

# ISMD2021

50th International Symposium on  
Multiparticle Dynamics (ISMD2021)

## Novel multi-particle correlations for the heavy-ion study: tools for the new decade



You Zhou

*Niels Bohr Institute*

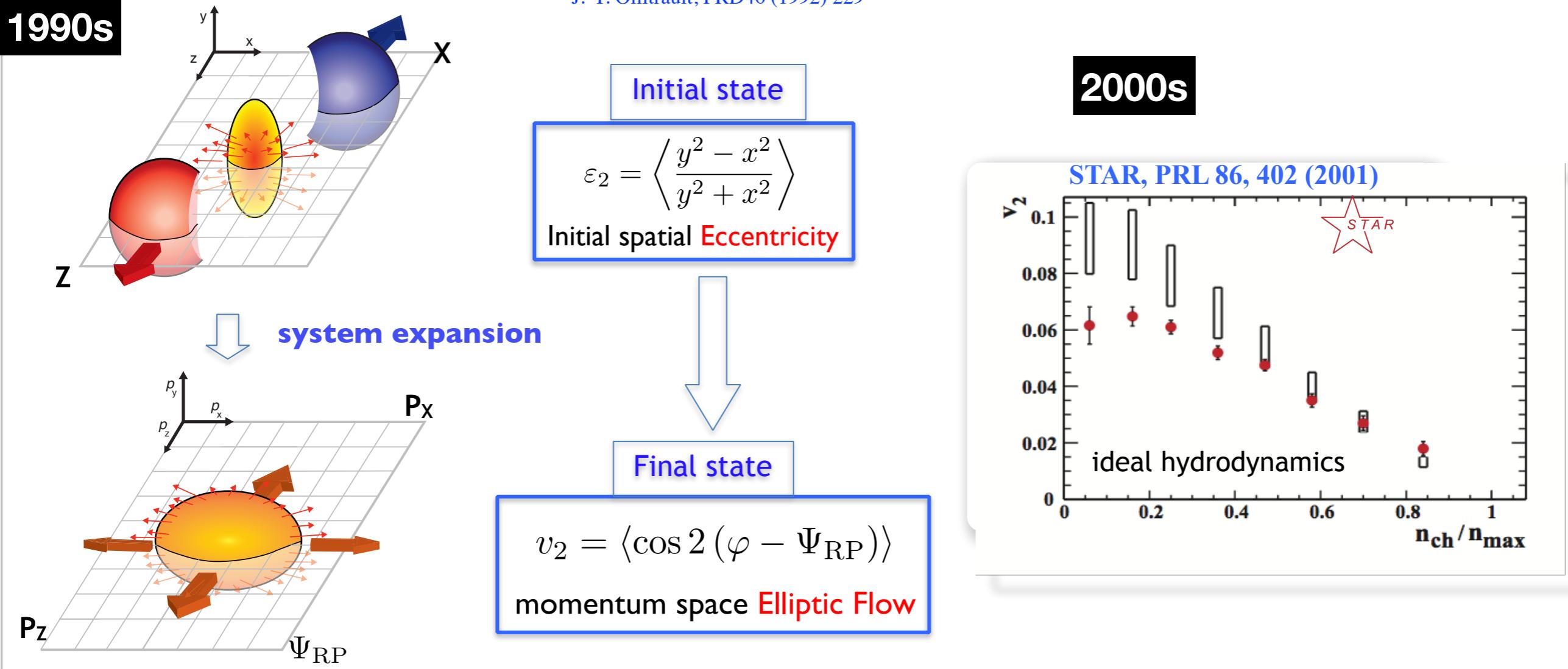


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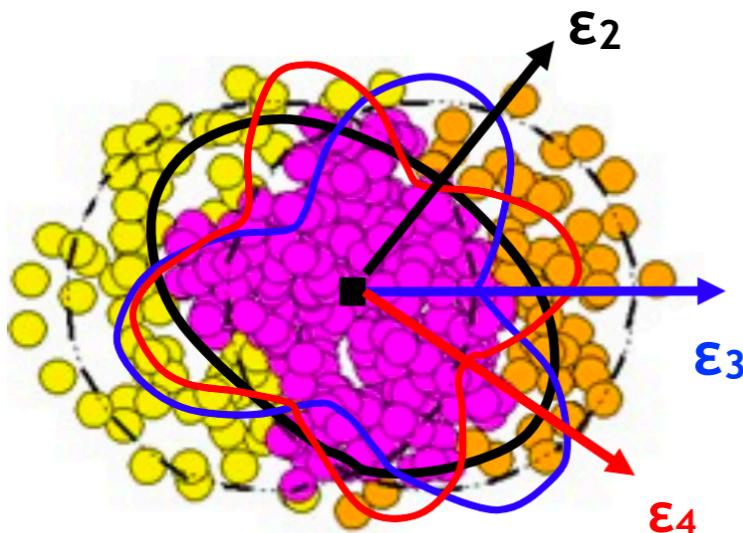
# Studying QGP with flow

- ❖ Spatial eccentricity in the initial state converted to momentum anisotropic particle distributions
  - known as **elliptic flow**
  - reflect initial **eccentricity** and **transport properties** of QGP

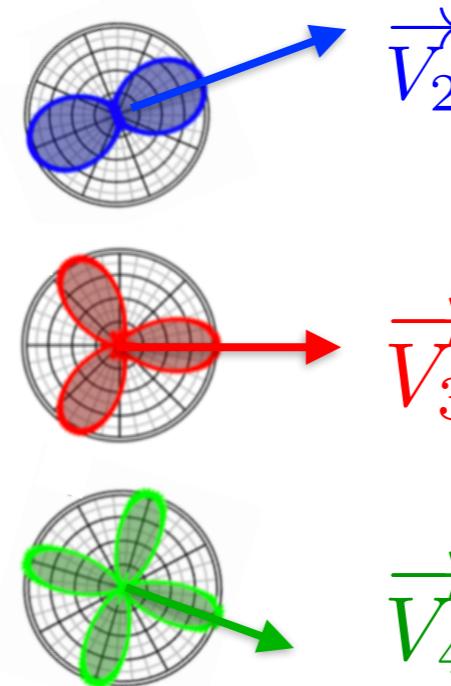


# From initial anisotropy to anisotropic flow

Initial state

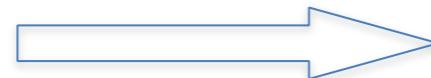


Final state



2010s

System expansion



$$\vec{V}_n = v_n e^{-in\Psi_n}$$

- $v_n$  : Anisotropic flow
- $\Psi_n$  : Flow symmetry plane

$P(\varepsilon_m, \varepsilon_n, \varepsilon_k, \dots, \Phi_m, \Phi_n, \Phi_k, \dots)$

$P(v_m, v_n, v_k, \dots, \Psi_m, \Psi_n, \Psi_k, \dots)$

How does  $v_n$  fluctuate

How does  $\Psi_n$  fluctuate

How do  $\Psi_n$  and  $\Psi_m$  correlate

How do  $v_n$  and  $v_m$  correlate

ALICE, JHEP 07 (2018) 103

ALICE, JHEP 09 (2017) 032

ALICE, JHEP05 (2020) 085  
JHEP06 (2020) 147

ALICE, PRL117, (2016) 182301  
PRC97, (2018) 024906  
PLB818 (2021) 136354



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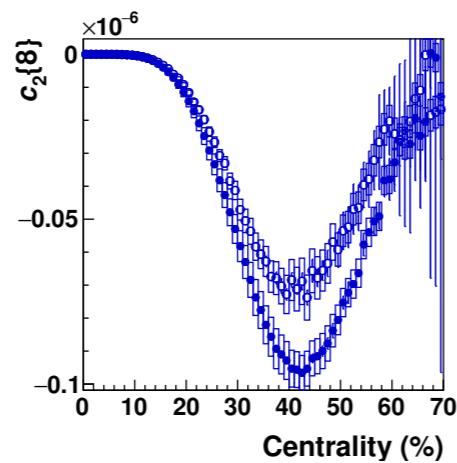
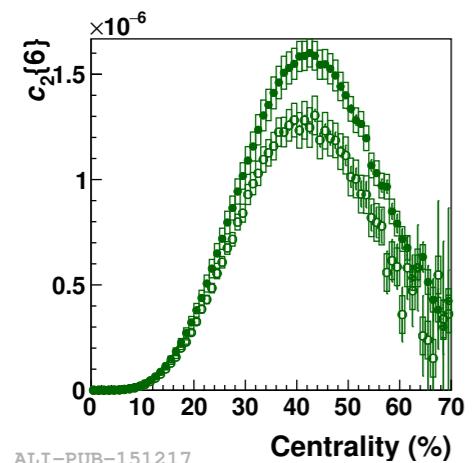
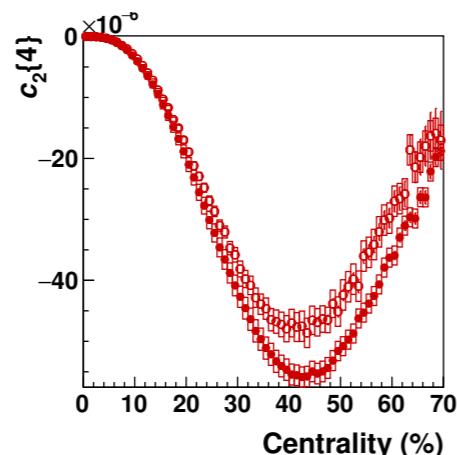
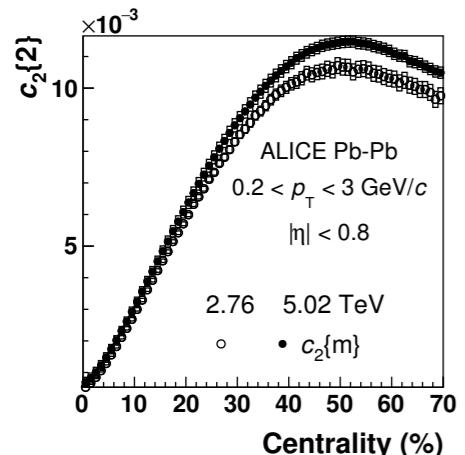
You Zhou (NBI)



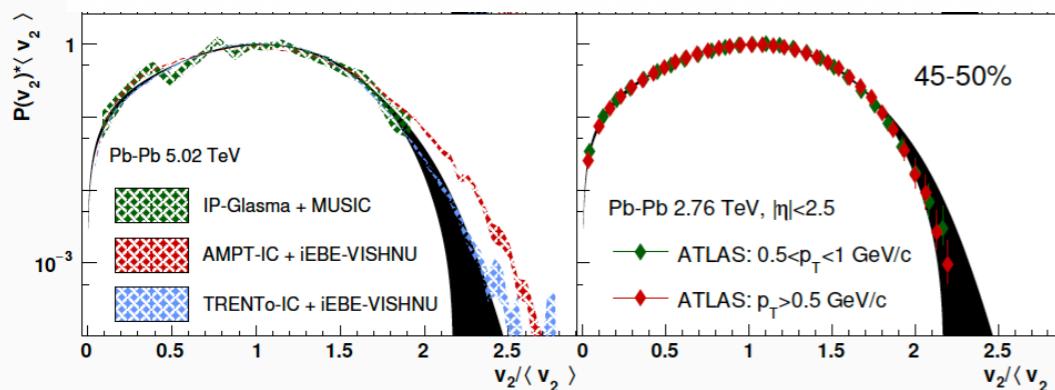
ISMD2021

# $P(v_n)$ with multi-particle cumulants of $v_n$

ALICE, JHEP 07 (2018) 103



Unfolding



$p(\epsilon_n)$  → System Expansion →  $p(v_n)$

$v_n\{2\}, v_n\{4\}, v_n\{6\}, v_n\{8\}, v_n\{10\}, v_n\{12\} \dots$

Multi-particle **correlations** of single harmonic  $v_n$

$$\langle\langle \cos(n\phi_1 - n\phi_2 + n\phi_3 - n\phi_4) \rangle\rangle = \langle v_n^4 \cos(n\Phi_n - n\Phi_n + n\Phi_n - n\Phi_n) \rangle = \langle v_n^4 \rangle$$

Multi-particle **cumulants** of single harmonic  $v_n$

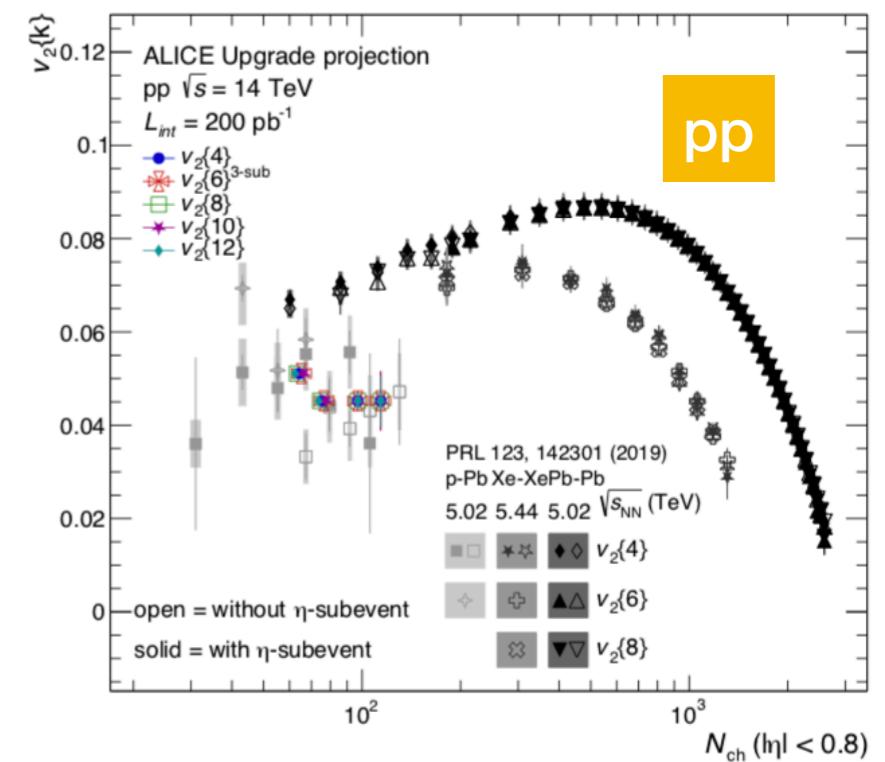
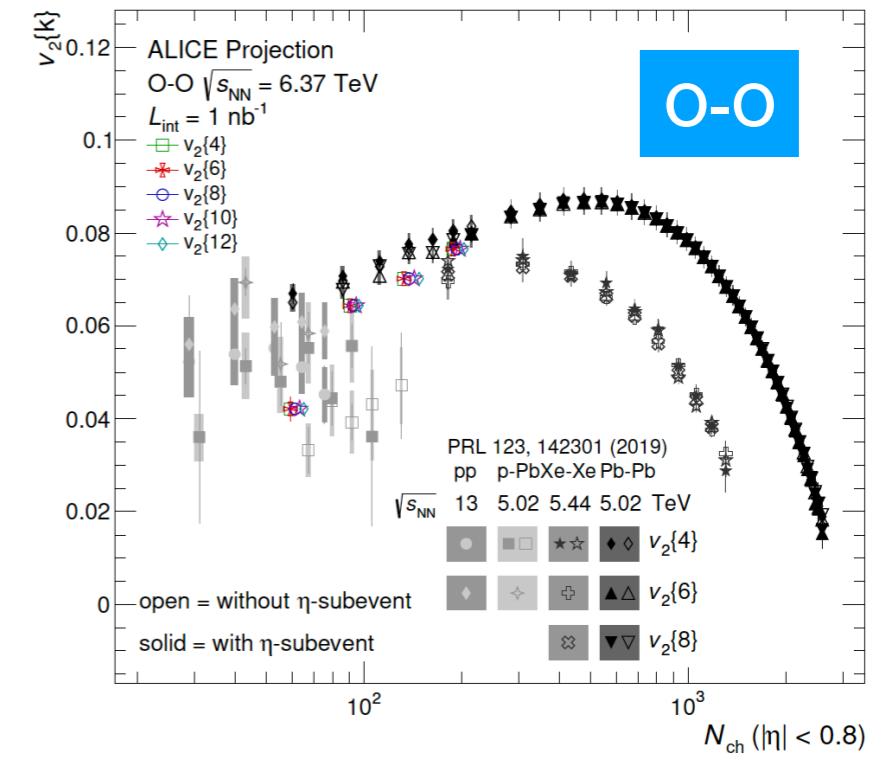
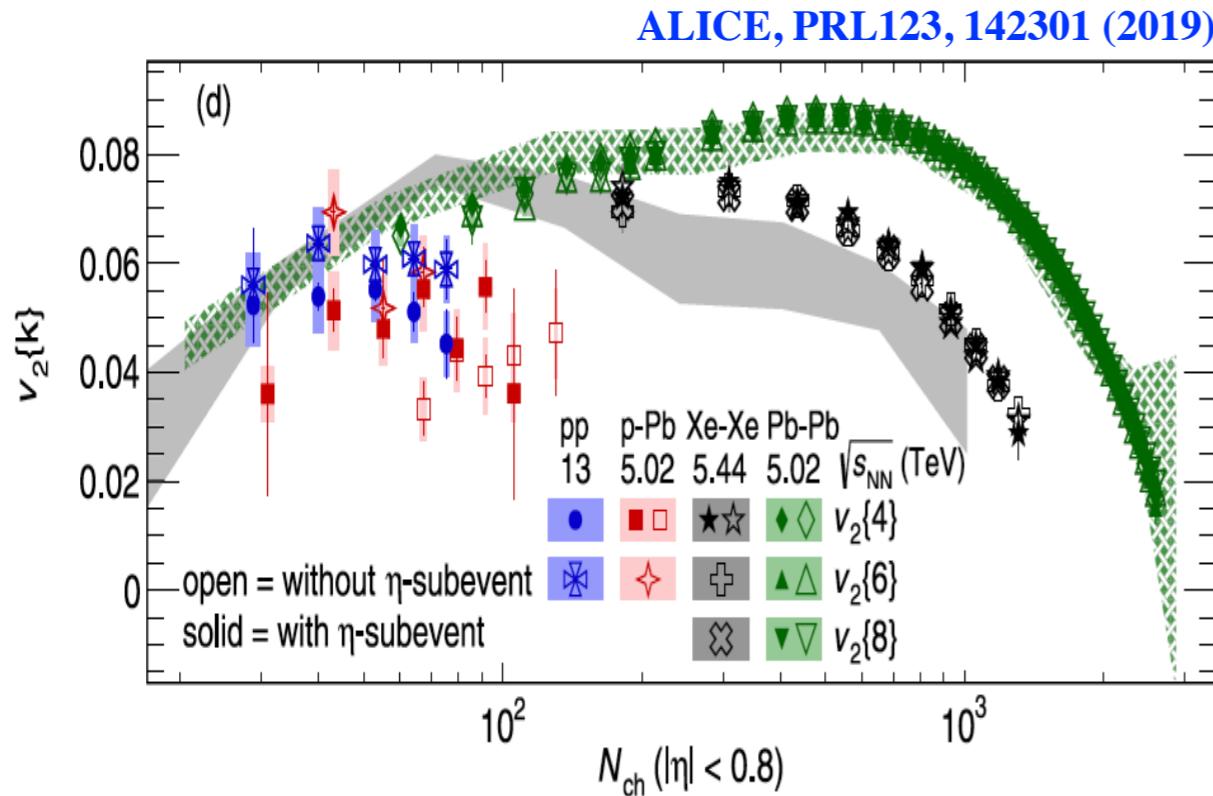
$$\begin{aligned} \langle\langle \cos(n\phi_1 - n\phi_2 + n\phi_3 - n\phi_4) \rangle\rangle_c &= \langle \cos(n\phi_1 - n\phi_2 + n\phi_3 - n\phi_4) \rangle \\ &- \langle\langle \cos(n\phi_1 - n\phi_2) \rangle\rangle \langle\langle \cos(n\phi_3 - n\phi_4) \rangle\rangle \\ &- \langle\langle \cos(n\phi_1 - n\phi_4) \rangle\rangle \langle\langle \cos(n\phi_2 - n\phi_3) \rangle\rangle \\ &= \langle v_n^4 \rangle - 2 \langle v_n^2 \rangle^2 \end{aligned}$$

$$\begin{aligned} v_n\{2\} &= \sqrt[2]{\langle v_n^2 \rangle}, \\ v_n\{4\} &= \sqrt[4]{2\langle v_n^2 \rangle^2 - \langle v_n^4 \rangle}, \\ v_n\{6\} &= \sqrt[6]{\langle v_n^6 \rangle - 9\langle v_n^2 \rangle \langle v_n^4 \rangle + 12\langle v_n^2 \rangle^3}, \\ v_n\{8\} &= \sqrt[8]{\langle v_n^8 \rangle - 16\langle v_n^2 \rangle \langle v_n^6 \rangle - 18\langle v_n^4 \rangle^2 + 144\langle v_n^2 \rangle^2 \langle v_n^4 \rangle - 144\langle v_n^2 \rangle^4}. \end{aligned}$$



# $P(v_n)$ in Xe-Xe, O-O, p-Pb and pp

ALICE-PUBLIC-2020-005, ALICE-PUBLIC-2021-004

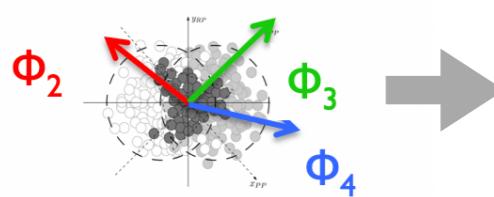


- ❖ New possibility of m-particle correlations ( $m > 8$ ) with existing RHIC & LHC heavy-ion data, and also in the future HL-LHC program.

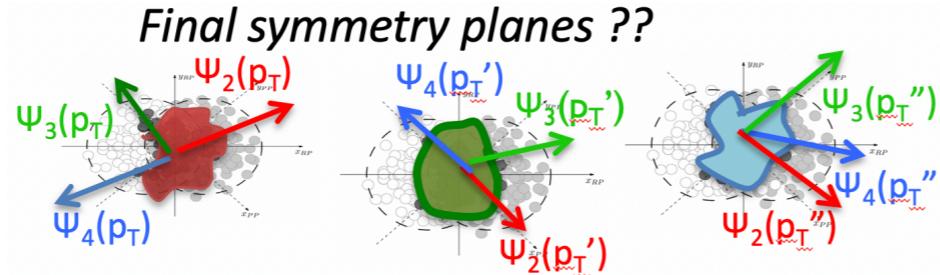


# $\Psi_n$ fluctuations $P(\Psi_n)$

Initial symmetry planes

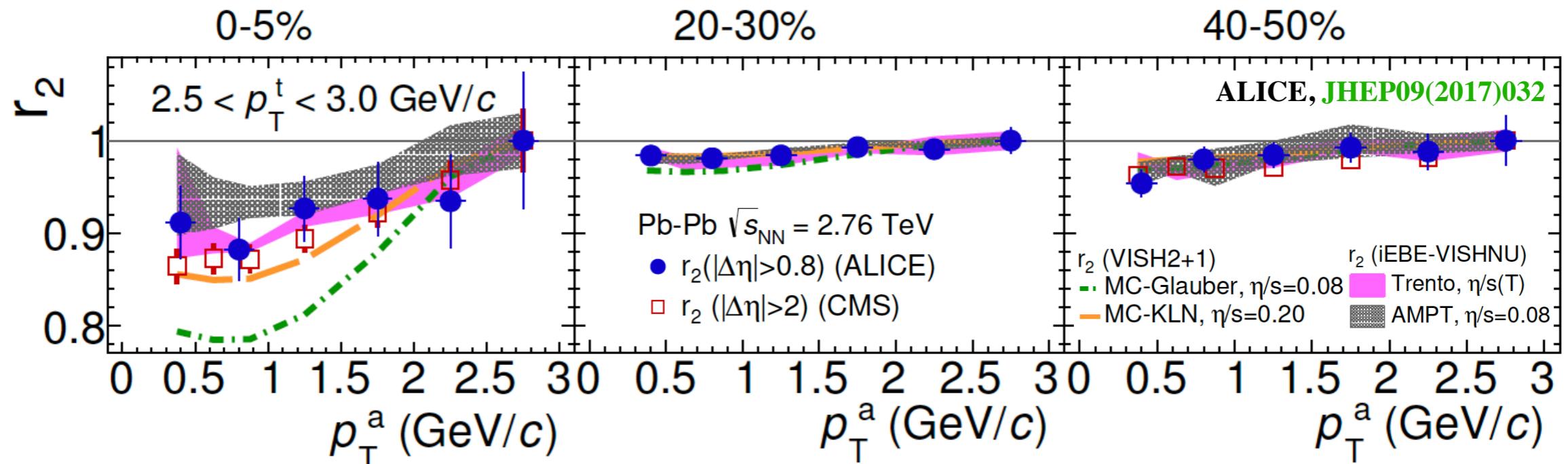


Final symmetry planes ??



$$r_n = \frac{V_n \Delta(p_T^a, p_T^b)}{\sqrt{V_n \Delta(p_T^a, p_T^a) \cdot V_n \Delta(p_T^b, p_T^b)}}$$

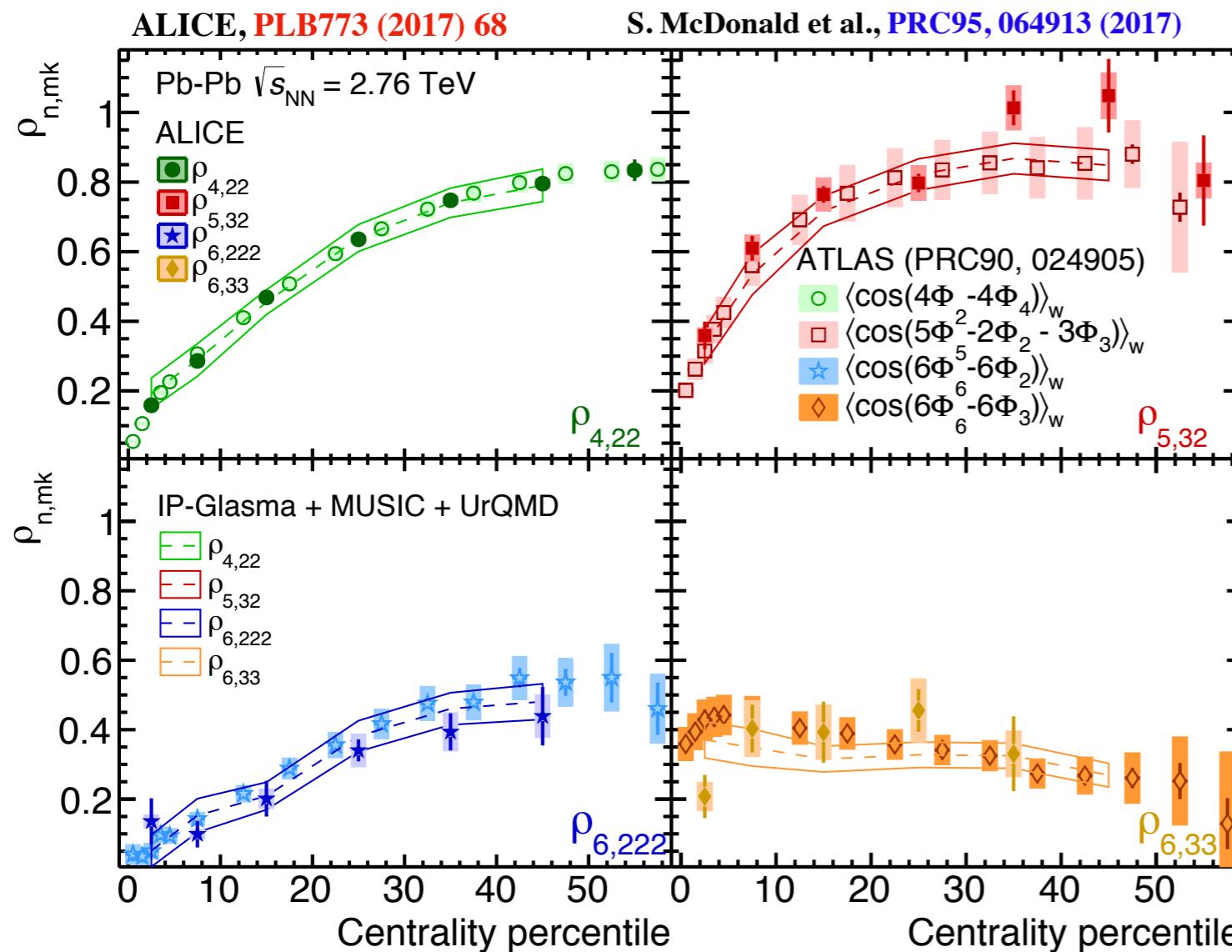
- $r_n$  probes  $\langle a, b \rangle \rightarrow \langle a, a \rangle \& \langle b, b \rangle$
- $r_n < 1$ , Factorization broken



- ❖ Breakdown of factorization more pronounced in central collisions.
- ❖ Hydrodynamic reproduce the factorization broken
  - Indication of  $p_T$  dependent flow angle (and magnitude) fluctuations
- ❖ Using novel multi-particle correlations, both flow-angle and flow magnitude fluctuations are observed in experiments (see backup [slide 21](#))



# $\Psi_n$ correlations: $P(\Psi_m, \Psi_n, \Psi_k)$



$$\begin{aligned} \rho_{4,22} &= \frac{\langle \cos(4\varphi_1 - 2\varphi_2 - 2\varphi_3) \rangle}{\sqrt{\langle \cos(4\varphi_1 - 4\varphi_2) \rangle \langle \cos(2\varphi_1 + 2\varphi_2 - 2\varphi_3 - 2\varphi_4) \rangle}} \\ &= \frac{\langle v_4 v_2^2 \cos(4\Psi_4 - 4\Psi_2) \rangle}{\sqrt{\langle v_4^2 \rangle \langle v_2^4 \rangle}} \\ &\approx \langle \cos(4\Psi_4 - 4\Psi_2) \rangle \end{aligned}$$

$\rho_{422}$   $\approx \langle \cos(4\Psi_4 - 4\Psi_2) \rangle$

$\rho_{532}$   $\approx \langle \cos(5\Psi_5 - 3\Psi_3 - 2\Psi_2) \rangle$

$\rho_{6222}$   $\approx \langle \cos(6\Psi_6 - 6\Psi_2) \rangle$

$\rho_{633}$   $\approx \langle \cos(6\Psi_6 - 6\Psi_3) \rangle$

- ❖  $\rho_{mn}$  (probes the symmetry plane correlations)
  - Agreement between ALICE and ATLAS (different eta coverage)
  - Results are compatible with hydrodynamic calculations using IP-Glasma &  $\eta/s=0.095$ ,
  - calculations using other initial conditions have difficulties to quantitatively describe the data.
- ❖ The next: test  $\rho_{5432} = \rho_{532} * \rho_{422}$  (answer see [here](#))



# Generic algorithm of multi-particle cumulants

❖ 2021, Generic algorithm of multi-particle cumulant, the most efficient and precise method

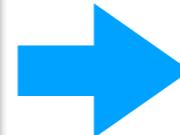
PHYSICAL REVIEW C 103, 024913 (2021)

## Generic algorithm for multiparticle cumulants of azimuthal correlations in high energy nucleus collisions

Zuzana Moravcova , Kristjan Gulbrandsen , \* and You Zhou   
Niels Bohr Institute, Blegdamsvej 17, 2100 Copenhagen, Denmark

```
complex Cumulant(int* harmonic, int n, bool remove_zeros=true, int negsplit=-1,
    int mult = 1, int skip = 0)
{
    bool remove_term = false;
    if (remove_zeros)
    {
        int har_sum = 0;
        for (int i = 0; i<mult; ++i) har_sum += harmonic[n-1+i];
        if (har_sum != 0) remove_term = true;
    }
    complex c = 0;
    if (!remove_term)
    {
        c = Corr(harmonic+(n-1), mult);
        if (n == 1) return c;
        c *= negsplit*Cumulant(harmonic, n-1, remove_zeros, negsplit-1);
    }

    int h_hold = harmonic[n-2];
    for (int counter = 0; counter <= n-2-skip; ++counter)
    {
        harmonic[n-2] = harmonic[counter];
        harmonic[counter] = h_hold;
        c += Cumulant(harmonic, n-1, remove_zeros, negsplit, mult+1, n-2-counter);
        harmonic[counter] = harmonic[n-2];
    }
    harmonic[n-2] = h_hold;
    return c;
}
```



Jiangyong Jia, J.Phys.G 41 (2014) 12

	pdfs	cumulants
	$p(v_n)$	$v_n\{2k\}, k = 1,2,\dots$
	$p(v_n, v_m)$	$\langle v_n^2 v_m^2 \rangle - \langle v_n^2 \rangle \langle v_m^2 \rangle, n \neq m$ ...
Flow-amplitudes	$p(v_n, v_m, v_l)$	$\langle v_n^2 v_m^2 v_l^2 \rangle + 2\langle v_n^2 \rangle \langle v_m^2 \rangle \langle v_l^2 \rangle -$ $\langle v_n^2 v_m^2 \rangle \langle v_l^2 \rangle - \langle v_n^2 v_l^2 \rangle \langle v_m^2 \rangle - \langle v_l^2 v_n^2 \rangle \langle v_m^2 \rangle$ $n \neq m \neq l$ ...
	...	Obtained recursively as above
EP-correlation	$p(\Phi_n, \Phi_m, \dots)$	$\langle v_n^{ c_n } v_m^{ c_m } \dots \cos(c_n n \Phi_n + c_m m \Phi_m + \dots) \rangle$ $\sum_k k c_k = 0$
Mixed-correlation	$p(v_l, \Phi_n, \Phi_m, \dots)$	$\langle v_l^2 v_n^{ c_n } v_m^{ c_m } \dots \cos(c_n n \Phi_n + c_m m \Phi_m + \dots) \rangle -$ $\langle v_l^2 \rangle \langle v_n^{ c_n } v_m^{ c_m } \dots \cos(c_n n \Phi_n + c_m m \Phi_m + \dots) \rangle$ $\sum_k k c_k = 0, n \neq m \neq l \dots$

- Few lines of code, for **any** multi-particle cumulants -> no need to download any package for cumulant calculations
- Much faster than existing framework (much shorter CPU times)



# P(v<sub>m</sub>, v<sub>n</sub>, v<sub>k</sub>, ...)

PHYSICAL REVIEW C 103, 024913 (2021)

## Generic algorithm for multiparticle cumulants of azimuthal correlations in high energy nucleus collisions

Zuzana Moravcova , Kristjan Gulbrandsen , \* and You Zhou 

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### Mixed harmonic cumulants with 4-particles

$$\text{MHC}(v_m^2, v_n^2) = \text{SC}(m, n) = \langle v_m^2 v_n^2 \rangle - \langle v_m^2 \rangle \langle v_n^2 \rangle$$

### Mixed harmonic cumulants with 6-particles

$$\begin{aligned} \text{MHC}(v_2^4, v_3^2) &= \langle\langle e^{i(2\varphi_1+2\varphi_2+3\varphi_3-2\varphi_4-2\varphi_5-3\varphi_6)} \rangle\rangle_c \\ &= \langle v_2^4 v_3^2 \rangle - 4 \langle v_2^2 v_3^2 \rangle \langle v_2^2 \rangle - \langle v_2^4 \rangle \langle v_3^2 \rangle \\ &\quad + 4 \langle v_2^2 \rangle^2 \langle v_3^2 \rangle. \end{aligned}$$

$$\begin{aligned} \text{MHC}(v_2^2, v_3^4) &= \langle\langle e^{i(2\varphi_1+3\varphi_2+3\varphi_3-2\varphi_4-3\varphi_5-3\varphi_6)} \rangle\rangle_c \\ &= \langle v_2^2 v_3^4 \rangle - 4 \langle v_2^2 v_3^2 \rangle \langle v_3^2 \rangle - \langle v_2^2 \rangle \langle v_3^4 \rangle \\ &\quad + 4 \langle v_2^2 \rangle \langle v_3^2 \rangle^2. \end{aligned}$$

$$\begin{aligned} \text{MHC}(v_2^2, v_3^2, v_4^2) &= \langle\langle e^{i(2\varphi_1+3\varphi_2+4\varphi_3-2\varphi_4-3\varphi_5-4\varphi_6)} \rangle\rangle_c \\ &= \langle v_2^2 v_3^2 v_4^2 \rangle - \langle v_2^2 v_3^2 \rangle \langle v_4^2 \rangle - \langle v_2^2 v_4^2 \rangle \langle v_3^2 \rangle \\ &\quad - \langle v_3^2 v_4^2 \rangle \langle v_2^2 \rangle + 2 \langle v_2^2 \rangle \langle v_3^2 \rangle \langle v_4^2 \rangle. \end{aligned}$$

### ❖ Multi-particle mixed harmonic cumulants

- correlation between v<sub>m</sub><sup>k</sup>, v<sub>n</sub><sup>l</sup> and v<sub>p</sub><sup>q</sup>
- correlation between v<sub>m</sub><sup>k</sup> and v<sub>n</sub><sup>l</sup>

### Mixed harmonic cumulants with 8-particles

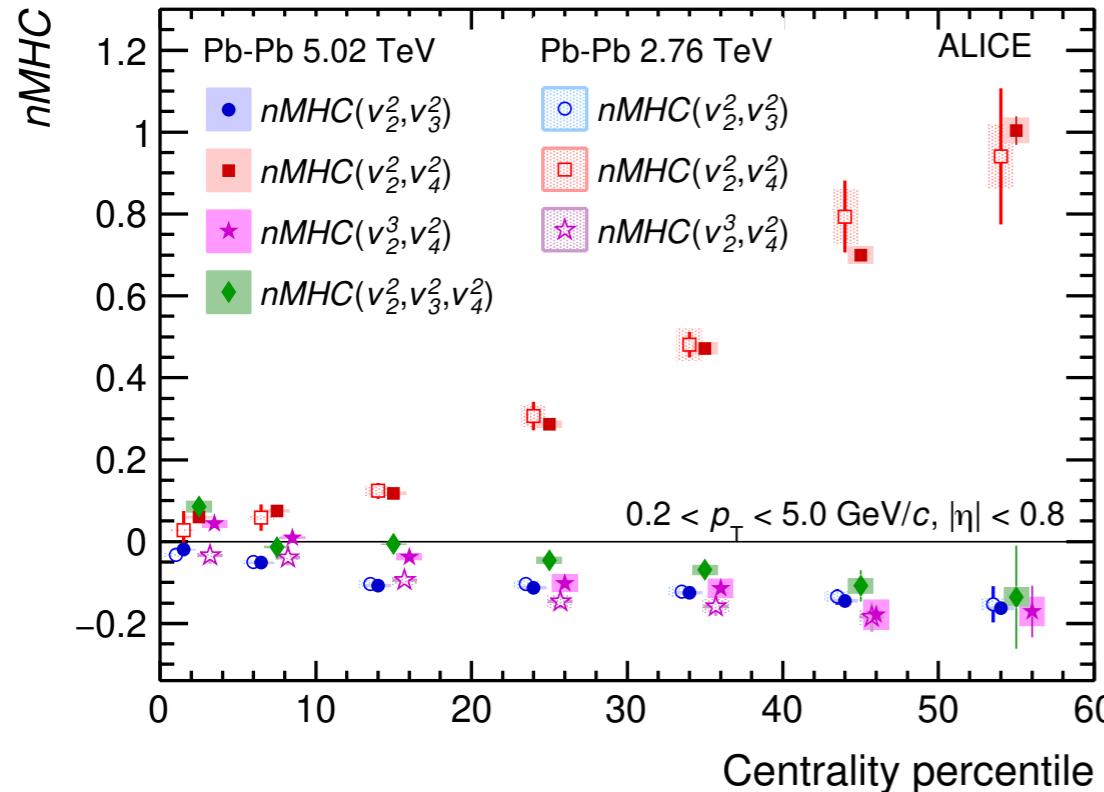
$$\begin{aligned} \text{MHC}(v_2^6, v_3^2) &= \langle\langle e^{i(2\varphi_1+2\varphi_2+2\varphi_3+3\varphi_4-2\varphi_5-2\varphi_6-2\varphi_7-3\varphi_8)} \rangle\rangle_c \\ &= \langle v_2^6 v_3^2 \rangle - 9 \langle v_2^4 v_3^2 \rangle \langle v_2^2 \rangle - \langle v_2^6 \rangle \langle v_3^2 \rangle \\ &\quad - 9 \langle v_2^4 \rangle \langle v_2^2 v_3^2 \rangle - 36 \langle v_2^2 \rangle^3 \langle v_3^2 \rangle \\ &\quad + 18 \langle v_2^2 \rangle \langle v_3^2 \rangle \langle v_2^4 \rangle + 36 \langle v_2^2 \rangle^2 \langle v_2^2 v_3^2 \rangle. \end{aligned}$$

$$\begin{aligned} \text{MHC}(v_2^4, v_3^4) &= \langle\langle e^{i(2\varphi_1+2\varphi_2+3\varphi_3+3\varphi_4-2\varphi_5-2\varphi_6-3\varphi_7-3\varphi_8)} \rangle\rangle_c \\ &= \langle v_2^4 v_3^4 \rangle - 4 \langle v_2^4 v_3^2 \rangle \langle v_3^2 \rangle \\ &\quad - 4 \langle v_2^2 v_3^4 \rangle \langle v_2^2 \rangle - \langle v_2^4 \rangle \langle v_3^4 \rangle \\ &\quad - 8 \langle v_2^2 v_3^2 \rangle^2 - 24 \langle v_2^2 \rangle^2 \langle v_3^2 \rangle^2 \\ &\quad + 4 \langle v_2^2 \rangle^2 \langle v_3^4 \rangle + 4 \langle v_2^4 \rangle \langle v_3^2 \rangle^2 \\ &\quad + 32 \langle v_2^2 \rangle \langle v_3^2 \rangle \langle v_2^2 v_3^2 \rangle. \end{aligned}$$

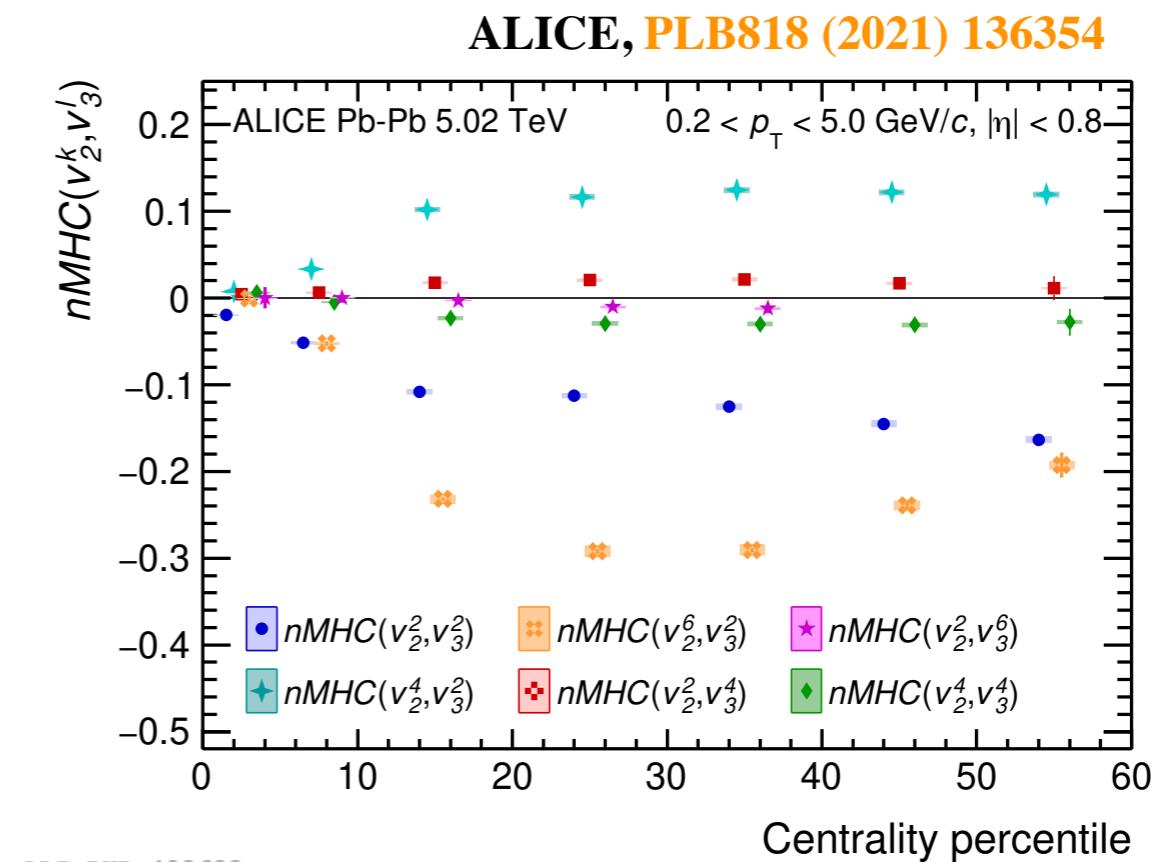
$$\begin{aligned} \text{MHC}(v_2^2, v_3^6) &= \langle\langle e^{i(2\varphi_1+3\varphi_2+3\varphi_3+3\varphi_4-2\varphi_5-3\varphi_6-3\varphi_7-3\varphi_8)} \rangle\rangle_c \\ &= \langle v_2^2 v_3^6 \rangle - 9 \langle v_2^2 v_3^4 \rangle \langle v_3^2 \rangle - \langle v_3^6 \rangle \langle v_2^2 \rangle \\ &\quad - 9 \langle v_3^4 \rangle \langle v_2^2 v_3^2 \rangle - 36 \langle v_2^2 \rangle \langle v_3^2 \rangle^3 \\ &\quad + 18 \langle v_2^2 \rangle \langle v_3^2 \rangle \langle v_3^4 \rangle + 36 \langle v_3^2 \rangle^2 \langle v_2^2 v_3^2 \rangle. \end{aligned}$$



# Multi-particle cumulants of mixed harmonics



ALI-PUB-482613



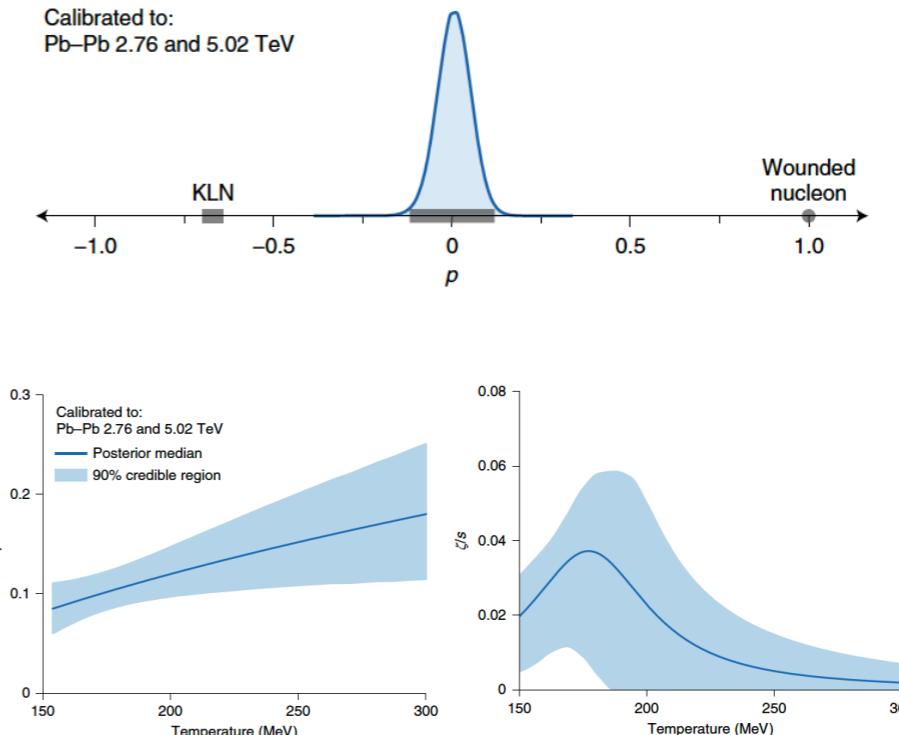
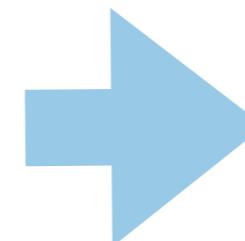
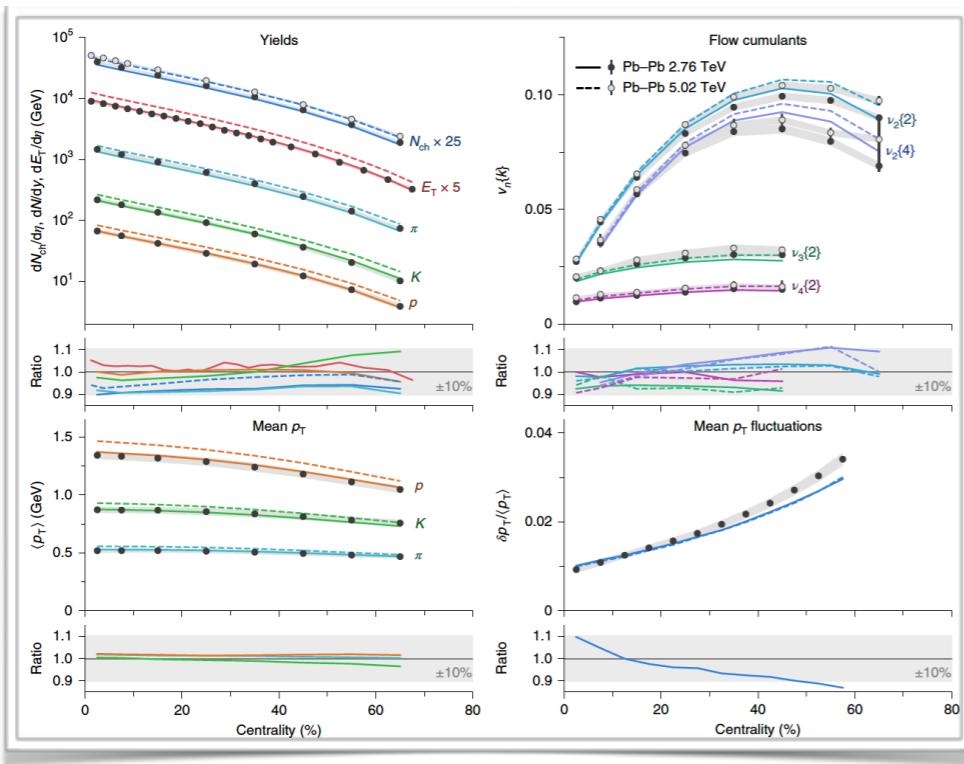
ALI-PUB-482633

- ❖ Non-zero value of  $nMHC(v_2^2, v_3^2, v_4^2)$  in Pb-Pb collisions
  - ▶ Highly non-trivial correlations among three flow coefficients
- ❖ First measurement of correlations between higher order moments of  $v_2$  and  $v_3$ 
  - ▶ characteristic -, +, - signs observed for 4-, 6- and 8-particle cumulants of *mixed harmonic*
  - ▶ Comparisons to hydrodynamic model calculations show new opportunity to constrain initial conditions and QGP properties (more in slides 22-23)

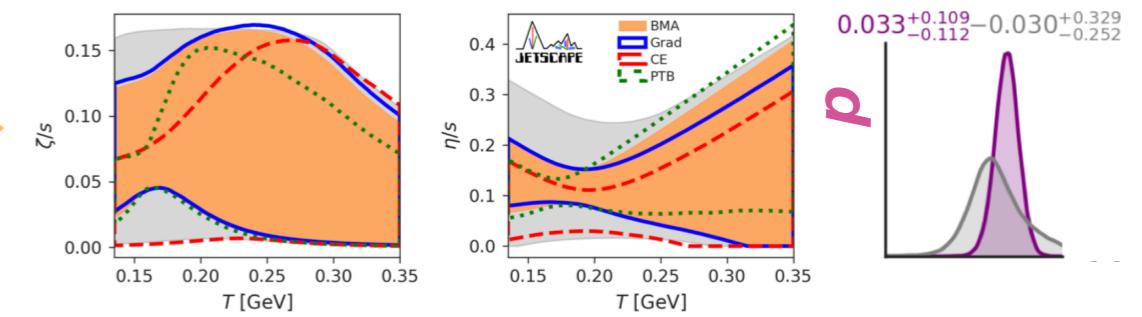
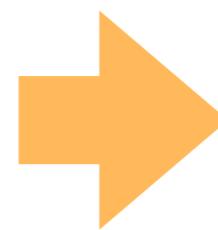
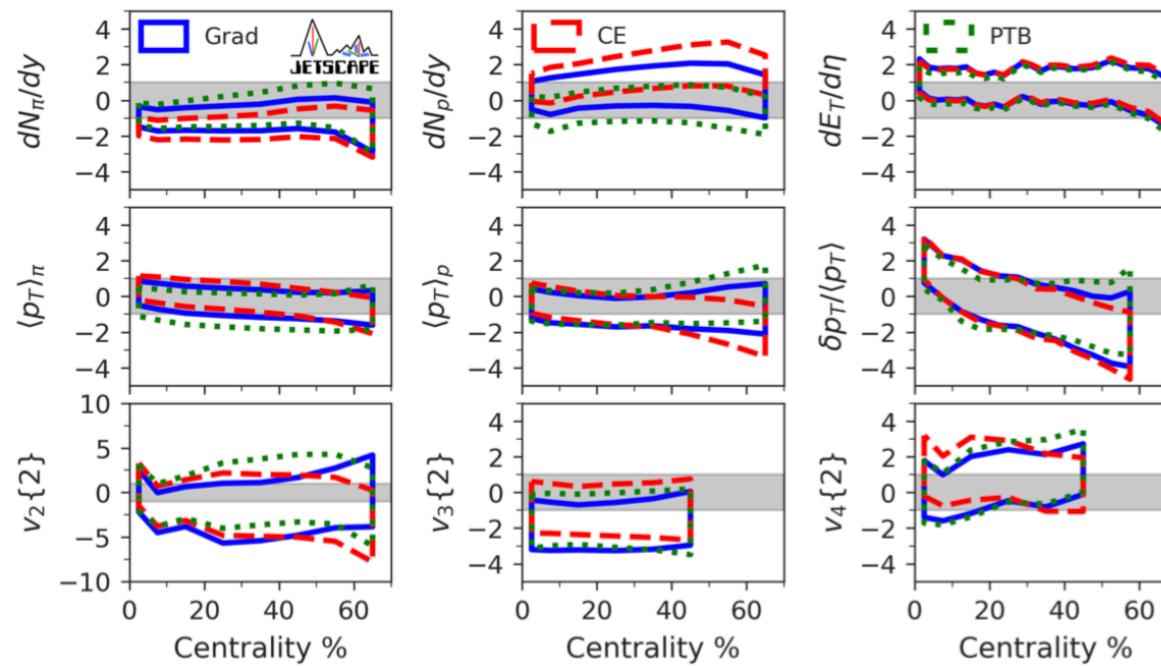


# With Bayesian analyses — End Game?

J.E. Bernhard etc, Nature Physics, 15, 1113 (2019)



JETSCAPE, Phys. Rev. Lett. 126, 242301 (2021)



End Game?



# $\langle p_T \rangle$ - $v_n$ correlations

- ❖ Anisotropic flow  $\rightarrow$  Shape of the fireball
- ❖ radial flow,  $[p_T]$   $\rightarrow$  Size of the fireball
- ❖ Final state: correlation between  $v_n$  and  $[p_T]$   
 $\rightarrow$  shape and size of the initial geometry

P. Bozek, PRC93 (2016) 044908

$$\rho(v_n^2, [p_T]) = \frac{cov(v_n^2, [p_T])}{\sqrt{var(v_n^2)}\sqrt{var([p_T])}}$$

- ★  $cov(v_n^2, [p_T])$ : **3-particle correlation** (2 azimuthal, 1  $[p_T]$ )

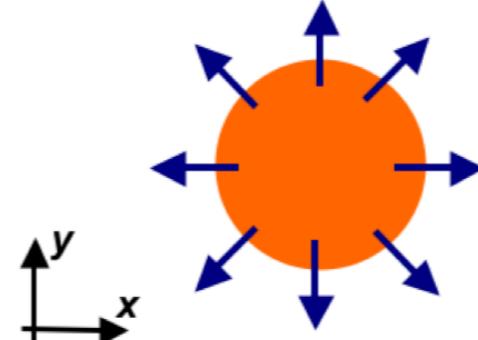
$$\left\langle \frac{\sum_{i \neq j \neq k} w_i w_j w_k e^{in\phi_i} e^{-in\phi_j} (p_{T,k} - \langle \langle p_T \rangle \rangle)}{\sum_{i \neq j \neq k} w_i w_j w_k} \right\rangle_{\text{evt}}$$

- ★  $\sqrt{var(v_n^2)}$ : **2 and 4-particle azimuthal correlations**  
 $= v_n \{2\}^4 - v_n \{4\}^4$

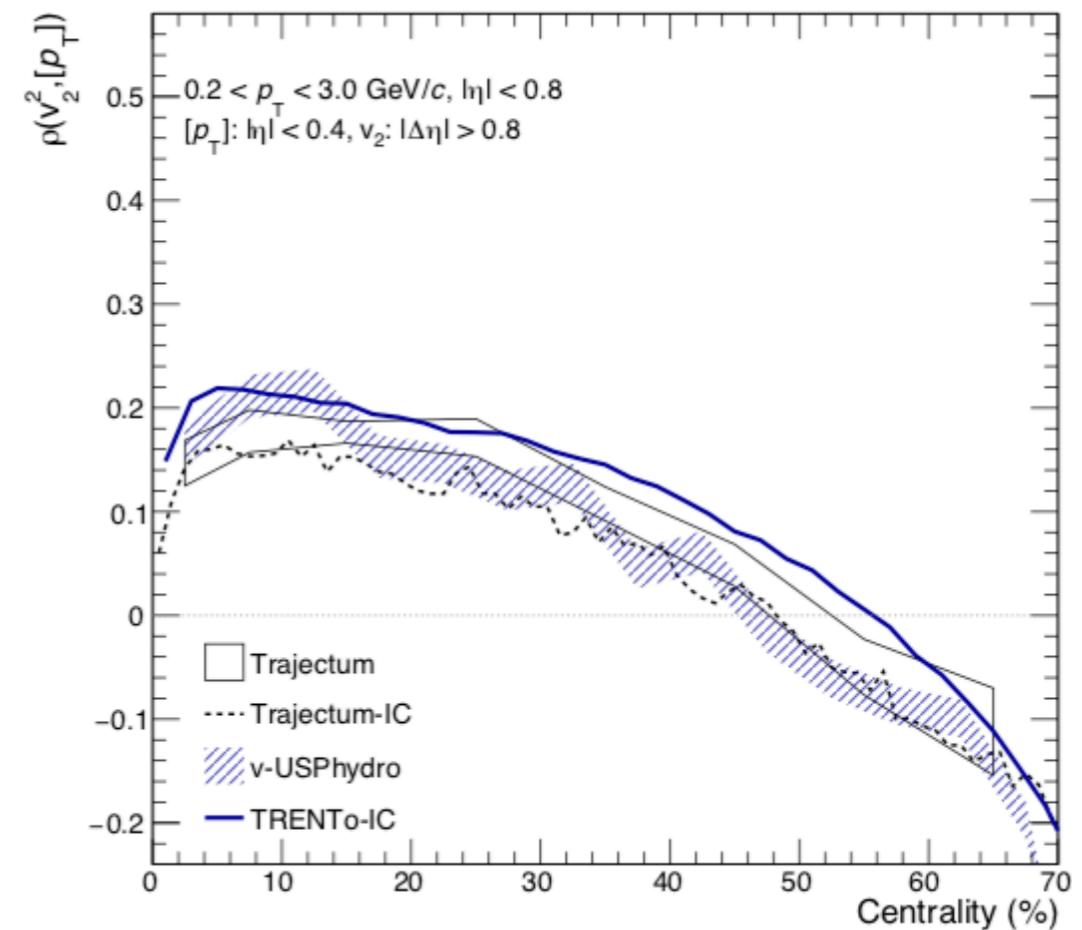
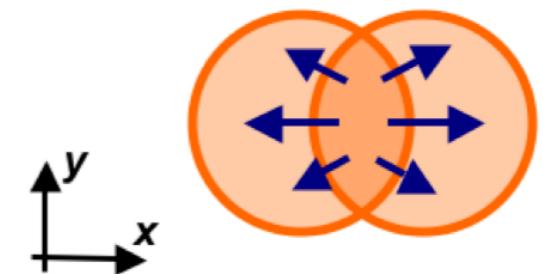
- ★  $\sqrt{var([p_T])}$  : **2-particle  $[p_T]$  correlations**

$$\left\langle \frac{\sum_{i \neq j} w_i w_j (p_{T,i} - \langle \langle p_T \rangle \rangle)(p_{T,j} - \langle \langle p_T \rangle \rangle)}{\sum_{i \neq j} w_i w_j} \right\rangle_{\text{evt}}$$

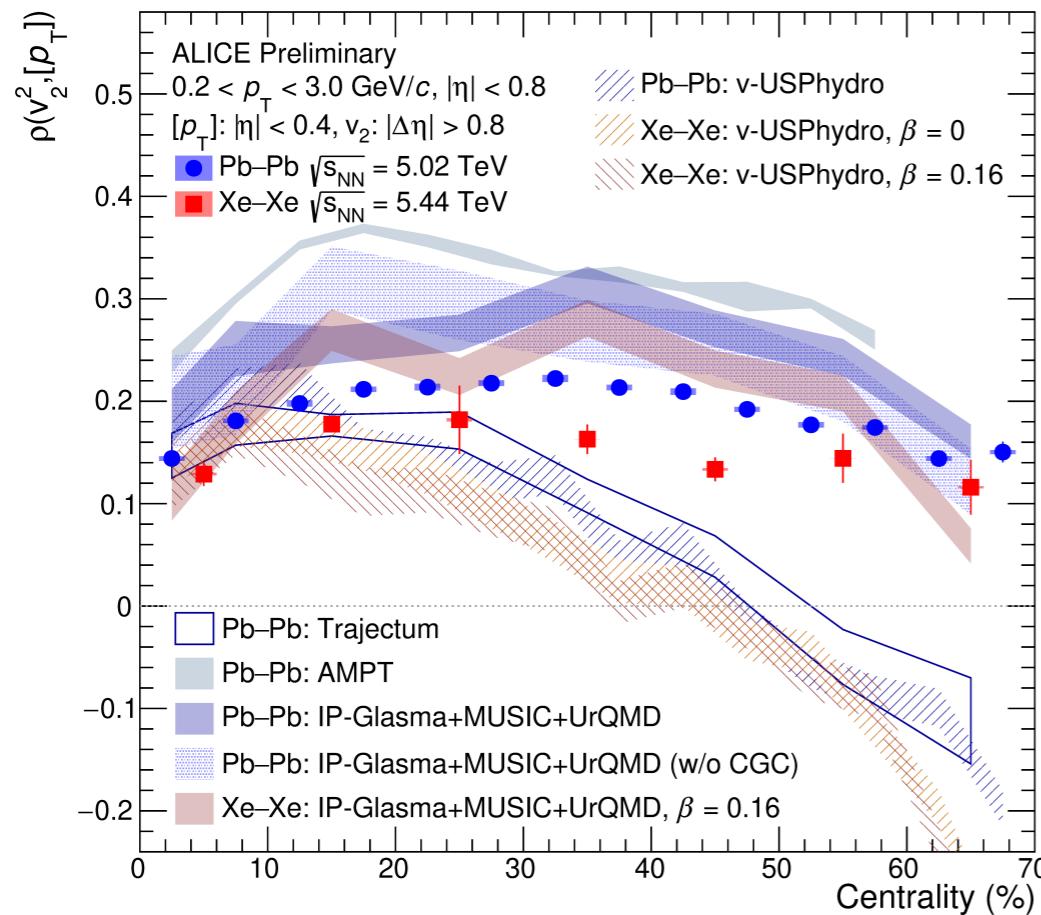
Radial flow



Anisotropic flow



# New measurements of $\langle p_T \rangle - v_n$ correlations

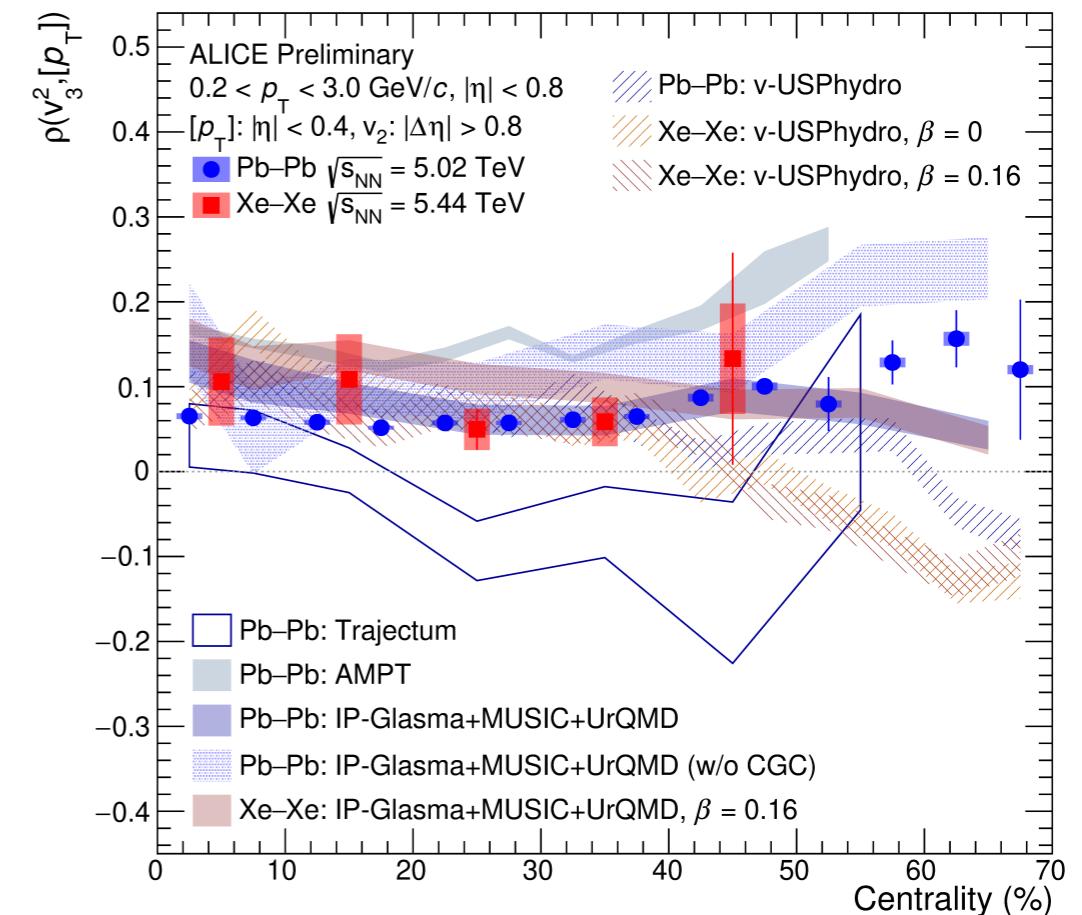


ALI-PREL-494367

## ❖ Comparison with hydro models:

- IP-Glasma + MUSIC + UrQMD works well for Pb-Pb, overestimates Xe-Xe
- TRENTo-IC based calculations with strong centrality dependence and negative values for centrality above 40-50%

## ❖ Expectations: all current Bayesian analyses will fail to describe the ALICE data



ALI-PREL-494374

- ❖ Pb-Pb and Xe-Xe results are compatible
- ❖  $\rho_3$  results are positive and have a modest centrality dependence for the presented centralities,
  - better described by IP-Glasma,
  - TRENTo predicts negative  $\rho_3$  in peripheral collisions, which is not seen in data

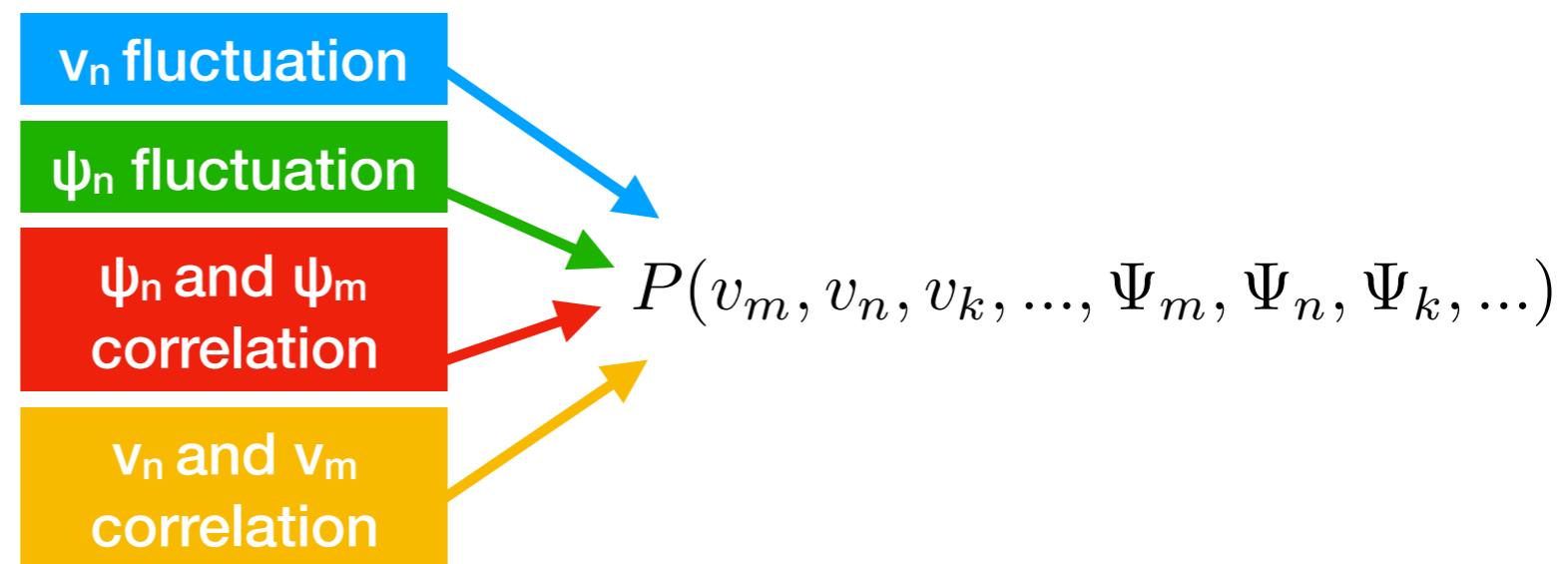
**More see: E. G. Nielsen @ EPS-HEP21**



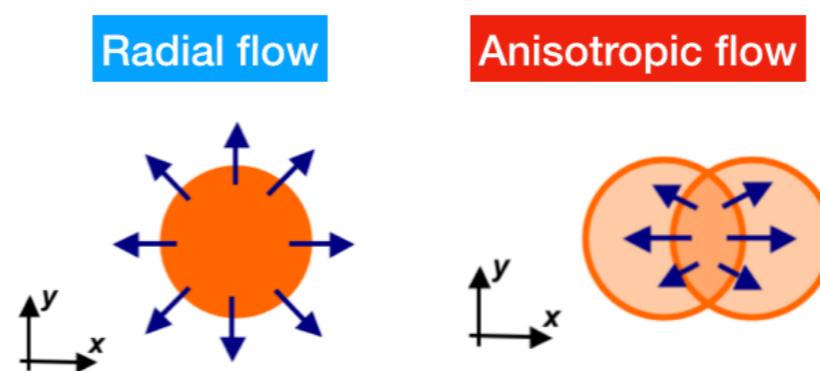
# Summary

## For the studies of multi-particle correlations

### ❖ Last decade



### ❖ New decade



C. Zhang @ ISMD2021  
S. Huang @ ISMD2021

Thanks for your attention!



# Backup



# Multi-particle correlations

- ❖ Multi-particle correlations with **generating function** method, popular at RHIC

Flow analysis from multiparticle azimuthal correlations

Nicolas Borghini, Phuong Mai Dinh, and Jean-Yves Ollitrault  
Phys. Rev. C **64**, 054901 – Published 25 September 2001

408 citations

- ❖ Multi-particle correlations with **Q-cumulant** method, popular at earlier LHC

Flow analysis with cumulants: Direct calculations

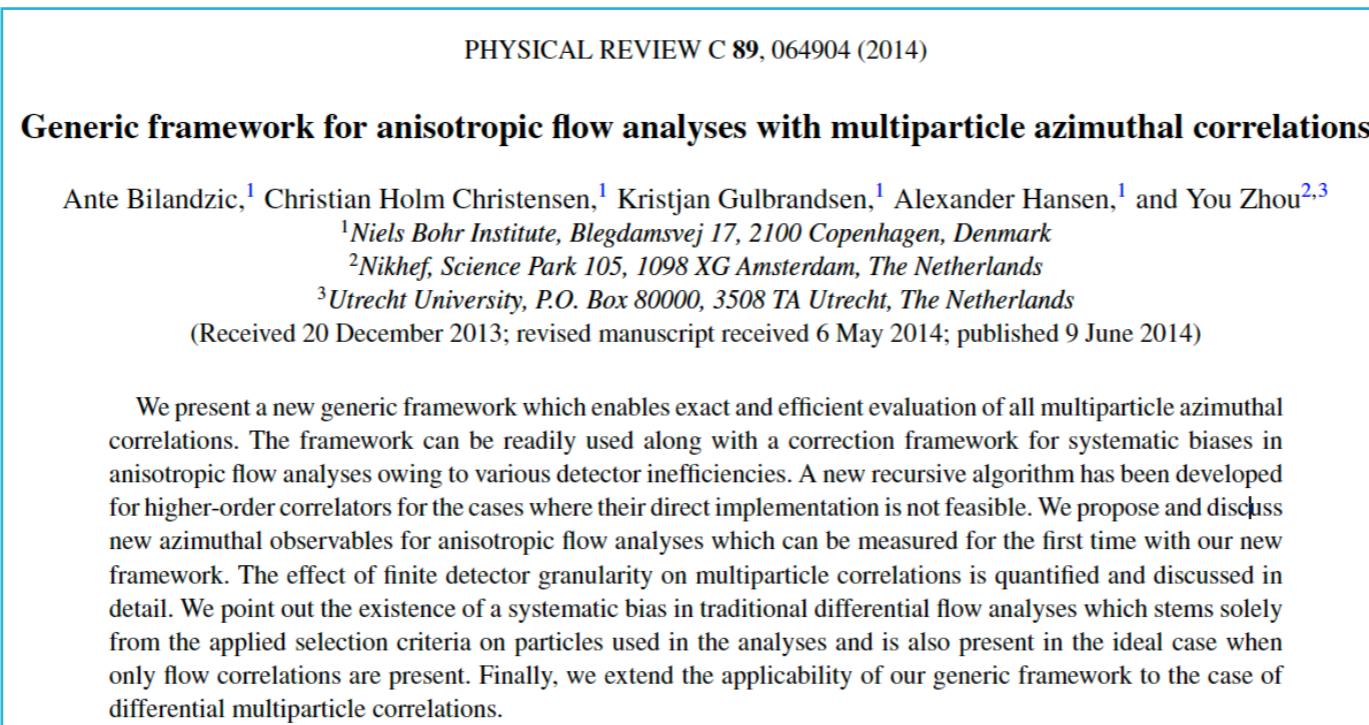
Ante Bilandzic, Raimond Snellings, and Sergei Voloshin  
Phys. Rev. C **83**, 044913 – Published 26 April 2011

317 citations



# Generic framework (2014)

❖ 2014, Generic framework method, the most popular approach at the LHC



154 citations

Step1 → Step2 → Step3

$$Q_{n,p} \equiv \sum_{k=1}^M w_k^p e^{in\varphi_k}$$

$$\begin{aligned} N\langle m \rangle_{n_1, n_2, \dots, n_m} &\equiv \sum_{\substack{k_1, k_2, \dots, k_m = 1 \\ k_1 \neq k_2 \neq \dots \neq k_m}}^M w_{k_1} w_{k_2} \cdots w_{k_m} \\ &\times e^{i(n_1\varphi_{k_1} + n_2\varphi_{k_2} + \dots + n_m\varphi_{k_m})}, \\ D\langle m \rangle_{n_1, n_2, \dots, n_m} &\equiv \sum_{\substack{k_1, k_2, \dots, k_m = 1 \\ k_1 \neq k_2 \neq \dots \neq k_m}}^M w_{k_1} w_{k_2} \cdots w_{k_m} \\ &= N\langle m \rangle_{0,0,\dots,0}. \end{aligned}$$

$$\langle m \rangle_{n_1, n_2, \dots, n_m} \equiv \langle e^{i(n_1\varphi_{k_1} + n_2\varphi_{k_2} + \dots + n_m\varphi_{k_m})} \rangle \equiv \frac{\sum_{\substack{k_1, k_2, \dots, k_m = 1 \\ k_1 \neq k_2 \neq \dots \neq k_m}}^M w_{k_1} w_{k_2} \cdots w_{k_m} e^{i(n_1\varphi_{k_1} + n_2\varphi_{k_2} + \dots + n_m\varphi_{k_m})}}{\sum_{\substack{k_1, k_2, \dots, k_m = 1 \\ k_1 \neq k_2 \neq \dots \neq k_m}}^M w_{k_1} w_{k_2} \cdots w_{k_m}}.$$



# Generic algorithm (2021)

❖ 2021, Generic algorithm method, the most *efficient, precise* and *reliable* method

arXiv: 2005.07974

PHYSICAL REVIEW C 103, 024913 (2021)

## Generic algorithm for multiparticle cumulants of azimuthal correlations in high energy nucleus collisions

Zuzana Moravcova<sup>✉</sup>, Kristjan Gulbrandsen<sup>✉,\*</sup> and You Zhou<sup>✉†</sup>

Niels Bohr Institute, Blegdamsvej 17, 2100 Copenhagen, Denmark

(Received 23 November 2020; accepted 11 February 2021; published 26 February 2021)

- Few lines of code, for **any** multi-particle correlations
- Much faster than generic framework (much shorter CPU times)

```
complex Correlator(int* harmonic, int n, int mult = 1, int skip = 0)
{
    int har_sum = 0;
    for (int i = 0; i<mult; ++i) har_sum += harmonic[n-1+i];
    complex c(Q(har_sum, mult));
    if (n == 1) return c;
    c *= Correlator(harmonic, n-1);
    if (n == 1+skip) return c;

    complex c2 = 0;
    int h_hold = harmonic[n-2];
    for (int counter = 0; counter <= n-2-skip; ++counter)
    {
        harmonic[n-2] = harmonic[counter];
        harmonic[counter] = h_hold;
        c2 += Correlator(harmonic, n-1, mult+1, n-2-counter);
        harmonic[counter] = harmonic[n-2];
    }
    harmonic[n-2] = h_hold;
    return c-mult*c2;
}
```

Feel free to contact [you.zhou@cern.ch](mailto:you.zhou@cern.ch)  
if you have any technical question



## THERE ARE CURRENTLY THREE CATEGORIES OF MODELS.

- “sharp” models: IP-GLASMA and TRENTo 2016

[Schenke, Shen, Tribedy [2005.14682](#)]

[Bass, Bernhard, Moreland [1605.03954](#)]

Nucleons have a width of ~0.5fm (trento), 3 sub-nucleons with size ~0.2fm (IP-Glasma). Trento is used for the entropy density at the beginning of hydro. IP-Glasma is the only model which incorporates a realistic pre-equilibrium evolution with longitudinal cooling.

- “fat” models: TRENTo 2019 and JETSCAPE

[Bass, Bernhard, Moreland [Nature Phys. 15 \(2019\)](#)]

[JETSCAPE Collaboration [2011.01430](#), [2010.03928](#) ]

The Trento parametrization is now used for the energy density at tau=0+. There is no substructure. The nucleon width is now ~1fm. Very smooth profiles.

- “lumpy fat” models: TRENTo 2018 and Trajectum

[Bass, Bernhard, Moreland [1808.02106](#)]

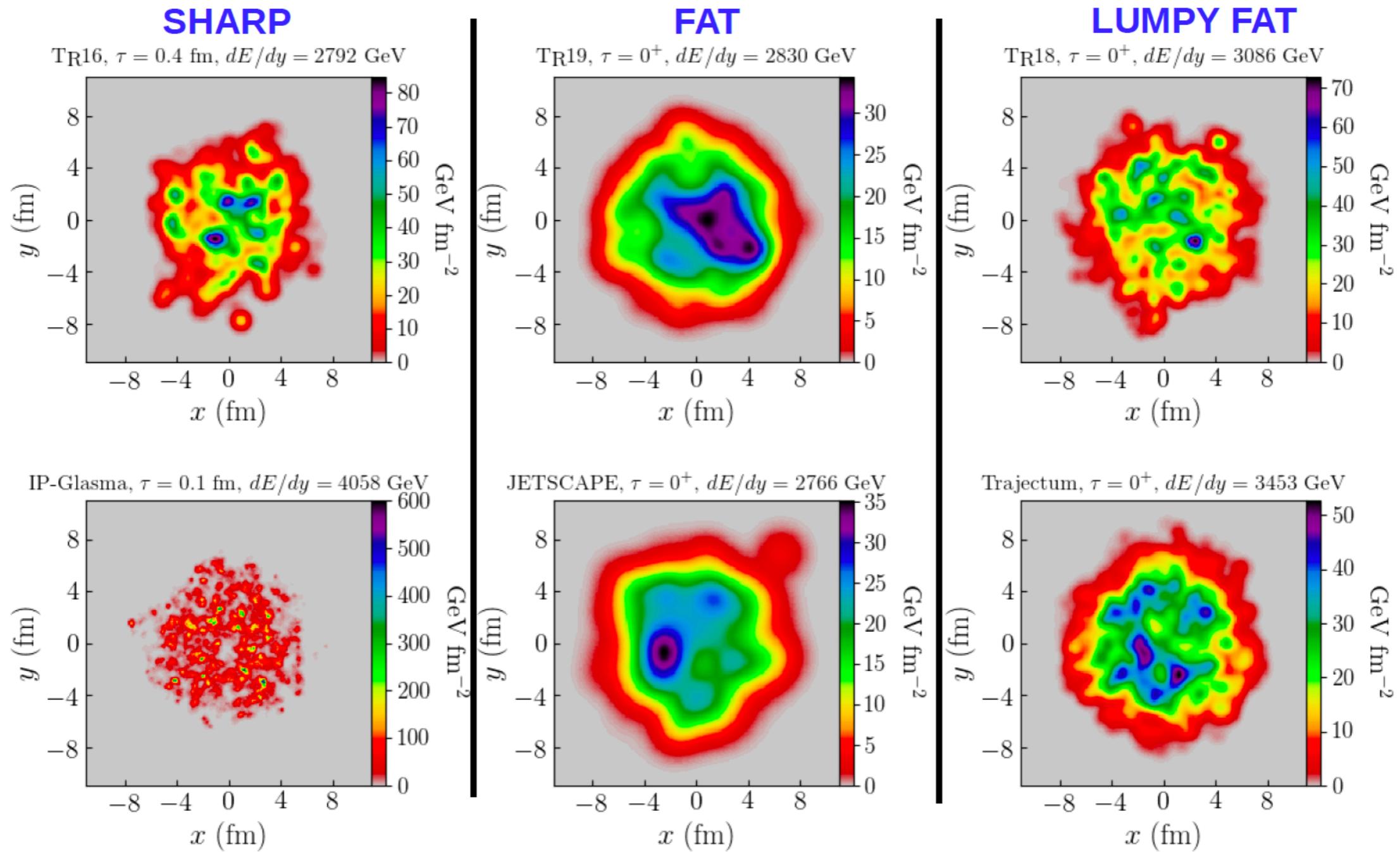
[Nijs, van der Schee, Gürsoy, Snellings [2010.15130](#), [2010.15134](#)]

The Trento parametrization is the energy density at tau=0+. Substructure is included: 4-6 constituents with width ~0.5fm. Profiles with some ‘old school’ lumpiness.



## Example of profiles for Pb-Pb @ 2.76 TeV at b=0

G. Giacalone @ IS2021



# Flow angle and flow magnitude fluctuations

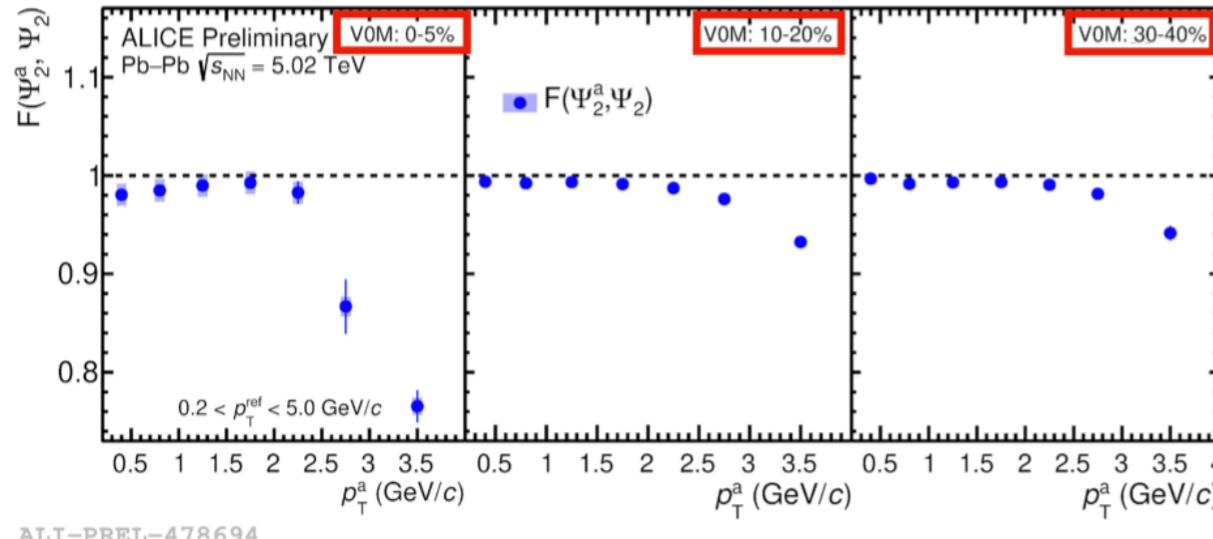
Emil Gorm Nielsen @ IS2021

## Flow angle and magnitude fluctuations

Flow angle fluctuations

$$F(\Psi_2^a, \Psi_2) = \langle \cos 2n[\Psi_2(p_T^a) - \Psi_2] \rangle$$

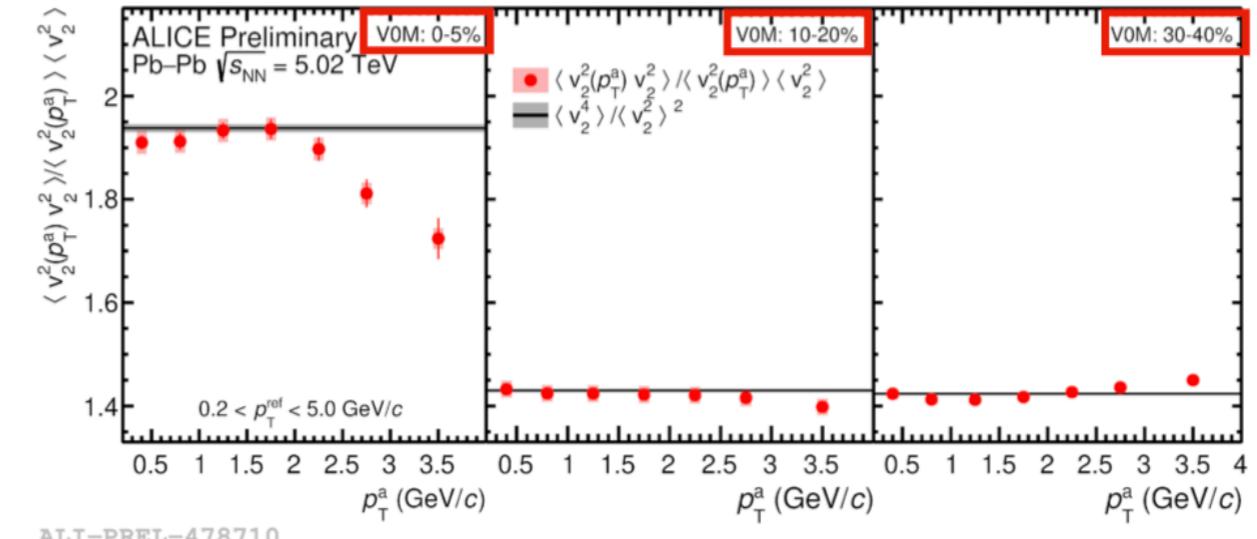
**NEW**



Flow magnitude fluctuations

$$\langle v_n^2(p_T^a) v_n^2 \rangle / \langle v_n^2(p_T^a) \rangle \langle v_n^2 \rangle$$

**NEW**



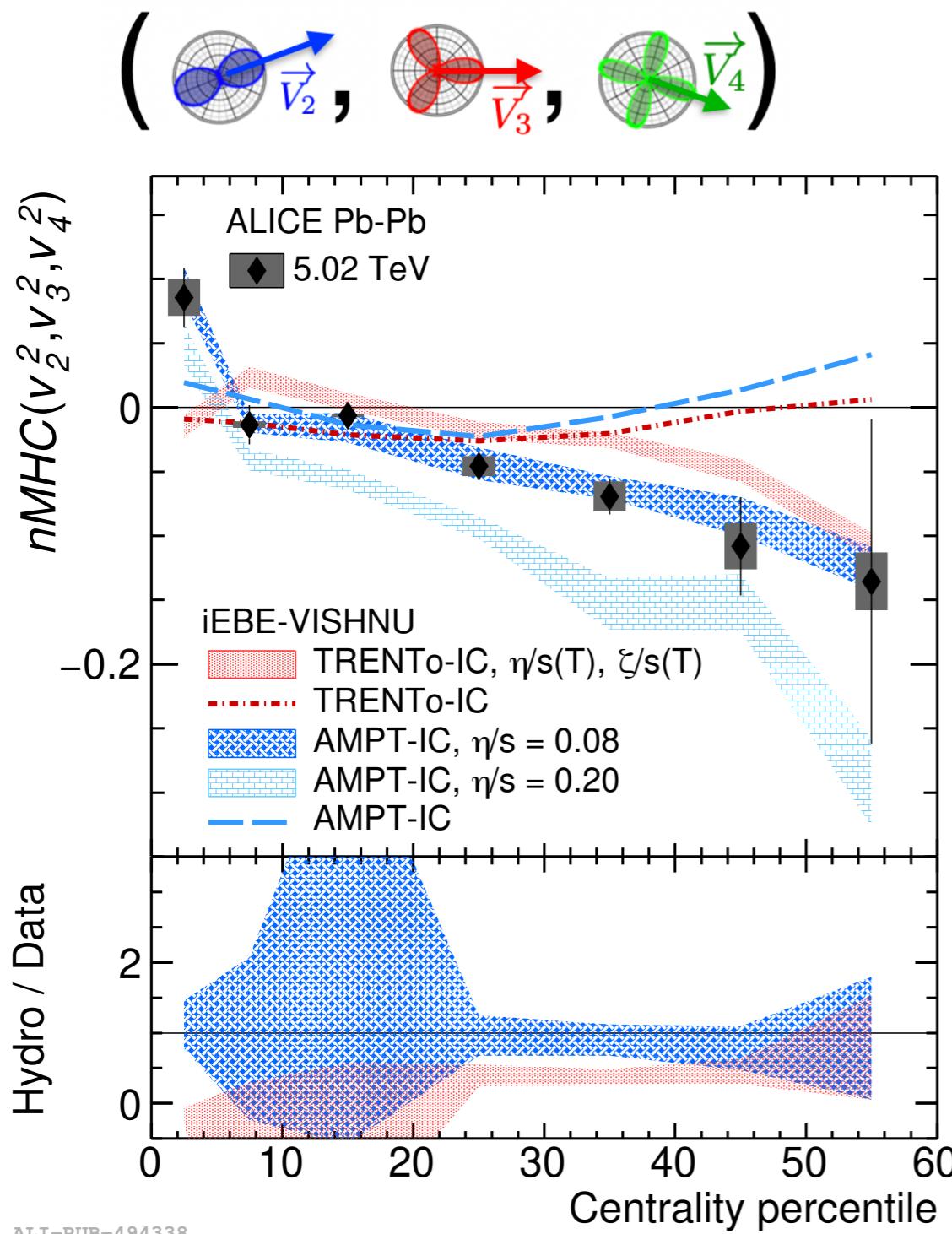
- Deviation from unity  $\rightarrow p_T$ -dependent flow angle fluctuations
- $> 5\sigma$  significance in most centralities

- Deviation from  $\langle v_n^4 \rangle / \langle v_n^2 \rangle^2 \rightarrow p_T$ -dependent flow magnitude fluctuations
- $\sim 5\sigma$  significance in most central collisions

**Discovery of both flow angle and flow magnitude fluctuations in most central collisions!**



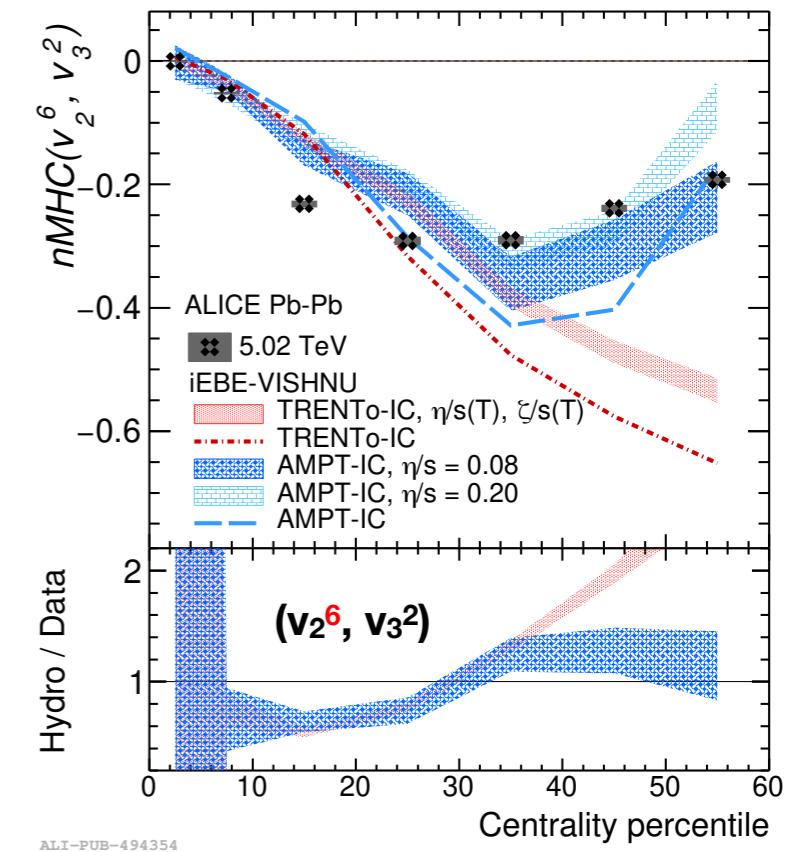
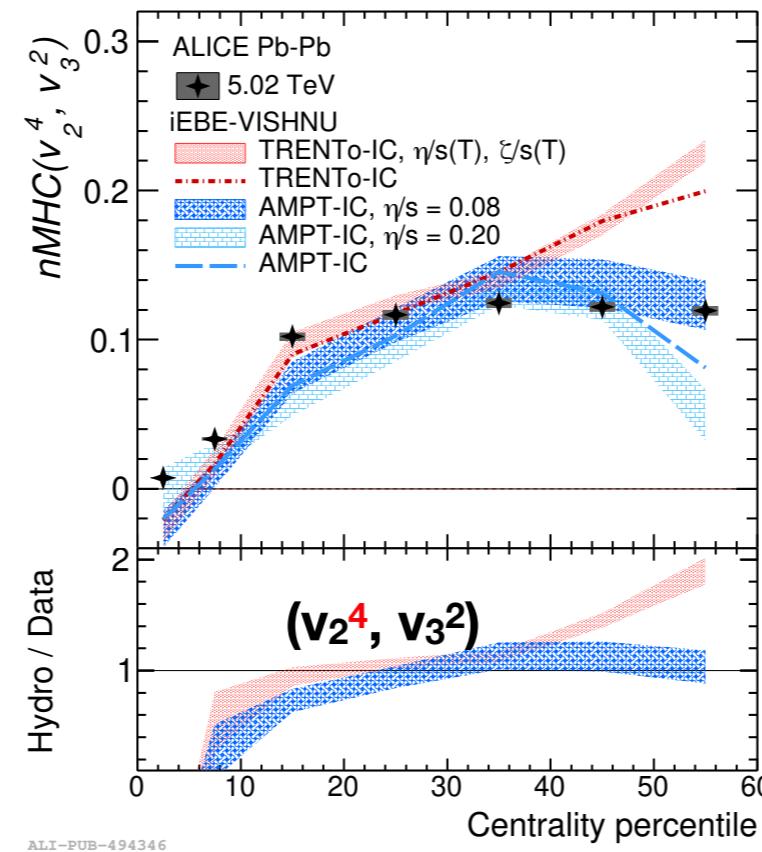
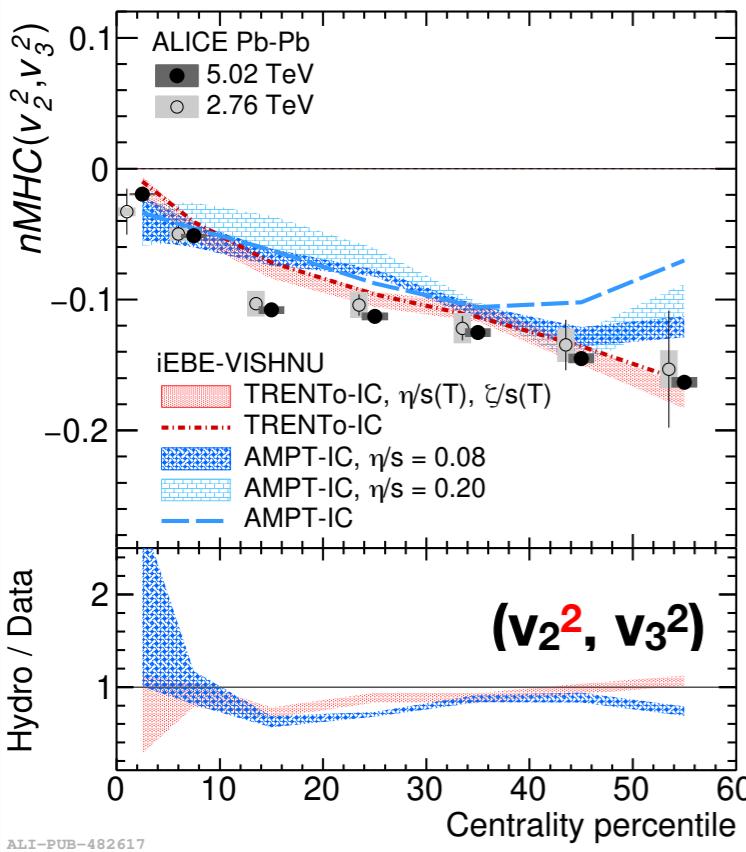
# Data vs hydro: $v_m^2$ , $v_n^2$ and $v_p^2$ correlations



- ❖ TRENTo+iEBE-VISHNU and AMPT+iEBE-VISHNU work fairly well in describing  $nMHC(v_m^2, v_n^2)$
- ❖  $nMHC(v_2^2, v_3^2, v_4^2) \neq nMHC(\varepsilon_2^2, \varepsilon_3^2, \varepsilon_4^2)$ 
  - ▶ Non-linear response
  - ▶  $nMHC(v_2^2, v_3^2, v_4^2)$  is sensitive to  $\eta/s$  of QGP
- ❖ AMPT+iEBE-VISHNU calculations quantitatively agree with the ALICE data
- ❖ TRENTo+iEBE-VISHNU calculations underestimate the data by 50%
- ❖ A new challenge for the current understanding of initial conditions and QGP properties from Bayesian analysis (TRENTo+iEBE-VISHNU) with the presented  $nMHC(v_m^2, v_n^2, v_p^2)$  data



# $v_2^k$ and $v_3^2$ correlations



ALICE, PLB818 (2021)  
Hydro, arXiv:2104.10422

- ❖ Good agreement between initial eccentricity estimations and final  $nMHC(v_2^k, v_3^2)$  in central collisions
- ❖ Deviations are getting larger in more peripheral collisions and/or for higher order  $v_2$ 
  - ▶ Non-linear response of  $v_2$  in non-central Pb-Pb collisions
- ❖ AMPT+iEBE-VISHNU calculations work better in peripheral collisions

