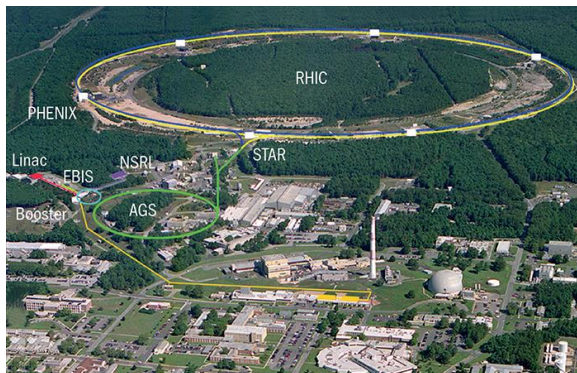


# First indication on self-similarity of strangeness production in **Au+Au** collisions at **RHIC** and critical phenomena in nuclear matter

**M. Tokarev\*** & **I. Zborovsky\*\***

\*JINR, Dubna, Russia

\*\*NPI, Řež, Czech Republic



# Contents

---

- Introduction
- $z$ -Scaling (ideas, definitions,...)
- Properties of data  $z$ -presentation
- Self-similarity of strange particle production in  $p+p$  collisions at RHIC
- Self-similarity of  $K_S^0$  meson production in Au+Au collisions at RHIC
- Momentum fractions, recoil mass and constituent energy loss vs.  $\sqrt{s_{NN}}$ , centrality,  $p_T$
- Summary



# Motivation & Goals

## Search for new symmetries in Nature

Systematic analysis of inclusive cross sections of particle production in  $p+p$ ,  $p+A$  and  $A+A$  collisions to search for general features of hadron and nucleus structure, constituent interaction and fragmentation process over a wide scale range

## $z$ -Scaling is a tool in high energy physics

Development of  $z$ -scaling approach for description of processes with strange particle production in inclusive reactions and verification of self-similarity principle

Analysis of STAR data on  $K_S^0$  meson spectra in Au+Au collisions

## The suggested approach can be used to study

- Origin of strangeness
- Symmetry of constituent interactions at small scales
- Similarity and difference of  $u, d, s, c, b, t$  quark fragmentation
- Strangeness as probe to search for new physics
- New phenomena in  $A+A$  in comparison with  $p+p$



# Fundamental principles and symmetries

---



"Fundamental symmetry principles dictate the basic laws of physics, control the structure of matter, and define the fundamental forces in Nature."

Leon M. Lederman

Self-similarity is a property of physical phenomena  
and the principle to construct theories.

Flavor is one of mystery property of quarks.

Special topic:

Self-similarity of strangeness production in  $p+p$  and  $A+A$

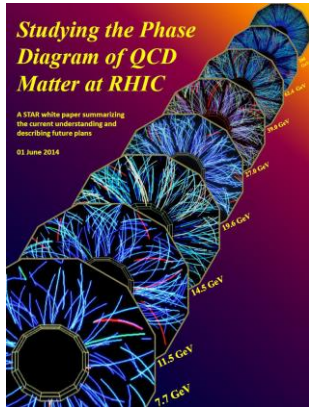


## Phase transition and critical phenomena in usual matter (gas, liquid, solid)

“Scaling” and “Universality” are concepts developed to understanding critical phenomena. Scaling means that systems near the critical points exhibiting self-similar properties are invariant under transformation of a scale. According to universality, quite different systems behave in a remarkably similar fashion near the respective critical points. Critical exponents are defined only by symmetry of interactions and dimension of the space.

H.Stanley, G.Barenblatt,...

## Phase transition and critical phenomena in nuclear matter



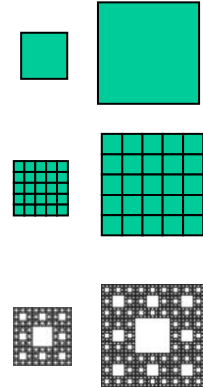
- The idea is to vary the collision energy and look for the signatures of QCD **phase boundary** and QCD **critical point** i.e. to span the phase diagram from the top RHIC energy (lower  $\mu_B$ ) to the lowest possible energy (higher  $\mu_B$ ).
- To look for the phase boundary, we would study the established signatures of QGP at 200 GeV as a function of beam energy. Turn-off of these signatures at particular energy would suggest the crossing of phase boundary.
- Similarly, near critical point, there would be **enhanced fluctuations** in multiplicity distributions of conserved quantities (net-charge, net-baryon).

STAR collaboration



# Self-similarity

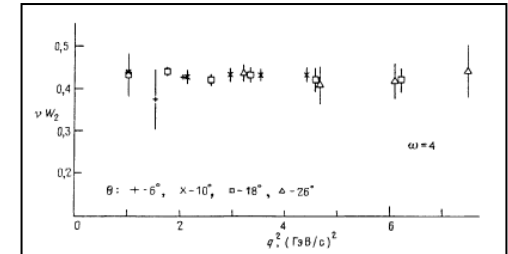
- A self-similar object is exactly or approximately **similar** to a part of itself (i.e. the whole has the same shape as one or more of the parts).
- **Self-similarity** is a typical property of **fractals**.
- **Scale invariance** is an exact form of self-similarity where at any magnification there is a smaller piece of the object that **is similar** to the whole.



## Dimensionless dynamical function vs. self-similarity parameter

- Drag force vs. Reynolds number  $Re = \rho VD/\eta$  hydrodynamics
- Drag force vs. Mach number  $Ma = v/c$  aerodynamics
- Structure function  $F(x)$  vs. Bjorken variable  $x = -q^2/2(pq)$  deep-inelastic scattering

.....



laminar & turbulent flow

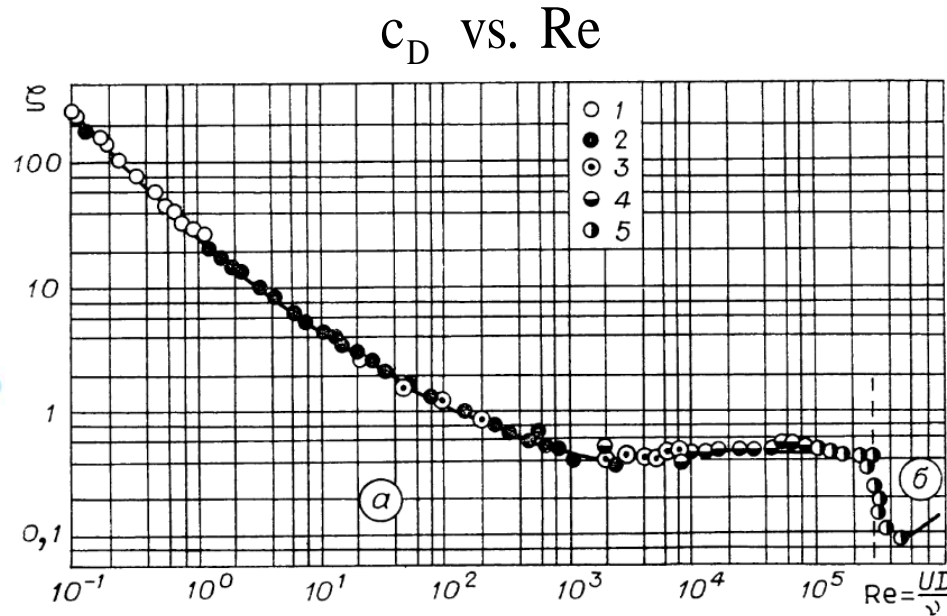
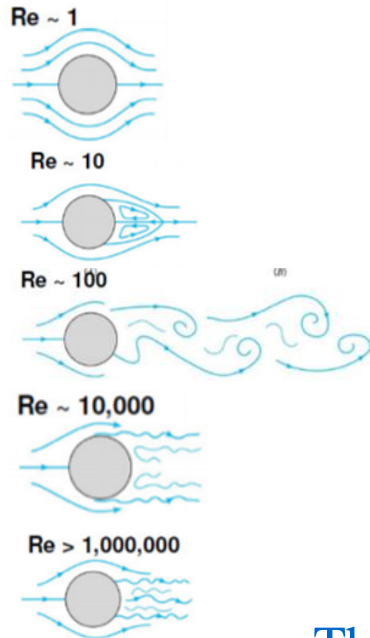
subsonic & supersonic wave

low x & high x





# Drag coefficient $c_D$ for a circular sphere in flow



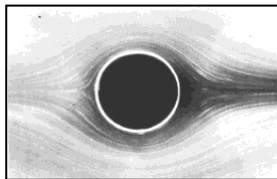
S.S.Kutateladze  
(1986)

$$C_D = \frac{F_D}{\rho v^2 d^2}$$

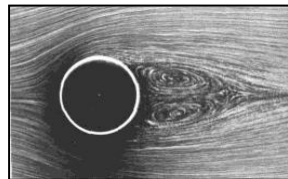
$$Re = \frac{\rho v d}{\eta}$$

The uniform flow passes over the circular cylinder

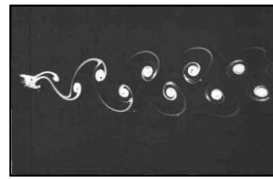
M. Van Dyke  
(1982)



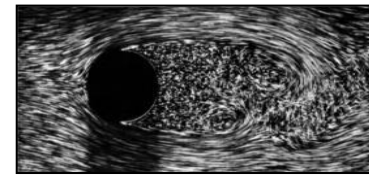
$Re = 0,16$



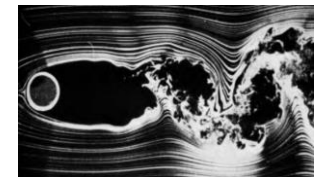
$Re = 26$



$Re = 105$



$Re = 2000$

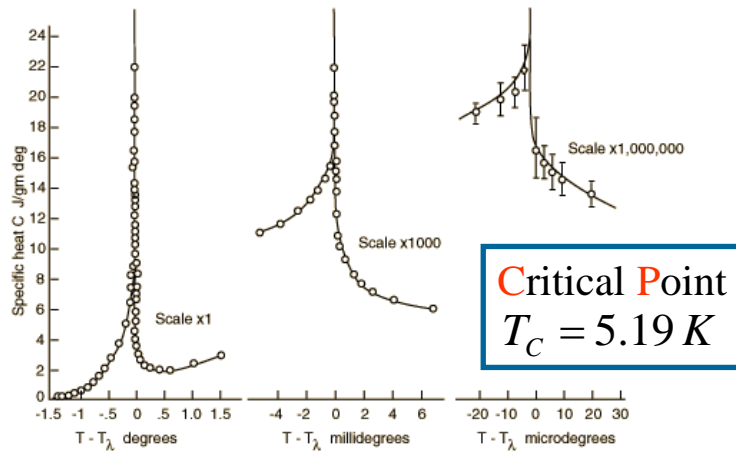


$Re = 10000$

- Self-similarity of both, in laminar and turbulent flow
- Smooth behavior of transition from the laminar to turbulent flow

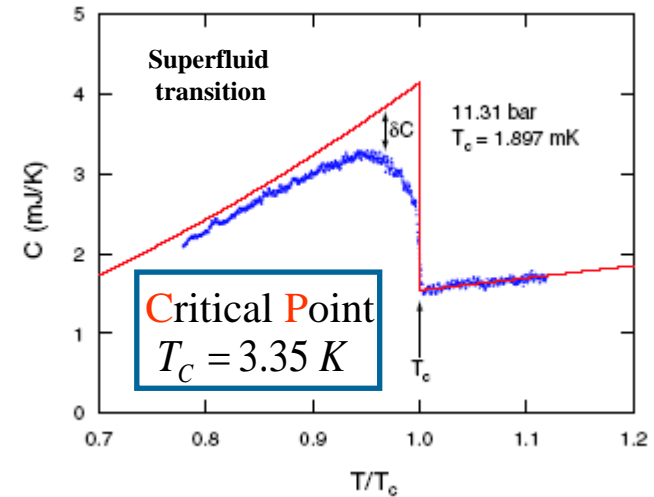
# Discontinuity of specific heat near a Critical Point

## Specific heat of liquid $^4\text{He}$



M. J. Buckingham and W. M. Fairbank, 1961  
H.E. Stanley, 1971

## Heat capacity of liquid $^3\text{He}$



H. Choi et al., PRL 96, 125301 (2006)

- Near a critical point the singular part of thermodynamic potentials is a Generalized Homogeneous Function (GHF).
- The Gibbs potential  $G(\lambda^{a_\varepsilon} \varepsilon, \lambda^{a_p} p) = \lambda G(\varepsilon, p)$  is GHF of  $(\varepsilon, p)$ .

$$c_p \sim |\varepsilon|^{-\alpha} \quad \varepsilon \equiv (T - T_c)/T_c \quad c_p = -T(d^2G / dT^2)$$

Critical exponents define the behavior  
of thermodynamic quantities nearby the Critical Point.



# Phase transitions & Critical phenomena

- Critical phenomena are unusual phenomena that reveal characteristic behavior of substances in the vicinity of phase transition points.
- They are observed due to an increase in the characteristic sizes of different fluctuations.
- In these phenomena, the self-similarity of the system arises spontaneously. This is a scale property that is characteristic for fractal structures.
- Second order transition is accompanied by a spontaneous symmetry breaking.

## Signatures of these phenomena:

- increase in compressibility (liquid-vapor equilibrium)
- increase in magnetic and dielectric susceptibility in the vicinity of the Curie points of ferromagnets and ferroelectrics
- anomaly in heat capacity at the point of transition of helium to the superfluid state
- slowing of the mutual diffusion of substances near the critical points of mixtures of stratifying liquids
- anomaly in the propagation of ultrasound (absorption of sound and an increase in its dispersion)
- anomalies in viscosity, thermal conductivity, a slowdown in the establishment of thermal equilibrium, etc.

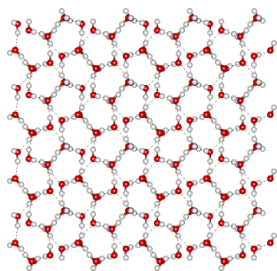
These anomalies are described by power laws with critical indices.  
Strong fluctuations with an infinite correlation radius appear in systems.



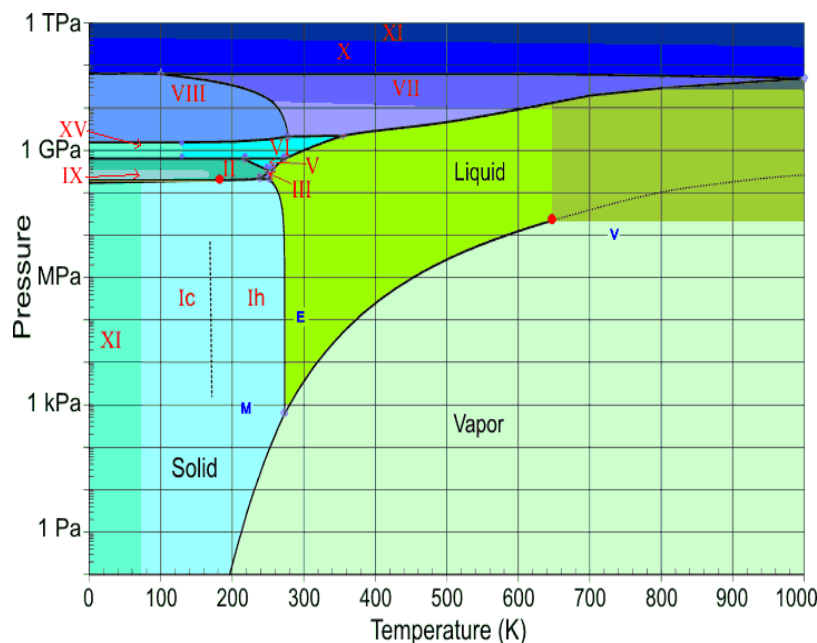
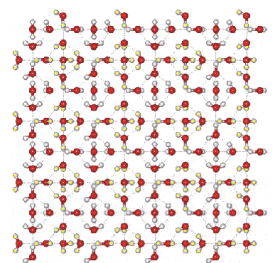
# The phase diagram of water $\text{H}_2\text{O}$

- Self-similarity as a symmetry principle is confirmed
- The law of corresponding states, equation of state are found
- Phase diagram – boundaries, triple and critical points,... is established
- Properties of phases are investigated

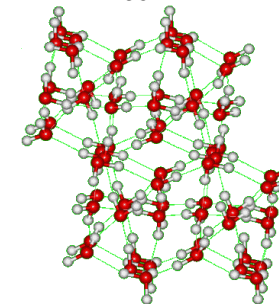
Ice III



Ice VI



Ice XIII



- Phases (ice I-XVIII, liquid, vapor)
- Phase boundaries
- Phase transitions
- Triple Point (17)
- Critical Point (1)

What one can say about phase diagram of nuclear matter ?

---

Self-similarity in inclusive reactions  
of hadron production in  $p+p$  and  $A+A$  collisions  
at high energies



# Self-similarity & $z$ -scaling

Inclusive cross sections of  $\pi^-, K^-, \bar{p}, \Lambda$  in pp collisions

FNAL:

PRD 75 (1979) 764

ISR:

NPB 100 (1975) 237

PLB 64 (1976) 111

NPB 116 (1976) 77

(low  $p_T$ )

NPB 56 (1973) 333

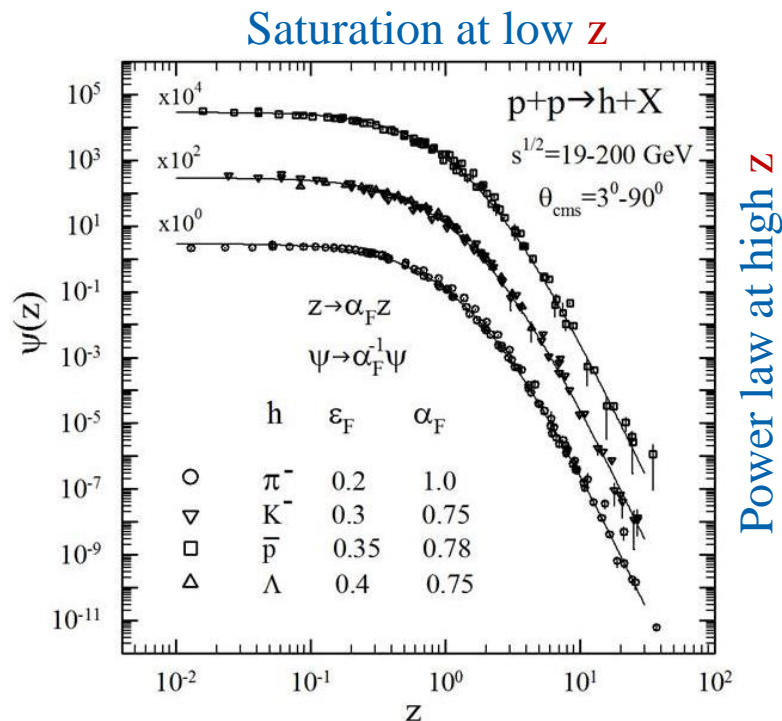
(small angles)

STAR:

PLB 616 (2005) 8

PLB 637 (2006) 161

PRC 75 (2007) 064901



Energy scan of spectra at U70, ISR, S $\bar{p}$ pS, SPS, HERA, FNAL(fixed target), Tevatron, RHIC, LHC

MT & I.Zborovsky  
T.Dedovich

Phys.Rev.D75,094008(2007)

Int.J.Mod.Phys.A24,1417(2009)

J. Phys.G: Nucl.Part.Phys.

37,085008(2010)

Int.J.Mod.Phys.A27,1250115(2012)

J.Mod.Phys.3,815(2012)

Int.J.Mod.Phys. A32,1750029(2017)

Nucl. Phys. A993 (2020) 121646

- Energy & angular independence
- Flavor independence ( $\pi, K, \bar{p}, \Lambda$ )
- Saturation for  $z < 0.1$
- Power law  $\Psi(z) \sim z^{-\beta}$  for high  $z > 4$

Scaling – “collapse” of data points onto a single curve.

Universality classes – hadron species ( $\varepsilon_F, \alpha_F$ ).



# Self-similarity at RHIC

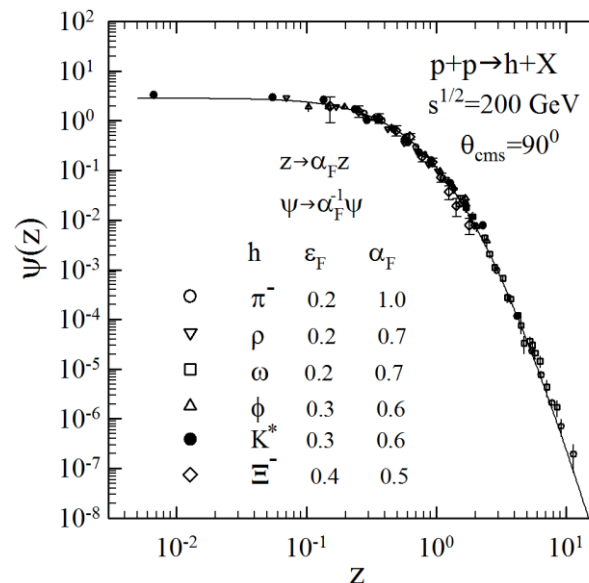
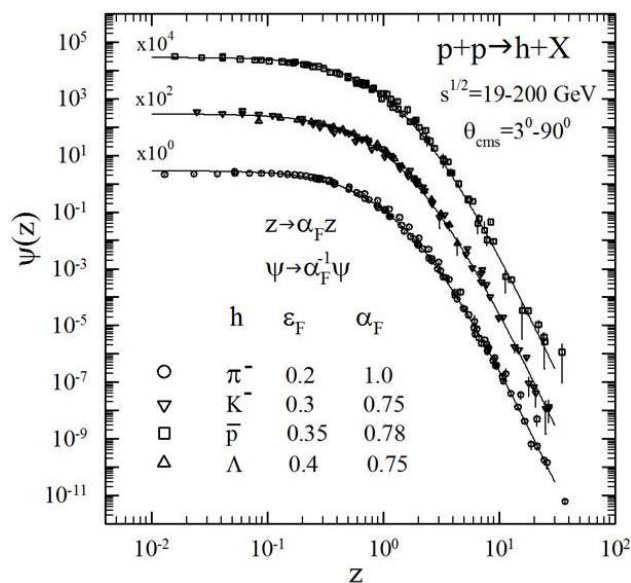
## Flavor independence of scaling function

M.T. & I.Zborovský

Int.J.Mod.Phys.

A24,1417(2009)

$\pi^-, \rho, \omega, \phi, K^*, \Lambda, \Xi, J/\psi$



STAR:

PRL 92 (2004) 092301

PLB 612 (2005) 181

PRC 71 (2005) 064902

PRC 75 (2007) 064901

PHENIX:

PRC 75 (2007) 051902

- Energy independence
- Angular independence
- Flavor independence
- Saturation for  $z < 0.01$

- Power law  $\Psi(z) \sim z^{-\beta}$  at large  $z$
- $\varepsilon_F, \alpha_F$  independent of  $p_T, s^{1/2}$

Self-similarity of particle production with various flavor content.



# Properties of $\Psi(z)$ in p+p collisions

- Energy independence of  $\Psi(z)$  ( $s^{1/2} > 20 \text{ GeV}$ )
- Angular independence of  $\Psi(z)$  ( $\theta_{\text{cms}} = 3^\circ - 90^\circ$ )
- Multiplicity independence of  $\Psi(z)$  ( $dN_{\text{ch}}/d\eta = 1.5 - 26$ )
- Saturation of  $\Psi(z)$  at low  $z$  ( $z < 0.1$ )
- Power law,  $\Psi(z) \sim z^{-\beta}$ , at high  $z$  ( $z > 4$ )
- Flavor independence of  $\Psi(z)$  ( $\pi, K, \phi, \Lambda, \dots, D, J/\psi, B, Y, \dots, \text{top}$ )

These properties reflect **self-similarity**, **locality**, and **fractality** of hadron interactions at a constituent level.

It concerns the **structure** of the colliding objects, constituent **interactions** and **fragmentation** process.





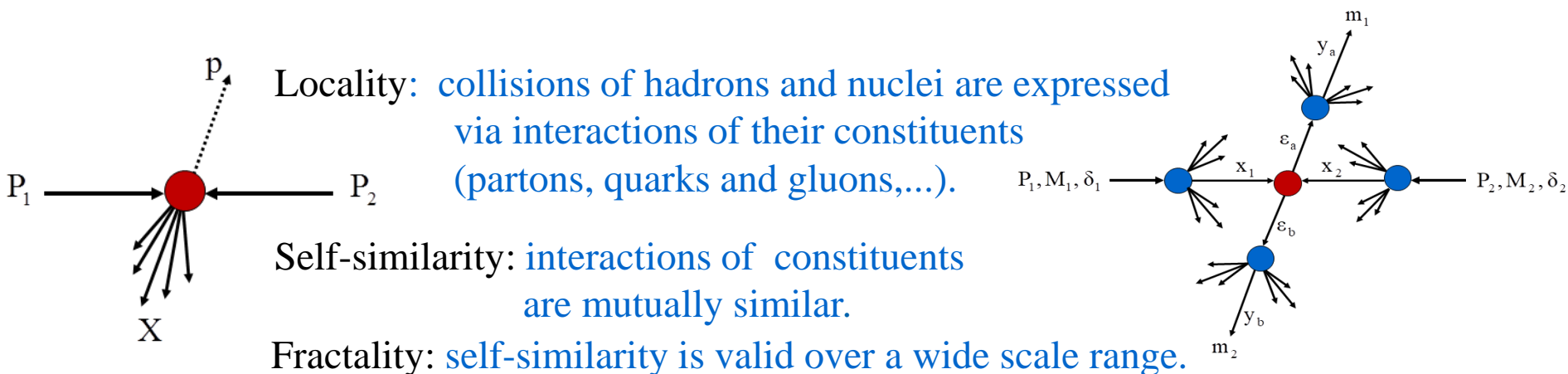
---

**z-Scaling:**  
ideas, definitions, hypothesis,...

**Basic principles:**  
locality, self-similarity, fractality,...



Principles: locality, self-similarity, fractality



Hypothesis of **Z**-scaling :

$$s^{1/2}, p_T, \theta_{\text{cms}}$$

Inclusive particle distributions can be described in terms of constituent sub-processes and parameters characterizing bulk properties of the system.

$$x_1, x_2, y_a, y_b$$

$$\delta_1, \delta_2, \epsilon_a, \epsilon_b, c$$

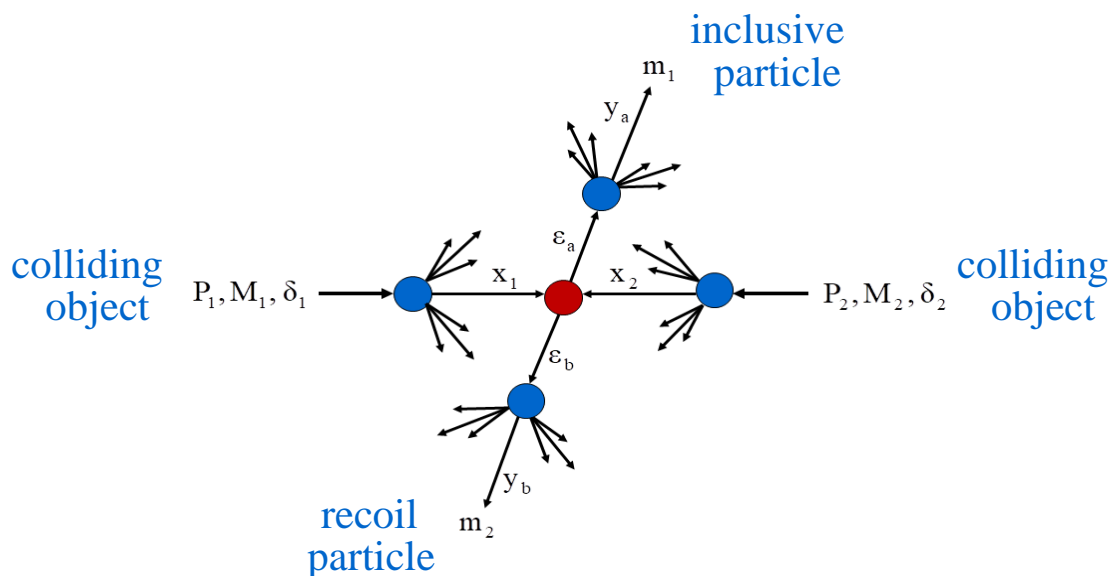
$$Ed^3\sigma/dp^3$$

Scaled inclusive cross section of particles depends in a self-similar way on a single scaling variable **Z**.

$$\Psi(Z)$$



Collisions of colliding objects  
are expressed via interactions of their constituents



$P_1, P_2, p$  – momenta of colliding and produced particles

$M_1, M_2, m_1$  – masses of colliding and produced particles

$x_1, x_2$  – momentum fractions of colliding particles carried by constituents

$y_a, y_b$  – momentum fractions of scattered constituents carried by inclusive particle and its recoil

$\delta_1, \delta_2$  – fractal dimensions of colliding particles

$\epsilon_a, \epsilon_b$  – fractal dimensions of scattered constituents (fragmentation dimensions)

$m_2$  – mass of recoil particle

Elementary sub-process:

$$(x_1 M_1) + (x_2 M_2) \rightarrow (m_1 / y_a) + (x_1 M_1 + x_2 M_2 + m_2 / y_b)$$

Momentum conservation law for sub-process

$$(x_1 P_1 + x_2 P_2 - p / y_a)^2 = M_X^2$$

Mass of recoil system

$$M_X = x_1 M_1 + x_2 M_2 + m_2 / y_b$$

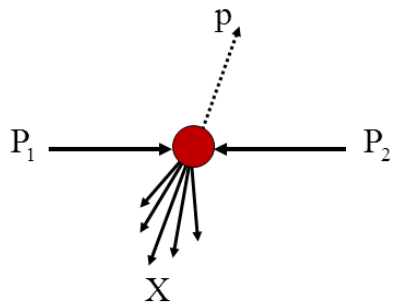
M.T., I.Zborovský  
Yu.Panebratsev, G.Skoro  
Phys.Rev.D54 5548 (1996)  
Int.J.Mod.Phys.A16 1281 (2001)



# Self-similarity

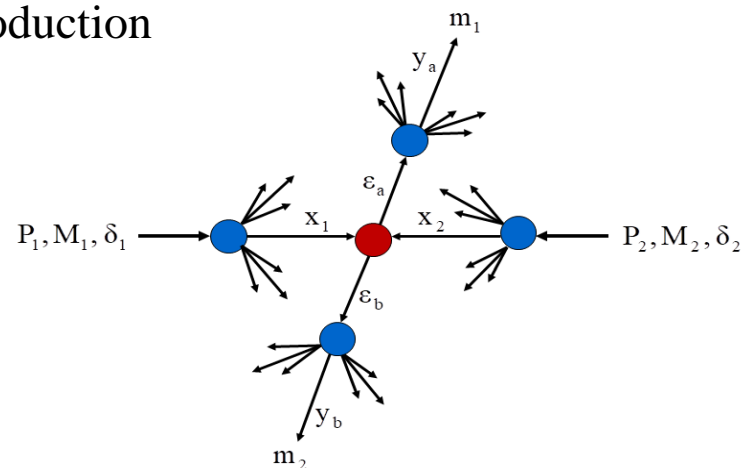
## Interactions of constituents are mutually similar

The self-similarity parameter  $z$  is a dimensionless quantity, expressed through the dimensional values  $P_1, P_2, p, M_1, M_2, m_1, m_2$ , characterizing the process of inclusive particle production



$$z = z_0 \cdot \Omega^{-1}$$

$$z_0 = \frac{s_{\perp}^{1/2}}{(dN_{ch}/d\eta|_0)^c m_N}$$



- $\Omega^{-1}$  is the minimal resolution at which a constituent sub-process can be singled out of the inclusive reaction
- $s_{\perp}^{1/2}$  is the transverse kinetic energy of the sub-process consumed on production of  $m_1$  &  $m_2$
- $dN_{ch}/d\eta|_0$  is the multiplicity density of charged particles at  $\eta = 0$
- $c$  is a parameter interpreted as a “specific heat” of created medium
- $m_N$  is an arbitrary constant (fixed at the value of nucleon mass)



## Self-similarity over a wide scale range

## Fractal measure

$$z = z_0 \cdot \Omega^{-1}$$

$$\Omega = (1 - x_1)^{\delta_1} (1 - x_2)^{\delta_2} (1 - y_a)^{\varepsilon_a} (1 - y_b)^{\varepsilon_b}$$

$$0 < x_1, x_2 < 1$$

$$0 < y_a, y_b < 1$$

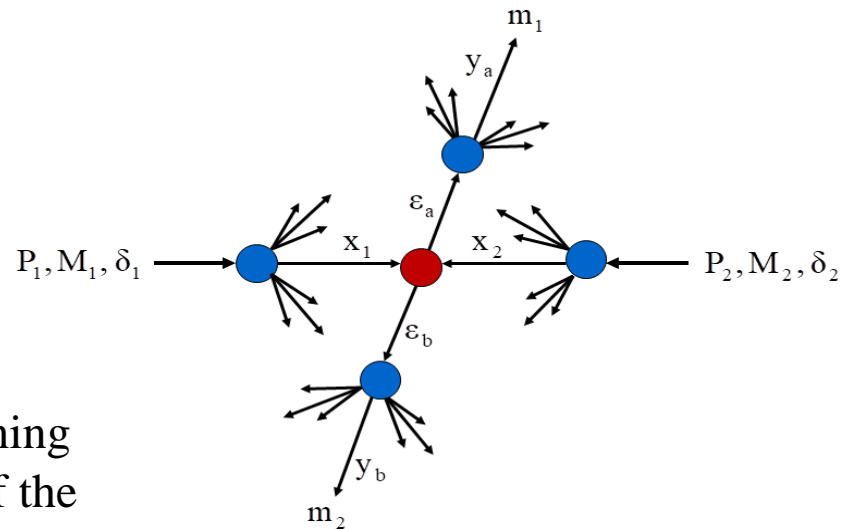
$\Omega$  is relative number of configurations containing a sub-process with fractions  $x_1, x_2, y_a, y_b$  of the corresponding 4-momenta

$\delta_1, \delta_2, \varepsilon_a, \varepsilon_b$  are parameters characterizing structure of the colliding objects and fragmentation process, respectively

$\Omega^{-1}(x_1, x_2, y_a, y_b)$  characterizes resolution at which a constituent sub-process can be singled out of the inclusive reaction

The fractal measure  $z$  diverges as the resolution  $\Omega^{-1}$  increases.

$$z(\Omega) \Big|_{\Omega^{-1} \rightarrow \infty} \rightarrow \infty$$



# Momentum fractions: $x_1, x_2, y_a, y_b$

**Principle of minimal resolution:** The momentum fractions  $x_1, x_2$  and  $y_a, y_b$  are determined in a way to minimize the resolution  $\Omega^{-1}$  of the fractal measure  $z$  with respect to all constituent sub-processes taking into account 4-momentum conservation law:

## Momentum conservation law

$$(x_1 P_1 + x_2 P_2 - p/y_a)^2 = M_X^2$$

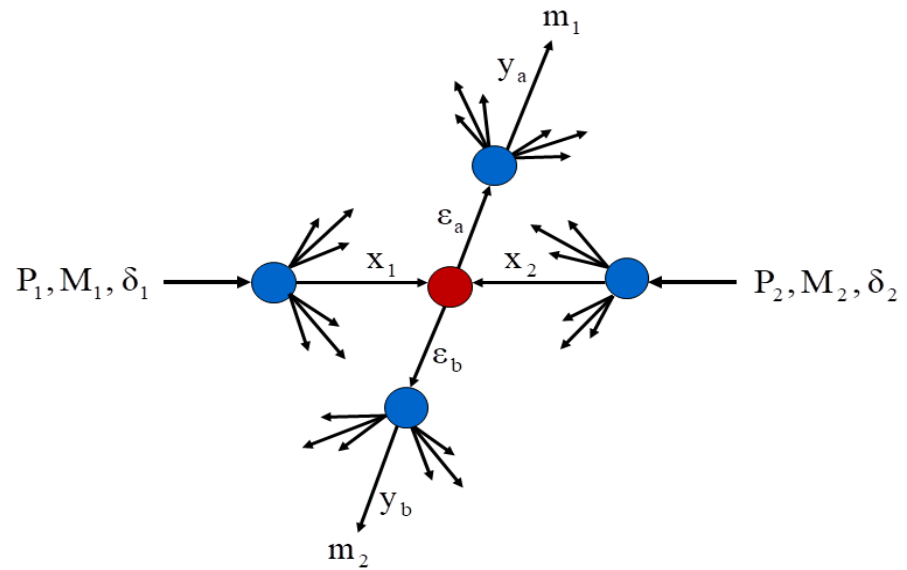
$$\begin{cases} \partial\Omega / \partial x_1 |_{y_a=y_a(x_1, x_2, y_b)} = 0 \\ \partial\Omega / \partial x_2 |_{y_a=y_a(x_1, x_2, y_b)} = 0 \\ \partial\Omega / \partial y_b |_{y_a=y_a(x_1, x_2, y_b)} = 0 \end{cases}$$

## Resolution of sub-process

$$\Omega^{-1} = (1 - x_1)^{-\delta_1} (1 - x_2)^{-\delta_2} (1 - y_a)^{-\varepsilon_a} (1 - y_b)^{-\varepsilon_b}$$

## Mass of recoil system

$$M_X = x_1 M_1 + x_2 M_2 + m_2 / y_b$$

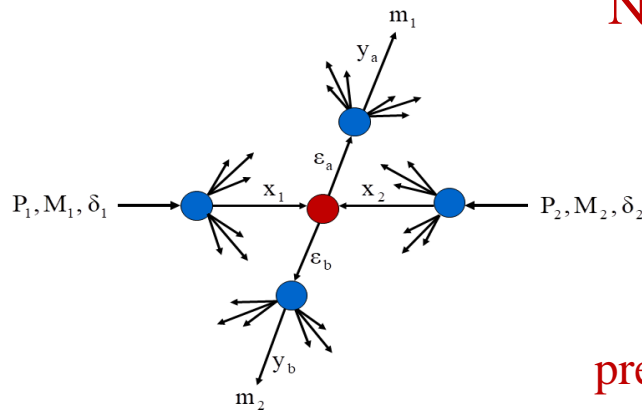


Fractions  $x_1, x_2, y_a, y_b$  are expressed via Lorentz invariants – scalar products of 4-D momenta and particle masses.





# Scaling function $\Psi(z)$



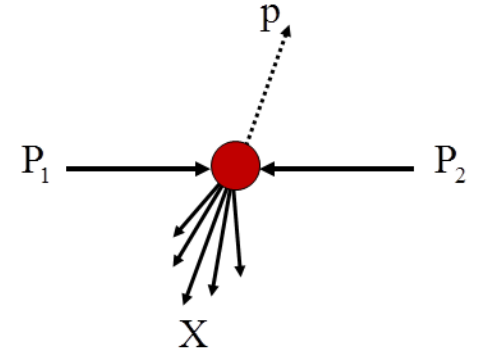
Normalization condition

$$\int_0^{\infty} \Psi(z) dz = 1$$

Scale transformation

$$z \rightarrow \alpha_F \cdot z \quad \Psi \rightarrow \alpha_F^{-1} \cdot \Psi$$

preserves the normalization condition



$$\Psi(z) = \frac{\pi}{(dN/d\eta) \cdot \sigma_{\text{inel}}} \cdot J^{-1} \cdot E \frac{d^3\sigma}{dp^3} \quad \longleftrightarrow \quad \int E \frac{d^3\sigma}{dp^3} dy d^2p_{\perp} = \sigma_{\text{inel}} \cdot \langle N \rangle$$

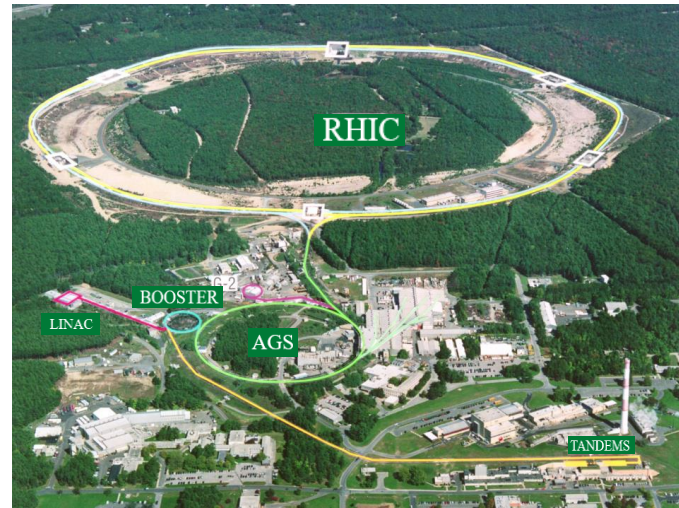
- $\sigma_{\text{in}}$  - the inelastic cross section
- $\langle N \rangle$  - the average multiplicity
- $dN/d\eta$  - the multiplicity density
- $J(z, \eta; p_T^2, y)$  - the Jacobian
- $Ed^3\sigma/dp^3$  - the inclusive cross section

The scaling function  $\Psi(z)$  is a probability density to produce the inclusive particle with the corresponding  $z$ .



# Strange particle production in $p+p$ from RHIC

$p+p$  @ RHIC



$K_S^0, K^-, K^*, \phi, \Lambda, \Xi, \Omega, \Sigma^*, \Lambda^*$

$p+p$  is a benchmark for strangeness production in  $A+A$  collisions

M.T.& I.Zborovský

Int.J.Mod.Phys.

A32,1750029(2017)



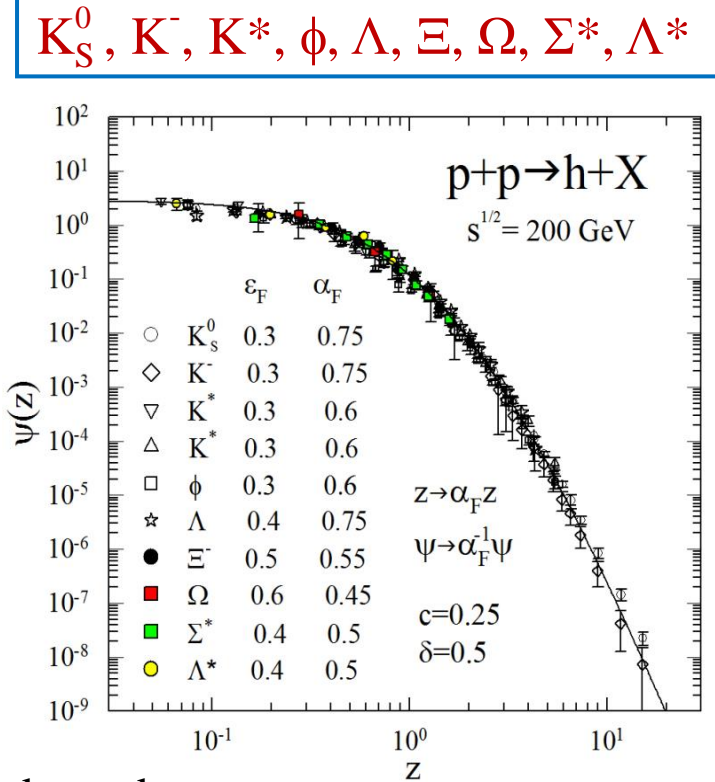
# Self-similarity of strangeness production in p+p

Universality: flavor independence of the scaling function

M.T.& I.Zborovský  
Int.J.Mod.Phys.  
A24,1417(2009)

Solid line for  $\pi^-$  meson  
is a reference frame

$$\varepsilon_\pi = 0.2, \quad \alpha_\pi = 1$$



STAR:

PRL 92 (2004) 092301  
PRL 97 (2006) 132301  
PLB 612 (2005) 181  
PRC 71 (2005) 064902  
PRC 75 (2007) 064901  
PRL 108 (2012) 072302

PHENIX:

PRC 75 (2007) 051902  
PRD 83 (2011) 052004  
PRC 90 (2014) 054905

- Energy independence
- Angular independence
- Flavor independence
- Saturation for  $z < 0.01$

- Power law  $\Psi(z) \sim z^{-\beta}$  at large  $z$
- $\varepsilon_F, \alpha_F$  independent of  $p_T, s^{1/2}$

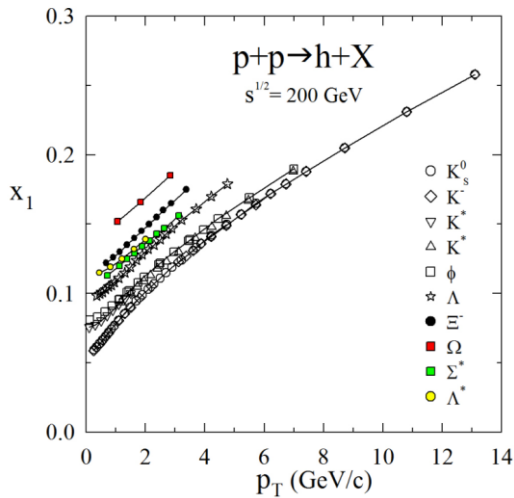


# Self-similarity of strangeness production in p+p

$$K_S^0, K^-, K^*, \phi, \Lambda, \Xi, \Omega, \Sigma^*, \Lambda^*$$

Constituent sub-process in terms of

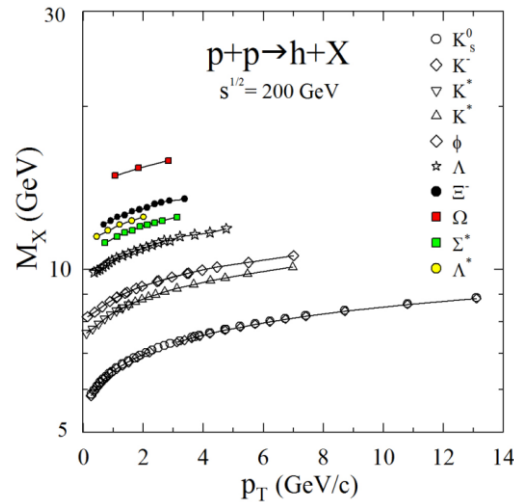
Momentum fraction



The more strangeness,  
the larger momentum fraction

$$x_1^\Omega > x_1^\Xi > x_1^\Sigma > x_1^K$$

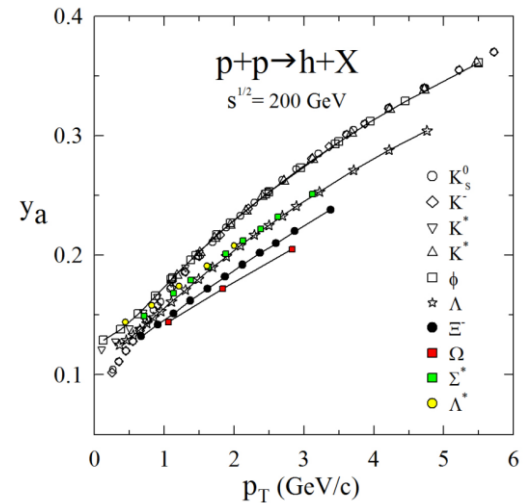
Recoil mass



The more strangeness,  
the larger recoil mass

$$M_X^\Omega > M_X^\Xi > M_X^\Sigma > M_X^K$$

Energy loss  $\Delta E/E \sim (1-y_a)$



The more strangeness,  
the larger energy loss

$$\epsilon_\Omega > \epsilon_\Xi > \epsilon_\Sigma > \epsilon_K$$

Self-similarity dictates the properties of constituent sub-process.

# Self-similarity of $K_S^0$ production in p+p

## Self-similarity parameter

$$z = z_0 \Omega^{-1}$$

$$z_0 = \frac{s_{\perp}^{1/2}}{(dN_{ch}/d\eta|_0)^c m_N}$$

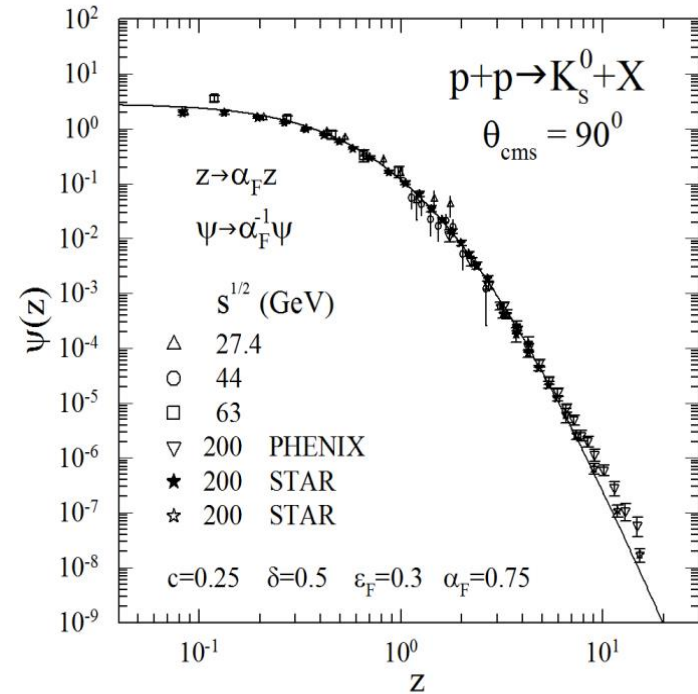
$$\Omega = (1-x_1)^\delta (1-x_2)^\delta (1-y_a)^{\varepsilon_F} (1-y_b)^{\varepsilon_F}$$

- $dN_{ch}/d\eta|_0$  - multiplicity density
- $c$  - “specific heat” of bulk matter
- $\delta$  - proton fractal dimension
- $\varepsilon_F$  - fragmentation fractal dimension

## Scaling function

$$\Psi(z) = \frac{\pi}{(dN/d\eta) \cdot \sigma_{inel}} \cdot J^{-1} \cdot E \frac{d^3\sigma}{dp^3}$$

## “Collapse” of data onto a single curve



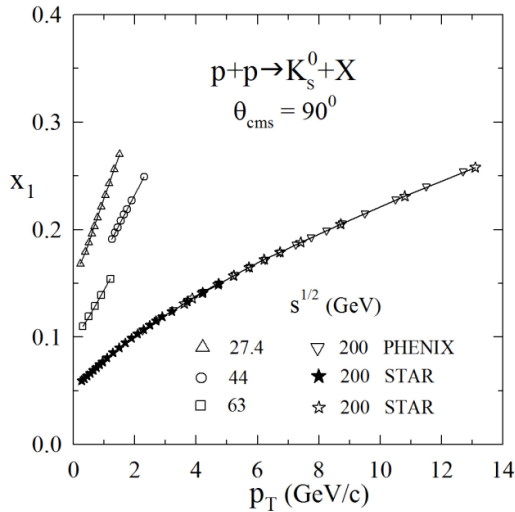
- Energy independence of  $\Psi(z)$
- Centrality independence of  $\Psi(z)$
- Power law at high  $z$
- Saturation at low  $z$

Universality: the same shape of  $\Psi$  both for  $K_S^0$  and  $\pi^-$  (solid line)

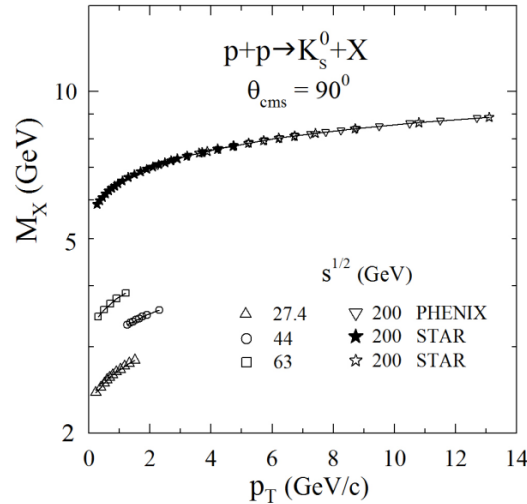
# Self-similarity of $K_s^0$ production in p+p

## Constituent sub-process in terms of

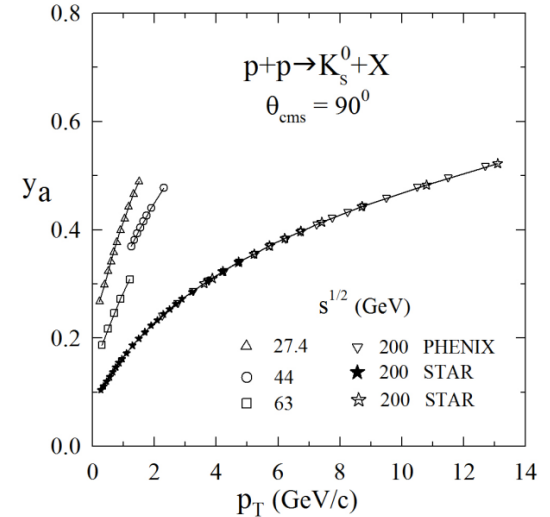
Momentum fraction



Recoil mass



Energy loss  $\Delta E/E \sim (1-y_a)$



Momentum fraction

- increases with  $p_T$
- decreases with  $\sqrt{s_{NN}}$

Recoil mass

- increases with  $p_T$
- increases with  $\sqrt{s_{NN}}$

Constituent energy loss

- decreases with  $p_T$
- increases with  $\sqrt{s_{NN}}$

- High  $x_1$  and  $p_T \rightarrow$  compressed matter
- Large  $M_X \rightarrow$  high density recoil system
- High  $y_a \rightarrow$  small energy loss, pure signatures of **PT** & **CP**





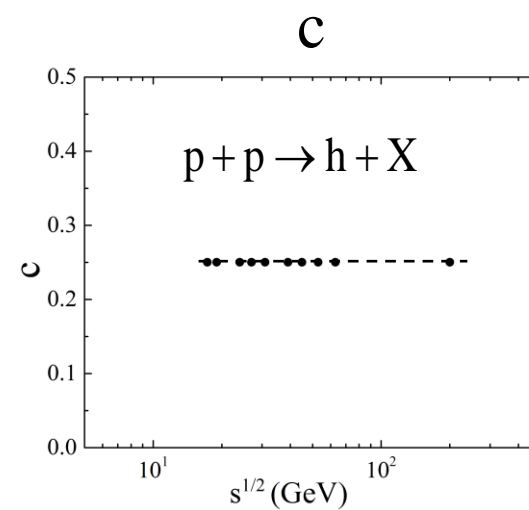
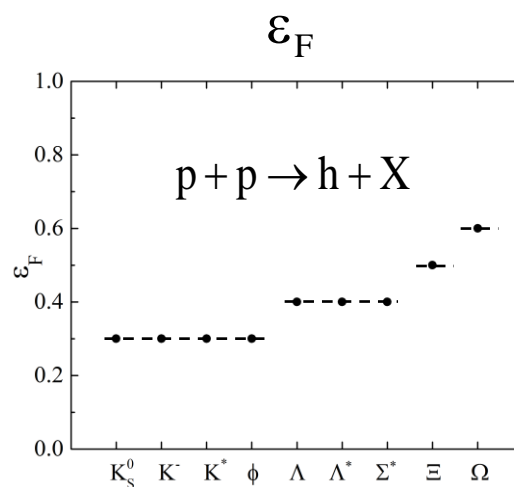
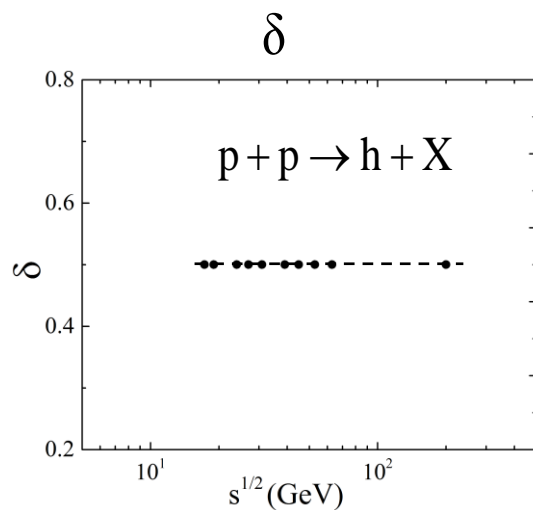
# Model parameters: $\delta$ , $\varepsilon_F$ , $c$

Parameters  $\delta$ ,  $\varepsilon_F$ ,  $c$  are found from the scaling behavior of  $\Psi$  as a function of self-similarity variable  $z$

Proton fractal dimension

Fragmentation dimension

“Specific heat”



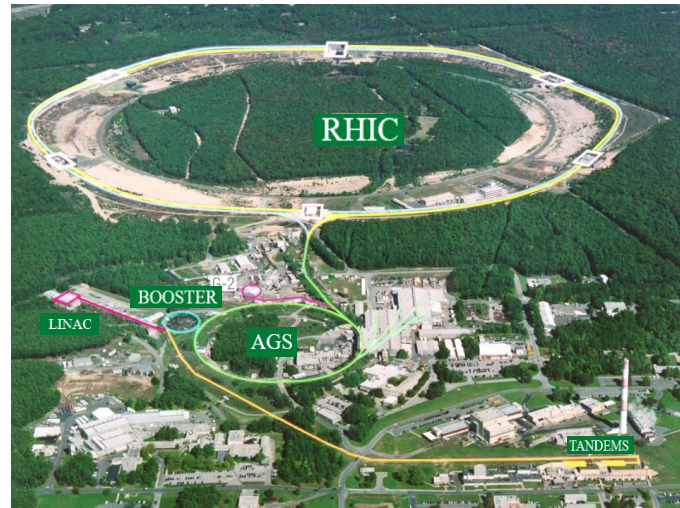
- $\delta$ ,  $\varepsilon_F$ ,  $c$  are independent of  $\sqrt{s}$ ,  $p_T$
- $\varepsilon_F$  depends on flavor

A discontinuity and strong correlation of the model parameters could give indication on new physics in p+p collisions:

Search for signatures of phase transition, critical point .... with strange probes.



# Strange particle production in Au+Au from RHIC



$K_S^0, K^-, K^*, \phi, \Lambda, \Xi, \Omega, \Sigma^*, \Lambda^*$

J. Adam et al. (STAR Collaboration)  
Phys. Rev. C 102 (2020) 034909



# Self-similarity of $K_S^0$ production in Au+Au

## Self-similarity parameter

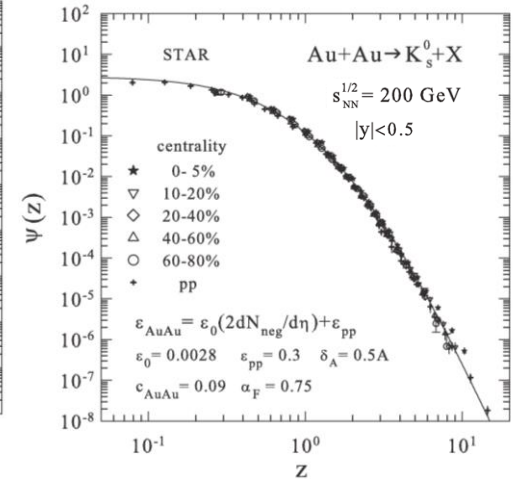
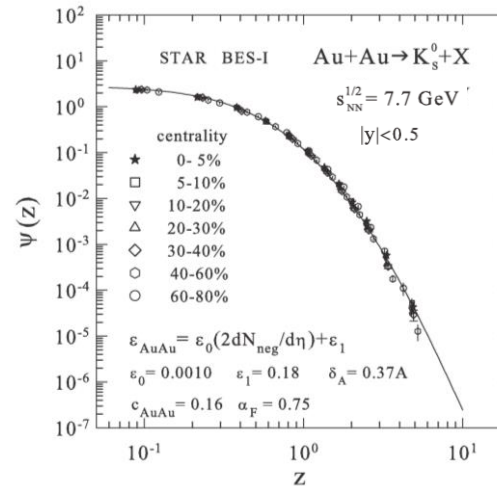
$$Z = Z_0 \Omega^{-1}$$

$$Z_0 = \frac{s_{\perp}^{1/2}}{(dN_{ch}/d\eta|_0)^c m_N}$$

$$\Omega = (1-x_1)^{\delta_{A1}} (1-x_2)^{\delta_{A2}} (1-y_a)^{\varepsilon} (1-y_b)^{\varepsilon}$$

- $dN_{ch}/d\eta|_0$  - multiplicity density
- $c_{AA}$  - “specific heat” of bulk matter
- $\delta_A$  - nucleus fractal dimension
- $\varepsilon_{AA}$  - fragmentation dimension

“Collapse” of data points onto a single curve



## AA collisions:

$$\delta_A = A\delta$$

$$\varepsilon_{AA} = \varepsilon_0 (dN_{AA}/d\eta) + \varepsilon_{pp}$$

$$\Psi(Z) = \frac{\pi}{(dN/d\eta) \sigma_{inel}} J^{-1} E \frac{d^3\sigma}{dp^3}$$

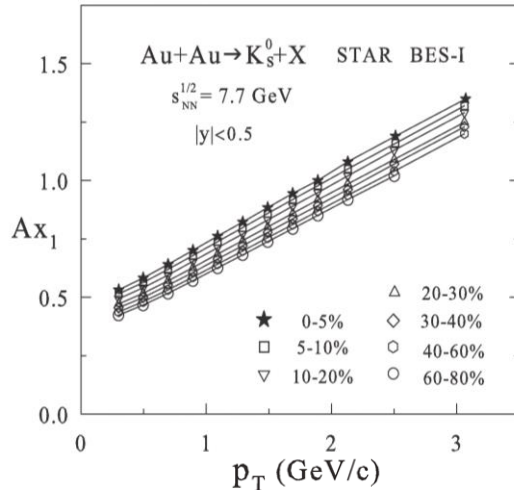
- Energy independence of  $\Psi(z)$
- Centrality independence of  $\Psi(z)$
- Dependence of  $\varepsilon_{AA}$  on multiplicity
- Power law at low- and high- $z$  regions

Indication of a decrease  
of  $\delta$  for  $\sqrt{s_{NN}} < 19.6$  GeV

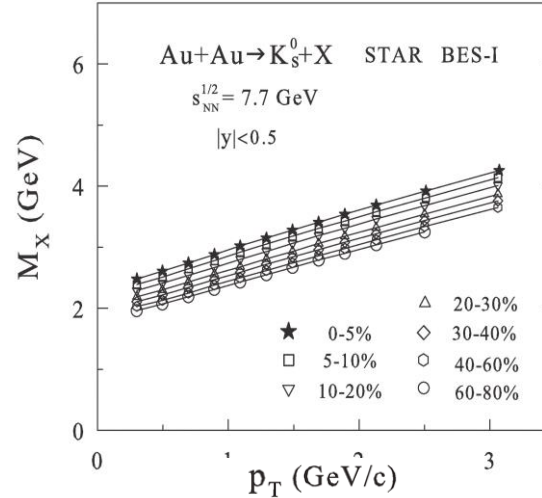
# $K_S^0$ production in Au+Au @ 7.7 GeV

## Constituent sub-process in terms of

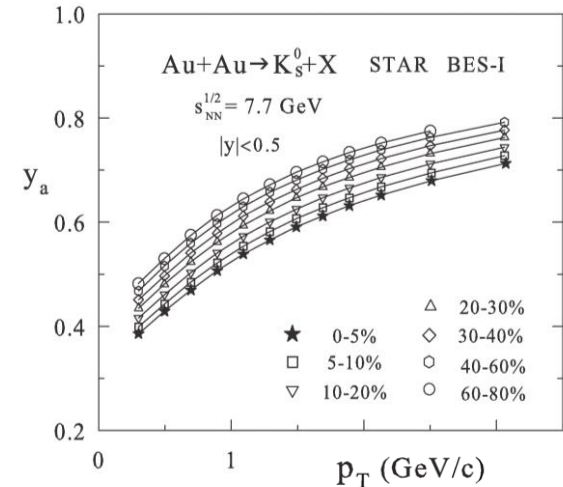
### Momentum fraction $Ax_1$



### Recoil mass $M_X$



### Energy loss $\Delta E/E \sim (1-y_a)$



### Momentum fraction

- increases with  $p_T$
- decreases with  $\sqrt{s_{NN}}$
- increases with centrality

### Recoil mass

- increases with  $p_T$
- increases with  $\sqrt{s_{NN}}$
- decreases with centrality

### Energy loss

- decreases with  $p_T$
- increases with  $\sqrt{s_{NN}}$
- increases with centrality

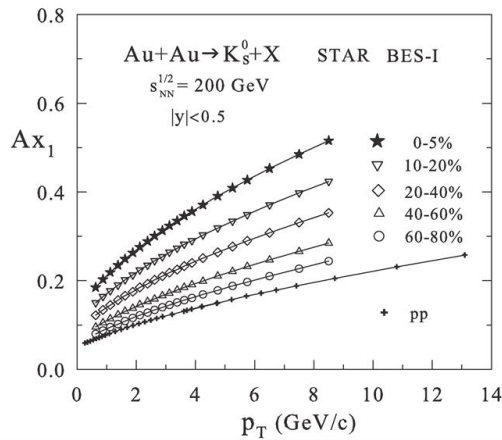
- High  $x_1$  and  $p_T \rightarrow$  compressed matter
- Large  $M_X \rightarrow$  high density recoil system
- High  $y_a \rightarrow$  small energy loss, pure signatures of **PT** & **CP**



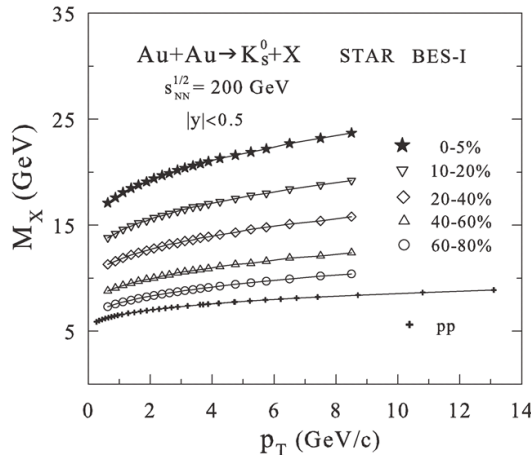
# $K_S^0$ production in Au+Au @ 200 GeV

## Constituent sub-process in terms of

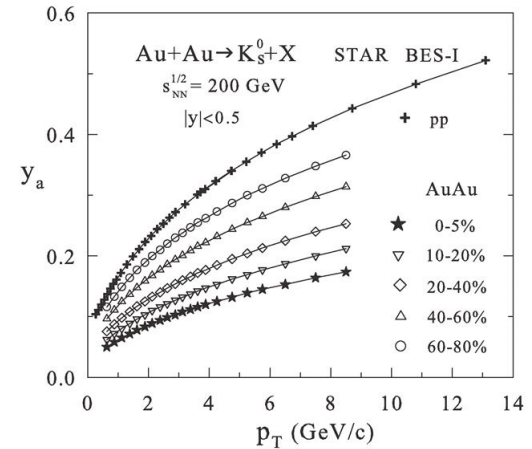
### Momentum fraction $Ax_1$



### Recoil mass $M_X$



### Energy loss $\Delta E/E \sim (1-y_a)$



### Momentum fraction

- increases with  $p_T$
- decreases with  $\sqrt{s_{NN}}$
- increases with centrality

### Recoil mass

- increases with  $p_T$
- increases with  $\sqrt{s_{NN}}$
- decreases with centrality

### Energy loss

- decreases with  $p_T$
- increases with  $\sqrt{s_{NN}}$
- increases with centrality

- High  $x_1$  and  $p_T$  → compressed matter
- Large  $M_X$  → high density recoil system
- High  $y_a$  → small energy loss, pure signatures of **PT** & **CP**



# Self-similarity of $K_S^0$ production in Au+Au

## Self-similarity parameter

$$z = z_0 \Omega^{-1}$$

$$z_0 = \frac{s_{\perp}^{1/2}}{(dN_{ch}/d\eta|_0)^c m_N}$$

$$\Omega = (1 - x_1)^{\delta_{A_1}} (1 - x_2)^{\delta_{A_2}} (1 - y_a)^{\varepsilon} (1 - y_b)^{\varepsilon}$$

- $dN_{ch}/d\eta|_0$  - multiplicity density
- $c_{AA}$  - “specific heat” of bulk matter
- $\delta_A$  - nucleus fractal dimension
- $\varepsilon_{AA}$  - fragmentation dimension

## AA collisions:

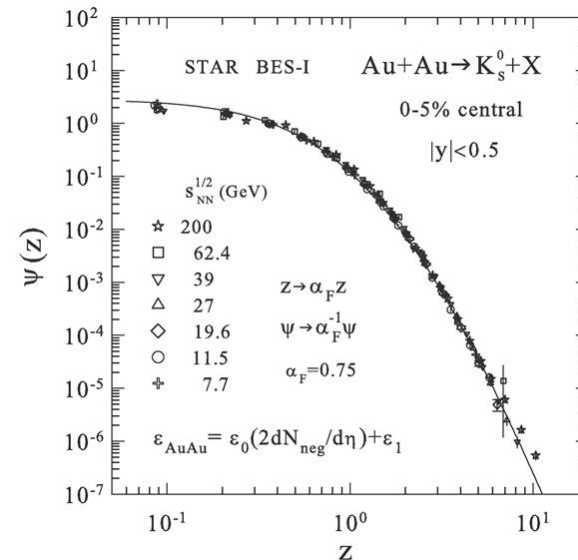
$$\delta_A = A\delta$$

$$\varepsilon_{AA} = \varepsilon_0 (dN_{AA}/d\eta) + \varepsilon_{pp}$$

$$\Psi(z) = \frac{\pi}{(dN/d\eta) \sigma_{inel}} J^{-1} E \frac{d^3\sigma}{dp^3}$$

M.T. & I.Zborovsky, Nucl. Phys. A993 (2020) 121646

“Collapse” of data points onto a single curve



- Energy independence of  $\Psi(z)$
- Centrality independence of  $\Psi(z)$
- Dependence of  $\varepsilon_{AA}$  on multiplicity
- Power law at low- and high- $z$  regions

Indication of a decrease  
of  $\delta$  for  $\sqrt{s_{NN}} < 19.6$  GeV

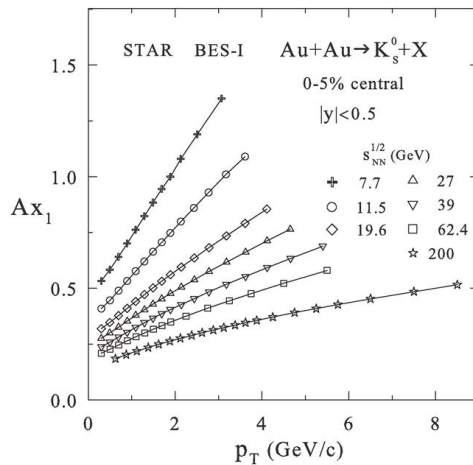




# $K_S^0$ production in central Au+Au @ 7.7-200 GeV

## Constituent sub-process in terms of

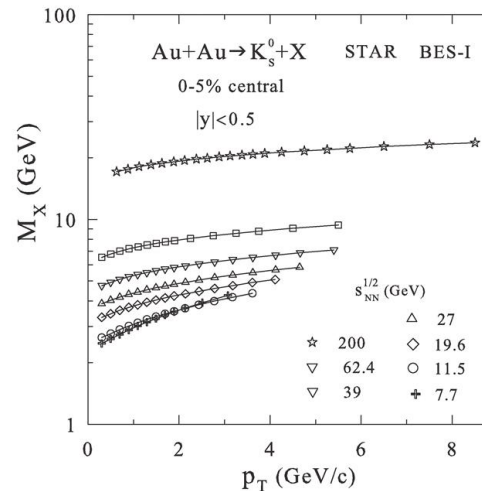
### Momentum fraction $Ax_1$



### Momentum fraction

- increases with  $p_T$
- decreases with  $\sqrt{s_{NN}}$

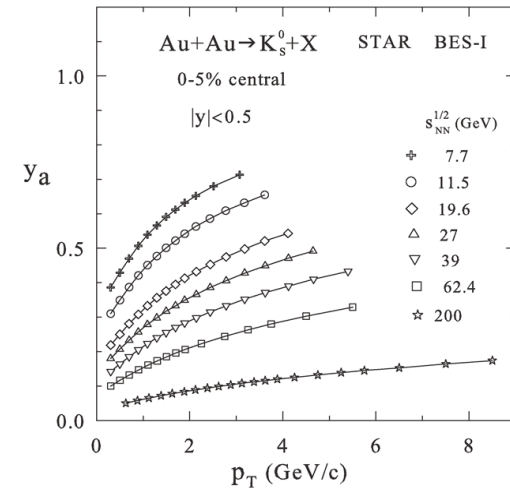
### Recoil mass $M_X$



### Recoil mass

- increases with  $p_T$
- increases with  $\sqrt{s_{NN}}$

### Energy loss $\Delta E/E \sim (1-y_a)$



### Energy loss

- decreases with  $p_T$
- increases with  $\sqrt{s_{NN}}$

- High  $x_1$  and  $p_T$  → compressed nuclear matter
- Large  $M_X$  → high density recoil system
- High  $y_a$  → small energy loss, pure signatures of **PT** & **CP**

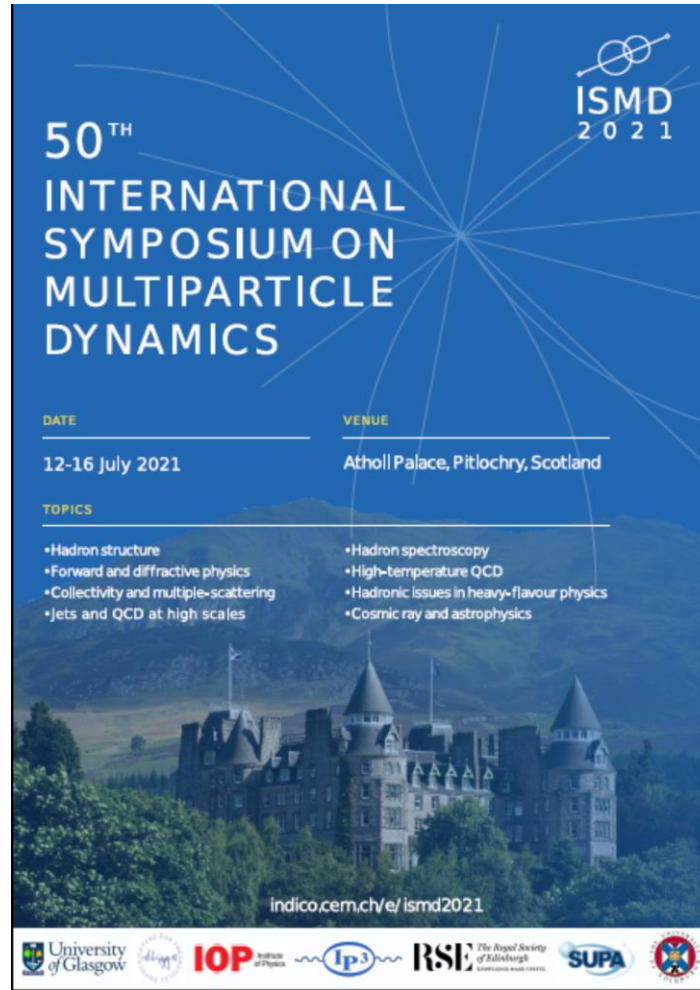


# Summary & Outlook

- STAR BES-I data on transverse momentum spectra of  $K_S^0$  mesons produced in Au+Au collisions at RHIC in mid-rapidity region were analyzed in the  $z$ -scaling approach.
- Self-similarity of strange  $K_S^0$  meson production in Au+Au collisions over a wide kinematical and centrality range was found.
- Constituent energy loss as a function of collision energy and centrality, and transverse momentum of  $K_S^0$  meson was estimated.
- Model parameters - fractal dimensions and “specific heat”, were found.
- The method of analysis is extended for systematical description of A+A collisions with production of identified hadrons.

Specific features of constituent sub-process with strange particles found in the  $z$ -scaling approach can be sensitive to critical phenomena in Strange Quark Matter created in A+A collisions.





50<sup>TH</sup>  
INTERNATIONAL  
SYMPOSIUM ON  
MULTIPARTICLE  
DYNAMICS

ISMD  
2021

DATE  
12-16 July 2021

VENUE  
Atholl Palace, Pitlochry, Scotland

TOPICS

- Hadron structure
- Forward and diffractive physics
- Collectivity and multiple-scattering
- Jets and QCD at high scales
- Hadron spectroscopy
- High-temperature QCD
- Hadronic issues in heavy-flavour physics
- Cosmic ray and astrophysics

[indico.cern.ch/e/ismd2021](https://indico.cern.ch/e/ismd2021)

University of Glasgow IOP Institute of Physics IP3 RSE The Royal Society of Edinburgh SUPA JINR

Thank You for Your Attention !