



ISMD2021

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A Fourier-Cumulant Analysis for Multiharmonic Flow Fluctuation

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Dense & Strange Hadronic Matter Group, Technical University of Munich

SFT, arXiv: 2005.04742, (accepted for publication in EPJC)

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15 July 2021



Current status of heavy-ion collision models

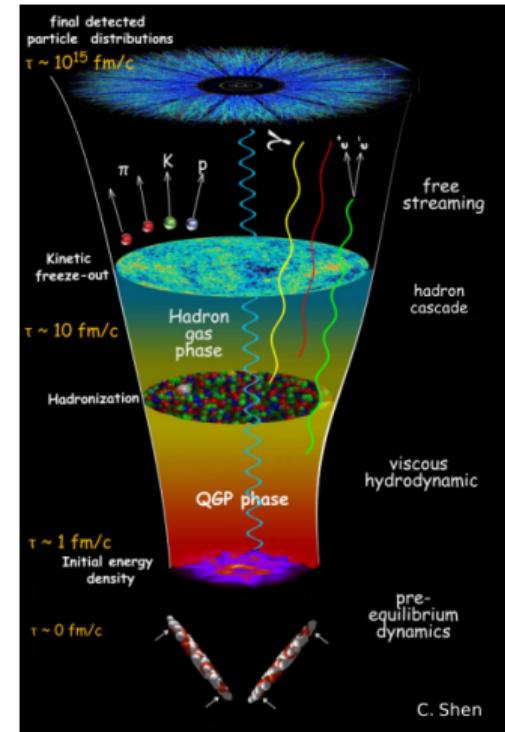
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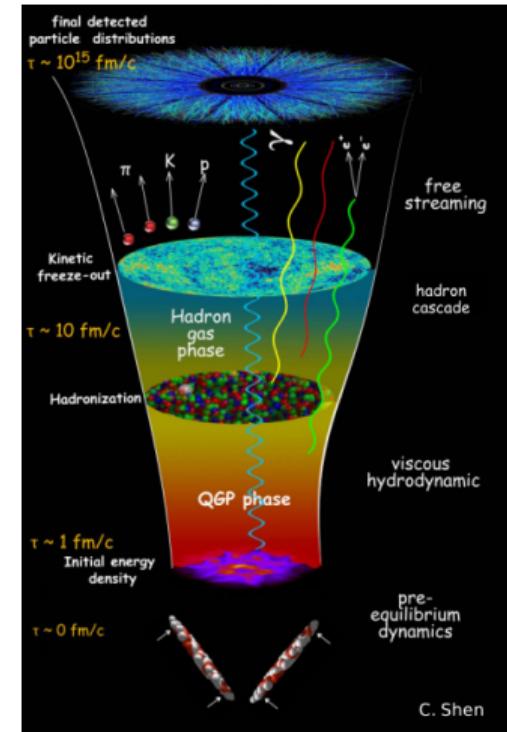




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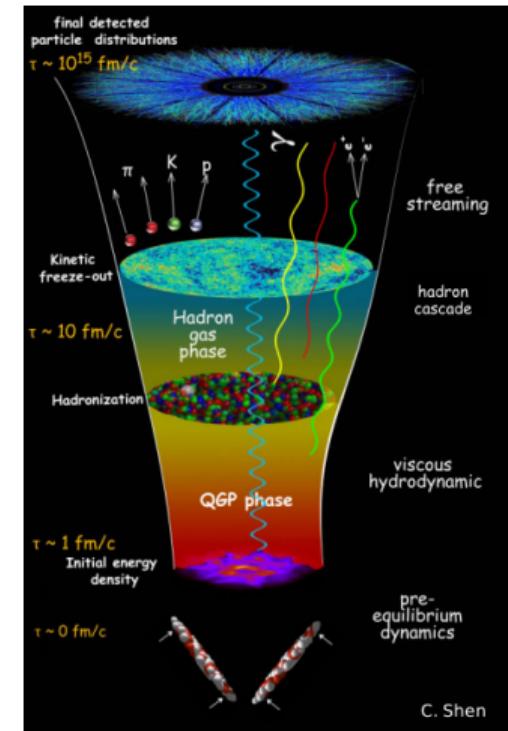


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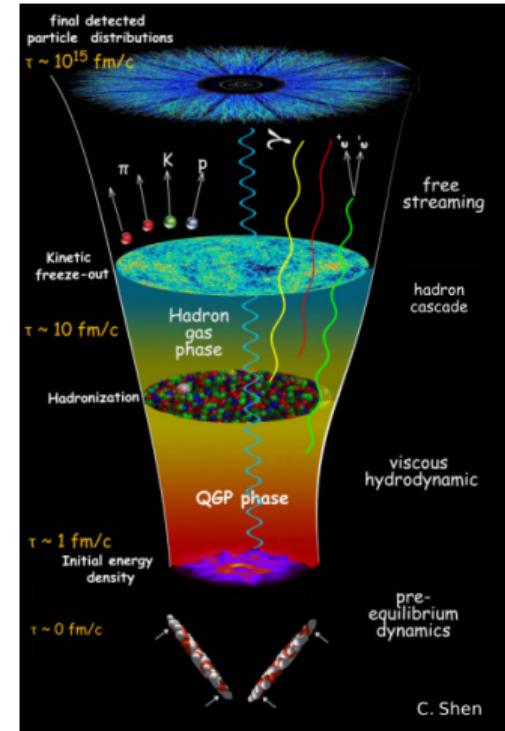


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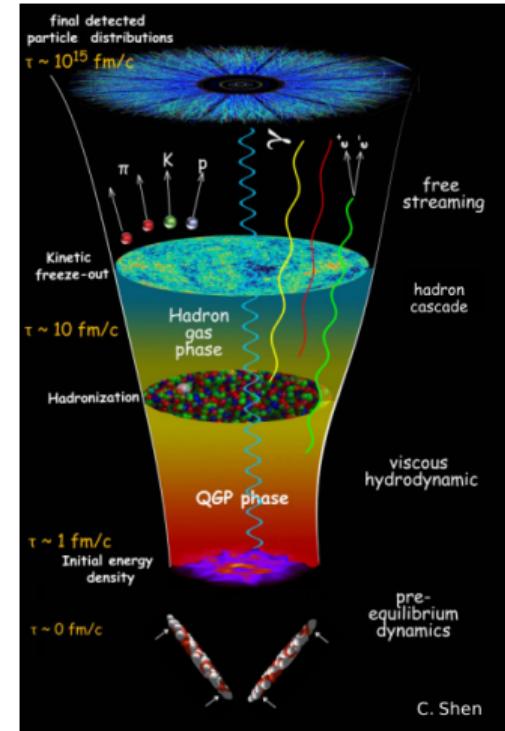
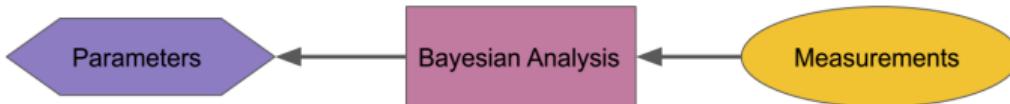


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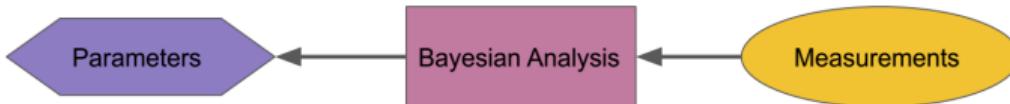


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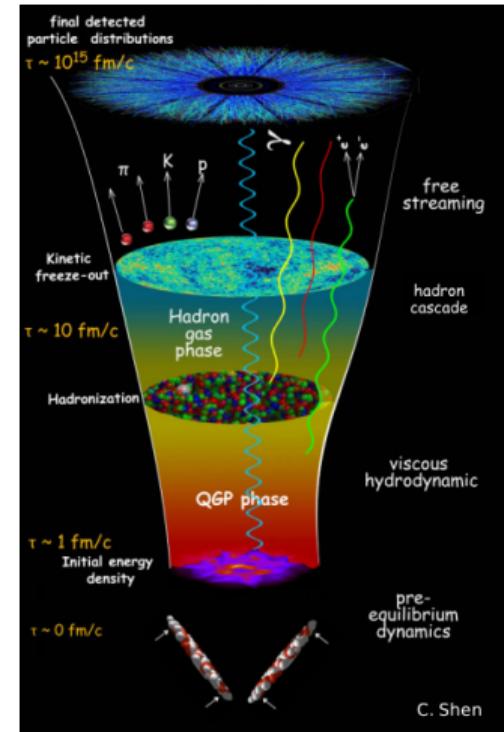
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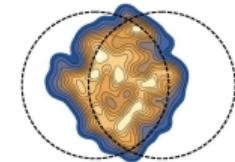
It is important to introduce experimental observables.



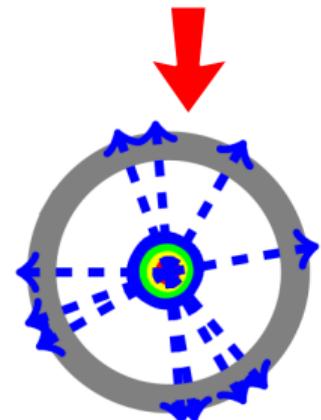
C. Shen



Flow harmonics in a nutshell!



collective evolution

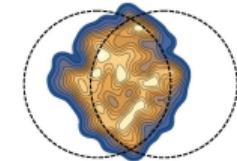


$$\frac{dN}{d\varphi} \propto 1 + \sum_{n=1}^{\infty} 2 \textcolor{red}{v}_n \cos [n(\varphi - \psi_n)]$$

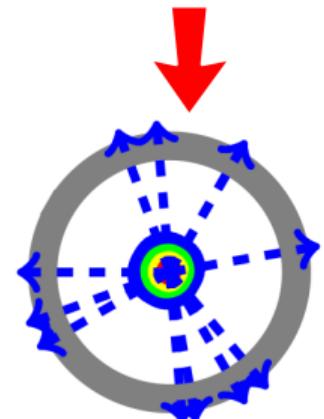


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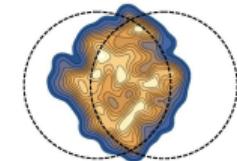


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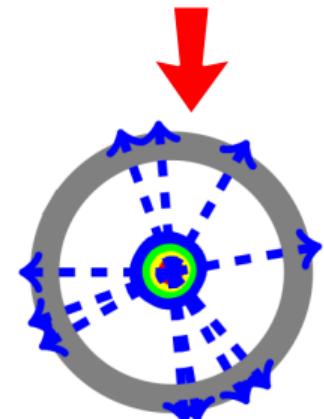
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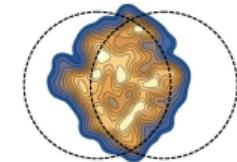


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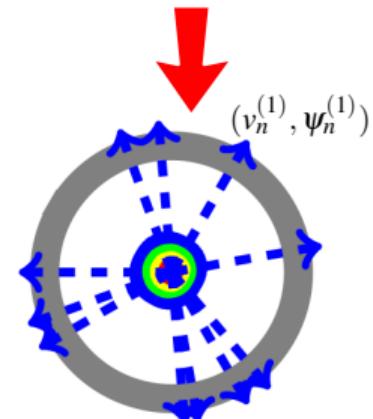
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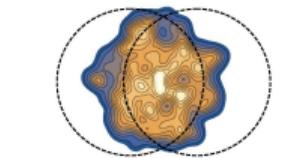
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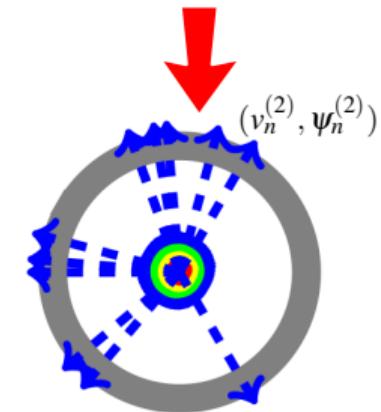
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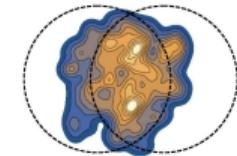
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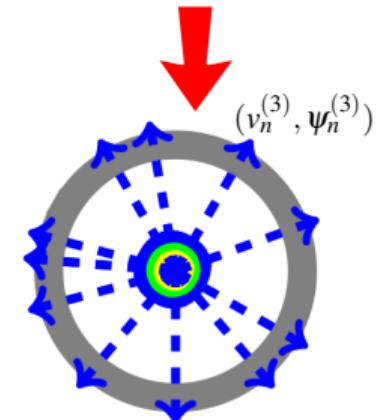
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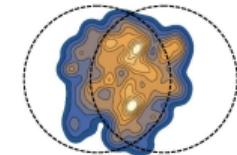


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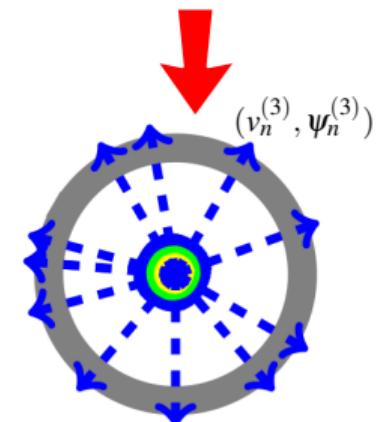
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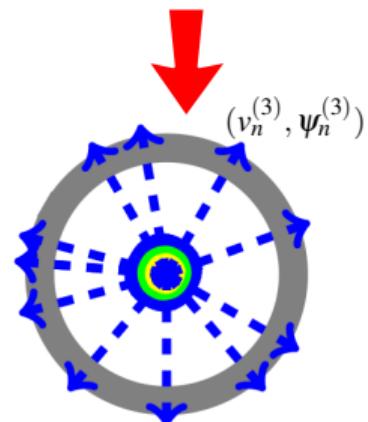
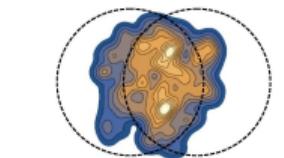
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[Borghini, Dinh, Ollitrault, PRC, '00, '01]

$$c_n\{2\} (\equiv v_n^2\{2\}), \quad c_n\{4\} (\equiv -v_n^4\{4\}), \quad \dots$$





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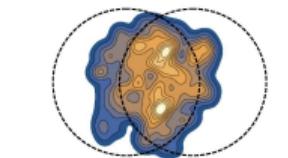
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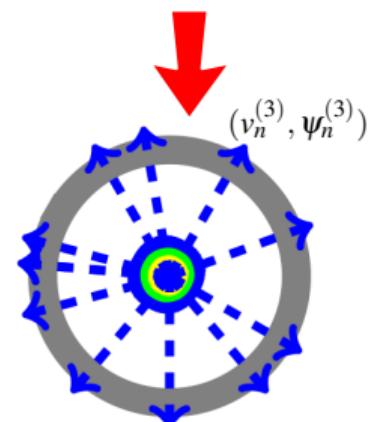
- ▶ Symmetric Cumulants:

[Bilandzic, Christensen, Gulbrandsen, Hansen, Zhou, PRC, '13]

$$\text{SC}(n, m) = \langle v_n^2 v_m^2 \rangle - \langle v_n^2 \rangle \langle v_m^2 \rangle$$



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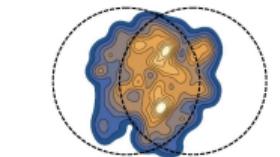
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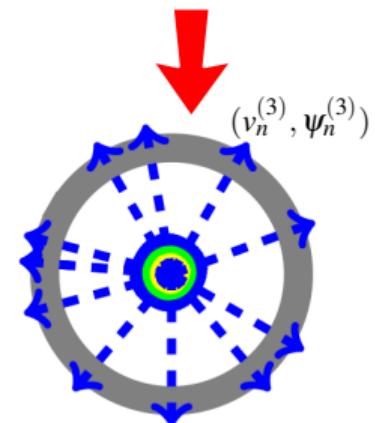
- ▶ Generalized symmetric cumulants:

[C. Mordasini, A. Bilandzic, D. Karakoc, SFT, PRC, '19],

$$\text{SC}(k, l, m) = \langle v_k^2 v_l^2 v_m^2 \rangle - \langle v_k^2 v_l^2 \rangle \langle v_m^2 \rangle - \langle v_k^2 v_m^2 \rangle \langle v_l^2 \rangle - \langle v_l^2 v_m^2 \rangle \langle v_k^2 \rangle + 2 \langle v_k^2 \rangle \langle v_l^2 \rangle \langle v_m^2 \rangle$$



collective evolution



$$(v_n^{(3)}, \psi_n^{(3)})$$



One package for all cumulants

[SFT, arXiv: 2005.04742, (accepted for publication in EPJC)]

The distribution $p_f(v_1, v_2, v_3, \dots, \psi_1 - \psi_2, \psi_2 - \psi_3, \dots)$ is very big!



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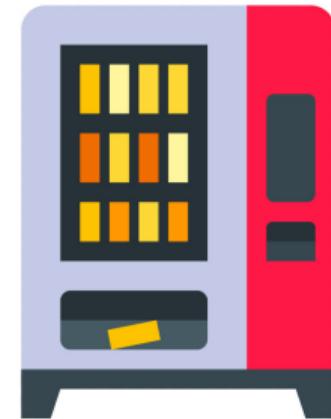
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- ▶ Mathematica package **MultiharmonicCumulants_v2_1.m**

<https://github.com/FaridTaghavi/MultiharmonicCumulants.git>

ORDER YOUR CUMULANT!





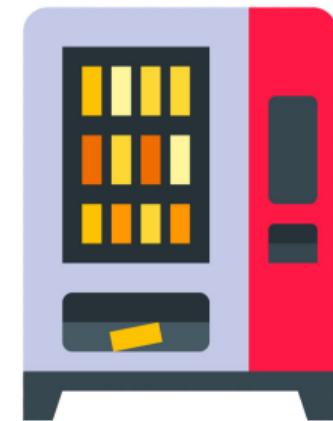
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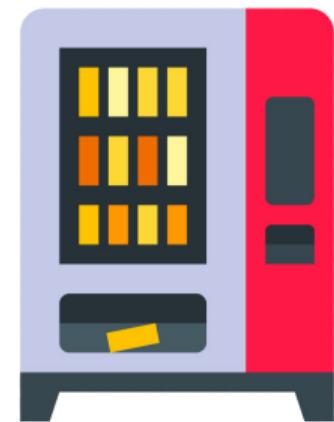
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- ▶ For instance, symmetric cumulant is obtained by

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In[1] := c[{2,2},{0},{2,3},v,\psi]  
Out[1] := \langle v_2^2 v_3^2 \rangle - \langle v_2^2 \rangle \langle v_3^2 \rangle
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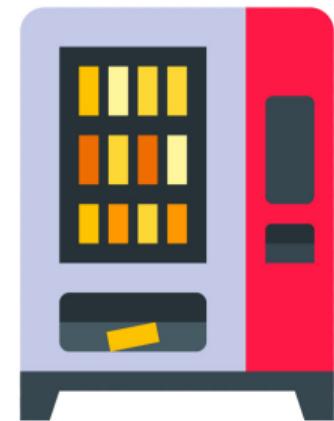
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```

- ▶ Or **statistical error** of $c_2\{2\}$ ($N * \sigma_{c_2\{2\}}^2$)

```
In[1] := Nsigma2[cCorr[{2}, {}, {2}, corr]]
Out[1] := \langle \langle 2 \rangle_{-2,2}^2 \rangle - \langle \langle 2 \rangle_{-2,2} \rangle^2
```

ORDER YOUR CUMULANT!





All cumulants involving harmonics 2,3,4,5 and orders 2,3,4,5

cumulant	order	cumulant expression
1 $c_2\{2\}$	2	$\langle v_2^2 \rangle$
2 $c_3\{2\}$	2	$\langle v_3^2 \rangle$
3 $c_4\{2\}$	2	$\langle v_4^2 \rangle$
4 $c_5\{2\}$	2	$\langle v_5^2 \rangle$
5 $c_2^{(4)}\{2, 1\}$	3	$\langle v_2^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle$
6 $c_{2,3,5}^{\{5,-5,1\}}\{1, 1, 1\}$	3	$\langle v_2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
7 $c_2\{4\}$	4	$\langle v_2^4 \rangle - 2\langle v_2^2 \rangle^2$
8 $c_3\{4\}$	4	$\langle v_3^4 \rangle - 2\langle v_3^2 \rangle^2$
9 $c_4\{4\}$	4	$\langle v_4^4 \rangle - 2\langle v_4^2 \rangle^2$
10 $c_5\{4\}$	4	$\langle v_5^4 \rangle - 2\langle v_5^2 \rangle^2$
11 $c_2^{(6)}\{2, 2\}$	4	$\langle v_2^2 v_2^2 \rangle - \langle v_2^2 \rangle \langle v_2^2 \rangle$
12 $c_{2,4}^{(6)}\{2, 2\}$	4	$\langle v_2^2 v_4^2 \rangle - \langle v_2^2 \rangle \langle v_4^2 \rangle$
13 $c_{2,4}^{(6)}\{2, 2\}$	4	$\langle v_2^2 v_2^2 \rangle - \langle v_2^2 \rangle \langle v_2^2 \rangle$
14 $c_{3,4}^{(6)}\{2, 2\}$	4	$\langle v_3^2 v_4^2 \rangle - \langle v_3^2 \rangle \langle v_4^2 \rangle$
15 $c_{3,4}^{(6)}\{2, 2\}$	4	$\langle v_3^2 v_2^2 \rangle - \langle v_3^2 \rangle \langle v_2^2 \rangle$
16 $c_{4,5}^{(6)}\{2, 2\}$	4	$\langle v_4^2 v_5^2 \rangle - \langle v_4^2 \rangle \langle v_5^2 \rangle$
17 $c_2^{(6,-4)}\{1, 2, 1\}$	4	$\langle v_2^2 v_2 v_4 \cos(2(\psi_2 - 3\psi_3 + 2\psi_4)) \rangle$
18 $c_{3,4,5}^{(8,-5)}\{1, 2, 1\}$	4	$\langle v_2^4 v_3 v_5 \cos(3\psi_2 - 8\psi_3 + 5\psi_5) \rangle$
19 $c_{2,3,4,5}^{(8,-4,-5)}\{1, 1, 1, 1\}$	4	$\langle v_2 v_3 v_4 v_5 \cos(2\psi_2 - 3\psi_3 - 4\psi_4 - 5\psi_5) \rangle$
20 $c_{2,3}^{(8)}\{3, 2\}$	5	$\langle v_2^3 v_3^2 \cos(6(\psi_2 - \psi_3)) \rangle$
21 $c_2^{(4)}\{2, 3\}$	5	$\langle v_2^2 v_2^3 \cos(4(\psi_2 - \psi_4)) \rangle - 2\langle v_2^2 \rangle \langle v_2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle$
22 $c_{2,4}^{(4)}\{4, 1\}$	5	$\langle v_2^4 v_4 \cos(4(\psi_2 - \psi_4)) \rangle - 3\langle v_2^2 \rangle \langle v_2^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle$
23 $c_{2,3,4}^{(-6,8)}\{1, 2, 2\}$	5	$\langle v_2^2 v_3^2 v_2 \cos(2(\psi_2 + 3\psi_3 - 4\psi_4)) \rangle$
24 $c_{2,3,4}^{(-6,8)}\{2, 1, 1\}$	5	$\langle v_2^2 v_3^2 v_2 \cos(4(\psi_2 - \psi_4)) \rangle - \langle v_2^2 \rangle \langle v_2^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle$
25 $c_{2,3,5}^{(-5,5)}\{1, 1, 3\}$	5	$\langle v_2^2 v_2 v_3 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle - 2\langle v_2^2 \rangle \langle v_2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
26 $c_{2,3,5}^{(-5,5)}\{1, 3, 1\}$	5	$\langle v_2^3 v_2 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle - 2\langle v_2^2 \rangle \langle v_2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
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28 $c_{2,4,5}^{(8,10)}\{1, 2, 2\}$	5	$\langle v_2^2 v_4^2 v_2 \cos(2(\psi_2 + 4\psi_4 - 5\psi_5)) \rangle$
29 $c_{2,4,5}^{(4,10)}\{2, 1, 2\}$	5	$\langle v_2^2 v_2^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle - \langle v_2^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle \langle v_2^2 \rangle$
30 $c_{2,4,5}^{(4,10)}\{2, 1, 2\}$	5	$\langle v_2^2 v_2^2 v_4 \cos(6\psi_2 + 4\psi_4 - 10\psi_5) \rangle$
31 $c_{2,4,5}^{(4,5)}\{3, 1, 1\}$	5	$\langle v_2^3 v_4 v_5 \cos(9\psi_2 - 4\psi_4 - 5\psi_5) \rangle$
32 $c_{2,3,4,5}^{(8,5,3)}\{1, 1, 2, 1\}$	5	$\langle v_2^4 v_2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle - \langle v_2^2 \rangle \langle v_2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
33 $c_{2,3,4,5}^{(8,-4,-5)}\{2, 1, 1, 1\}$	5	$\langle v_2^2 v_3 v_4 v_5 \cos(4\psi_2 - 3\psi_3 + 4\psi_4 - 5\psi_5) \rangle$



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5	3	$\langle v_2^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle$
6	3	$\langle v_2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
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9	4	$\langle v_4^4 \rangle - 2\langle v_4^2 \rangle^2$
10	4	$\langle v_5^4 \rangle - 2\langle v_5^2 \rangle^2$
11	4	$\langle v_2^2 v_3^2 \rangle - \langle v_2^2 \rangle \langle v_3^2 \rangle$
12	4	$\langle v_2^2 v_4^2 \rangle - \langle v_2^2 \rangle \langle v_4^2 \rangle$
13	4	$\langle v_2^2 v_5^2 \rangle - \langle v_2^2 \rangle \langle v_5^2 \rangle$
14	4	$\langle v_3^2 v_4^2 \rangle - \langle v_3^2 \rangle \langle v_4^2 \rangle$
15	4	$\langle v_3^2 v_5^2 \rangle - \langle v_3^2 \rangle \langle v_5^2 \rangle$
16	4	$\langle v_4^2 v_5^2 \rangle - \langle v_4^2 \rangle \langle v_5^2 \rangle$
17	4	$\langle v_2^2 v_3 v_4 \cos(2(\psi_2 - 3\psi_3 + 2\psi_4)) \rangle$
18	4	$\langle v_2^2 v_3 v_5 \cos(3\psi_2 - 8\psi_3 + 5\psi_5) \rangle$
19	4	$\langle v_2 v_3 v_4 v_5 \cos(2\psi_2 - 3\psi_3 - 4\psi_4 + 5\psi_5) \rangle$
20	5	$\langle v_2^3 v_3^2 \cos(6(\psi_2 - \psi_3)) \rangle$
21	5	$\langle v_2^2 v_4^2 \cos(4(\psi_2 - \psi_4)) \rangle - 2\langle v_2^2 \rangle \langle v_4^2 \cos(4(\psi_2 - \psi_4)) \rangle$
22	5	$\langle v_2^4 v_4 \cos(4(\psi_2 - \psi_4)) \rangle - 3\langle v_2^2 \rangle \langle v_2^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle$
23	5	$\langle v_2^2 v_3^2 v_2 \cos(2(\psi_2 + 3\psi_3 - 4\psi_4)) \rangle$
24	5	$\langle v_2^2 v_3^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle - \langle v_2^2 \rangle \langle v_3^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle$
25	5	$\langle v_2^2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle - 2\langle v_2^2 \rangle \langle v_2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
26	5	$\langle v_2^3 v_3^2 v_3 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle - 2\langle v_2^2 \rangle \langle v_2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
27	5	$\langle v_2^2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle - 2\langle v_2^2 \rangle \langle v_3 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
28	5	$\langle v_2^2 v_4^2 v_2 \cos(2(\psi_2 + 4\psi_4 - 5\psi_5)) \rangle$
29	5	$\langle v_2^2 v_2^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle - \langle v_2^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle \langle v_2^2 \rangle$
30	5	$\langle v_2^2 v_3^2 v_4 \cos(6\psi_2 + 4\psi_4 - 10\psi_5) \rangle$
31	5	$\langle v_2^2 v_4 v_5 \cos(9\psi_2 - 4\psi_4 - 5\psi_5) \rangle$
32	5	$\langle v_2^2 v_3 v_5 v_3 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle - \langle v_2^2 \rangle \langle v_2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
33	5	$\langle v_2^2 v_3 v_4 v_5 \cos(4\psi_2 - 3\psi_3 + 4\psi_4 - 5\psi_5) \rangle$



All cumulants involving harmonics 2,3,4,5 and orders 2,3,4,5

cumulant	order	cumulant expression
1 $c_2\{2\}$	2	$\langle v_2^2 \rangle$
2 $c_3\{2\}$	2	$\langle v_3^2 \rangle$
3 $c_4\{2\}$	2	$\langle v_4^2 \rangle$
4 $c_5\{2\}$	2	$\langle v_5^2 \rangle$
5 $c_2^{(4)}\{2, 1\}$	3	$\langle v_2^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle$
6 $c_{2,3,5}^{\{-5,5,1\}}\{1, 1, 1\}$	3	$\langle v_2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
7 $c_2\{4\}$	4	$\langle v_2^4 \rangle - 2\langle v_2^2 \rangle^2$
8 $c_3\{4\}$	4	$\langle v_3^4 \rangle - 2\langle v_3^2 \rangle^2$
9 $c_4\{4\}$	4	$\langle v_4^4 \rangle - 2\langle v_4^2 \rangle^2$
10 $c_5\{4\}$	4	$\langle v_5^4 \rangle - 2\langle v_5^2 \rangle^2$
11 $c_{2,3}^{(6)}\{2, 2\}$	4	$\langle v_2^2 v_3^2 \rangle - \langle v_2^2 \rangle \langle v_3^2 \rangle$
12 $c_{2,4}^{(6)}\{2, 2\}$	4	$\langle v_2^2 v_4^2 \rangle - \langle v_2^2 \rangle \langle v_4^2 \rangle$
13 $c_{2,5}^{(6)}\{2, 2\}$	4	$\langle v_2^2 v_5^2 \rangle - \langle v_2^2 \rangle \langle v_5^2 \rangle$
14 $c_{3,4}^{(6)}\{2, 2\}$	4	$\langle v_3^2 v_4^2 \rangle - \langle v_3^2 \rangle \langle v_4^2 \rangle$
15 $c_{3,5}^{(6)}\{2, 2\}$	4	$\langle v_3^2 v_5^2 \rangle - \langle v_3^2 \rangle \langle v_5^2 \rangle$
16 $c_{4,5}^{(6)}\{2, 2\}$	4	$\langle v_4^2 v_5^2 \rangle - \langle v_4^2 \rangle \langle v_5^2 \rangle$
17 $c_{2,3,4}^{(-8,-5)}\{1, 2, 1\}$	4	$\langle v_2^2 v_3 v_4 \cos(2(\psi_2 - 3\psi_3 + 2\psi_4)) \rangle$
18 $c_{2,3,4,5}^{(8,-5)}\{1, 2, 1\}$	4	$\langle v_2^2 v_3 v_4 v_5 \cos(3\psi_2 - 8\psi_3 + 5\psi_5) \rangle$
19 $c_{2,3,4,5}^{(3,4,-5)}\{1, 1, 1, 1\}$	4	$\langle v_2 v_3 v_4 v_5 \cos(2\psi_2 - 3\psi_3 - 4\psi_4 + 5\psi_5) \rangle$
20 $c_{2,3}^{(8)}\{2, 3\}$	5	$\langle v_2^3 v_3^2 \cos(6(\psi_2 - \psi_3)) \rangle$
21 $c_2^{(4)}\{2, 3\}$	5	$\langle v_2^2 v_4^2 \cos(4(\psi_2 - \psi_4)) \rangle - 2\langle v_2^2 \rangle \langle v_2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle$
22 $c_{2,4}^{(4)}\{4, 1\}$	5	$\langle v_2^4 v_4 \cos(4(\psi_2 - \psi_4)) \rangle - 3\langle v_2^2 \rangle \langle v_2^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle$
23 $c_{2,3,4}^{(-6,8)}\{1, 2, 2\}$	5	$\langle v_2^2 v_3^2 v_2 \cos(2(\psi_2 + 3\psi_3 - 4\psi_4)) \rangle$
24 $c_{2,3,4}^{(-8,4)}\{2, 1, 1\}$	5	$\langle v_2^2 v_3^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle - \langle v_2^2 \rangle \langle v_2^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle$
25 $c_{2,3,5}^{(-8,5)}\{1, 1, 3\}$	5	$\langle v_2^2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle - 2\langle v_2^2 \rangle \langle v_2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
26 $c_{2,3,5}^{(8,5)}\{1, 1, 3\}$	5	$\langle v_2^2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle - 2\langle v_2^2 \rangle \langle v_2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
27 $c_{2,3,5}^{(-8,5)}\{3, 1, 1\}$	5	$\langle v_2^2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle - 2\langle v_2^2 \rangle \langle v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
28 $c_{2,4,5}^{(8,10)}\{1, 2, 2\}$	5	$\langle v_2^2 v_4^2 v_2 \cos(2(\psi_2 + 4\psi_4 - 5\psi_5)) \rangle$
29 $c_{2,4,5}^{(4,0)}\{2, 1, 2\}$	5	$\langle v_2^2 v_3^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle - \langle v_2^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle \langle v_3^2 \rangle$
30 $c_{2,4,5}^{(-8,10)}\{2, 1, 2\}$	5	$\langle v_2^2 v_3^2 v_4 \cos(6\psi_2 + 4\psi_4 - 10\psi_5) \rangle$
31 $c_{2,4,5}^{(4,5)}\{3, 1, 1\}$	5	$\langle v_2^2 v_4 v_5 \cos(9\psi_2 - 4\psi_4 - 5\psi_5) \rangle$
32 $c_{2,3,4,5}^{(8,5,0)}\{1, 1, 2, 1\}$	5	$\langle v_2^2 v_3 v_4 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle - \langle v_2^2 \rangle \langle v_2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
33 $c_{2,3,4,5}^{(8,-4,5)}\{2, 1, 1, 1\}$	5	$\langle v_2^2 v_3 v_4 v_5 \cos(4\psi_2 - 3\psi_3 + 4\psi_4 - 5\psi_5) \rangle$



All cumulants involving harmonics 2,3,4,5 and orders 2,3,4,5

cumulant	order	cumulant expression
1	2	$\langle v_2^2 \rangle$
2	2	$\langle v_3^2 \rangle$
3	2	$\langle v_4^2 \rangle$
4	2	$\langle v_5^2 \rangle$
5	3	$\langle v_2^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle$
6	3	$\langle v_2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
7	4	$\langle v_2^4 \rangle - 2\langle v_2^2 \rangle^2$
8	4	$\langle v_3^4 \rangle - 2\langle v_3^2 \rangle^2$
9	4	$\langle v_4^4 \rangle - 2\langle v_4^2 \rangle^2$
10	4	$\langle v_5^4 \rangle - 2\langle v_5^2 \rangle^2$
11	4	$\langle v_2^2 v_3^2 \rangle - \langle v_2^2 \rangle \langle v_3^2 \rangle$
12	4	$\langle v_2^2 v_4^2 \rangle - \langle v_2^2 \rangle \langle v_4^2 \rangle$
13	4	$\langle v_2^2 v_5^2 \rangle - \langle v_2^2 \rangle \langle v_5^2 \rangle$
14	4	$\langle v_3^2 v_4^2 \rangle - \langle v_3^2 \rangle \langle v_4^2 \rangle$
15	4	$\langle v_3^2 v_5^2 \rangle - \langle v_3^2 \rangle \langle v_5^2 \rangle$
16	4	$\langle v_4^2 v_5^2 \rangle - \langle v_4^2 \rangle \langle v_5^2 \rangle$
17	4	$\langle v_5^2 v_2 v_4 \cos(2(\psi_2 - 3\psi_3 + 2\psi_5)) \rangle$
18	4	$\langle v_2^4 v_5 v_5 \cos(3\psi_2 - 8\psi_4 + 5\psi_5) \rangle$
19	4	$\langle v_2 v_3 v_4 v_5 \cos(2\psi_2 - 3\psi_3 - 4\psi_4 + 5\psi_5) \rangle$
20	5	$\langle v_3^2 v_2^2 v_5 \cos(6(\psi_2 - \psi_3)) \rangle$
21	5	$\langle v_2^2 v_4^2 \cos(4(\psi_2 - \psi_4)) \rangle - 2\langle v_2^2 \rangle \langle v_2^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle$
22	5	$\langle v_2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle - 3\langle v_2 \rangle \langle v_2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle$
23	5	$\langle v_2^2 v_5^2 v_2 \cos(2(\psi_2 + 3\psi_3 - 4\psi_4)) \rangle$
24	5	$\langle v_3^2 v_2^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle - \langle v_3^2 \rangle \langle v_3^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle$
25	5	$\langle v_2^2 v_2 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle - 2\langle v_2^2 \rangle \langle v_2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
26	5	$\langle v_3^2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle - 2\langle v_3^2 \rangle \langle v_2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
27	5	$\langle v_2^2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle - 2\langle v_2^2 \rangle \langle v_3 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
28	5	$\langle v_2^2 v_4^2 v_2 \cos(2(\psi_2 + 4\psi_4 - 5\psi_5)) \rangle$
29	5	$\langle v_2^2 v_2^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle - \langle v_2^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle \langle v_2^2 \rangle$
30	5	$\langle v_2^2 v_5^2 v_2 \cos(6\psi_2 + 4\psi_4 - 10\psi_5) \rangle$
31	5	$\langle v_3^2 v_4 v_5 \cos(9\psi_2 - 4\psi_4 - 5\psi_5) \rangle$
32	5	$\langle v_2^2 v_2 v_5 v_3 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle - \langle v_2^2 \rangle \langle v_2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
33	5	$\langle v_2 v_3 v_4 v_5 \cos(4\psi_2 - 3\psi_3 + 4\psi_4 - 5\psi_5) \rangle$



All cumulants involving harmonics 2,3,4,5 and orders 2,3,4,5

cumulant	order	cumulant expression
1	2	$\langle v_2^2 \rangle$
2	2	$\langle v_3^2 \rangle$
3	2	$\langle v_4^2 \rangle$
4	2	$\langle v_5^2 \rangle$
5	3	$\langle v_2^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle$
6	3	$\langle v_2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
7	4	$\langle v_2^4 \rangle - 2\langle v_2^2 \rangle^2$
8	4	$\langle v_3^4 \rangle - 2\langle v_3^2 \rangle^2$
9	4	$\langle v_4^4 \rangle - 2\langle v_4^2 \rangle^2$
10	4	$\langle v_5^4 \rangle - 2\langle v_5^2 \rangle^2$
11	4	$\langle v_2^2 v_3^2 \rangle - \langle v_2^2 \rangle \langle v_3^2 \rangle$
12	4	$\langle v_2^2 v_4^2 \rangle - \langle v_2^2 \rangle \langle v_4^2 \rangle$
13	4	$\langle v_2^2 v_5^2 \rangle - \langle v_2^2 \rangle \langle v_5^2 \rangle$
14	4	$\langle v_3^2 v_4^2 \rangle - \langle v_3^2 \rangle \langle v_4^2 \rangle$
15	4	$\langle v_3^2 v_5^2 \rangle - \langle v_3^2 \rangle \langle v_5^2 \rangle$
16	4	$\langle v_4^2 v_5^2 \rangle - \langle v_4^2 \rangle \langle v_5^2 \rangle$
17	4	$\langle v_2^2 v_3 v_4 \cos(2(\psi_2 - 3\psi_3 + 2\psi_4)) \rangle$
18	4	$\langle v_2^2 v_3 v_5 \cos(3\psi_2 - 8\psi_3 + 5\psi_5) \rangle$
19	4	$\langle v_2 v_3 v_4 v_5 \cos(2\psi_2 - 3\psi_3 - 4\psi_4 + 5\psi_5) \rangle$
20	5	$\langle v_2^3 v_3^2 \cos(6(\psi_2 - \psi_3)) \rangle$
21	5	$\langle v_2^2 v_4^2 \cos(4(\psi_2 - \psi_4)) \rangle - 2\langle v_2^2 \rangle \langle v_2^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle$
22	5	$\langle v_2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle - 3\langle v_2 \rangle \langle v_2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle$
23	5	$\langle v_2^2 v_5^2 v_2 \cos(2(\psi_2 + 3\psi_3 - 4\psi_4)) \rangle$
24	5	$\langle v_2^2 v_5^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle - \langle v_2^2 \rangle \langle v_2^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle$
25	5	$\langle v_2^2 v_2 v_3 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle - 2\langle v_2^2 \rangle \langle v_2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
26	5	$\langle v_2^2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle - 2\langle v_2^2 \rangle \langle v_2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
27	5	$\langle v_2^2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle - 2\langle v_2^2 \rangle \langle v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
28	5	$\langle v_2^2 v_4 v_2 \cos(2(\psi_2 + 4\psi_4 - 5\psi_5)) \rangle$
29	5	$\langle v_2^2 v_2^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle - \langle v_2^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle \langle v_2^2 \rangle$
30	5	$\langle v_2^2 v_5^2 v_2 \cos(6\psi_2 + 4\psi_4 - 10\psi_5) \rangle$
31	5	$\langle v_2^2 v_4 v_5 \cos(9\psi_2 - 4\psi_4 - 5\psi_5) \rangle$
32	5	$\langle v_2^2 v_2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle - \langle v_2^2 \rangle \langle v_2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
33	5	$\langle v_2 v_3 v_4 v_5 \cos(4\psi_2 - 3\psi_3 + 4\psi_4 - 5\psi_5) \rangle$

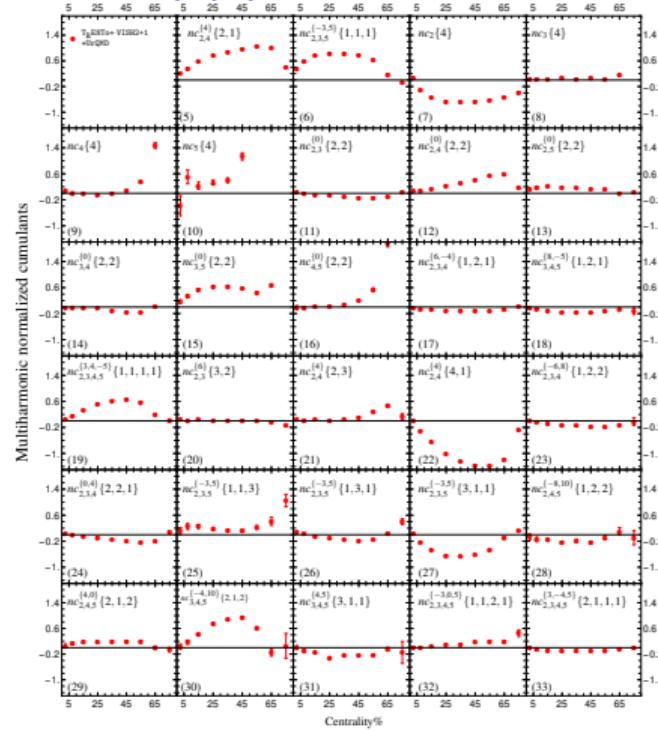
$$nc_{n_1, \dots, n_k}^{\{\alpha_1, \dots, \alpha_{k-1}\}} \{m_1, \dots, m_k\} = \frac{c_{n_1, \dots, n_k}^{\{\alpha_1, \dots, \alpha_{k-1}\}} \{m_1, \dots, m_k\}}{\sqrt{c_{n_1}^{m_1} \{2\} \cdots c_{n_k}^{m_k} \{2\}}}$$

- T_RENTo + free streaming + VISH(2+1) + UrQMD
Maximum A Posteriori (MAP) tuning.
[Bernhard et al, Nature Phys., 2019]



All cumulants involving harmonics 2,3,4,5 and orders 2,3,4,5

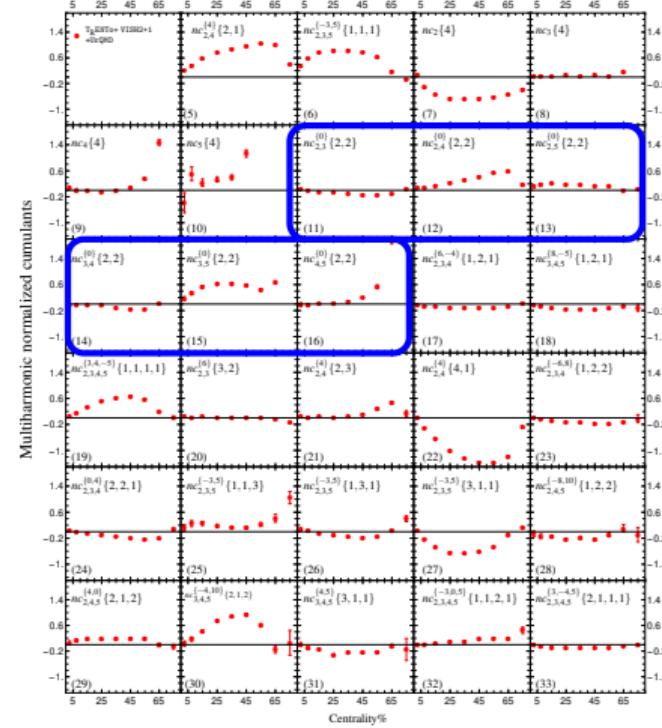
cumulant	order	cumulant expression
1	2	$\langle v_2^2 \rangle$
2	2	$\langle v_3^2 \rangle$
3	2	$\langle v_4^2 \rangle$
4	2	$\langle v_5^2 \rangle$
5	3	$\langle v_2^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle$
6	3	$\langle v_2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
7	4	$\langle v_2^4 \rangle - 2\langle v_2^2 \rangle^2$
8	4	$\langle v_3^4 \rangle - 2\langle v_3^2 \rangle^2$
9	4	$\langle v_4^4 \rangle - 2\langle v_4^2 \rangle^2$
10	4	$\langle v_5^4 \rangle - 2\langle v_5^2 \rangle^2$
11	4	$\langle v_2^2 v_3^2 \rangle - \langle v_2^2 \rangle \langle v_3^2 \rangle$
12	4	$\langle v_2^2 v_4^2 \rangle - \langle v_2^2 \rangle \langle v_4^2 \rangle$
13	4	$\langle v_2^2 v_5^2 \rangle - \langle v_2^2 \rangle \langle v_5^2 \rangle$
14	4	$\langle v_3^2 v_4^2 \rangle - \langle v_3^2 \rangle \langle v_4^2 \rangle$
15	4	$\langle v_3^2 v_5^2 \rangle - \langle v_3^2 \rangle \langle v_5^2 \rangle$
16	4	$\langle v_4^2 v_5^2 \rangle - \langle v_4^2 \rangle \langle v_5^2 \rangle$
17	4	$\langle v_5^2 v_2 v_4 \cos(2(\psi_2 - 3\psi_3 + 2\psi_4)) \rangle$
18	4	$\langle v_2^4 v_5 \cos(3\psi_2 - 8\psi_4 + 5\psi_5) \rangle$
19	4	$\langle v_2 v_3 v_4 v_5 \cos(2\psi_2 - 3\psi_3 - 4\psi_4 + 5\psi_5) \rangle$
20	5	$\langle v_2^3 v_3^2 \cos(6(\psi_2 - \psi_3)) \rangle$
21	5	$\langle v_2^2 v_4^2 \cos(4(\psi_2 - \psi_4)) \rangle - 2\langle v_2^2 \rangle \langle v_4^2 \cos(4(\psi_2 - \psi_4)) \rangle$
22	5	$\langle v_2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle - 3\langle v_2 \rangle \langle v_4 \cos(4(\psi_2 - \psi_4)) \rangle$
23	5	$\langle v_2^2 v_3^2 v_2 \cos(2(\psi_2 + 3\psi_3 - 4\psi_4)) \rangle$
24	5	$\langle v_2^3 v_3^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle - \langle v_2^3 \rangle \langle v_3^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle$
25	5	$\langle v_2^2 v_2 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle - 2\langle v_2^2 \rangle \langle v_2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
26	5	$\langle v_3^2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle - 2\langle v_3^2 \rangle \langle v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
27	5	$\langle v_2^2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle - 2\langle v_2^2 \rangle \langle v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
28	5	$\langle v_2^2 v_4 v_2 \cos(2(\psi_2 + 4\psi_4 - 5\psi_5)) \rangle$
29	5	$\langle v_2^2 v_2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle - \langle v_2^2 \rangle \langle v_4 \cos(4(\psi_2 - \psi_4)) \rangle \langle v_2^2 \rangle$
30	5	$\langle v_2^2 v_5^2 \cos(6\psi_2 + 4\psi_4 - 10\psi_5) \rangle$
31	5	$\langle v_2^4 v_4 \cos(9\psi_2 - 4\psi_4 - 5\psi_5) \rangle$
32	5	$\langle v_2^2 v_3 v_5 v_3 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle - \langle v_2^2 \rangle \langle v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
33	5	$\langle v_2^2 v_3 v_4 v_5 \cos(4\psi_2 - 3\psi_3 + 4\psi_4 - 5\psi_5) \rangle$





All cumulants involving harmonics 2,3,4,5 and orders 2,3,4,5

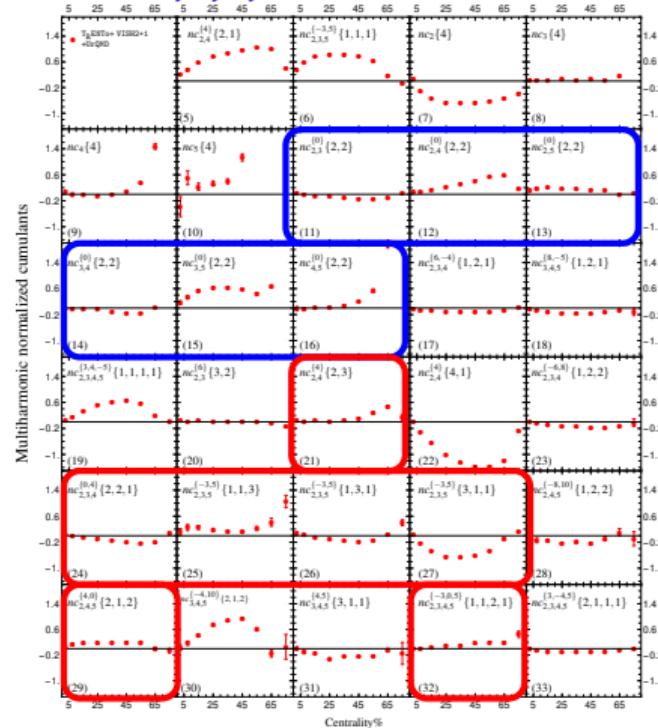
cumulant	order	cumulant expression
1	2	$\langle v_2^2 \rangle$
2	2	$\langle v_3^2 \rangle$
3	2	$\langle v_4^2 \rangle$
4	2	$\langle v_5^2 \rangle$
5	3	$\langle v_2^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle$
6	3	$\langle v_2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
7	4	$\langle v_2^4 \rangle - 2\langle v_2^2 \rangle^2$
8	4	$\langle v_3^4 \rangle - 2\langle v_3^2 \rangle^2$
9	4	$\langle v_4^4 \rangle - 2\langle v_4^2 \rangle^2$
10	4	$\langle v_5^4 \rangle - 2\langle v_5^2 \rangle^2$
11	4	$\langle v_2^2 v_3^2 \rangle - \langle v_2^2 \rangle \langle v_3^2 \rangle$
12	4	$\langle v_2^2 v_4^2 \rangle - \langle v_2^2 \rangle \langle v_4^2 \rangle$
13	4	$\langle v_2^2 v_5^2 \rangle - \langle v_2^2 \rangle \langle v_5^2 \rangle$
14	4	$\langle v_3^2 v_4^2 \rangle - \langle v_3^2 \rangle \langle v_4^2 \rangle$
15	4	$\langle v_3^2 v_5^2 \rangle - \langle v_3^2 \rangle \langle v_5^2 \rangle$
16	4	$\langle v_4^2 v_5^2 \rangle - \langle v_4^2 \rangle \langle v_5^2 \rangle$
17	4	$\langle v_5^2 v_2 v_4 \cos(2(\psi_2 - 3\psi_3 + 2\psi_4)) \rangle$
18	4	$\langle v_2^4 v_5 \sin(3\psi_2 - 8\psi_4 + 5\psi_5) \rangle$
19	4	$\langle v_2 v_3 v_4 v_5 \cos(2\psi_2 - 3\psi_3 - 4\psi_4 + 5\psi_5) \rangle$
20	5	$\langle v_2^3 v_3^2 \cos(6(\psi_2 - \psi_3)) \rangle$
21	5	$\langle v_2^2 v_4^2 \cos(4(\psi_2 - \psi_4)) \rangle - 2\langle v_2^2 \rangle \langle v_2^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle$
22	5	$\langle v_2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle - 3\langle v_2 \rangle \langle v_2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle$
23	5	$\langle v_2^2 v_3^2 v_2 \cos(2(\psi_2 + 3\psi_3 - 4\psi_4)) \rangle$
24	5	$\langle v_2^3 v_3^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle - \langle v_2^3 \rangle \langle v_2^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle$
25	5	$\langle v_2^2 v_2 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle - 2\langle v_2^2 \rangle \langle v_2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
26	5	$\langle v_2^3 v_3^2 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle - 2\langle v_2^3 \rangle \langle v_2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
27	5	$\langle v_2^2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle - 2\langle v_2^2 \rangle \langle v_2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
28	5	$\langle v_2^3 v_4^2 v_2 \cos(2(\psi_2 + 4\psi_4 - 5\psi_5)) \rangle$
29	5	$\langle v_2^2 v_3^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle - \langle v_2^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle \langle v_2^2 \rangle$
30	5	$\langle v_2^2 v_5^2 v_4 \cos(6\psi_2 + 4\psi_4 - 10\psi_5) \rangle$
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33	5	$\langle v_2^2 v_3 v_4 v_5 \cos(4\psi_2 - 3\psi_3 + 4\psi_4 - 5\psi_5) \rangle$





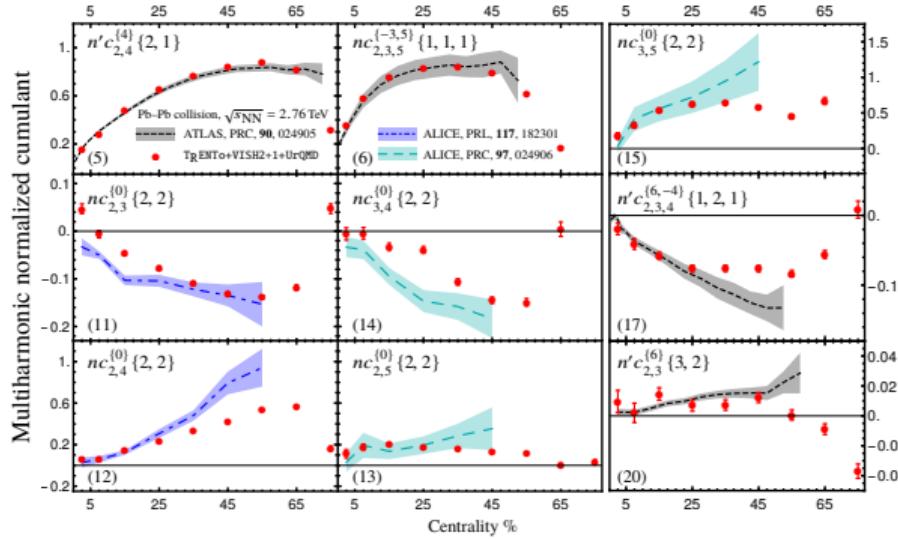
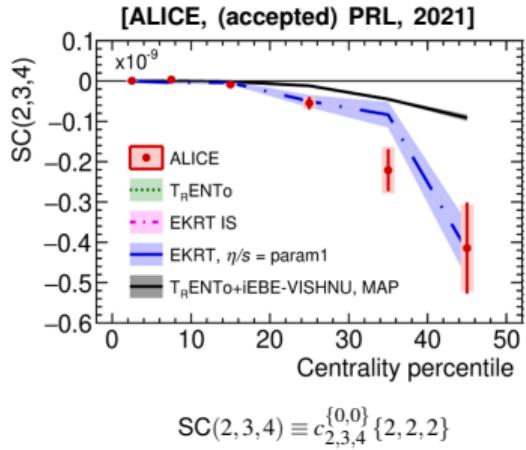
All cumulants involving harmonics 2,3,4,5 and orders 2,3,4,5

cumulant	order	cumulant expression
1	2	$\langle v_2^2 \rangle$
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6	3	$\langle v_2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
7	4	$\langle v_2^4 \rangle - 2\langle v_2^2 \rangle^2$
8	4	$\langle v_3^4 \rangle - 2\langle v_3^2 \rangle^2$
9	4	$\langle v_4^4 \rangle - 2\langle v_4^2 \rangle^2$
10	4	$\langle v_5^4 \rangle - 2\langle v_5^2 \rangle^2$
11	4	$\langle v_2^2 v_3^2 \rangle - \langle v_2^2 \rangle \langle v_3^2 \rangle$
12	4	$\langle v_2^2 v_4^2 \rangle - \langle v_2^2 \rangle \langle v_4^2 \rangle$
13	4	$\langle v_2^2 v_5^2 \rangle - \langle v_2^2 \rangle \langle v_5^2 \rangle$
14	4	$\langle v_3^2 v_4^2 \rangle - \langle v_3^2 \rangle \langle v_4^2 \rangle$
15	4	$\langle v_3^2 v_5^2 \rangle - \langle v_3^2 \rangle \langle v_5^2 \rangle$
16	4	$\langle v_4^2 v_5^2 \rangle - \langle v_4^2 \rangle \langle v_5^2 \rangle$
17	4	$\langle v_2^2 v_3 v_4 \cos(2(\psi_2 - 3\psi_3 + 2\psi_4)) \rangle$
18	4	$\langle v_2^2 v_3 v_5 \cos(3\psi_2 - 8\psi_3 + 5\psi_5) \rangle$
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23	5	$\langle v_2^2 v_3^2 v_2 \cos(2(\psi_2 + 3\psi_3 - 4\psi_4)) \rangle$
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25	5	$\langle v_2^2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle - 2\langle v_2^2 \rangle \langle v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
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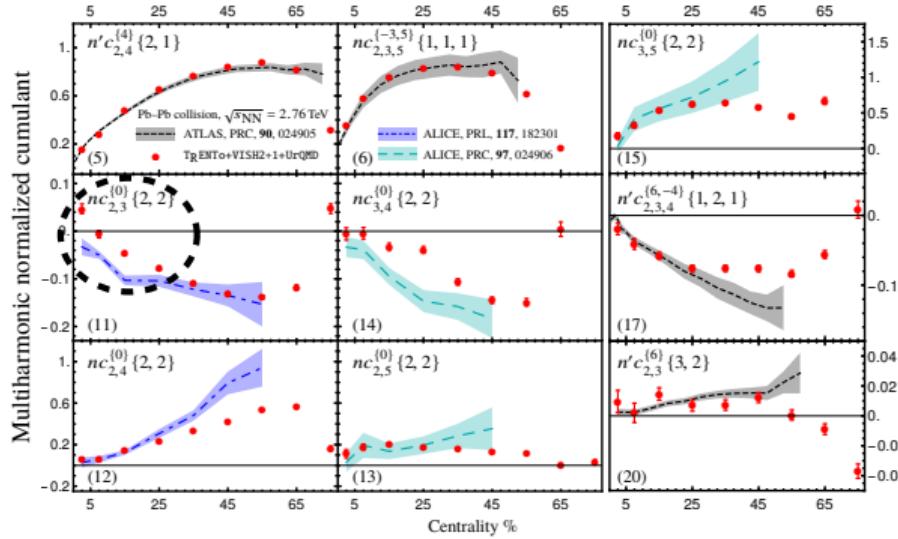
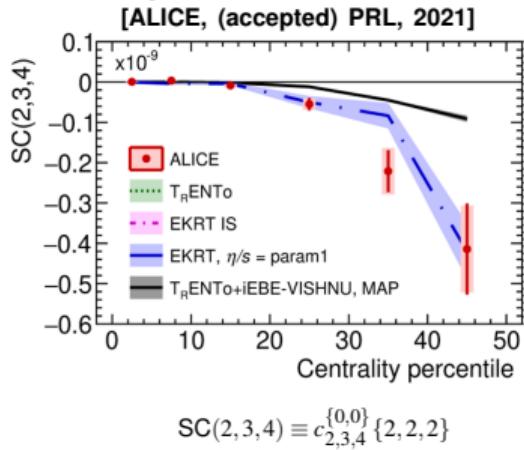


(Normalized) multiharmonic cumulants at the LHC



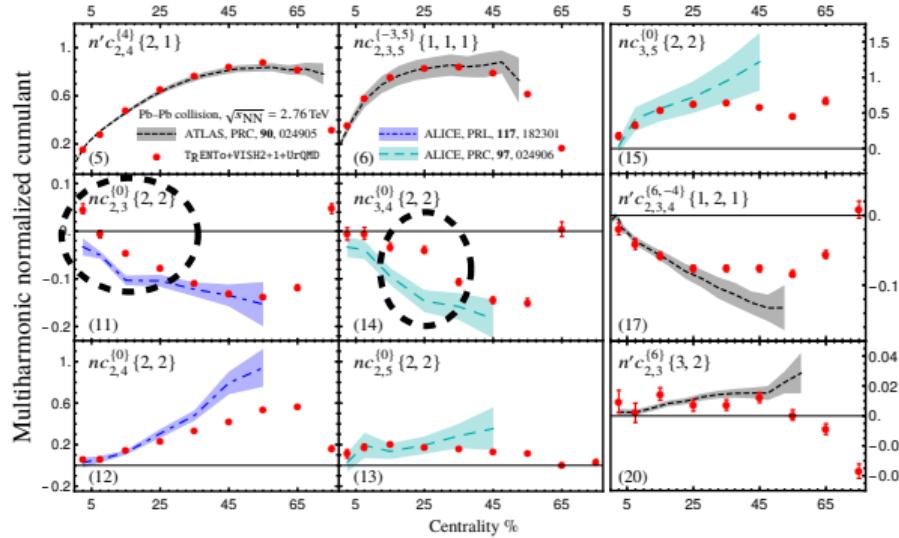
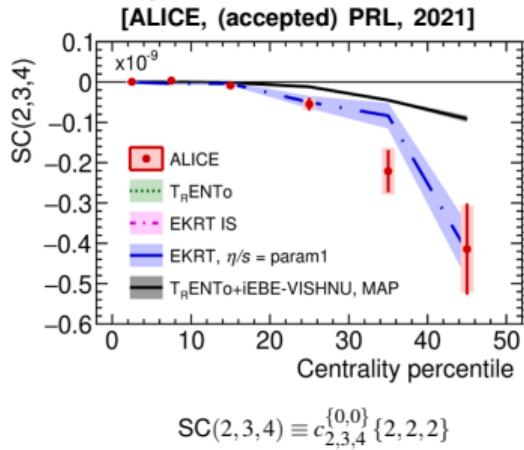


(Normalized) multiharmonic cumulants at the LHC



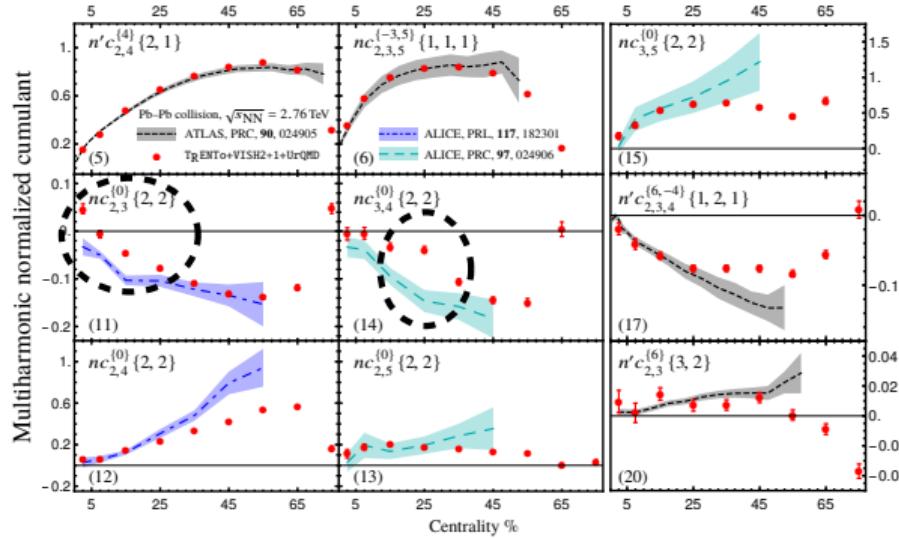
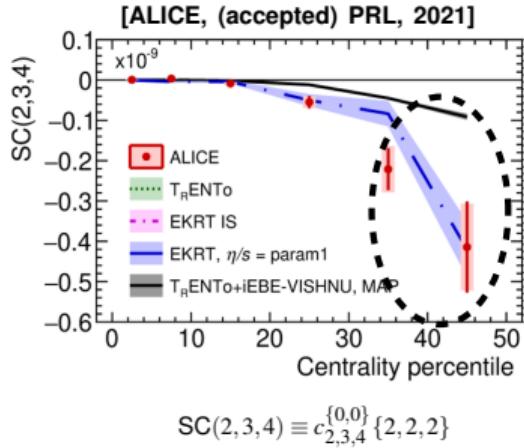


(Normalized) multiharmonic cumulants at the LHC

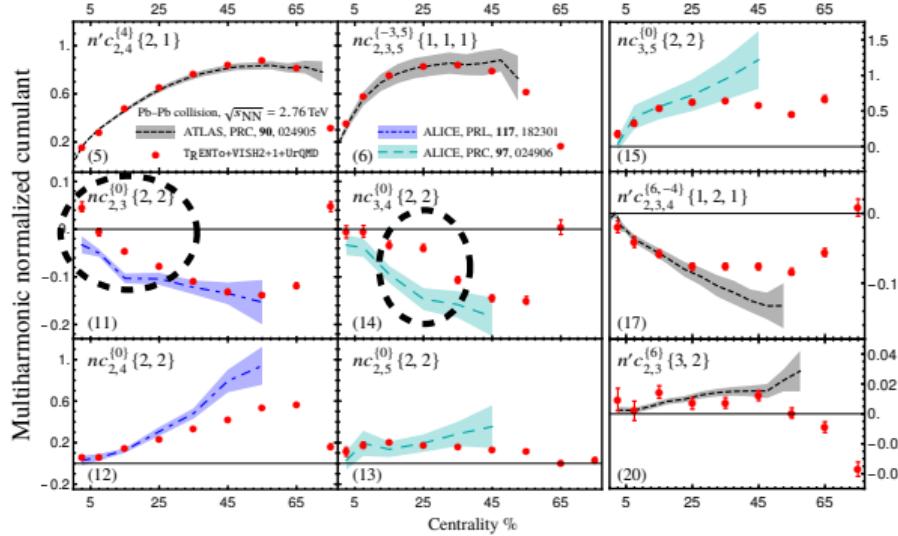
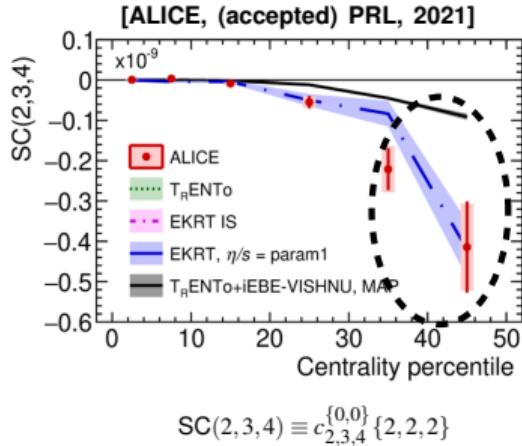




(Normalized) multiharmonic cumulants at the LHC

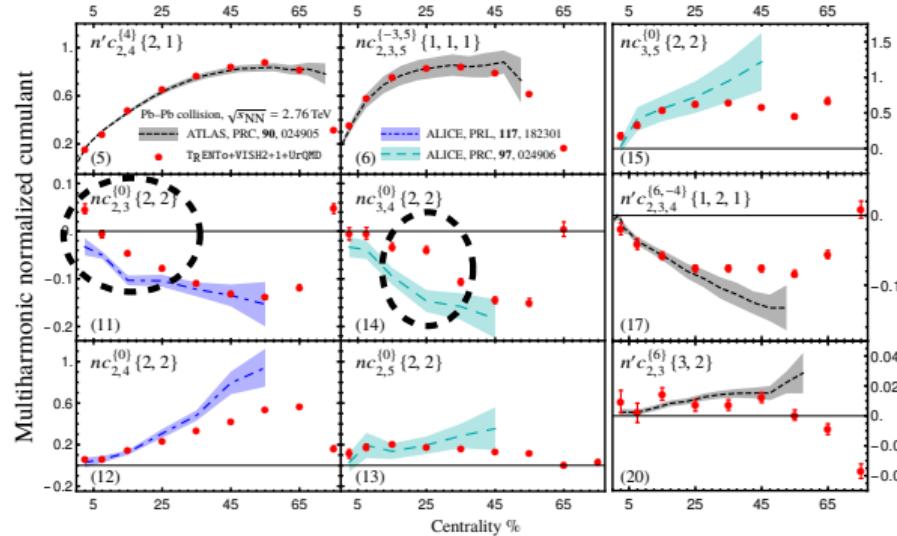
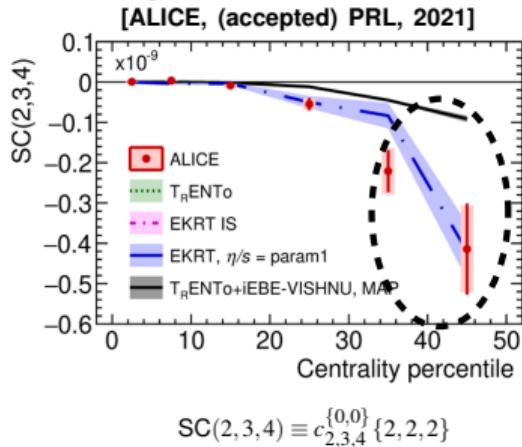


(Normalized) multiharmonic cumulants at the LHC



- ▶ The discrepancies need more investigation.

(Normalized) multiharmonic cumulants at the LHC



- The discrepancies need more investigation.

Summary and outlook: We introduced a systematic method to extract all flow harmonic cumulants and their statistical uncertainties up to a given order. What about other unmeasured normalized cumulants? Can the new cumulants help the Bayesian analysis? **Thank you and see you in room**