Ab Initio Coupling of Jets to Collective Flow in the Opacity Expansion Approach

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in collaboration with M. Sievert and I. Vitev
based on 2104.09513

Jet Tomography

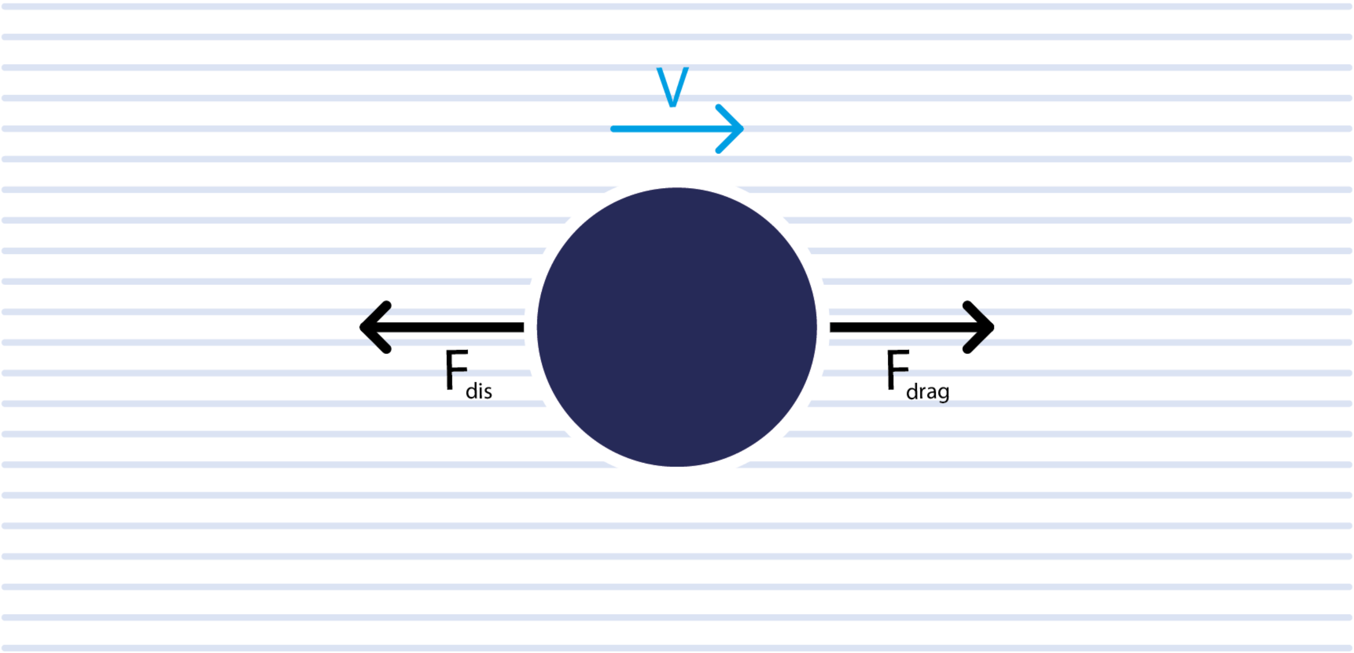
- The hot nuclear matter in HIC undergoes multi-phase evolution and its details are hard to access through the soft sector. In turn, jets see the matter at multiple scales, and essentially X-ray it;

- However, most approaches to the jet-medium interaction are either empirical or based on multiple simplifying assumptions – static matter, no fluctuations, etc;

- In what follows I will highlight our recent progress on the medium motion effects in the QCD calculations for jet broadening and gluon emission;

- The developed formalism can be also applied to include orbital motion of nucleons and some of in-medium fluctuations (e.g. spatial inhomogeneities) in the DIS context;

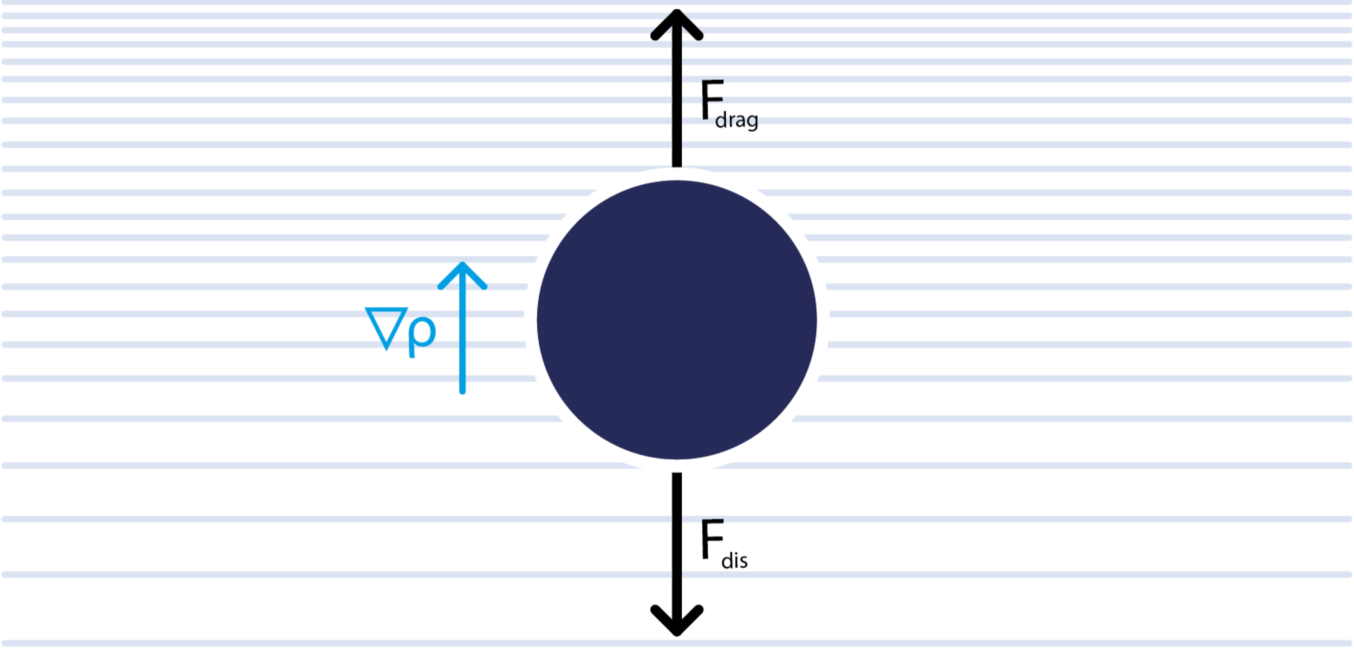
Drag Force



$$\vec{f} \sim T^2 \vec{v}$$

M. Lekaveckas, K. Rajagopal, JHEP, 2014
K. Rajagopal, AS, JHEP, 2017
J. Reiten, AS, JHEP, 2020

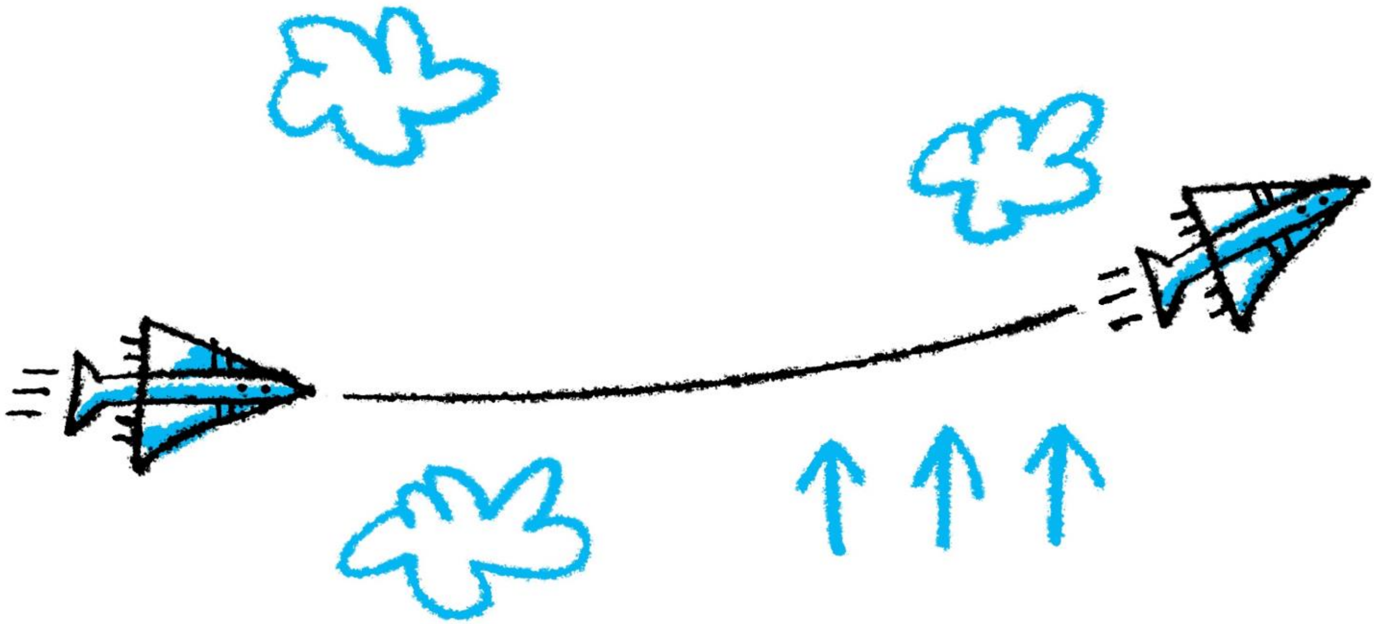
Drag Force

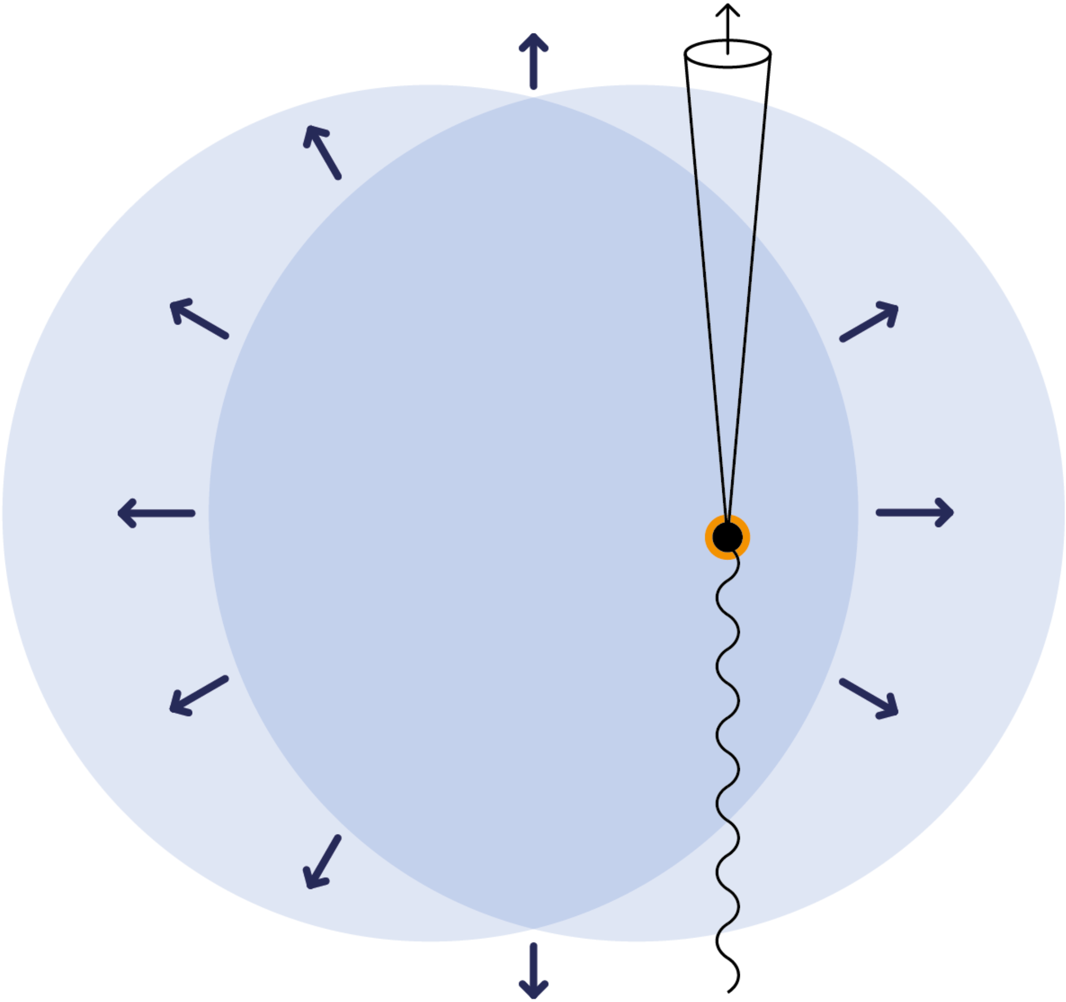


$$\vec{f} \sim \vec{\nabla} \rho$$

Jets

Does a jet feel the flow?

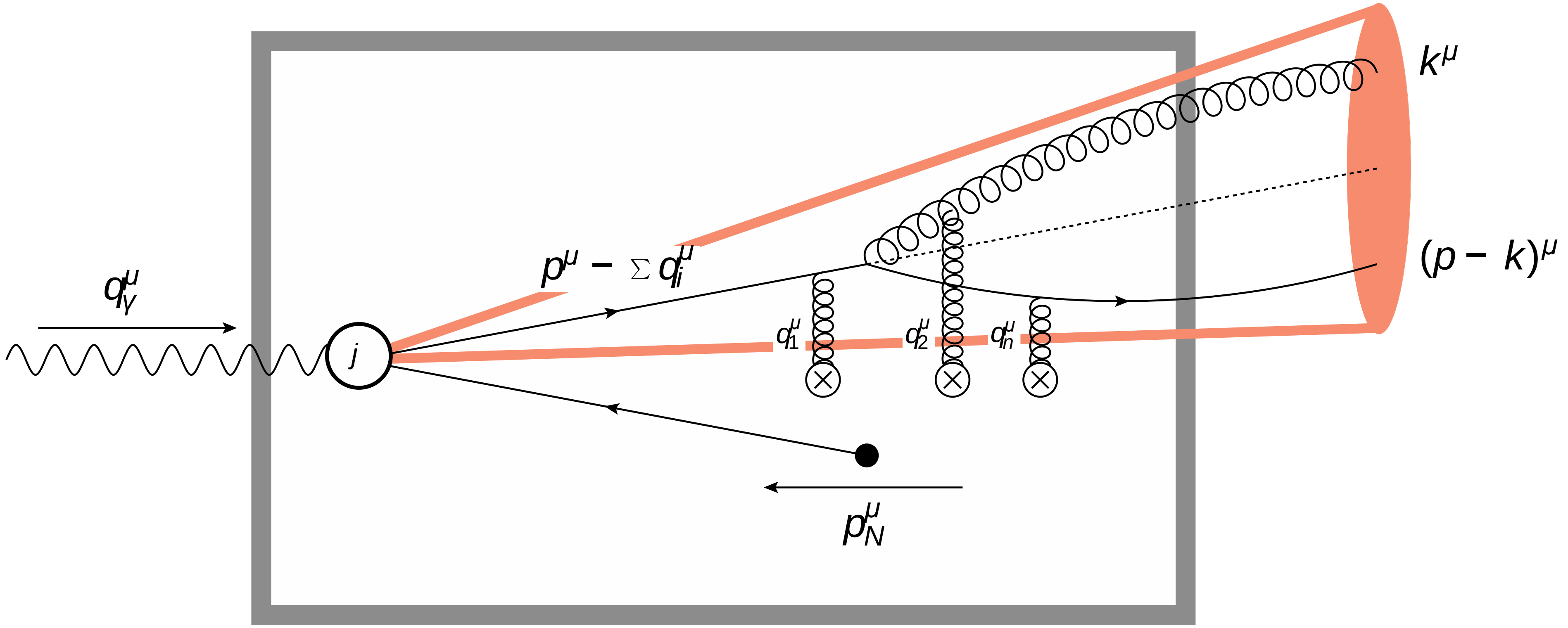




R. Baier et al, NPB, 1997
B. G. Zakharov, JETP, 1997
R. Baier et al, NPB, 1998
M. Gyulassy et al, NPB, 2000
M. Gyulassy et al, NPB, 2001
…

Jets

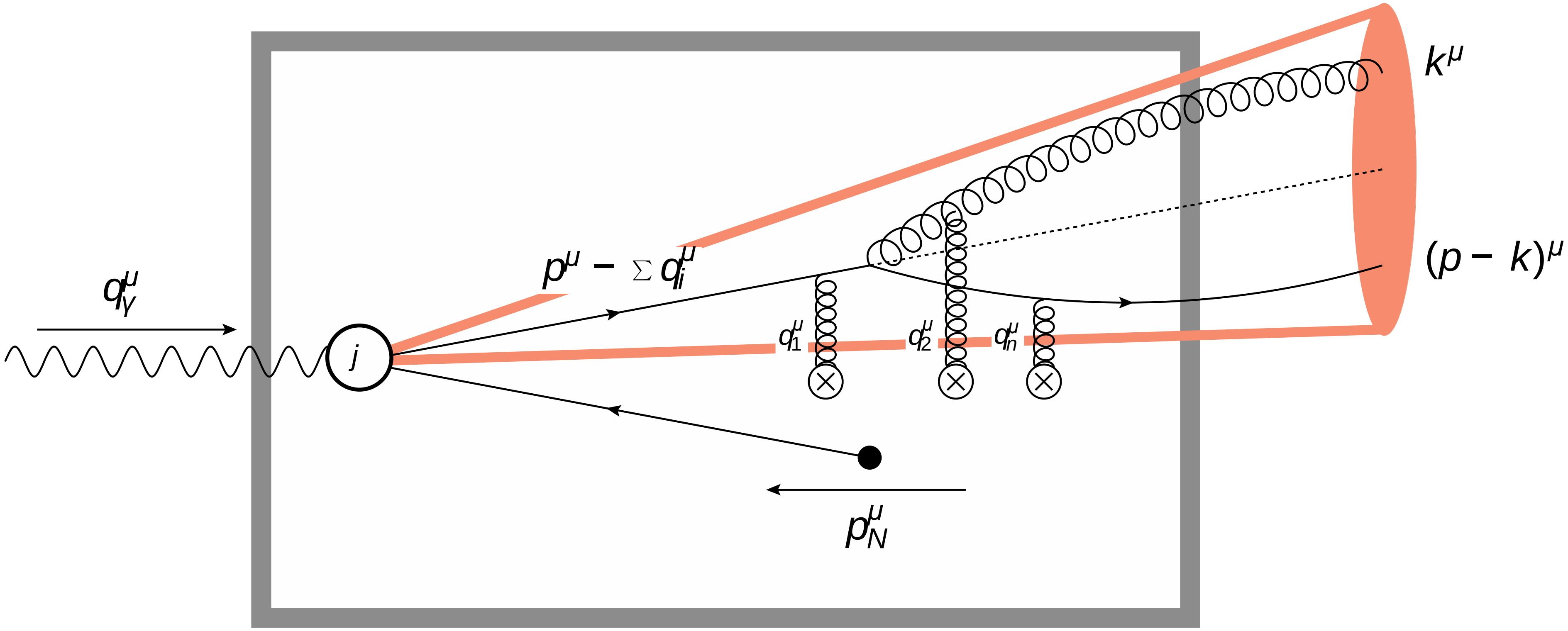
QCD broadening and gluon emission
(GLV/BDMPS-Z) with flow



R.B. Neufeld et al, PRC, 2011
R.B. Neufeld, I. Vitev, PRL, 2012
Y.T. Chien, I. Vitev, JHEP, 2016
Y.T. Chien, I. Vitev, PRL, 2017
Z.B. Kang et al, JHEP, 2017
Z.B. Kang et al, PLB, 2017
…

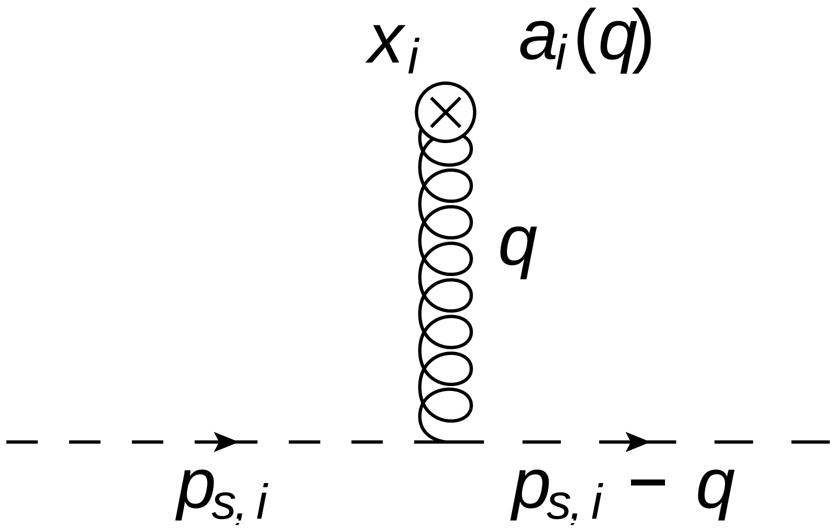
Jets

QCD broadening and gluon emission (GLV/BDMPS-Z) with flow



M. Sievert, I.Vitev, PRD, 2018

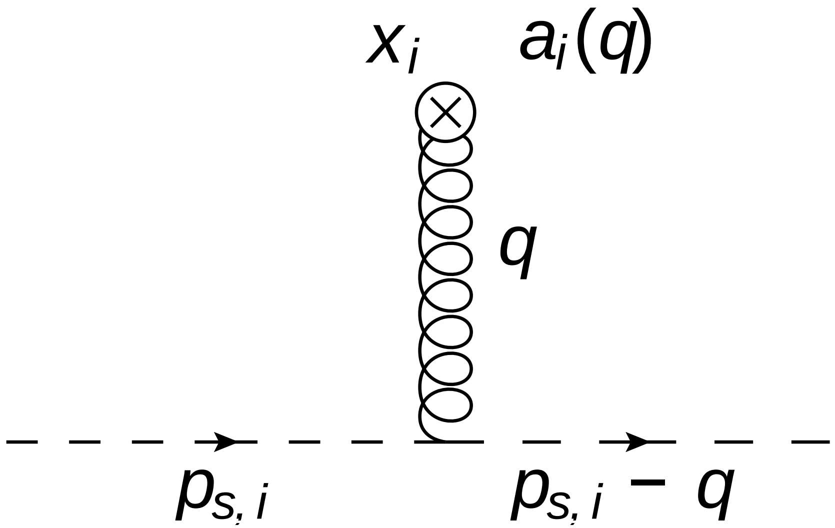
Color Potential



$$a_i^{\mu a}(q) = (ig\, t_i^a)\,(2p_{s\,i} - q)_\nu \left(\frac{-ig^{\mu\nu}}{q^2 - \mu^2 + i\epsilon} \right)\,(2\pi)\,\delta\Big((p_{s\,i} - q)^2 - M^2\Big)$$

$v(q^2)$ -- **the Gyulassy-Wang potential**

Color Potential



the four-vector of the fluid non-relativistic velocity

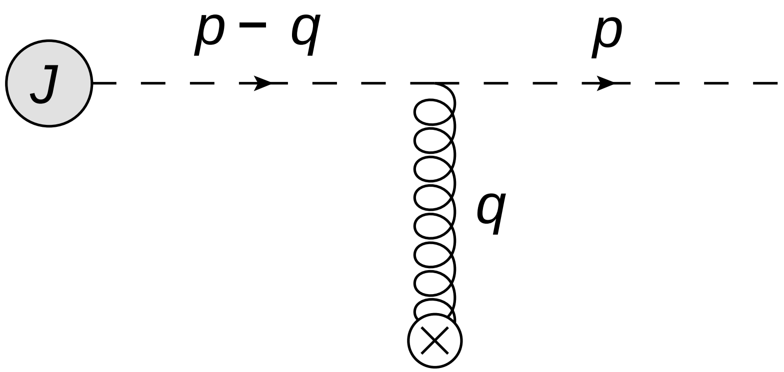
$$a_i^{\mu a}(q) = g\, t_i^a \left(\frac{u_i^\mu}{q^2 - \mu^2 + i\epsilon} \right) (2\pi)\, \delta\left(q^0 - \vec{u}_i \cdot \vec{q}\right)$$

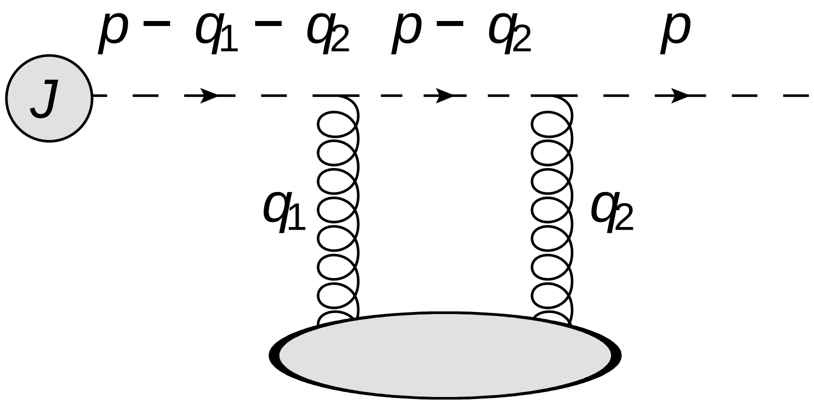
finite energy transfer!!!

$$\langle t_i^a t_j^b \rangle = \mathcal{C}\delta_{ij}\delta^{ab}$$

color neutrality

Jet Broadening





the static case:

the 1st order in opacity

$$E\frac{dN^{(1)}}{d^3p} = \int dz\, d^2q_\perp\, \rho(z)\, \frac{d\sigma}{d^2q_\perp}\left[\left(E\frac{dN^{(0)}}{d^2(p-q)_\perp\, dE}\right) - \left(E\frac{dN^{(0)}}{d^2p_\perp\, dE}\right)\right]$$

elastic scattering cross section

Jet Broadening

with flow:

$$E\frac{dN^{(1)}}{d^3p} = \int dz\, d^2q_\perp\, \rho(z)\, \frac{d\sigma}{d^2q_\perp}\Bigg[\left(E\frac{dN^{(0)}}{d^2(p-q)_\perp\, dE}\right)\left(1+\vec{u}_\perp(z)\cdot\vec{\Gamma}(\vec{q}_\perp)\right)$$
$$-\left(E\frac{dN^{(0)}}{d^2p_\perp\, dE}\right)\left(1+\vec{u}_\perp(z)\cdot\vec{\Gamma}_{DB}(\vec{q}_\perp)\right)\Bigg] + \mathcal{O}\left(\partial_\perp\right)$$

$$\vec{\Gamma}(\vec{q}_\perp) = -2\frac{\vec{p}_\perp - \vec{q}_\perp}{(1-u_{i\,z})E} + \frac{2\vec{q}_\perp}{(1-u_{i\,z})E}\left(\frac{(p-q)_\perp^2 - p_\perp^2}{v(q_\perp^2)}\right)\frac{\partial v}{\partial q_\perp^2}$$
$$-\frac{\vec{q}_\perp}{1-u_z}\left(\frac{1}{\bar{N}_0(E,\vec{p}_\perp - \vec{q}_\perp)}\frac{\partial \bar{N}_0}{\partial E}\right)$$

Jet Broadening

with flow:

$$\vec{\Gamma}(\vec{q}_\perp) = -2\frac{\vec{p}_\perp - \vec{q}_\perp}{(1-u_{i\,z})E} + \frac{2\vec{q}_\perp}{(1-u_{i\,z})E}\left(\frac{(p-q)_\perp^2 - p_\perp^2}{v(q_\perp^2)}\right)\frac{\partial v}{\partial q_\perp^2}$$
$$-\frac{\vec{q}_\perp}{1-u_z}\left(\frac{1}{\bar{N}_0(E,\vec{p}_\perp - \vec{q}_\perp)}\frac{\partial \bar{N}_0}{\partial E}\right)$$

- The finite collisional energy transfer q^0 to the jet results in a small shift in the energy of the initial jet distribution and in a shift of the transverse momentum spectrum of $\frac{d\sigma}{d^2q}$ leading to the two last terms above;

- The first term in Γ, in turn, appears due to a sub-eikonal correction to the vertex, a penalty for bending the jet, and the modification of the propagator due to the energy transfer, which can increase the scattering amplitude;

Jet Broadening

An illustration:

assuming a model source

$$E\frac{dN^{(0)}}{d^3p} \propto \delta^{(2)}(\vec{p}_\perp)$$

and ignoring z-dependence, we find

the distribution in energies

$$\langle \boldsymbol{p}_\perp (p_\perp^2)^k \rangle = -\frac{\boldsymbol{u}_\perp}{(1-u_z)}\frac{L}{2\lambda}(\mu^2)^{k+1}\left(-\frac{2}{E}\int_0^\infty d\xi \frac{\xi^{k+2}}{(1+\xi)^3} + \frac{1}{f(E)}\frac{\partial f}{\partial E}\int_0^\infty d\xi \frac{\xi^{k+1}}{(1+\xi)^2}\right)$$

$1/\rho\sigma_0$ **the mean free path**

while even moments are unmodified

A family of velocity-sensitive observables!

Jet Broadening

An illustration:

assuming a model source

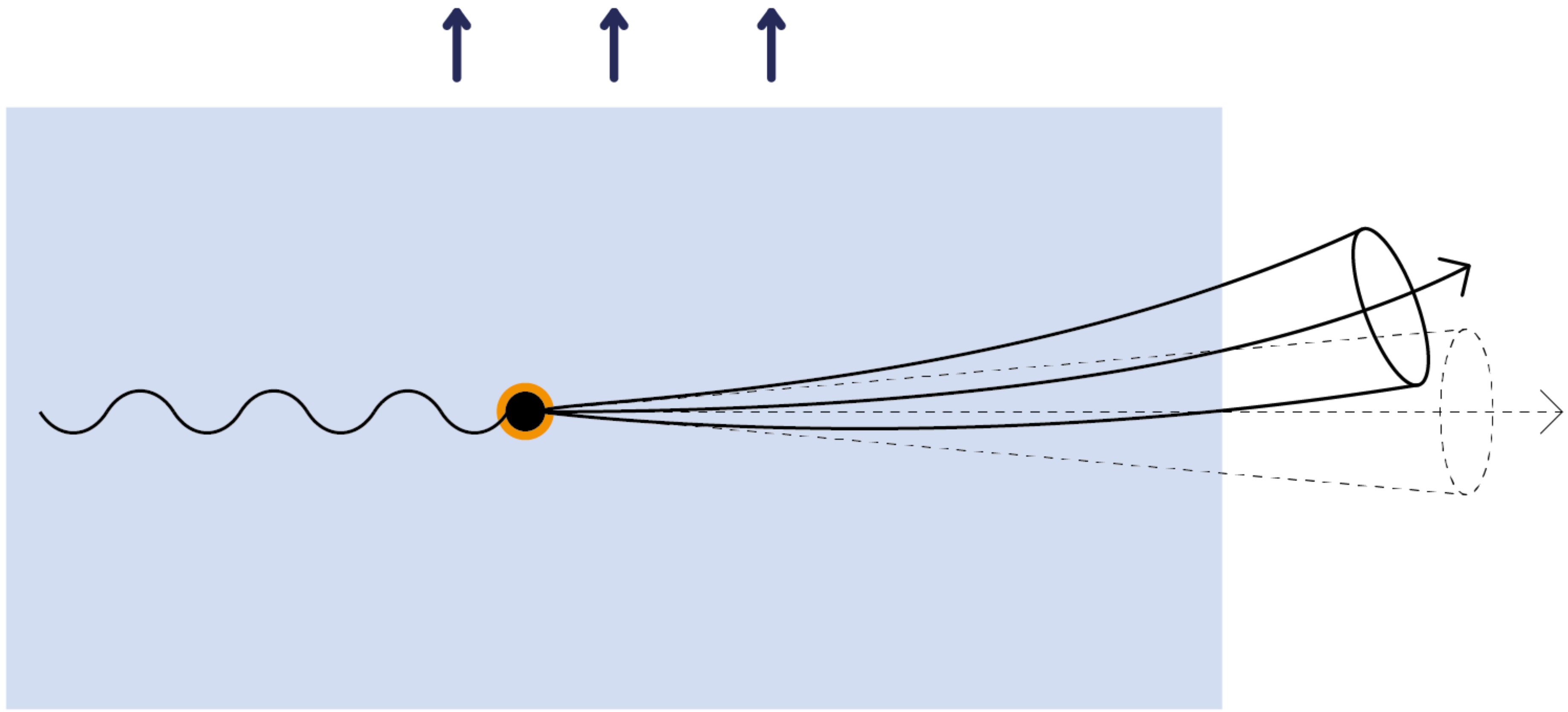
$$E\frac{dN^{(0)}}{d^3p} \propto E^{-4}\,\delta^{(2)}(\vec{p}_\perp)$$

and ignoring z-dependence, we find

$$\left\langle \frac{\vec{p}_\perp}{p_\perp^2} \right\rangle = \frac{5}{2}\frac{\vec{u}_\perp}{(1-u_z)}\frac{L}{\lambda}\,\frac{1}{E}\quad , \quad \frac{\langle \vec{p}_\perp \rangle}{\mu^2} = 3\frac{\vec{u}_\perp}{(1-u_z)}\frac{L}{\lambda}\,\frac{1}{E}\ln\frac{E}{\mu}$$

while even moments are unmodified

A family of velocity-sensitive observables!



M. Lekaveckas, K. Rajagopal, JHEP, 2014
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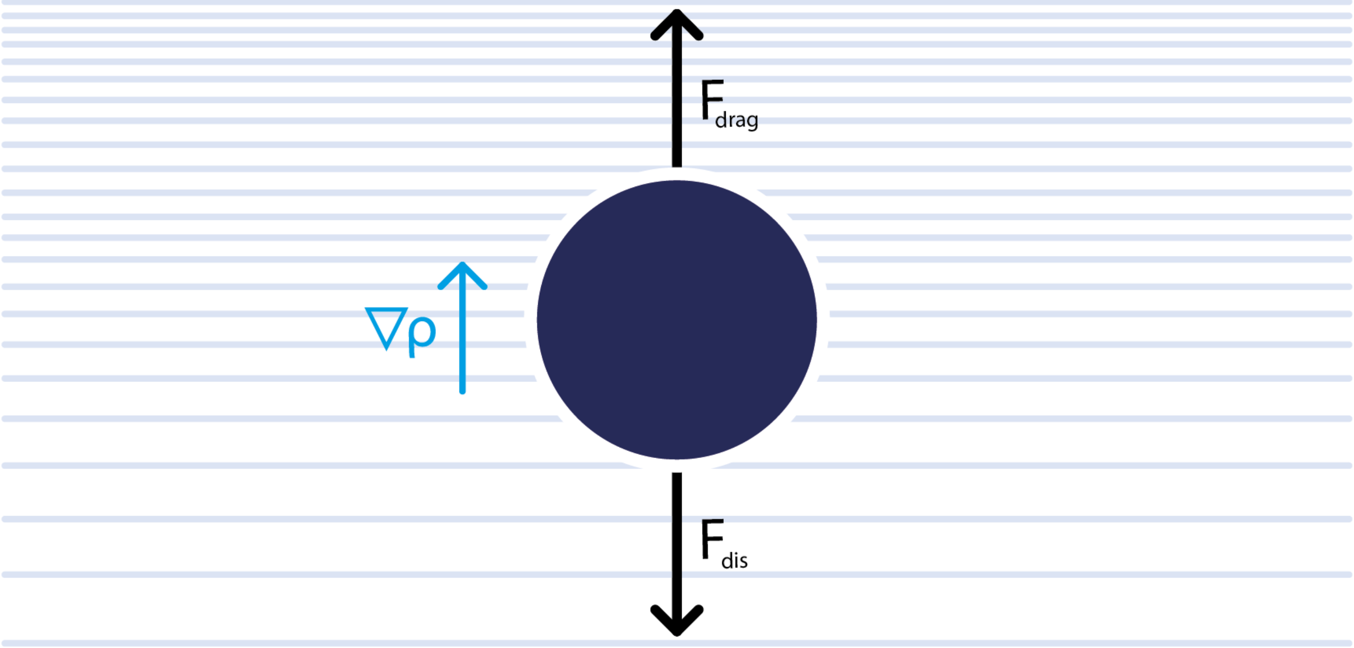


$$\vec{f} \sim \vec{\nabla}\rho$$

Jet Broadening

inhomogeneous matter

The amplitude squared should be averaged over the sources and their colors:

$$\langle t_i^a t_j^b \rangle = \mathcal{C} \delta_{ij} \delta^{ab}$$

-- color neutrality

$$\sum_i f_i = \int d^3x \, \rho(\vec{x}) \, f(\vec{x})$$

-- source average

and in static GLV calculation

$$\int d^2x_\perp \, e^{-i(\boldsymbol{q}_\perp - \boldsymbol{q}'_\perp)\cdot \boldsymbol{x}_\perp} \rightarrow (2\pi)^2 \delta^{(2)}(\boldsymbol{q}_\perp - \boldsymbol{q}'_\perp)$$

Jet Broadening

inhomogeneous matter

The amplitude squared should be averaged over the sources and their colors:

$$\langle t_i^a t_j^b \rangle = \mathcal{C}\delta_{ij}\delta^{ab}$$ -- color neutrality

$$\sum_i f_i = \int d^3x\, \rho(\vec{x})\, f(\vec{x})$$ -- source average

but if the sources are different in their properties,
then the final result is more involved.

Jet Broadening

gradient expansion

$$\rho(\vec{x}_\perp, z) \approx \rho_0(z) + \partial^j \rho(z)\, x_\perp^j \qquad \mu^2(\vec{x}_\perp, z) \approx \mu_0^2(z) + \partial^j \mu^2(z)\, x_\perp^j$$

$$\left(E\frac{dN^{(1)}}{d^3p}\right)^{(\text{linear})} = \int dz \int d^2q_\perp\, \bar{\sigma}(q_\perp^2) \left(\partial^j \rho + \rho\, \frac{1}{\bar{\sigma}(q_\perp^2)} \frac{\partial \bar{\sigma}}{\partial \mu^2}\, \partial^j \mu^2\right) \times \left\{ \left(E\frac{dN^{(0)}}{d^2(p-q)_\perp\, dE}\right) \left[\frac{(p-q)_\perp^j}{E} z\right] - \left(E\frac{dN^{(0)}}{d^2p_\perp\, dE}\right) \left[\frac{p_\perp^j}{E} z\right] \right\}$$

the leading gradient correction at the first order in opacity

Jet Broadening

gradient expansion

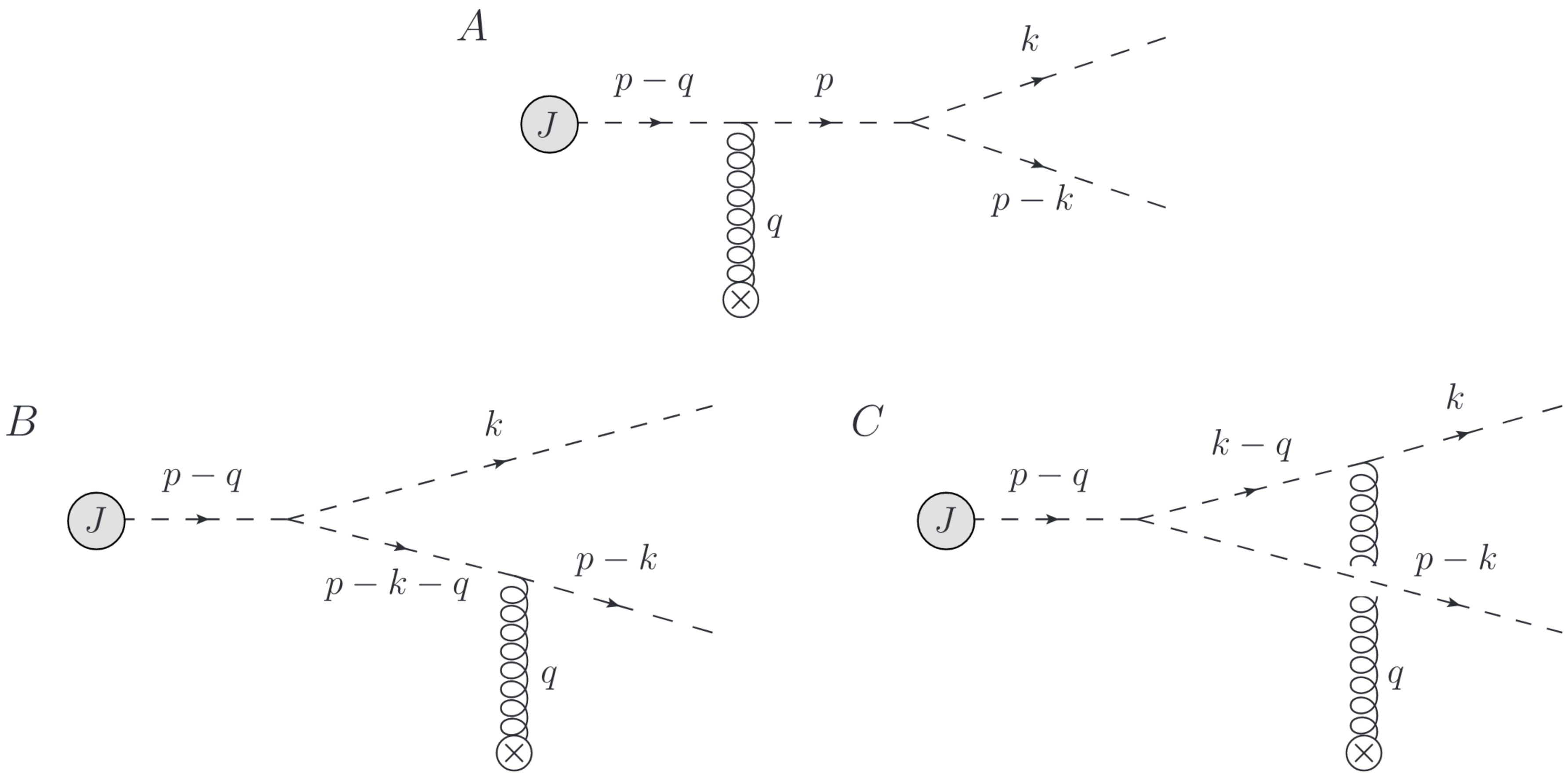
$$E\frac{dN^{(0)}}{d^3p} = \frac{1}{2(2\pi)^3}|J(p)|^2 = \frac{f(E)}{2\pi w^2}\, e^{-\frac{p_\perp^2}{2w^2}}$$

$$\downarrow$$

$$\langle \vec{p}_\perp\, p_\perp^2 \rangle^{(linear)} \simeq \frac{L}{\lambda}\frac{L}{E}\, w^2 \mu^2 \frac{\vec{\nabla}_\perp \rho}{\rho}\, \ln\frac{E}{\mu}$$

the leading gradient correction at the first order in opacity

“Gluon” Emission



$$E\frac{dN^{(1)}}{d^2k_\perp\, dx\, d^2p_\perp\, dE} = \frac{1}{2(2\pi)^3 x(1-x)}\int_0^L dz\,\rho \int d^2q_\perp \frac{d\sigma}{d^2q_\perp}\Bigg\{\left(E\frac{dN^{(0)}}{d^2(p-q)_\perp\, dE}\right)$$

$$\times\Bigg[\frac{\mathcal{C}_{(A,A)}}{\mathcal{C}}|\psi_A|^2\left(1+2\vec{u}_\perp\cdot\vec{\Omega}_A\right)+\frac{2\mathcal{C}_{(B,B)}}{\mathcal{C}}|\psi_B|^2\left(1+\vec{u}_\perp\cdot(\vec{\Omega}_{IB}+\vec{\Omega}_{IIB})\right)\left(1-\cos\left((q^-_{p-k-q}-q^-_{p-q})z\right)\right)$$

$$+\frac{2\mathcal{C}_{(C,C)}}{\mathcal{C}}|\psi_C|^2\left(1+\vec{u}_\perp\cdot(\vec{\Omega}_{IC}+\vec{\Omega}_{IIC})\right)\left(1-\cos\left((q^-_{k-q}-q^-_{p-q})z\right)\right)$$

$$+\frac{2\mathcal{C}_{(A,B)}}{\mathcal{C}}(\psi_A\psi_B^*)\left[\left(1+\vec{u}_\perp\cdot(\vec{\Omega}_A+\vec{\Omega}_{IB})\right)\cos\left((q^-_{p-k-q}-q^-_{p-q})z\right)-\left(1+\vec{u}_\perp\cdot(\vec{\Omega}_A+\vec{\Omega}_{IIB})\right)\right]$$

$$+\frac{2\mathcal{C}_{(A,C)}}{\mathcal{C}}(\psi_A\psi_C^*)\left[\left(1+\vec{u}_\perp\cdot(\vec{\Omega}_A+\vec{\Omega}_{IC})\right)\cos\left((q^-_{k-q}-q^-_{p-q})z\right)-\left(1+\vec{u}_\perp\cdot(\vec{\Omega}_A+\vec{\Omega}_{IIC})\right)\right]$$

$$+\frac{2\mathcal{C}_{(B,C)}}{\mathcal{C}}(\psi_B\psi_C^*)\Bigg[\left(1+\vec{u}_\perp\cdot(\vec{\Omega}_{IB}+\vec{\Omega}_{IC})\right)\cos\left((q^-_{p-k-q}-q^-_{k-q})z\right)+\left(1+\vec{u}_\perp\cdot(\vec{\Omega}_{IIB}+\vec{\Omega}_{IIC})\right)$$

$$-\left(1+\vec{u}_\perp\cdot(\vec{\Omega}_{IB}+\vec{\Omega}_{IIC})\right)\cos\left((q^-_{p-k-q}-q^-_{p-q})z\right)-\left(1+\vec{u}_\perp\cdot(\vec{\Omega}_{IIB}+\vec{\Omega}_{IC})\right)\cos\left((q^-_{k-q}-q^-_{p-q})z\right)\Bigg]\Bigg]$$

$$+\left(E\frac{dN^{(0)}}{d^2p_\perp\, dE}\right)\Bigg[-\frac{\mathcal{C}_{(D,0)}}{\mathcal{C}}|\psi_A|^2\cos\left(q^-_p z\right)\left(1+\vec{u}_\perp\cdot\vec{\Gamma}_{DB}\right)$$

$$-\frac{\mathcal{C}_{(E,0)}}{\mathcal{C}}|\psi_A|^2\left(1-\cos\left(q^-_p z\right)\right)\left(1+\vec{u}_\perp\cdot\vec{\Gamma}^{(p-k)}_{DB}\right)-\frac{\mathcal{C}_{(F,0)}}{\mathcal{C}}|\psi_A|^2\left(1-\cos\left(q^-_p z\right)\right)\left(1+\vec{u}_\perp\cdot\vec{\Gamma}^{(k)}_{DB}\right)$$

$$+\frac{2\mathcal{C}_{(G,0)}}{\mathcal{C}}(\psi_G\psi_A^*)\left(1+\vec{u}_\perp\cdot\vec{\Gamma}_G\right)\left(\cos\left(q^-_p z\right)-\cos\left((q^-_{k+q}+q^-_{p-k-q})z\right)\right)\Bigg]\Bigg\}$$

$$q^-_p \equiv \frac{(k-xp)^2_\perp}{2x(1-x)E}$$

$$q^-_{p-q} \equiv \frac{(k-xp)^2_\perp}{2x(1-x)E} - \frac{(p-q)^2_\perp - p^2_\perp}{2E}$$

$$q^-_{p-k-q} \equiv -\frac{(p-k-q)^2_\perp - (p-k)^2_\perp}{2(1-x)E}$$

$$q^-_{k-q} \equiv -\frac{(k-q)^2_\perp - k^2_\perp}{2xE}$$

$$\psi_A \equiv \psi(x, \vec{k}_\perp - x\vec{p}_\perp)$$

$$\psi_B \equiv \psi(x, \vec{k}_\perp - x\vec{p}_\perp + x\vec{q}_\perp)$$

$$\psi_C \equiv \psi(x, \vec{k}_\perp - x\vec{p}_\perp - (1-x)\vec{q}_\perp)$$

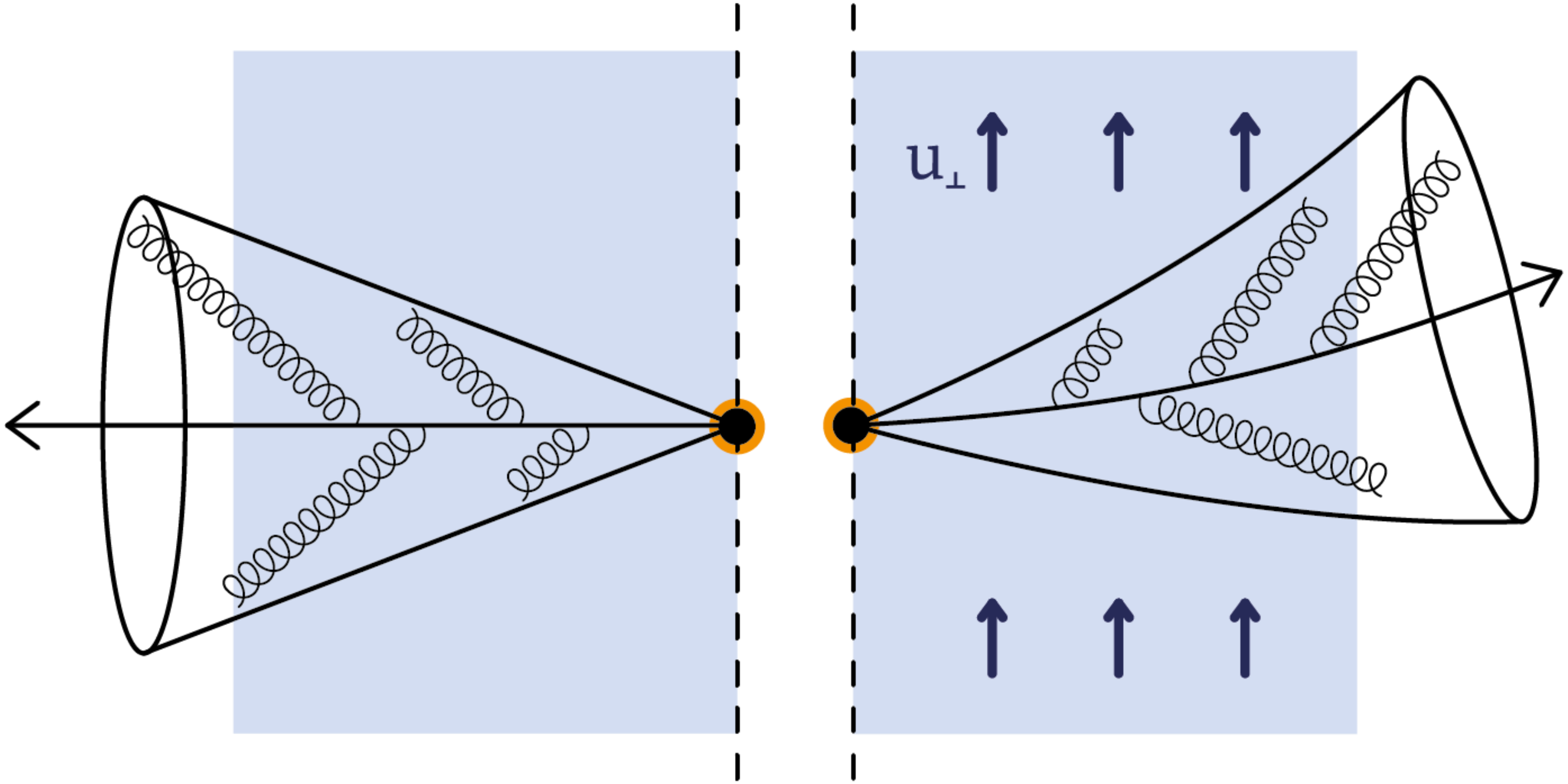
$$\psi_G \equiv \psi(x, \vec{k}_\perp - x\vec{p}_\perp + \vec{q}_\perp)$$

In the small-x limit, and for broad source, the gluon emission spectrum reduces to

$$E\frac{dN^{(1)}}{d^2k_\perp\, dx\, d^2p_\perp\, dE} = \frac{\alpha_s N_c}{\pi^2 x}\left(E\frac{dN^{(0)}}{d^2p_\perp\, dE}\right)\int_0^L dz\,\rho\int d^2q_\perp\bar{\sigma}(q_\perp^2)$$
$$\times\left\{\frac{2\vec{k}_\perp\cdot\vec{q}_\perp}{k_\perp^2(k-q)_\perp^2}\left(1-\cos\left(\frac{(k-q)_\perp^2}{2xE(1-u_z)}z\right)\right)+\frac{q_\perp^2}{k_\perp^2(q_\perp^2+\mu^2)}\frac{\vec{u}_\perp\cdot\vec{k}_\perp}{2(1-u_z)xE}\right\}$$

where the QCD LFWFs were substituted

$$\left\langle\frac{\vec{k}_\perp}{k_\perp^2}\right\rangle = \frac{N_c}{C_F}\frac{L}{\lambda}\frac{\vec{u}_\perp}{8(1-u_z)xE}$$



$$\left\langle \frac{\vec{k}_\perp}{k_\perp^2} \right\rangle = \frac{N_c}{C_F}\frac{L}{\lambda}\frac{\vec{u}_\perp}{8(1-u_z)xE}$$

Summary

- We have constructed a generalization of the GLV approach which includes the medium motion effects. With this tool one can study general flow, temperature, and source density profiles in the context of HIC;
- It is shown that the odd moments of the jet momentum are modified by the medium motion, and the jet is bended by the flow and gradients;
- We have derived the “gluon” emission spectrum in the case of uniformly flowing matter. The resulting answer, while obtained for scalar interactions, can be compared to the QCD results when written in the terms of LFWFs;
- In the context of DIS our formalism can be used to study nucleon orbital motion and spatial inhomogeneities in the system (a relation to GPDs and TMDs?);
- These results open multiple opportunities to include the medium motion and in-medium fluctuation effects into studies of other hard probes of nuclear matter;