

Logarithmic corrections for Jet Production at the LHC

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Outline

Introduction

High-energy logarithms

The High Energy Jets framework

H+jets [1]

Top quark mass effects to all-orders

Impact of VBF cuts to GF

[1] arXiv:1812.08072 Jeppe Andersen, James Cockburn, Marian Heil, Andreas Maier and Jennifer Smillie

W+jets [2]

New NLL components

NLO Matching

[2] arXiv:2012.10310 Jeppe Andersen, James Black, Helen Brooks, EB, Andreas Maier and Jennifer Smillie

Some ongoing work

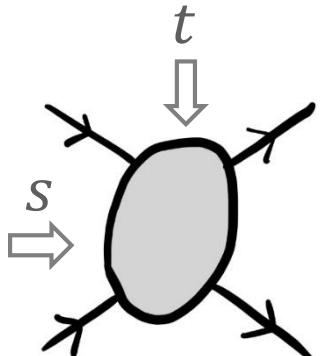
WW+jets

HEJ+Pythia

High-energy logarithms

At each order in perturbative QCD, large logarithms arise when the centre of mass energy is much greater than the transverse momenta of the produced partons.

For $2 \rightarrow 2$ scattering we can write the cross section as



$$L \equiv \log\left(\frac{S}{-t}\right) \gg 1$$

$$\alpha_s \ll 1$$

$$\alpha_s L \sim 1$$

$$\begin{aligned} \sigma^{(0)}/\sigma^{(0)} &= \boxed{\text{LL}} \\ &\quad 1 \\ \sigma^{(1)}/\sigma^{(0)} &= \alpha_s L c_0^{(1)} + \boxed{\text{NLL}} \\ &\quad \alpha_s c_1^{(1)} \\ \sigma^{(2)}/\sigma^{(0)} &= \alpha_s^2 L^2 c_0^{(2)} + \boxed{\text{NNLL}} \\ &\quad \alpha_s^2 c_1^{(2)} L \\ \sigma^{(3)}/\sigma^{(0)} &= \alpha_s^3 L^3 c_0^{(3)} + \dots \\ &\quad \vdots \end{aligned}$$

Let's begin with a description of QCD amplitudes which include the leading logarithmic terms at all orders in α_s .

QCD amplitudes at LL

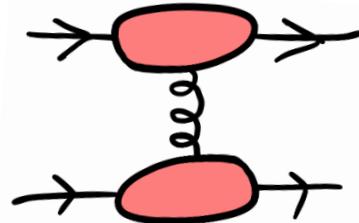
Lipatov, Fadin, Kuraev et. al. [3] found that to LL accuracy QCD amplitudes take the following factorised form:

$$\begin{aligned}
 M_{qQ \rightarrow 2+n}^{LL} &= i g_s \Gamma_{q \rightarrow q \mu}^{C_1} \\
 &\times \prod_{i=1}^{n-1} \left(\left(\frac{-i}{t_i} \right) e^{\alpha(q_i) \log\left(\frac{s_{i,i+1}}{-t_i}\right)} \right) \\
 &\times \prod_{i=1}^n g_s f^{C_i C_{i+1} G_i} V_g^\gamma(q_i, q_{i+1}) \varepsilon_\gamma(p_i) \\
 &\times i g_s \Gamma_{Q \rightarrow Q}^{C_{i-1} \mu}
 \end{aligned}$$

\$\rightarrow\$ [red box] \$\rightarrow\$ $\Gamma_{q \rightarrow q \mu}^A = 2 T^A p_{a \mu} \delta^{\lambda_a \lambda_1}$
\$\rightarrow\$ [purple box] \$\rightarrow\$ $\alpha(q_i) = -g_s^2 C_A \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{1}{\epsilon} \left(\frac{q_{i\perp}^2}{\mu^2} \right)^\epsilon$
\$\rightarrow\$ [purple box] \$\rightarrow\$ $V_g^\gamma(q_1, q_2) = -(q_1 + q_2)_\perp^\gamma + \frac{p_a^\gamma}{2} \left(\frac{q_1^2}{s_{a2}} + \frac{s_{b2}}{s_{ab}} + \frac{s_{23}}{s_{a3}} \right) - \frac{p_b^\gamma}{2} \left(\frac{q_2^2}{s_{b2}} + \frac{s_{a2}}{s_{ab}} + \frac{s_{12}}{s_{1b}} \right)$

Return to LO $qQ \rightarrow qQ$

At LO the process is *already factorised*:



$$M_{qQ \rightarrow qQ} = [ig_s T^A \bar{u}^{\lambda_a}(p_a) \gamma^\mu u^{\lambda_1}(p_1)] \left(\frac{-i}{t} \right) [ig_s T^A \bar{u}^{\lambda_b}(p_b) \gamma_\mu u^{\lambda_2}(p_2)]$$

This motivates us to take the quark current as our impact factor

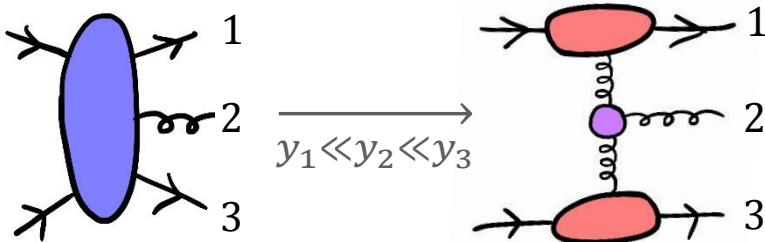
$$\rightarrow \text{red oval} \rightarrow j_{q \rightarrow q}^\alpha(p_a, p_b) = \bar{u}^{\lambda_a}(p_a) \gamma^\alpha u^{\lambda_1}(p_1)$$

rather than the more severe approximation $\rightarrow 2p_a^\mu \delta^{\lambda_a \lambda_1} \rightarrow$

Aside: The dominant helicity configurations of LO $qg \rightarrow qg$ can also be written exactly as a contraction of currents over a t-channel pole. See [4] arXiv:0910.5113 Andersen, Smillie.

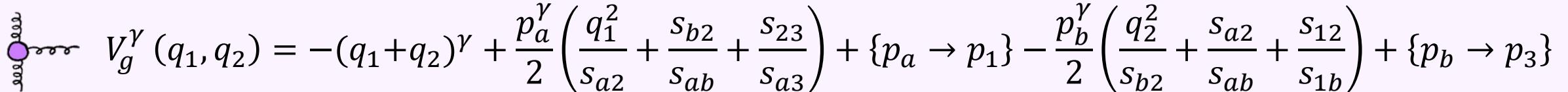
Real emissions

The process $qQ \rightarrow qgQ$ factorises in the MRK limit. However, we make minimal approximations to the LO amplitude in order to retain as much of the LO process as possible



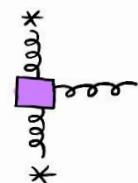
$$M_{qQ \rightarrow qgQ} \xrightarrow{y_1 \ll y_2 \ll y_3} [ig_s T^A j_{q \rightarrow q}^\mu] \left(\frac{-i}{t_1} \right) [f^{ABG} V_g^\gamma \varepsilon_\gamma^*] \left(\frac{-i}{t_2} \right) [ig_s T^B j_{Q \rightarrow Q}^\mu]$$

This lets us define the HEJ Lipatov vertex



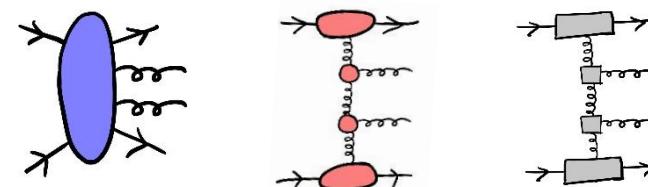
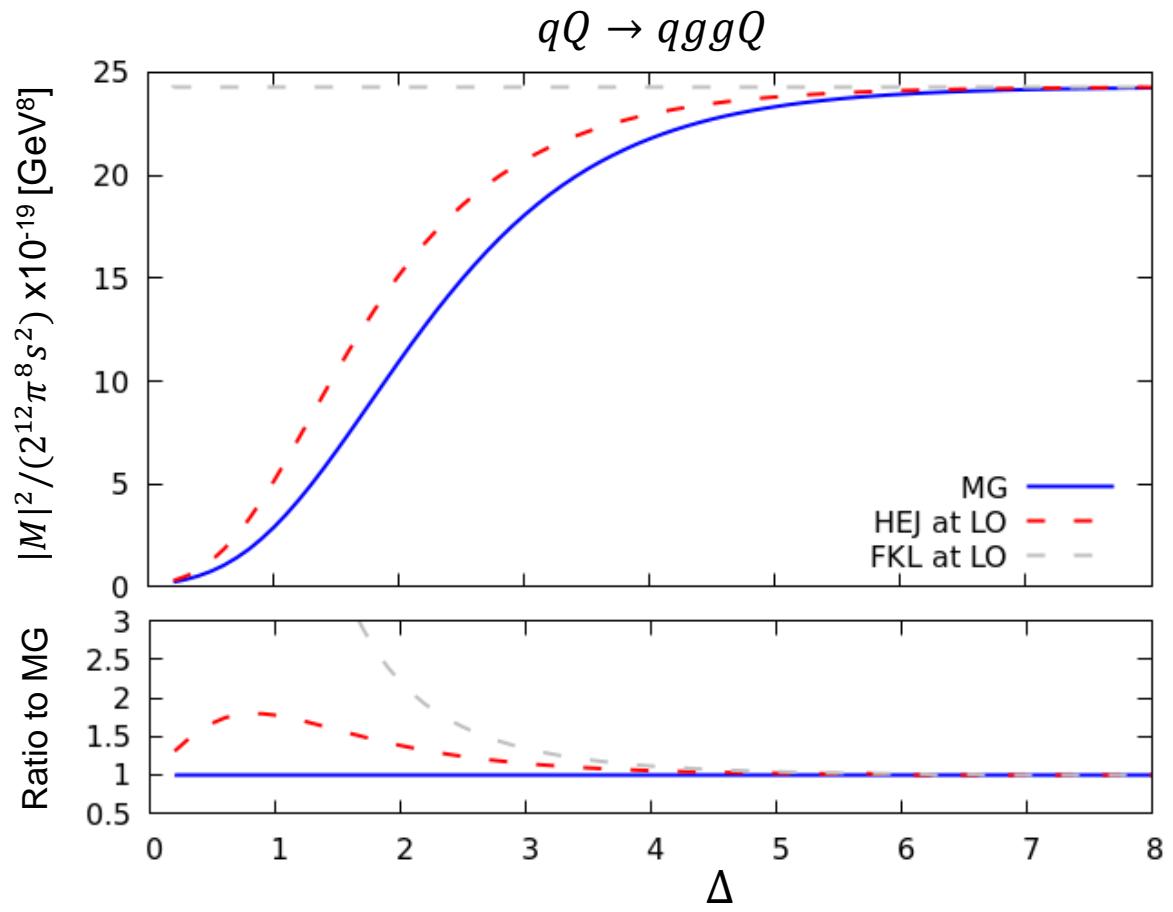
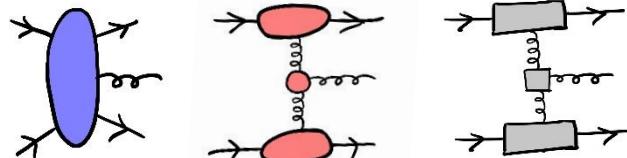
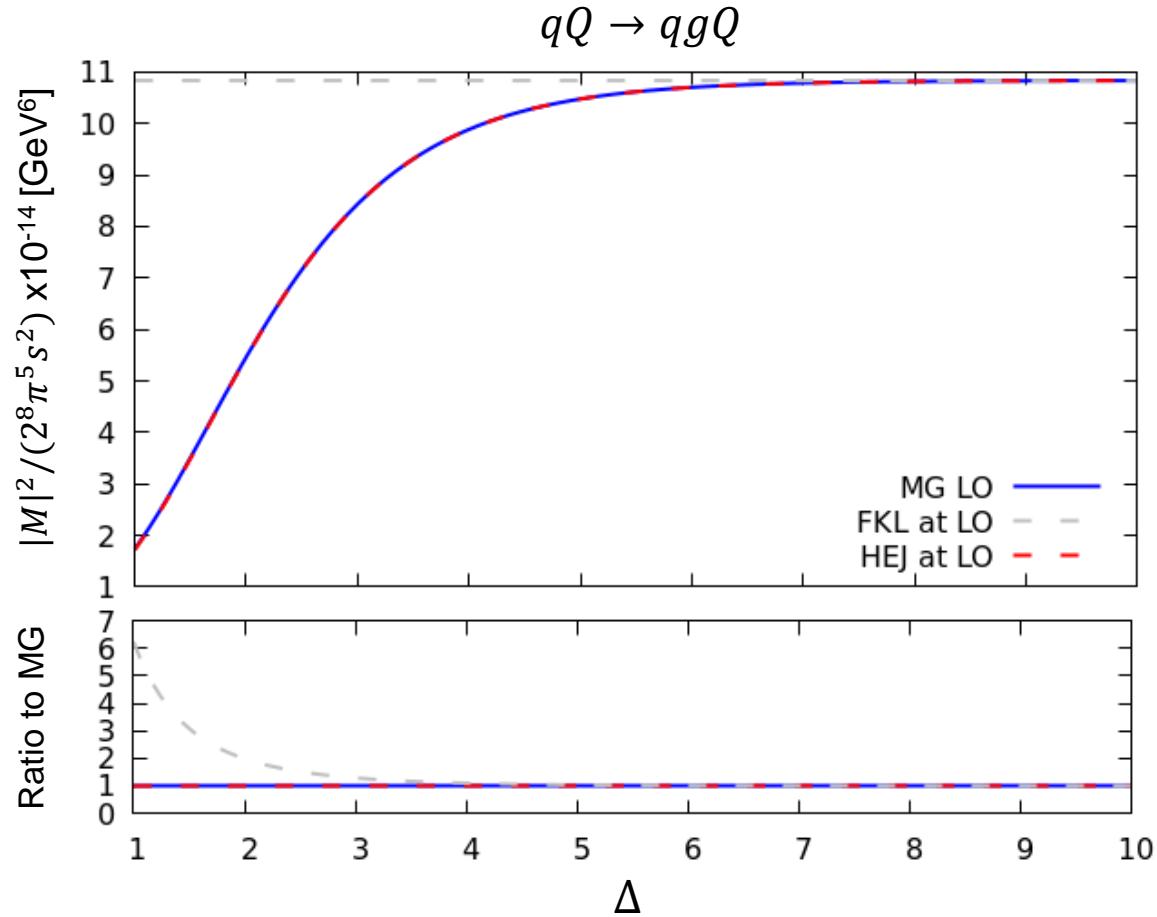
$$V_g^\gamma(q_1, q_2) = -(q_1 + q_2)^\gamma + \frac{p_a^\gamma}{2} \left(\frac{q_1^2}{s_{a2}} + \frac{s_{b2}}{s_{ab}} + \frac{s_{23}}{s_{a3}} \right) + \{p_a \rightarrow p_1\} - \frac{p_b^\gamma}{2} \left(\frac{q_2^2}{s_{b2}} + \frac{s_{a2}}{s_{ab}} + \frac{s_{12}}{s_{1b}} \right) + \{p_b \rightarrow p_3\}$$

which agrees with the FKL form upon further approximation

$$\rightarrow -(q_1 + q_2)_\perp^\gamma + p_a^\gamma \left(\frac{q_{1\perp}^2}{s_{a2}} + \frac{s_{b2}}{s_{ab}} + \frac{s_{23}}{s_{a3}} \right) - p_b^\gamma \left(\frac{q_{2\perp}^2}{s_{b2}} + \frac{s_{a2}}{s_{ab}} + \frac{s_{12}}{s_{1b}} \right)$$


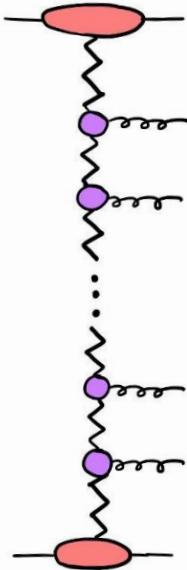
Comparison at LO

The HEJ amplitudes capture more of the LO physics relevant to LHC phase space than the strictly LL amplitudes:



LL HEJ amplitude for QCD

With these building blocks we can write the LL HEJ QCD amplitudes:



$$\begin{aligned} \left| M_{f_a f_b \rightarrow f_a (n-2) g f_b}^{HEJ} \right|^2 &= \frac{1}{4(N_C^2 - 1)} |j_{f_a \rightarrow f_a} \cdot j_{f_b \rightarrow f_b}|^2 (g_s^2 K_{f_a} \frac{1}{t_1}) (g_s^2 K_{f_b} \frac{1}{t_{n-1}}) \\ &\times \prod_{i=1}^{n-1} e^{\omega(t_i)(y_{j+1} - y_j)} \\ &\times \prod_{i=1}^n \left(\frac{-g_s^2 C_A}{t_i t_{i+1}} |V_g(q_i, q_{i+1})|^2 \right) \end{aligned}$$

Approximations are only performed at amplitude level.

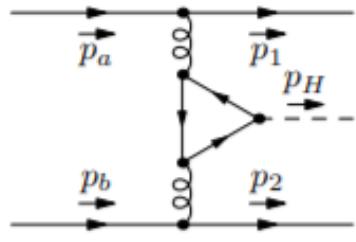
Phase space integration is performed via Monte Carlo sampling.

As of HEJ 2.0, LO matched by default: LO events can be used as input where they are available.
HEJ Fixed Order Generator can be used where these are unavailable or computationally expensive.

What about beyond pure QCD?

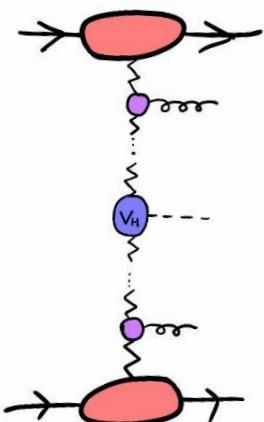
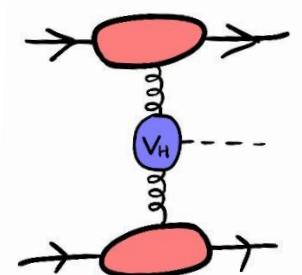
HEJ amplitude for $H+ \geq 2j$

For $qQ \rightarrow qHQ$ the process again factorises at LO:



$$M_{qQ \rightarrow qHQ} \propto j_{q \rightarrow q}^\mu V_{H \mu\nu}(q_1, q_2) j_{Q \rightarrow Q}^\nu$$

$$V_H^{\mu\nu}(q_1, q_2) = \frac{\alpha_s m^2}{\pi\nu} [g^{\mu\nu} T_1(q_1, q_2) - q_2^\mu q_1^\nu T_2(q_1, q_2)]$$



The structure of LL resummation is unchanged by the presence of this colour singlet

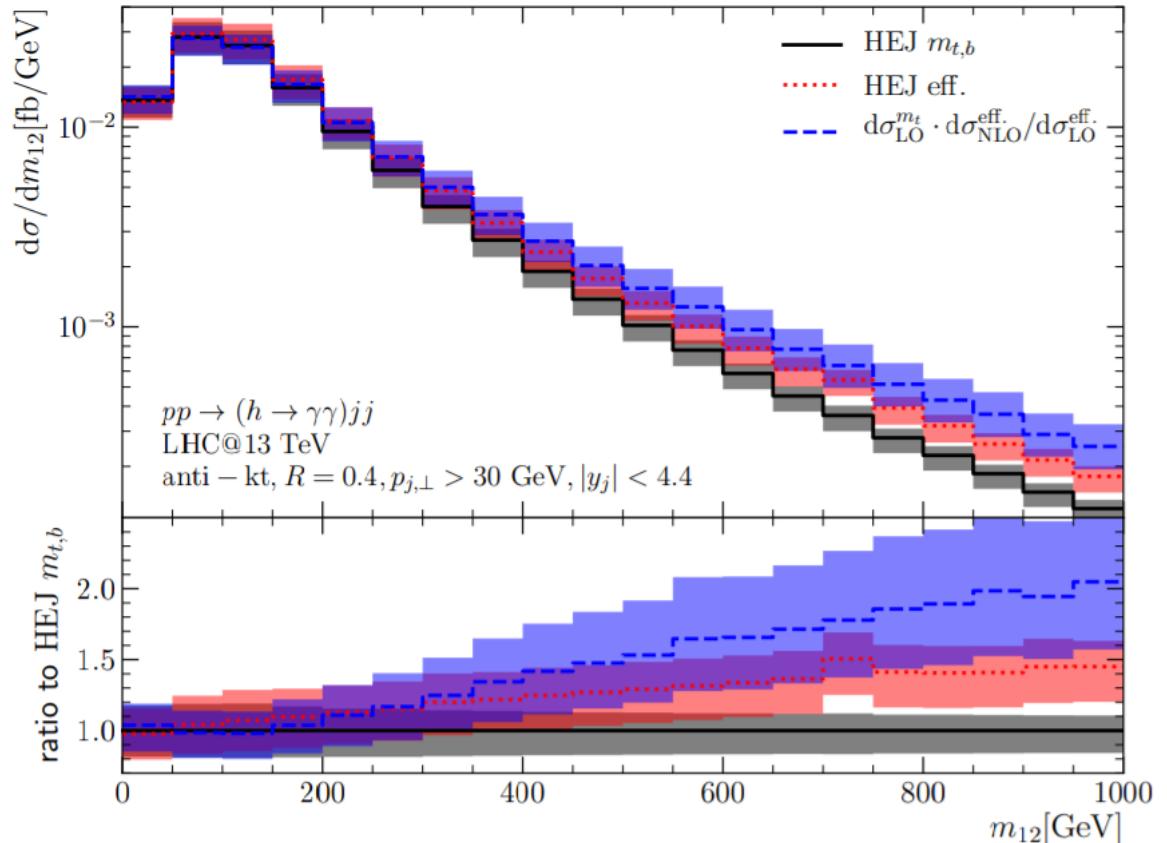
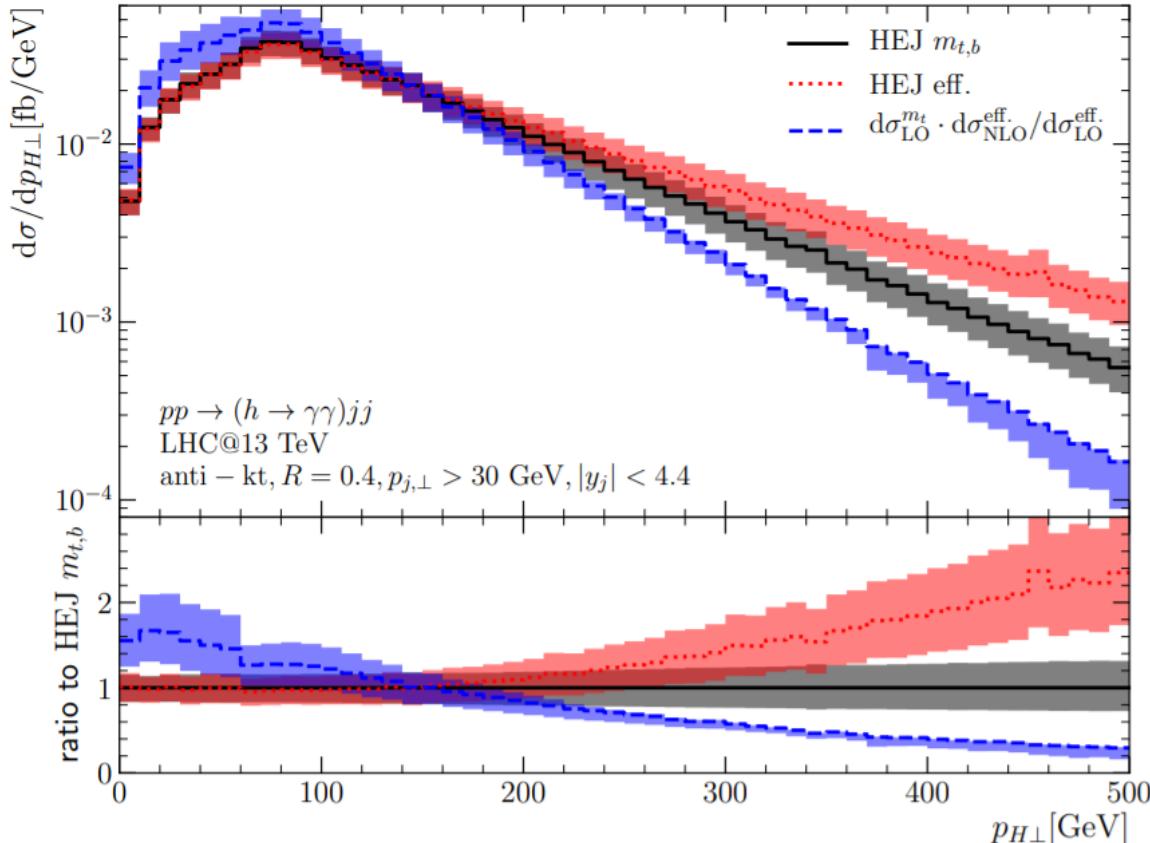
We can sum over top and bottom contributions at amplitude level to study interference effects

The structure of the amplitude is unchanged if the infinite mass limit is taken. The only further approximation is to this factorised expression:

$$V_H^{\mu\nu}(q_1, q_2) \xrightarrow[m \rightarrow \infty]{} \frac{\alpha_s}{3\pi\nu} [g^{\mu\nu} q_1 \cdot q_2 - q_2^\mu q_1^\nu]$$

Other partonic channels are described in detail in [1] arXiv:1812.08072.

Finite Quark-Mass Effects in H+2j



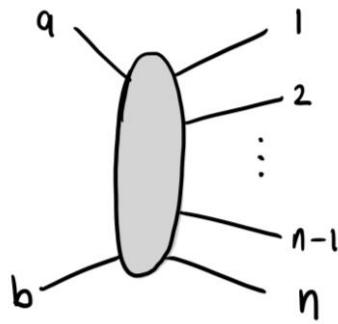
FO predictions suggest finite top mass effects have little impact on the total cross section. An all-orders treatment shows a $\sim 9\%$ effect, correlated with the harder p_\perp distribution of the Higgs.

An all-orders treatment shows there is significantly less GF contamination within VBF cuts than suggested by FO.

Regge Scaling

We can use ideas from Regge theory to anticipate the importance of a given rapidity ordering of an event.

This gives us a useful classification of events. This classification is not altered by production of colour singlets.



$$|M_{LO}|^2 \propto$$

	$s_{12}^2 s_{23}^2$
	$s_{12}^1 s_{23}^2$
	$s_{12}^1 s_{23}^1$

Multi Regge Kinematics [4]:

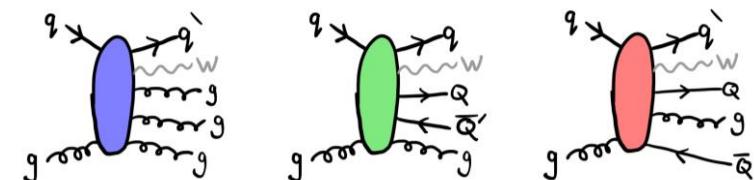
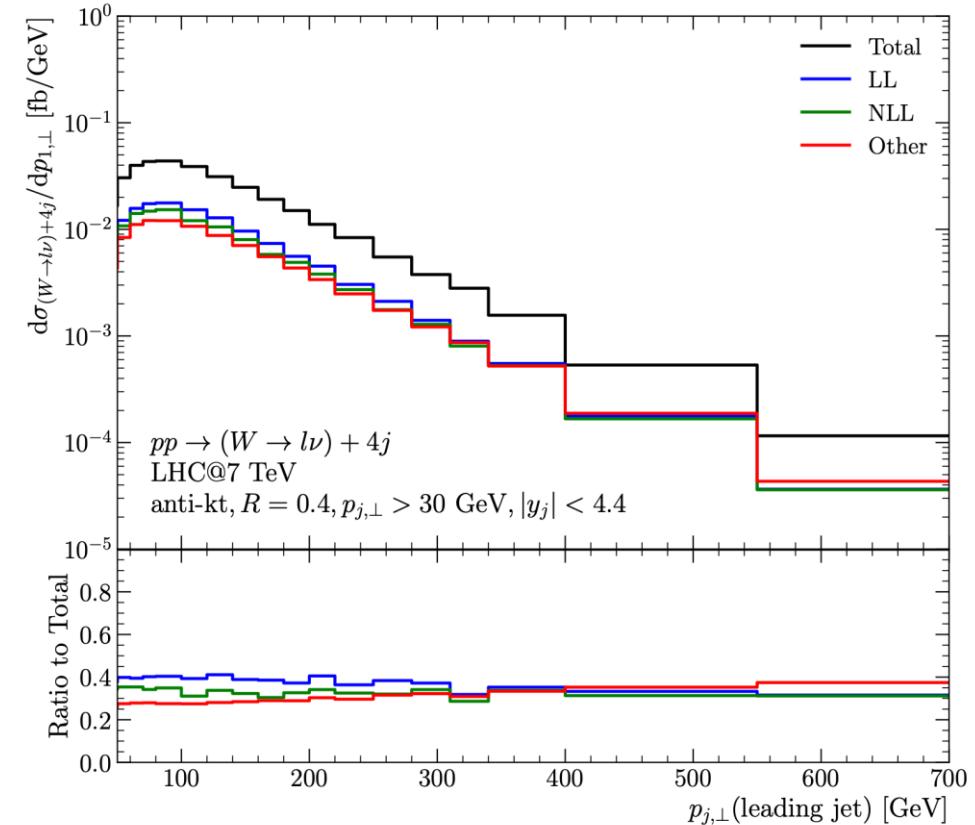
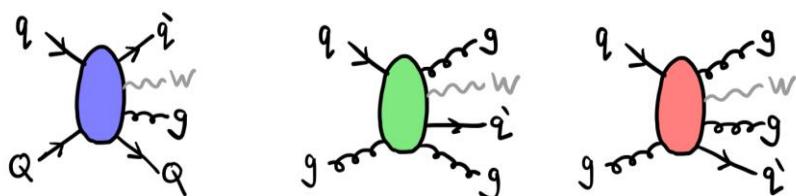
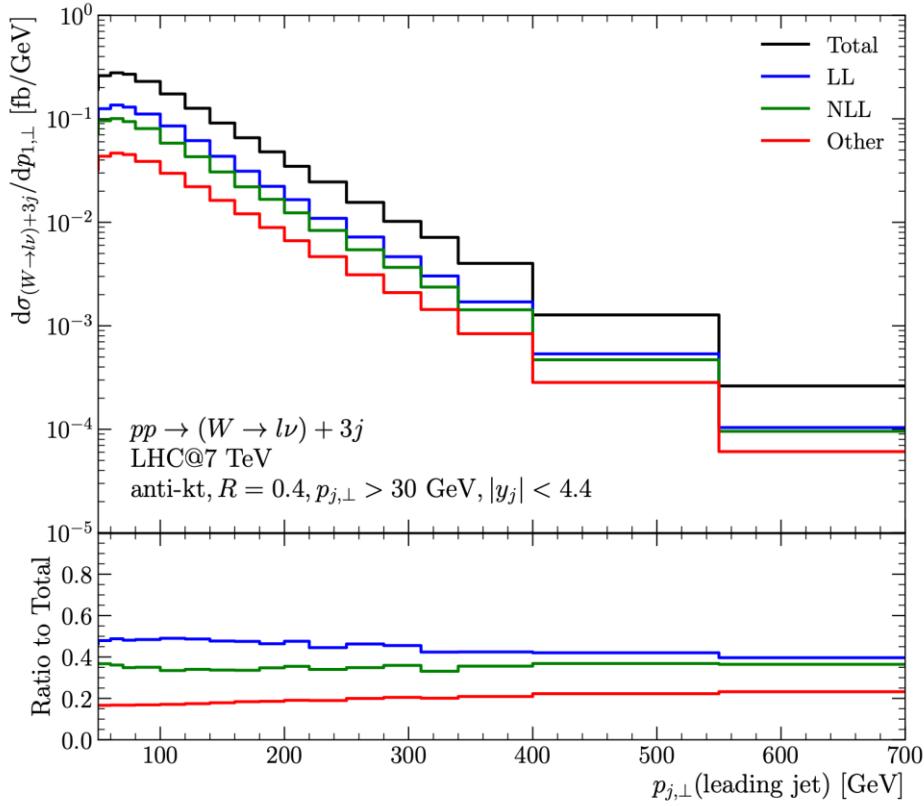
$y_1 \ll y_2 \ll \dots \ll y_{n-1} \ll y_n$,
 $|p_{i,\perp}|$ finite.

$$M^{MRK} \propto s_{12}^{J_1(t_1)} s_{23}^{J_2(t_2)} \dots s_{n-1,n}^{J_{n-1}(t_{n-1})} \gamma$$

LL config.	NLL config.	NNLL config.

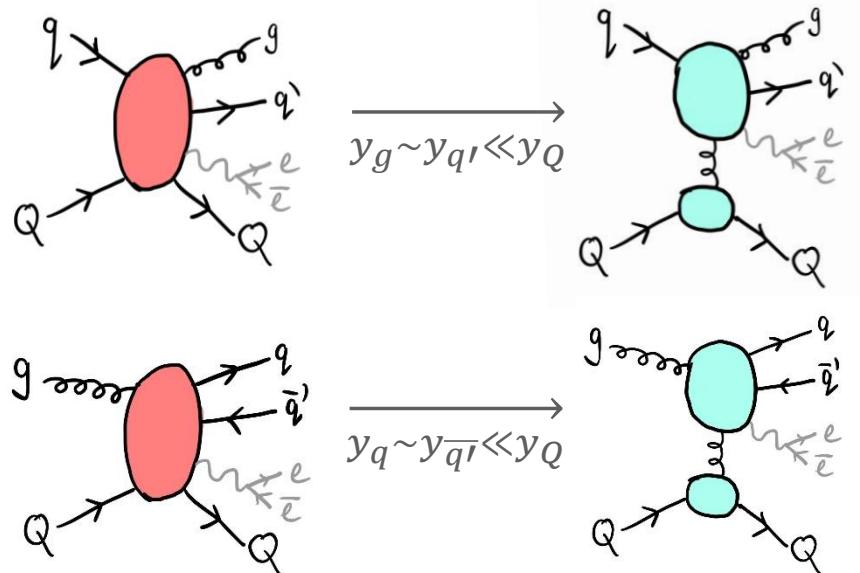
Impact of NLL configurations at LO

A LO analysis demonstrates the importance of also including LL resummation for NLL configurations:



New factorised components for W+3j

In order to apply LL resummation to all NLL configurations for W+2j, we need several new factorised expressions. For 3j events we can extract the required pieces from LO amplitudes:



$$j_{q \rightarrow W g q'}^\mu(p_q; p_l, p_{\bar{l}}, p_g, p_{q'})$$

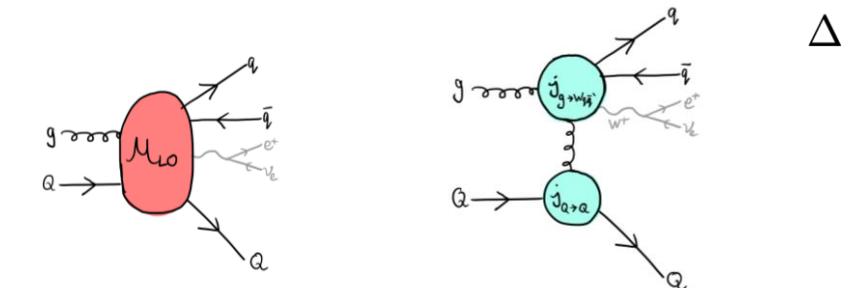
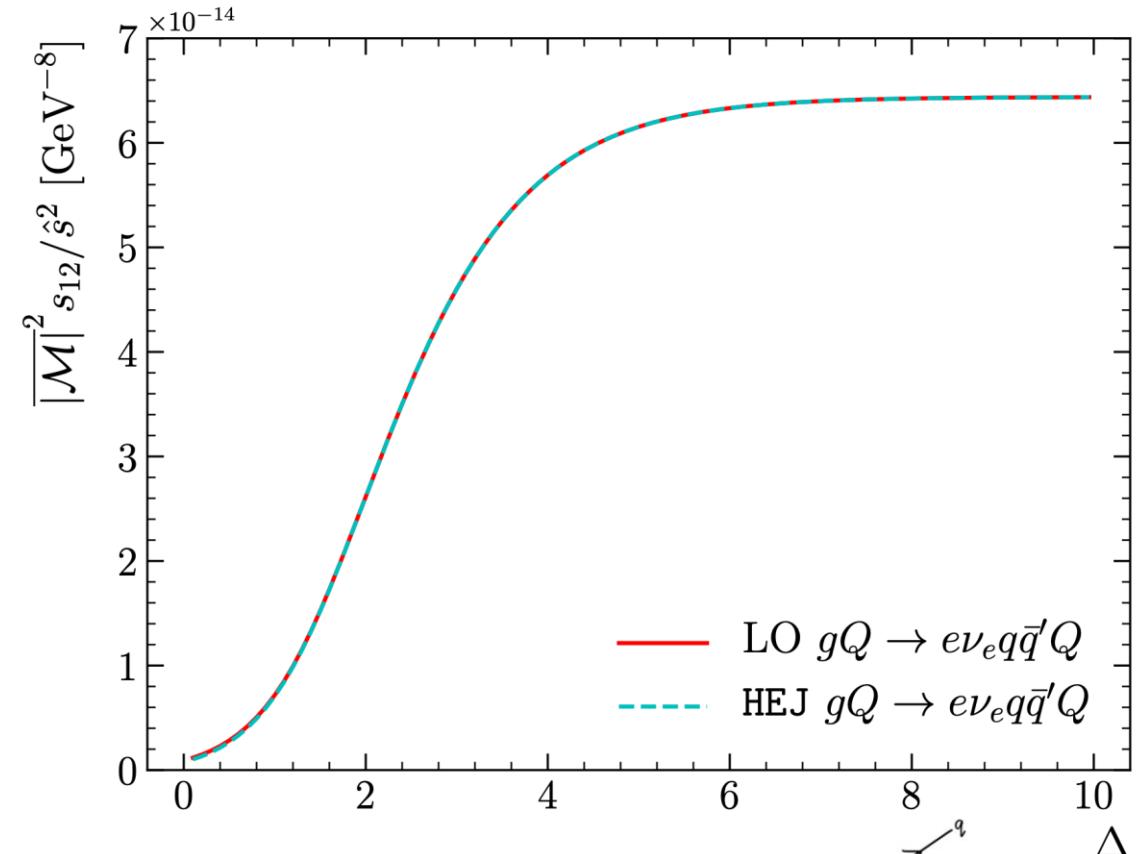
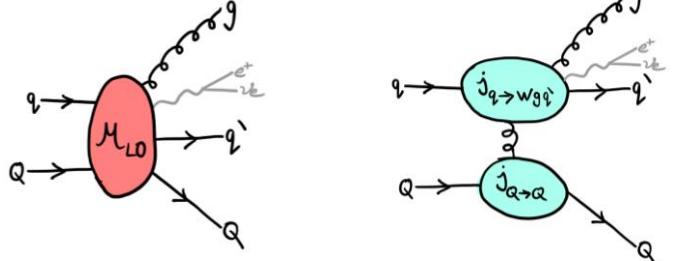
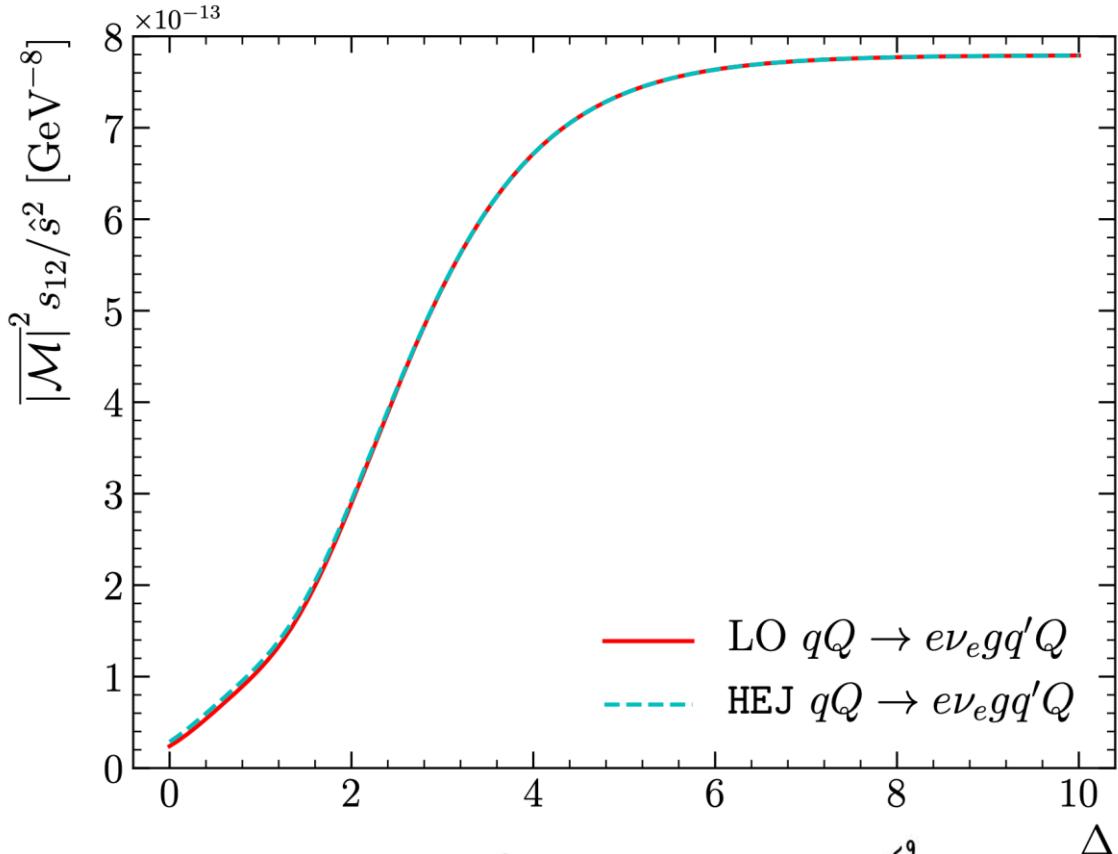
$$j_{g \rightarrow W q \bar{q}'}^\mu(p_g; p_l, p_{\bar{l}}, p_q, p_{\bar{q}'})$$

The approximations required to factorise a quark current from these LO amplitudes are so mild that the expressions derived still possess crossing symmetry:

$$j_{q \rightarrow W g q'}^\mu(p_q; p_l, p_{\bar{l}}, p_g, p_{q'}) = j_{g \rightarrow W q \bar{q}'}^\mu(p_g; p_l, p_{\bar{l}}, p_q, p_{\bar{q}'})$$

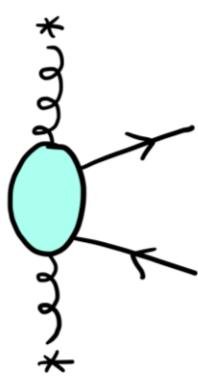
New factorised components for W+3j

These expressions obey the expected scaling, and provide a good approximation to the LO amplitude:

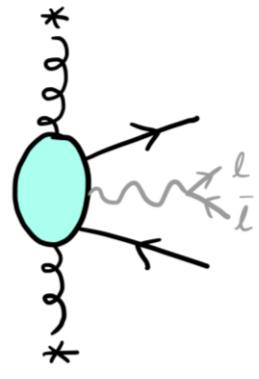


New factorised components for W+4j

Finally, we need effective vertices for the emission of a central $Q\bar{Q}$ pair in order to describe all NLL configurations. These are required for $\geq 4j$ events.

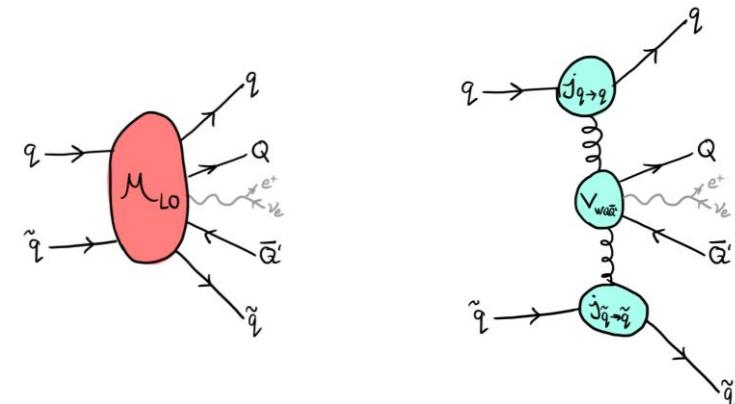
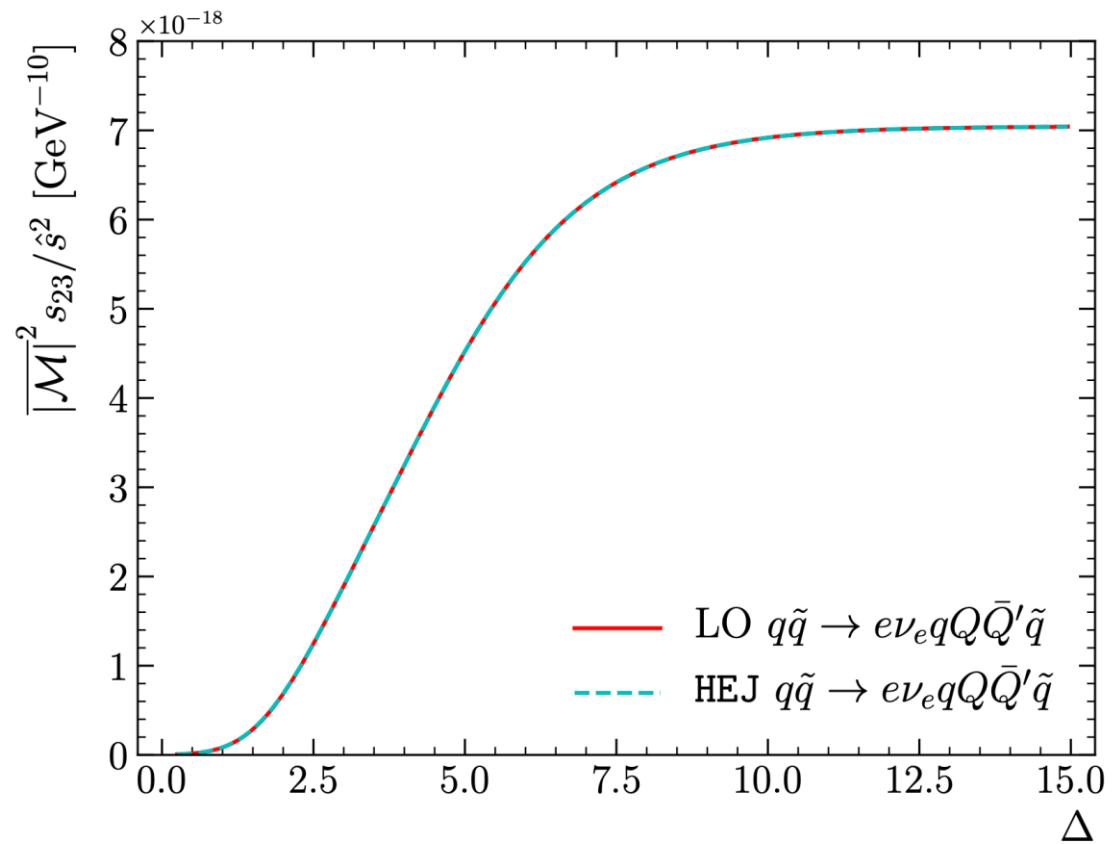


$$V_{Q\bar{Q}}^{\mu\nu}$$



$$V_{WQ\bar{Q'}}^{\mu\nu}$$

See appendix of [2] arXiv:2012.10310 for the derivation of these factorised expressions.



NLO matching procedure

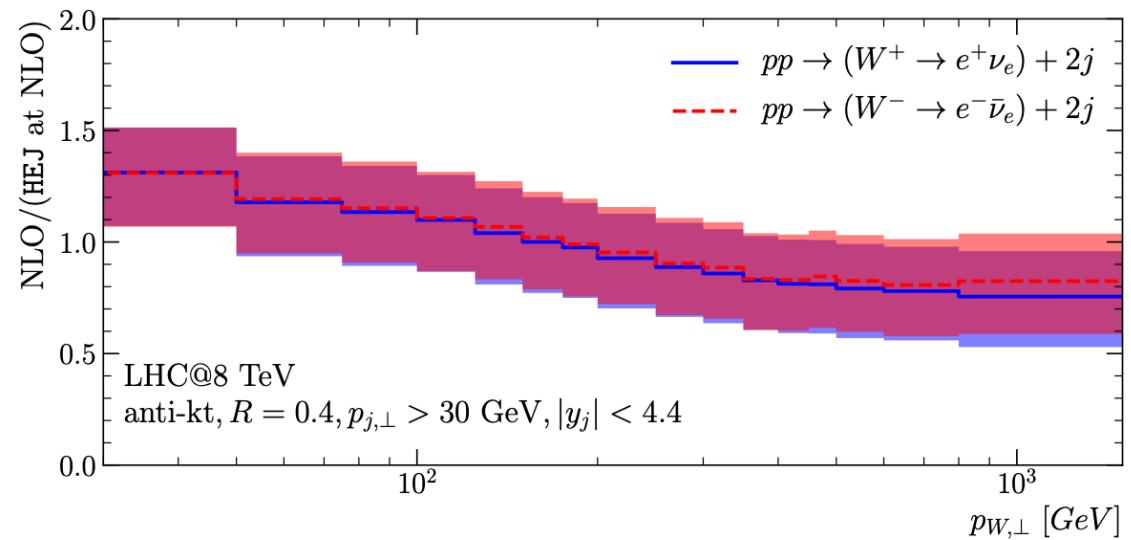
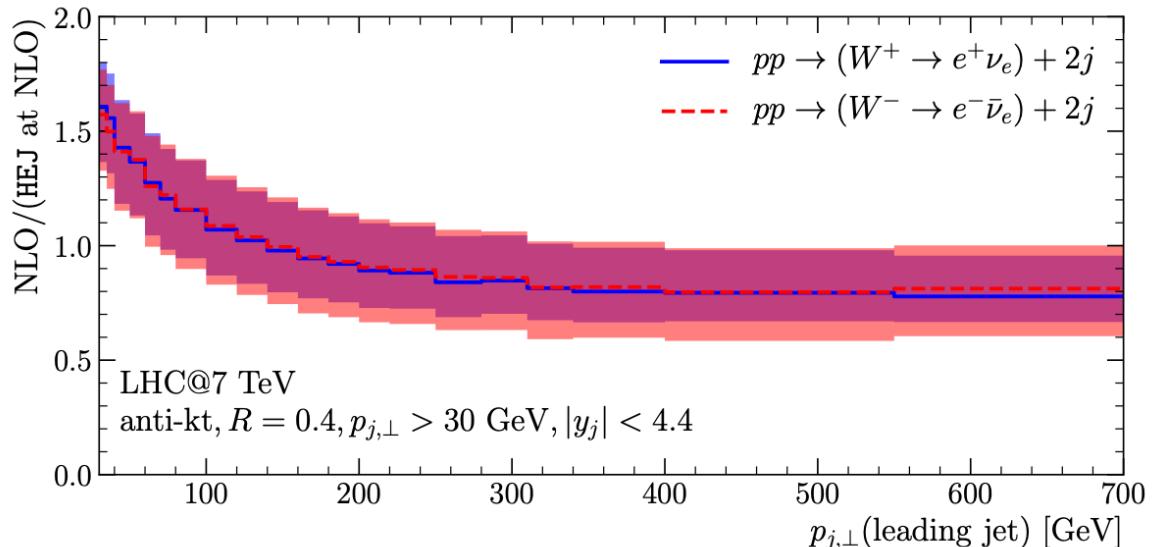
HEJ provides an all-orders description which may be truncated to any desired order in α_s

In particular we can obtain HEJ@NLO by restricting to a single real emission, and truncating the all-orders virtual corrections:

$$\frac{1}{t} e^{\tilde{\alpha}(q)\Delta y} \rightarrow \frac{1}{t} (1 + \tilde{\alpha}(q)\Delta y)$$

We can match HEJ bin-by-bin to W+2j NLO results to obtain NLO accuracy while maintaining the logarithmic accuracy of HEJ

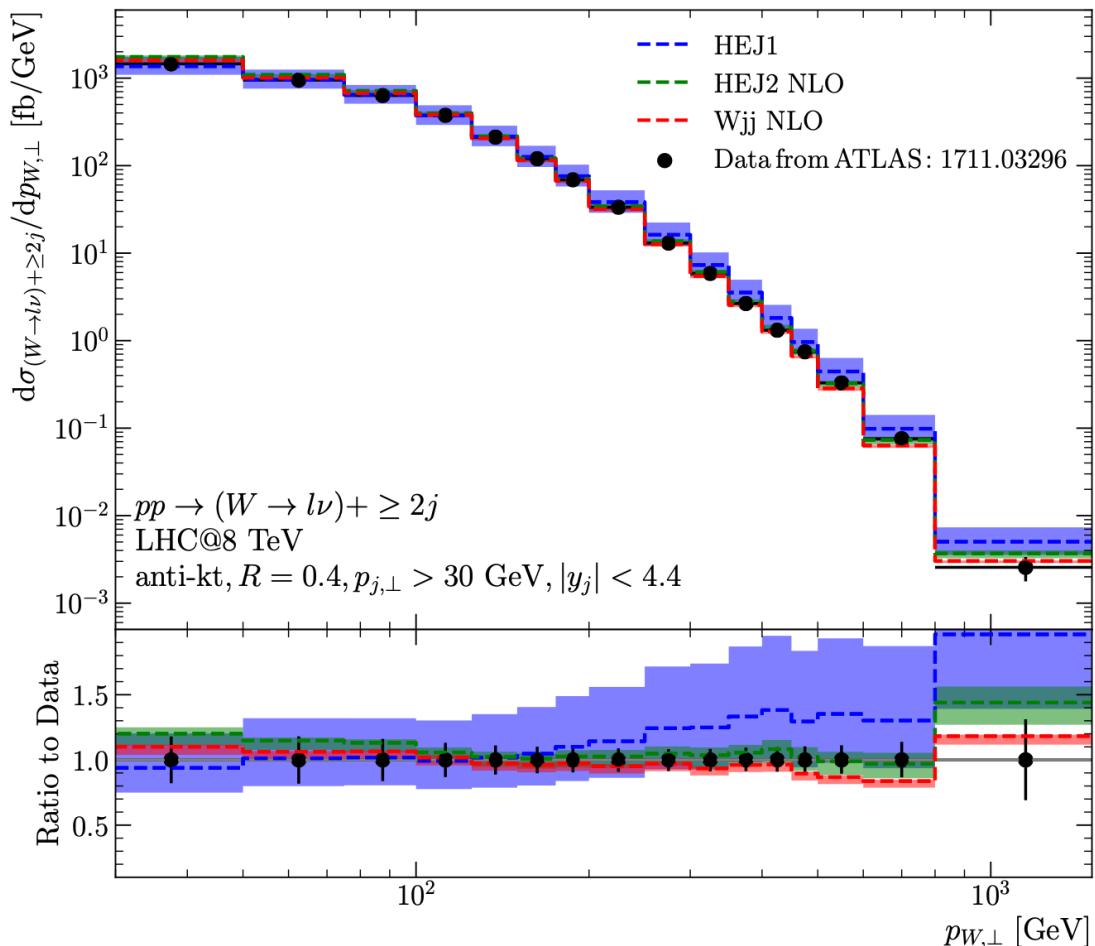
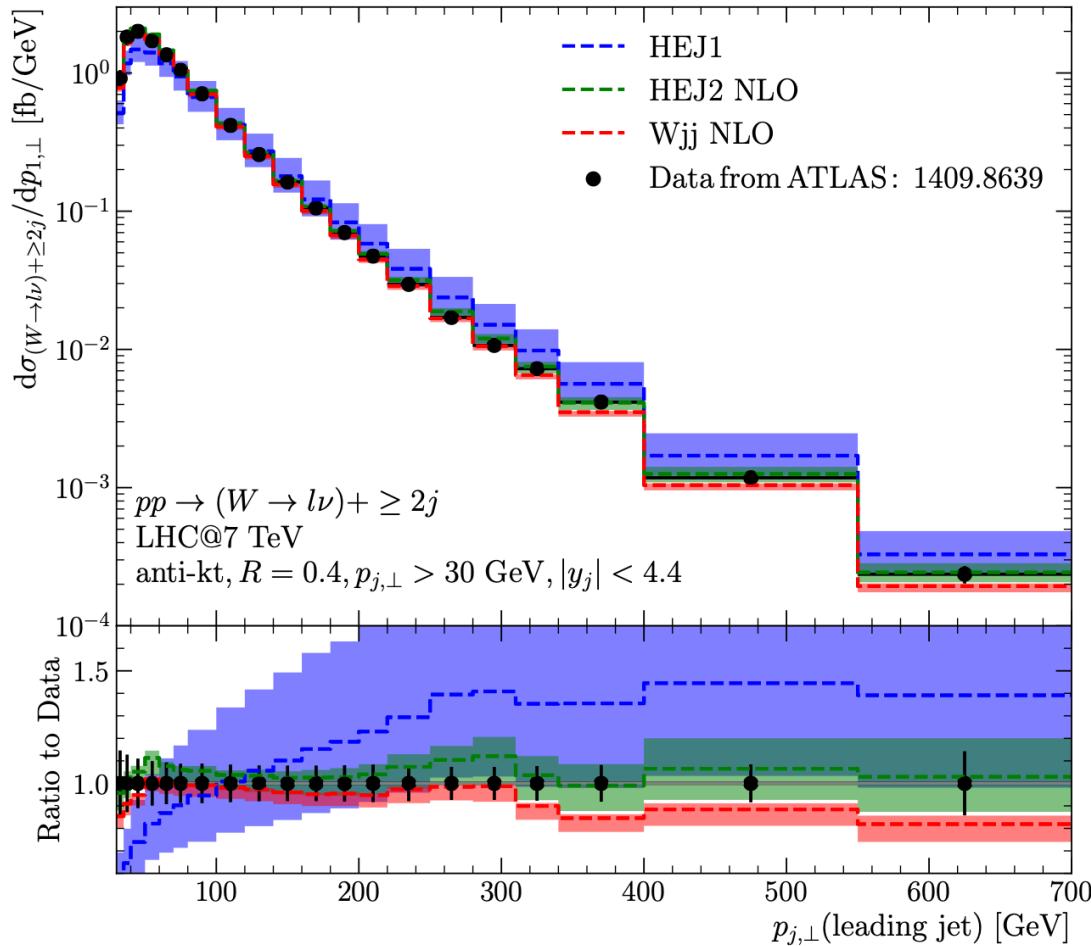
$$R = \frac{d\sigma_{NLO}}{d\sigma_{HEJ@NLO}}$$



Comparison to W+2j data

[5] arXiv:1409.8639 ATLAS W+jets study at 7TeV
 [6] arXiv:1711.03296 ATLAS W+jets study at 8TeV

LL HEJ ([HEJ1](#)) does not give a good description of observables uncorrelated with the high energy limit, as we expect: This breaks the MRK condition upon which they were based.



HEJ with the inclusion of the new NLL configurations, and bin-by-bin matching to NLO ([HEJ2](#)), gives a good description of data.

Ongoing work

- Full NLL accuracy for pure jets which involves one-loop corrections and a new treatment of colour.
- Same-sign WW+ $\geq 2j$ has been included in the HEJ framework and a study is to be published very soon.
- Work on merging HEJ+Pythia is ongoing, which will include the towers of leading logarithms from both high-energy and soft-collinear regions.
- The HEJ treatment of Higgs production is being extended to also provide predictions for H+1j.

Thanks for your attention!