The 16th International Workshop on Tau Lepton

Sep 29, 2021, Indiana (Virtual Edition)

Radiative two-pion tau decays and the T-odd asymmetries

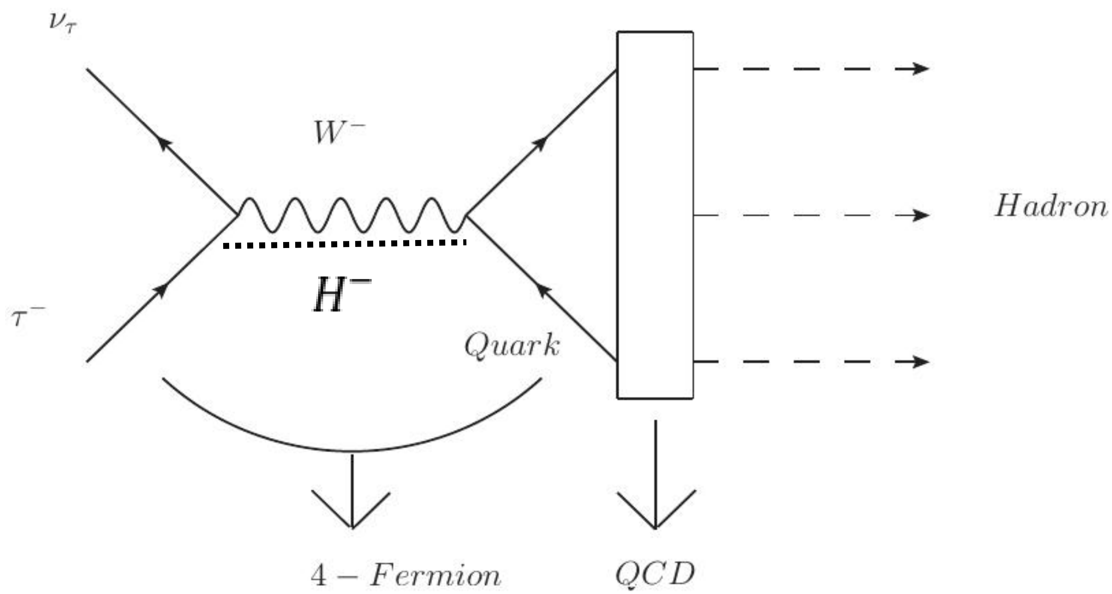
Zhi-Hui Guo

Southeast University

Outline:

1. Introduction
2. Resonance chiral theory in the $\tau \to \pi\pi\gamma\nu_\tau$ process
3. Prelimenary phenomenological results
4. Summary

Introduction



(SM, BSM)

Exclusive tau decay processes encode rich phenomenologies:

important observables for hadron physics and also possible new physics phenomenons

CP violation in tau decay: rate asymmtry

$$A_{CP} = \frac{\Gamma(\tau^- \to \nu_\tau H) - \Gamma(\tau^+ \to \nu_\tau \bar{H})}{\Gamma(\tau^- \to \nu_\tau H) + \Gamma(\tau^+ \to \nu_\tau \bar{H})}$$

Intensive discussions on tau -> Ks pi nu

$$A_Q = \frac{\Gamma\left(\tau^+ \to \pi^+ K_S^0 \bar{\nu}_\tau\right) - \Gamma\left(\tau^- \to \pi^- K_S^0 \nu_\tau\right)}{\Gamma\left(\tau^+ \to \pi^+ K_S^0 \bar{\nu}_\tau\right) + \Gamma\left(\tau^- \to \pi^- K_S^0 \nu_\tau\right)}$$

$$\approx (0.36 \pm 0.01)\%$$

SM prediction

$$\left(-0.36 \pm 0.23_{\text{stat}} \pm 0.11_{\text{syst}}\right)\%$$

BaBar

[Bigi et al., PLB'05] [Grossman et al., JHEP'12] [Lees et al., PRD'12]

[Cirigliano et al., PRL'18] [Rendo et al., PRD'19][Chen et al.,PRD'19 JHEP'20]

Alternative way to probe CPV: triple-product T-odd asymmetries

$$\xi \equiv \varepsilon_{\mu\nu\rho\sigma} a^\mu b^\nu c^\rho d^\sigma \xlongequal[\text{of particle } a]{\text{rest frame}} \vec{b} \cdot (\vec{c} \times \vec{d})\, m_a / s_a$$

a*, *b*, *c*, *d*: either *momentum* or *spin

- **When spin is involved, measurement of polarization is needed.**

[Nelson, et al., PRD'94] [Tsai, PRD'95] [Datta, PRD'07]

- **We focus on the situation with four momenta, *i.e.***

$$\xi = \varepsilon_{\mu\nu\rho\sigma} p_1^\mu p_2^\nu p_3^\rho p_4^\sigma$$

$$\xlongequal[\text{of particle 1}]{\text{rest frame}} \vec{p_2} \cdot (\vec{p_3} \times \vec{p_4})\, m_1$$

There should be at least four particles in the final state !

$$A_\xi = \frac{\Gamma(\xi > 0) - \Gamma(\xi < 0)}{\Gamma(\xi > 0) + \Gamma(\xi < 0)} \qquad \overline{A}_{\overline{\xi}} = \frac{\overline{\Gamma}(\overline{\xi} > 0) - \overline{\Gamma}(\overline{\xi} < 0)}{\overline{\Gamma}(\overline{\xi} > 0) + \overline{\Gamma}(\overline{\xi} < 0)}$$

- **Both CPC (final state interactions) and CPV effects can cause nonzero A_ξ.**

- $\mathcal{A}_\xi = A_\xi - \overline{A}_{\overline{\xi}}$ **: only includes the CPV contributions.**

- **Up to now, none of A_ξ in the tau decays have been measured.**

- **Early proposals to study the T-odd asymmetry in $\tau \to K\pi\pi\nu_\tau$, $KK\pi\nu_\tau$** [Kilian, et al., ZPC'94]

- **Here we take an exploring study of the T-odd asymmetry in the $\tau \to \pi\pi\gamma\nu_\tau$ process.**

Remarks about A_ξ in the $K_{l3\gamma}$, i.e. $K \to \pi\gamma l\nu_l$

[Braguta, et al., PRD'02 '03] [Rudenko, PRD'11] [Muller, et al., EPJC'06]

$$A_\xi(K^+ \to \pi e^+ \nu_e \gamma) = -0.59 \times 10^{-4} \qquad A_\xi(K^+ \to \pi \mu^+ \nu_\mu \gamma) = 1.14 \times 10^{-4}$$

TABLE II. A_ξ in $K^0 \to \pi^- l^+ \nu_l \gamma$ decays ($\omega \geq 30$ MeV and $\theta_{l\gamma} \geq 20°$).

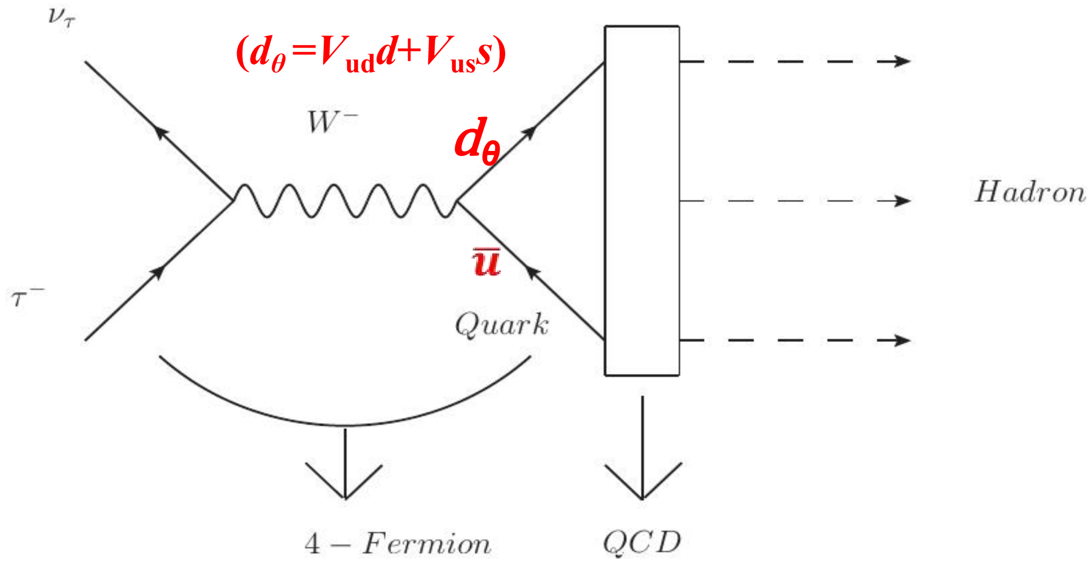
	$l = \mu$	$l = e$
Group I ($l - \gamma$)	-0.54×10^{-4}	-1.32×10^{-4}
Group II ($\pi - \gamma$)	-3.6×10^{-4}	-3.2×10^{-4}
Group III ($\pi - l$)	1.73×10^{-3}	8.6×10^{-4}
Group IV ($\pi - l - \gamma$)	-1.41×10^{-3}	-8.6×10^{-4}
Total	-1×10^{-4}	-4.5×10^{-4}

- **Hadronic contributions to A_ξ are negligible in $K_{l3\gamma}$: the inner bremsstrahlung (model independent) FF is real & the structure-dependent parts are suppressed due to kinematics.**

- **The EM effects (photon loops) give the most imporatnt contributions to the T-odd asymmetries in the $K_{l3\gamma}$ decays**

- **The situation is rather different in $\tau \to \pi\pi\gamma\nu_\tau$: hadronic effects are expected to dominate !**

Resonance chiral theory in $\tau \to \pi\pi\gamma\nu_\tau$ decay



Hadronic V-A currents

$$\mathbf{H}_\mu = \langle H^- | \bar{u}\gamma_\mu(1-\gamma_5)d_\theta\, e^{iL_{QCD}} | 0 \rangle$$

Chiral EFT is the low energy realization of QCD:

$$e^{i\mathcal{Z}(v_\mu, a_\mu, s, p)} = \int \mathcal{D}q\mathcal{D}\bar{q}\mathcal{D}G_\mu\; e^{i\int d^4x \mathcal{L}_{QCD}(v_\mu, a_\mu, s, p)} = \int \mathcal{D}u\; e^{i\int d^4x \mathcal{L}_{EFT}(v_\mu, a_\mu, s, p)}$$

$$\mathcal{L}^{QCD} = \mathcal{L}_0^{QCD} + \bar{q}\gamma^\mu(v_\mu + a_\mu\gamma_5)q - \bar{q}(s - i\gamma_5 p)q$$

***v_μ, a_μ, s, p* are the external source fields .**

Leading order [Weinberg, '79]

$$\mathcal{L}_2 = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+\rangle$$

Higher orders [Gasser,Leutwyler,'83 '84] [Bijnens et al., '99]

$\mathcal{O}(p^4)$:

$$\mathcal{L}_4^{\chi PT} = L_1\langle u_\mu u^\mu \rangle^2 + L_2\langle u_\mu u^\nu \rangle\langle u^\mu u_\nu \rangle + L_3\langle u_\mu u^\mu u_\nu u^\nu \rangle + L_4\langle u_\mu u^\mu \rangle\langle \chi_+ \rangle$$
$$+L_5\langle u_\mu u^\mu \chi_+ \rangle + L_6\langle \chi_+ \rangle^2 + L_7\langle \chi_- \rangle^2 + \frac{L_8}{2}\langle \chi_+^2 + \chi_-^2 \rangle + \cdots$$

$\mathcal{O}(p^6)$:

$$\mathcal{L}_6^{\chi PT} = C_1\langle u_\rho u^\rho h_{\mu\nu} h^{\mu\nu} \rangle + C_2\langle u_\beta u^\beta \rangle\langle h_{\mu\nu} h^{\mu\nu} \rangle + C_3\langle h_{\mu\nu} u_\rho h^{\mu\nu} u^\rho \rangle$$
$$+\ldots 94 \text{ terms in total in SU}(3) \text{ case}$$

Alternatively, one could explicitly introduce heavy dynamical degrees of freedom, i.e., resonances, in the chiral EFT.

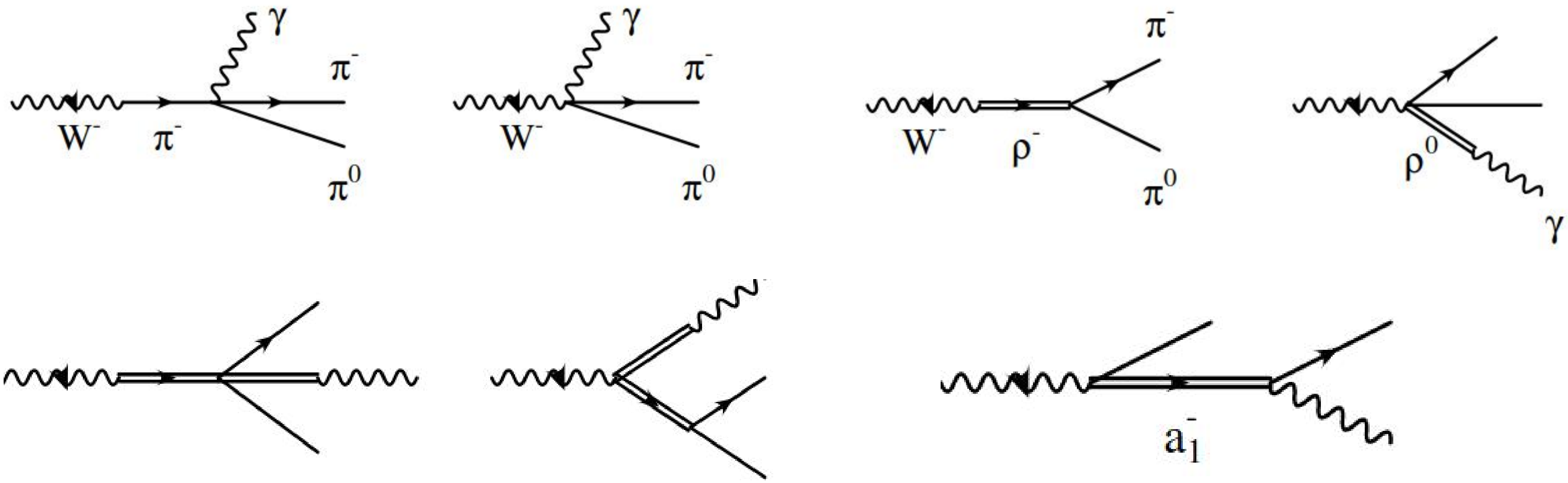
Minimal RχT Lagrangian [Ecker, *et al.*, '89]

$$\mathcal{L}_{kin}(V) = -\frac{1}{2} < \nabla^\lambda V_{\lambda\mu}\nabla_\nu V^{\nu\mu} - \frac{1}{2} V^{\mu\nu}V_{\mu\nu} >$$

$$\mathcal{L}_{2V} = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{iG_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle \qquad \mathcal{L}_{2A} = \frac{F_A}{2\sqrt{2}}\langle A_{\mu\nu} f_-^{\mu\nu}\rangle$$

Minimal RChT contributions to $\tau \to \pi\pi\gamma\nu_\tau$

[Cirigliano et al., JHEP'02]



- **Other extentions by including anomalous vertices, such as the $\rho\omega\pi$ types, and even-parity vertices of the $a_1\rho\pi$, are also studied.**
  [Flores-Tlalpa, et al., PRD'05] [Miranda, Roig, PRD'20]

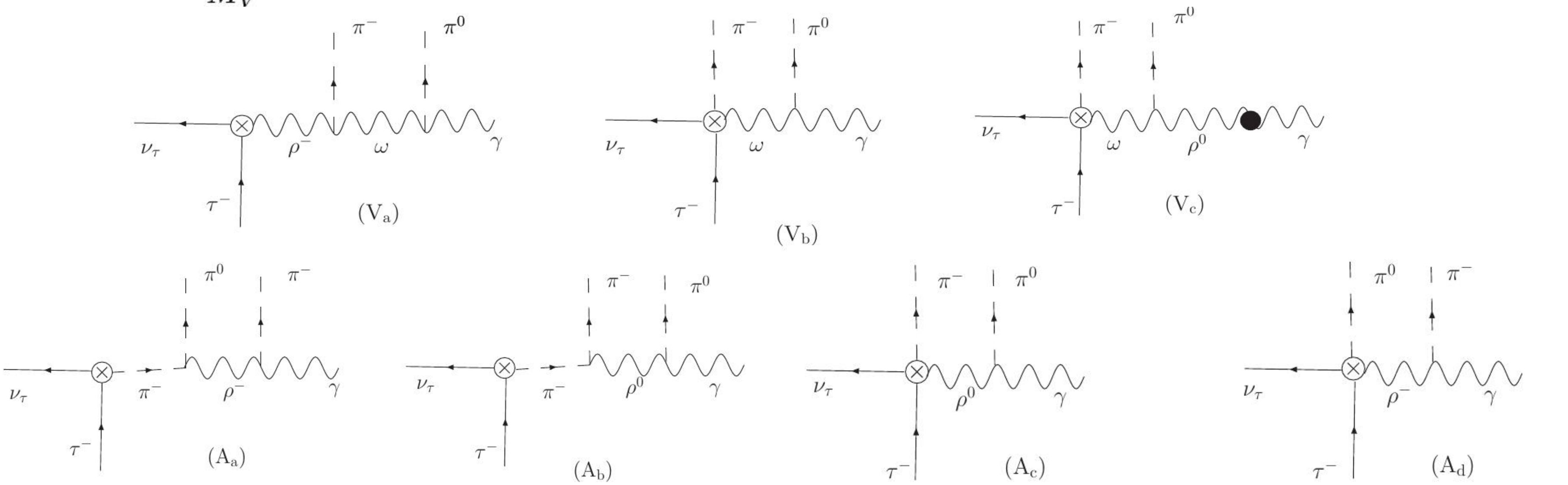
- **Dedicated study of the isospin-breaing effect is considered to calibrate the tau data in the estimation of muon g-2.**
  [Cirigliano,et al.,JHEP'02][Flores-Baez,et al.,PRD'06][Davier,et al.,EPJC'10][Miranda,Roig,PRD'20]

Contributions from VVP and VJP operators in RChT

[Ruiz-Femenia, Pich, Portoles, JHEP'03]

$$\mathcal{L}_{VVP} = d_1\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu},V^{\rho\alpha}\}\nabla_\alpha u^\sigma\rangle + id_2\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu},V^{\rho\sigma}\}\chi_-\rangle + d_3\varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla_\alpha V^{\mu\nu},V^{\rho\alpha}\}u^\sigma\rangle + d_4\varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla^\sigma V^{\mu\nu},V^{\rho\alpha}\}u_\alpha\rangle$$

$$\mathcal{L}_{VJP} = \frac{c_1}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu},f_+^{\rho\alpha}\}\nabla_\alpha u^\sigma\rangle + \frac{c_2}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\alpha},f_+^{\rho\sigma}\}\nabla_\alpha u^\nu\rangle + \frac{ic_3}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu},f_+^{\rho\sigma}\}\chi_-\rangle + \frac{ic_4}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle V^{\mu\nu}[f_-^{\rho\sigma},\chi_+]\rangle + \frac{c_5}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla_\alpha V^{\mu\nu},f_+^{\rho\alpha}\}u^\sigma\rangle + \frac{c_6}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla_\alpha V^{\mu\alpha},f_+^{\rho\sigma}\}u^\nu\rangle + \frac{c_7}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla^\sigma V^{\mu\nu},f_+^{\rho\alpha}\}u_\alpha\rangle\,.$$



[Chen, Duan, ZHG , in preparation]

High energy contraints to the resonance couplings

$$\int d^4x \int d^4y e^{i(p\cdot x+q\cdot y)} \langle 0|T[V_\mu^a(x)V_\nu^b(y)P^c(0)]|0\rangle = d^{abc}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta \Pi_{\text{VVP}}(p^2,q^2,r^2),$$

$$\lim_{\lambda\to\infty}\Pi^{(8)}_{\text{VVP}}[(\lambda p)^2,(\lambda q)^2,(\lambda p+\lambda q)^2] = \lim_{\lambda\to\infty}\Pi^{(0)}_{\text{VVP}}[(\lambda p)^2,(\lambda q)^2,(\lambda p+\lambda q)^2] = -\frac{\langle\bar\psi\psi\rangle_0}{2\lambda^4}\frac{p^2+q^2+r^2}{p^2q^2r^2}[1+\mathcal{O}(\alpha_s)] + O\left(\frac{1}{\lambda^6}\right).$$

$$c_1 + 4c_3 = 0 \qquad c_1 - c_2 + c_5 = 0 \qquad c_5 - c_6 = \frac{N_C M_V}{64\sqrt{2}\pi^2 F_V}$$

$$d_1 + 8d_2 = -\frac{N_c M_V^2}{(8\pi F_V)^2} + \frac{F^2}{4F_V^2} \qquad d_3 = -\frac{N_c M_V^2}{(8\pi F_V)^2} + \frac{F^2}{8F_V^2}$$

Other constraints from scattering and form factors

$$F_A = F_\pi, \quad F_V = \sqrt{2}F_\pi, \quad G_V = F_\pi/\sqrt{2}.$$

Or

$$F_A = \sqrt{2}F_\pi, \quad F_V = \sqrt{3}F_\pi, \quad G_V = F_\pi/\sqrt{3}$$

On-shell approxiamtion to the $J\omega\pi$ vertex and additional input from the $\omega \to \pi^0\pi^0\gamma$ decay width

$$T_{\omega\to\pi^0\pi^0\gamma} = \frac{2}{F}\Big\{ d_1(\epsilon_{\lambda\delta\mu\sigma}p_{1\nu}p_1^\sigma + \epsilon_{\mu\nu\lambda\sigma}p_{1\delta}p_1^\sigma) + 4d_2 m_\pi^2 \epsilon_{\mu\nu\lambda\delta}$$
$$+d_3[\,\epsilon_{\lambda\delta\mu\sigma}(k+p_2)_\nu p_1^\sigma - \epsilon_{\mu\nu\lambda\sigma}q_\delta p_1^\sigma\,] + d_4[\,\epsilon_{\lambda\delta\mu\sigma}(k+p_2)^\sigma p_1^\nu - \epsilon_{\mu\nu\lambda\sigma}q^\sigma p_{1\delta}]\Big\}$$
$$D^{\lambda\delta,\beta\theta}(k+p_2, M_V^2) g_{\rm v} \epsilon_{\beta\theta\rho\alpha} k^\rho \varepsilon_\gamma^\alpha(k)\, \frac{q^\mu \varepsilon_\omega^\nu(q) - q^\nu \varepsilon_\omega^\mu(q)}{M_\omega} \quad + \Big(p_1 \leftrightarrow p_2\Big)$$

$$\Gamma^{Exp}_{\omega\to\pi^0\pi^0\gamma} = (5.8 \pm 1.0) \times 10^{-5}\ \text{MeV}$$

$\Longrightarrow$

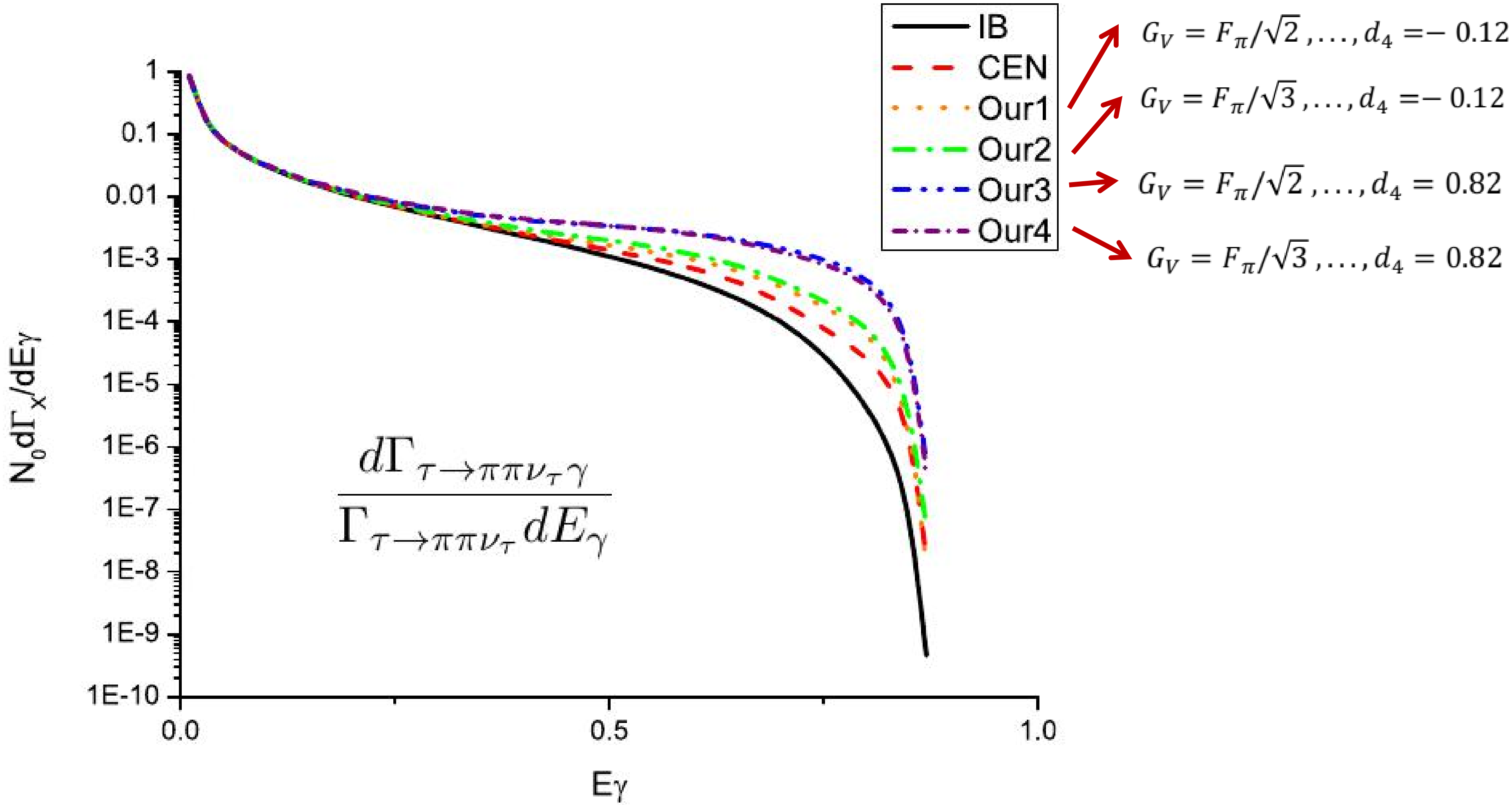
$$d_4 = -0.12 \pm 0.05\,,$$
$$d_4 = 0.82 \pm 0.05\,.$$

Important: **we are left with a parameter free theoretical amplitude for the $\tau \to \pi\pi\gamma\nu_\tau$ process !**

Prelimenary phenomenological discussions

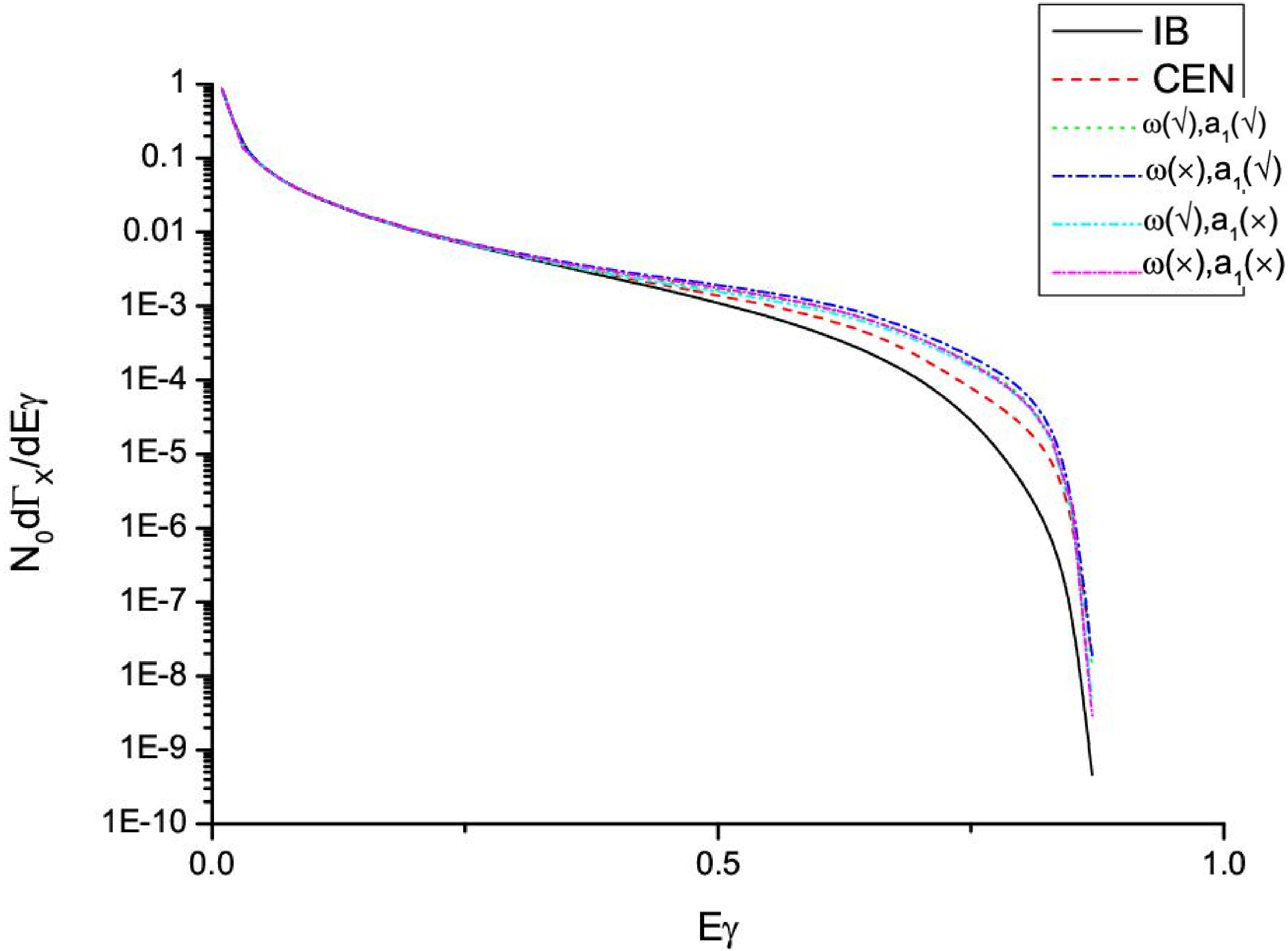
[Chen, Duan, ZHG , in preparation]

Differential decay widths as a function of photon energies

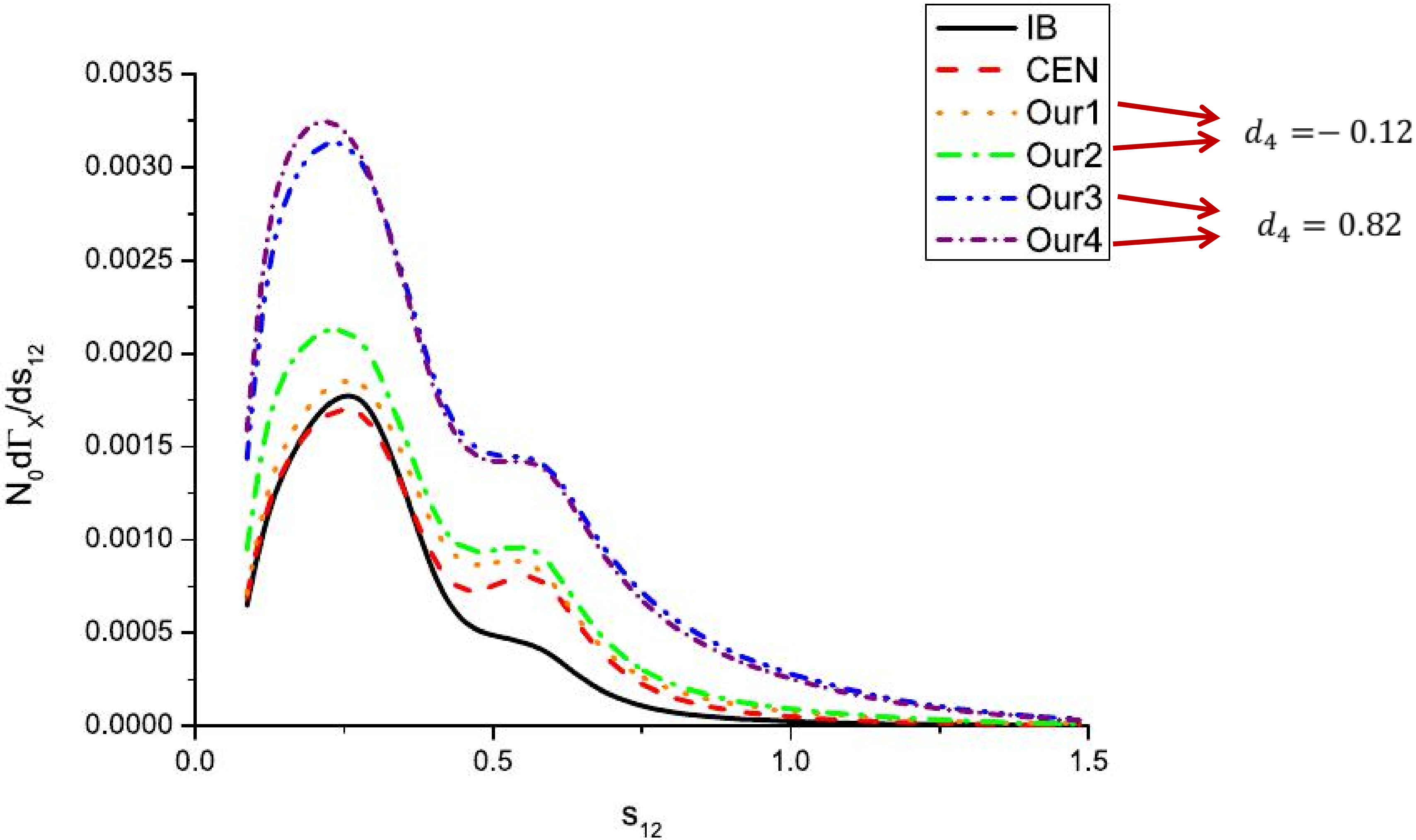


When the photon energy cutoff is around 300 MeV, the absolute branching ratio is predicted to be around 10^{-4}.

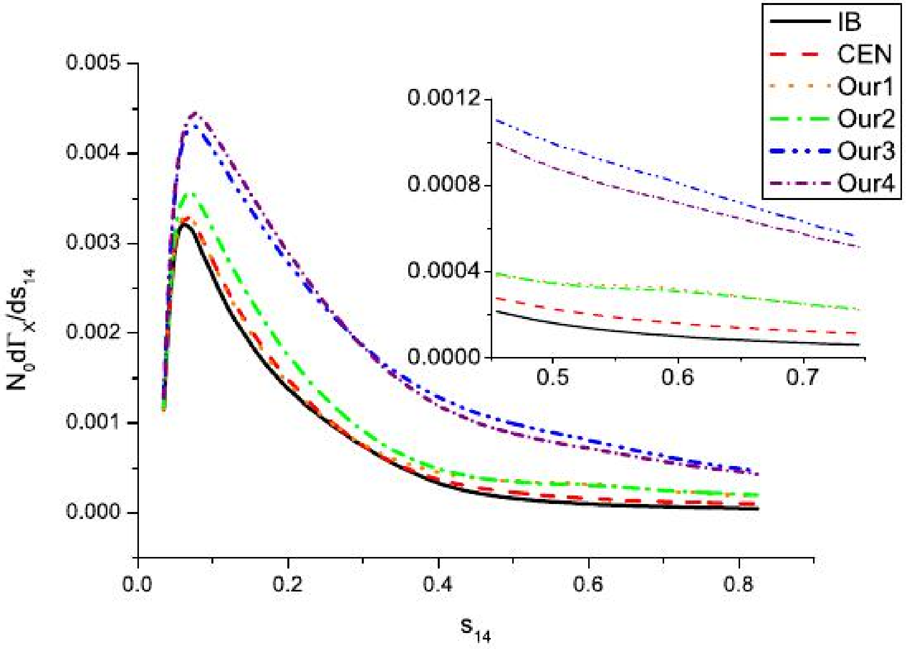
Anatomy of different contributions in the photon spectra



Invariant-mass distributions of the $\pi\pi$ system

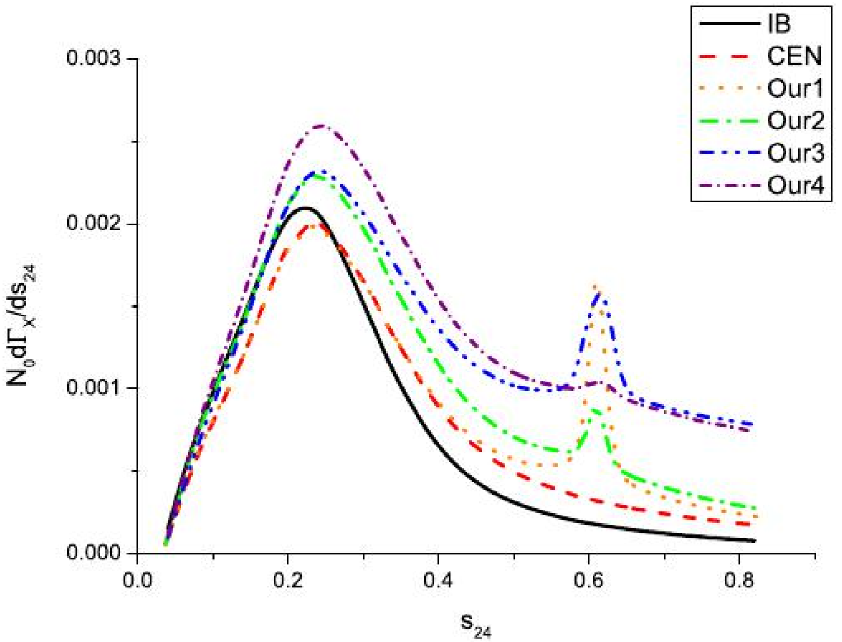


Invariant-mass distributions of the $\pi^- \gamma$ and $\pi^0 \gamma$ systems



$\pi^- \gamma$

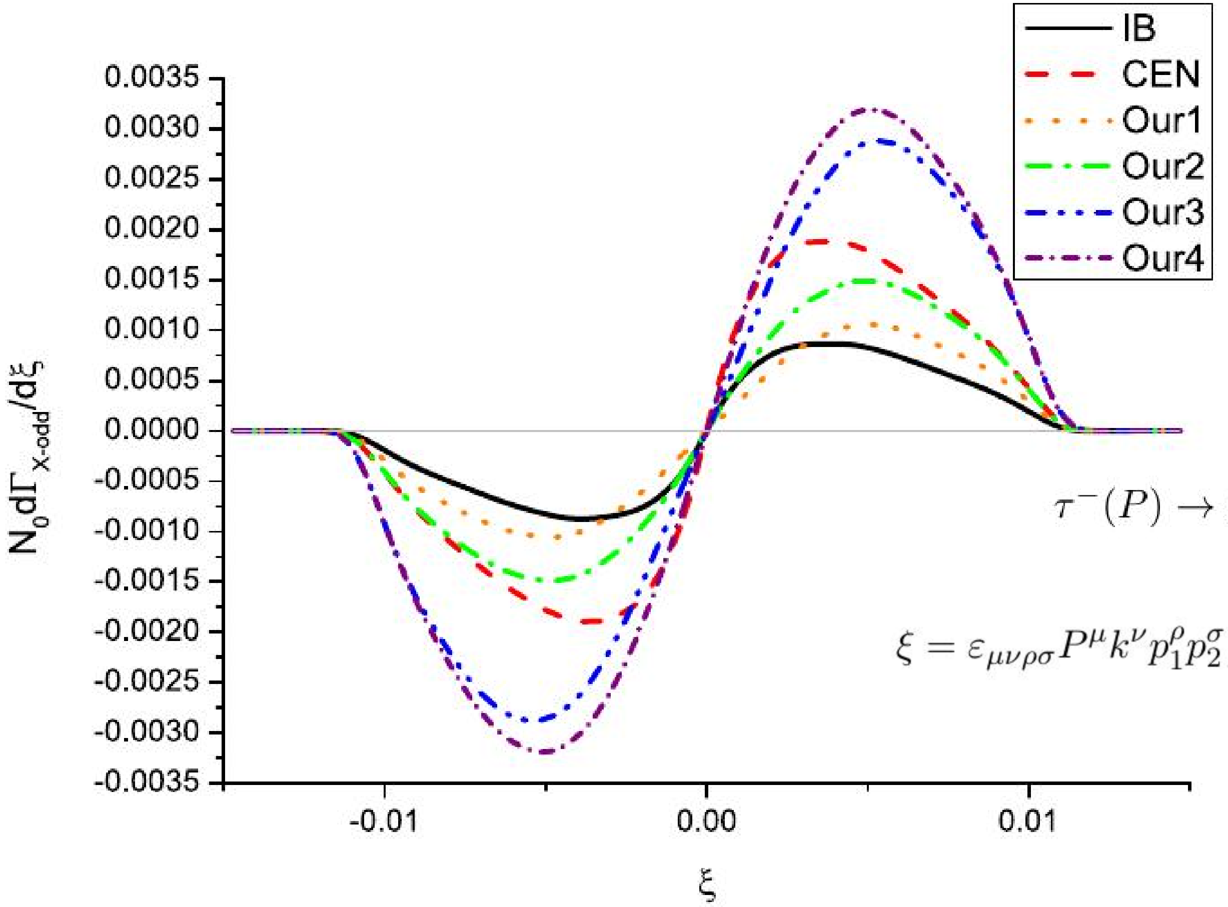
(ρ)



$\pi^0 \gamma$

(ω)

Predicitons of the T-odd asymmetry distributions with respect to ξ

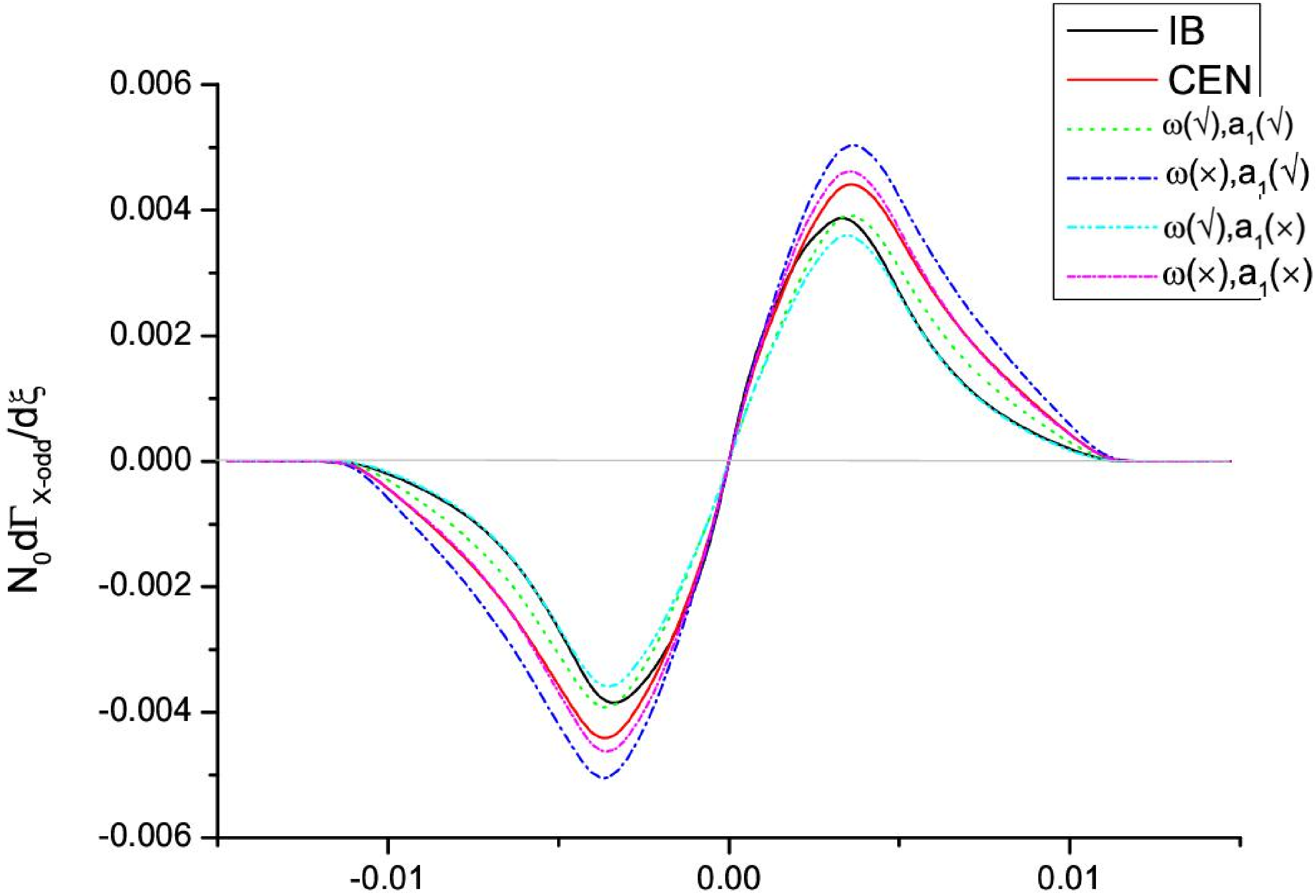


$$\tau^-(P) \to \pi^-(p_1)\pi^0(p_2)\nu_\tau(q)\gamma(k)$$

$$\xi = \varepsilon_{\mu\nu\rho\sigma}P^\mu k^\nu p_1^\rho p_2^\sigma / m_\tau^4 \xrightarrow[\text{of }\tau]{\text{rest frame}} \vec{k}\cdot(\vec{p}_1 \times \vec{p}_2)/m_\tau^3$$

With photon energy cutoff around 300 MeV, A_ξ is around the order of 10^{-2} (compared to the 10^{-4} in $K_{l3\gamma}$), which has the good chance to be measured in Belle-II and super tau-charm facilities.

Anatomy of different effects in the T-odd asymmetric distributions



Summary

- **Rich phenomenologies in $\tau^- \to \pi^-\pi^0\gamma\nu_\tau$: photon spectrum (useful inputs for the estimation of muon g-2 from tau data), different resonance interactions in the $\pi^-\pi^0$, $\pi^-\gamma$, $\pi^0\gamma$ spectra**

- **We give a promising prediction of the triple-product T-odd asymmetry in tau decay. It could provide a useful guide for future experimental measurements, especially in Belle-II and super-charm facilities.**

Thanks for your attention!