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# Applications of $p_T$ - $x_R$ Variables in Describing Inclusive Cross Sections at the LHC

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*LHCb*  
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# Applications of $p_T$ - $x_R$ Variables in Describing Inclusive Cross Sections at the LHC\*

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**Abstract:** Invariant inclusive single-particle/jet cross sections in p–p collisions can be factorized in terms of two separable  $p_T$  dependences, a  $[p_T - \sqrt{s}]$  sector and an  $[x_R - p_T - \sqrt{s}]$  sector. We have analyzed data from ATLAS, CMS and LHCb to explore various s-dependent attributes and other systematics of inclusive jet, photon and single particle productions. Approximate power laws in  $\sqrt{s}$ ,  $p_T$  and  $x_R$  are found when we boost the kinematics by  $\sqrt{s} \rightarrow \infty$  for finite  $p_T$  using the radial scaling variable,  $x_R \equiv \sqrt{x_\perp^2 + x_\parallel^2}$ . We show that the  $A(\sqrt{s}, p_T)$  function, introduced in our earlier publication to describe the  $p_T$  dependence of the inclusive cross section[1], is directly related to the underlying hard parton–parton scattering for jet production, with little influence from soft physics. In addition to the  $A$ -function, we introduce the  $F(\sqrt{s}, x_R)$  function that obeys radial scaling for inclusive jets and offers another test of the underlying parton physics. An application to heavy ion physics is given, where we use our variables to determine the transparency of cold nuclear matter to penetrating heavy mesons through the lead nucleus in pA collisions.

\* This work has been published: <https://www.mdpi.com/2218-1997/7/6/196/pdf> in Universe | Special Issue : Analysis Techniques and Algorithms for QCD Studies (mdpi.com)

[1] Taylor, F.E. Radial scaling in inclusive jet production at hadron colliders. *Phys. Rev. D* **2018**, *97*, 054016, doi:10.1103/physrevd.97.054016.

# Single Particle & Jet Inclusive Production

Particles & Jets

Hard Scattering of partons +  
Fragmentation & Hadronization

LO Dimension 2→2 parton  
scattering

$p + p \rightarrow \text{single jet/particle} + X$

$$E \frac{d^3\sigma}{dp^3} = \sum_{a,b} \int dx_1 dx_2 f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) \frac{d\hat{\sigma}_{ab}(\alpha_s(\mu_R^2), s/\mu_R^2, s/\mu_F^2)}{dt} \otimes \text{Frag} \otimes \text{Had}$$

$$\left[ \begin{array}{l} E \frac{d^3\sigma}{dp^3} \sim \frac{d^2\sigma}{dp_T^2 dy} \sim \frac{d\hat{\sigma}_{ab}(\alpha_s(\mu_R^2), s/\mu_R^2, s/\mu_F^2)}{dt} \\ \sim \frac{cm^2}{GeV^2} \sim \frac{1}{GeV^4} \end{array} \right]$$

QCD Factorization  
Theorem

# $p_T$ , $y$ and $x_R \equiv E^*/E_{\max}^*$

- p-p COM  $\sqrt{s} = 13$  TeV

- $p+p \rightarrow \text{jet}+x$

$$x_R = \frac{2(p_T^2 + M^2)^{1/2} \cosh(y)}{\sqrt{s}}$$

$$y = \frac{1}{2} \ln \left[ \frac{E + p_Z}{E - p_Z} \right]$$

$$R = (\Delta\phi^2 + \Delta\eta^2)^{1/2}$$

$$M(\text{jet}) / p_T \approx R / \sqrt{2}$$

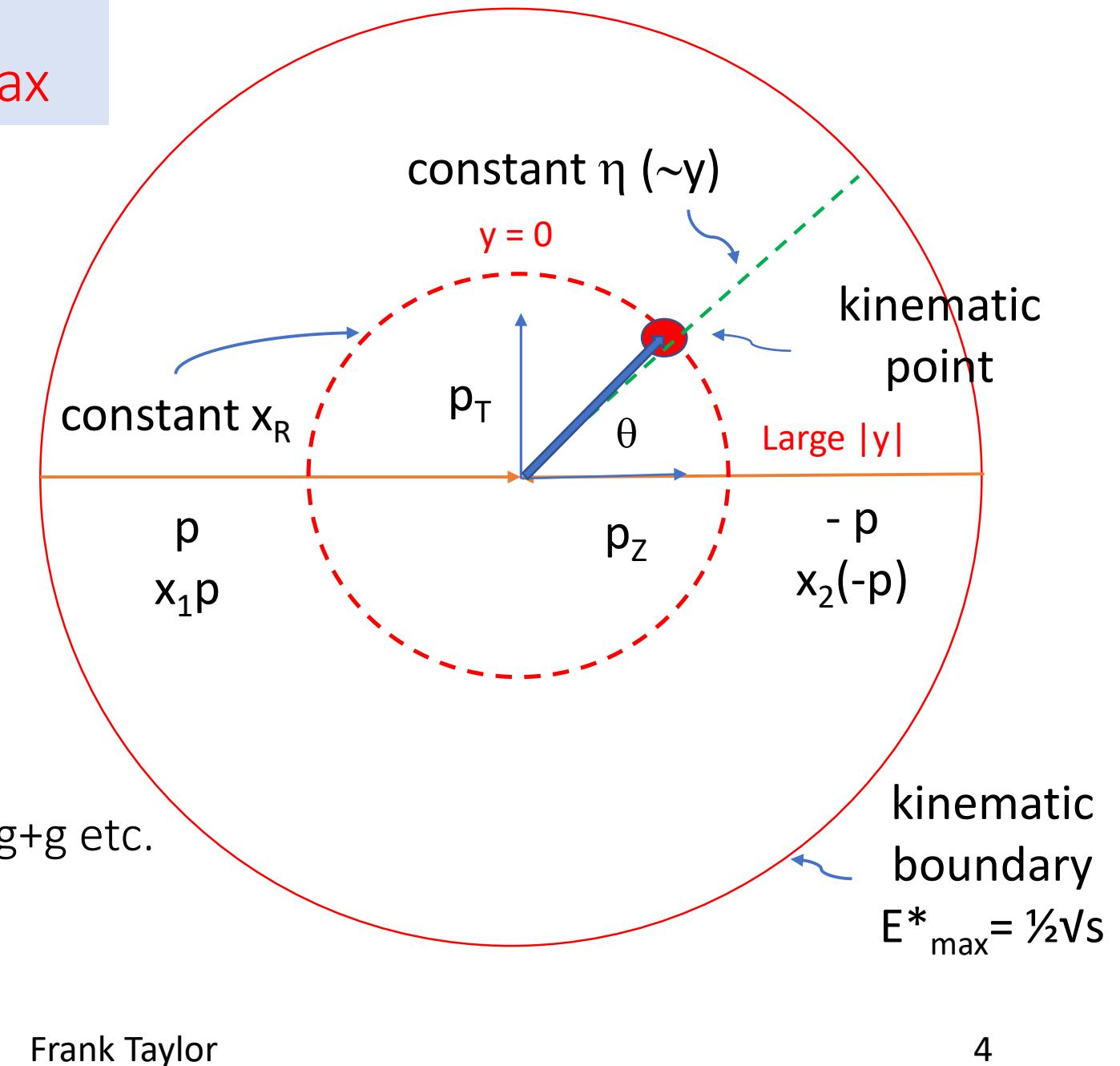
$$\eta = -\ln \left[ \tan \left( \frac{\theta}{2} \right) \right] \approx y$$

- parton-parton COM

- elastic scattering:  $q+q \rightarrow q+q$ ,  $g+g \rightarrow g+g$  etc.

$$\beta = \frac{x_1 - x_2}{x_1 + x_2}$$

$$x_R \approx \frac{\sqrt{s}}{\sqrt{s}} = \sqrt{x_1 x_2}$$

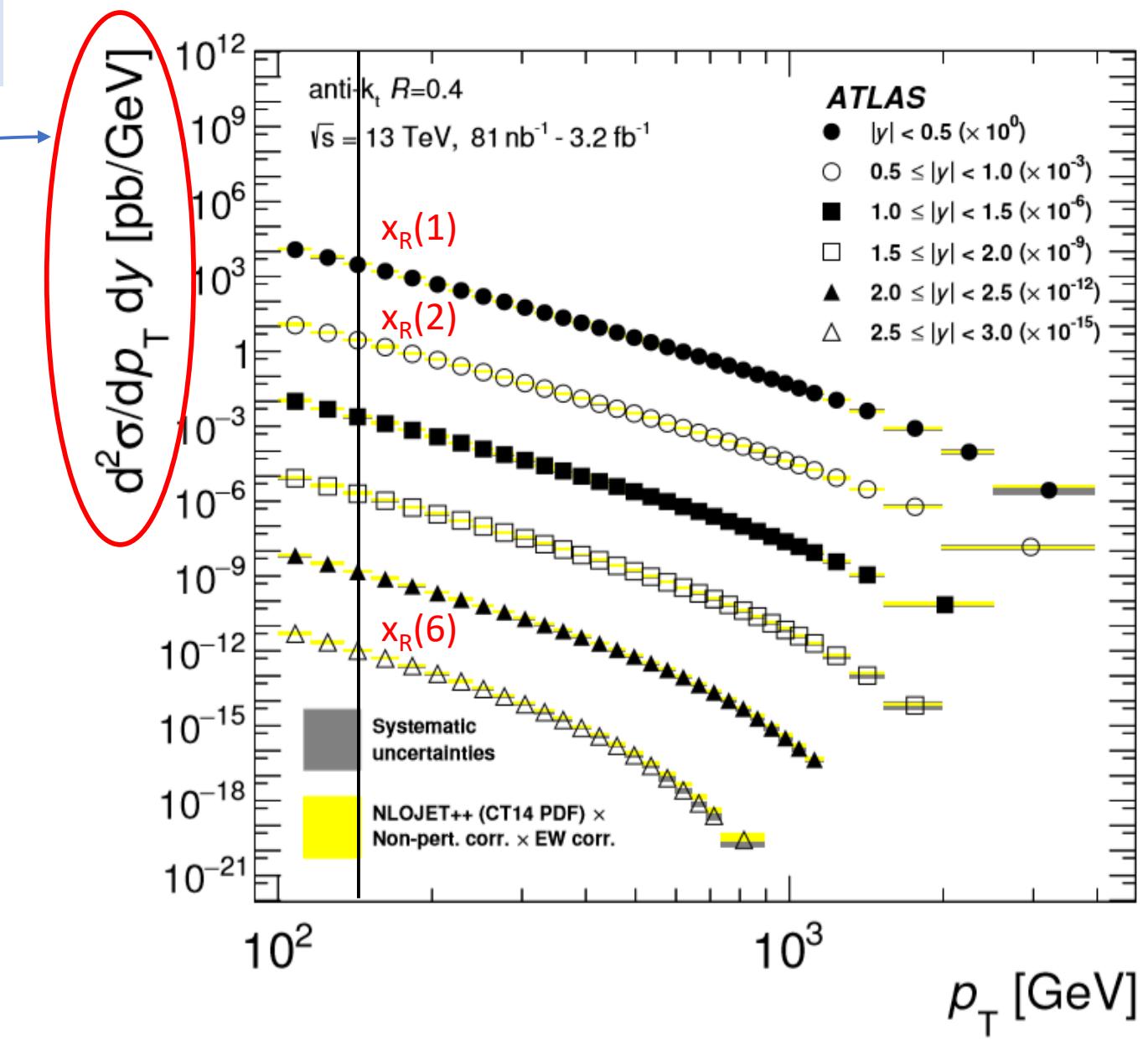


# A Different Look at Jets

- Odd Dimensions  $\sim 1/(\text{GeV}/c)^3$
- Convert to same dimensions as  $\frac{d\sigma}{d\hat{t}}$

$$\frac{Ed^3\sigma}{dp^3} \rightarrow \frac{d^2\sigma}{2\pi p_T dp_T dy} \sim \frac{d\sigma}{d\hat{t}} \left[ \frac{1}{(GeV/c)^4} \right]$$

- Project data onto lines of constant  $p_T$  and convert  $(p_T, y)$  to  $x_R$
- Plot the so-arranged constant  $p_T$  reformatted-data vs.  $(1-x_R)^{nxR}$  as suggested by proton structure functions



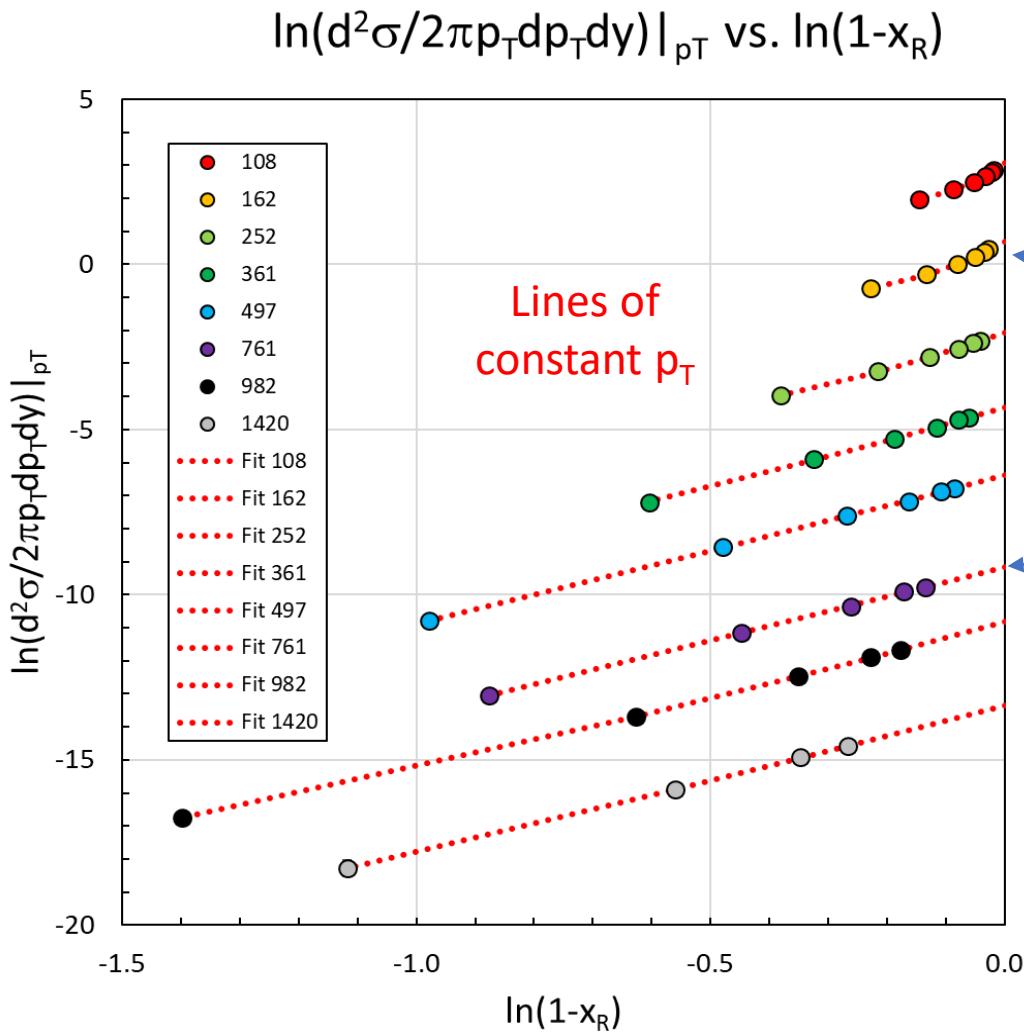
# Ansatz of Inclusive Jet Cross section

- The invariant cross section can be **factorized** into a  $p_T$ - $\sqrt{s}$  sector independent of  $y$  (**A-function**) and a sector dependent of everything else (**f-function**) [FET, PRD 2018, 97, 054016, and earlier]
- Analysis goal is to separate the two sectors (**A, f**) and study them
- Expect that there are **underlying power laws** since jets originate from hard scattering of point-like particles

$$\frac{d^2\sigma}{2\pi p_T dp_T dy} = C(\sqrt{s}, p_T, x_R(p_T, m, y, \sqrt{s})) = A(\sqrt{s}, p_T, \Lambda) f(\sqrt{s}, p_T, x_R)$$

$$A \sim \frac{1}{p_T^{n_{pT}}} \quad f \sim (1-x_R)^{n_{xR}} \quad n_{pT} \text{ and } n_{xR} \text{ are power indices}$$

# Analysis of 13 TeV R=0.4 ATLAS Jets

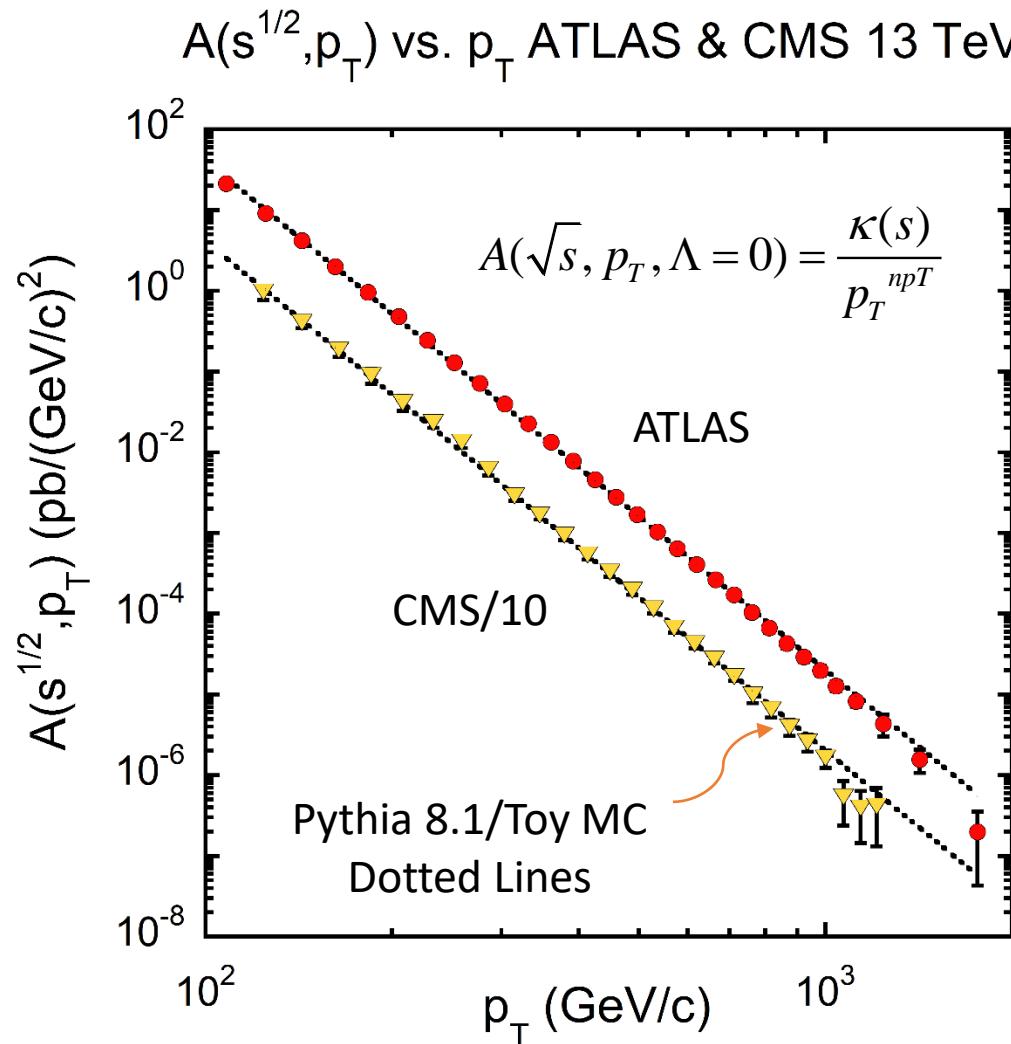


$$\ln\left(\frac{d^2\sigma}{2\pi p_T dp_T dy}\right)_{p_T} = \ln(A) + n_{xR} \ln(1 - x_R) + n_{xRQ} \ln^2(1 - x_R)$$

$\ln(A(p_T))$  values are the intercepts

- Extrapolation to  $x_R = 0$  (factor of 1.06 to 2.3) evaluates the cross section in the limit of no ‘soft physics’ – hence probes the parton-parton hard scattering
- Determine the amplitude function  $A(p_T)$  and two power indices  $n_{xR}$  and  $n_{xRQ}$  by fits

# ATLAS and CMS Inclusive Jets $R = 0.4$ , $\sqrt{s} = 13 \text{ TeV}$

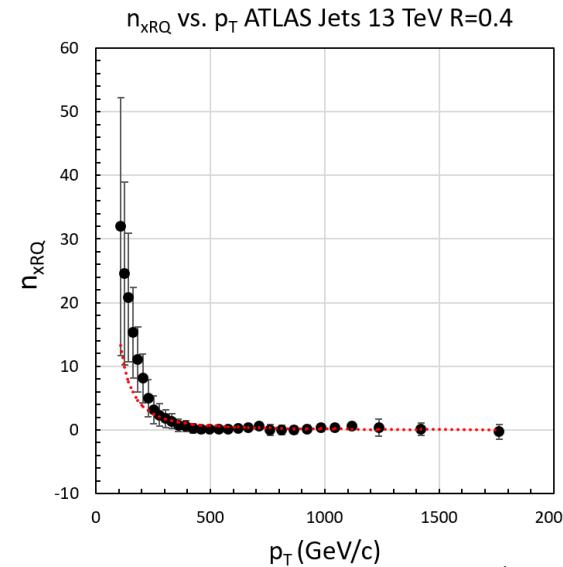
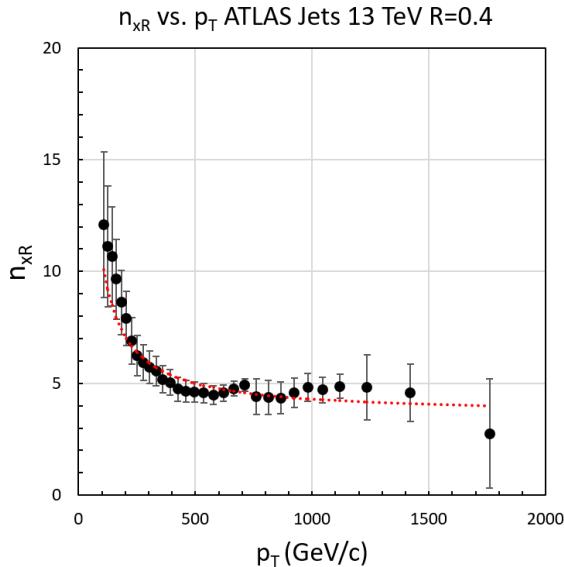


Experiment	Parameter	Value	$\chi^2$	Degrees of freedom
ATLAS	$\kappa$ $n_{pT}$	$(2.1 \pm 0.3) \times 10^{14} \text{ pb}/(\text{GeV}/c)^{(2-n_{pT})}$ $6.35 \pm 0.02$	33	29
CMS	$\kappa$ $n_{pT}$	$(3.1 \pm 1.0) \times 10^{14} \text{ pb}/(\text{GeV}/c)^{(2-n_{pT})}$ $6.41 \pm 0.05$	15	25

A power law !  
 Residuals  $\sim + 10\%, - 30\%$  over  $10^8$   
 But power index  $\approx 6.4$  rather than the expected  $\approx 4$

# The $p_T$ - $x_R$ - $y$ Sector

- Quadratic log fits determine  $A(p_T)$ ,  $n_{xR}$  and  $n_{xRQ}$  – now examine the  $n_{xR}$ 's

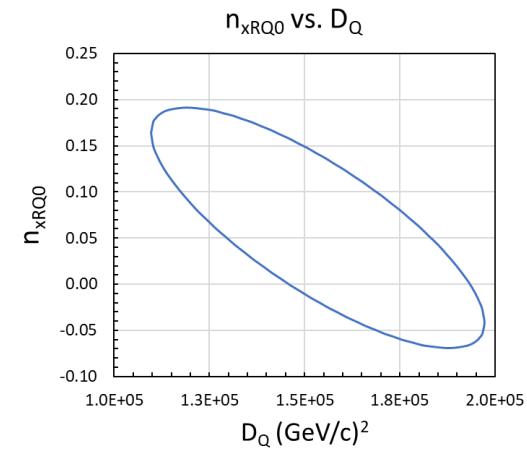
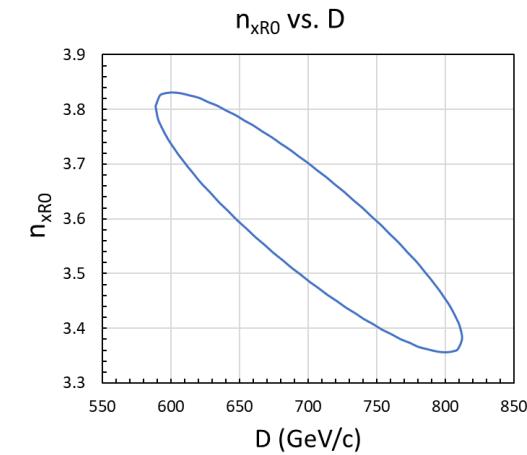


$$n_{xR}(\sqrt{s}, p_T) = \frac{D(\sqrt{s})}{p_T} + n_{xR0}$$

$$n_{xRQ}(\sqrt{s}, p_T) = \frac{D_Q(\sqrt{s})}{p_T^2} + n_{xRQ0}$$

D and  $D_Q$  are  
“Distortion”  
parameters

ATLAS	D $n_{xR0}$	$(7.0 \pm 1.1) \times 10^2$ GeV/c $3.6 \pm 0.2$	14	29
CMS	D $n_{xR0}$	$(7.5 \pm 3.1) \times 10^2$ GeV/c $3.3 \pm 0.6$	7	25
ATLAS	$D_Q$ $n_{xRQ0}$	$(1.5 \pm 0.4) \times 10^5$ (GeV/c) <sup>2</sup> $0.06 \pm 0.1$	24	29
CMS	$D_Q$ $n_{xRQ0}$	$(2.0 \pm 1.3) \times 10^5$ (GeV/c) <sup>2</sup> $0.08 \pm 0.4$	8	25



# Extract the $x_R$ -Dependence

- Determine  $x_R$  dependence by:

$$f(\sqrt{s}, p_T, x_R) = \frac{1}{A(\sqrt{s}, p_T, \Lambda)} \frac{d^2\sigma}{2\pi p_T dp_T dy} = \exp\left(n_{xR} \ln(1-x_R) + n_{xRQ} \ln^2(1-x_R)\right)$$

- Pure “radial scaling” would mean RHS would be independent of  $p_T$  – **clearly violated** because distortion parameters  $D, D_Q \neq 0$  !
- But by  $D$  and  $D_Q$   $p_T$ -dependences a **factorization** is possible:

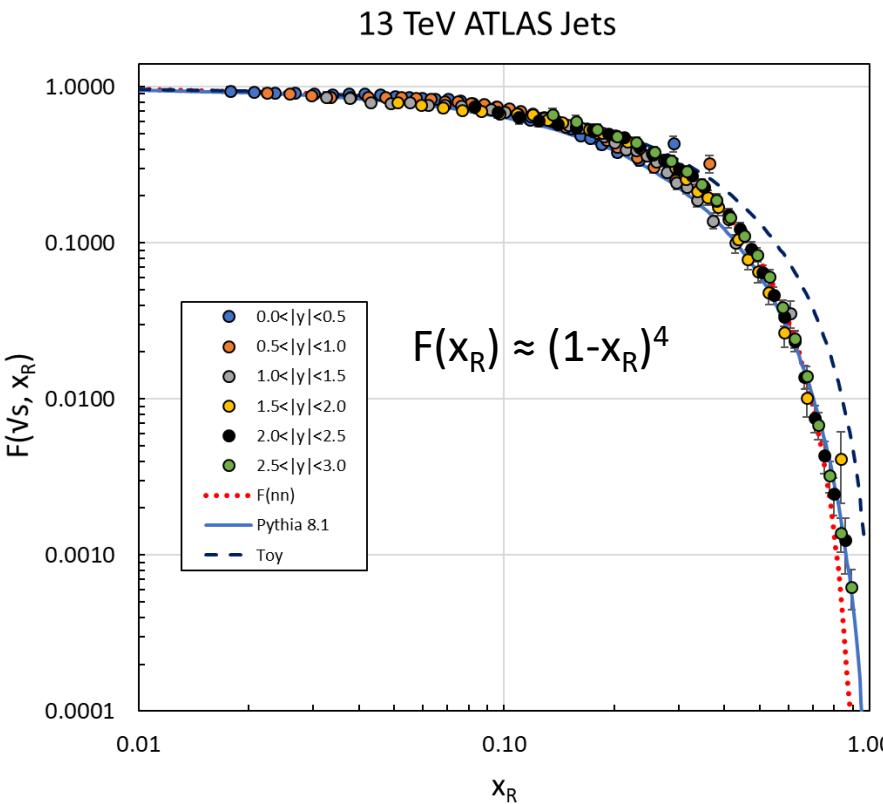
$$f(\sqrt{s}, p_T, x_R) = \exp\left(\frac{D}{p_T} \zeta + \frac{D_Q}{p_T^2} \zeta^2\right) \exp\left(n_{xR0} \zeta + n_{xRQ0} \zeta^2\right)$$

$$\zeta = \ln(1-x_R)$$

# Radial Scaling Function for Inclusive Jets

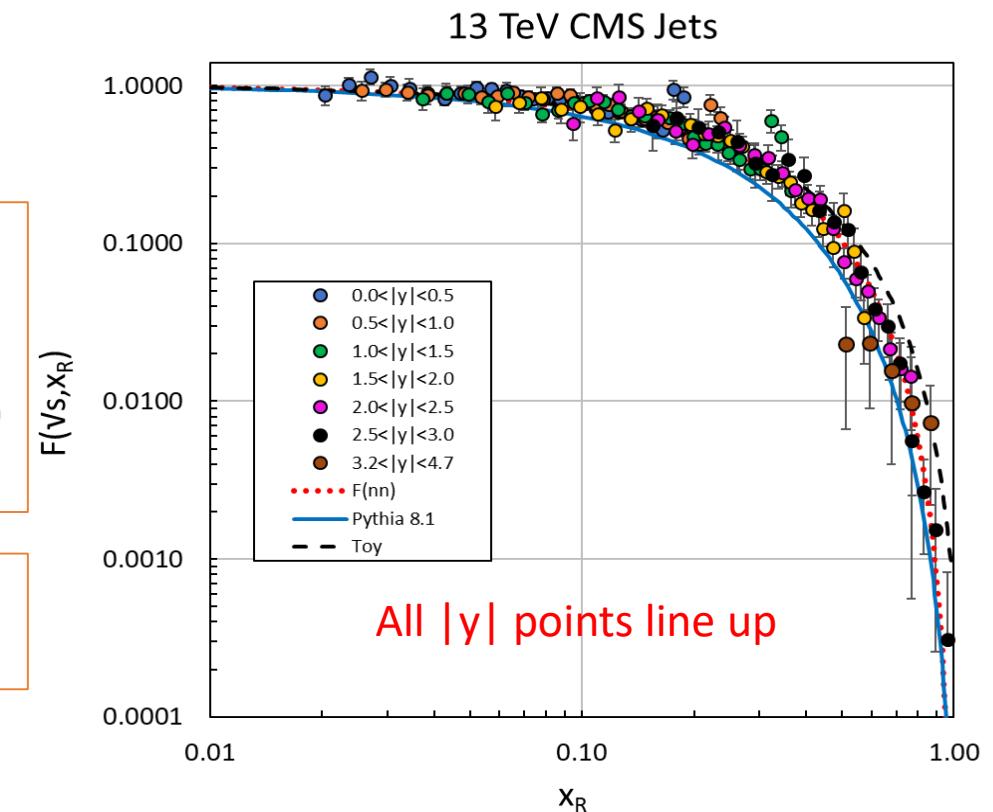
- We already know  $F(\sqrt{s}, x_R)$  by previous factorization:

$$F(\sqrt{s}, x_R) = \exp\left(n_{xR0} \ln(1-x_R) + n_{xRQ0} \ln^2(1-x_R)\right)$$



Red-dotted lines  
derived/  $n_{xR0}$  &  $n_{xRQ0}$   
ATLAS  
 $\chi^2/df = 1.06$  for 170  
points

Pythia better  
simulation than Toy



# Toy MC & Pythia 8.1

- Use MC simulations to understand  $A(p_T)$  and  $F(x_R)$ 
  - $\Lambda = 0$  for jets

$$\frac{d^2\sigma(p + p \rightarrow jet + x)}{2\pi p_T dp_T dy} = A(\sqrt{s}, p_T, \Lambda) Y(\sqrt{s}, y) F(\sqrt{s}, x_R)$$

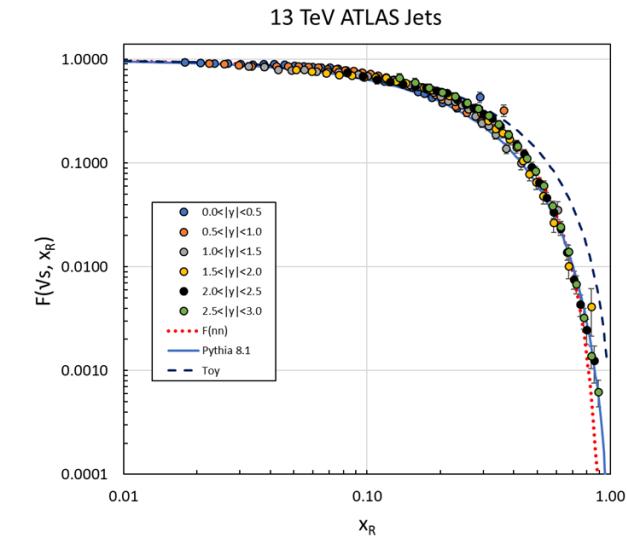
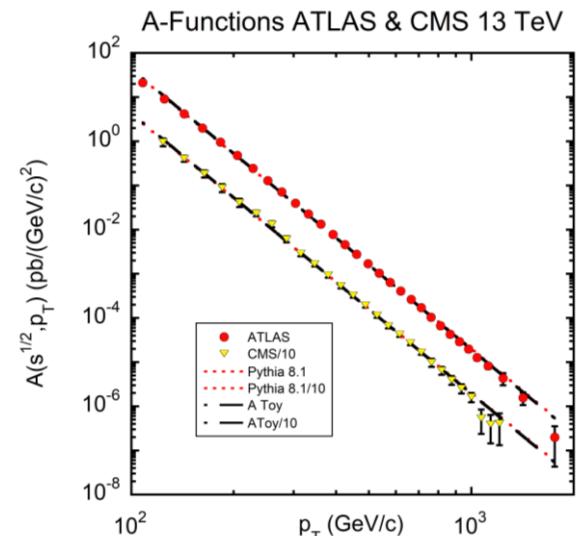
$$A(\sqrt{s}, p_T, \Lambda) = \frac{\kappa(s)}{(p_T^2 + \Lambda^2)^{\frac{npT}{2}}} = \frac{\kappa(s)}{P_T^{npT}}$$

$$Y(\sqrt{s}, y) = \exp\left(\frac{D}{p_T} \zeta + \frac{D_Q}{p_T^2} \zeta^2\right)$$

$$F(\sqrt{s}, x_R) = \exp(n_{xR0}\zeta + n_{xRQ0}\zeta^2)$$

Process	$np_T$
ATLAS	$6.35 \pm 0.02$
CMS	$6.41 \pm 0.05$
Pythia 8.1	$6.308 \pm 0.005$
All Toy	$6.351 \pm 0.017$

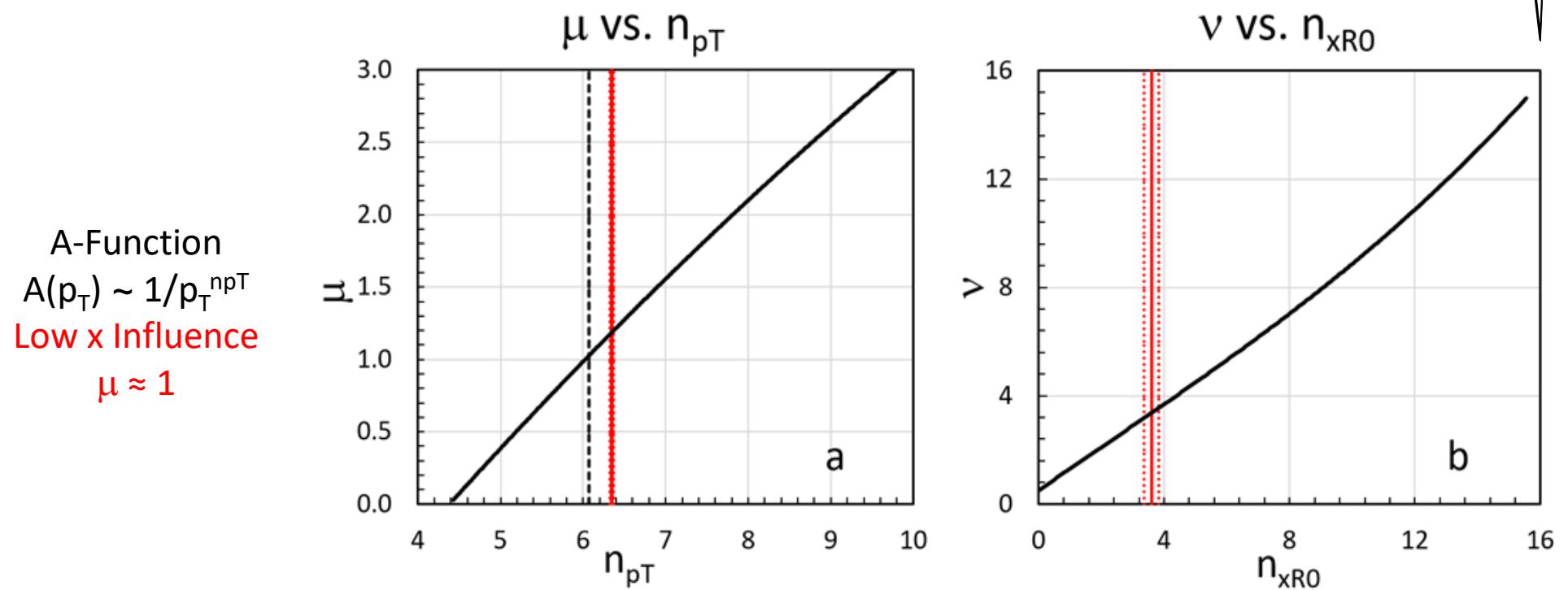
Data and Simulation of  $p_T$ -dependence  
of  $A(p_T)$  in good agreement



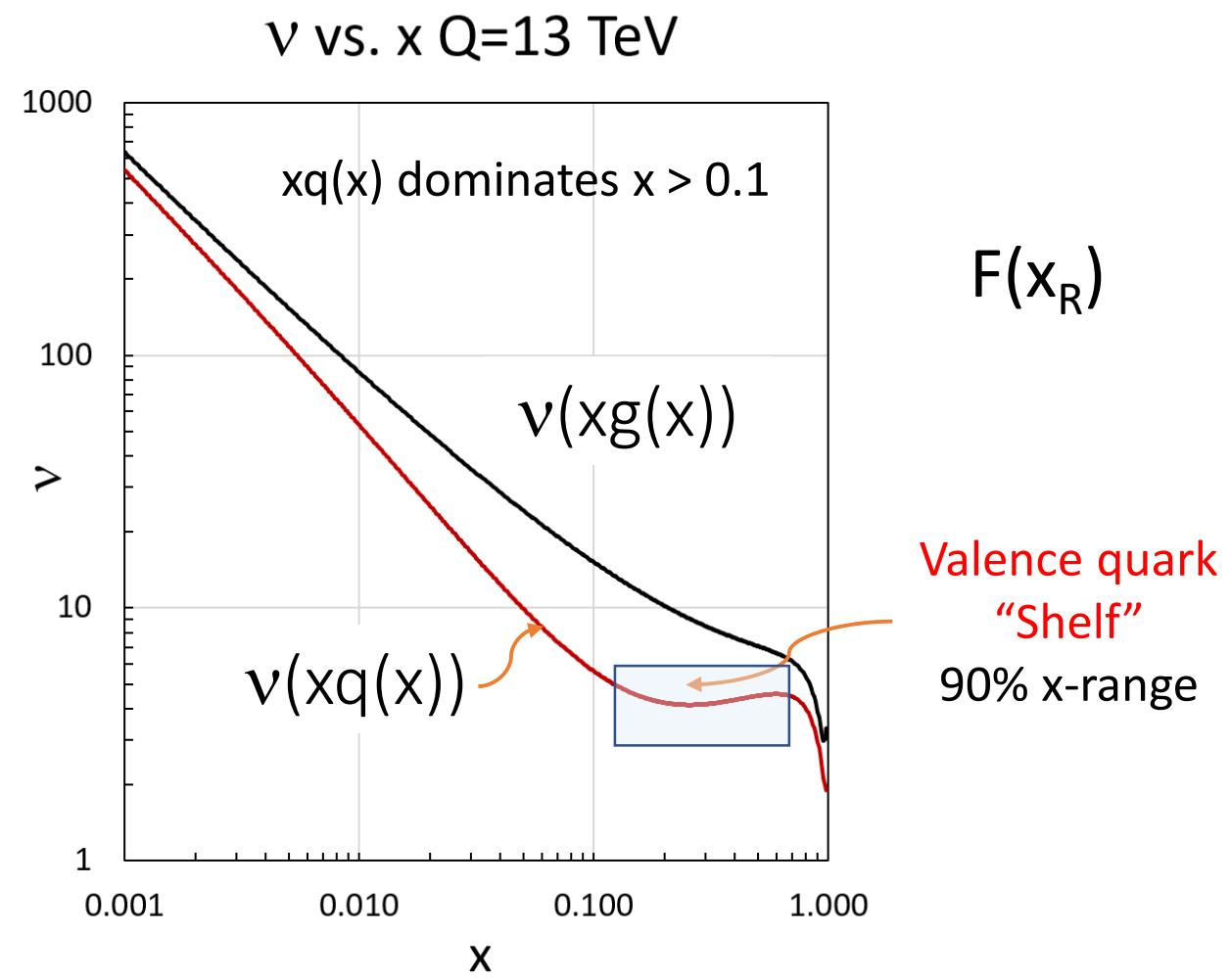
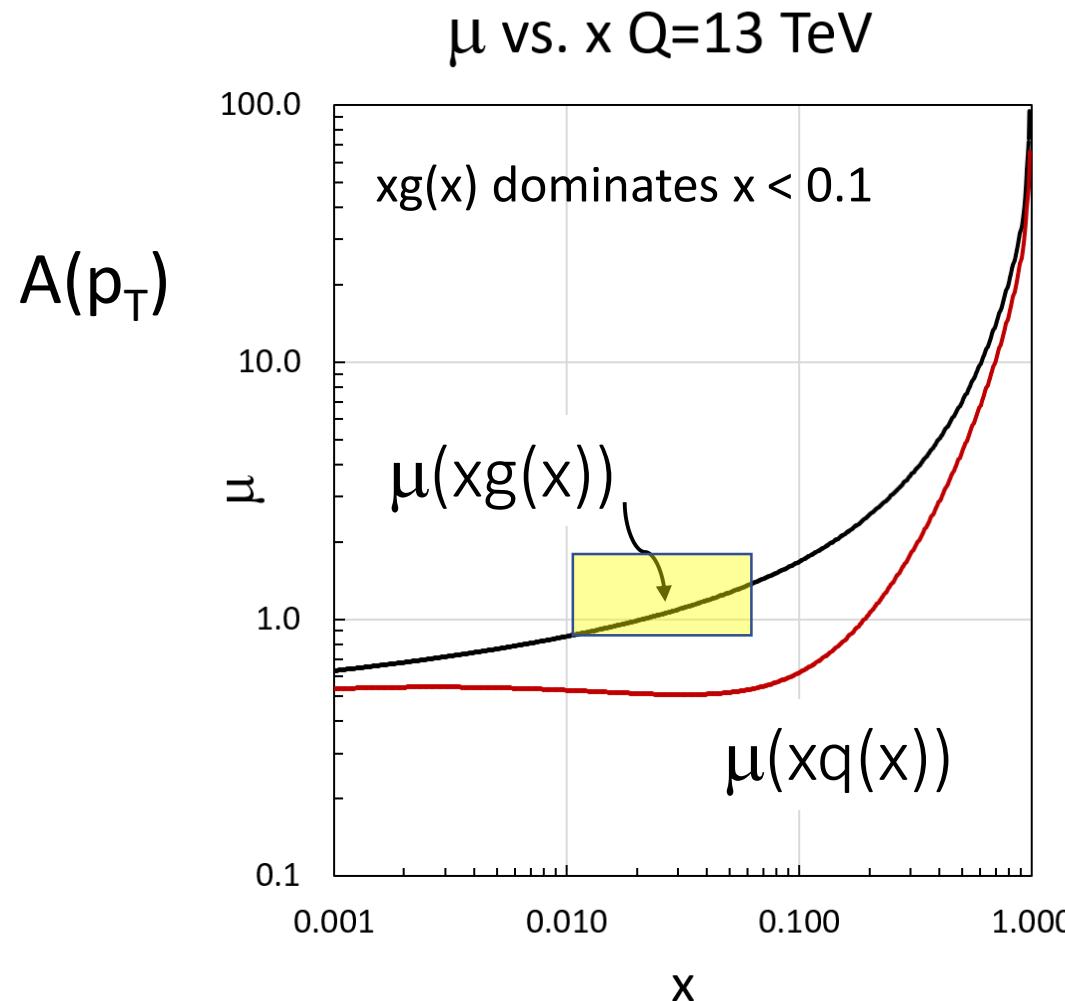
# Shapes of PDFs Determine Shape of $A(p_T)$ and $F(x_R)$

- Consider two extreme forms PDFs

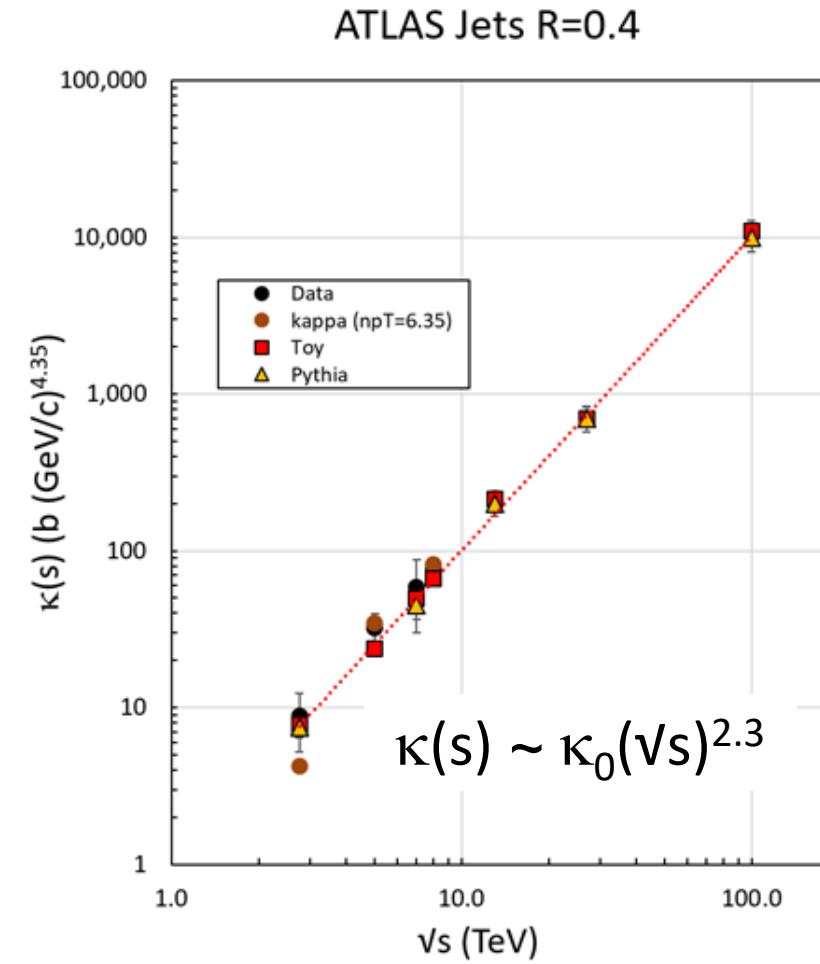
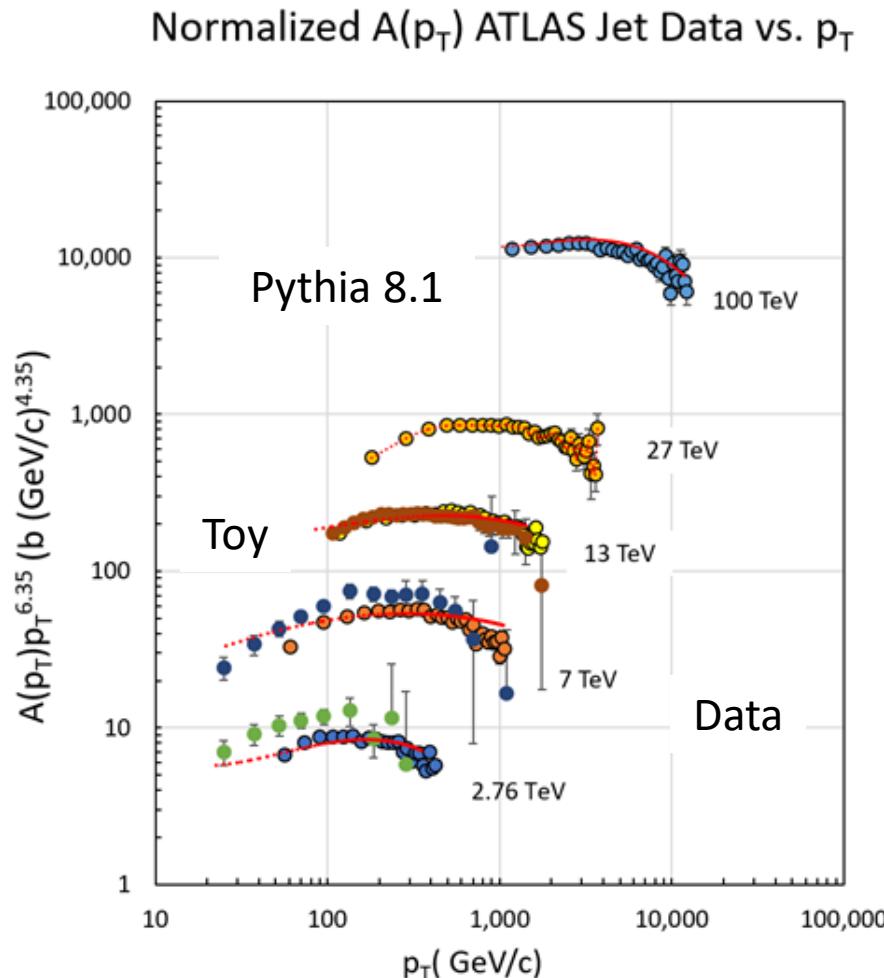
- $xg(x) = 1/x^\mu$  (Pomeron-like) and  $xq(x) = (1-x)^\nu$  (valence-like)
- Study behavior of  $A(p_T)$  and  $F(x_R)$  as a function of  $\mu$  and  $\nu$



# Log-derivatives of $xg(x)$ and $xq(x)$ PDFs (CTEQ10) to find $\mu$ and $\nu$



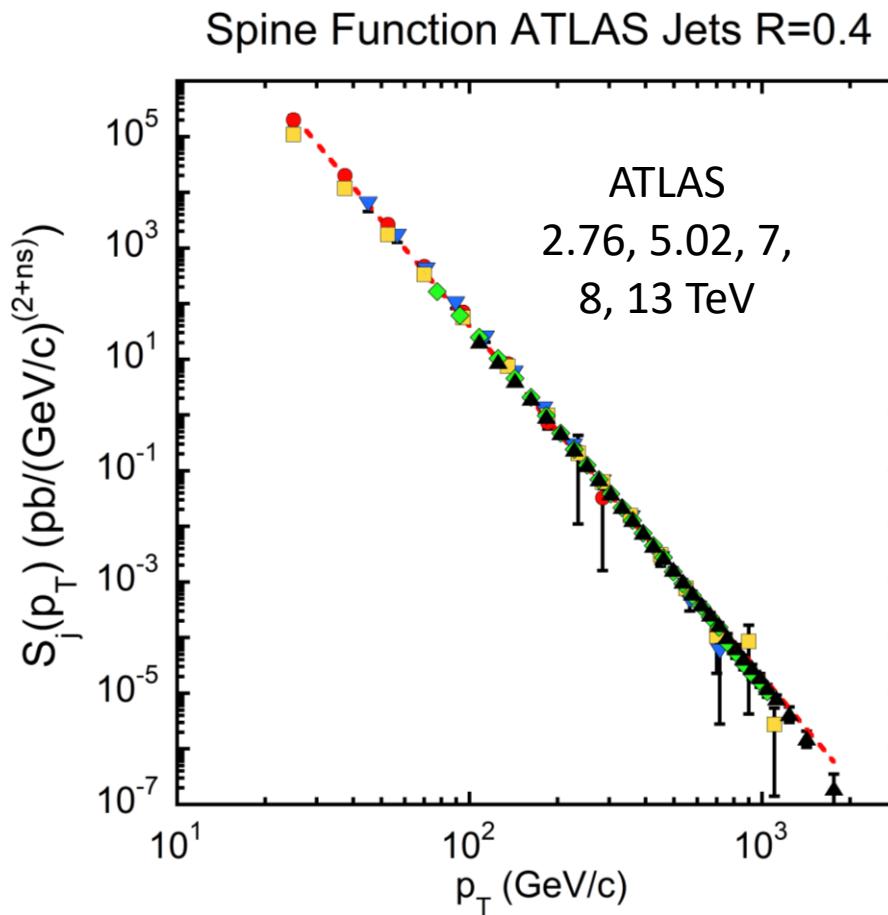
$A(\sqrt{s}, p_T)$  grows as  $(\sqrt{s})^{ns}$ , where  $ns = 2.3 \pm 0.7$



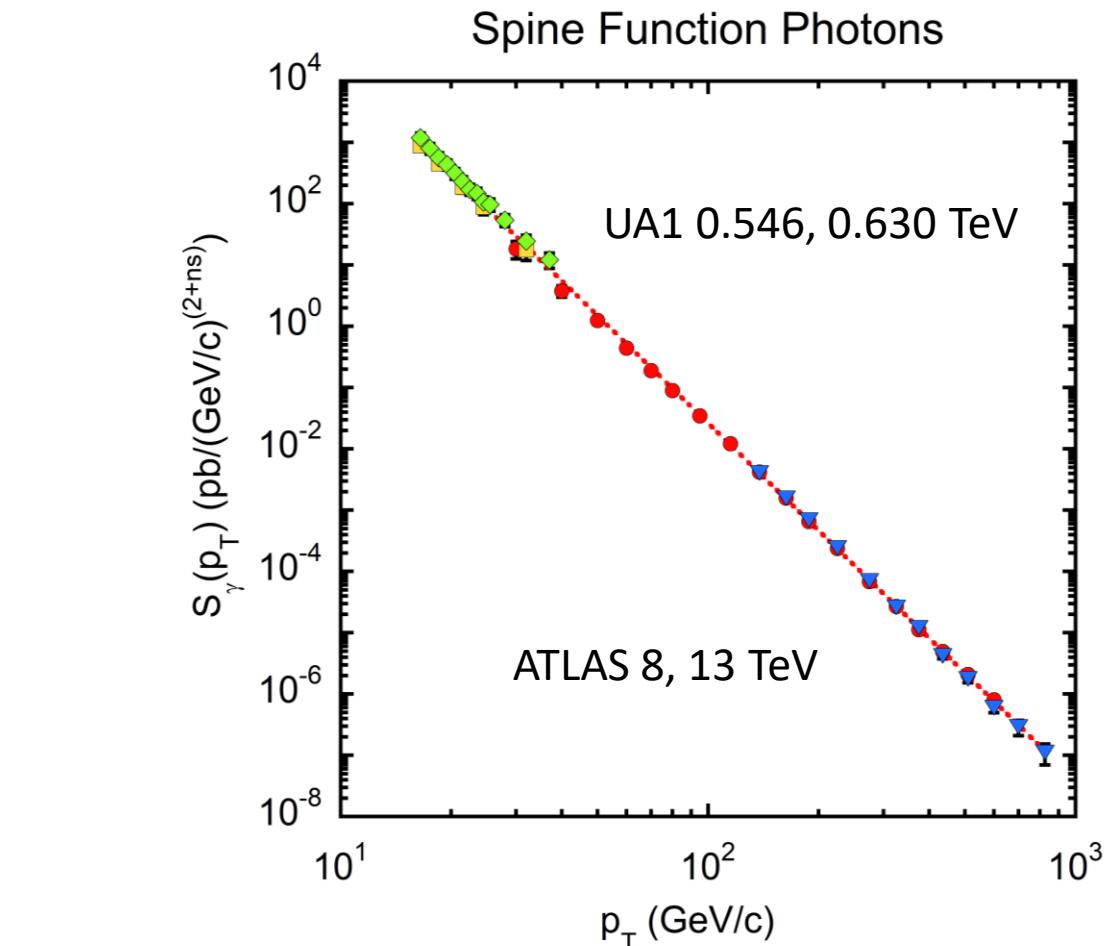
# Spine Functions Jets and Photons

$$S_{j,\gamma}(p_T) \equiv \frac{A(\sqrt{s}, p_T)}{(\sqrt{s})^{ns}} = \frac{\kappa_0}{p_T^{npT}}$$

Remove the s-dependence in the  $A(p_T)$  functions – then all jet (photon) data should follow same power law if  $n_{pT}$  = constant for jets and for photons



12/07/2021



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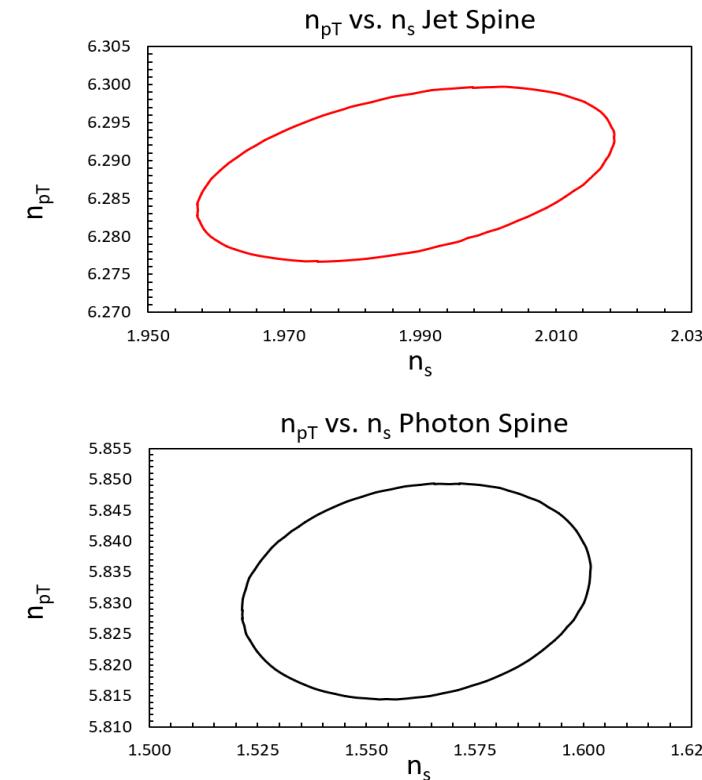
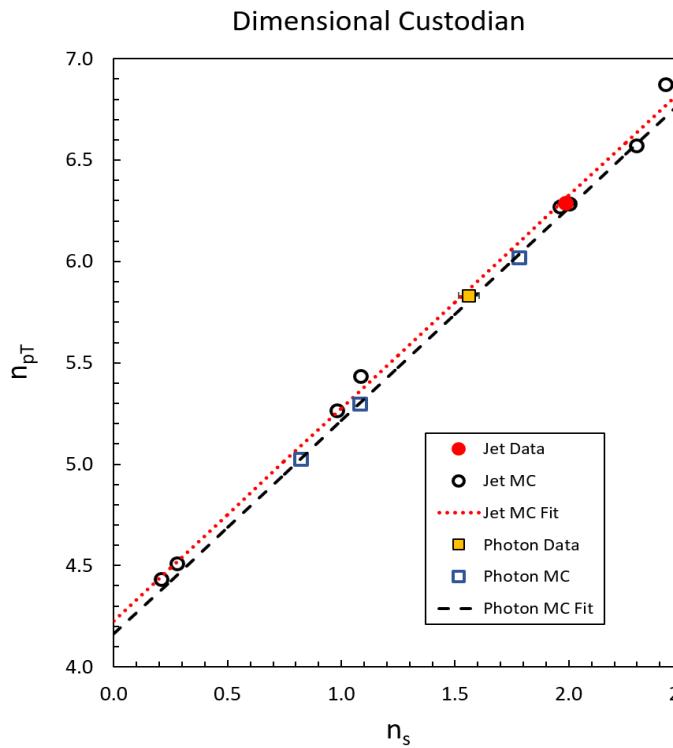
16

# Dimensional Custodian

$$\left[ \frac{d^2\sigma}{2\pi p_T dp_T dy} \right] = \left[ \frac{d\sigma}{d\hat{t}} \right] = \left[ A(\sqrt{s}, p_T) \right] = \left[ \frac{cm^2}{(GeV/c)^2} \right] = \left[ \frac{1}{(GeV/c)^4} \right]$$

- Therefore expect:  $n_{pT} - n_s - 4 = 0$   $\sqrt{s}$ -dependence of A compensates for  $1/p_T^{\text{npT}}$
- Jets:  $(6.29 \pm 0.02) - (1.99 \pm 0.04) - 4 = 0.30 \pm 0.03$  ( $7.5 \sigma$ )
- Photons:  $(5.83 \pm 0.02) - (1.56 \pm 0.04) - 4 = 0.27 \pm 0.05$  ( $5.4 \sigma$ )

From Spine Function



$$n_{pT}(\text{jets}) = (1.00 \pm 0.02)n_s + 4.28 \pm 0.03$$

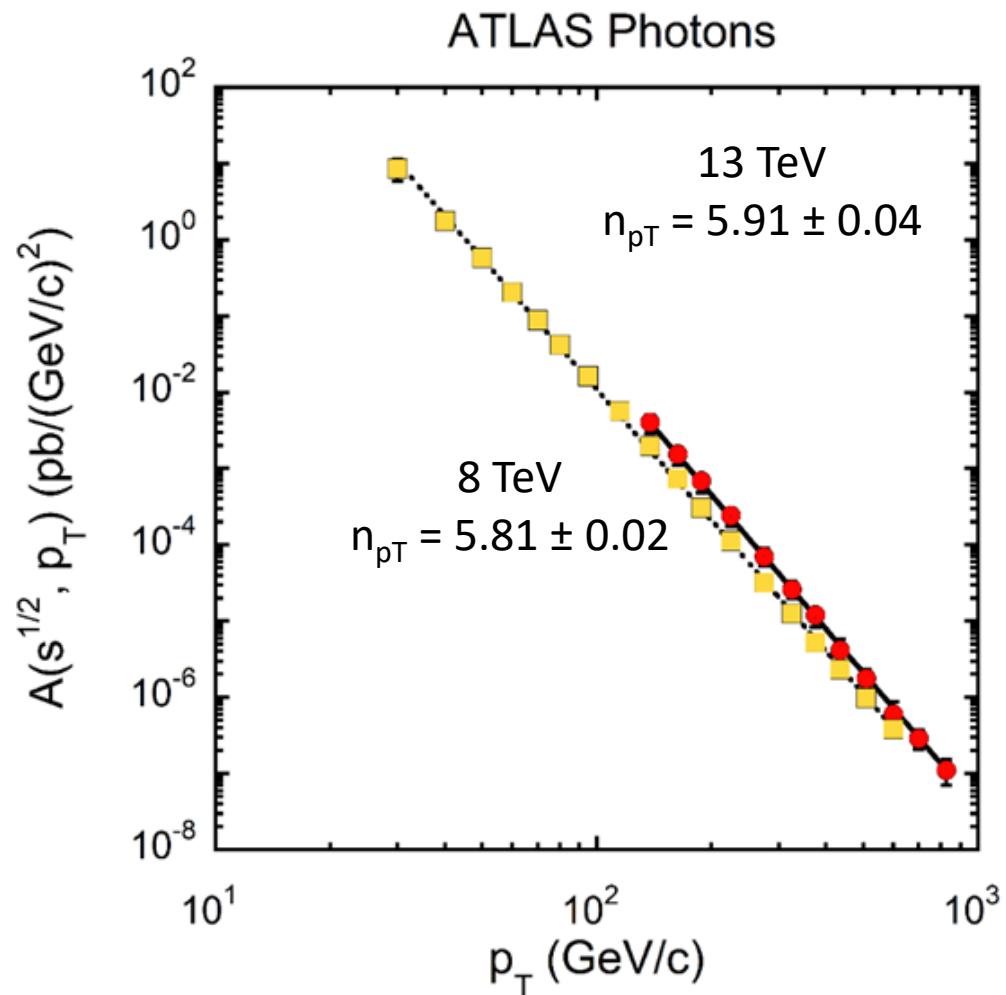
$$n_{pT}(\gamma) = (1.05 \pm 0.03)n_s + 4.16 \pm 0.04$$

Residual power (intercept - 4) from evolution of  $\alpha_s^2(Q^2)$  for jets and  $\alpha_s(Q^2)$  for photons and evolution of PDFs

$$n_r \sim \left( \frac{2p_{T\min}}{\alpha_s(p_{T\min})} \right) \frac{d\alpha_s}{dp_T} \sim 2.42 \alpha_s(p_{T\min})$$

$\sim 0.3$  for  $p_{T\min} = 70 \text{ GeV}/c$

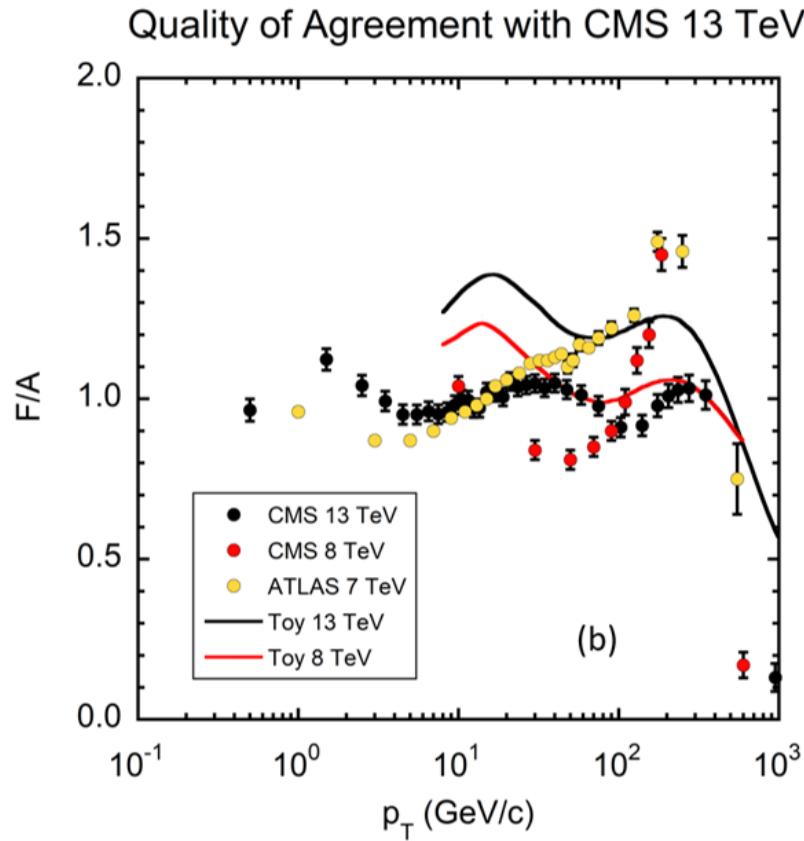
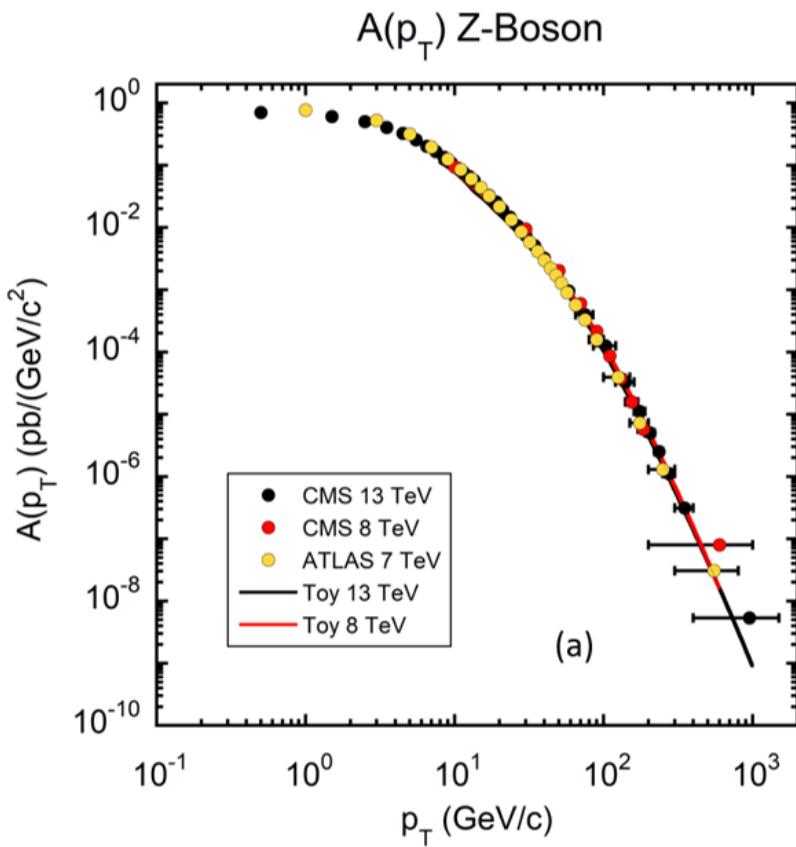
# Inclusive Photons



- Bremsstrahlung channels dominate
  - Hard scattering  $\sim 1/p_T^2 \rightarrow$  expect  $n_{pT}$  to be smaller than  $n_{pT}$  for jets
- Toy MC  $n_{pT} = 5.3$  smaller than data ( $5.91 \pm 0.04$ )  
– but no isolation cuts in Toy MC, etc.

Process	Leading $p_T$	Value at $p_T = \sqrt{s}/2$
$gq \rightarrow \gamma q$	$\frac{d\sigma}{dt} \approx \frac{\pi \alpha_e \alpha_s}{s} \left(\frac{e_q^2}{3}\right) \frac{1}{p_T^2}$	$\frac{d\sigma}{dt} = \frac{\pi \alpha_e \alpha_s}{p_T^4} \left(\frac{5e_q^2}{96}\right)$
$q\bar{q} \rightarrow \gamma g$	$\frac{d\sigma}{dt} \approx \frac{\pi \alpha_e \alpha_s}{s} \left(\frac{8e_q^2}{9}\right) \frac{1}{p_T^2}$	$\frac{d\sigma}{dt} = \frac{\pi \alpha_e \alpha_s}{p_T^4} \left(\frac{e_q^2}{18}\right)$
$q\bar{q} \rightarrow \gamma\gamma$	$\frac{d\sigma}{dt} \approx \frac{\pi \alpha_e^2}{s} \left(\frac{e_q^4}{2}\right) \frac{1}{p_T^2}$	$\frac{d\sigma}{dt} = \frac{\pi \alpha_e^2}{p_T^4} \left(\frac{e_q^4}{24}\right)$
$gg \rightarrow \gamma\gamma$	$\frac{d\sigma}{dt} \sim \frac{\alpha_s^2}{8\pi^2} \frac{\pi \alpha_e^2}{s^2} \left(\sum_{i=1}^{nf} e q_i^2\right)^2 \sum_i T_i$ $\sim (1 \times 10^{-3}) \frac{d\sigma(q\bar{q} \rightarrow \gamma g)}{dt}$	See Owens [30] (neglected)

# Inclusive Z-Bosons



- Analyze Z data in the same manner: determine  $A(p_T)$  by  $x_R \rightarrow 0$  extrapolation, study  $n_{xR}(p_T)$  and compare data at different  $\nu$ s.
- Used Toy simulation of inclusive photons by considering the Z as a heavy photon – adequate fit of  $A(p_T)$  for  $p_T > 7 \text{ GeV}/c$ .
- Toy MC does not simulate the  $n_{xR}(p_T)$  behavior very well – in fact there seems to be a lot of s - and  $p_T$ - dependent structure.

# Production of Heavy Mesons/Baryons

- LHCb Collaboration provides a rich trove of low  $p_T$ , high  $|y|$  data for heavy mesons
- Analyze in same manner as jets and photon: log-log fits, determine the A and F functions
- Expect  $n_{pT}$  in modified transverse momentum,  $P_T$ , to be smaller than that for jets

Process	Leading $p_T$	Value at $p_T = \frac{1}{2}\sqrt{\hat{s} - 4m^2}$ $\hat{s} = 4P_T^2$
$gg \rightarrow Q\bar{Q}$	$\frac{d\sigma}{dt} = \frac{\pi\alpha_s^2}{s^2} \left( \frac{s}{6(m^2 + p_T^2)} - \frac{3}{8} \right) \left[ 1 - 2 \frac{m^4 + p_T^4}{s(m^2 + p_T^2)} \right]$ $\frac{d\sigma}{dt} \approx \frac{\pi\alpha_s^2}{s} \left( \frac{1}{6(m^2 + p_T^2)} \right)$	$\frac{d\sigma}{dt} = \frac{7}{768} \frac{\pi\alpha_s^2}{P_T^4} \left( 1 + \frac{2m^2}{P_T^2} - \frac{2m^4}{P_T^4} \right)$
$q\bar{q} \rightarrow Q\bar{Q}$	$\frac{d\sigma}{dt} = \frac{4\pi\alpha_s^2}{9} \frac{1}{s^2} \left( 1 - \frac{2p_T^2}{s} \right)$	$\frac{d\sigma}{dt} = \frac{\pi\alpha_s^2}{72P_T^4} \left( 1 + \frac{m^2}{P_T^2} \right)$

Modified transverse momentum is the key variable

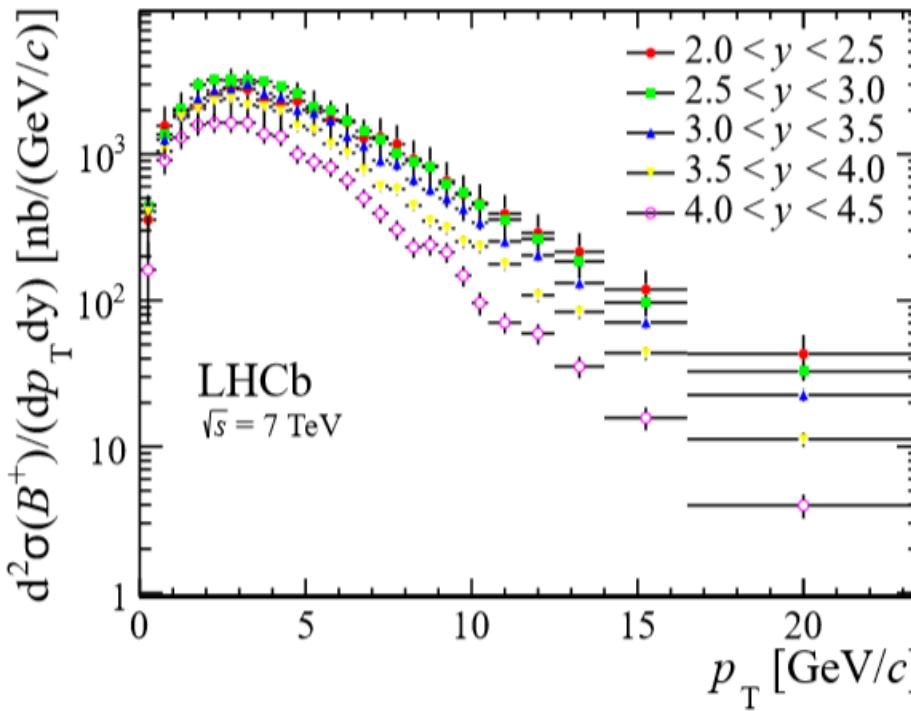
$$\Lambda \sim m$$

$$A(\sqrt{s}, p_T, \Lambda) = \frac{\kappa(s)}{(p_T^2 + \Lambda^2)^{\frac{npT}{2}}} = \frac{\kappa(s)}{P_T^{npT}}$$

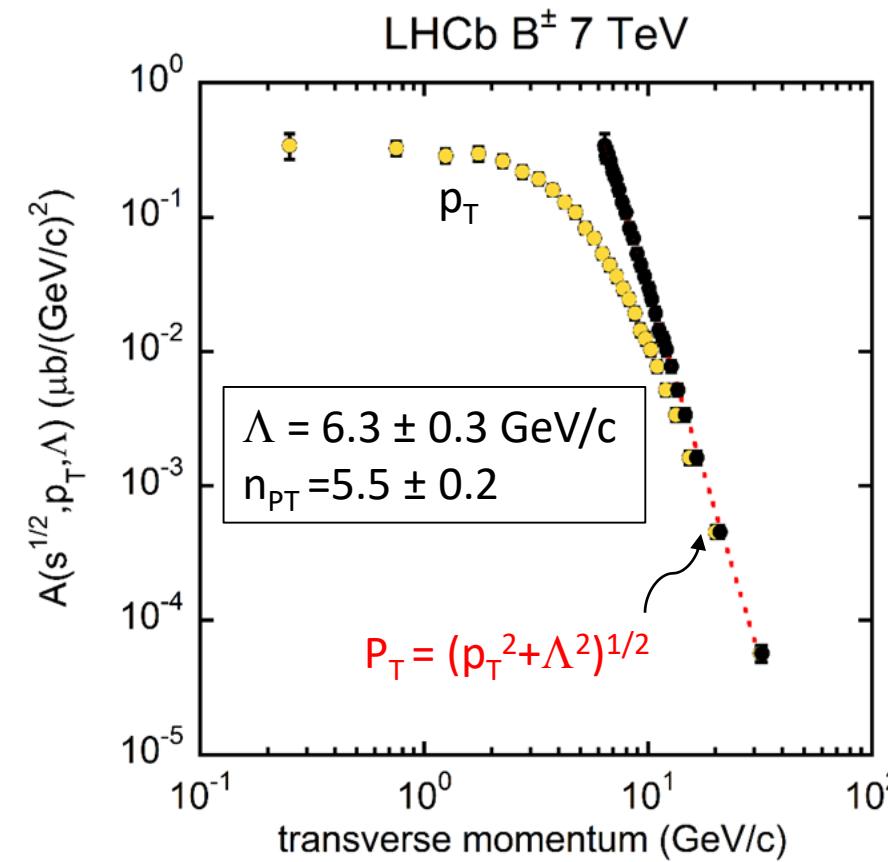
Toy MC only roughly works  
FONNL MC agrees

# LHCb Example B-meson production

- Collaboration uses  $d^2\sigma/dp_T dy$  convention → have to convert to  $d^2\sigma/2\pi p_T dp_T dy$  to directly connect to the underlying hard scattering



arXiv:1306.3663v2 [hep-ex] 1 Oct 2013



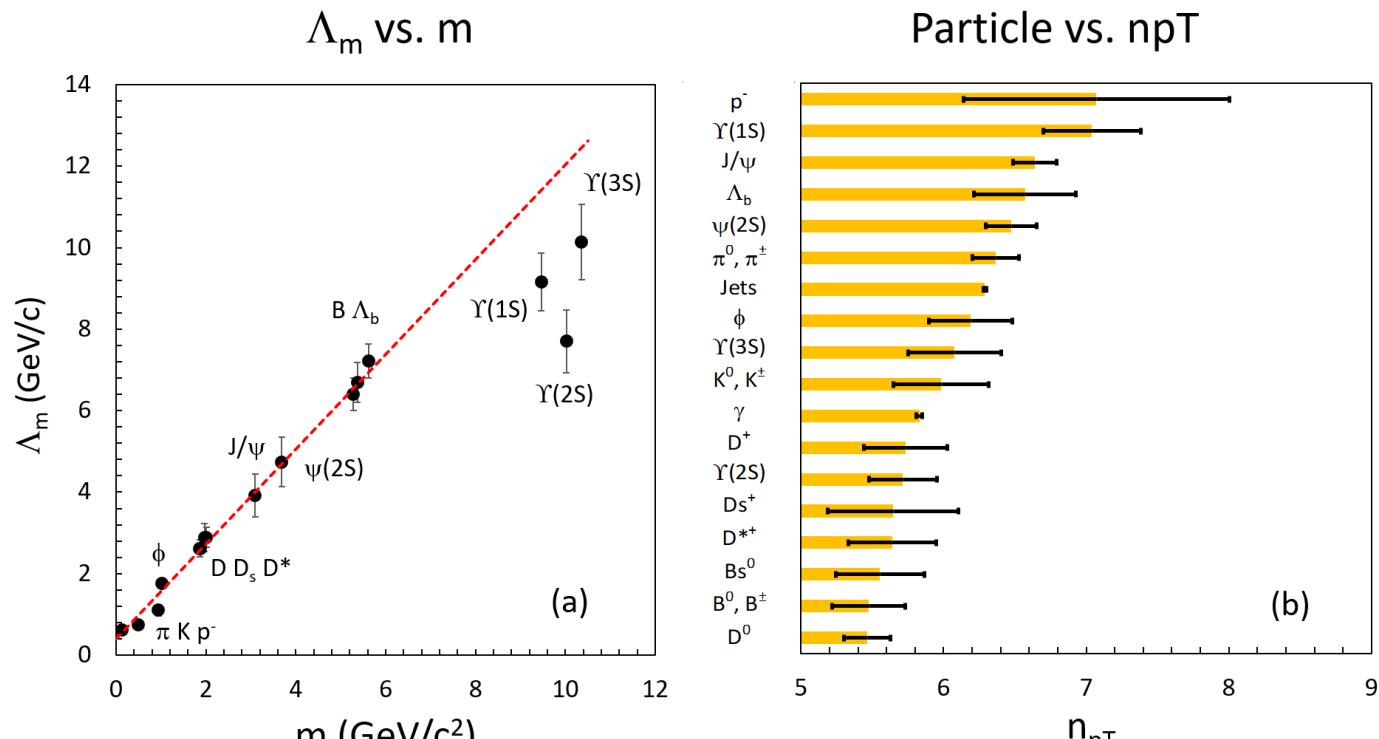
Data follow a  
**power law** in  $P_T =$   
 $(p_T^2 + \Lambda^2)^{1/2}$   
 $A(p_T)$  determines  
both  $\Lambda$  and  $n_{pT}$

# The $\Lambda$ - $m$ Relation

Data points are average of similar measurements and errors are average errors  
Used 'old' data vs ranges from 63 GeV to 13 TeV

Relation sensitive to intrinsic transverse momentum of partons - for  $m = 0$

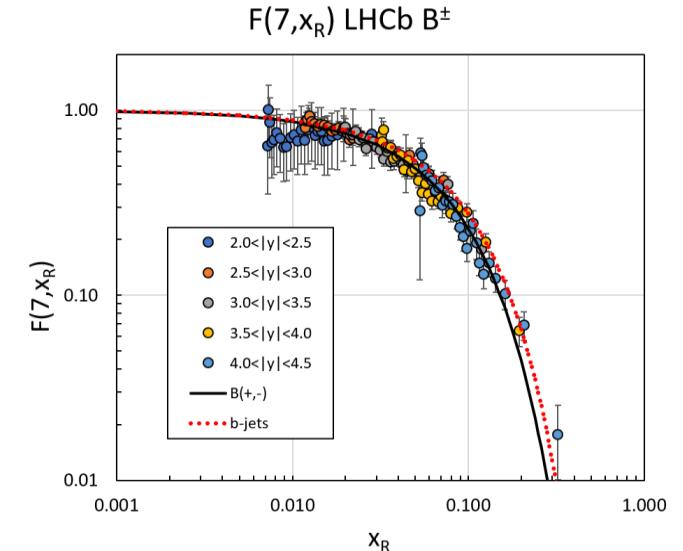
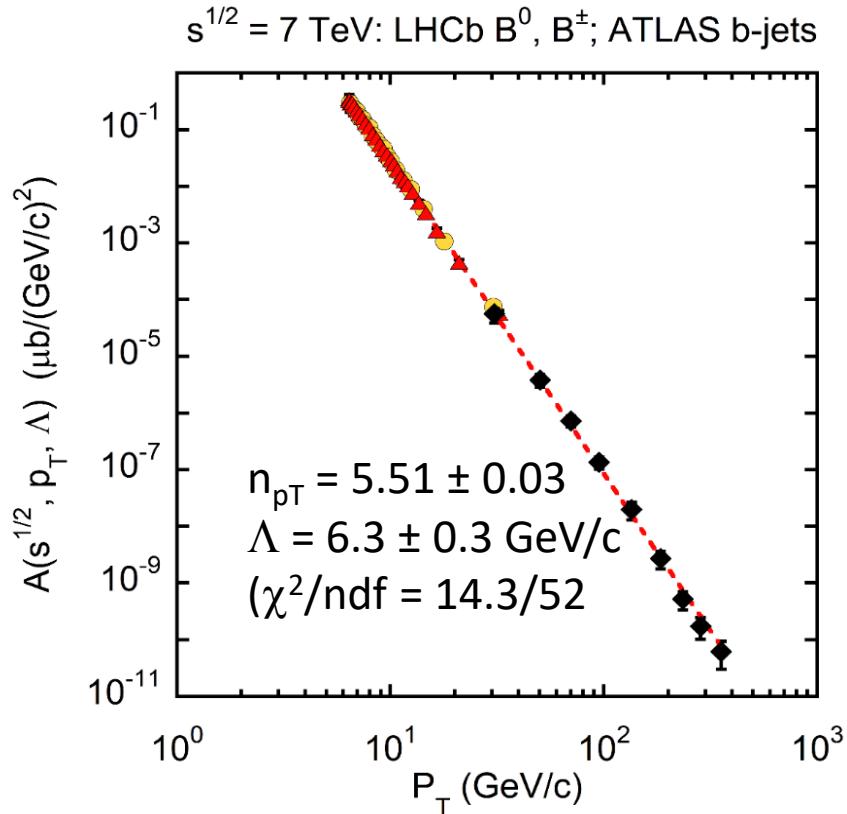
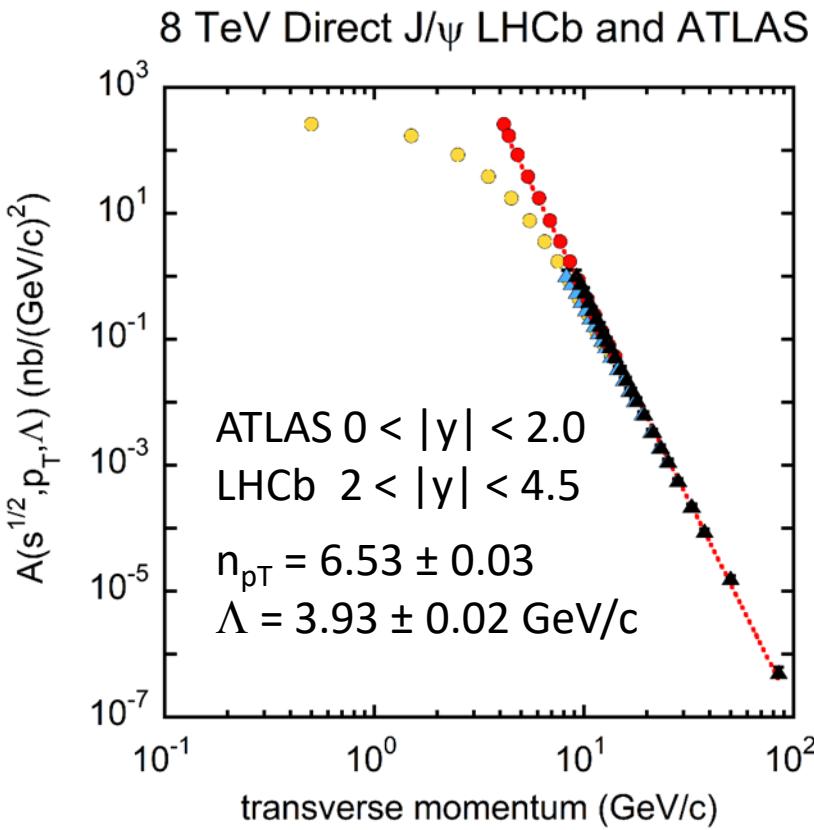
$$\Lambda_0 = 0.40 \pm 0.04 \text{ GeV}$$



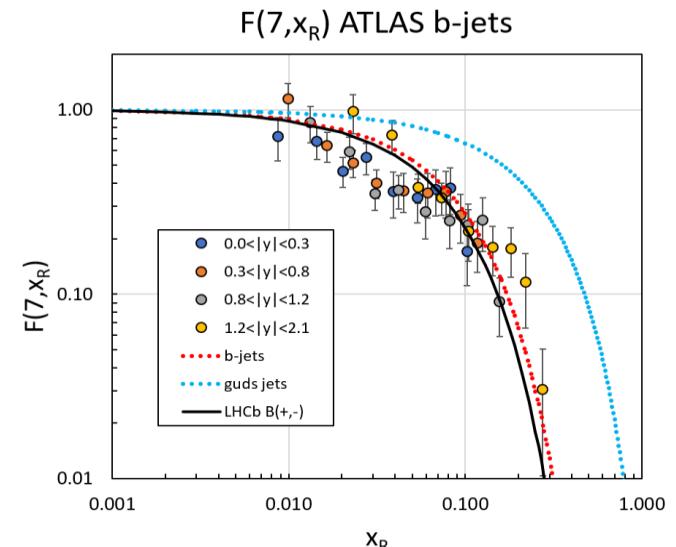
$$\Lambda = (1.17 \pm 0.04)m + (0.40 \pm 0.04)$$

FET, PRD 97,054016(2018)

# A( $p_T$ ) and F( $x_R$ ) useful in Comparison/Contrast

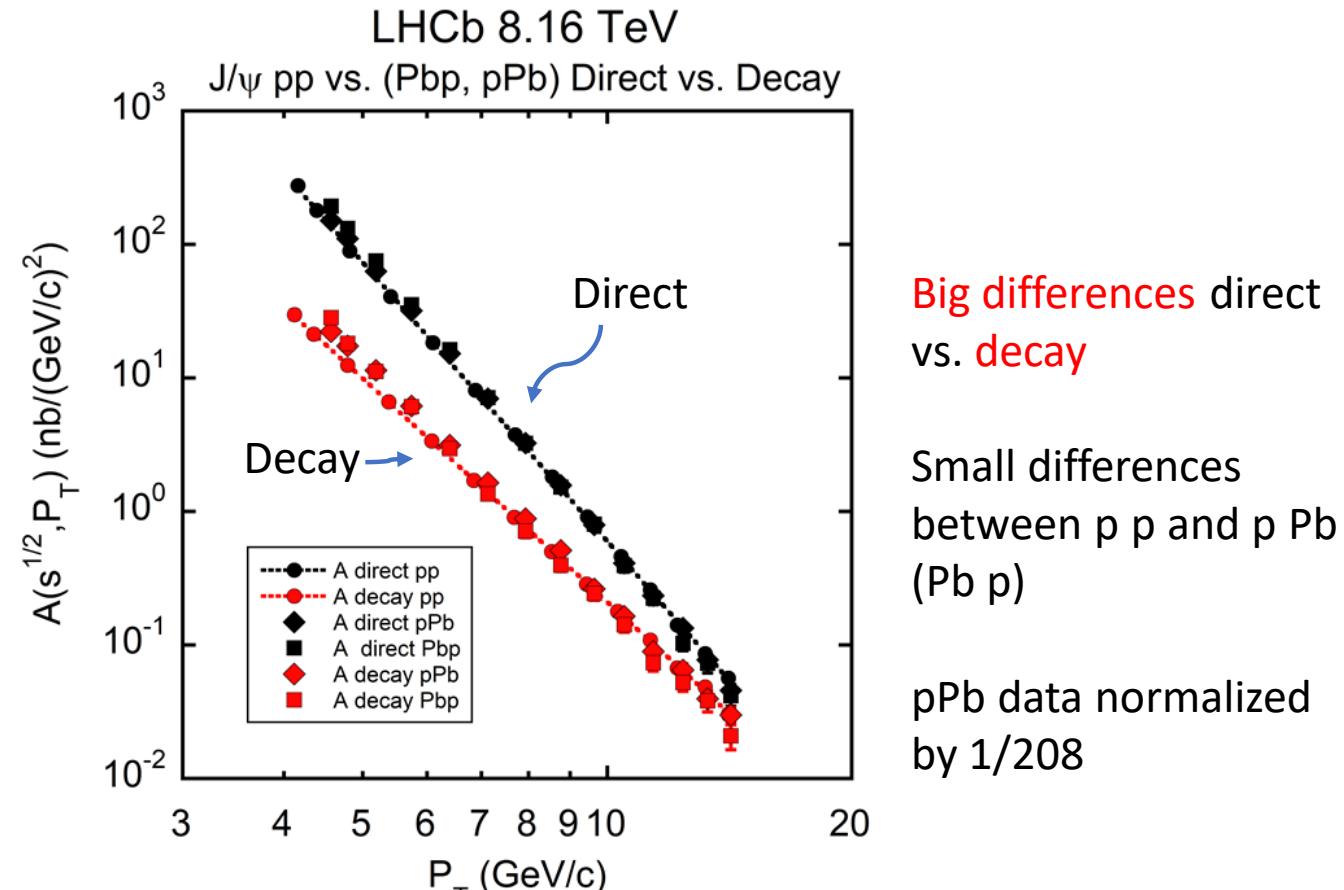
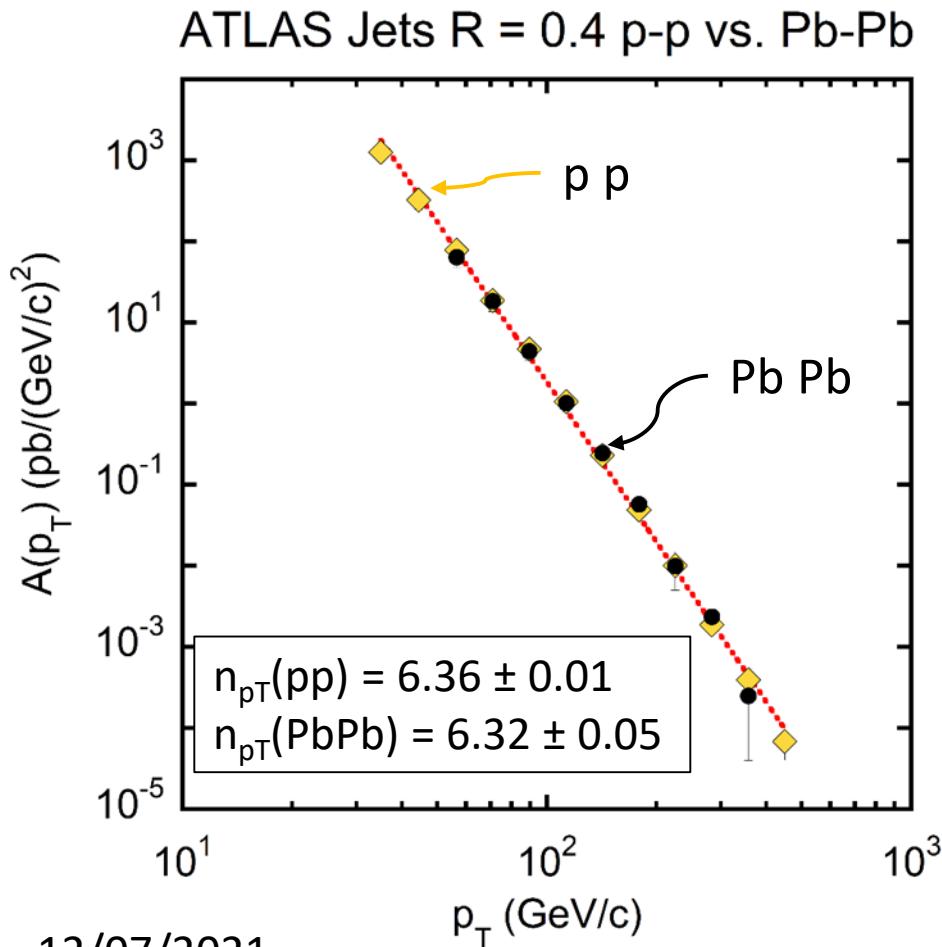


The same !

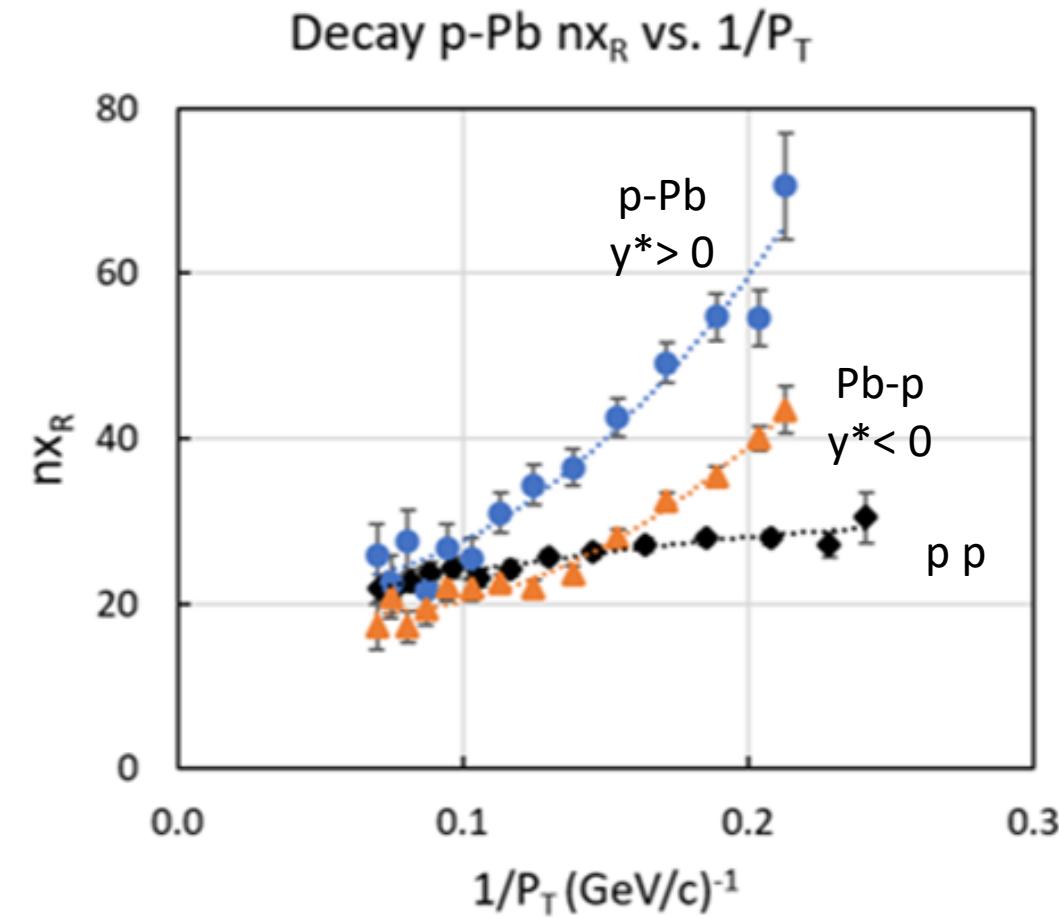
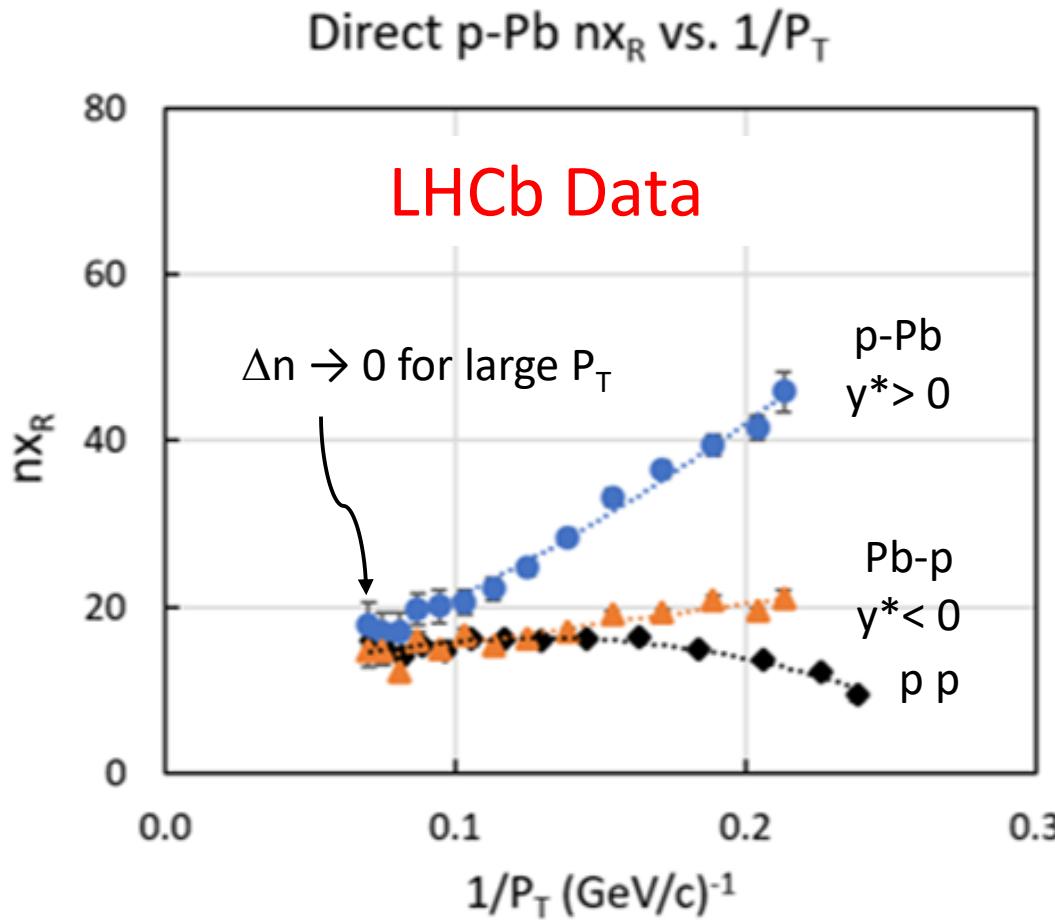


# Applications to Heavy Ions

- Brings two new tools: A-function and the  $x_R$  dependence (F-function)
  - A-function not very sensitive but  $x_R$  dependence very sensitive to HI effects



# Concentrate on Differences p-p vs. p-Pb and Pb-p

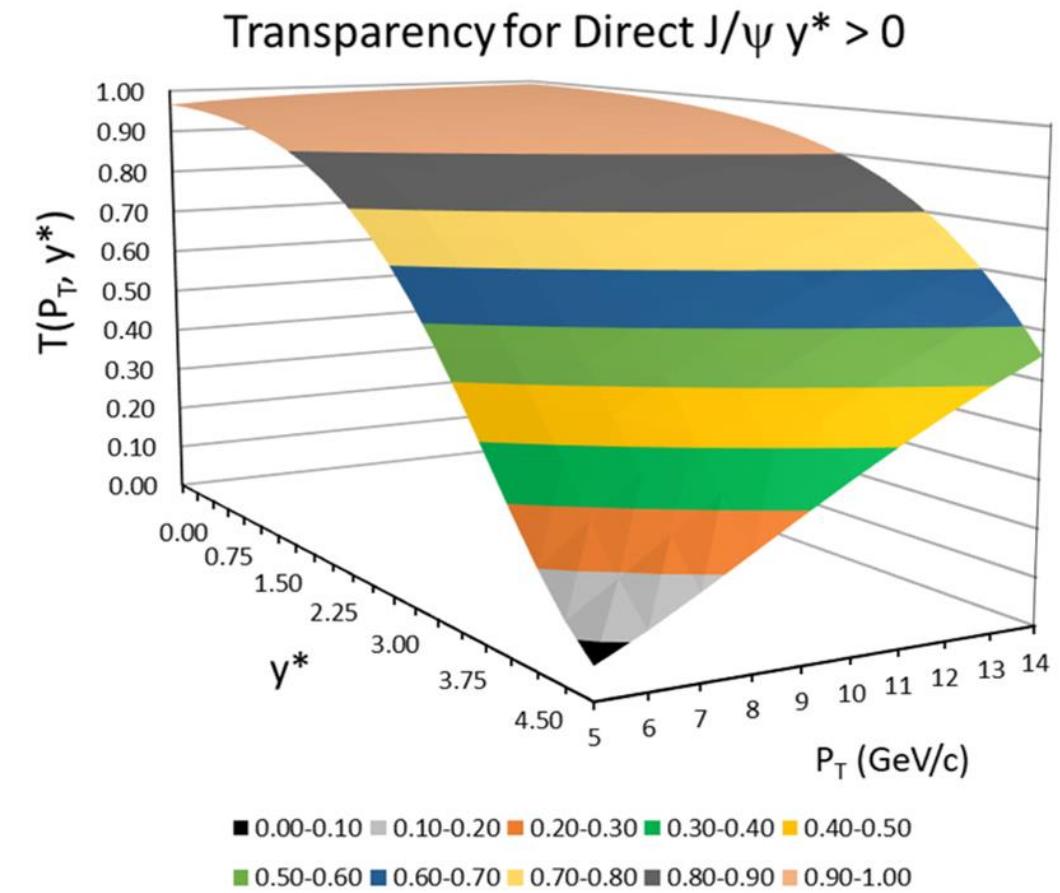
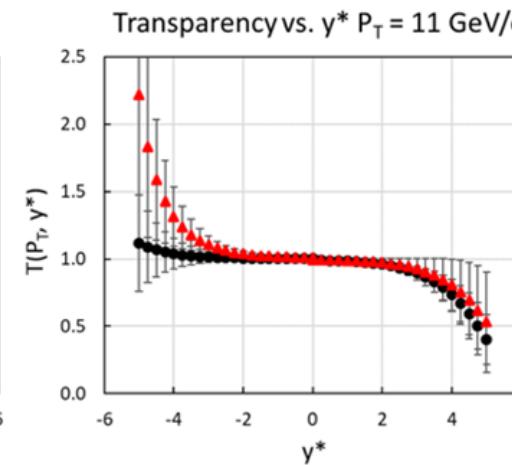
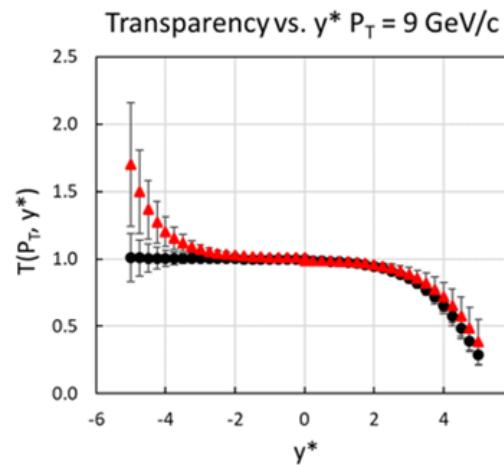
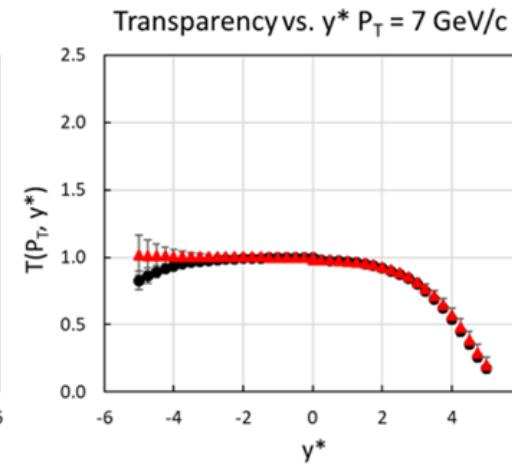
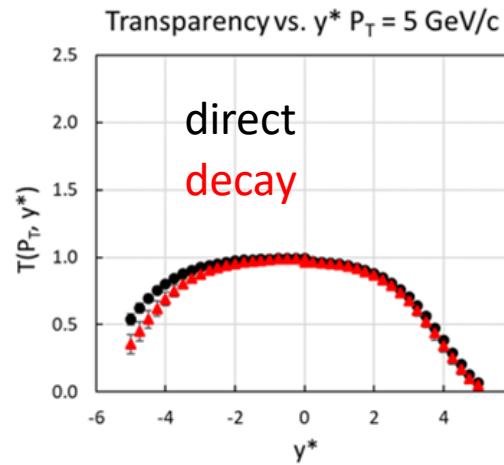


- $n_{x_R}$  are power indices of  $(1-x_R)^{nx_R}$   $y^* > 0$  ( $< 0$ ) proton (Pb) fragmentation region

# CNM Transparency

$$T(P_T, y^*) = (1 - x_R)^{\Delta n x R(P_T)} = \left(1 - \frac{2P_T \cosh(y^*)}{\sqrt{s}}\right)^{\Delta n x R(P_T)} \approx \exp(-\Delta n x_R(P_T) x_R)$$

- Compute transparency of nuclear matter by  $\Delta n = n_{xR}(pPb) - n_{xR}(pp)$



# Summary

Abstract: <https://www.mdpi.com/2218-1997/7/6/196>

HTML Version: <https://www.mdpi.com/2218-1997/7/6/196/htm>

PDF Version: <https://www.mdpi.com/2218-1997/7/6/196/pdf>

Special Issue: [https://www.mdpi.com/journal/universe/special\\_issues/Analysis\\_techniques\\_algorithms\\_QCD\\_studies](https://www.mdpi.com/journal/universe/special_issues/Analysis_techniques_algorithms_QCD_studies)

Published here

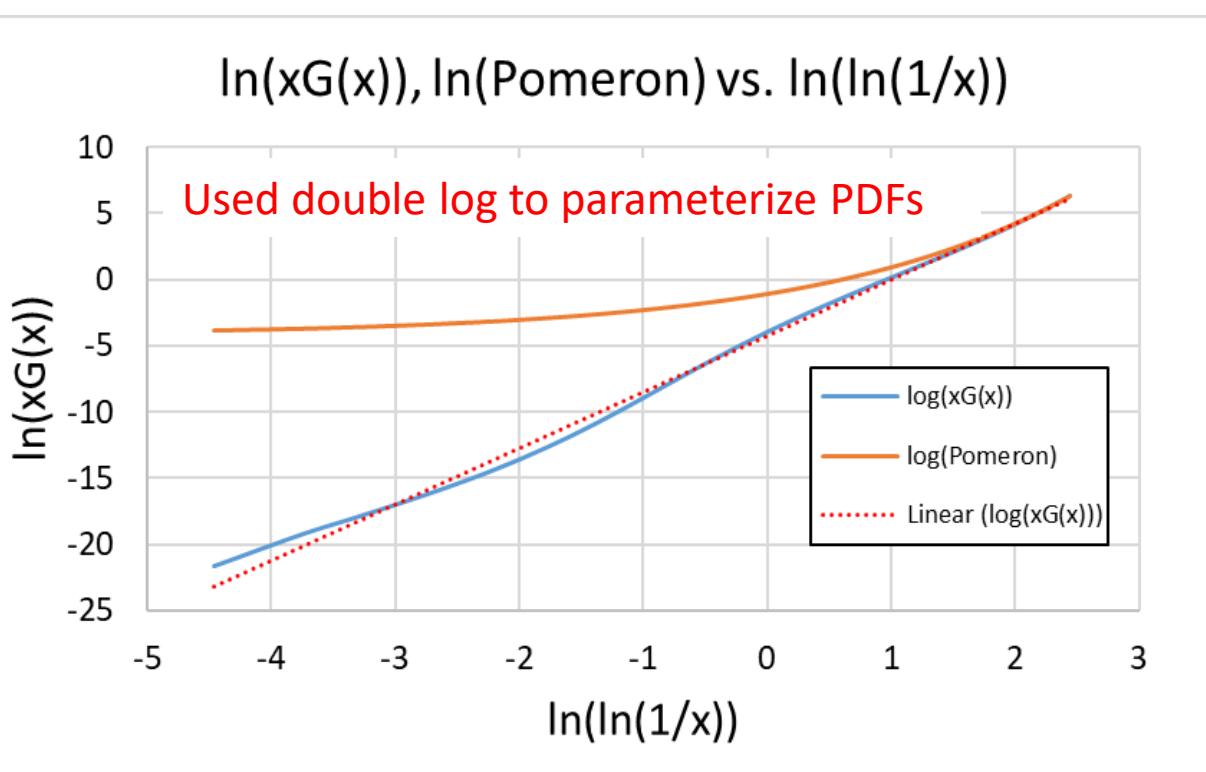
- Single particle and Jet Inclusive production in p-p collisions when described in terms of  $p_T$  &  $x_R$  reveal many simplicities
  - *A-function* is sensitive to primordial hard scattering at low  $x$  and reveals underlying power laws, is useful in comparing and contrasting cross sections and is independent of  $y$
  - *F-function* and  $x_R$  dependence sensitive to fragmentation, hadronization and higher  $x$  and is similar to structure functions in  $e$ ,  $\mu$ ,  $\nu$  – proton scattering
  - The *Dimensional Custodian* relates power laws in  $p_T$  &  $s$ -dependences with QCD:
    - Jets:  $(6.29 \pm 0.02) - (1.99 \pm 0.04) - 4 = 0.30 \pm 0.03$  ( $7.5\sigma$ )
    - Photons:  $(5.83 \pm 0.02) - (1.56 \pm 0.04) - 4 = 0.27 \pm 0.05$  ( $5.4\sigma$ )
  - The *A-m Relation* is a window into heavy object production
  - Applications to **heavy ion physics** suggest that most of the ‘physics’ is in the  $[x_R - p_T]$  sector

Evolution of  $(\alpha_s(Q^2))^2$   
responsible for small  
difference:  $0.30 \pm 0.03$

# Backup

# Toy Monte Carlo Simulation

- Wrote in Root framework
- Limited goal to only simulating parton-parton hard scattering with known parton distribution functions (PDFs) and the kinematic factors transforming from L-frame of parton-parton scattering to p-p COM → **every event is a “dijet”**



- Parton-parton elastic scattering in parton-parton COM weighted by PDFs (CT10) and cross sections – Field & Feynman

$$\hat{s} = s x_1 x_2$$

$$\hat{t} = -\frac{\hat{s}}{2}(1 - \cos \theta)$$

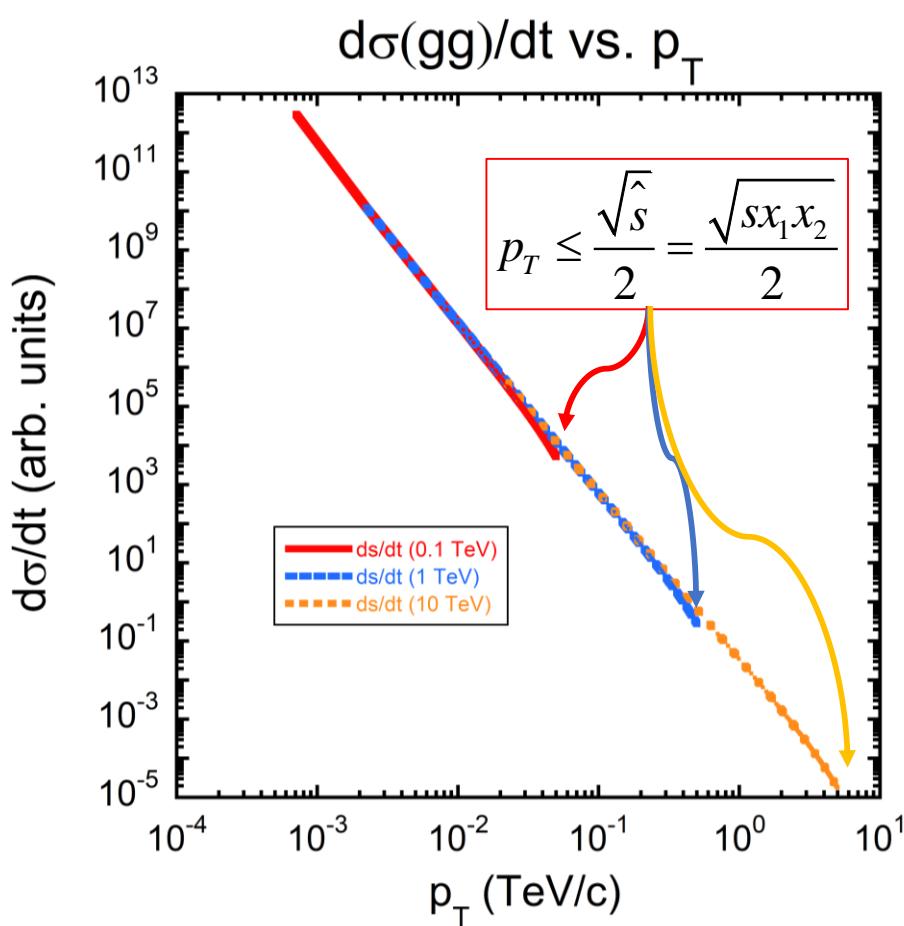
$$\hat{u} = -\frac{\hat{s}}{2}(1 + \cos \theta)$$

$$\frac{d\sigma(gg \rightarrow gg)}{dt} = \frac{\pi \alpha_s^2}{s^2} \frac{9}{2} \left( 3 - \frac{tu}{s^2} - \frac{su}{t^2} - \frac{st}{u^2} \right)$$

# Find a “Nice” Simplicity

Aside: Evolution of  $\alpha_s(Q^2)$  modifies power 4  $\rightarrow \approx 4.3$

- Cross sections very simple  $\sim 1/p_T^4$  and finite at maximum  $p_T = (sx_1x_2)^{1/2}/2$



Process	Leading $p_T$ Behavior	Value at $p_T = \sqrt{s}/2$
$gg \rightarrow gg$	$\frac{d\sigma}{dt} \approx \pi\alpha_s^2 \frac{9}{2} \left(\frac{1}{p_T^4}\right)$	$\frac{d\sigma}{dt} = \pi\alpha_s^2 \frac{243}{128} \left(\frac{1}{p_T^4}\right)$
$g\bar{q} \rightarrow g\bar{q}$	$\frac{d\sigma}{dt} \approx 2\pi\alpha_s^2 \left(\frac{1}{p_T^4}\right)$	$\frac{d\sigma}{dt} = \pi\alpha_s^2 \frac{55}{144} \left(\frac{1}{p_T^4}\right)$
$q\bar{q} \rightarrow q\bar{q}$ $\bar{q}\bar{q} \rightarrow \bar{q}\bar{q}$	$\frac{d\sigma}{dt} \approx \pi\alpha_s^2 \frac{8}{9} \left(\frac{1}{p_T^4}\right)$	$\frac{d\sigma}{dt} = \pi\alpha_s^2 \frac{99}{486} \left(\frac{1}{p_T^4}\right)$
$q_a q_b \rightarrow q_a q_b$ $\bar{q}_a \bar{q}_b \rightarrow \bar{q}_a \bar{q}_b$	$\frac{d\sigma}{dt} \approx \pi\alpha_s^2 \frac{8}{9} \left(\frac{1}{p_T^4}\right)$	$\frac{d\sigma}{dt} = \pi\alpha_s^2 \frac{5}{36} \left(\frac{1}{p_T^4}\right)$
$q\bar{q} \rightarrow gg$	$\frac{d\sigma}{dt} \approx \pi\alpha_s^2 \frac{32}{27s} \left(\frac{1}{p_T^2}\right)$	$\frac{d\sigma}{dt} = \pi\alpha_s^2 \frac{21}{324} \left(\frac{1}{p_T^4}\right)$
$q\bar{q} \rightarrow q\bar{q}$	$\frac{d\sigma}{dt} \approx \pi\alpha_s^2 \frac{8}{9} \left(\frac{1}{p_T^4}\right)$	$\frac{d\sigma}{dt} = \pi\alpha_s^2 \frac{315}{1944} \left(\frac{1}{p_T^4}\right)$
$q_a \bar{q}_a \rightarrow q_b \bar{q}_b$	$\frac{d\sigma}{dt} = \pi\alpha_s^2 \frac{4}{9s^2} \left(1 - \frac{2p_T^2}{s}\right)$	$\frac{d\sigma}{dt} = \pi\alpha_s^2 \frac{1}{72} \left(\frac{1}{p_T^4}\right)$
$gg \rightarrow q\bar{q}$	$\frac{d\sigma}{dt} \approx \pi\alpha_s^2 \frac{1}{6s} \left(\frac{1}{p_T^2}\right)$	$\frac{d\sigma}{dt} = \pi\alpha_s^2 \frac{7}{768} \left(\frac{1}{p_T^4}\right)$

# Simulations

11 processes considered

Process	$n_{p_T}$
ATLAS	$6.35 \pm 0.02$
CMS	$6.41 \pm 0.05$
Pythia 8.1	$6.308 \pm 0.005$
All Toy	$6.351 \pm 0.017$

A-function parameters  
*Toy MC works very well  
because the A-function  
is a measure of hard  
scattering*

Process	$\sigma/\sigma(\text{all})$	$n_{p_T}$
ATLAS	1	$6.35 \pm 0.02$
CMS	1	$6.41 \pm 0.05$
Pythia 8.1	1	$6.308 \pm 0.005$
All Toy	100%	$6.351 \pm 0.017$

Process	D (GeV/c)	$n_{xR0}$	$D_Q (\text{GeV}/c)^2$	$n_{xRQ0}$
ATLAS	$700 \pm 110$	$3.6 \pm 0.2$	$(1.5 \pm 0.4) \times 10^5$	$0.06 \pm 0.1$
CMS	$750 \pm 307$	$3.3 \pm 0.6$	$(2.0 \pm 1.3) \times 10^5$	$0.08 \pm 0.4$
Pythia 8.1	$322 \pm 30$	$4.31 \pm 0.06$	$(6.5 \pm 1.3) \times 10^4$	$0.43 \pm 0.04$
All Toy	$1170 \pm 92$	$3.14 \pm 0.15$	$(2.0 \pm 0.3) \times 10^5$	$0.35 \pm 0.07$
$gg \rightarrow gg$	$969 \pm 17$	$7.10 \pm 0.05$	$(3.20 \pm 0.03) \times 10^5$	$1.04 \pm 0.05$
$gq \rightarrow gq$	$-25 \pm 46$	$4.2 \pm 0.1$	$(8.9 \pm 0.9) \times 10^4$	$-0.2 \pm 0.1$
$qq \rightarrow qq$	$-34 \pm 55$	$2.7 \pm 0.2$	$(5.4 \pm 1.1) \times 10^4$	$-1.1 \pm 0.2$
$q_a q_b \rightarrow q_a q_b$	$-65 \pm 52$	$3.3 \pm 0.1$	$(7.3 \pm 1.1) \times 10^4$	$-0.82 \pm 0.2$
$q\bar{q} \rightarrow gg$	$253 \pm 28$	$4.74 \pm 0.08$	$(1.18 \pm 0.05) \times 10^5$	$0.26 \pm 0.07$

Process	D (GeV/c)	$n_{xR0}$	$D_Q (\text{GeV}/c)^2$	$n_{xRQ0}$
$gg \rightarrow q\bar{q}$				
$q_a \bar{q}_a \rightarrow q_b \bar{q}_b$				
$q\bar{q} \rightarrow q\bar{q}$				
$q_a \bar{q}_b \rightarrow q_a \bar{q}_b$				
$g\bar{q} \rightarrow g\bar{q}$				
ATLAS	$700 \pm 110$	$3.6 \pm 0.2$	$(1.5 \pm 0.4) \times 10^5$	$0.06 \pm 0.1$
CMS	$750 \pm 307$	$3.3 \pm 0.6$	$(2.0 \pm 1.3) \times 10^5$	$0.08 \pm 0.4$
Pythia 8.1	$322 \pm 30$	$4.31 \pm 0.06$	$(6.5 \pm 1.3) \times 10^4$	$0.43 \pm 0.04$
All Toy	$1170 \pm 92$	$3.14 \pm 0.15$	$(2.0 \pm 0.3) \times 10^5$	$0.35 \pm 0.07$

f-function  
parameters

# Spine and Dimensional Custodian Parameters

Inclusive Jets	Parameter	Value
Data	$\kappa_0$	$(10 \pm 3) \times 10^5 \text{ (pb GeV}^{(npT-ns-2)})$
Data	$n_s$	$1.99 \pm 0.04$
Toy MC	$n_s$	$2.084 \pm 0.004$
Pythia 8.1 MC	$n_s$	$2.028 \pm 0.005$
Data	$np_T$	$6.29 \pm 0.01$
Toy MC	$np_T$	$6.286 \pm 0.002$
Pythia 8.1 MC	$np_T$	$6.243 \pm 0.003$
Data	$n_r$	$0.28 \pm 0.03$
Toy MC	$n_r$	$0.203 \pm 0.004$
Pythia 8.1 MC	$n_r$	$0.216 \pm 0.005$
Data	$\chi^2/ndf$	221/94

Inclusive Photons	Parameter	Value
Data	$\kappa_0$	$(5 \pm 2) \times 10^3 \text{ (pb (GeV/c}^{(npT-ns-2)})$
Data	$n_s$	$1.56 \pm 0.04$
Toy MC	$n_s$	$1.08 \pm 0.01$
Data	$np_T$	$5.83 \pm 0.02$
Toy MC	$np_T$	$5.30 \pm 0.01$
Data	$n_r$	$0.27 \pm 0.05$
Toy MC	$n_r$	$0.22 \pm 0.01$
Data	$\chi^2/ndf$	44/27

# Photons Ratio 13 TeV / 8 TeV

- ATLAS measured ratio 13 TeV to 8 TeV and compared with MC (JETPHOX 1.3.1\_2) which is a NLO QCD treatment of both direct production and fragmentation photons
- Toy MC considers only direct production at tree level but captures the  $p_T$ -dependence of ratio – especially at high  $|\eta|$

$$R'_{13/8}(p_T, \eta) = R_A(p_T) R(p_T, x_R) R_Q(p_T, x_R)$$

$$R_A(p_T) = \left( \frac{\kappa(13)}{\kappa(8)} \right) \left( p_T^{\Delta npT} \right),$$

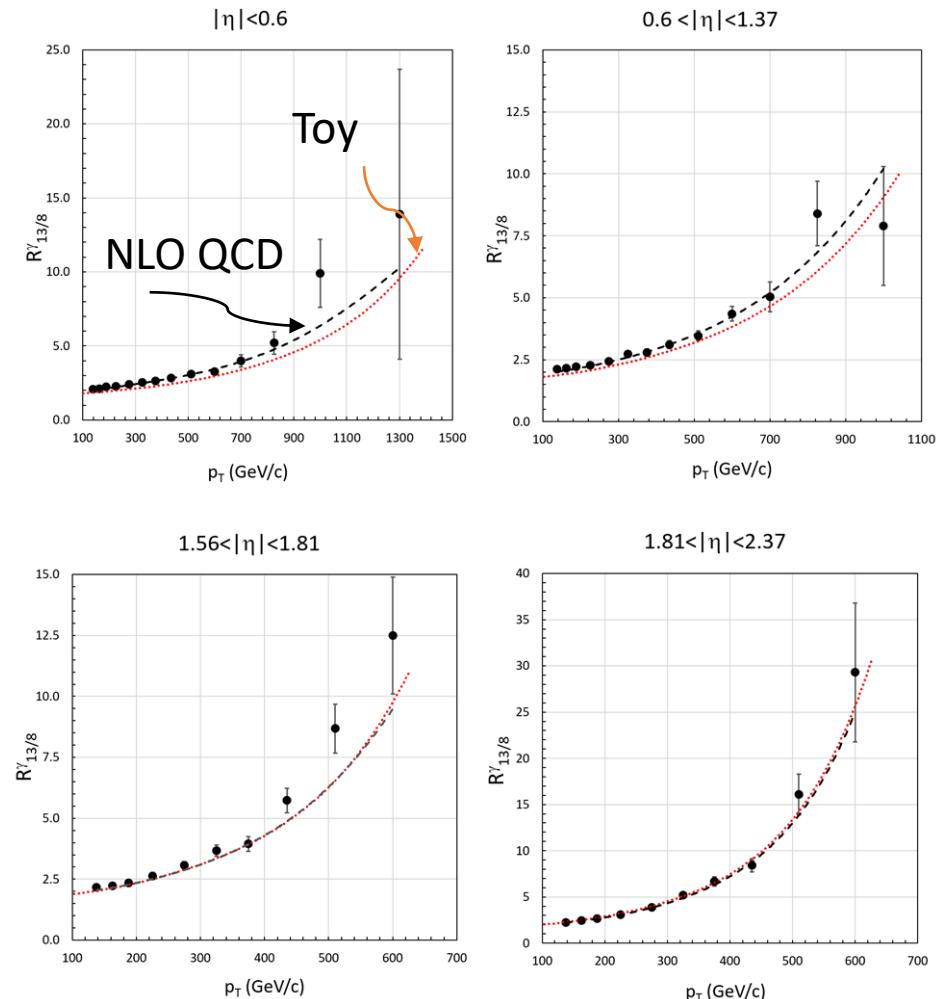
$$R(p_T, x_R) = \left( \frac{(1-x_R)^{nx_R(13, p_T)}}{(1-13/8 x_R)^{nx_R(8, p_T)}} \right),$$

$$R_Q(p_T, x_R) = \exp \left( n_{xRQ}(13, p_T) \ln^2(1-x_R) - n_{xRQ}(8, p_T) \ln^2(1-13/8 x_R) \right)$$

The  $p_T$ -dependence for same  
 $(p_T, \eta) \times_R \rightarrow (13/8)x_R$

$$\chi^2/\text{ndf NLO} = 31/47$$

$$\chi^2/\text{ndf Toy} = 175/47$$



# Cronin Effect – Another probe by ratio of A-functions

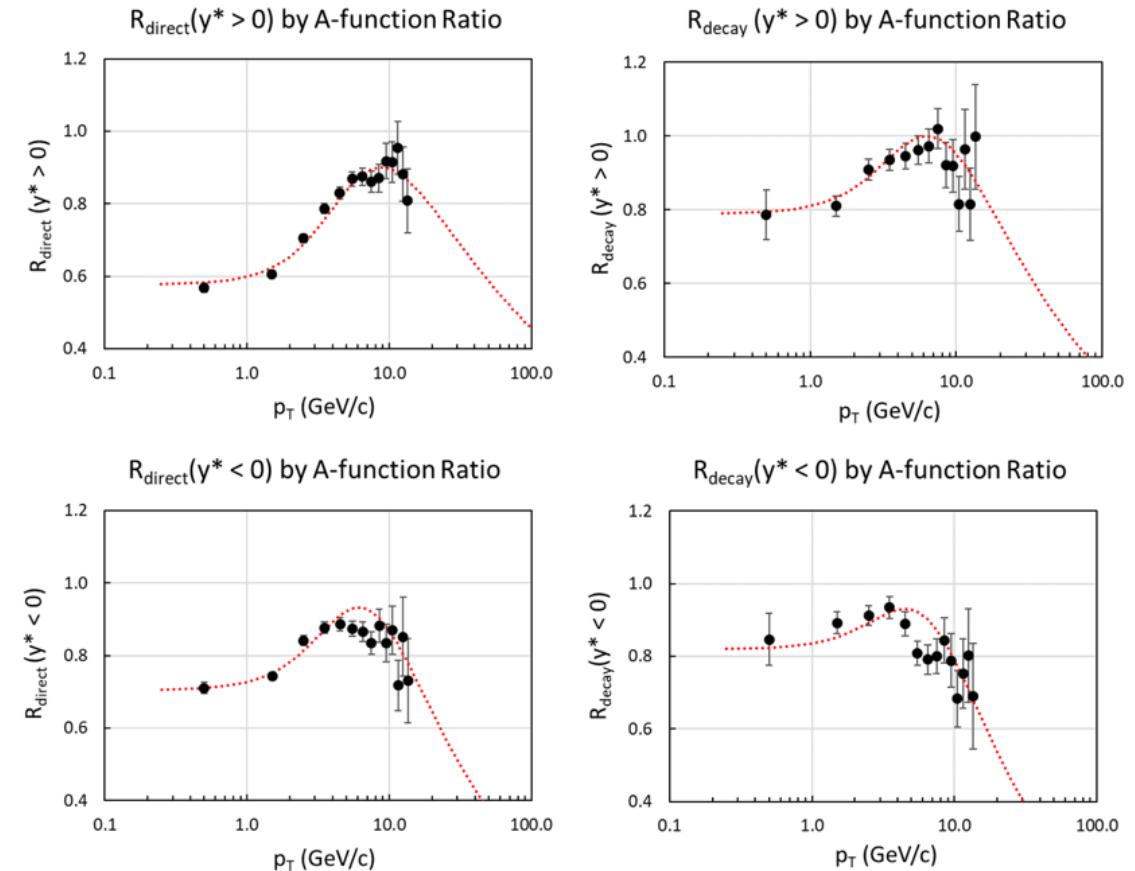
- The effect is a bump in  $\pi$ -production  $\sigma(pA)/\sigma(pp)$  as function of  $p_T$  at low  $p_T$
- Could be due to multiple scattering of partons and other final state interactions as  $\pi$  moves through nucleus
- Compute  $A(J/\psi, pPb)/A(J/\psi, pp)$  from LHCb data
- Calling pPb data “i” and pp data “j” where  $n, \Lambda, \kappa$  parameters of A-function

$$R_{i/j}(p_T) = \frac{\kappa_i}{\kappa_j} \frac{(p_T^2 + \Lambda_j^2)^{n_j/2}}{(p_T^2 + \Lambda_i^2)^{n_i/2}}$$

$$p_{T\max} = \sqrt{\frac{\Lambda_j^2 n_i - \Lambda_i^2 n_j}{n_j - n_i}}$$

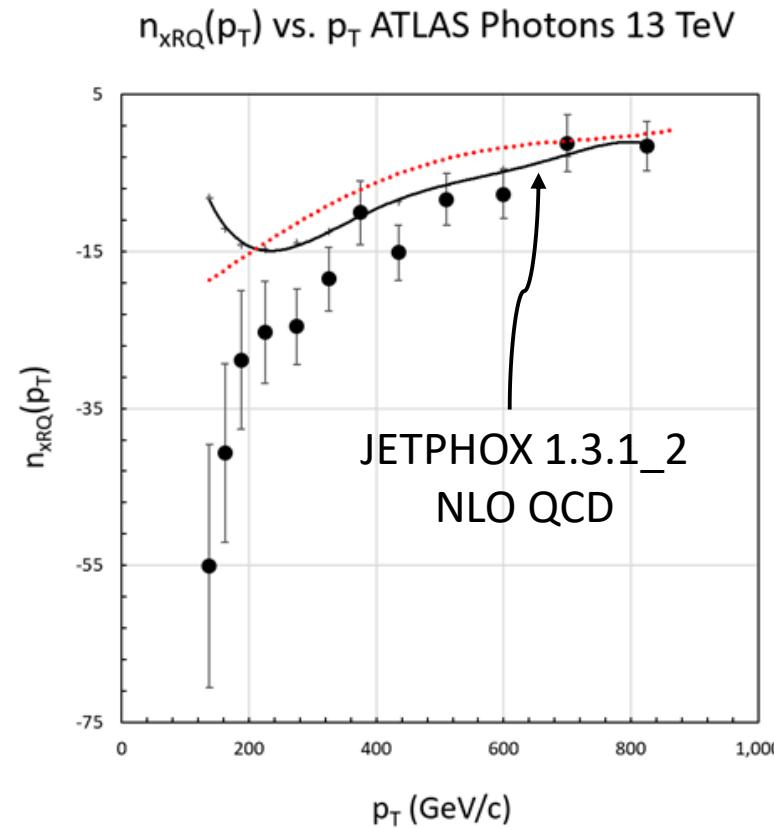
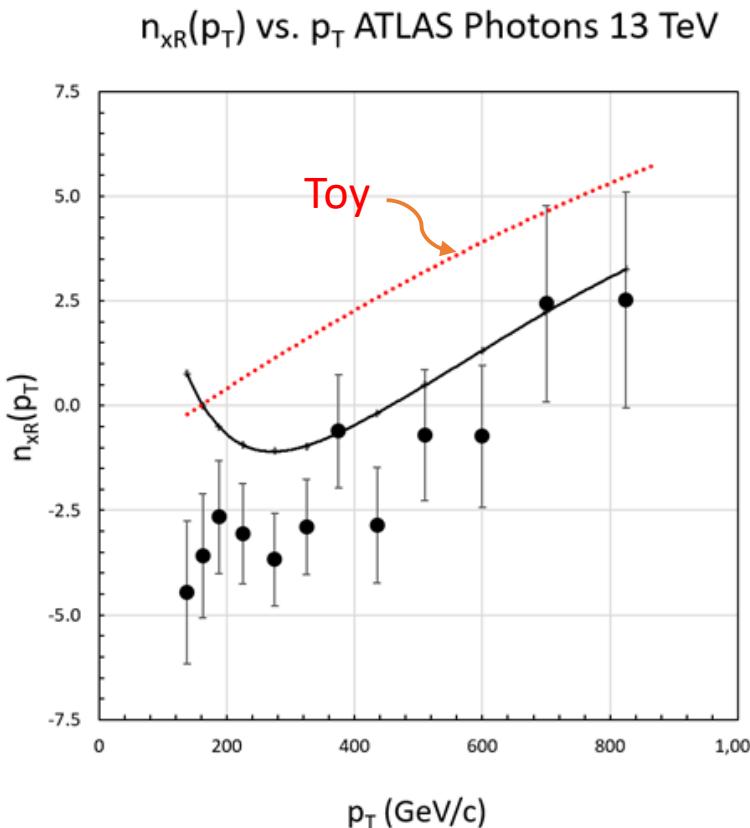
$$R_{i/j}(p_{T\max}) = \frac{\kappa_i}{\kappa_j} \left( \frac{\Lambda_j^2 - \Lambda_i^2}{n_j - n_i} \right)^{(n_j - n_i)/2} \frac{n_j^{n_j/2}}{n_i^{n_i/2}}$$

Cronin effect is  
then due to  
different values  
of  $\Lambda$  and  $n$



# Photons x<sub>R</sub>-p<sub>T</sub> Sector

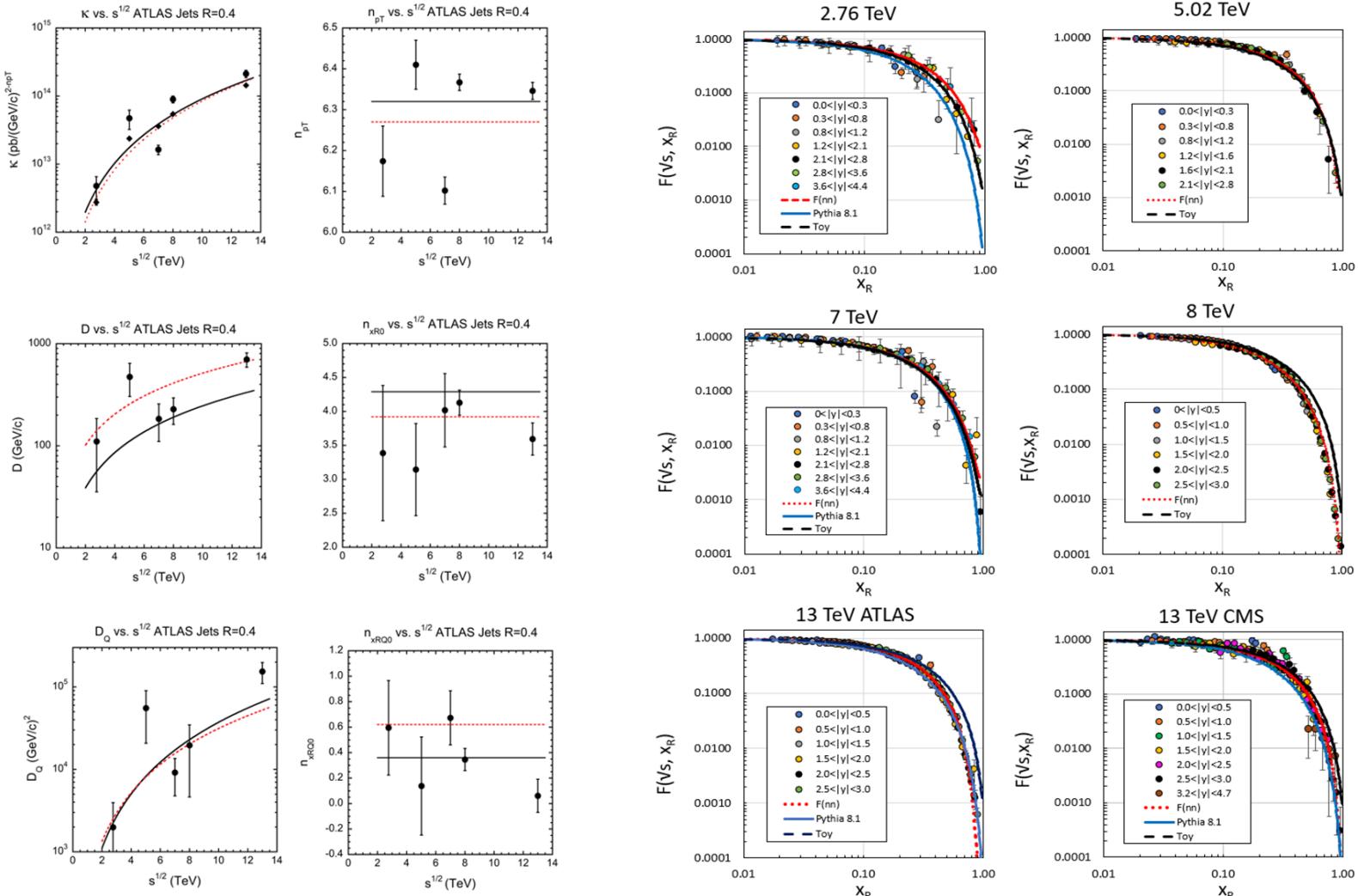
- Different from jets – n<sub>xR</sub> & n<sub>xRQ</sub> increase with p<sub>T</sub>



Toy MC qualitatively captures the smaller value of n<sub>pT</sub> of A-function and the rise in n<sub>xR</sub> and n<sub>xRQ</sub> with p<sub>T</sub>.

But Toy MC has only hard scattering at a fixed renormalization scale, whereas JETPHOX has full NLO QCD, fragmentation, radiative effects, isolation cuts, etc.

# s-dependence of Inclusive Jets



- LHC ran at various values of  $\sqrt{s}$  during the commissioning phase of operation as well as for data for the Heavy Ion program
- Some of data ‘early’ – suspect that not all systematics have been accounted

$2.76 \text{ TeV} \leq \sqrt{s} \leq 13 \text{ TeV}$

# Jet Parameters Obey Power Laws in $\sqrt{s}$

Parameter	Power Index	Constant Term
$\kappa(s)$	$n_s$	$\ln(k_0)$
Data	$2.3 \pm 0.7$	$11 \pm 6$
Pythia 8.1	$2.4 \pm 0.2$	$10.2 \pm 1.7$
Toy	$2.5 \pm 0.3$	$9 \pm 3$
$D(s)$	$n_D$	$\ln(D_0)$
Data	$0.9 \pm 0.5$	$-2 \pm 5$
Pythia 8.1	$1.1 \pm 0.1$	$-5.1 \pm 0.8$
Toy	$1.0 \pm 0.1$	$-3.0 \pm 0.1$
$D_Q(s)$	$n_{DQ}$	$\ln(D_{Q0})$
Data	$2.3 \pm 1.0$	$-10 \pm 9$
Pythia 8.1	$2.2 \pm 0.1$	$-9.6 \pm 0.9$
Toy	$2.0 \pm 0.5$	$-8 \pm 5$

$$\ln(\kappa(s)) \sim n_s \ln(\sqrt{s}) + \ln(k_0),$$

$$\ln(D(s)) \sim n_D \ln(\sqrt{s}) + \ln(D_0),$$

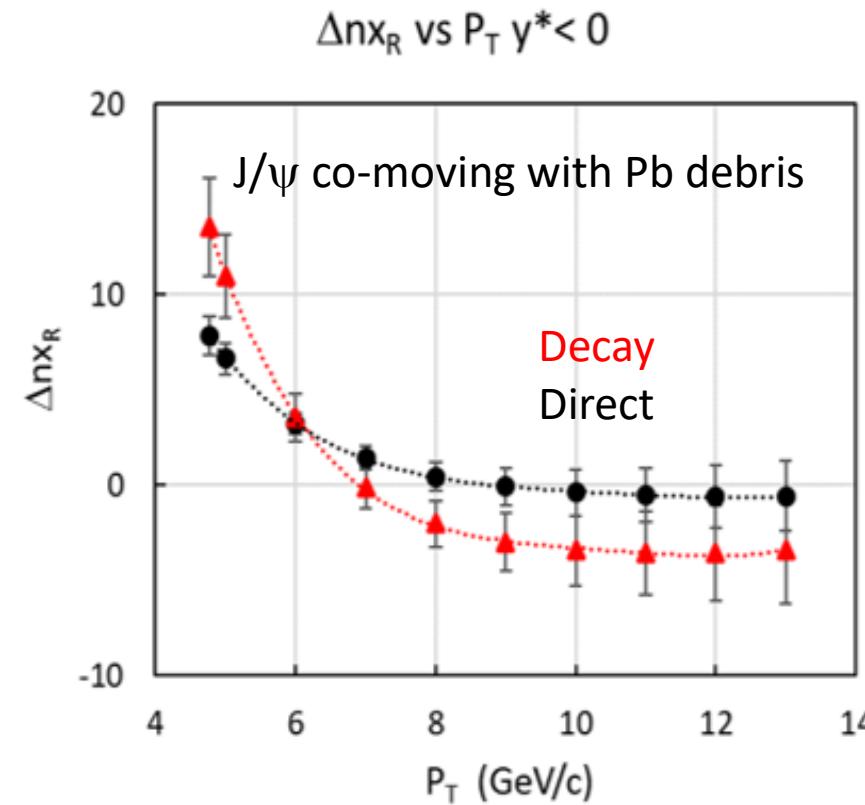
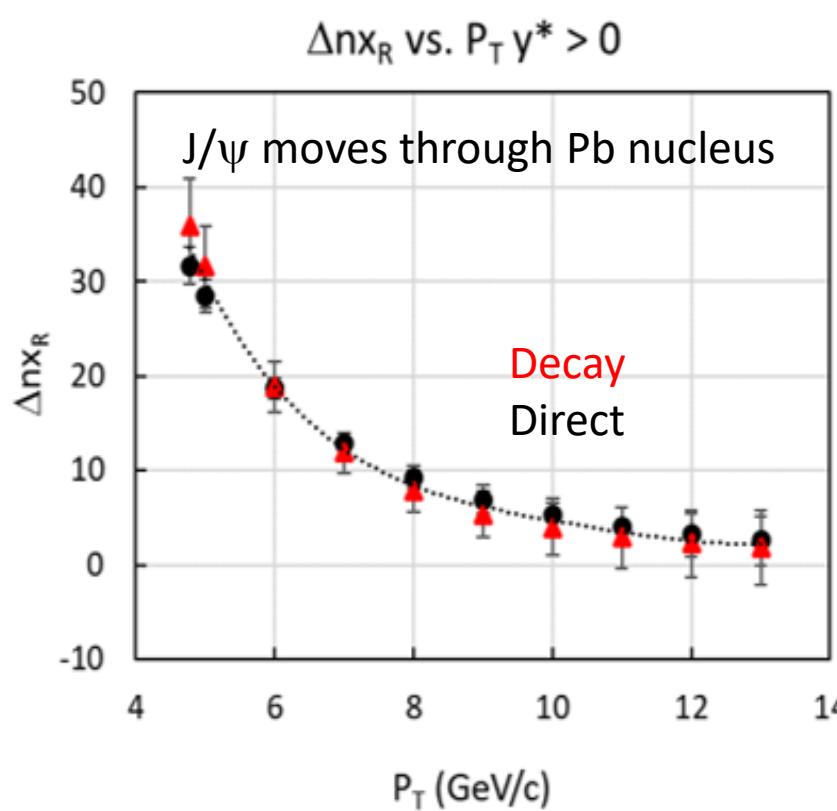
$$\ln(D_Q(s)) \sim n_{DQ} \ln(\sqrt{s}) + \ln(D_{Q0})$$

Expect:

$$D(s) \sim -\frac{\sqrt{s}}{2} \left( \frac{d \ln(\sigma(p_{T \min}, y))}{d \cosh(y)} \right)$$

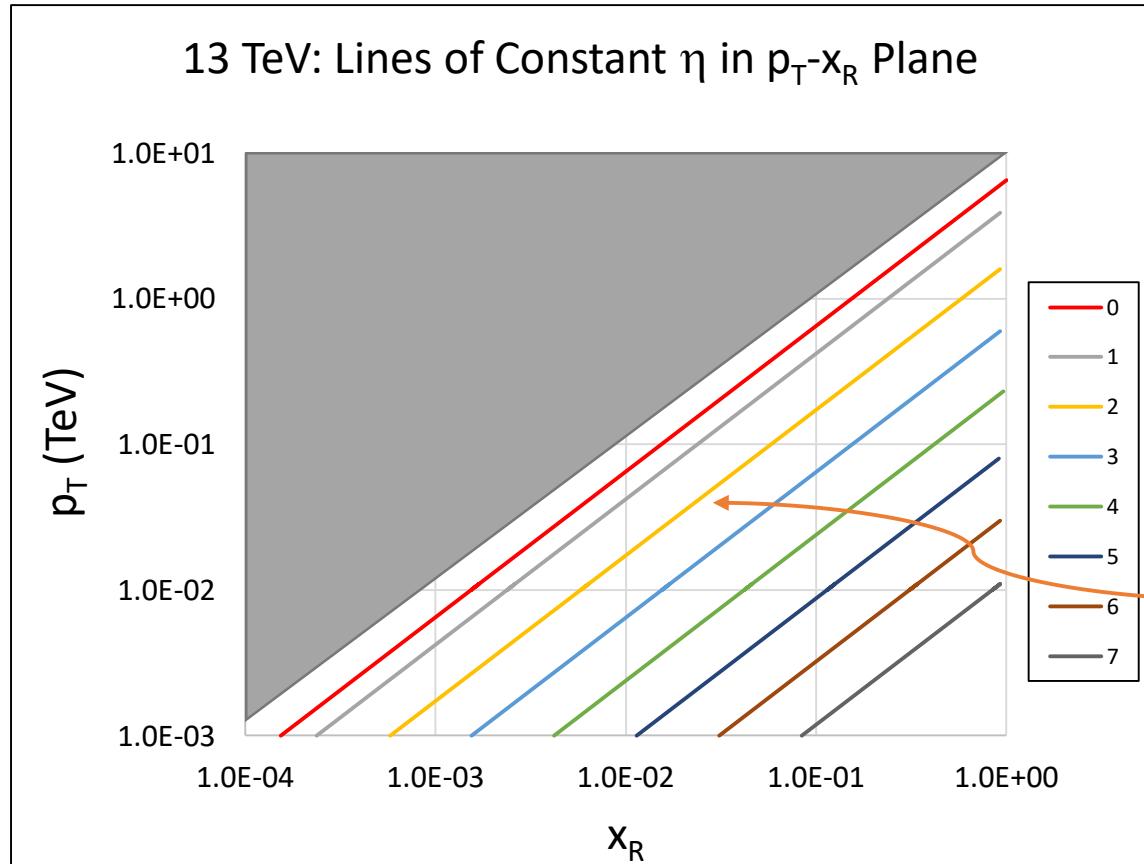
$$\Delta n_{xR}(\text{direct, decay}) = n_{xR}(p\text{-Pb, Pb-p}) - n_{xR}(pp)$$

- Assert that  $\Delta n$  is a measure of attenuation of  $J/\psi$  as it moves through the Pb nucleus (Cold Nuclear Matter CNM)



# $\eta$ verses $x_R$

$$|y| = \ln \left[ \frac{\sqrt{s}}{2} \frac{x_R}{\sqrt{p_T^2 + m^2}} + \sqrt{\frac{s}{4} \frac{x_R^2}{p_T^2 + m^2} - 1} \right] = \ln \left[ \frac{E}{\sqrt{p_T^2 + m^2}} + \sqrt{\frac{E^2}{p_T^2 + m^2} - 1} \right]$$

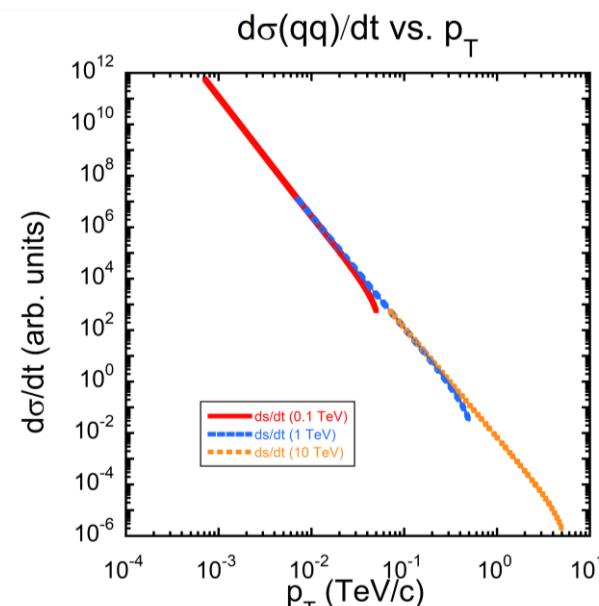
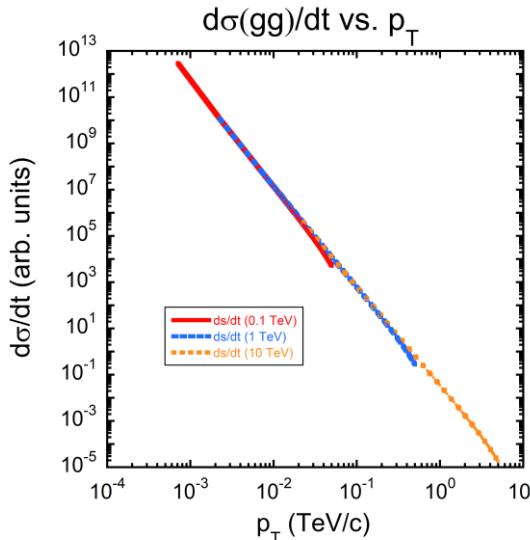


$$\eta(x_R, s, p_T) = \ln \left( \frac{x_R \sqrt{s}}{2 p_T} + \sqrt{\frac{x_R s}{4 p_T^2} - 1} \right)$$

$$\eta_{\max} = \ln \left( \frac{\sqrt{s}}{2 p_T} + \sqrt{\frac{s}{4 p_T^2} - 1} \right)$$

Analyses in constant  $\eta$  ( $y$ ) couples  $p_T$  and  $x_R$  so that the hard scattering part of  $d^2\sigma/p_T dp_T d\eta$  that is characterized by  $p_T$  is entangled with the influence of  $x_R$  – the kinematic boundary parameter

# “Hard Physics is Easy – Soft Physics is Hard”, BJ



Process	Leading $p_T$ Behavior	Value at $p_T = \sqrt{s}/2$
$gg \rightarrow gg$	$\frac{d\sigma}{dt} \approx \pi\alpha_s^2 \frac{9}{2} \left( \frac{1}{p_T^4} \right)$	$\frac{d\sigma}{dt} = \pi\alpha_s^2 \frac{243}{128} \left( \frac{1}{p_T^4} \right)$
$g\bar{q} \rightarrow g\bar{q}$ $g\bar{q} \rightarrow g\bar{q}$	$\frac{d\sigma}{dt} \approx 2\pi\alpha_s^2 \left( \frac{1}{p_T^4} \right)$	$\frac{d\sigma}{dt} = \pi\alpha_s^2 \frac{55}{144} \left( \frac{1}{p_T^4} \right)$
$q\bar{q} \rightarrow q\bar{q}$ $q\bar{q} \rightarrow q\bar{q}$	$\frac{d\sigma}{dt} \approx \pi\alpha_s^2 \frac{8}{9} \left( \frac{1}{p_T^4} \right)$	$\frac{d\sigma}{dt} = \pi\alpha_s^2 \frac{99}{486} \left( \frac{1}{p_T^4} \right)$
$q_a q_b \rightarrow q_a q_b$ $\bar{q}_a \bar{q}_b \rightarrow \bar{q}_a \bar{q}_b$	$\frac{d\sigma}{dt} \approx \pi\alpha_s^2 \frac{8}{9} \left( \frac{1}{p_T^4} \right)$	$\frac{d\sigma}{dt} = \pi\alpha_s^2 \frac{5}{36} \left( \frac{1}{p_T^4} \right)$
$q\bar{q} \rightarrow gg$	$\frac{d\sigma}{dt} \approx \pi\alpha_s^2 \frac{32}{27s} \left( \frac{1}{p_T^2} \right)$	$\frac{d\sigma}{dt} = \pi\alpha_s^2 \frac{21}{324} \left( \frac{1}{p_T^4} \right)$
$q\bar{q} \rightarrow q\bar{q}$	$\frac{d\sigma}{dt} \approx \pi\alpha_s^2 \frac{8}{9} \left( \frac{1}{p_T^4} \right)$	$\frac{d\sigma}{dt} = \pi\alpha_s^2 \frac{315}{1944} \left( \frac{1}{p_T^4} \right)$
$q_a \bar{q}_a \rightarrow q_b \bar{q}_b$	$\frac{d\sigma}{dt} = \pi\alpha_s^2 \frac{4}{9s^2} \left( 1 - \frac{2p_T^2}{s} \right)$	$\frac{d\sigma}{dt} = \pi\alpha_s^2 \frac{1}{72} \left( \frac{1}{p_T^4} \right)$
$gg \rightarrow q\bar{q}$	$\frac{d\sigma}{dt} \approx \pi\alpha_s^2 \frac{1}{6s} \left( \frac{1}{p_T^2} \right)$	$\frac{d\sigma}{dt} = \pi\alpha_s^2 \frac{7}{768} \left( \frac{1}{p_T^4} \right)$

# J/ $\psi$ Parameters from LHCb

Process	y range	$\Lambda$ (GeV/c)	$n p_T$	$\kappa$ (nb/(GeV/c) <sup>2</sup> )
Direct p-p	$2.0 < y^* < 4.5$	$4.1 \pm 0.2$	$6.9 \pm 0.3$	$(4.7 \pm 0.2) \times 10^6$
Direct p-Pb	$1.5 < y^* < 4.0$	$4.8 \pm 0.2$	$7.5 \pm 0.2$	$(1.2 \pm 0.1) \times 10^7$
Direct Pb-p	$-5.0 < y^* < -2.5$	$4.6 \pm 0.1$	$7.5 \pm 0.2$	$(2.3 \pm 0.2) \times 10^7$
Decay p-p	$2.0 < y^* < 4.5$	$4.1 \pm 0.1$	$5.6 \pm 0.1$	$(8.0 \pm 0.3) \times 10^4$
Decay p-Pb	$1.5 < y^* < 4.0$	$4.6 \pm 0.3$	$6.0 \pm 0.3$	$(2.7 \pm 0.2) \times 10^5$
Decay Pb-p	$-5.0 < y^* < -2.5$	$4.3 \pm 0.3$	$5.9 \pm 0.3$	$(4.2 \pm 0.6) \times 10^5$

# Eliminating $|y|$

- We could measure:

$$\frac{d^2\sigma}{2\pi p_T dp_T dx_R} = G(\sqrt{s}, p_T, x_R) \quad x_R \leq 1; p_{T\min} \leq p_T \leq p_{T\max}; x_R \leq \frac{2p_T}{\sqrt{s}} \cosh(\eta_{\max}); x_R \geq \frac{2p_T}{\sqrt{s}}$$

Since  $x_R$  and  $p_T$  are sufficient to specify a kinematic point in p-p collisions within the kinematic boundary

Toy simulation of g g  
→ g g channel only  
 $n_{xRQ0}$  differs

Cross section definition	With $x_R$	With $y$
$n_{pT}$	$6.76 \pm 0.03$	$6.59 \pm 0.02$
$n_{xR0}$	$6.83 \pm 0.04$	$7.3 \pm 0.1$
$n_{xRQ0}$	$1.04 \pm 0.04$	$0.1 \pm 0.1$
$D (\text{GeV}/c)$	$948 \pm 17$	$950 \pm 37$
$D_0 (\text{GeV}/c)^2$	$(3.01 \pm 0.03) \times 10^5$	$(2.98 \pm 0.08) \times 10^5$