

A Fourier-Cumulant Analysis for Multiharmonic Flow Fluctuation

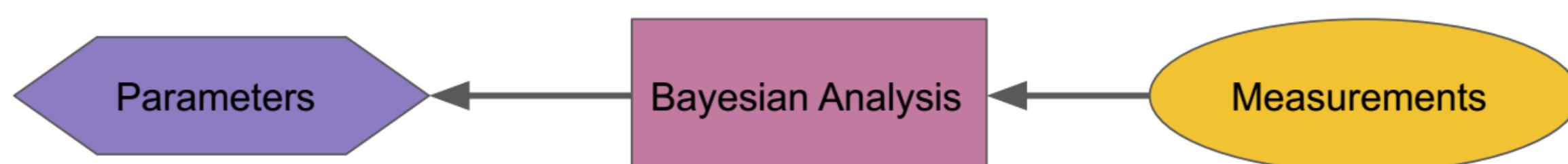
arXiv: 2005.04742 [nucl-th], (accepted for publication in EPJC)

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INTRODUCTION

- Significant progress has happened in modeling heavy-ion collisions.
 - A typical model contains 10 to 20 parameters. Two most interesting ones are η/s and ζ/s .



- It is important to introduce new experimental observables.

FLOW HARMONICS IN A NUTSHELL!

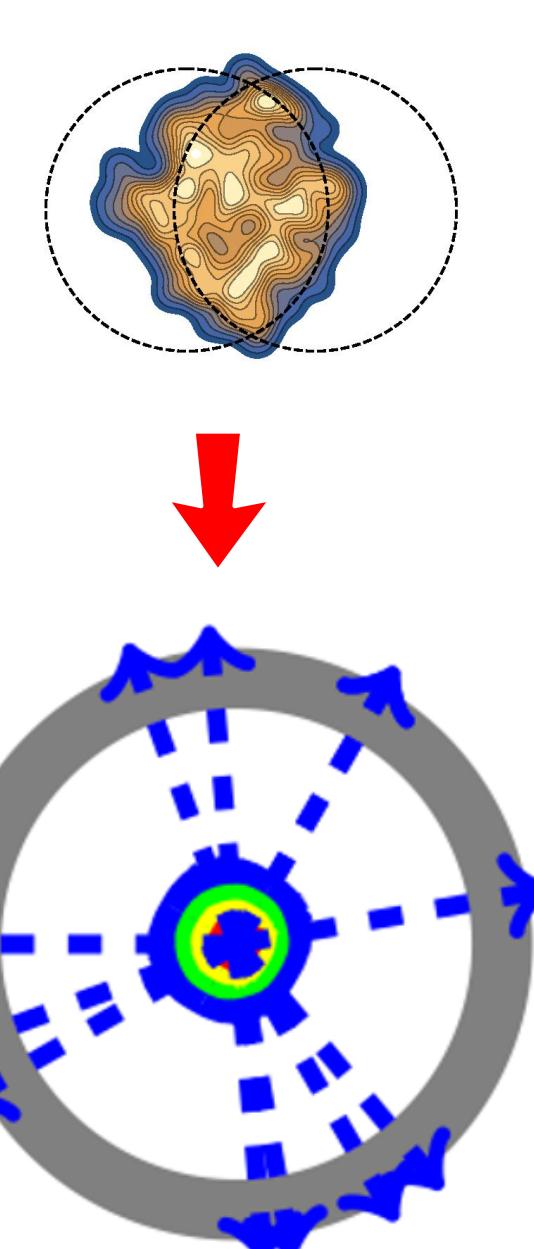
$$\frac{dN}{d\varphi} \propto 1 + \sum_{n=1}^{\infty} 2 \textcolor{red}{v}_n \cos [n(\varphi - \textcolor{blue}{\psi}_n)]$$

Flow harmonic fluctuation distribution:

$$p_f(v_1, v_2, v_3, \dots, \psi_1 - \psi_2, \psi_2 - \psi_3, \dots)$$

Some examples for the distribution's cumulant: [1]

$c_n\{2k\}$, SC(n, m), SC(n, m, ℓ)

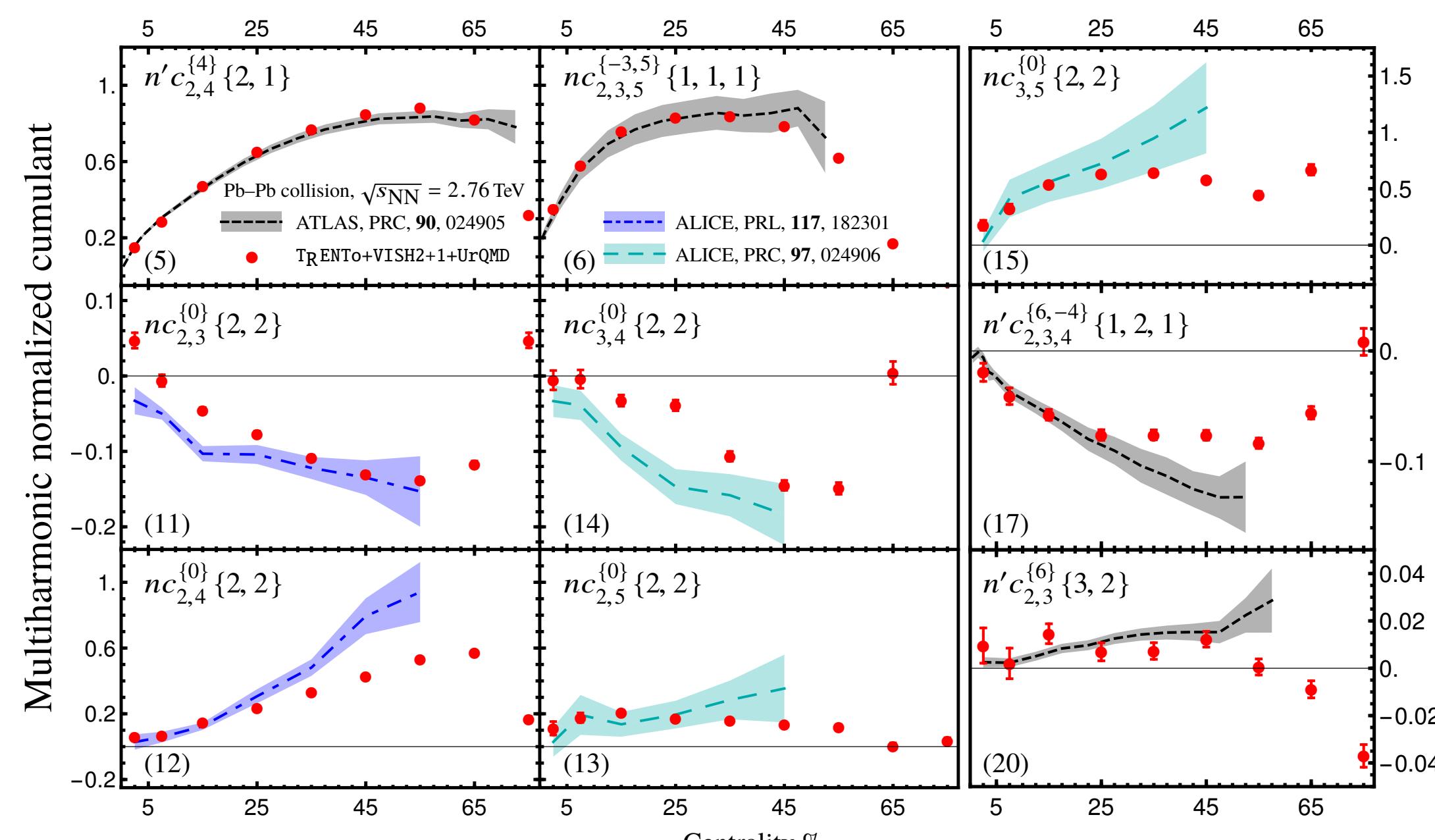


ONE PACKAGE FOR ALL CUMULANTS

- Mathematica package **MultiharmonicCumulants_v2_1.m**
<https://github.com/FaridTaghavi/MultiharmonicCumulants.git>
 - Returns the cumulants in terms of correlation functions, Q -vectors, etc, and their statistical uncertainty relations.
 - Examples: SC(2,3), statistical error of $c_2\{2\}$

```
In[1] := c[{2,2},{0},{2,3},v,ψ], In[2] := Nsigma2[cCorr[{2},{}, {2},corr]]
Out[1] := <v₂² v₃²> - <v₂²><v₃²>, Out[2] := <⟨2⟩₂⁻₂,₂> - <⟨2⟩⁻₂,₂>²
```

NORMALIZED CUMULANTS AT THE LHC

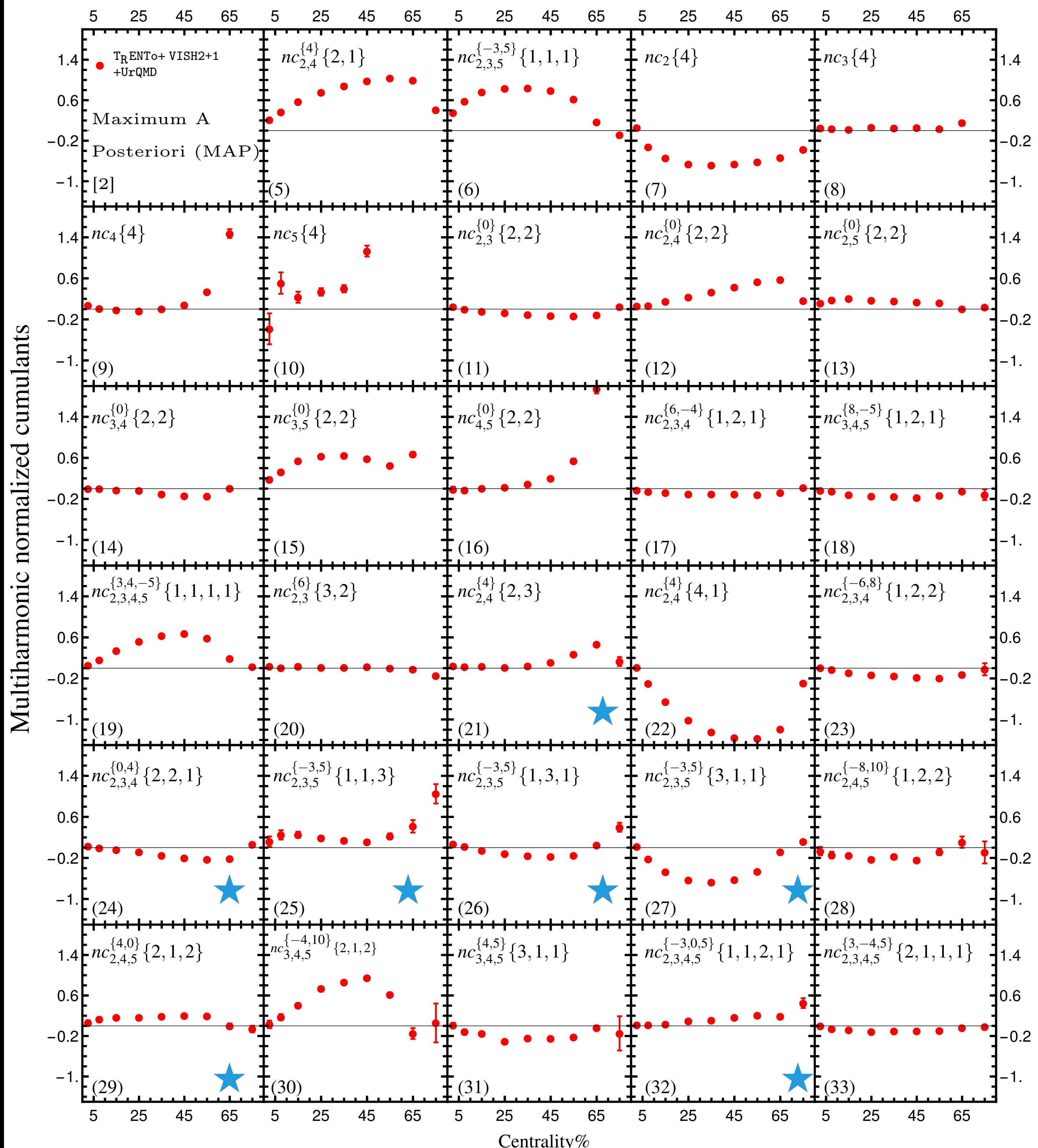


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- [1] N. Borghini, P. M. Dinh and J. Y. Ollitrault, PRC **64**, 054901 (2001); A. Bilandzic, C. H. Christensen, K. Gulbrandsen, A. Hansen and Y. Zhou, PRC **89**, no.6, 064904 (2014); C. Mordasini, A. Bilandzic, D. Karakoç and S. F. Taghavi, PRC **102**, no.2, 024907 (2020); S. Acharya *et al.* [ALICE], [arXiv:2101.02579 [nucl-ex]].
 - [2] J. E. Bernhard, J. S. Moreland and S. A. Bass, Nature Phys. **15**, no.11, 1113-1117 (2019)

ALL CUMULANTS: HARMONICS 2.3.4.5 AND ORDERS 2.3.4.5

cumulant	order	cumulant expression
1 $c_2\{2\}$	2	$\langle v_2^2 \rangle$
2 $c_3\{2\}$	2	$\langle v_3^2 \rangle$
3 $c_4\{2\}$	2	$\langle v_4^2 \rangle$
4 $c_5\{2\}$	2	$\langle v_5^2 \rangle$
5 $c_{2,4}^{\{4\}}\{2,1\}$	3	$\langle v_2^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle$
6 $c_{2,3,5}^{\{-3,5\}}\{1,1,1\}$	3	$\langle v_2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
7 $c_2\{4\}$	4	$\langle v_2^4 \rangle - 2\langle v_2^2 \rangle^2$
8 $c_3\{4\}$	4	$\langle v_3^4 \rangle - 2\langle v_3^2 \rangle^2$
9 $c_4\{4\}$	4	$\langle v_4^4 \rangle - 2\langle v_4^2 \rangle^2$
10 $c_5\{4\}$	4	$\langle v_5^4 \rangle - 2\langle v_5^2 \rangle^2$
11 $c_{2,3}^{\{0\}}\{2,2\}$	4	$\langle v_2^2 v_3^2 \rangle - \langle v_2^2 \rangle \langle v_3^2 \rangle$
12 $c_{2,4}^{\{0\}}\{2,2\}$	4	$\langle v_2^2 v_4^2 \rangle - \langle v_2^2 \rangle \langle v_4^2 \rangle$
13 $c_{2,5}^{\{0\}}\{2,2\}$	4	$\langle v_2^2 v_5^2 \rangle - \langle v_2^2 \rangle \langle v_5^2 \rangle$
14 $c_{3,4}^{\{0\}}\{2,2\}$	4	$\langle v_3^2 v_4^2 \rangle - \langle v_3^2 \rangle \langle v_4^2 \rangle$
15 $c_{3,5}^{\{0\}}\{2,2\}$	4	$\langle v_3^2 v_5^2 \rangle - \langle v_3^2 \rangle \langle v_5^2 \rangle$
16 $c_{4,5}^{\{0\}}\{2,2\}$	4	$\langle v_4^2 v_5^2 \rangle - \langle v_4^2 \rangle \langle v_5^2 \rangle$
17 $c_{2,3,4}^{\{6,-4\}}\{1,2,1\}$	4	$\langle v_3^2 v_2 v_4 \cos(2(\psi_2 - 3\psi_3 + 2\psi_4)) \rangle$
18 $c_{3,4,5}^{\{8,-5\}}\{1,2,1\}$	4	$\langle v_4^2 v_3 v_5 \cos(3\psi_3 - 8\psi_4 + 5\psi_5) \rangle$
19 $c_{2,3,4,5}^{\{3,4,-5\}}\{1,1,1,1\}$	4	$\langle v_2 v_3 v_4 v_5 \cos(2\psi_2 - 3\psi_3 - 4\psi_4 + 5\psi_5) \rangle$
20 $c_{2,3}^{\{6\}}\{3,2\}$	5	$\langle v_2^3 v_3^2 \cos(6(\psi_2 - \psi_3)) \rangle$
21 $c_{2,4}^{\{4\}}\{2,3\}$	5	$\langle v_2^2 v_4^3 \cos(4(\psi_2 - \psi_4)) \rangle - 2\langle v_4^2 \rangle \langle v_2^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle$
22 $c_{2,4}^{\{4\}}\{4,1\}$	5	$\langle v_2^4 v_4 \cos(4(\psi_2 - \psi_4)) \rangle - 3\langle v_2^2 \rangle \langle v_2^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle$
23 $c_{2,3,4}^{\{-6,8\}}\{1,2,2\}$	5	$\langle v_4^2 v_3^2 v_2 \cos(2(\psi_2 + 3\psi_3 - 4\psi_4)) \rangle$
24 $c_{2,3,4}^{\{0,4\}}\{2,2,1\}$	5	$\langle v_3^2 v_2^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle - \langle v_3^2 \rangle \langle v_2^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle$
25 $c_{2,3,5}^{\{-3,5\}}\{1,1,3\}$	5	$\langle v_5^3 v_2 v_3 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle - 2\langle v_5^2 \rangle \langle v_2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
26 $c_{2,3,5}^{\{-3,5\}}\{1,3,1\}$	5	$\langle v_3^3 v_2 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle - 2\langle v_3^2 \rangle \langle v_2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
27 $c_{2,3,5}^{\{-3,5\}}\{3,1,1\}$	5	$\langle v_2^3 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle - 2\langle v_2^2 \rangle \langle v_2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
28 $c_{2,4,5}^{\{-8,10\}}\{1,2,2\}$	5	$\langle v_5^2 v_4^2 v_2 \cos(2(\psi_2 + 4\psi_4 - 5\psi_5)) \rangle$
29 $c_{2,4,5}^{\{4,0\}}\{2,1,2\}$	5	$\langle v_5^2 v_2^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle - \langle v_2^2 v_4 \cos(4(\psi_2 - \psi_4)) \rangle \langle v_5^2 \rangle$
30 $c_{3,4,5}^{\{-4,10\}}\{2,1,2\}$	5	$\langle v_5^2 v_3^2 v_4 \cos(6\psi_3 + 4\psi_4 - 10\psi_5) \rangle$
31 $c_{3,4,5}^{\{4,5\}}\{3,1,1\}$	5	$\langle v_3^3 v_4 v_5 \cos(9\psi_3 - 4\psi_4 - 5\psi_5) \rangle$
32 $c_{2,3,4,5}^{\{-3,0,5\}}\{1,1,2,1\}$	5	$\langle v_4^2 v_2 v_5 v_3 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle - \langle v_4^2 \rangle \langle v_2 v_3 v_5 \cos(2\psi_2 + 3\psi_3 - 5\psi_5) \rangle$
33 $c_{2,3,4,5}^{\{3,-4,5\}}\{2,1,1,1\}$	5	$\langle v_2^2 v_3 v_4 v_5 \cos(4\psi_2 - 3\psi_3 + 4\psi_4 - 5\psi_5) \rangle$



SUMMARY AND OUTLOOK

- We introduced a systematic method to extract flow harmonic cumulants and their statistical uncertainties up to a given order.
 - The experimental values for unmeasured normalized cumulants? Can the new cumulants help the Bayesian analysis?