Matching event generators and N3LO QCD calculations

based on arXiv:2106.03206

ISMD 2021, July 12, 2021

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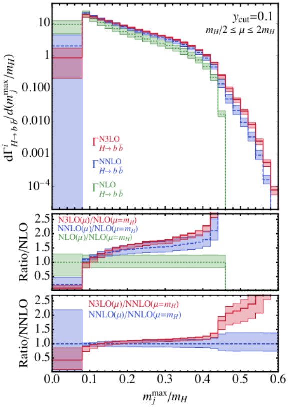
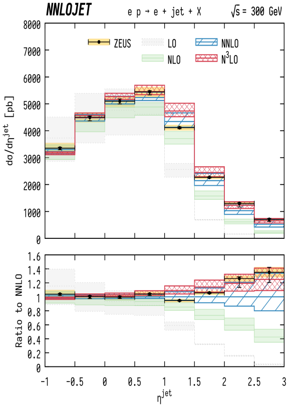
Why precision QCD?

more data ~~,better theory~~ → inconclusive analyses

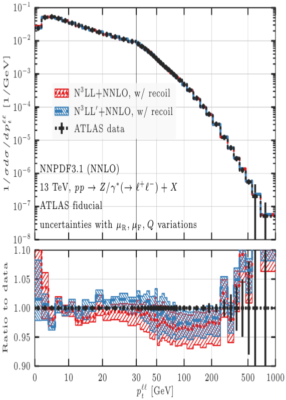
more data, better theory → conclusive i.e. better analyses



arXiv:1904.08960



arXiv:1803.09973



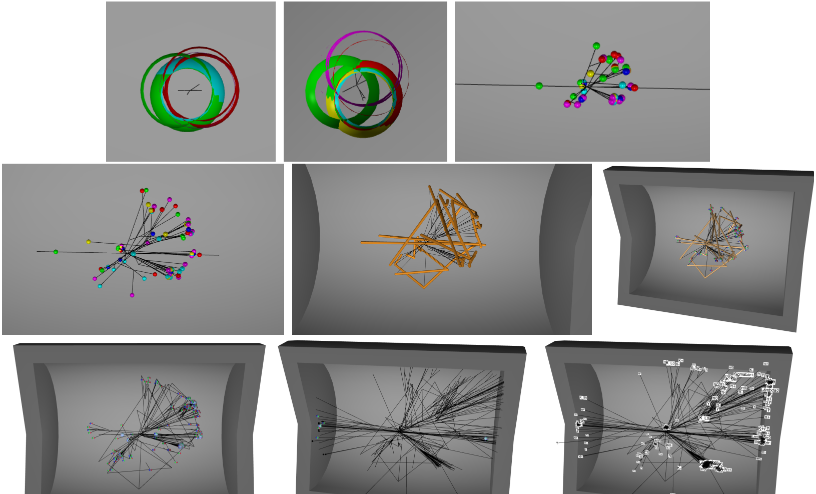
arXiv:2104.07509

We don't want to be left with inconclusive measurements!

Scattering events

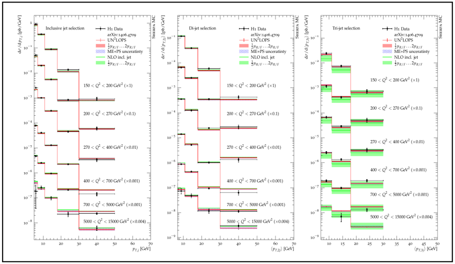
"Events" allow to disentangle calculation and analysis ...but are naively not IR-safe

For "safe" events from higher-order calculations, matching (to shower) required. This even offers physical final states:



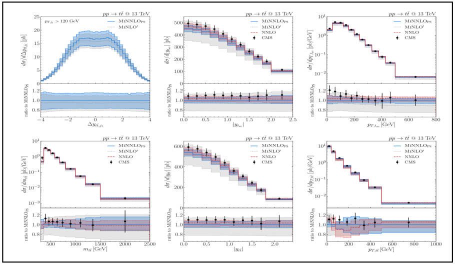
Matching record so far: NNLO+PS

NNLO+PS achieved for $pp \to$ singlet(s): Precision for fiducial "standard candles". Impressive exceptions beyond singlet production:



DIS NNLO+PS (`arXiv:1809.04192`):

Has "hard" light jet in final state, and complex relation between "natural scale" and available phase space. Available in Sherpa.



$t\bar{t}$ NNLO+PS (`arXiv:2012.14267`):

First pp collider process with colored final state @ NNLO+PS. Employs recent MINNLO$_{PS}$ scheme of Powheg-Box

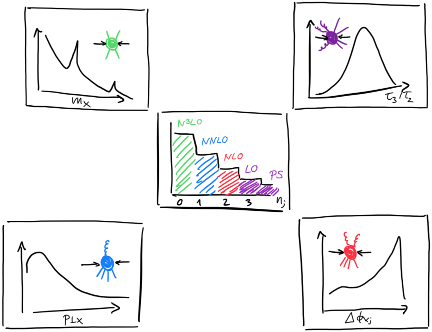
The goals of N3LO+PS matching

Why are V+j@NNLO+PS and H+j@NNLO+PS (etc) not available?

Strong $p_{\perp j}$ cut dependence problematic? Calculation w/o cut $\subset$ N3LO

N3LO+PS matching!

- 3rd-order precision for inclusive observables
- 2nd-order precision for one-jet observables, resummation when the jet becomes unresolved
- 1st-order precision for two-jet observables, resumm. when jet turns unresolved individually
- 0th-order precision for three-jet observables, resumm. when jet turns unresolved individually
- PS resummation of any observable sensitive to unresolved partons should not be impaired



N3LO+PS: Basic idea

arXiv:2106.03206 introduced a viable N3LO+PS matching scheme:

The ThirdOrderMatchedTransitionEvents = TOMTE method

Basic idea:

$\mathrm{N}^3\mathrm{LO}$ exclusive zero-jet x-section $\oplus$ NNLO+PS matched 1-jet x-section

$$\overset{massage}{=}$$

$\mathrm{N}^3\mathrm{LO}$+PS matching

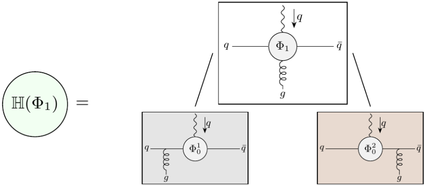
Run down:

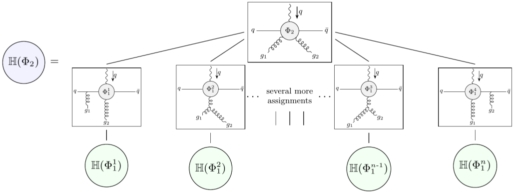
- Regularize 1-jet x-section with Sudakovs, so that the hardest jet may turn unresolved
- Remove unwanted NNLO terms
- Unitarize and complement (i.e. subtract projected one-jet bin from zero-jet bin)
- Include $\mathrm{N}^3\mathrm{LO}$ jet-vetoed zero-jet cross section

A word on "shower accuracy"

Preserving the "shower accuracy":
If the fixed-order calculations were to employ the shower (=soft/collinear) approximation, then the result of the matched calculation has to be indistinguishable from the PS prediction.

$\rightarrow$ Application of Sudakovs needs to take into account that PS all-order factors are mixtures of different "production histories".



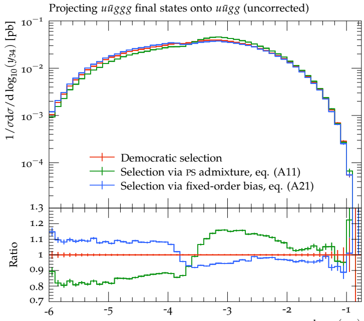


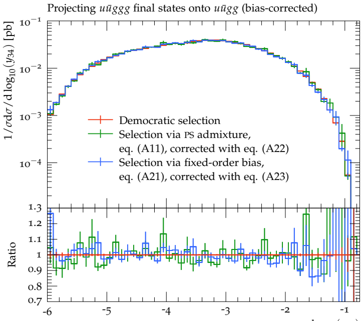
PS splits inclusive calculations into exclusive components. Inclusive rates are preserved by demanding [no-emission factor]$\mathcal{O}_n$ $=(1-\int$[emission rate]$)\mathcal{O}_n$

$\rightarrow$ Exclusive n-jet rate also depends on an admixture of histories.

A problem with unitarization and "shower bias"

- Matching improves emission rate $d\sigma_{n+1}$ beyond shower approximation
- Inclusive n-jet rate broken, unless no-emission-rate for n-jet state $=1 - \int d\Phi_1 d\sigma_{n+1}$
- The phase-space dependent admixture of emission histories has to apply to subtraction $\int d\Phi_1 d\sigma_{n+1}$. Otherwise inclusive n-jet rate broken
- PS admixture deforms n-jet distributions due to phase-space dependence of PS history mixing weight $\rightarrow$ exclusive n-jet rate broken $\rightarrow$ Need to introduce <u>bias correction</u>





Final N3LO+PS formula

↪more details
↪details on 3 jets ↪details on 1 jets
↪details on 2 jets ↪details on 0 jets

Carefully matching terms (by reweighting/expanding), we find the <u>TOMTE formula</u>

$$
\begin{aligned}
&\mathcal{F}_n^{(\infty)[\mathrm{TOMTE}]}(\Phi_n, t_+, t_-) \\
&:= \mathrm{O}_n \Bigg\{ d\sigma_n^{(0+1+2+3)[\mathrm{EXC}]}(\Phi_n) \\
&\qquad + \oint_{t^-}^{t^+} d\sigma_{n+1}^{(0)}(\Phi_{n+1}) \left[\mathbb{1}_n^{n+1} - \boxed{\mathrm{w}_{n+1}(\Phi_{n+1}) \left(1 - \mathrm{w}_{n+1}(\Phi_{n+1})\Big|_{\mathcal{O}(\alpha_s^2)}\right)} \right] \\
&\qquad + \oint_{t^-}^{t^+} d\sigma_{n+1}^{(1)[\mathrm{EXC}]}(\Phi_{n+1}) \left[\mathbb{1}_n^{n+1} - \boxed{\mathrm{w}_{n+1}(\Phi_{n+1}) \left(1 - \mathrm{w}_{n+1}(\Phi_{n+1})\Big|_{\mathcal{O}(\alpha_s^1)}\right)} \right] + \oint_{t^-}^{t^+} d\sigma_{n+1}^{(2)[\mathrm{EXC}]}(\Phi_{n+1}) \left[\mathbb{1}_n^{n+1} - \boxed{\mathrm{w}_{n+1}(\Phi_{n+1})} \right] \\
&\qquad + \oiint_{t^-}^{t^+} d\sigma_{n+2}^{(0)}(\Phi_{n+2}) \left[\mathbb{1}_n^{n+2} - \boxed{\mathrm{w}_{n+1}(\Phi_{n+2}) \left(1 - \mathrm{w}_{n+1}(\Phi_{n+2})\Big|_{\mathcal{O}(\alpha_s^1)}\right) \mathbb{1}_{n+1}^{n+2}} \right] \\
&\qquad + \oiint_{t^-}^{t^+} d\sigma_{n+2}^{(1)[\mathrm{EXC}]}(\Phi_{n+2}) \left[\mathbb{1}_n^{n+2} - \boxed{\mathrm{w}_{n+1}(\Phi_{n+2}) \mathbb{1}_{n+1}^{n+2}} \right] + \oiiint_{t^-}^{t^+} d\sigma_{n+3}^{(0)}(\Phi_{n+3}) \left[\mathbb{1}_n^{n+3} - \boxed{\mathrm{w}_{n+1}(\Phi_{n+3}) \mathbb{1}_{n+1}^{n+3}} \right] \Bigg\} \\
&+ \mathrm{O}_{n+1} \Bigg\{ d\sigma_{n+1}^{(0)}(\Phi_{n+1}) \boxed{\mathrm{w}_{n+1}(\Phi_{n+1}) \left(1 - \mathrm{w}_{n+1}(\Phi_{n+1})\Big|_{\mathcal{O}(\alpha_s^2)}\right)} \\
&\qquad + d\sigma_{n+1}^{(1)[\mathrm{EXC}]}(\Phi_{n+1}) \boxed{\mathrm{w}_{n+1}(\Phi_{n+1}) \left[1 - \mathrm{w}_{n+1}(\Phi_{n+1})\Big|_{\mathcal{O}(\alpha_s^1)}\right]} + d\sigma_{n+1}^{(2)[\mathrm{EXC}]}(\Phi_{n+1}) \boxed{\mathrm{w}_{n+1}(\Phi_{n+1})} \\
&\qquad + \oint_{t^-}^{t^+} d\sigma_{n+2}^{(0)}(\Phi_{n+2}) \left[\boxed{\mathrm{w}_{n+1}(\Phi_{n+2}) \left(1 - \mathrm{w}_{n+1}(\Phi_{n+2})\Big|_{\mathcal{O}(\alpha_s^1)}\right) \mathbb{1}_{n+1}^{n+2}} - \boxed{\mathrm{w}_{n+2}(\Phi_{n+2}) \left(1 - \mathrm{w}_{n+2}(\Phi_{n+2})\Big|_{\mathcal{O}(\alpha_s^1)}\right)} \right] \\
&\qquad + \oint_{t^-}^{t^+} d\sigma_{n+2}^{(1)[\mathrm{EXC}]}(\Phi_{n+2}) \left[\boxed{\mathrm{w}_{n+1}(\Phi_{n+2}) \mathbb{1}_{n+1}^{n+2}} - \boxed{\mathrm{w}_{n+2}(\Phi_{n+2})} \right] + \oiint_{t^-}^{t^+} d\sigma_{n+3}^{(0)}(\Phi_{n+3}) \left[\boxed{\mathrm{w}_{n+1}(\Phi_{n+3}) \mathbb{1}_{n+1}^{n+3}} - \boxed{\mathrm{w}_{n+2}(\Phi_{n+3}) \mathbb{1}_{n+2}^{n+3}} \right] \Bigg\} \\
&+ \mathrm{O}_{n+2} \Bigg\{ d\sigma_{n+2}^{(0)}(\Phi_{n+2}) \boxed{\mathrm{w}_{n+2}(\Phi_{n+2}) \left[1 - \mathrm{w}_{n+2}(\Phi_{n+2})\Big|_{\mathcal{O}(\alpha_s^1)}\right]} + d\sigma_{n+2}^{(1)[\mathrm{EXC}]}(\Phi_{n+2}) \boxed{\mathrm{w}_{n+2}(\Phi_{n+2})} \\
&\qquad + \oint_{t^-}^{t^+} d\sigma_{n+3}^{(0)}(\Phi_{n+3}) \left[\boxed{\mathrm{w}_{n+2}(\Phi_{n+3}) \mathbb{1}_{n+2}^{n+3}} - \boxed{\mathrm{w}_{n+3}(\Phi_{n+3})} \right] \Bigg\} + d\sigma_{n+3}^{(0)}(\Phi_{n+3}) \boxed{\mathrm{w}_{n+3}(\Phi_{n+3})} \mathcal{F}_{n+3}^{(\infty)}(\Phi_{n+3}, t_{n+3}, t_-) \,.
\end{aligned}
$$

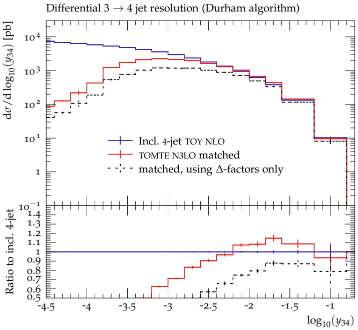
Lengthy, but *in principle* straight-forward to implement.

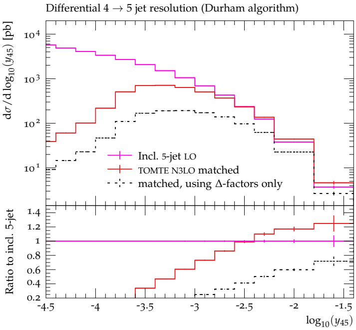
New w.r.t. NNLO+PS: 2nd-order PS expansion. Fix w.r.t. NNLO+PS: bias correction

Closure test

Construct toy N3LO calculation $(e^+e^- \to 2 \text{ jets})$ for closure testing:

- Approximate logarithmic virtual/real corr^s by clustered n^kLO events ($\sim$ LoopSim)
- Add (arbitrary) finite x-section changes by scaling LO events with α_s-polynomial
- Define result as "n^{k+1}LO calculation" and reiterate





Comparison [Toy Fixed-Order] $\leftrightarrow$ [TOMTE]: Five and four jet obs^s correctly handled: TFO result recovered at large jet separation, PS resummation when jets turn unresolved

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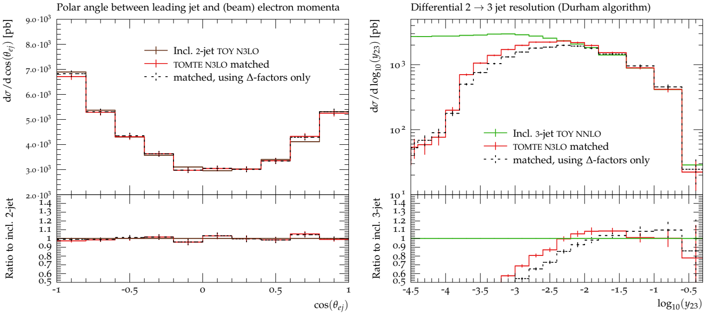

Comparison [Toy Fixed-Order] $\leftrightarrow$ [TOMTE]: Three and two jet obs^s correctly handled: TFO result recovered at large jet separation, PS resummation when jet turn unresolved

Looking ahead

Precision event generators are crucial when searching for physics beyond the SM – or simply beyond collinear factorization.

N3LO+PS is possible.

arXiv:2106.03206 introduced the TOMTE scheme, and performed closure tests for $e^+e^- \rightarrow 2$ jets

This may only be a beginning. Next steps could be

- extend implementation to hadron collisions: no in-principle bottlenecks, but some efficient expansions of PDF evolution required
- work together with developers of real N3LO calculations to produce real N3LO event generators

Feedback very welcome!

Backup

Final TOMTE formula

↪back to full formula

A more detailed version of the Tomte matching formula is

$$
\begin{aligned}
&\mathcal{F}_n^{(\infty)[\mathrm{TOMTE}]}(\Phi_n, t_+, t_-) \\
&:= \mathrm{O}_n \Big\{ d\sigma_n^{(0+1+2+3)[\mathrm{EXC}]}(\Phi_n) \\
&+ \oint_{t^-}^{t^+} d\sigma_{n+1}^{(2)\left[Q_{n+2} < Q_c \wedge Q_{n+3} < Q_c\right]}(\Phi_{n+1}) \left[\mathbb{1}_n^{n+1} - \boxed{\Delta_n(t_+, t_{n+1}) w_{n+1}^{(\infty)}(\Phi_{n+1})} \right] \\
&+ \oint_{t^-}^{t^+} d\sigma_{n+1}^{(0)}(\Phi_{n+1}) \left[\mathbb{1}_n^{n+1} - \boxed{\begin{aligned} &\Delta_n(t_+, t_{n+1}) w_{n+1}^{(\infty)}(\Phi_{n+1}) \\ &\quad \cdot \Big(1 - w_{n+1}^{(1)}(\Phi_{n+1}) - w_{n+1}^{(2)}(\Phi_{n+1}) - \Delta_n^{(1)}(t_+, t_{n+1}) - \Delta_n^{(2)}(t_+, t_{n+1}) \\ &\quad + \left[\Delta_n^{(1)}(t_+, t_{n+1})\right]^2 + \left[w_{n+1}^{(1)}(\Phi_{n+1})\right]^2 + w_{n+1}^{(1)}(\Phi_{n+1}) \Delta_n^{(1)}(t_+, t_{n+1})\Big) \end{aligned}} \right] \\
&+ \oint_{t^-}^{t^+} d\sigma_{n+1}^{(1)\left[Q_{n+2} < Q_c\right]}(\Phi_{n+1}) \left[\mathbb{1}_n^{n+1} - \boxed{\Delta_n(t_+, t_{n+1}) w_{n+1}^{(\infty)}(\Phi_{n+1}) \left(1 - w_{n+1}^{(1)}(\Phi_{n+1}) - \Delta_n^{(1)}(t_+, t_{n+1})\right)} \right] \\
&+ \oiint_{t^-}^{t^+} d\sigma_{n+2}^{(0)\left[Q_{n+2} > Q_c\right]}(\Phi_{n+2}) \left[\mathbb{1}_n^{n+2} - \boxed{\Delta_n(t_+, t_{n+1}) w_{n+1}^{(\infty)}(\Phi_{n+1}) \left(1 - w_{n+1}^{(1)}(\Phi_{n+1}) - \Delta_n^{(1)}(t_+, t_{n+1})\right) \mathbb{1}_{n+1}^{n+2}} \right] \\
&+ \oiint_{t^-}^{t^+} d\sigma_{n+2}^{(1)\left[Q_{n+2} > Q_c \wedge Q_{n+3} < Q_c\right]}(\Phi_{n+2}) \left[\mathbb{1}_n^{n+2} - \boxed{\Delta_n(t_+, t_{n+1}) w_{n+1}^{(\infty)}(\Phi_{n+1}) \mathbb{1}_{n+1}^{n+2}} \right] \\
&+ \oiiint_{t^-}^{t^+} d\sigma_{n+3}^{(0)\left[Q_{n+3} > Q_c\right]}(\Phi_{n+3}) \left[\mathbb{1}_n^{n+3} - \boxed{\Delta_n(t_+, t_{n+1}) w_{n+1}^{(\infty)}(\Phi_{n+1}) \mathbb{1}_{n+1}^{n+3}} \right] \Big\} \\
&+ \mathrm{O}_{n+1} \Big\{ d\sigma_{n+1}^{(2)\left[Q_{n+2} < Q_c \wedge Q_{n+3} < Q_c\right]}(\Phi_{n+1}) \boxed{\Delta_n(t_+, t_{n+1}) w_{n+1}^{(\infty)}(\Phi_{n+1})} \\
&+ d\sigma_{n+1}^{(0)}(\Phi_{n+1}) \otimes \boxed{\begin{aligned} &\Delta_n(t_+, t_{n+1}) w_{n+1}^{(\infty)}(\Phi_{n+1}) \\ &\quad \cdot \Big(1 - w_{n+1}^{(1)}(\Phi_{n+1}) - w_{n+1}^{(2)}(\Phi_{n+1}) - \Delta_n^{(1)}(t_+, t_{n+1}) - \Delta_n^{(2)}(t_+, t_{n+1}) \\ &\quad + \left[\Delta_n^{(1)}(t_+, t_{n+1})\right]^2 + \left[w_{n+1}^{(1)}(\Phi_{n+1})\right]^2 + w_{n+1}^{(1)}(\Phi_{n+1}) \Delta_n^{(1)}(t_+, t_{n+1})\Big) \end{aligned}} \\
&+ d\sigma_{n+1}^{(1)\left[Q_{n+2} < Q_c\right]}(\Phi_{n+1}) \\
&\quad \otimes \boxed{\left[1 - w_{n+1}^{(1)}(\Phi_{n+1}) - \Delta_n^{(1)}(t_+, t_{n+1})\right] \Delta_n(t_+, t_{n+1}) w_{n+1}^{(\infty)}(\Phi_{n+1})} \\
&+ \oint_{t^-}^{t^+} d\sigma_{n+2}^{(0)\left[Q_{n+2} > Q_c\right]}(\Phi_{n+2}) \\
&\quad \otimes \Delta_n(t_+, t_{n+1}) w_{n+1}^{(\infty)}(\Phi_{n+1}) \\
&\quad \otimes \left[\boxed{\left(1 - w_{n+1}^{(1)}(\Phi_{n+1}) - \Delta_n^{(1)}(t_+, t_{n+1})\right) \mathbb{1}_{n+1}^{n+2}} \right. \\
&\quad - \Delta_{n+1}(t_{n+1}, t_{n+2}) w_{n+2}^{(\infty)}(\Phi_{n+2}) \\
&\quad\quad \left. \otimes \boxed{\left(1 - w_{n+1}^{(1)}(\Phi_{n+1}) - w_{n+2}^{(1)}(\Phi_{n+2}) - \Delta_n^{(1)}(t_+, t_{n+1}) - \Delta_{n+1}^{(1)}(t_{n+1}, t_{n+2})\right)} \right] \\
&+ \oint_{t^-}^{t^+} d\sigma_{n+2}^{(1)\left[Q_{n+2} > Q_c \wedge Q_{n+3} < Q_c\right]}(\Phi_{n+2}) \\
&\quad \otimes \left[\boxed{\Delta_n(t_+, t_{n+1}) w_{n+1}^{(\infty)}(\Phi_{n+1}) \mathbb{1}_{n+1}^{n+2}} \right. \\
&\quad \left. - \boxed{\Delta_n(t_+, t_{n+1}) \Delta_{n+1}(t_{n+1}, t_{n+2}) w_{n+1}^{(\infty)}(\Phi_{n+1}) w_{n+2}^{(\infty)}(\Phi_{n+2})} \right] \\
&+ \oiint_{t^-}^{t^+} d\sigma_{n+3}^{(0)\left[Q_{n+3} > Q_c\right]}(\Phi_{n+3}) \\
&\quad \otimes \left[\boxed{\Delta_n(t_+, t_{n+1}) w_{n+1}^{(\infty)}(\Phi_{n+1}) \mathbb{1}_{n+1}^{n+3}} \right. \\
&\quad \left. - \boxed{\Delta_n(t_+, t_{n+1}) \Delta_{n+1}(t_{n+1}, t_{n+2}) w_{n+1}^{(\infty)}(\Phi_{n+1}) w_{n+2}^{(\infty)}(\Phi_{n+2}) \mathbb{1}_{n+2}^{n+3}} \right] \Big\} \\
&+ \mathrm{O}_{n+2} \Big\{ d\sigma_{n+2}^{(0)\left[Q_{n+2} > Q_c\right]}(\Phi_{n+2}) \\
&\quad \otimes \Delta_n(t_+, t_{n+1}) \Delta_{n+1}(t_{n+1}, t_{n+2}) w_{n+1}^{(\infty)}(\Phi_{n+1}) w_{n+2}^{(\infty)}(\Phi_{n+2}) \\
&\quad \otimes \boxed{\left[1 - w_{n+1}^{(1)}(\Phi_{n+1}) - w_{n+2}^{(1)}(\Phi_{n+2}) - \Delta_n^{(1)}(t_+, t_{n+1}) - \Delta_{n+1}^{(1)}(t_{n+1}, t_{n+2})\right]} \\
&+ d\sigma_{n+2}^{(1)\left[Q_{n+2} > Q_c \wedge Q_{n+3} < Q_c\right]}(\Phi_{n+2}) \\
&\quad \otimes \boxed{\Delta_n(t_+, t_{n+1}) \Delta_{n+1}(t_{n+1}, t_{n+2}) w_{n+1}^{(\infty)}(\Phi_{n+1}) w_{n+2}^{(\infty)}(\Phi_{n+2})} \\
&+ \oint_{t^-}^{t^+} d\sigma_{n+3}^{(0)\left[Q_{n+3} > Q_c\right]}(\Phi_{n+3}) \left[\boxed{\Delta_n(t_+, t_{n+1}) \Delta_{n+1}(t_{n+1}, t_{n+2}) w_{n+1}^{(\infty)}(\Phi_{n+1}) w_{n+2}^{(\infty)}(\Phi_{n+2}) \mathbb{1}_{n+2}^{n+3}} \right. \\
&\left. - \boxed{\Delta_n(t_+, t_{n+1}) \Delta_{n+1}(t_{n+1}, t_{n+2}) \Delta_{n+2}(t_{n+2}, t_{n+3}) w_{n+1}(\Phi_{n+1}) w_{n+2}(\Phi_{n+2}) w_{n+3}(\Phi_{n+3})} \right] \Big\} \\
&+ d\sigma_{n+3}^{(0)\left[Q_{n+3} > Q_c\right]}(\Phi_{n+3}) \boxed{\Delta_n(t_+, t_{n+1}) \Delta_{n+1}(t_{n+1}, t_{n+2}) \Delta_{n+2}(t_{n+2}, t_{n+3}) w_{n+1}(\Phi_{n+1}) w_{n+2}(\Phi_{n+2}) w_{n+3}(\Phi_{n+3})} \\
&\quad \otimes \mathcal{F}_{n+3}^{(\infty)}(\Phi_{n+3}, t_{n+3}, t_-) \,.
\end{aligned}
$$

Note the cuts on the contributions indicated as superscripts, which separate higher-order real contributions into complementary samples.

Three-jet observables in TOMTE

↪back to full formula

$$d\sigma_{n+3}^{(0)\left[Q_{n+3} > Q_c\right]}(\Phi_{n+3})\ \boxed{\Delta_n(t_+, t_{n+1})\Delta_{n+1}(t_{n+1}, t_{n+2})\Delta_{n+2}(t_{n+2}, t_{n+3})w_{n+1}(\Phi_{n+1})w_{n+2}(\Phi_{n+2})w_{n+3}(\Phi_{n+3})}$$
$$\otimes\ \mathcal{F}_{n+3}^{(\infty)}(\Phi_{n+3}, t_{n+3}, t_-)\ .$$

- Reweighting as in CKKW-L merging
- Events regularized by disallowing PS evolution variable below PS cut-off ~ 0.5 GeV
- Events showered to allow producing extra partons

Two-jet observables in TOMTE

↪back to full formula

$$
\begin{aligned}
&+ \mathcal{O}_{n+2} \Bigg\{ d\sigma^{(0)\left[Q_{n+2} > Q_c\right]}_{n+2}(\Phi_{n+2}) \\
&\quad \otimes \Delta_n(t_+, t_{n+1}) \Delta_{n+1}(t_{n+1}, t_{n+2}) w^{(\infty)}_{n+1}(\Phi_{n+1}) w^{(\infty)}_{n+2}(\Phi_{n+2}) \\
&\quad \otimes \left[1 - w^{(1)}_{n+1}(\Phi_{n+1}) - w^{(1)}_{n+2}(\Phi_{n+2}) - \Delta^{(1)}_n(t_+, t_{n+1}) - \Delta^{(1)}_{n+1}(t_{n+1}, t_{n+2})\right] \\
&+ d\sigma^{(1)\left[Q_{n+2} > Q_c \wedge Q_{n+3} < Q_c\right]}_{n+2}(\Phi_{n+2}) \\
&\quad \otimes \Delta_n(t_+, t_{n+1}) \Delta_{n+1}(t_{n+1}, t_{n+2}) w^{(\infty)}_{n+1}(\Phi_{n+1}) w^{(\infty)}_{n+2}(\Phi_{n+2}) \\
&+ \oint_{t^-}^{t^+} d\sigma^{(0)\left[Q_{n+3} > Q_c\right]}_{n+3}(\Phi_{n+3}) \Big[\Delta_n(t_+, t_{n+1}) \Delta_{n+1}(t_{n+1}, t_{n+2}) w^{(\infty)}_{n+1}(\Phi_{n+1}) w^{(\infty)}_{n+2}(\Phi_{n+2}) \mathbb{1}^{n+3}_{n+2} \\
&- \Delta_n(t_+, t_{n+1}) \Delta_{n+1}(t_{n+1}, t_{n+2}) \Delta_{n+2}(t_{n+2}, t_{n+3}) w_{n+1}(\Phi_{n+1}) w_{n+2}(\Phi_{n+2}) w_{n+3}(\Phi_{n+3}) \Big] \Bigg\} \\
&+ d\sigma^{(0)\left[Q_{n+3} > Q_c\right]}_{n+3}(\Phi_{n+3}) \, \Delta_n(t_+, t_{n+1}) \Delta_{n+1}(t_{n+1}, t_{n+2}) \Delta_{n+2}(t_{n+2}, t_{n+3}) w_{n+1}(\Phi_{n+1}) w_{n+2}(\Phi_{n+2}) w_{n+3}(\Phi_{n+3}) \\
&\otimes \mathcal{F}^{(\infty)}_{n+3}(\Phi_{n+3}, t_{n+3}, t_-) \,.
\end{aligned}
$$

- Handling $\sim$ UNLOPS-PC merging + properly removing clustering bias
- Events would only need to be showered below PS cut-off $\Rightarrow$ no showering applied (same for one-jet and zero-jet terms)

One-jet observables in TOMTE

$\hookrightarrow$back to full formula

$$+\,\mathrm{O}_{n+1}\left\{d\sigma^{(2)\left[Q_{n+2}<Q_c\,\wedge\,Q_{n+3}<Q_c\right]}_{n+1}(\Phi_{n+1})\;\boxed{\Delta_n(t_+,t_{n+1})w^{(\infty)}_{n+1}(\Phi_{n+1})}\right.$$

$$+\,d\sigma^{(0)}_{n+1}(\Phi_{n+1})\otimes\boxed{\begin{aligned}&\Delta_n(t_+,t_{n+1})w^{(\infty)}_{n+1}(\Phi_{n+1})\\&\cdot\Big(1-w^{(1)}_{n+1}(\Phi_{n+1})-w^{(2)}_{n+1}(\Phi_{n+1})-\Delta^{(1)}_n(t_+,t_{n+1})-\Delta^{(2)}_n(t_+,t_{n+1})\\&+\left[\Delta^{(1)}_n(t_+,t_{n+1})\right]^2+\left[w^{(1)}_{n+1}(\Phi_{n+1})\right]^2+w^{(1)}_{n+1}(\Phi_{n+1})\Delta^{(1)}_n(t_+,t_{n+1})\Big)\end{aligned}}$$

$$+\,d\sigma^{(1)\left[Q_{n+2}<Q_c\right]}_{n+1}(\Phi_{n+1})$$

$$\otimes\;\boxed{\left[1-w^{(1)}_{n+1}(\Phi_{n+1})-\Delta^{(1)}_n(t_+,t_{n+1})\right]\Delta_n(t_+,t_{n+1})w^{(\infty)}_{n+1}(\Phi_{n+1})}$$

$$+\oint_{t^-}^{t^+}d\sigma^{(0)\left[Q_{n+2}>Q_c\right]}_{n+2}(\Phi_{n+2})$$

$$\otimes\;\Delta_n(t_+,t_{n+1})w^{(\infty)}_{n+1}(\Phi_{n+1})$$

$$\otimes\Big[\;\boxed{\left(1-w^{(1)}_{n+1}(\Phi_{n+1})-\Delta^{(1)}_n(t_+,t_{n+1})\right)\mathbb{1}^{n+2}_{n+1}}$$

$$-\;\Delta_{n+1}(t_{n+1},t_{n+2})w^{(\infty)}_{n+2}(\Phi_{n+2})$$

$$\otimes\;\boxed{\left(1-w^{(1)}_{n+1}(\Phi_{n+1})-w^{(1)}_{n+2}(\Phi_{n+2})-\Delta^{(1)}_n(t_+,t_{n+1})-\Delta^{(1)}_{n+1}(t_{n+1},t_{n+2})\right)}\Big]$$

$$+\oint_{t^-}^{t^+}d\sigma^{(1)\left[Q_{n+2}>Q_c\,\wedge\,Q_{n+3}<Q_c\right]}_{n+2}(\Phi_{n+2})$$

$$\otimes\Big[\;\boxed{\Delta_n(t_+,t_{n+1})w^{(\infty)}_{n+1}(\Phi_{n+1})\mathbb{1}^{n+2}_{n+1}}$$

$$-\;\boxed{\Delta_n(t_+,t_{n+1})\Delta_{n+1}(t_{n+1},t_{n+2})w^{(\infty)}_{n+1}(\Phi_{n+1})w^{(\infty)}_{n+2}(\Phi_{n+2})}\;\Big]$$

$$+\oiint_{t^-}^{t^+}d\sigma^{(0)\left[Q_{n+3}>Q_c\right]}_{n+3}(\Phi_{n+3})$$

$$\otimes\Big[\;\boxed{\Delta_n(t_+,t_{n+1})w^{(\infty)}_{n+1}(\Phi_{n+1})\mathbb{1}^{n+3}_{n+1}}$$

$$-\;\boxed{\Delta_n(t_+,t_{n+1})\Delta_{n+1}(t_{n+1},t_{n+2})w^{(\infty)}_{n+1}(\Phi_{n+1})w^{(\infty)}_{n+2}(\Phi_{n+2})\mathbb{1}^{n+3}_{n+2}}\;\Big]\Big\}$$

$$+\,\mathrm{O}_{n+2}\left\{\;d\sigma^{(0)\left[Q_{n+2}>Q_c\right]}_{n+2}(\Phi_{n+2})\right.$$

$$\otimes\;\Delta_n(t_+,t_{n+1})\Delta_{n+1}(t_{n+1},t_{n+2})w^{(\infty)}_{n+1}(\Phi_{n+1})w^{(\infty)}_{n+2}(\Phi_{n+2})$$

$$\otimes\;\boxed{\left[1-w^{(1)}_{n+1}(\Phi_{n+1})-w^{(1)}_{n+2}(\Phi_{n+2})-\Delta^{(1)}_n(t_+,t_{n+1})-\Delta^{(1)}_{n+1}(t_{n+1},t_{n+2})\right]}$$

$$+\,d\sigma^{(1)\left[Q_{n+2}>Q_c\,\wedge\,Q_{n+3}<Q_c\right]}_{n+2}(\Phi_{n+2})$$

- Mildly $\sim$ UN2LOPS
- Three-jet contribution with bias correction trickles down
- Weight of LO two-jet ($d\sigma^{(0)}_{n+2}$) somewhat subtle
- Weight of LO one-jet requires 2nd-order PS expansion

Zero-jet observables in TOMTE

↪back to full formula

$$
\begin{aligned}
:= \mathrm{O}_n \Bigg\{ & d\sigma_n^{(0+1+2+3)[\mathrm{EXC}]}(\Phi_n) \\
+ & \oint_{t^-}^{t^+} d\sigma_{n+1}^{(2)\left[Q_{n+2}<Q_c \,\wedge\, Q_{n+3}<Q_c\right]}(\Phi_{n+1}) \left[\mathbb{1}_n^{n+1} - \boxed{\Delta_n(t_+,t_{n+1}) w_{n+1}^{(\infty)}(\Phi_{n+1})} \right] \\
+ & \oint_{t^-}^{t^+} d\sigma_{n+1}^{(0)}(\Phi_{n+1}) \left[\mathbb{1}_n^{n+1} - \boxed{\begin{aligned} & \Delta_n(t_+,t_{n+1}) w_{n+1}^{(\infty)}(\Phi_{n+1}) \\ & \cdot \Big(1 - w_{n+1}^{(1)}(\Phi_{n+1}) - w_{n+1}^{(2)}(\Phi_{n+1}) - \Delta_n^{(1)}(t_+,t_{n+1}) - \Delta_n^{(2)}(t_+,t_{n+1}) \\ & \quad + \left[\Delta_n^{(1)}(t_+,t_{n+1})\right]^2 + \left[w_{n+1}^{(1)}(\Phi_{n+1})\right]^2 + w_{n+1}^{(1)}(\Phi_{n+1}) \Delta_n^{(1)}(t_+,t_{n+1})\Big) \end{aligned}} \right] \\
+ & \oint_{t^-}^{t^+} d\sigma_{n+1}^{(1)\left[Q_{n+2}<Q_c\right]}(\Phi_{n+1}) \left[\mathbb{1}_n^{n+1} - \boxed{\Delta_n(t_+,t_{n+1}) w_{n+1}^{(\infty)}(\Phi_{n+1}) \left(1 - w_{n+1}^{(1)}(\Phi_{n+1}) - \Delta_n^{(1)}(t_+,t_{n+1})\right)} \right] \\
+ & \oiint_{t^-}^{t^+} d\sigma_{n+2}^{(0)\left[Q_{n+2}>Q_c\right]}(\Phi_{n+2}) \left[\mathbb{1}_n^{n+2} - \boxed{\Delta_n(t_+,t_{n+1}) w_{n+1}^{(\infty)}(\Phi_{n+1}) \left(1 - w_{n+1}^{(1)}(\Phi_{n+1}) - \Delta_n^{(1)}(t_+,t_{n+1})\right) \mathbb{1}_{n+1}^{n+2}} \right] \\
+ & \oiint_{t^-}^{t^+} d\sigma_{n+2}^{(1)\left[Q_{n+2}>Q_c \,\wedge\, Q_{n+3}<Q_c\right]}(\Phi_{n+2}) \left[\mathbb{1}_n^{n+2} - \boxed{\Delta_n(t_+,t_{n+1}) w_{n+1}^{(\infty)}(\Phi_{n+1}) \mathbb{1}_{n+1}^{n+2}} \right] \\
+ & \oiiint_{t^-}^{t^+} d\sigma_{n+3}^{(0)\left[Q_{n+3}>Q_c\right]}(\Phi_{n+3}) \left[\mathbb{1}_n^{n+3} - \boxed{\Delta_n(t_+,t_{n+1}) w_{n+1}^{(\infty)}(\Phi_{n+1}) \mathbb{1}_{n+1}^{n+3}} \right] \Bigg\}
\end{aligned}
$$

$$
+ \mathrm{O}_{n+1} \Big\{ d\sigma_{n+1}^{(2)\left[Q_{n+2}<Q_c \,\wedge\, Q_{n+3}<Q_c\right]}(\Phi_{n+1}) \; \boxed{\Delta_n(t_+,t_{n+1}) w_{n+1}^{(\infty)}(\Phi_{n+1})}
$$

$$
\Delta_n(t_+,t_{n+1}) w_{n+1}^{(\infty)}(\Phi_{n+1})
$$

- PS effects removed identically
- bias-corrected ($\mathbb{1}$) contributions complement exc. σ precisely