Two-pion contribution to hadronic vacuum polarization

Peter Stoffer
Physik-Institut, University of Zurich
and Paul Scherrer Institut

September 30, 2021

The 16th International Workshop on Tau Lepton Physics (TAU2021)

University of Zurich UZH
PAUL SCHERRER INSTITUT PSI
funded by Swiss National Science Foundation

$(g-2)_\mu$: theory vs. experiment

- discrepancy between SM theory white paper and experiment (E821 and FNAL combined) $\boldsymbol{4.2\sigma}$

  $\rightarrow$ talk by J. Stapleton

- hint to new physics?
- size of discrepancy points at **electroweak scale**
  $\Rightarrow$ heavy new physics needs some enhancement mechanism
- theory error completely dominated by **hadronic effects** $\rightarrow$ talk by G. Colangelo

Muon anomalous magnetic moment $(g-2)_\mu$

recent and future experimental progress:

- FNAL will improve precision further: **factor of 4 wrt E821**
- theory still needs to reduce **SM uncertainty**!

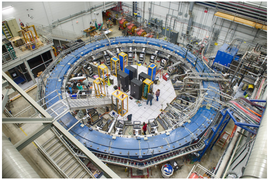
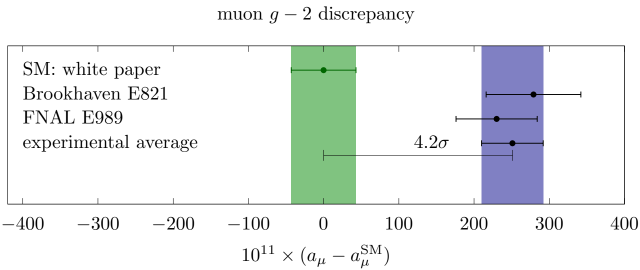


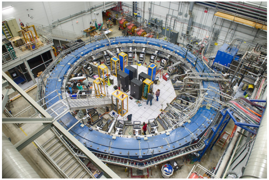
Photo: Glukicov (License: CC-BY-SA-4.0)

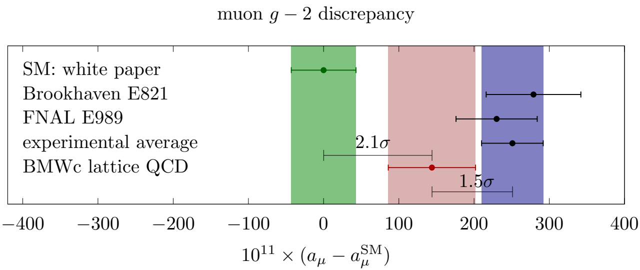


Muon anomalous magnetic moment $(g-2)_\mu$

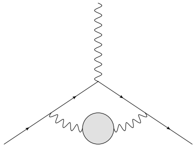
recent and future experimental progress:

- FNAL will improve precision further: **factor of 4 wrt E821**
- theory still needs to reduce **SM uncertainty**!





Hadronic vacuum polarization (HVP)



- at present evaluated via **dispersion relations** and cross-section input from $e^+e^- \to$ hadrons
- intriguing discrepancies between e^+e^- experiments $\Rightarrow$ treated as additional systematic uncertainty
- lattice QCD making fast progress $\to$ talk by A. El-Khadra
- $\mathbf{2.1\sigma}$ **tension** between dispersion relations and latest lattice results $\to$ S. Borsanyi *et al.*, Nature (2021)

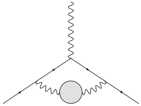
HVP contribution to $(g-2)_\mu$

$$a_\mu^{\mathrm{HVP}} = \frac{m_\mu^2}{12\pi^3} \int_{s_{\mathrm{thr}}}^{\infty} ds \, \frac{\hat{K}(s)}{s} \, \sigma(e^+e^- \to \mathrm{hadrons})$$

- basic principles: unitarity and analyticity
- direct **relation to data**: total hadronic cross section $\sigma(e^+e^- \to \mathrm{hadrons})$
- dedicated e^+e^- program (BaBar, Belle, BESIII, CMD3, KLOE, SND)

  $\to$ talks by B. Shwartz, T. Dimova, P. Lukin, K. Todyshev, A. Denig

Hadronic vacuum polarization



- final white paper number: data-driven evaluation

$$a_\mu^{\text{LO HVP, pheno}} = 6\,931(40) \times 10^{-11}$$

- previous average of published lattice-QCD results

$$a_\mu^{\text{LO HVP, lattice average}} = 7\,116(184) \times 10^{-11}$$

- newest lattice-QCD result

→ S. Borsanyi *et al.*, Nature (2021)

$$a_\mu^{\text{LO HVP, lattice}} = 7\,075(55) \times 10^{-11}$$

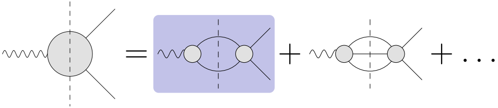
Two-pion contribution to HVP

- $\pi\pi$ contribution amounts to more than 70% of HVP contribution
- responsible for a similar fraction of HVP uncertainty
- can be expressed in terms of **pion vector form factor** $\Rightarrow$ constraints from analyticity and unitarity

  $\rightarrow$ Colangelo, Hoferichter, Stoffer, JHEP **02** (2019) 006

Dispersive representation of pion VFF

→ Colangelo, Hoferichter, Stoffer, JHEP **02** (2019) 006



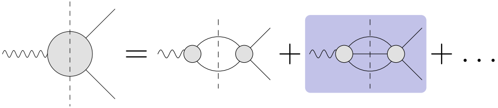
$$F_\pi^V(s) = \Omega_1^1(s) \times G_\omega(s) \times G_{\text{in}}^N(s)$$

- **Omnès function** with elastic $\pi\pi$-scattering P-wave phase shift $\delta_1^1(s)$ as input:

$$\Omega_1^1(s) = \exp\left\{ \frac{s}{\pi} \int_{4M_\pi^2}^{\infty} ds' \frac{\delta_1^1(s')}{s'(s'-s)} \right\}$$

Dispersive representation of pion VFF

→ Colangelo, Hoferichter, Stoffer, JHEP **02** (2019) 006



$$F_\pi^V(s) = \Omega_1^1(s) \times G_\omega(s) \times G_{\text{in}}^N(s)$$

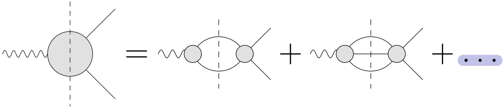
- isospin-breaking 3π intermediate state: negligible apart from ω **resonance** (ρ–ω interference effect)

$$G_\omega(s) = 1 + \frac{s}{\pi}\int_{9M_\pi^2}^\infty ds' \frac{\text{Im} g_\omega(s')}{s'(s'-s)} \left(\frac{1-\frac{9M_\pi^2}{s'}}{1-\frac{9M_\pi^2}{M_\omega^2}}\right)^4,$$

$$g_\omega(s) = 1 + \epsilon_\omega \frac{s}{(M_\omega - \frac{i}{2}\Gamma_\omega)^2 - s}$$

Dispersive representation of pion VFF

→ Colangelo, Hoferichter, Stoffer, JHEP **02** (2019) 006



$$F_\pi^V(s) = \Omega_1^1(s) \times G_\omega(s) \times G_{\text{in}}^N(s)$$

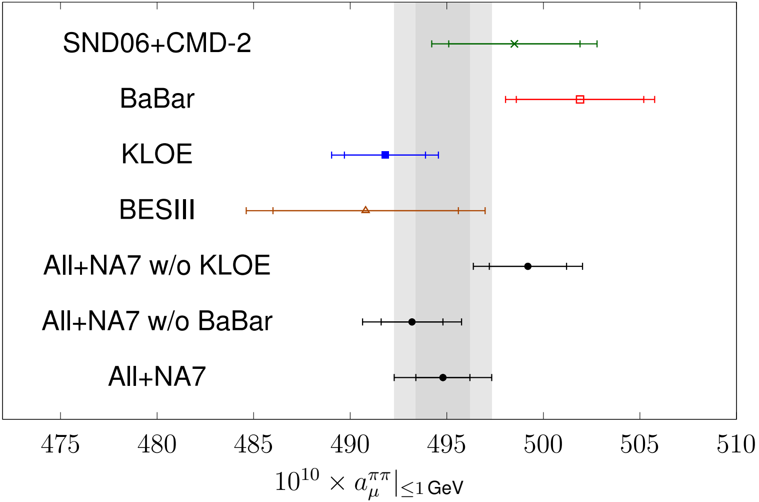
- heavier intermediate states: 4π (mainly $\pi^0\omega$), $\bar{K}K, \ldots$
- described in terms of a **conformal polynomial** with cut starting at $\pi^0\omega$ threshold

$$G_{\text{in}}^N(s) = 1 + \sum_{k=1}^{N} c_k (z^k(s) - z^k(0))$$

- correct P-wave threshold behavior imposed

Result for $a_\mu^{\mathrm{HVP},\pi\pi}$ below 1 GeV

→ Colangelo, Hoferichter, Stoffer, JHEP **02** (2019) 006



Tension with lattice QCD

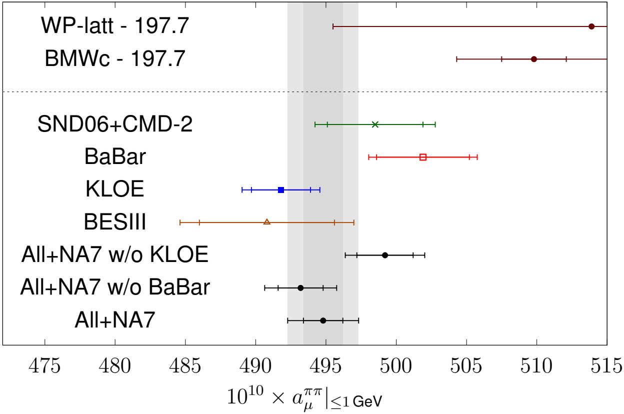
→ Colangelo, Hoferichter, Stoffer, PLB **814** (2021) 136073

- implications of changing HVP?
- modifications at high energies affect **hadronic running of** $\alpha_{\mathrm{QED}}^{\mathrm{eff}}$ $\Rightarrow$ clash with global EW fits

  → Passera, Marciano, Sirlin (2008), Crivellin, Hoferichter, Manzari, Montull (2020), Keshavarzi, Marciano, Passera, Sirlin (2020), Malaescu, Schott (2020)

  → talk by A. Keshavarzi
- lattice studies point at region < 2 GeV
- $\pi\pi$ **channel** dominates

Result for $a_\mu^{\mathrm{HVP},\pi\pi}$ below 1 GeV



assumption: suppose all changes occur in $\pi\pi$ channel < 1 GeV

$$\Rightarrow a_\mu^{\mathrm{total}}[\mathrm{WP20}] - a_\mu^{2\pi,<1\,\mathrm{GeV}}[\mathrm{WP20}] = 197.7 \times 10^{-10}$$

Tension with lattice QCD

→ Colangelo, Hoferichter, Stoffer, PLB **814** (2021) 136073

- force a different HVP contribution in VFF fits by including "lattice" datum with tiny uncertainty
- three different scenarios:
  - "low-energy" physics: $\pi\pi$ phase shifts
  - "high-energy" physics: inelastic effects, c_k
  - all parameters free
- study effects on pion charge radius, hadronic running of $\alpha_{\text{QED}}^{\text{eff}}$, phase shifts, cross sections

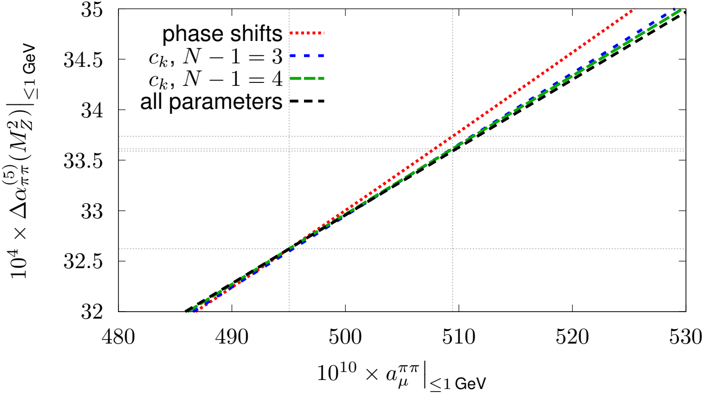
Modifying $a_\mu^{\pi\pi}|_{\leq 1\,\text{GeV}}$

→ Colangelo, Hoferichter, Stoffer, PLB **814** (2021) 136073

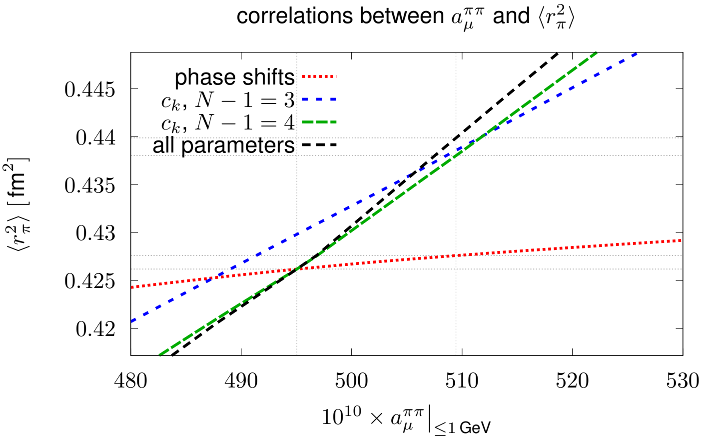
- “low-energy” scenario requires large local changes in the cross section in the ρ region
- “high-energy” scenario has an impact on **pion charge radius** and the space-like VFF ⇒ chance for independent lattice-QCD checks

Modifying $a_\mu^{\pi\pi}|_{\leq 1\,\text{GeV}}$

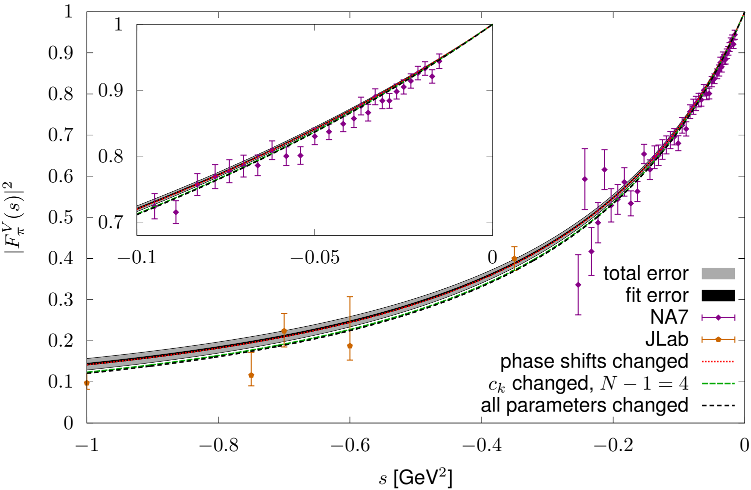
correlations between $a_\mu^{\pi\pi}$ and $\Delta\alpha_{\pi\pi}^{(5)}(M_Z^2)$



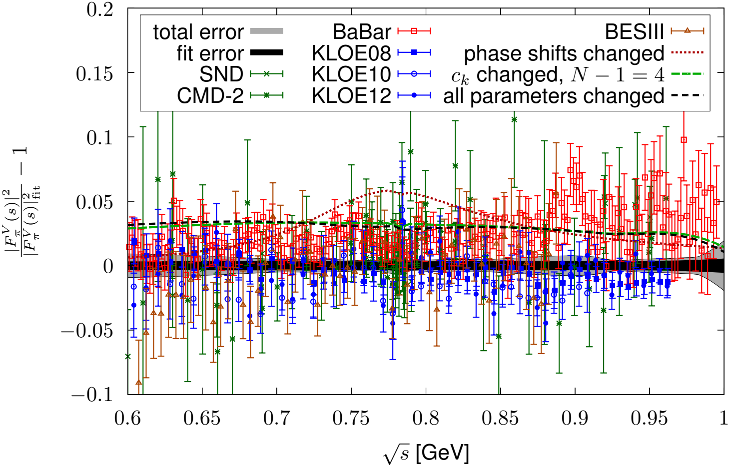
Modifying $a_\mu^{\pi\pi}|_{\leq 1\,\text{GeV}}$



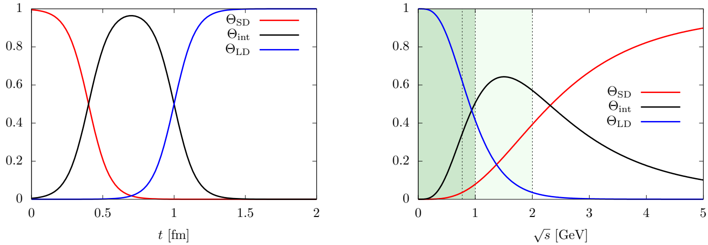
Modifying $a_\mu^{\pi\pi}|_{\leq 1\,\text{GeV}}$



Modifying $a_\mu^{\pi\pi}|_{\leq 1\,\text{GeV}}$



Some insights from the window quantities



- smooth window weight functions in Euclidean time

  $\rightarrow$ Blum et al. [RBC/UKQCD] (2018)

- out of a_μ[BMWc] $- a_\mu$[WP20] $= 14.4 \times 10^{-10}$, maybe $6 - 7 \times 10^{-10}$ from intermediate window
- using form of weight functions and assuming rather uniform shifts in low-energy region: at least 5×10^{-10} from **above 1 GeV**

Summary

- long-standing discrepancy between BaBar/KLOE
  $\Rightarrow$ wait for new e^+e^- data
- intriguing tension with BMWc
  $\Rightarrow$ unitarity/analyticity enable **independent checks** via pion VFF and $\langle r_\pi^2 \rangle$
- **window quantities** and **analyticity constraints** point at an effect $\leq 9 \times 10^{-10}$ below 1 GeV, $\geq 5 \times 10^{-10}$ above 1 GeV