

HADRON STRUCTURE FROM BASIS LIGHT-FRONT QUANTIZATION



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in collaboration with

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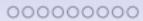
Introduction



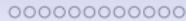
BLFQ



light



nucleon



Conclusions



Overview



Introduction

Basis light-front quantization (BLFQ)

Application of BLFQ to light mesons

PRL 122, 172001 (2019) and arXiv: 2106.04954

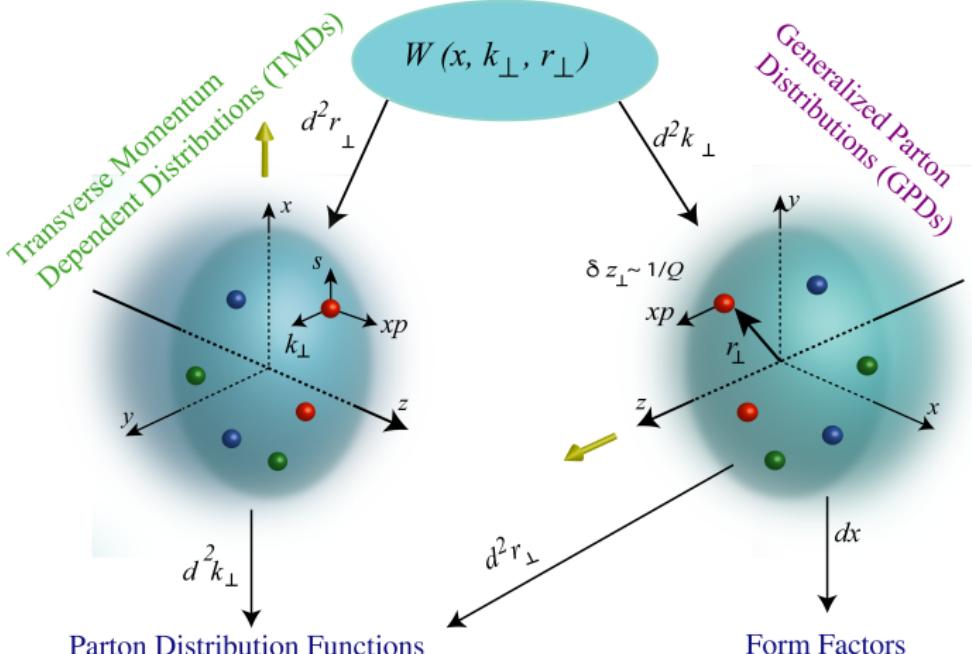
Application of BLFQ to nucleon

PRD 102, 016008 (2020) and (work in progress)

Conclusions

Hadron tomography

Wigner Distributions



- $x \rightarrow$ longitudinal momentum fraction; $k_{\perp} \rightarrow$ parton transverse momentum; $r_{\perp} \rightarrow$ transverse distance from the center.

Basis Light-Front Quantization (BLFQ)



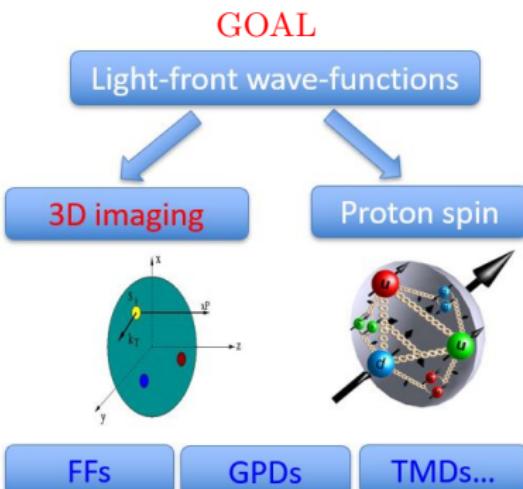
A computational framework for solving relativistic many-body bound state problems in quantum field theories¹

$$P^- P^+ |\Psi\rangle = M^2 |\Psi\rangle$$

- P^- : light-front Hamiltonian
- P^+ : longitudinal momentum
- $|\Psi\rangle$ mass eigenstate
- M^2 : mass squared eigenvalue for eigenstate $|\Psi\rangle$
- First-principles / effective Hamiltonian as input
- Evaluate observables

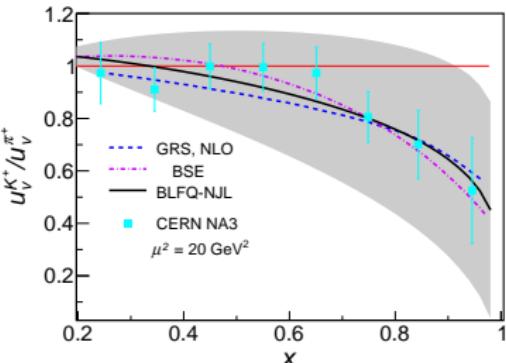
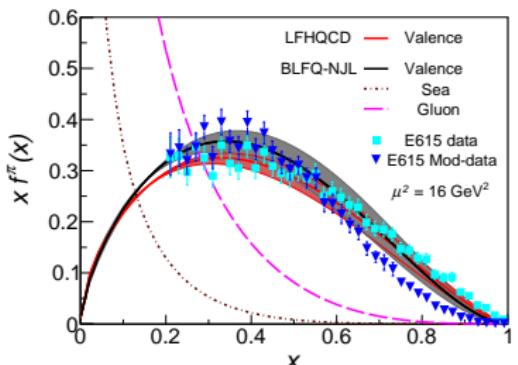
$$O \sim \langle \Psi | \hat{O} | \Psi \rangle$$

- direct access to light-front wavefunction of bound states



¹Vay *et. al.* PRC 81 (2010).

Application: light meson PDFs



Light front effective Hamiltonian, H_{eff} : ($\mu_{0\pi}^2 = 0.240 \pm 0.024 \text{ GeV}^2$)

$$H_{\text{eff}} = \underbrace{\frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x}}_{\text{LF Kinetic energy}} + \underbrace{\kappa^4 x(1-x)\vec{r}_\perp^2}_{\text{Transverse}} - \underbrace{\frac{\kappa^4}{(m_q + m_{\bar{q}})^2} \partial_x(x(1-x)\partial_x)}_{\text{Longitudinal}} + H_{\text{NJL}}^{\text{eff}},$$

Diagonalizing $H_{\text{eff}} \Rightarrow$ LF wavefunction \Rightarrow Initial PDFs \Rightarrow Scale evolution ¹.

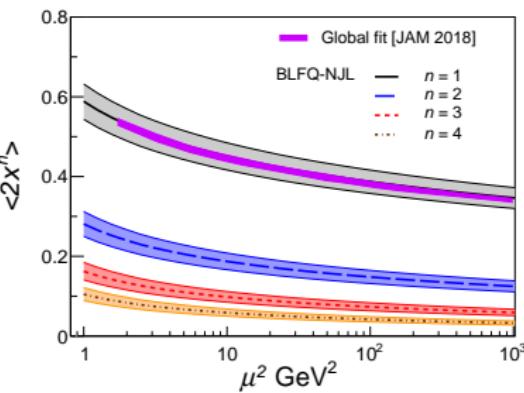
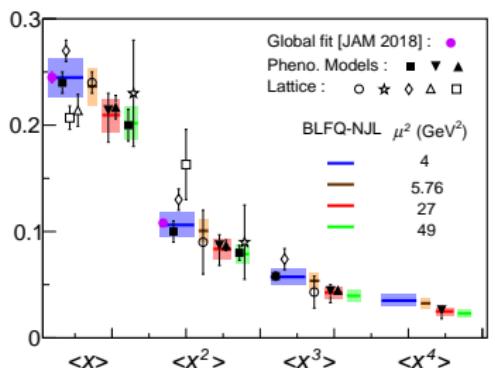
¹ Lan, CM, Jia, Zhao, Vary: PRL 122 (2019)

Moments of Pion PDF

Moments of the valence quark PDF



$$\langle x^n \rangle = \int_0^1 dx \ x^n f_v^\pi(x, \mu^2), \ n = 1, 2, 3, 4.$$



Consistent with global fit, lattice, and phenomenological models.

¹ Lan, CM, Jia, Zhao, Vary: PRL 122 (2019)

Distribution Amplitude and Transition Form Factor

$\pi \rightarrow \gamma^* \gamma$ TFF:¹

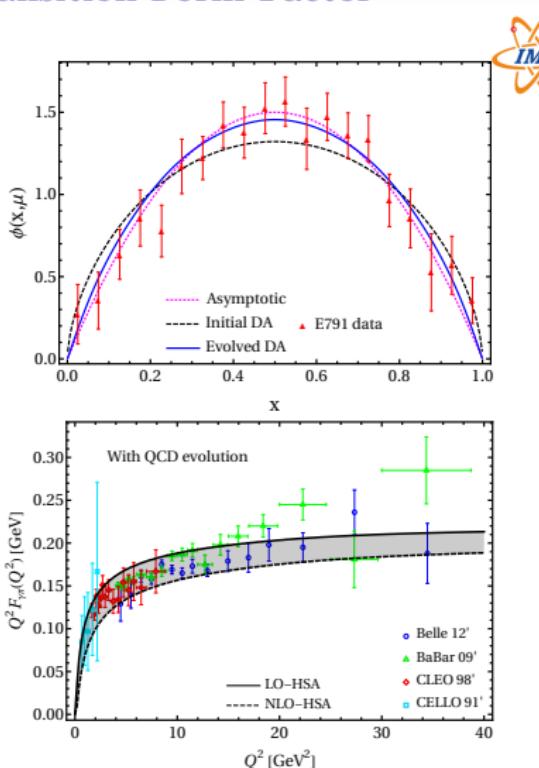
$$F_{\pi\gamma}(Q^2) = \frac{\sqrt{2}}{3} f_\pi \int_0^1 dx \frac{\phi_\pi(x, xQ)}{Q^2 x}$$

- DA evolution: ERBL evol.
(Gegenbauer basis)
Ruiz, et. al. PRD 66, (2002)
- Our DA is close to Asymptotic DA
- Our prediction agrees well with data reported by Belle Collaboration.
- It deviates from the rapid growth of the large Q^2 data reported by BaBar Collaboration.

Decay constant f_π :

BLFQ: 142.9 MeV

Experimental data: 130.2 ± 1.7 MeV



¹ CM et. al., in preparation



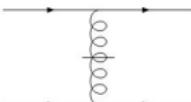
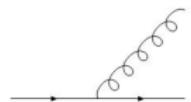
$$|\pi\rangle = |q\bar{q}\rangle \boxed{|} + \dots$$



$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle \boxed{|} + \dots$$

Interaction Part of Hamiltonian

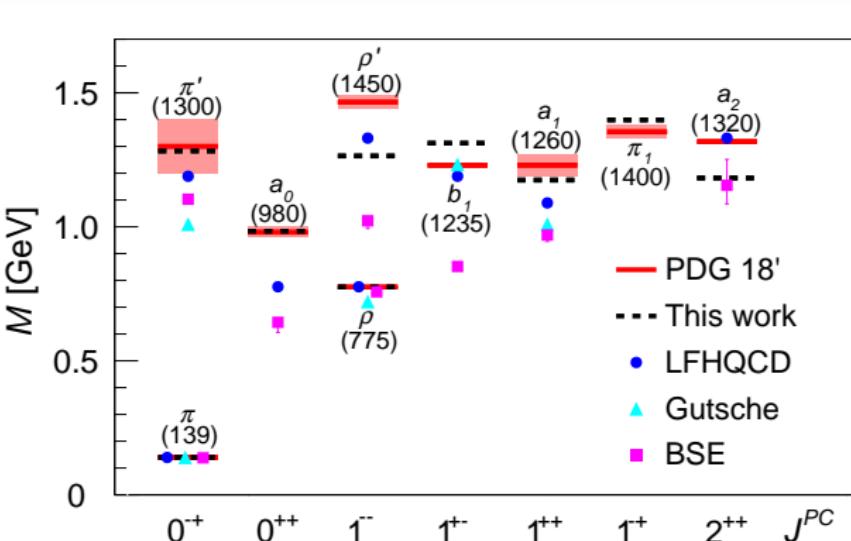
$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle^{\textcolor{red}{1}} + \dots$$

H_{int}	$ q\bar{q}\rangle$	$ q\bar{q}g\rangle$
$\langle q\bar{q} $		
$\langle q\bar{q}g $		0

$$\begin{aligned} P^- = & \frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x} + \kappa^4 x(1-x) \vec{r}_\perp^2 \\ & - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2} \partial_x(x(1-x)\partial_x) + H_{\text{int}} \end{aligned}$$



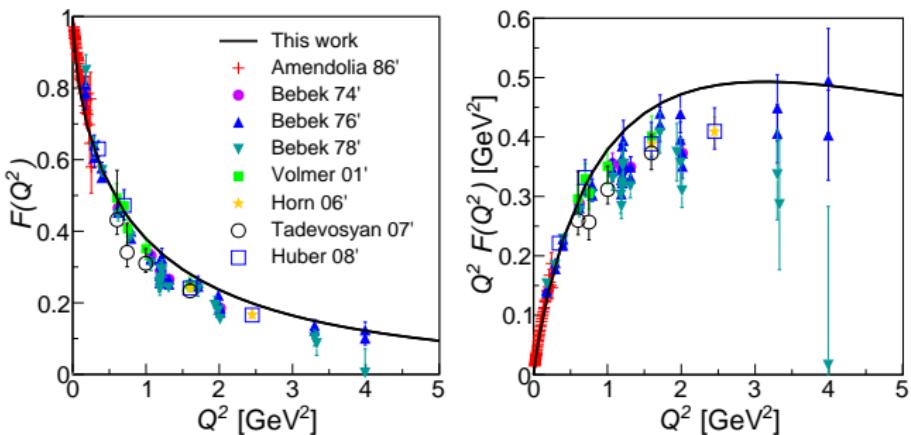
¹ J. Lan, K. Fu, CM, X. Zhao and j. P. Vary, arXiv:2106.04954 [hep-ph]



- Mass spectra for unflavored light mesons
- We are able to fit the hybrid state $\pi_1(1400)$, which may be viewed as $q\bar{q}$ meson with a vibrating gluon flux tube.

¹J. Lan, K. Fu, CM, X. Zhao and j. P. Vary, arXiv:2106.04954 [hep-ph]

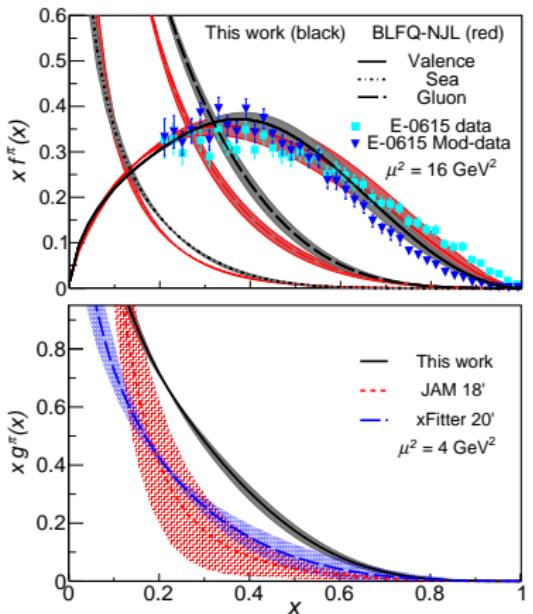
Pion Electromagnetic Form Factor



- Our choice of $N_{\max} = 14$ (BLFQ basis), implies the UV regulator $\Lambda_{\text{UV}} \approx 1$ GeV.
- The agreement with precise low Q^2 EMFF data is consistent with our expectation that our predictions are most reliable in the low Q^2 regime.

¹ J. Lan, K. Fu, CM, X. Zhao and j. P. Vary, arXiv:2106.04954 [hep-ph]

Valence quark and gluon PDFs in Pion

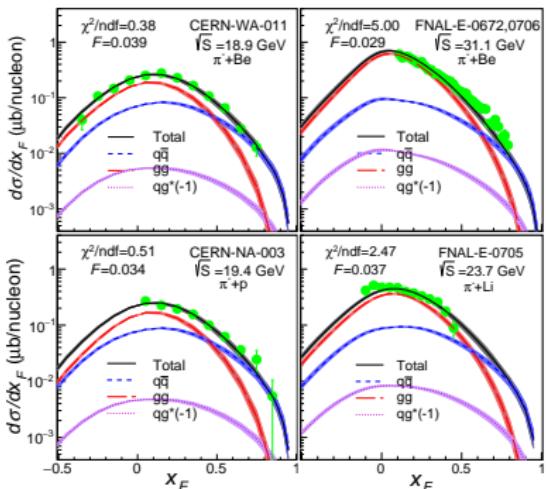


- Our model includes a dynamical gluon at the initial scale and results in a larger gluon PDF at large- x (> 0.2).

¹J. Lan, K. Fu, CM, X. Zhao and j. P. Vary, arXiv:2106.04954 [hep-ph]

Pion-nucleus induced J/ψ production

$$\frac{d\sigma}{dx_F} \Big|_{J/\psi} = F \sum_{i,j=q,\bar{q},g} \int_{2m_c}^{2m_D} dM_{c\bar{c}} \frac{2M_{c\bar{c}}}{S\sqrt{x_F^2 + 4M_{c\bar{c}}^2/S}} \\ \times \hat{\sigma}_{ij}(s, m_c^2, \mu_F^2, \mu_R^2) f_i^{\pi^\pm}(x_1, \mu_F^2) f_j^N(x_2, \mu_F^2)$$



- Color evaporation model framework at NLO → Our pion PDFs → obtain good agreement with available data.

¹ J. Lan, K. Fu, CM, X. Zhao and j. P. Vary, arXiv:2106.04954 [hep-ph]

Nucleon within BLFQ



- The LF eigenvalue equation: $H_{\text{eff}}|\Psi\rangle = M^2|\Psi\rangle$

$$H_{\text{eff}} = \sum_a \frac{\vec{p}_{\perp a}^2 + m_a^2}{x_a} + \frac{1}{2} \sum_{a \neq b} \kappa^4 \left[x_a x_b (\vec{r}_{\perp a} - \vec{r}_{\perp b})^2 - \frac{\partial_{x_a} (x_a x_b \partial_{x_b})}{(m_a + m_b)^2} \right]$$

$$+ \frac{1}{2} \sum_{a \neq b} \frac{C_F 4\pi \alpha_s}{Q_{ab}^2} \bar{u}_{s'_a}(k'_a) \gamma^\mu u_{s_a}(k_a) \bar{u}_{s'_b}(k'_b) \gamma^\nu u_{s_b}(k_b) g_{\mu\nu}$$

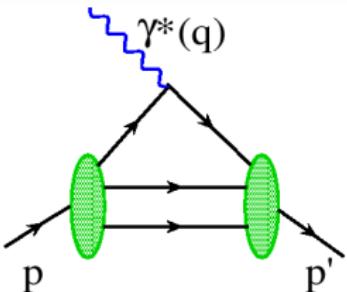
- For the first Fock sector:

$$|qqq\rangle = |n_{q_1}, m_{q_1}, k_{q_1}, \lambda_{q_1}\rangle \otimes |n_{q_2}, m_{q_2}, k_{q_2}, \lambda_{q_2}\rangle \otimes |n_{q_3}, m_{q_3}, k_{q_3}, \lambda_{q_3}\rangle$$

- Transverse : 2D harmonic oscillator basis $\phi_{nm}(\vec{p}_\perp)$;
Plane wave basis in longitudinal direction.
- The valence wavefunction in momentum space ¹:

$$\Psi_{\{x_i, \vec{p}_{\perp i}, \lambda_i\}}^{M_J} = \sum_{n_i, m_i} \left[\psi(\alpha_i) \prod_{i=1}^3 \phi_{n_i m_i}(\vec{p}_{\perp i}) \right]$$

Proton Form Factors



$$\langle P'; \uparrow | \frac{J^+(0)}{2P^+} | P; \uparrow \rangle = F_1(q^2)$$

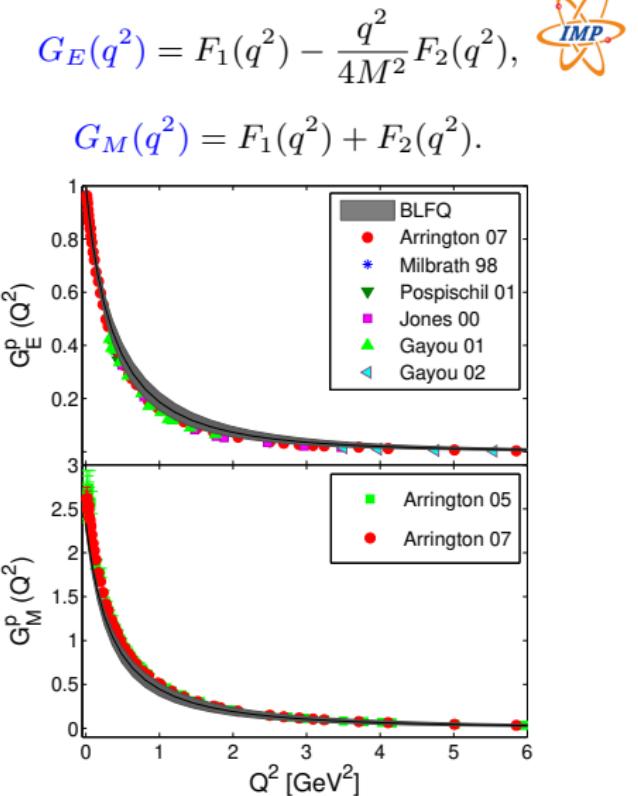
$$\langle P'; \uparrow | \frac{J^+(0)}{2P^+} | P; \downarrow \rangle = -(q^1 - iq^2) \frac{F_2(q^2)}{2M}$$

Drell & Yan (PRL, 70); West (PRL, 70)

$$N_{\max} = 10, K = 16$$

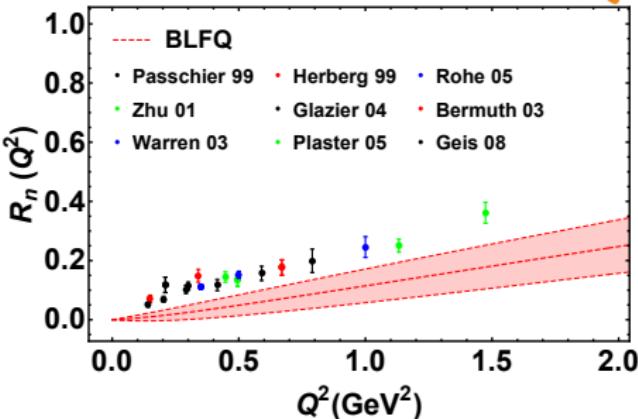
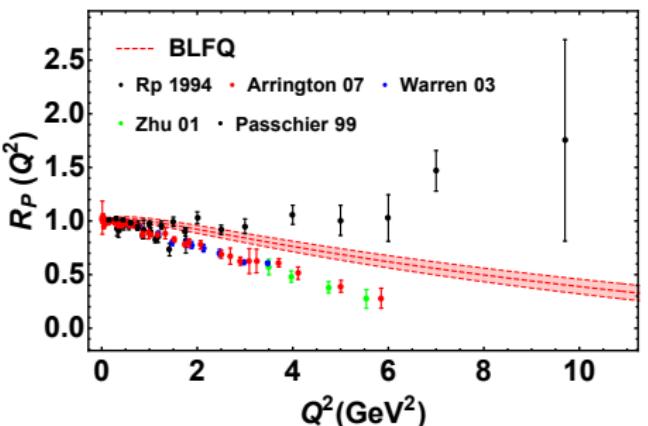
Basis truncation :

$$\sum_i (2n_i + |m_i| + 1) \leq N_{\max}; \quad K = \sum_i k_i$$



¹CM, Siqi Xu *et. al.*, Phys. Rev. D **102**, 016008 (2020)

Ratio of Form Factors



- Only valence quarks contributions
- Missing meson-cloud effects
- $|qqqq\bar{q}\rangle$ has a significant effect on Pauli FF: 30% in proton; 40% in neutron

Sufian *et. al.* PRD 95 (2017)

$$R_{p/n} = \mu_{p/n} \frac{G_E^{p/n}}{G_M^{p/n}}$$

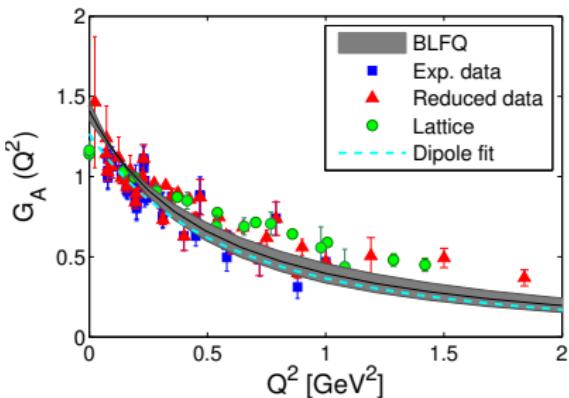
¹ Siqi Xu, CM *et. al.*, in preparation

Nucleon Axial Form Factor



$$\langle N(p) | A^\mu | N(p') \rangle = \bar{u}(p') \left[\gamma^\mu G_A(t) + \frac{(p' - p)^\mu}{2m} G_p(t) \right] \gamma_5 u(p)$$

- Axial vector current:
 $A^\mu = \bar{q} \gamma^\mu \gamma_5 q$
- Axial form factor can be measured by ordinary muon capture (OMC)
 $\mu^-(l) + p(r) \rightarrow \nu_\mu(l') + n(r')$
- Provide information on spin-isospin distributions



$$G_A(Q^2) = G_u(Q^2) - G_d(Q^2)$$

¹CM, Siqi Xu *et. al.*, Phys. Rev. D **102**, 016008 (2020)

TMDs of spin-1/2 target



- The k^\perp -dependent fermion-fermion correlator:

$$\Phi^{[\Gamma]}(x, k^\perp) = \int \frac{dz^- d^2 z^\perp}{2(2\pi)^3} e^{ik \cdot z} \langle P, S | \bar{\Psi}(0) \mathcal{U}(0, z) \Gamma \Psi(z) | P, S \rangle|_{z^+ = 0}$$

- At the leading-twist,

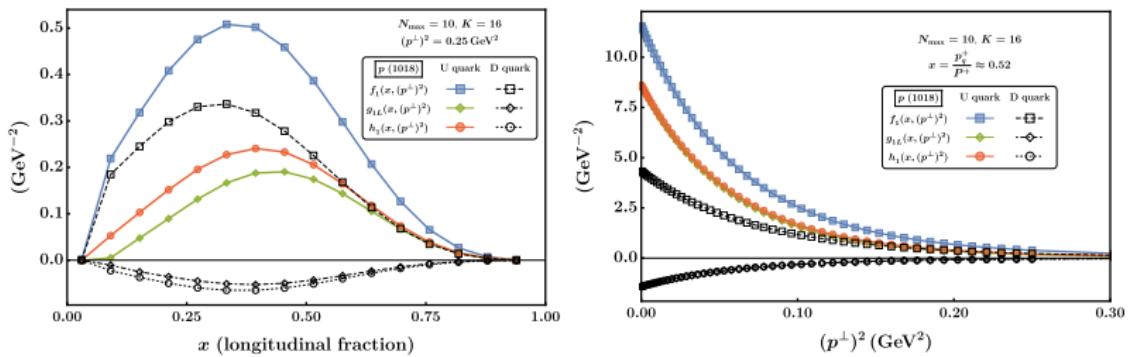
$$\begin{aligned} \Phi^{[\gamma^+]}(x, k^\perp) &= f_1^e - \frac{\epsilon^{ij} k^i S^j}{M} f_{1T}^{\perp e}; \\ \Phi^{[\gamma^+ \gamma_5]}(x, k^\perp) &= S^3 g_{1L}^e + \frac{S^\perp \cdot k^\perp}{M} g_{1T}^e \quad ; \quad \Phi^{[i\sigma^j + \gamma^5]}(x, k^\perp) = \dots \end{aligned}$$

- Spin 4-vector of the physical state:

$$S = \left(S^3 \frac{P^+}{M_e}, -S^3 \frac{M_e}{P^+}, S^\perp \right)$$



Quark TMDs in the Proton



f_1 , g_{1L} and h_1 TMDs for proton with $N_{\max} = 10$ and $K = 16$ ¹

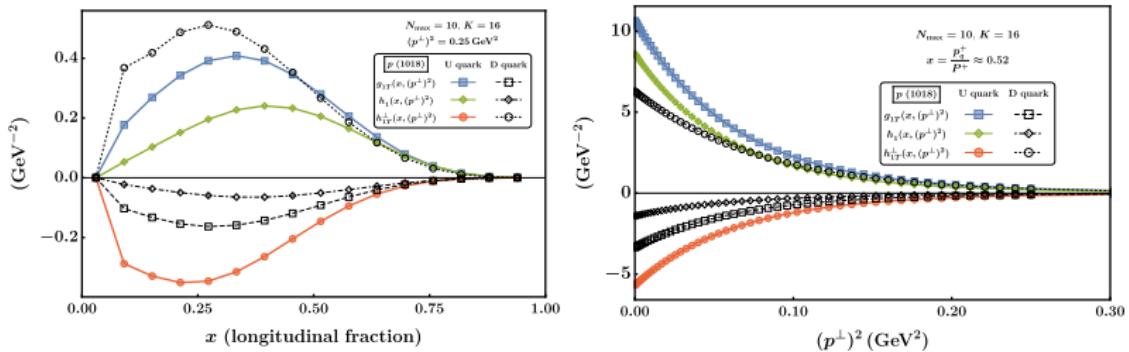
- Qualitative nature consistent with CQM ² / quark-diquark models

¹ Zhi Hu, *et. al.*, In preparation .

² B. Pasquini, S. Cazzaniga, and S. Boffi, PRD 78, 034025 (2008).



Quark TMDs in the Proton



g_{1T} , h_{1L}^\perp and h_{1T}^\perp TMDs for proton with $N_{\max} = 10$ and $K = 16$ ¹

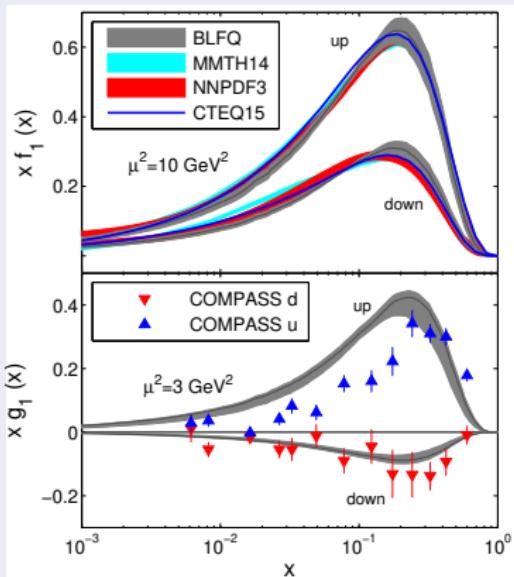
- Qualitative nature consistent with CQM ² / quark-diquark models

¹ Zhi Hu, *et. al.*, In preparation .

² B. Pasquini, S. Cazzaniga, and S. Boffi, PRD 78, 034025 (2008).

Parton Distribution Functions

CM, Siqi Xu et. al., PRD 102, 016008 (2020)



$$f_1(x, k^\perp) \rightarrow f_1(x), \\ g_{1L}(x, k^\perp) \rightarrow g_1(x)$$

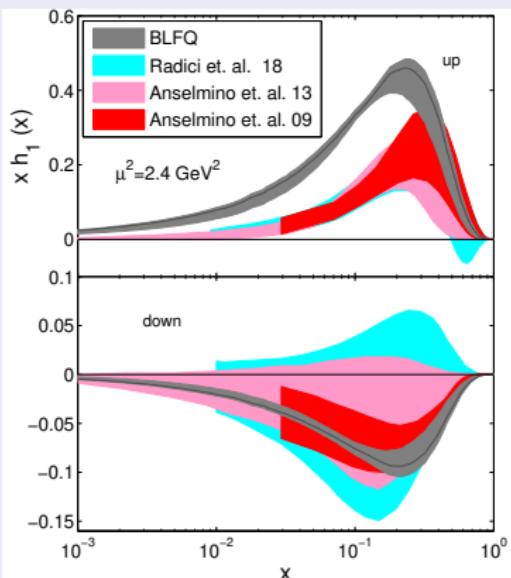
- Unpolarized PDFs $f_1(x)$: longitudinal momentum distribution of unpolarized quark in unpolarized proton.
- Helicity PDFs $g_1(x)$: longitudinal momentum distribution of the polarized quark
- Results correspond to leading Fock sector only, missing higher Fock sectors.

¹ NNPDF, EPJC 77, 663 (2017); HMMT, EPJC 75, 204 (2015); CTEQ, PRD 93, 033006 (2016).

² COMPASS Collaboration, Phys. Lett. B 693, 227 (2010).

Transversity Distribution

CM, Siqi Xu *et. al.*, PRD 102, 016008 (2020)



$$h_1(x, k^\perp) \rightarrow h_1(x)$$

- Transversity PDFs describe correlation between the transverse polarization of the nucleon and the transverse polarization of the parton.

- Satisfy Soffer Bound:

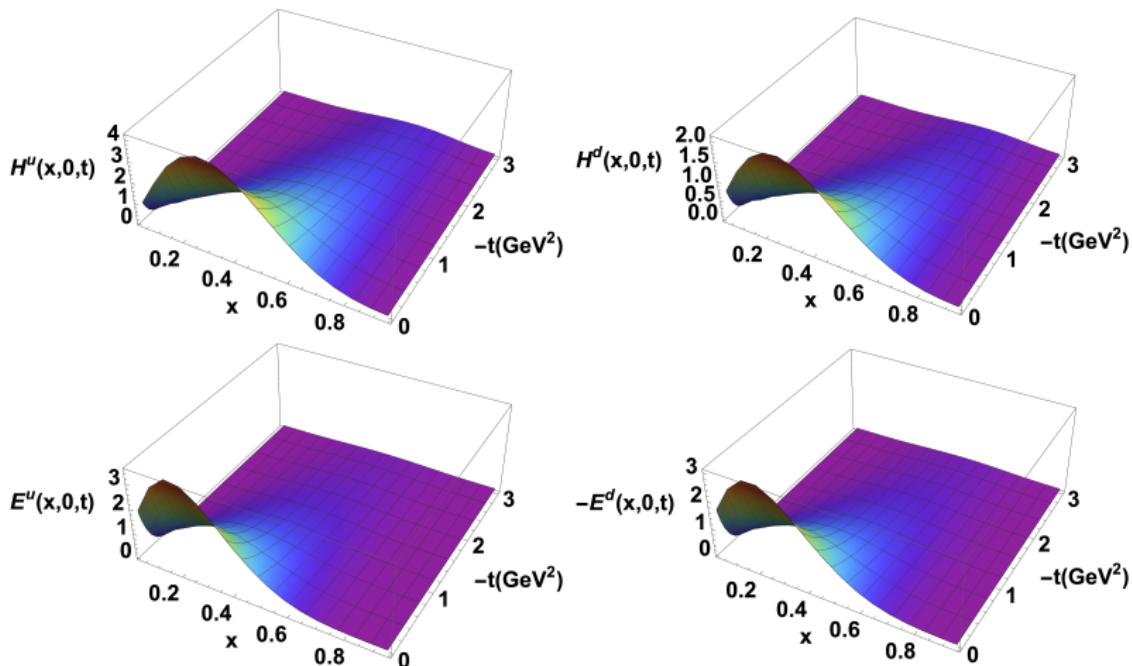
$$|h_1(x)| \leq \frac{1}{2} |f_1(x) + g_1(x)|$$

- Results correspond to leading Fock sector only, missing higher Fock sectors.

¹ M. Radici and A. Bacchetta, Phys. Rev. Lett. 120, 192001 (2018).

² M. Anselmino, *et. al.*, Phys. Rev. D 87, 094019 (2013).

Quark GPDs in the Proton



Unpolarized quark GPDs in the proton with $N_{\max} = 10$ and $K = 16$ ¹

- Qualitative nature consistent with phenomenological models²

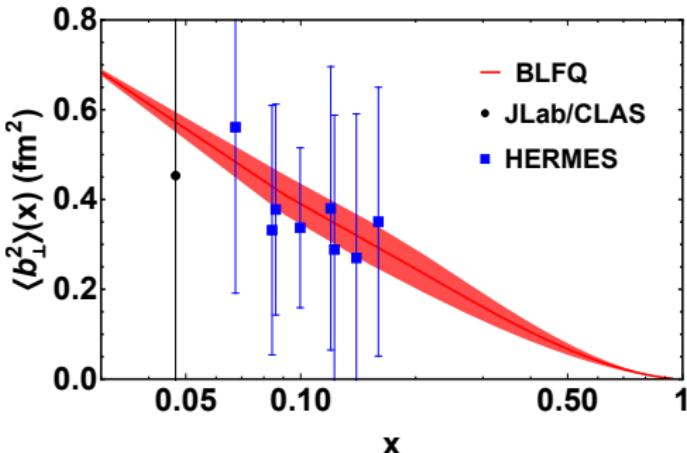
¹ Siqi Xu, CM *et. al.*, in preparation .

² CM, D. Chakrabarti, EPJC 75, 261 (2015); PRD 88, 073006 (2013)

The x dependent squared radius



$$\langle b_\perp^2 \rangle^q(x) = \frac{\int d^2 \vec{b}_\perp b_\perp^2 \mathcal{H}^q(x, b_\perp)}{\int d^2 \vec{b}_\perp \mathcal{H}^q(x, b_\perp)},$$



- We obtain for proton, $\langle b_\perp^2 \rangle = 0.36 \pm 0.04$ fm 2 , while experimental data ²: $\langle b_\perp^2 \rangle_{\text{exp}} = 0.43 \pm 0.01$ fm 2 . ($\langle b_\perp^2 \rangle = e_u \langle b_\perp^2 \rangle^u + e_d \langle b_\perp^2 \rangle^d$)

¹ Siqi Xu, CM *et. al.*, in preparation

² R. Dupre, M. Guidal and M. Vanderhaeghen, PRD 95, 011501 (2017).

Other Observables

Proton radii, axial and tensor charges, first moments of transversity



Quantity	BLFQ	Extracted data	Lattice
r_E fm	$0.802^{+0.042}_{-0.040}$	0.833 ± 0.010 [98]	$0.742(13)$ [27]
r_M fm	$0.834^{+0.029}_{-0.029}$	0.851 ± 0.026 [99]	$0.710(26)$ [27]
r_A fm	$0.680^{+0.070}_{-0.073}$	0.667 ± 0.12 [100]	$0.512(34)$ [101]
g_A	$1.41^{+0.06}_{-0.06}$	1.2723 ± 0.0023 [99]	$1.237(74)$ [101]
g_T^d	$-0.20^{+0.02}_{-0.04}$	$-0.25^{+0.30}_{-0.10}$ [74]	$-0.204(11)$ [102]
g_T^u	$0.94^{+0.06}_{-0.15}$	$0.39^{+0.18}_{-0.12}$ [74]	$0.784(28)$ [102]
$\langle x \rangle_T^{u-d}$	$0.229^{+0.019}_{-0.048}$	—	$0.203(24)$ [103]

- [99]: M. Anselmino *et. al.* PRD 87, 094019 (2013).
- [102]: Gupta *et. al.*, PRD D 98, 091501 (2018).

¹CM, Siqi Xu *et. al.*, PRD 102, 016008 (2020)

Conclusions & Outlook



- We discussed structure of light mesons and nucleon from the eigenstates of light front effective Hamiltonian
- $|q\bar{q}\rangle$ and $|q\bar{q}g\rangle$ for pion, $|qqq\rangle$ for the nucleon.
- **LF Hamiltonian \Rightarrow Wavefunctions \Rightarrow Observables.**
- we discussed the sensitivity of J/Ψ production to our pion PDFs especially on gluon PDF.
- BLFQ provides a good description of the experimental data/Global fits for various observables: form factors, PDFs, GPDs, etc.,
- **This is not the complete picture ... long way to go.**

Outlook:

- Study meson-cloud effects and gluon content (in baryons).
- GPDs \rightarrow gravitational form factors \rightarrow Mechanical properties, spin structure of proton, mass decomposition.

Enormous amount of possibilities Thank You