Determination of α_S beyond $NNLO$ using the event shape averages

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Introduction and goals

The accurate knowledge of the QCD coupling constant α_S is a key to precise Standard Model (SM) physics and tests of models beyond the SM. The factors contributing to the precise measurements of α_S are the precision of data, the accuracy of the perturbative QCD (pQCD) predictions and the accuracy of the selected models for non-perturbative effects. It is widely accepted that the accuracy of pQCD predictions has the largest impact on the precision of α_S and obtaining the pQCD predictions with higher order expansion terms (Next-to-Next-to-…-Leading order expansion) automatically leads to increased precision of α_S determination.

In this work we consider if the availability of N^3LO perturbative calculations would improve the accuracy of α_S determination. We check this scenario using the available high-precision $NNLO$ calculations for the event shape averages in the $e^+e^- \to Z/\gamma \to partons$ process and estimating the presently unknown N^3LO contributions from the data.

Methodology and the challenges of analysis

The basic idea is to use pQCD predictions for the event shape averages $\langle O\rangle$ at different scales (centre-of-mass energies) $\mu_0 = \sqrt{s}$

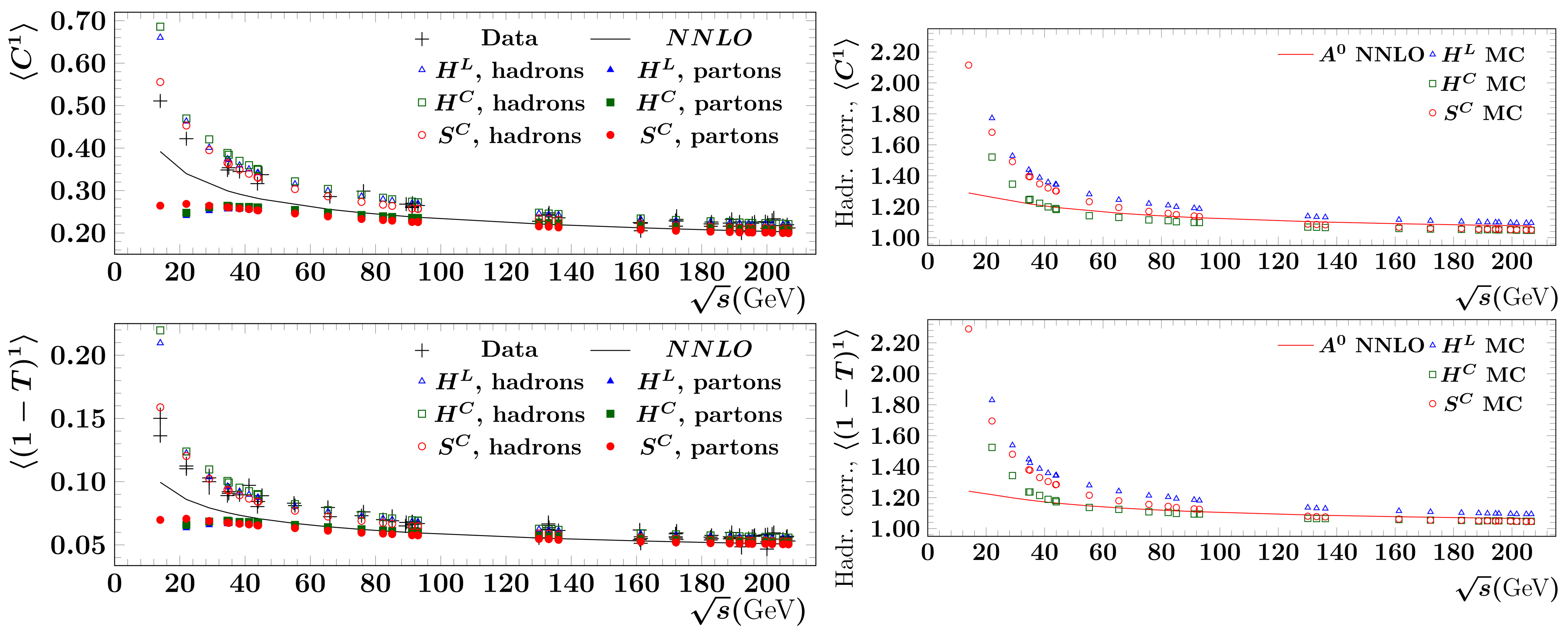
$$\langle O\rangle = \frac{\alpha_S(\mu_0)}{2\pi}\bar{A}_0^{\langle O\rangle} + \left(\frac{\alpha_S(\mu_0)}{2\pi}\right)^2\bar{B}_0^{\langle O\rangle} + \left(\frac{\alpha_S(\mu_0)}{2\pi}\right)^3\bar{C}_0^{\langle O\rangle} + \left(\frac{\alpha_S(\mu_0)}{2\pi}\right)^4\bar{D}_0^{\langle O\rangle} + \mathcal{O}(\alpha_S^5).$$

and with an assumption of running α_S extract simultaneously the α_S and the **single** higher-order coefficient $\bar{D}_0^{\langle O\rangle}$ not known at present. The correction of the pQCD predictions to hadron level to make them comparable to the data can be done in several ways. The presence of an additional parameter in the fit automatically rises the requirements to

- The amount of the data to be used. The main requirement for the selection of the event shape for the studies was the availability of large number of measurements. → The analysis was performed using the data on $1-T$ and C-parameter averages.
- The precision of the determination of the numerically known perturbative coefficients. → The most precise pQCD calculation with the ColorfulNNLO framework +fully analytic expressions for the C-parameter.
- The perturbative precision of the analytic hadronisation models. → an extension of analytic models to higher orders in α_S.
- The precision of the MC hadronisation models. → usage of modern MC event generators.

Analysis components

- The measurements of average values of $1-T$ and C from ALEPH, AMY, DELPHI, HRS, JADE, L3, MARK, MARKII, OPAL, TASSO were used, see Ref. [1] for details. The simultaneous fit benefits from the large span of centre-of-mass-energies $30 < \sqrt{s} < 205 GeV$.



- Well-tested MC hadronisation models combined with modern MC event generators describe the data for $30GeV < \sqrt{s}$ well. See details on SHERPA+Lund hadronisation (S^L), SHERPA+Cluster S^C, Herwig+Lund H^L, Herwig+Cluster H^C in Ref. [1].
- For **the first time**, using recent advances in theory [3], the analytic hadronisation models were extended to α_S^3, e.g.

$$\langle O\rangle_{\text{hadrons}} = \langle O\rangle_{\text{partons}} + a_O\mathcal{P}(\alpha_S^3).$$

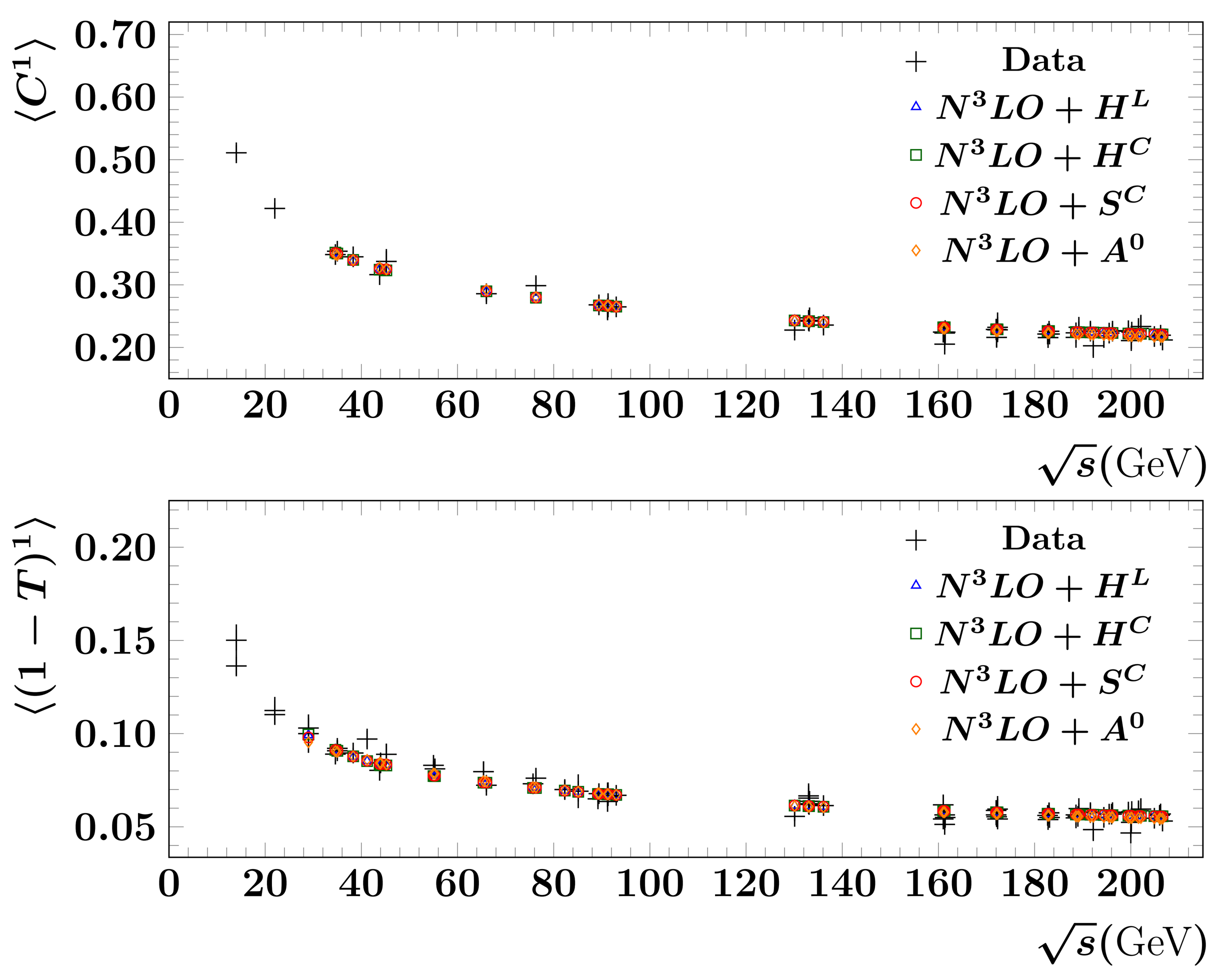
for $O = 1-T, C$ with the correction $\mathcal{P}$ considered in different models of gluon radiation treatment (A^0, A^T and A^M below).

- The work also presents for **the first time** high precision massless NNLO pQCD predictions for event shape momenta in $e^+e^- \to Z/\gamma \to partons$ from CoLoRFulNNLO [5]. And **for the first time** the analytic expression for the NLO coefficient $B_0^{\langle C^1\rangle}$ was derived.

$$B_0^{\langle C^1\rangle} = C_F N_F T_R\left(\frac{18759}{140} - 7\pi^2 - \frac{2728\zeta_3}{35}\right) + C_F^2\left(-\frac{8947}{224} + \frac{101\pi^2}{24} + \frac{2\pi^4}{15} - \frac{201\zeta_3}{7}\right) + C_A C_F\left(-\frac{209821}{840} + \frac{247\pi^2}{18} - \frac{8\pi^4}{15} + \frac{7057\zeta_3}{35}\right) = 172.85901\ldots$$

- Quark mass effects treated @NLO with Zbb4 [4].

Results of global fits



The NNLO results are close to those form previous analyses [2], e.g. $\alpha_S(MC\ hadr., 1-T\ data) = 0.11459 \pm 0.00022(exp.) \pm 0.00024(hadr.) \pm 0.0025(scale)$. The N^3LO results are:

- $\alpha_S(MC\ hadr., 1-T\ data) = 0.14092 \pm 0.00116(exp.) \pm 0.00111(hadr.) \pm 0.0090(scale)$, $D^{\langle 1-T\rangle} = -7.51\times 10^4 \pm 1.14\times 10^3(exp.)$
- $\alpha_S(A^0\ an.\ hadr., 1-T\ data) = 0.11927 \pm 0.00125(exp.)$, $D^{\langle 1-T\rangle} = -9.36\times 10^4 \pm 1.33\times 10^4(exp.)$
- $\alpha_S(MC\ hadr., C\ data) = 0.14120 \pm 0.00096(exp.) \pm 0.00097(hadr.) \pm 0.0100(scale)$, $D^{\langle C\rangle} = -3.10\times 10^5 \pm 3.21\times 10^3(exp.)$
- $\alpha_S(A^0\ an.\ hadr., C\ data) = 0.11958 \pm 0.00120(exp.)$, $D^{\langle C\rangle} = -4.12\times 10^5 \pm 4.21\times 10^4(exp.)$

show high degree of consistency in the D coefficients and show the same pattern of discrepancy between the analytic methods and the MC approach as seen in earlier analyses. **Despite quite large uncertainties it is possible to extract the α_S at N^3LO with this approach. The pQCD part of the analysis has a very good precision and the large uncertainties and the discrepancies between different approaches originate in the lack of data and the understating of hadronisation.**

The lack of data and the improvement in understanding of hadronisation can be addressed in future, e.g. at FCC. It would be crucial to obtain the $e^+e^- \to hardons$ data on the widest possible $\sqrt{s}$ range and going to the lowest possible FCC energies of $\sqrt{s} \approx 30\text{GeV}^2$. **Thanks to the high cross-section, the needed data can be collected within days in dedicated datataking runs.**

Conclusions

- The analysis shows that the extraction of α_S at N^3LO is technically possible now.
- The modern pQCD predictions are precise and future improvements, while useful, will likely not have dominant effect on the reducing the α_S extraction uncertainties.
- For a breakthrough improvement of α_S measurements new precise data and/or breakthrough in the understanding of hadronisation is needed. Both options can be pursued simultaneously at (low-energy) FCC-ee.

Thanks

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