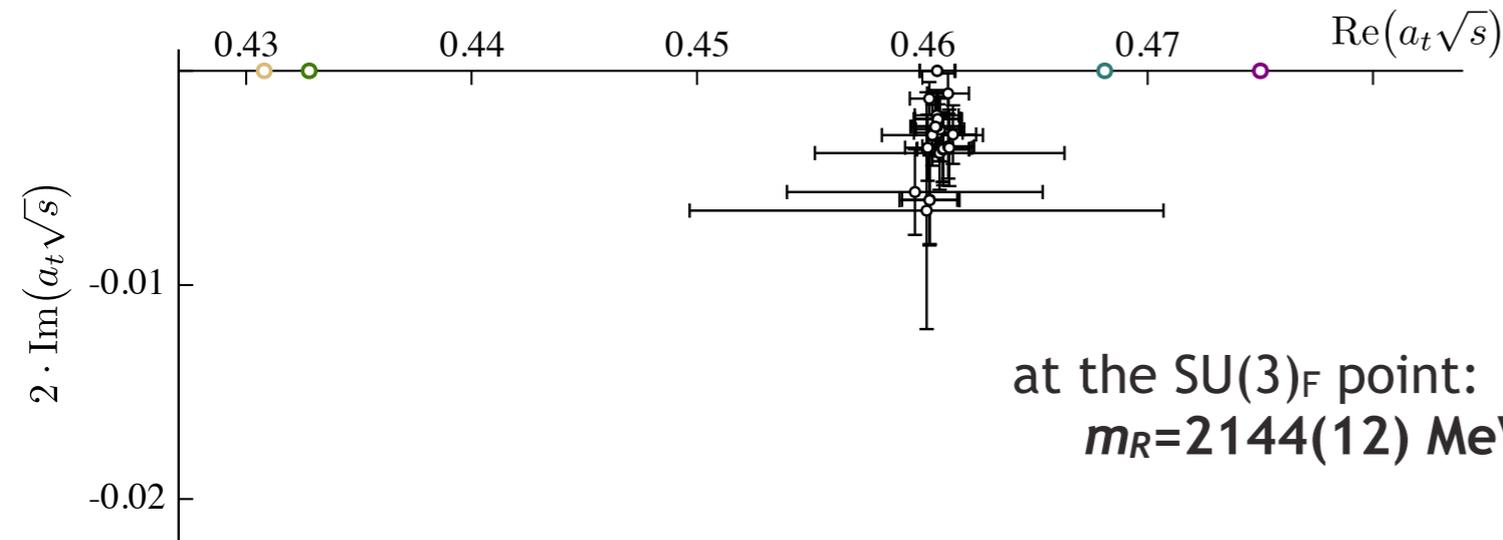


a good description of the spectrum ...

- $\eta^1\eta^8 \rightsquigarrow \pi\eta'$
- $\omega^8\eta^8 \rightsquigarrow \pi\rho$
- $f_1^8\eta^8 \rightsquigarrow \pi f_1, \eta a_1$
- $h_1^8\eta^8 \rightsquigarrow \pi b_1$





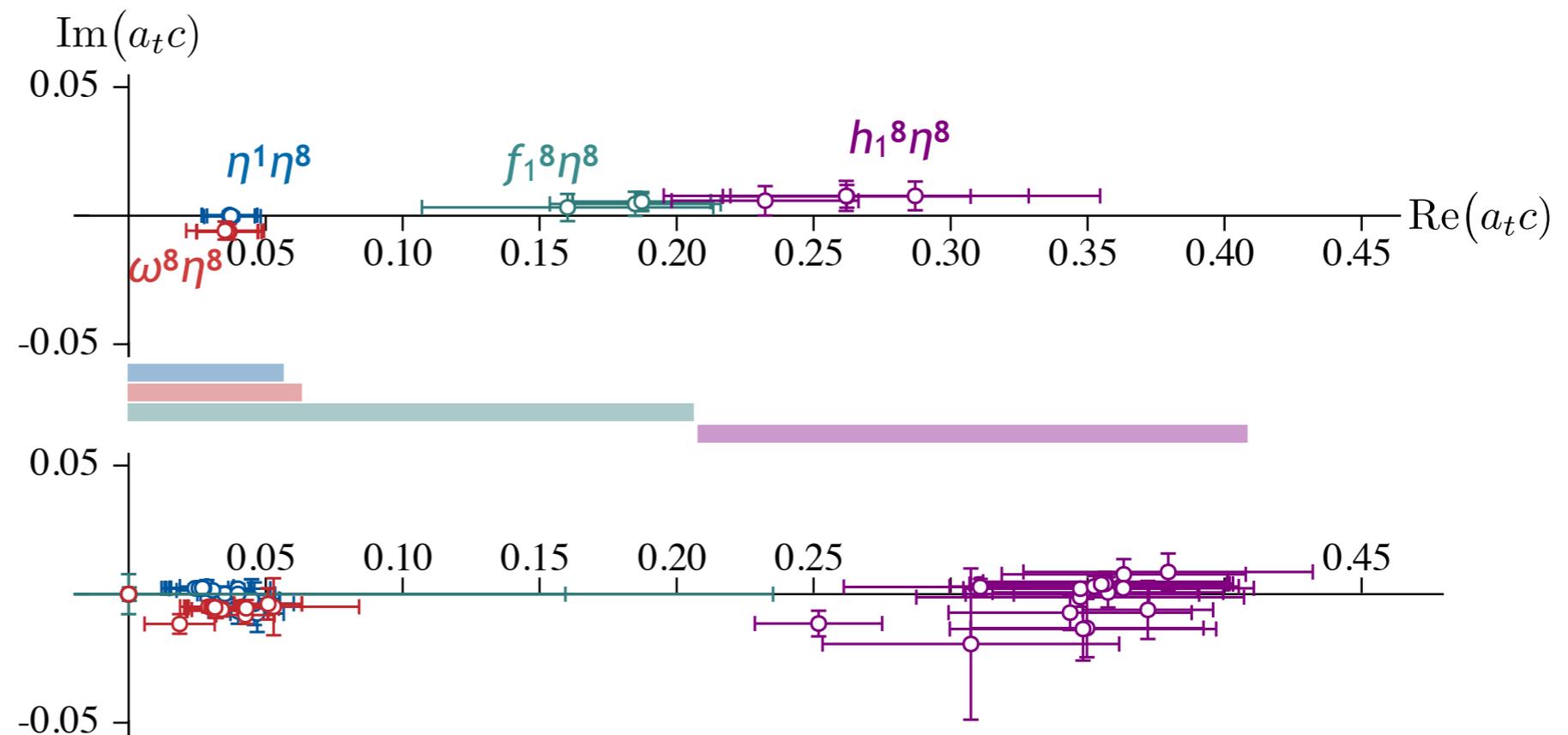


$$t_{ab}(s) \sim \frac{C_a C_b}{s_0 - s}$$

$$\sqrt{s_0} = m_R - i \frac{1}{2} \Gamma_R$$

at the $SU(3)_F$ point:

$m_R = 2144(12)$ MeV, $\Gamma_R = 21(21)$ MeV (a narrow resonance)



resonance below $h_1^8 \eta^8$ threshold, but with a large coupling