

Limiting fragmentation at RHIC and LHC energies

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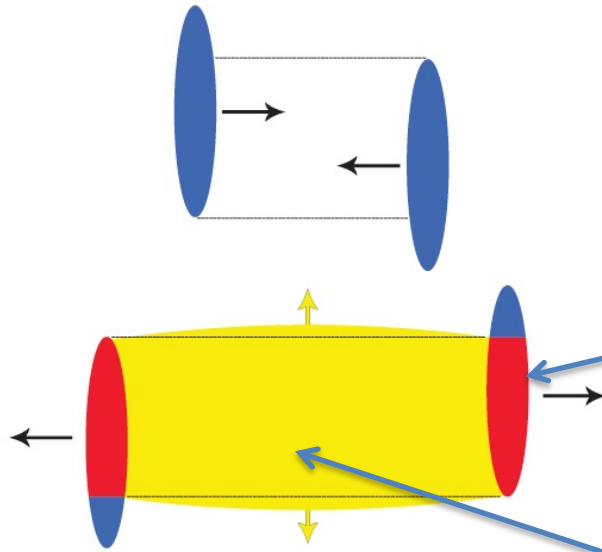
*collaboration with B. Kellers, now PhD student at Ulm University

Topics

1. Introduction
2. A three-source diffusion model for particle production in relativistic collisions
3. Application of the model with linear and sinh-drift to RHIC/LHC data
4. Limiting fragmentation and its centrality dependence
5. Number of produced charged hadrons in fragmentation and fireball sources
6. Summary and Conclusion

1. Introduction

Relativistic heavy-ion collisions (schematic)

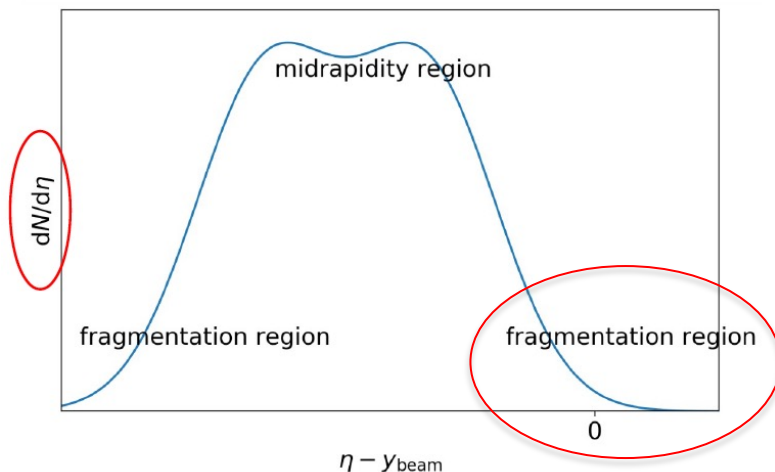


Local equilibration of **quarks** in the fragmentation sources towards the FD-distribution

Fast local equilibration of **gluons** in the **fireball** towards the BE-distribution (global thermodynamic equilibrium is usually not attained)

Most produced particles in the fragmentation region (large pseudorapidities: $\tilde{\eta} \approx \eta - y_{\text{beam}} \approx 0$) arise from the fragmentation sources

Limiting fragmentation (LF): The charged-particle yield does not depend on energy over a large (pseudo-)rapidity range $\eta - y_{\text{beam}}$



2. A three-source relativistic diffusion model (RDM)

The Lorentz-invariant cross section for produced particles is

$$E \frac{d^3 N}{d^3 p} = \frac{d^2 N}{2\pi p_T dp_T dy} = \frac{d^2 N}{2\pi m_T dm_T dy}$$

$$E = m_T \cosh(y), \quad p_T = \sqrt{p_x^2 + p_y^2},$$

$$m_T = \sqrt{m^2 + p_T^2},$$

$$y = 0.5 \ln[(E + p_{||}) / (E - p_{||})]$$

In a three-source model, the partial rapidity distributions dN_k/dy ($k=1,2,3$) are obtained by integrating over the transverse mass

$$\frac{dN_k}{dy}(y, t) = c_k^b \int m_T E \frac{d^3 N_k}{d^3 p} dm_T.$$

The rapidity distributions are calculated in a phenomenological **relativistic diffusion model (RDM)** as $dN_k/dy = N_k R(y, t=\tau_f)$ from the solution of a **Fokker-Planck equation** and then added incoherently to obtain the total distribution

$$\frac{dN_{\text{ch}}}{dy}(y, t = \tau_f) = \frac{dN_1}{dy}(y, \tau_f) + \frac{dN_2}{dy}(y, \tau_f) + \frac{dN_{\text{gg}}}{dy}(y, \tau_f)$$

with the freeze-out time τ_f and the particle numbers N_k in the respective sources.

The underlying (linear) transport equation* is of the **Fokker-Planck form**

$$\frac{\partial R_k(y, t)}{\partial t} = -\frac{\partial}{\partial y} [J_k(y, t) R_k(y, t)] + \frac{\partial^2}{\partial y^2} [D_k(y, t) R_k(y, t)]$$

with the **drift functions** $J_k(y, t)$ that account for dissipative effects, and the **diffusion functions** $D_k(y, t)$ that cause the broadening of the rapidity distributions due to scatterings and particle creations.

Its stationary limit for $t \rightarrow \infty$ and sufficiently high temperatures T should give rise to the **Maxwell-Juettner distribution**

$$E \frac{d^3 N}{d^3 p} \Big|_{\text{eq}} \propto E \exp(-E/T) = m_T \cosh(y) \exp(-m_T \cosh(y)/T).$$

In a simplified approach that provides **analytical solutions** of the FPE, the diffusion coefficients $D_k(y, t)$ are assumed to be constants, and the drifts depend linearly on the rapidity, $J_k(y, t) = (y_{\text{eq}} - y)/\tau_y$. The (gaussian) stationary limit then deviates slightly from Maxwell-Juettner, but the time-dependent FPE solutions still provide a good representation of the data.

* For a derivation see J. Hoelck and GW, Phys. Rev. Res. 2, 033409 (2020)

In order to reach the correct stationary solution, the drift terms become

$$J_k(y, t) = -A_k \sinh(y)$$

with drift amplitudes

$$A_k = m_T^k D_k / T .$$

In **thermal equilibrium**, the three subdistributions merge and the rapidity distribution attains the overall Maxwell-Juettner form

$$\frac{dN_{\text{eq}}}{dy} = C_b \left(m_T^2 T + \frac{2m_T T^2}{\cosh y} + \frac{2T^3}{\cosh^2 y} \right) \times \exp \left(-\frac{m_T \cosh y}{T} \right)$$

with the normalization constant C_b determined by the number of produced charged hadrons as function of centrality, or impact parameter b . However, the actual time-dependent rapidity distribution functions remain far from thermal equilibrium.

The **drift amplitudes A_k** at each centrality are determined from the **positions of the fragmentation peaks**. Four centrality bins are investigated.

Numerical solution of the FPE with sinh-drift

Transform the equations for $R_k(y, t)$ by introducing a dimensionless time $\tau = t / t_c$ (t_c a fixed timescale)

$$\frac{\partial R_k}{\partial \tau}(y, \tau) = t_c A_k \frac{\partial}{\partial y} [\sinh(y) R_k(y, \tau)] + t_c D_k \frac{\partial^2}{\partial y^2} R_k(y, \tau)$$

with $A_k = m_T^k D_k / T$ and $t_c = T / (m_T^k D_k) = A_k^{-1}$

\Rightarrow the dimensionless FPE contains only the ratio of temperature T and transverse masses m_T^k , characterizing the **diffusion strengths**

$\gamma_k = T / m_T^k$; its solutions depend on y and τ

$$\frac{\partial R_k}{\partial \tau}(y, \tau) = \frac{\partial}{\partial y} [\sinh(y) R_k(y, \tau)] + \gamma_k \frac{\partial^2}{\partial y^2} R_k(y, \tau)$$

The three partial y -distributions in each centrality bin are determined through the time parameter τ and the diffusion strengths γ_k .

The numerical integration is performed using matlab's pdep routine.

3. Application of the model to RHIC and LHC data

The FPE is solved numerically for each centrality class, and the results are transformed into rapidity distributions. The constant C_b is adjusted to the total number of produced charged hadrons in each centrality bin.

For unidentified particles, the rapidity distributions dN/dy are converted to **pseudorapidity space**. The scattering angle θ defines the pseudorapidity variable η

$$\eta = \frac{1}{2} \ln \frac{|\mathbf{p}| + p_{\parallel}}{|\mathbf{p}| - p_{\parallel}} = -\ln \left[\tan(\theta/2) \right]$$

and the pseudorapidity distribution of produced charged hadrons becomes

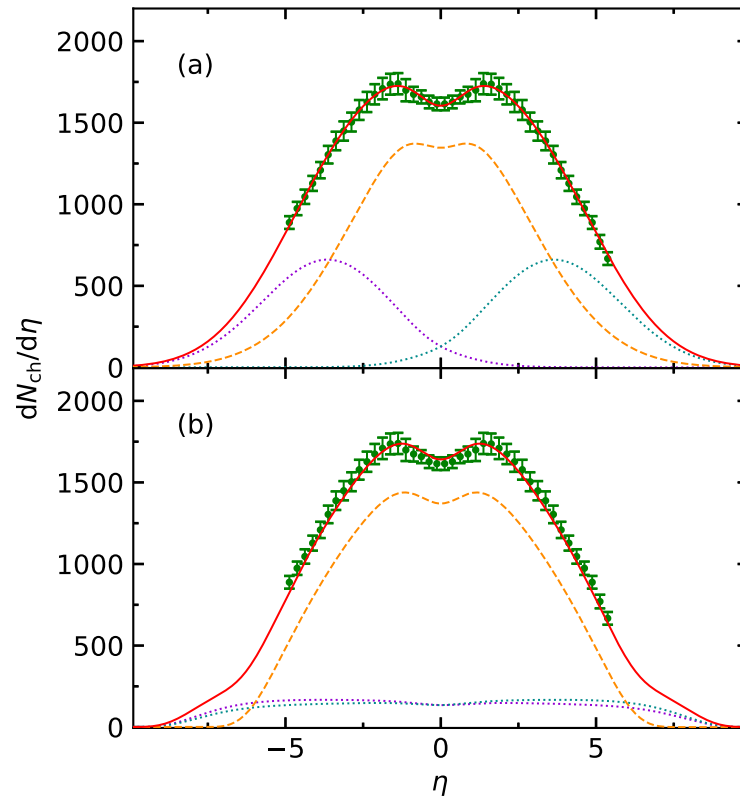
$$\frac{dN}{d\eta} \simeq \frac{dy}{d\eta} \frac{dN}{dy} = \mathcal{J} \left(\eta, \frac{m}{p_T} \right) \frac{dN}{dy}$$

with the Jacobian

$$\mathcal{J} \left(\eta, \frac{m}{p_T} \right) = \left[1 + \left(\frac{m}{p_T \cosh(\eta)} \right)^2 \right]^{-1/2}$$

The model parameters are then determined in χ^2 optimizations with respect to the data.

RDM with three sources, linear and nonlinear drift

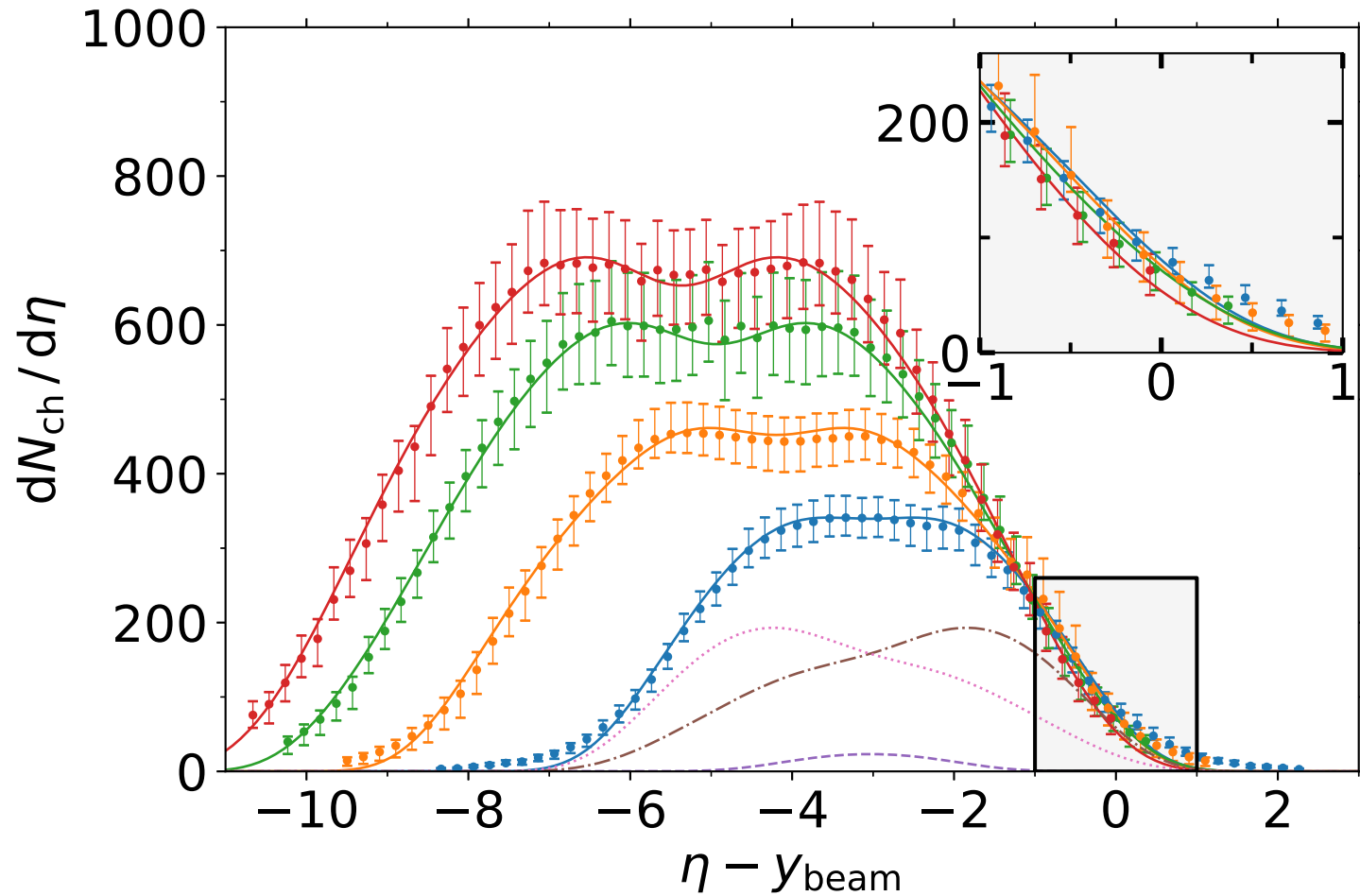


2.76 TeV Pb-Pb, central collisions

Linear drift;
analytical solutions of the FPE

Nonlinear drift: Fragmentation
sources more extended in η ;
numerical solutions of the FPE

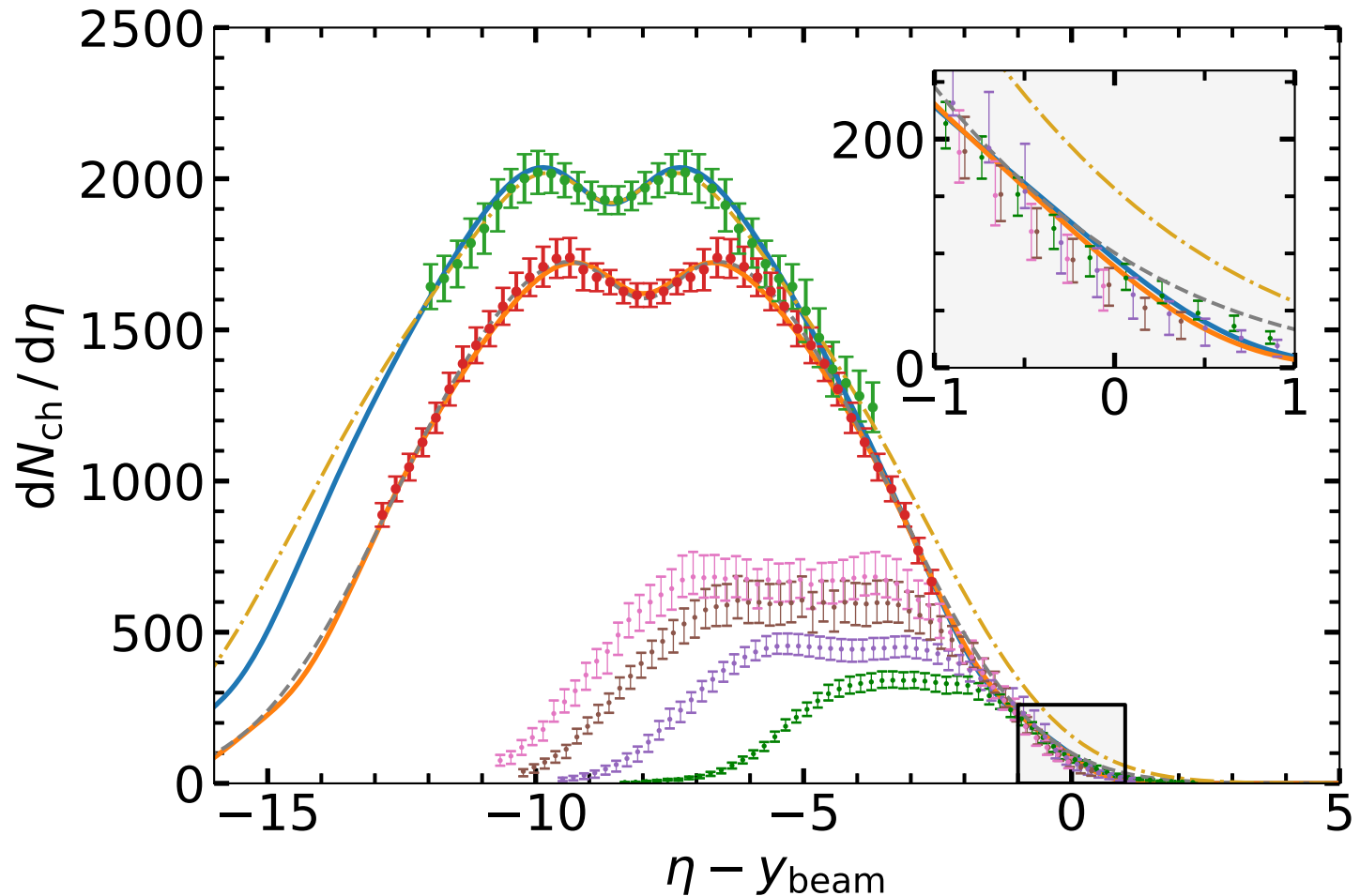
Pseudorapidity distributions of produced charged hadrons
at RHIC (PHOBOS Au-Au data @ 19.6, 62.4, 130 and 200 GeV)



RHIC: LF is fulfilled in the data, and in the RDM
with both, linear and nonlinear drift

B. Kellers and GW, PTEP 2019, 053D03 (2019)

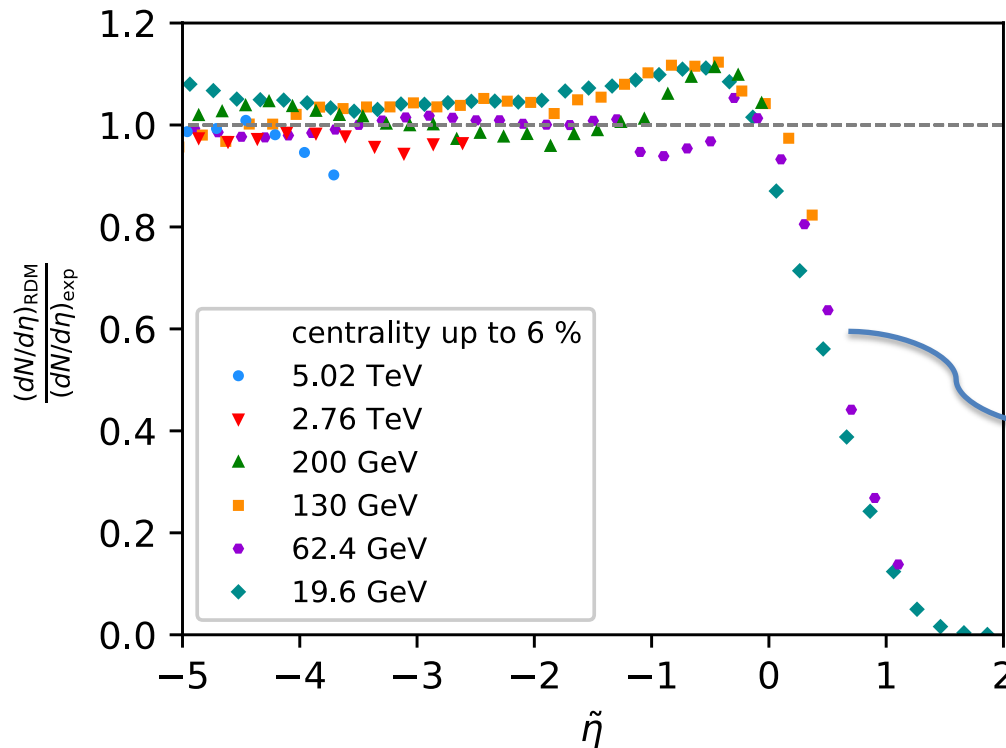
Limiting fragmentation at LHC energies?
Central Pb-Pb collisions @ 2.76 and 5.02 TeV



dashed, dot-dashed: linear drift
solid: sinh-drift

The RDM with sinh-drift is
consistent with LF in central collisions
at both LHC energies

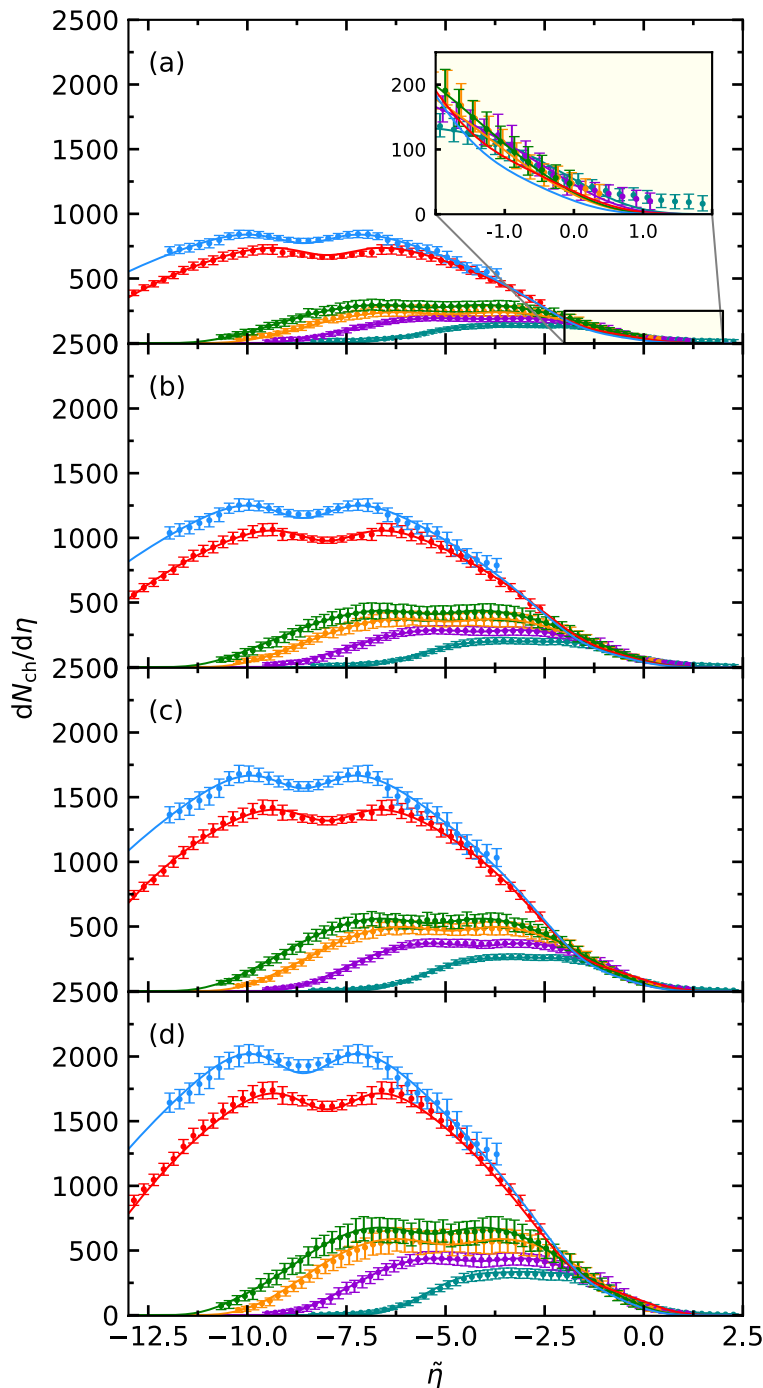
Ratio: Numerical RDM-solutions to data in the fragmentation region



LF is approximately
fulfilled (sinh-drift)

$(dN/d\eta)_{\text{RDM}}$ drops towards
zero faster than the data, to
fulfill the Dirichlet boundary
conditions imposed by the
numerical solver of the FPE

4. Centrality dependence of LF at RHIC and LHC



Centralities:

(a) 20–30 %

(b) 10–20 %

(c) 6–10 % at RHIC, 5–10 % at LHC

(d) 0–6 % at RHIC, 0–5 % at LHC

Numerical RDM-solutions with nonlinear drift (solid lines) compared to RHIC (PHOBOS) and LHC (ALICE) data:

Consistent with LF in all 4 centrality classes

B. Kellers and GW, EPJA 57, 47 (2021)

System and model parameters

Table 1 System and model parameters with sinh-drift in four centrality classes for Au–Au (RHIC) and Pb–Pb (LHC) at six incident energies. Listed are particle content N_{gg} and $N_{1,2}$ of the fireball and fragmenta-

tion sources, corresponding diffusion strengths γ_{gg} and $\gamma_{1,2}$, time-like variable τ (see text), χ^2 - and χ^2/ndf -values

$\sqrt{s_{NN}}$ (GeV)	$ y_{beam} $	N_{gg}	$N_{1,2}$	γ_{gg}	$\gamma_{1,2}$	τ	χ^2	χ^2/ndf
Centrality: 0–6% (RHIC) and 0–5% (LHC)								
19.6	3.037	20	830	0.214	6.01	0.800	239.1	4.78
62.4	4.197	435	1273	4.85	18.8	0.706	80.1	1.60
130	4.931	1276	1533	11.6	43.3	0.681	60.2	1.20
200	5.362	2013	1656	11.0	61.1	0.578	23.9	0.48
2760	7.987	12638	2309	115	1333	0.050	28.9	0.76
5023	8.585	16196	2548	232	1424	0.027	32.2	1.07
Centrality: 6–10% (RHIC) and 5–10% (LHC)								
19.6	3.037	20	700	0.274	6.66	0.800	237.7	4.75
62.4	4.197	439	1068	6.69	20.4	0.556	93.9	1.88
130	4.931	1228	1271	10.5	42.4	0.337	18.0	0.36
200	5.362	1871	1310	16.9	63.7	0.222	14.9	0.30
2760	7.987	10380	1987	117	1324	0.054	9.5	0.25
5023	8.585	13814	1926	237	1612	0.029	19.7	0.66
Centrality: 10–20% (RHIC and LHC)								
19.6	3.037	17	560	0.356	7.74	0.748	259.4	5.19
62.4	4.197	331	844	4.13	24.7	0.628	73.2	1.46
130	4.931	899	977	11.1	42.8	0.280	24.8	0.50
200	5.362	1403	1081	19.0	68.5	0.175	25.5	0.51
2760	7.987	7730	1573	141	955	0.043	5.66	0.15
5023	8.585	10153	1597	225	1565	0.030	18.1	0.60
Centrality: 20–30% (RHIC and LHC)								
19.6	3.037	18	391	0.539	9.22	0.792	277.1	5.54
62.4	4.197	273	553	6.21	25.0	0.297	78.3	1.56
130	4.931	639	598	11.9	42.2	0.404	93.5	1.87
200	5.362	1013	697	19.6	66.2	0.198	9.51	0.19
2760	7.987	5520	982	122	1073	0.050	13.5	0.36
5023	8.585	7027	959	227	1458	0.030	12.6	0.42

5. Number of produced charged hadrons in fragmentation and fireball sources

The total number of produced charged hadrons follows a power law

$$N_{\text{ch}} = \sum_i N_i \sim s^b$$

whereas the fragmentation sources contribute logarithmically according to

$$N_{1,2} \sim \ln(s).$$

In contrast, the fireball source behaves like a cubic-log

$$N_{\text{gg}} \sim \ln(s)^3$$

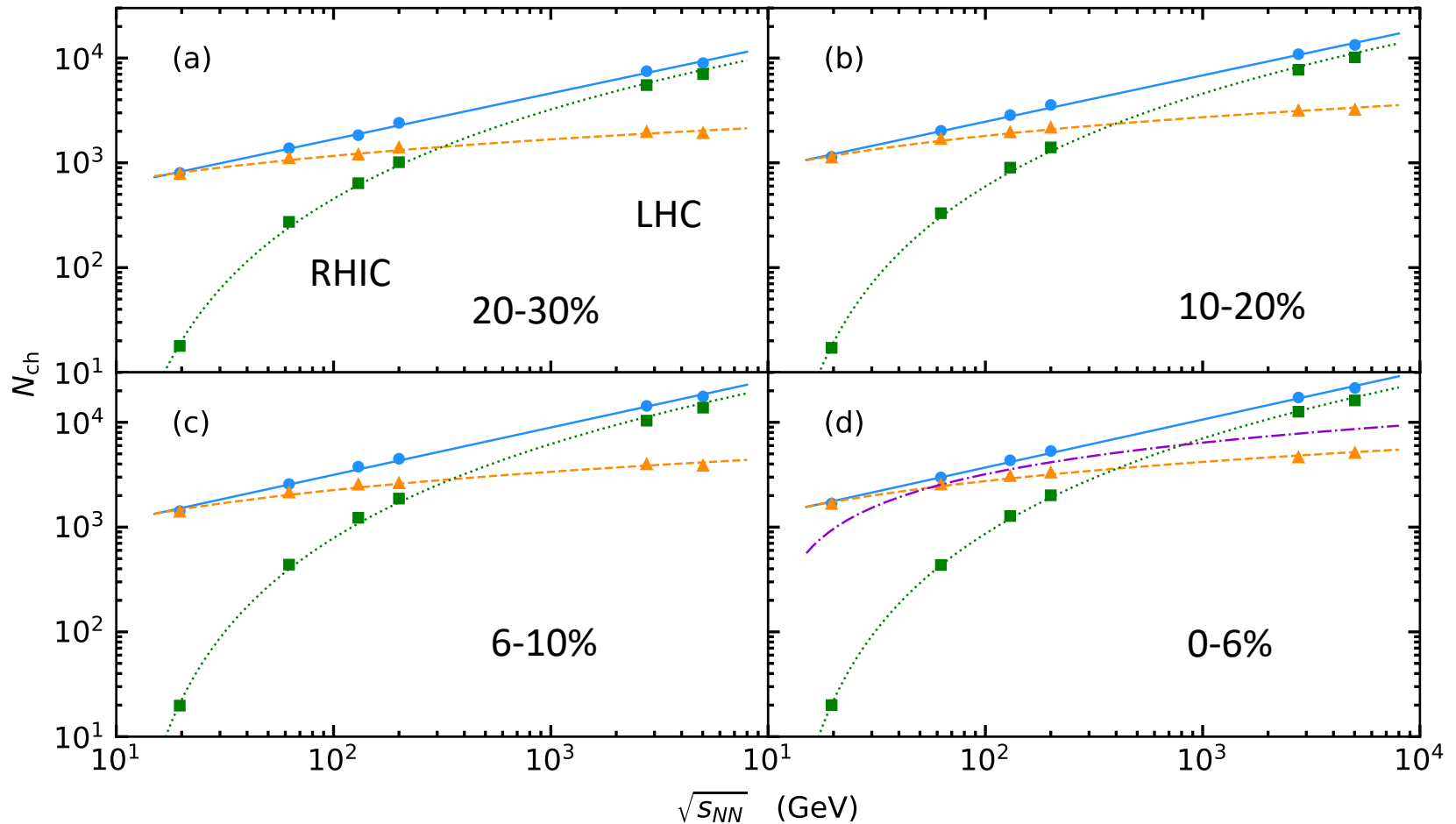
since its width Γ in rapidity space is a linear function of $\ln(s)$

$$\Gamma \sim y_{\text{beam}} \simeq \ln \frac{\sqrt{s_{NN}}}{m_p} = \ln(s)/2 + \text{const.}$$

and the predicted cross-section for low- x gluon-gluon interactions is $\propto \ln(s)^2$, consistent with the Froissart bound \Rightarrow cubic-log behaviour for N_{gg} .

To obtain equalities, the parameters are determined from the RDM results with sinh-drift.

Number of produced charged hadrons



Blue: total, $N_{ch} \sim s_{NN}^b$ (PHOBOS and ALICE data)

Yellow: fragmentation sources, $N_1+N_2 \sim \ln(s_{NN})$: RDM results, nonlinear drift

Green: fireball source, $N_{gg} \sim \ln^3(s_{NN})$. Its relevance rises rapidly with increasing c.m. energy.

6. Summary and conclusion

- The RDM solutions agree with the RHIC (PHOBOS) data on Au-Au at four different energies in the fragmentation region, where limiting fragmentation has been found
- The sinh- drift gives better results than the linear drift, consistent with the expectation from theory
- The centrality dependence of the data at RHIC and LHC (ALICE Pb-Pb) is well reproduced
- The RDM results with nonlinear drift are consistent with LF at LHC energies, but data are still missing
- The particle content in the fragmentation sources at the investigated centralities is $\sim \ln(s_{NN})$, in the fireball source $\sim \ln^3(s_{NN})$, consistent with expectations from theory

Thank you for your attention !

