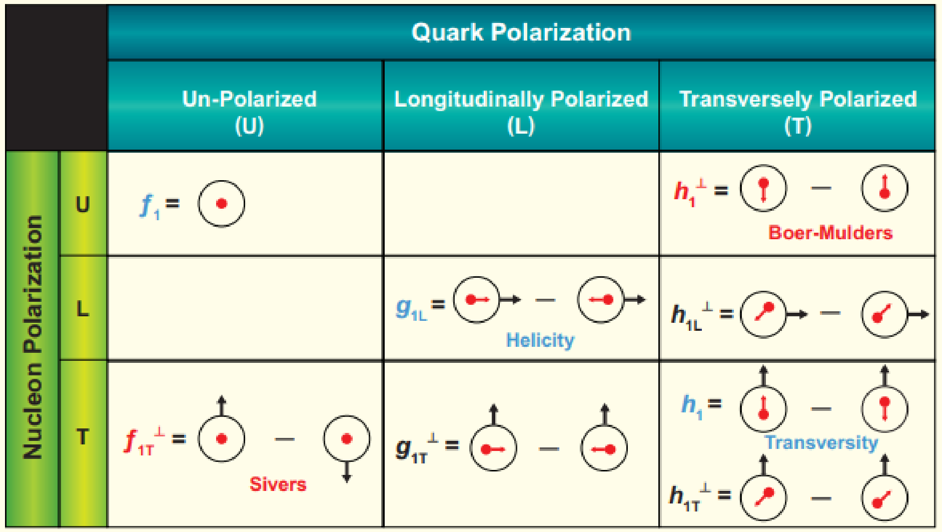
Quark Sivers Function at Small-x: Return of a Spin-Dependent Odderon

M. Gabriel Santiago

With Yuri Kovchegov

Quark Sivers function

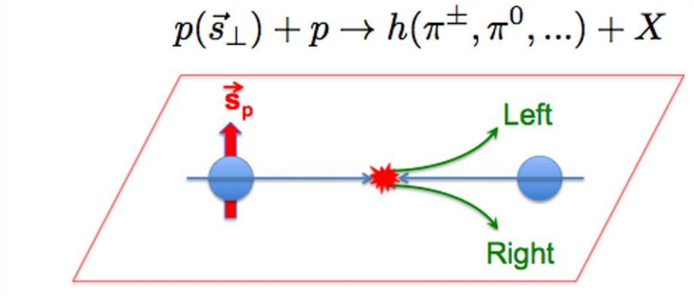
- TMDs give the transverse momentum part of the three-dimensional structure of hadrons



- Finding the small-x asymptotics of these functions is an ongoing effort
- The Sivers TMD is the density of unpolarized quarks with transverse momentum k_T and longitudinal momentum fraction x in a proton with transverse polarization $\underline{S}_P$

Quark Sivers function

- The Sivers function captures orbital angular momentum and spin-orbit coupling as seen in single spin asymmetries



$$\propto (S_P \times \vec{p}) \cdot \vec{k}_T$$

- The unintegrated quark density f_1^q and Sivers function $f_{1T}^{\perp q}$ are defined by the non-local operator product

$$f_1^q(x, k_T^2) - \frac{k_T \times \underline{S}_P}{M_P} f_{1\,T}^{\perp\,q}(x, k_T^2) = \int \frac{\mathrm{d}r^- \, \mathrm{d}^2 r_\perp}{(2\pi)^3} e^{ik\cdot r} \langle P, S | \bar{\psi}(0) \mathcal{U}[0, r] \frac{\gamma^+}{2} \psi(r) | P, S \rangle$$

- Analogous TMD for gluons has known small-x asymptotics!

Spin-Dependent Odderon

- *Boer et al* (2016) showed that at small-x the T-odd gluon TMDs are generated by a gauge link which couples an odderon exchange to the proton's spin

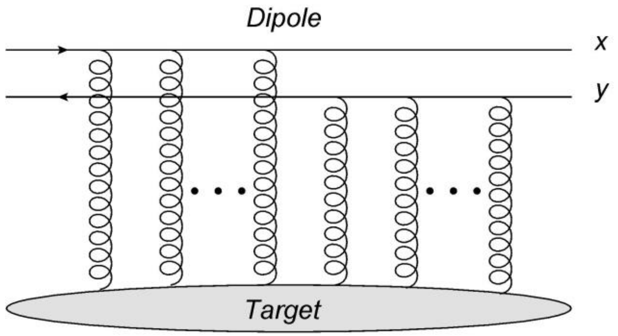
$$f_{1T}^{\perp g} = h_{1T}^{g} = h_{1T}^{\perp g} \propto \mathcal{O}_{1T}^{\perp} \,, \;\; x << 1$$

- The odderon is an elusive C-odd three gluon exchange
- *Szymanowski and Zhou* (2016) showed that the odderon contributes to cross sections when the proton's wave function is asymmetric in the transverse plane
- The D0 and TOTEM collaborations recently announced odderon in the asymmetry between pp and $p\bar{p}$ collisions [2012.03981 [hep-ex]]
- Does the quark Sivers function have this same contribution? *Zhou et al* (2019) showed that the answer is yes!

Small-x TMDs from polarized Wilson lines

- *Kovchegov and Sievert* (2019) constructed the helicity and quark transversity TMDs using a high-energy scattering operator formalism
- The strategy is to rewrite the TMD operator definitions as dipole correlators

$$\mathrm{tr}[V_x V_y^\dagger] =$$



$$V_x[b^-, a^-] = \mathcal{P} \exp \left[ig \int_{a^-}^{b^-} \mathrm{d}s^- \, A^+(s^-, x_\perp) \right]$$

*in $A^- = 0$ or $\partial_\mu A^\mu$ gauge

$$V_x = V_x[\infty, -\infty]$$

Small-x TMDs from polarized Wilson lines

- The quark correlator can be rewritten in terms of Wilson lines

$$f_1^q(x,k_T^2) - \frac{k_T \times S_P}{M_P} f_{1\,T}^{\perp\,q}(x,k_T^2) = \frac{2p^+}{(2\pi)^3} \sum_{\bar q} \int\limits_{-\infty}^{0} d\zeta^- \int\limits_{0}^{\infty} d\xi^- \int d^2\zeta_\perp\, d^2\xi_\perp\, e^{ik\cdot(\zeta-\xi)}$$

$$\times \left[\frac{\gamma^+}{2}\right]_{\alpha\beta} \left\langle \bar\psi_\alpha(\xi) V_{\underline{\xi}}[\xi^-,\infty]|\bar q\rangle\langle\bar q|V_{\underline{\zeta}}[\infty,\zeta^-]\psi_\beta(\zeta)\right\rangle + c.c.$$

$$= -\frac{2p^+}{(2\pi)^3}\int d^2\zeta_\perp\, d^2w_\perp\, \frac{d^2k_{1\perp}\, dk_1^-}{(2\pi)^3} e^{i(\underline{k}_1+\underline{k})\cdot(w-\zeta)}\theta(k_1^-) \sum_{\chi_1,\chi_2} \bar v_{\chi_2}(k_2)\frac{\gamma^+}{2} v_{\chi_1}(k_1)$$

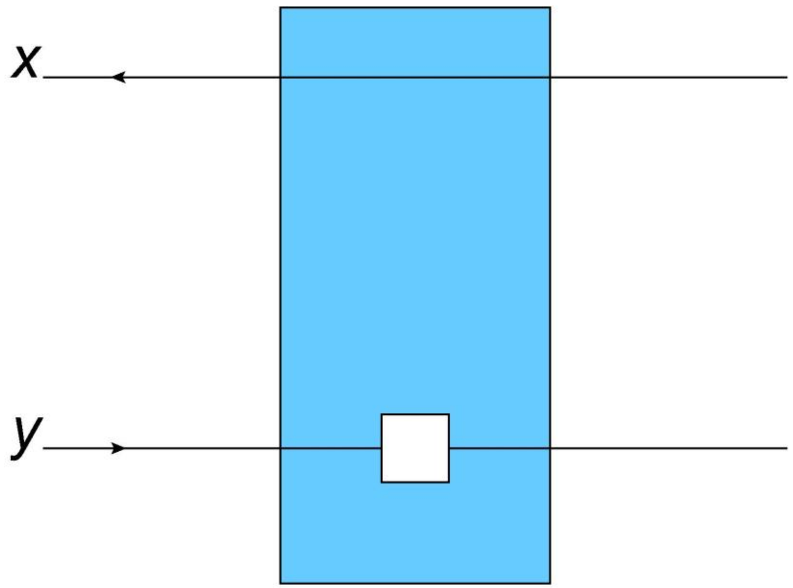
$$\times \left\langle \mathrm{T}\, V_{\underline{\zeta}}^{ij}[\infty,-\infty] \bar v_{\chi_1}(k_1) V_{\underline{w}}^{\dagger\,\mathrm{pol,\ T}\,ji} v_{\chi_2}(k_2)\right\rangle \frac{1}{(2xp^+k_1^- + \underline{k}_1^2)(2xp^+k_1^- + \underline{k}^2)} + c.c.$$

- The polarized Wilson line $V_{\underline{w}}^{\dagger\,\mathrm{pol,\ T}}$ makes the correlator a transverse polarized dipole

Small-x TMDs from polarized Wilson lines

- Define new polarized dipoles by adding spin-dependent interaction

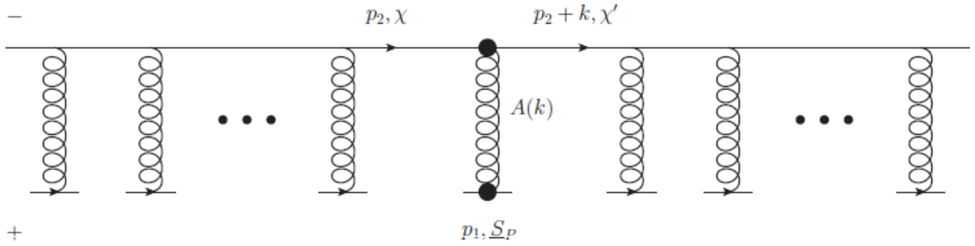
$$\mathrm{tr}[V_x V_y^{pol\,\dagger}] =$$



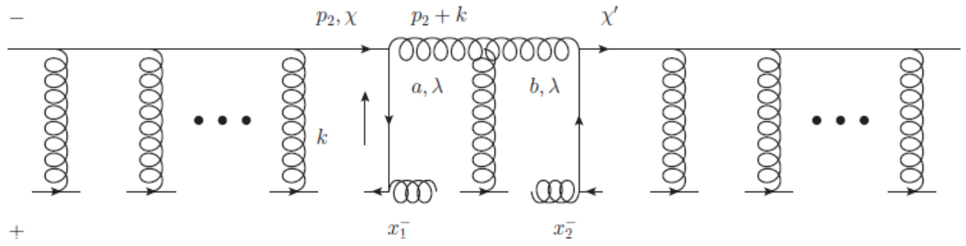
- Helicity interaction comes at sub-eikonal level, transversity at sub-sub-eikonal
- Define evolution for new polarized dipoles to get TMD evolution!

General sub-sub-eikonal Wilson line

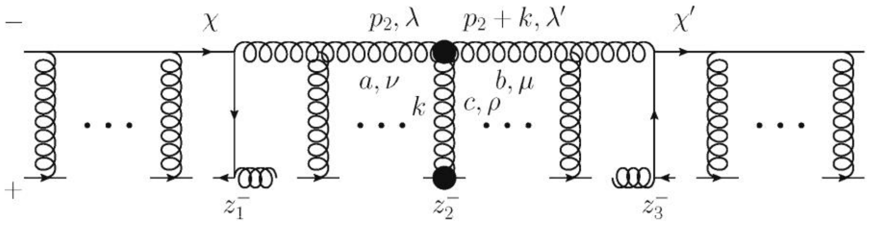
- Insert sub- or sub-sub-eikonal operator into ordinary Wilson lines



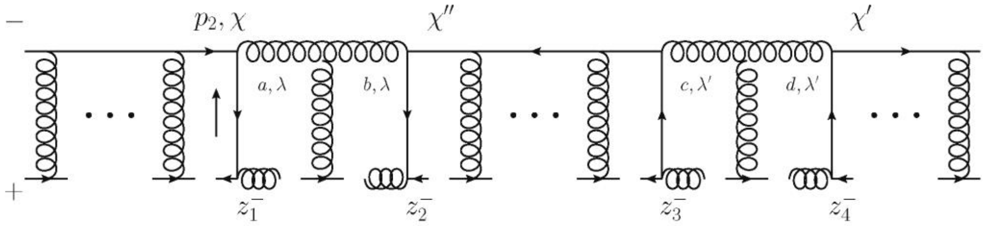
$\mathcal{O}^{\mathrm{pol\ G}}_{\chi,\chi'}$



$\mathcal{O}^{\mathrm{pol\ q\bar{q}}}_{\chi,\chi'}$



$\mathcal{O}^{\mathrm{pol\ qG\bar{q}}}_{\chi,\chi'}$

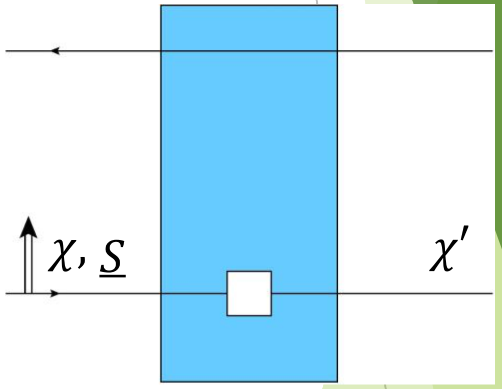


$\mathcal{O}^{\mathrm{pol\ q\bar{q}q\bar{q}}}_{\chi,\chi'}$

General polarized Wilson line

- Full sub-sub-eikonal fundamental Wilson line for TMDs which depend on the proton's transverse spin

$$
\begin{aligned}
V_{\underline{x},\underline{y};\chi',\chi} &= V_{\underline{x}}\, \delta^2(\underline{x}-\underline{y})\, \delta_{\chi,\chi'} + \int\limits_{-\infty}^{\infty} \mathrm{d}z^-\, d^2z\, V_{\underline{x}}[\infty, z^-]\, \delta^2(\underline{x}-\underline{z})\, \mathcal{O}^{\mathrm{pol\ G}}_{\chi',\chi}(z^-, \underline{z})\, V_{\underline{y}}[z^-, -\infty]\, \delta^2(\underline{y}-\underline{z}) \\
&+ \int\limits_{-\infty}^{\infty} \mathrm{d}z_1^-\, d^2z_1 \int\limits_{z_1^-}^{\infty} \mathrm{d}z_2^-\, d^2z_2 \sum_{\chi''=\pm 1} V_{\underline{x}}[\infty, z_2^-]\, \delta^2(\underline{x}-\underline{z}_2)\, \mathcal{O}^{\mathrm{pol\ G}}_{\chi',\chi''}(z_2^-, \underline{z}_2)\, V_{\underline{z}_1}[z_2^-, z_1^-]\, \delta^2(\underline{z}_2-\underline{z}_1) \\
&\times \mathcal{O}^{\mathrm{pol\ G}}_{\chi'',\chi}(z_1^-, \underline{z}_1)\, V_{\underline{y}}[z_1^-, -\infty]\, \delta^2(\underline{y}-\underline{z}_1) + \int\limits_{-\infty}^{\infty} \mathrm{d}z_1^- \int\limits_{z_1^-}^{\infty} dz_2^-\ V_{\underline{x}}[\infty, z_2^-]\, \mathcal{O}^{\mathrm{pol\ q\bar{q}}}_{\chi',\chi}(z_2^-, z_1^-; \underline{x}, \underline{y})\, V_{\underline{y}}[z_1^-, -\infty] \\
&+ \int\limits_{-\infty}^{\infty} \mathrm{d}z_1^- \int\limits_{z_1^-}^{\infty} \mathrm{d}z_2^- \int\limits_{z_2^-}^{\infty} \mathrm{d}z_3^- \int\limits_{z_3^-}^{\infty} \mathrm{d}z_4^-\, d^2z \sum_{\chi''=\pm 1} V_{\underline{x}}[\infty, z_4^-]\, \mathcal{O}^{\mathrm{pol\ q\bar{q}}}_{\chi',\chi''}(z_4^-, z_3^-; \underline{x}, \underline{z})\, V_{\underline{z}}[z_3^-, z_2^-]\, \mathcal{O}^{\mathrm{pol\ q\bar{q}}}_{\chi'',\chi}(z_2^-, z_1^-; \underline{z}, \underline{y})\, V_{\underline{y}}[z_1^-, -\infty] \\
&+ \int\limits_{-\infty}^{\infty} \mathrm{d}z_1^- \int\limits_{z_1^-}^{\infty} \mathrm{d}z_2^- \int\limits_{z_2^-}^{\infty} \mathrm{d}z_3^- \int\limits_{z_3^-}^{\infty} \mathrm{d}z_4^-\, V_{\underline{x}}[\infty, z_4^-]\, \mathcal{O}^{\mathrm{pol\ qq\bar{q}\bar{q}}}_{\chi',\chi'}(z_4^-, z_3^-, z_2^-, z_1^-; \underline{x})\, V_{\underline{x}}[z_1^-, -\infty] \\
&+ \int\limits_{-\infty}^{\infty} \mathrm{d}z_1^- \int\limits_{z_1^-}^{\infty} dz_2^-\, d^2z_2 \int\limits_{z_2^-}^{\infty} dz_3^-\ V_{\underline{x}}[\infty, z_3^-]\, \delta^2(\underline{x}-\underline{z}_2)\, \mathcal{O}^{\mathrm{pol\ qG\bar{q}}}_{\chi,\chi'}(z_3^-, z_2^-, z_1^-; \underline{x}, \underline{z}_2)\, V_{\underline{x}}[z_1^-, -\infty]\, \delta^2(\underline{z}-\underline{y}) \\
&+ \int\limits_{-\infty}^{\infty} \mathrm{d}z_1^- \int\limits_{z_1^-}^{\infty} \mathrm{d}z_2^- \int\limits_{z_2^-}^{\infty} \mathrm{d}z_3^-\, d^2z \sum_{\chi''=\pm 1} V_{\underline{x}}[\infty, z_3^-]\, \delta^2(\underline{x}-\underline{z})\, \mathcal{O}^{\mathrm{pol\ G}}_{\chi',\chi''}(z_3^-; \underline{z})\, V_{\underline{z}}[z_3^-, z_2^-]\, \mathcal{O}^{\mathrm{pol\ q\bar{q}}}_{\chi'',\chi}(z_2^-, z_1^-; \underline{z}, \underline{y})\, V_{\underline{y}}[z_1^-, -\infty] \\
&+ \int\limits_{-\infty}^{\infty} \mathrm{d}z_1^- \int\limits_{z_1^-}^{\infty} \mathrm{d}z_2^- \int\limits_{z_2^-}^{\infty} \mathrm{d}z_3^-\, d^2z \sum_{\chi''=\pm 1} V_{\underline{x}}[\infty, z_3^-]\, \mathcal{O}^{\mathrm{pol\ q\bar{q}}}_{\chi',\chi''}(z_3^-, z_2^-; \underline{x}, \underline{z})\, V_{\underline{z}}[z_2^-, z_1^-]\, \mathcal{O}^{\mathrm{pol\ G}}_{\chi'',\chi}(z_1^-; \underline{z})\, V_{\underline{y}}[z_1^-, -\infty]\, \delta^2(\underline{y}-\underline{z})
\end{aligned}
$$



cf. *Altinoluk et al* (2020), *Chirilli* (2021) sub-eikonal propagator

Eikonal Small-x Sivers function

- Apply the formalism to rewrite the Sivers function in terms of polarized transverse Wilson lines
- The quark-quark correlator which yields the leading order Sivers function is

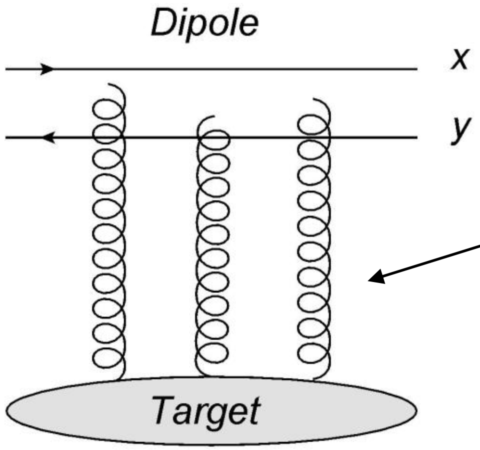
$$\left[f_1^q(x,k_T^2) - \frac{\underline{k}\times\underline{S}_P}{M_P} f_{1T}^{\perp q}(x,k_T^2)\right]_{\text{eikonal}} = \frac{4p_1^+}{(2\pi)^3}\int \mathrm{d}^2\zeta_\perp\,\mathrm{d}^2w_\perp\,\frac{\mathrm{d}^2k_{1\perp}\,\mathrm{d}k_1^-}{(2\pi)^3}e^{i(\underline{k}_1+\underline{k})\cdot(\underline{w}-\underline{\zeta})}\theta(k_1^-)$$
$$\times\left\{\frac{\underline{k}\cdot\underline{k}_1}{(xp_1^+k_1^- + \underline{k}_1^2)(xp_1^+k_1^- + \underline{k}^2)}\left\langle \mathrm{T}\,\mathrm{tr}\left[V_{\underline{\zeta}}\,V_{\underline{w}}^\dagger\right] + \bar{\mathrm{T}}\,\mathrm{tr}\left[V_{\underline{\zeta}}\,V_{\underline{w}}^\dagger\right]\right\rangle + \frac{\underline{k}_1^2}{(xp_1^+k_1^- + \underline{k}_1^2)^2}\left\langle \mathrm{T}\,\mathrm{tr}\left[V_{\underline{\zeta}}\,V_{\underline{w}}^\dagger\right]\right\rangle\right\}$$

- By symmetry, the Sivers function must come from the imaginary part of the Wilson line correlators

Dipole Odderon

- In the color dipole picture, it is the antisymmetric, imaginary piece of a dipole correlator

$$\mathcal{O}(x,y) \sim$$



$$d^{abc} = \mathrm{tr}[t^a\{t^b, t^c\}]$$

$$\mathcal{O}(x,y) = \frac{1}{2N_c}\,\mathrm{Im}\left[\left\langle \mathrm{T}\,\mathrm{tr}[V_x V_y^\dagger] + \bar{\mathrm{T}}\,\mathrm{tr}[V_x V_y^\dagger]\right\rangle\right]$$

Small-x Sivers = Odderon

- The imaginary correlator in the Sivers function is exactly the odderon amplitude, so we have

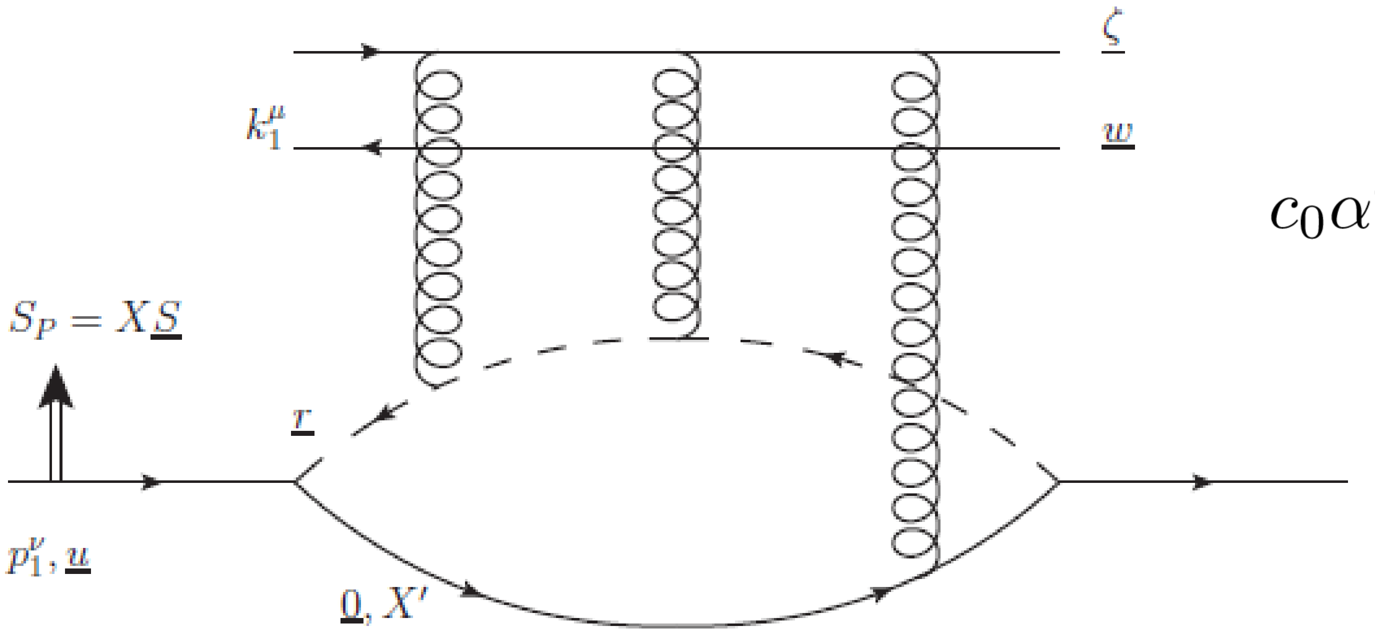
$$-\frac{\underline{k} \times \underline{S}_P}{M_P} f_{1\,T}^{\perp\, q}(x, k_T^2)\Big|_{\text{eikonal}} = \frac{4i\, N_c\, p_1^+}{(2\pi)^3} \int \mathrm{d}^2\zeta_\perp\, \mathrm{d}^2 w_\perp\, \frac{\mathrm{d}^2 k_{1\perp}\, \mathrm{d}k_1^-}{(2\pi)^3} e^{i(\underline{k}_1 + \underline{k})\cdot(\underline{w} - \underline{\zeta})} \theta(k_1^-)$$
$$\times \left[\frac{2\, \underline{k} \cdot \underline{k}_1}{(x p_1^+ k_1^- + \underline{k}_1^2)(x p_1^+ k_1^- + \underline{k}^2)} + \frac{\underline{k}_1^2}{(x p_1^+ k_1^- + \underline{k}_1^2)^2} \right] \mathcal{O}_{\underline{\zeta}\underline{w}}$$

- Agreement with the results of *Zhou et al* (2019) as well as the small-x gluon T-odd TMDs, spin-dependent odderon!
- Intrinsic asymmetry in the proton transverse momentum structure

Diquark model calculation

- We can estimate the small-x Sivers function in the diquark model, where the interaction between the point-particle proton ψ_P, the quark field ψ, and the scalar diquark φ is

$$\mathscr{L}_{int} = G\,\varphi^{*\,i}\,\bar{\psi}_q^i\,\psi_P + \text{c.c.}$$



$$c_0\alpha_s^3\ln^3\left(\frac{|\underline{\zeta}-\underline{0}||\underline{w}-\underline{r}|}{|\underline{w}-\underline{0}||\underline{\zeta}-\underline{r}|}\right)$$

Diquark model calculation

- Plugging the diquark model odderon into our Sivers function result gives

$$f_{1\,T}^{\perp\,q}(x,k_T^2)\Big|_{\text{eikonal}} = \frac{1}{x}\,\frac{N_c G^2\, c_0 \alpha_s^3}{2(2\pi)^5}\;\frac{M_P^2}{\underline{k}^2\Lambda^2}$$

- Nonzero spin-dependent odderon!
- Interesting behavior as $M_P, \Lambda, \Lambda_{QCD} \to 0$, Sivers function does not vanish

Outlook

- Small-x Sivers function is dominated by eikonal spin-dependent odderon

$$f_{1T}^{\perp q} \propto \frac{1}{x} \ , \ \ x << 1$$

- Odderon small-x evolution is known to have a zero intercept at LO and NLO in α_s (*Bartels, Lipatov, Vacca* (2000), *Kovchegov* (2013)), at large N_c (*Bartels and Vacca* (2013)), and in $\mathcal{N} = 4$ supersymmetric Yang-Mills (*Brower, Djuric, Tan* (2009), *Caron-Huot and Herranen* (2018)), so we expect no suppression of Sivers at very small-x
- Opens the possibility to see spin-dependent odderon in spin asymmetries of SIDIS (and DY) with future colliders such as EIC!
- Coming soon: evolution for sub-eikonal contributions!

References

- D. Boer, M. G. Echevarria, P. Mulders, and J. Zhou, Phys. Rev. Lett. 116, 122001 (2016), arXiv:1511.03485 [hep-ph].
- L. Szymanowski and J. Zhou, Phys. Lett. B760, 249 (2016), arXiv:1604.03207 [hep-ph].
- Y. V. Kovchegov and M. D. Sievert, Phys. Rev. D99, 054032 (2019), arXiv:1808.09010 [hep-ph].
- Y. V. Kovchegov and M. D. Sievert, Phys. Rev. D99, 054033 (2019), arXiv:1808.10354 [hep-ph].
- V. M. Abazov et al. (D0, TOTEM), (2020), arXiv:2012.03981 [hep-ex].
- H. Dong, D.-X. Zheng, and J. Zhou, Phys. Lett. B 788, 401 (2019), arXiv:1805.09479 [hep-ph].
- T. Altinoluk, G. Beuf, A. Czajka, and A. Tymowska, (2020), arXiv:2012.03886 [hep-ph].
- G. A. Chirilli, (2021), arXiv:2101.12744 [hep-ph].
- J. Bartels, L. Lipatov, and G. Vacca, Phys.Lett. B477, 178 (2000), arXiv:hep-ph/9912423 [hep-ph].
- Y. V. Kovchegov, AIP Conf. Proc. 1523, 335 (2013), arXiv:1212.2113 [hep-ph].
- J. Bartels and G. P. Vacca, Eur. Phys. J. C 73, 2602 (2013), arXiv:1307.3985 [hep-th].
- R. C. Brower, M. Djuric, and C.-I. Tan, JHEP 0907, 063 (2009), arXiv:0812.0354 [hep-th].
- S. Caron-Huot and M. Herranen, JHEP 02, 058 (2018), arXiv:1604.07417 [hep-ph].

Thank You