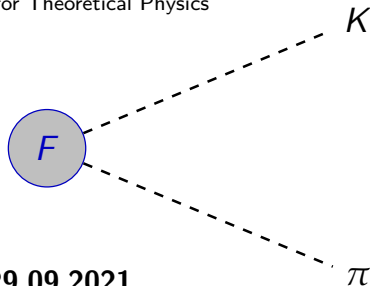


On the scalar πK form factor beyond the elastic region

Frederic Noël

Universität Bern
Institute for Theoretical Physics



29.09.2021

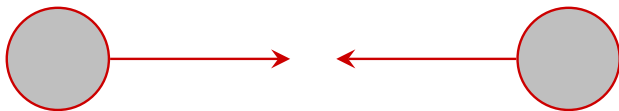
The 16th International Workshop on Tau Lepton Physics
(TAU2021)

based on

[Von Detten, Noël, Hanhart, Hoferichter and Kubis, 2021, Eur. Phys. J. C 81.5, p. 420]

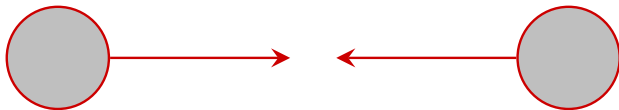
Frontiers in Particle Physics

Search for "new" physics via particle accelerators/colliders and detectors



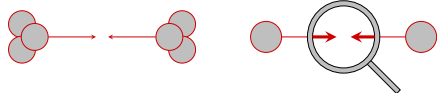
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energy frontier

- more energy
- more phase space
- more (massive) particles

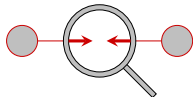
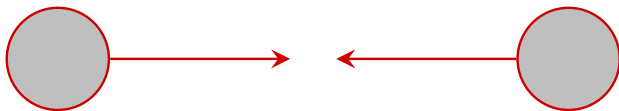


intensity/precision frontier

- more luminosity/measuring time
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- better/more precise detectors
- more precise quantities

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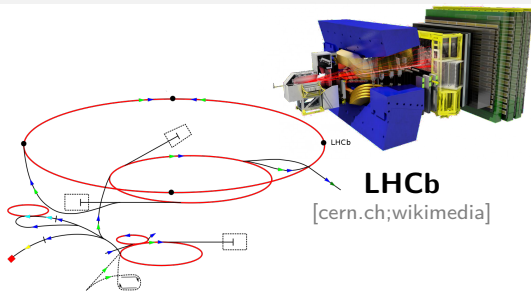
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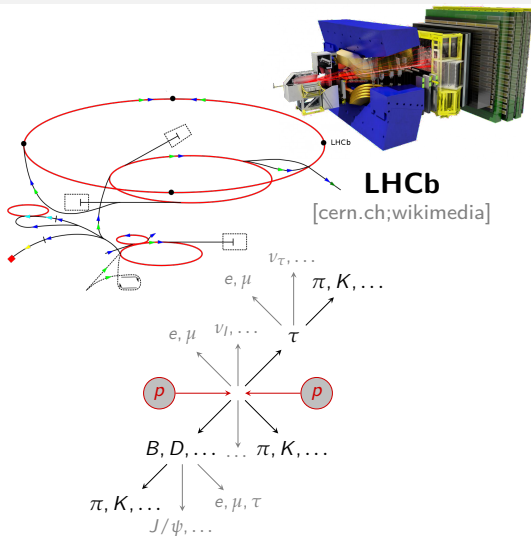
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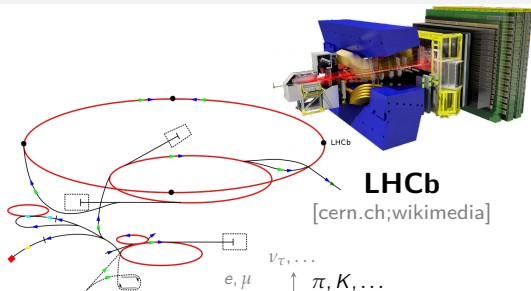
Precision Frontier - LHCb and Belle



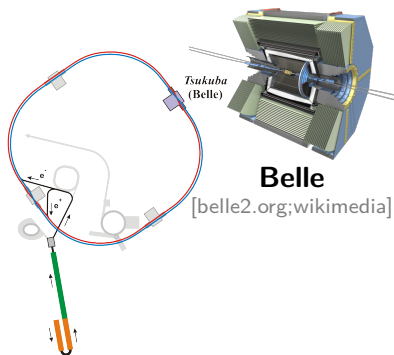
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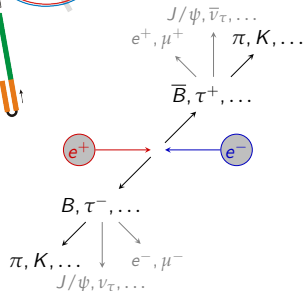
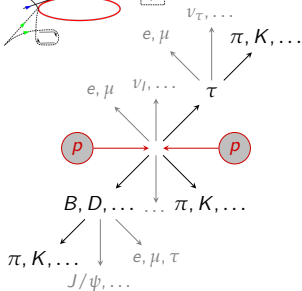
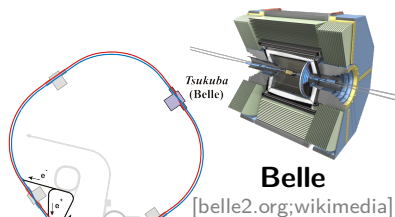
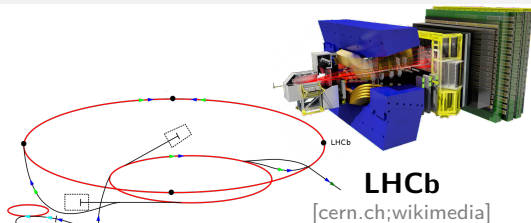


LHCb
[cern.ch;wikimedia]

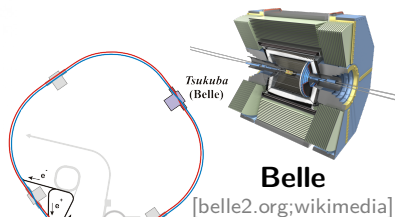
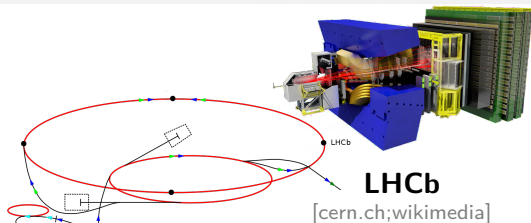


Belle
[belle2.org;wikimedia]

Precision Frontier - LHCb and Belle



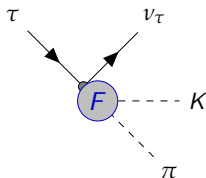
Precision Frontier - LHCb and Belle



Hadrons in final states are everywhere, especially π and K

Observables including πK final states

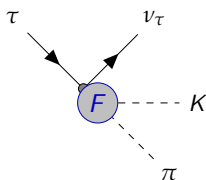
semi-leptonic decays



- **BSM CP violation** requires inelasticity [Cirigliano et al., 2018]

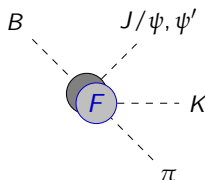
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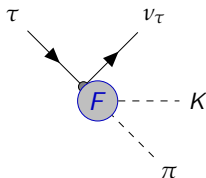
B & D decays



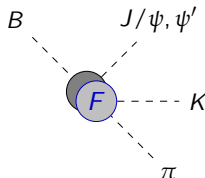
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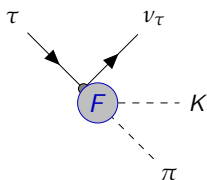


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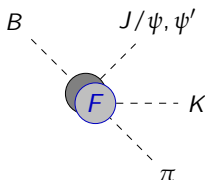
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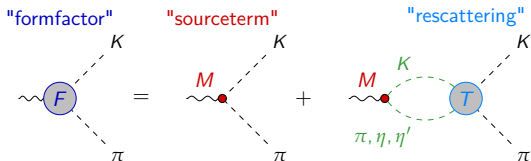


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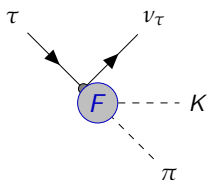
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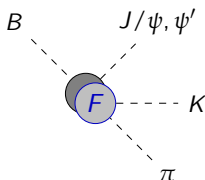


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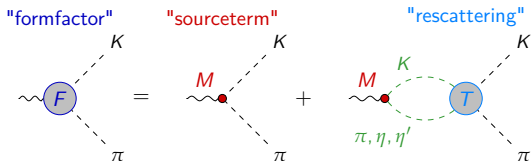


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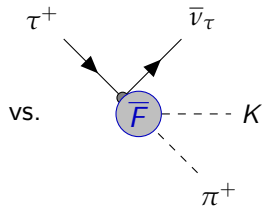
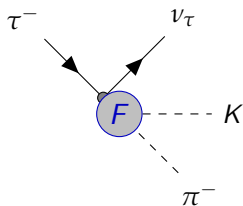
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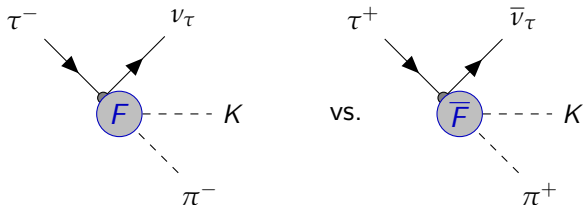


- requires **scattering matrix** T with **elastic** πK and **inelastic** $\eta K, \eta' K$

CP violation in τ decays



CP violation in τ decays

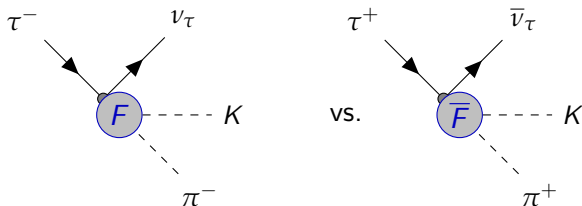


- BSM CP violation only via Tensor-Vector interference:

$$A_{CP}^{BSM} \sim \text{Re } F_V F_T^* - \text{Re } \bar{F}_V \bar{F}_T^* \sim \sin(\phi_V - \phi_T)$$

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- Watson's theorem states for elastic interactions:

$$\phi_V = \delta_{\ell=1}^{\pi K} \quad \text{and} \quad \phi_T = \delta_{\ell=1}^{\pi K} \quad (\delta_{\ell=1}^{\pi K} \text{ from elastic scattering})$$

- $\phi_T - \phi_V \neq 0$ needs inelastic effects [Cirigliano et al., 2018]

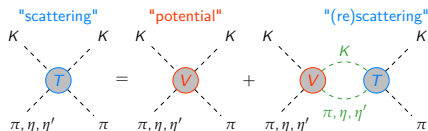
Inelastic effects are mandatory to give rise to BSM CP violation

Formalism - T -matrix

- Bethe-Salpeter (BS) equation:

$$T = V + VGT$$

- fulfils **unitarity** by construction if $V_{ij} \in \mathbb{R}$, $\text{Im } G_{ij} = \sigma_i \delta_{ij}$

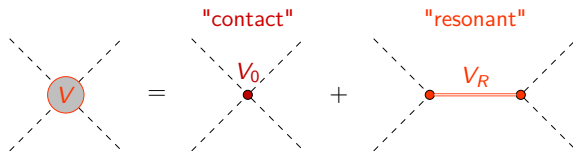
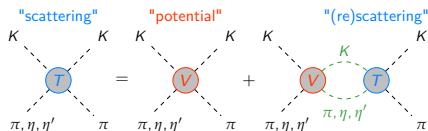


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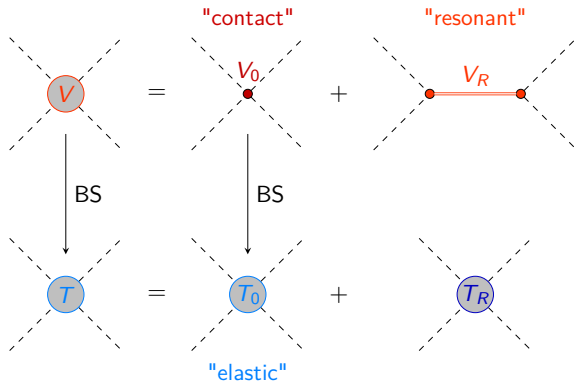
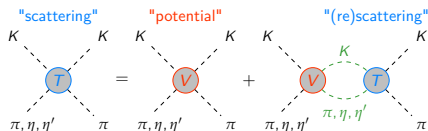


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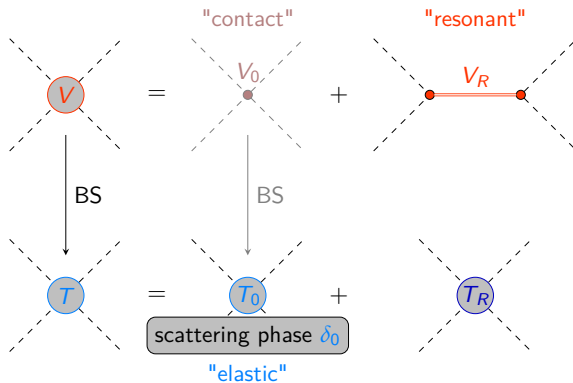
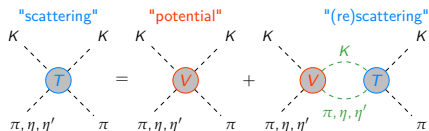


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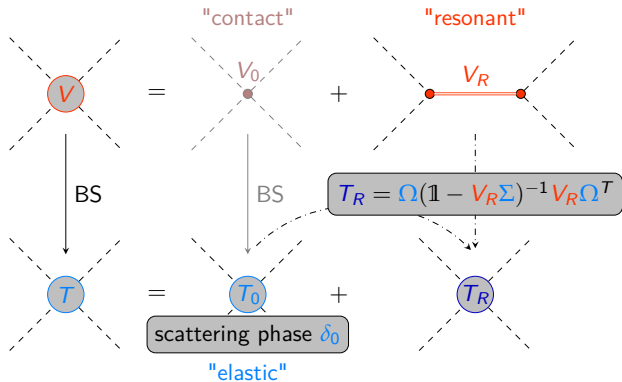
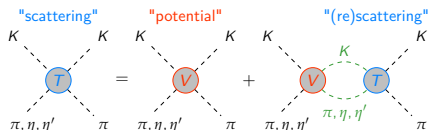


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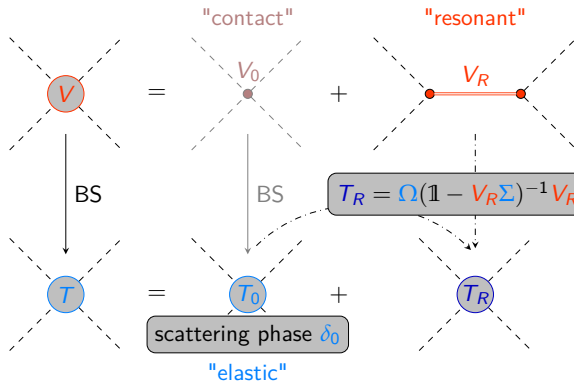
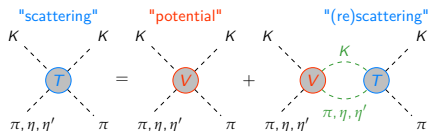
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parametrise V_R
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Fit to πK scattering data

application to $\pi K \rightarrow \pi K$:

- S -wave ($\ell = 0$)
- coupled channel $\eta' K$
- present resonances:
 - $\kappa / K_0^*(700)$
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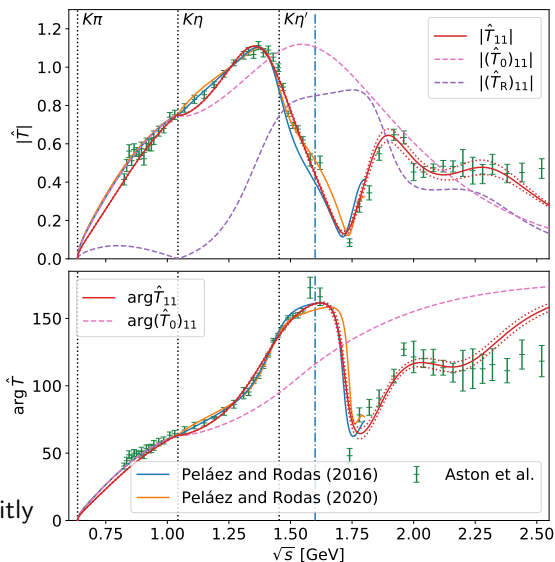
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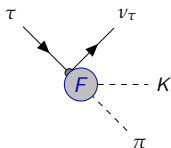
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[Aston et al., 1988; Peláez and Rodas, 2016; Peláez and Rodas, 2020]

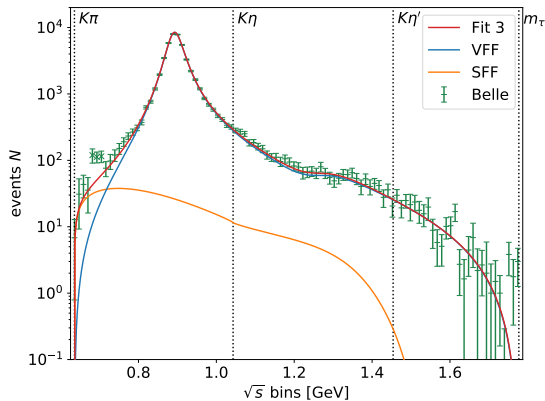
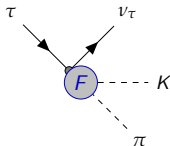
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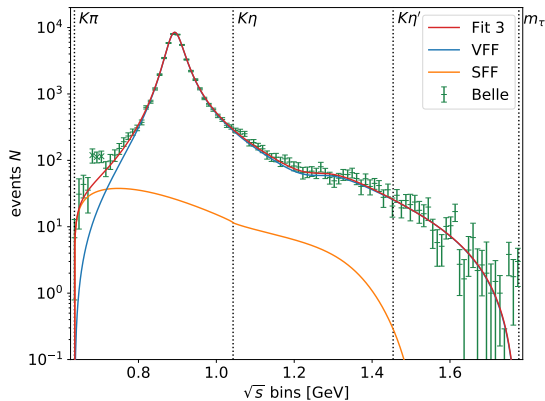
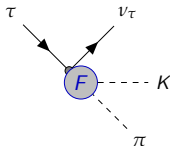


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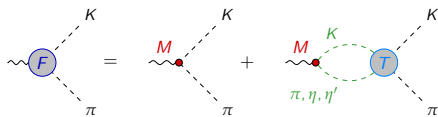


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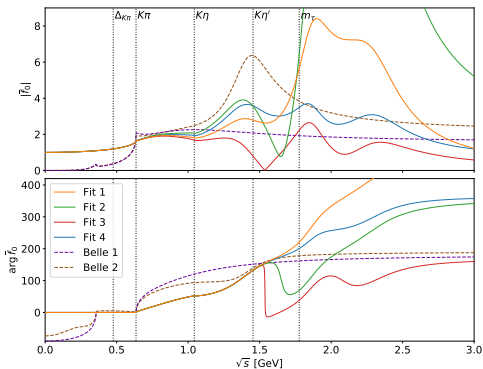
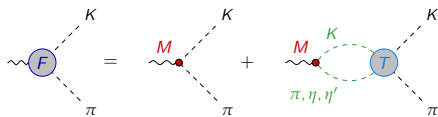
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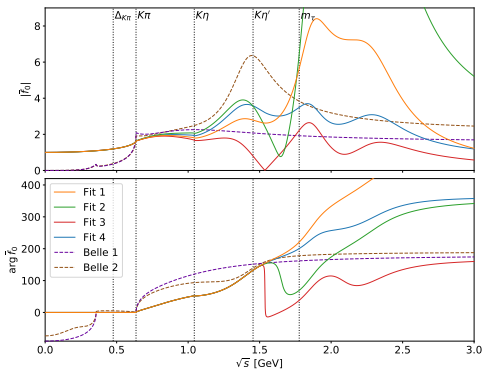
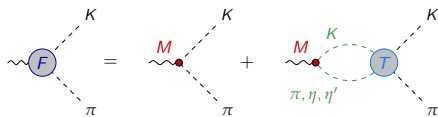
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Scalar form factor

- scalar form factor F fixed by T up to source term M :



- correct phase by construction
- fulfils unitarity & analyticity
- correct normalisation
- in contrast to Breit-Wigner
- further distinction requires additional data

Results

- estimate on **BSM CP asymmetry** supports [Cirigliano et al., 2018]

$$A_{CP}^{\tau, \text{BSM}} = -0.034(14) \text{ Im } c_T \quad (c_T, \text{ Tensor Wilson coefficient})$$

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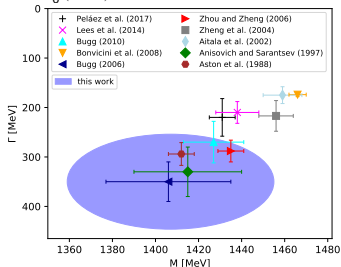
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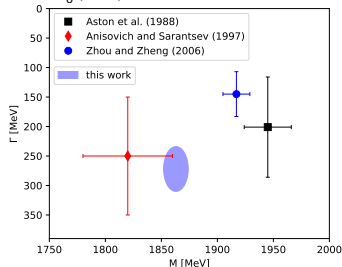
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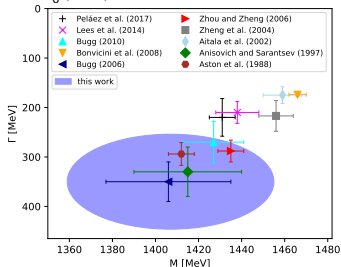
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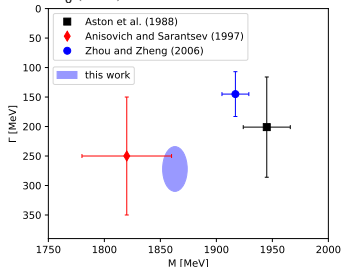
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- extracted **physical couplings** via residues:
scalar resonance coupling to $\bar{s}\gamma^\mu u$ gives $\text{BR}(\tau \rightarrow K_0^*(1430)\nu_\tau) < 1.6 \times 10^{-4}$

Conclusion/Outlook

Conclusion:

- πK form factor to describe πK final states in
 - semi-leptonic decays analysing BSM CP violation
 - B and D decays searching for exotica
- formalism combining low energy elastic description with high energy resonance exchange
- successful application to scalar form factor in $\tau \rightarrow \nu_\tau K \pi$

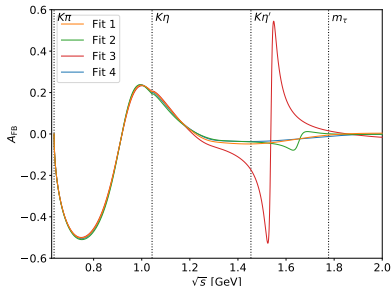
Conclusion/Outlook

Conclusion:

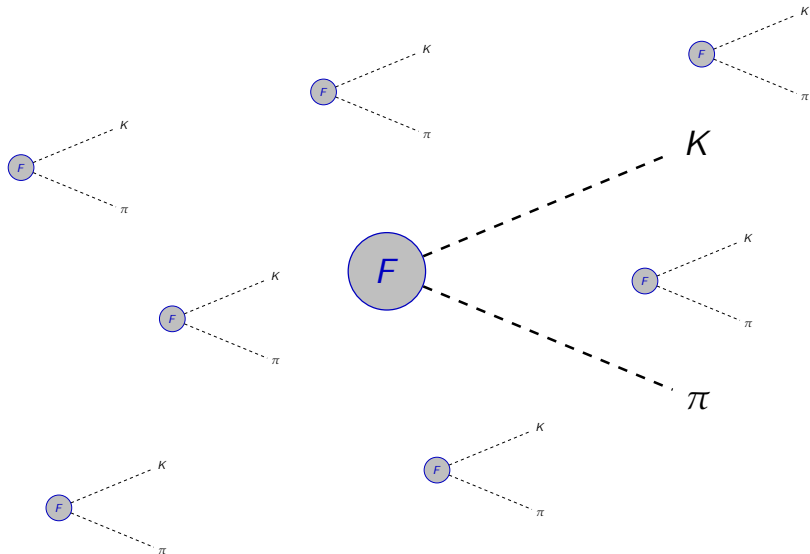
- πK form factor to describe πK final states in
 - semi-leptonic decays analysing BSM CP violation
 - B and D decays searching for exotica
- formalism combining low energy elastic description with high energy resonance exchange
- successful application to scalar form factor in $\tau \rightarrow \nu_\tau K \pi$

Outlook:

- application to other partial waves (i.e. P -wave)
- forward-backward asymmetry as potentially measured by Belle II \rightarrow separates S & P -wave
- exotica in B and D decays
- other semi-leptonic decays



Thank you for your attention!



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References III

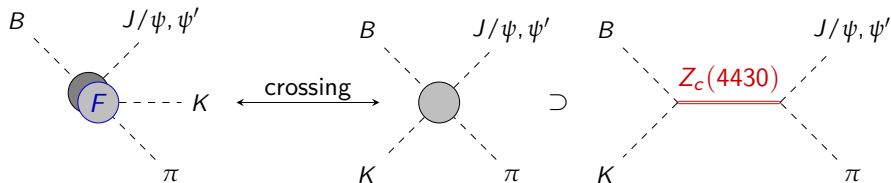
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Backup-Slides

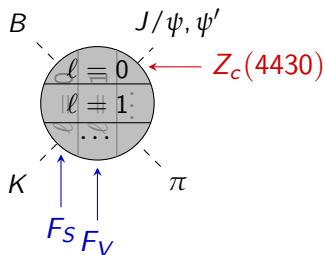
Comparison to other parametrisations/works

- as we have seen Breit-Wigners are a no-go, especially for S -wave
- Single-channel Omnès-function
(e.g. [Moussallam, 2008; Boito et al., 2009; Boito et al., 2010; Bernard et al., 2011; Antonelli et al., 2013,...])
 - phase input either ...
 - ... the pure scattering phase
→ no inelastic channels
 - ... the argument of a non-unitary model (e.g. RChPT)
→ no consistency with scattering
- Coupled-channel Omnès-matrix (e.g. [El-Bennich et al., 2009])
 - in principle does fulfil all constraints (unitarity, analyticity)
 - needs data for all scattering channels (only $\pi K \rightarrow \pi K$ atm)
 - can compensate this by using chiral constraints
 - but: would need more constraints to extend the model
- our model is fully consistent with unitarity and easier expandable

Exotica in B decays

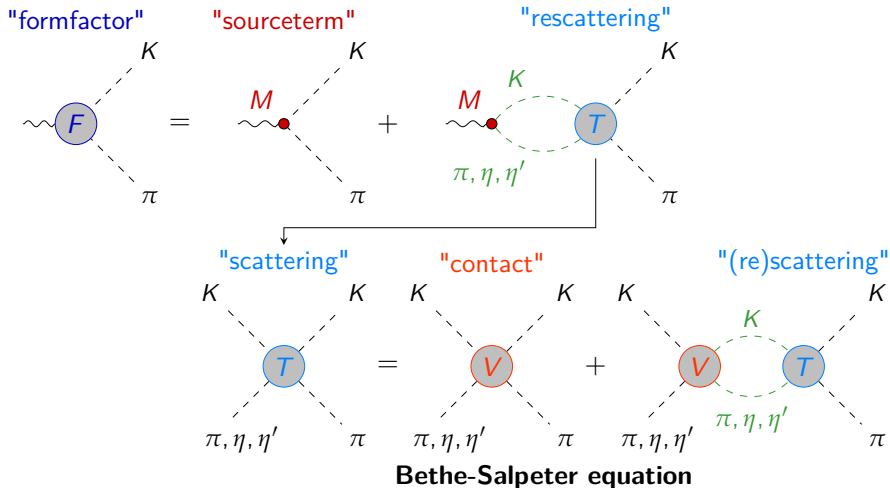


- exotic state in crossed channel:
 - $Z_c^-(4430)$, $J^P = 1^+$
 - supposedly: $c\bar{c}d\bar{u}$
- different πK partial waves interfere
- no proper phase behaviour for e.g. Breit-Wigner models by construction



Proper phases of F_S and F_V are essential to investigate these exotica

The πK form factor and scattering matrix

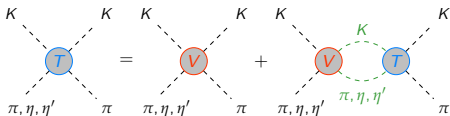


Proper description of πK form factor F
 requires proper description of scattering matrix T

Formalism - T -matrix

- describe $\pi K \rightarrow \pi K$ scattering T to describe πK form factor F
- Bethe-Salpeter (BS) equation:

$$T = V + VGT$$



- fulfils **unitarity** by construction if $V_{ij} \in \mathbb{R}$, $\text{Im } G_{ij} = \sigma_i \delta_{ij}$
- two-potential formalism: [Nakano, 1982; Hanhart, 2012; Ropertz et al., 2018]

$$V = V_0 + V_R \rightarrow T = T_0 + T_R$$

- T_0 elastic, given by **elastic scattering phase** δ_0
- Inelasticity driven by **resonance exchange** via V_R
- T_R given implicitly in terms of T_0 and V_R via BS equation

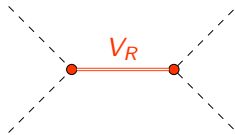
$$T_R = \Omega(\mathbb{1} - V_R \Sigma)^{-1} V_R \Omega^T$$

- Ω - **Omnès function**, given via dispersion relation from δ_0
- Σ - **Selfenergy**, given via dispersion relation from Ω and $\text{Im } G$

Formalism - T -matrix

- parametrisation of the resonance potential V_R

$$(V_R)_{ij} = \sum_r g_i^{(r)} \frac{s - s_0}{(s - m_{(r)}^2)(s_0 - m_{(r)}^2)} g_j^{(r)}$$



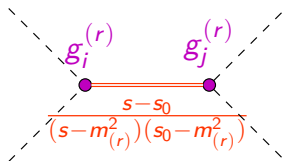
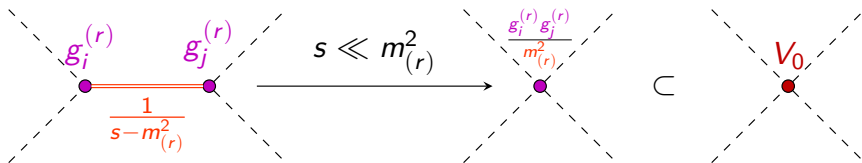
- subtracted at ηK threshold $s_0 = (m_\eta + m_K)^2$
- parameters: bare couplings $g_i^{(r)}$, bare resonance masses $m_{(r)}$

application to $\pi K \rightarrow \pi K$:

- S -wave ($\ell = 0$) with crossed channel $\eta' K$, ηK nearly elastic
- present resonances: $\kappa/K_0^*(700)$, $K_0^*(1430)$, $K_0^*(1950)$
- input phase δ_0 - non-resonant part of [Peláez and Rodas, 2016]
 \rightarrow includes $\kappa/K_0^*(700)$ implicitly
- two resonances explicitly via V_R : $K_0^*(1430)$ and $K_0^*(1950)$

Formalism - resonance potential

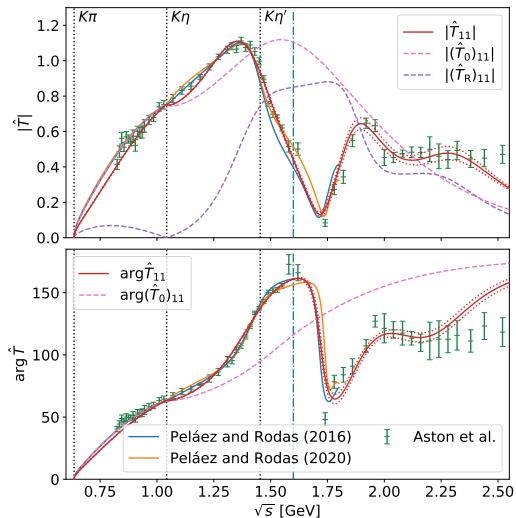
- parametrisation of the resonance potential V_R :



$$\begin{aligned}
 (V_R)_{ij} &= \tilde{V}_R(s)_{ij} - \tilde{V}_R(s_0)_{ij} \\
 &= \sum_r g_i^{(r)} \frac{s - s_0}{(s - m_{(r)}^2)(s_0 - m_{(r)}^2)} g_j^{(r)}
 \end{aligned}$$

- subtracted at ηK threshold $s_0 = (m_\eta + m_K)^2$:
- parameters: bare couplings $g_i^{(r)}$, bare resonance masses $m_{(r)}$

Fit to πK scattering data



$$\text{red. } \chi^2: \frac{370}{112 - 6} \approx 3.5$$

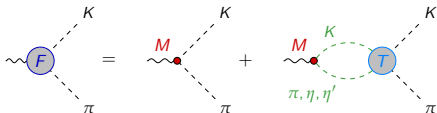
- contradiction between different data sets
[Estabrooks et al., 1978]
- **systematic uncertainties underestimated**
- covers the data till ≈ 2.3 GeV

The result is sufficient considering the modest data quality

Formalism - formfactor F

- application to the decay $\tau \rightarrow \nu_\tau K \pi$:
- need form factor:

$$F = M + MGT$$



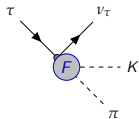
- with the parametrisation of T follows:

$$F = \Omega(\mathbb{1} - V_R \Sigma)^{-1} M$$

- M - reparametrised source term

$$M_i = \sum_k c_i^{(k)} s^k - \sum_r g_i^{(r)} \frac{s - s_0}{(s - m_{(r)}^2)(s_0 - m_{(r)}^2)} \alpha^{(r)}$$

- (additional) parameters: coefficients $c_i^{(k)}$, source couplings $\alpha^{(r)}$



Application to $\tau \rightarrow \nu_\tau K \pi$

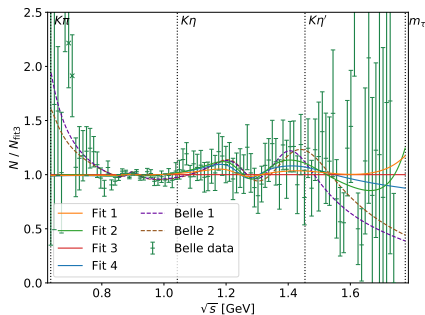
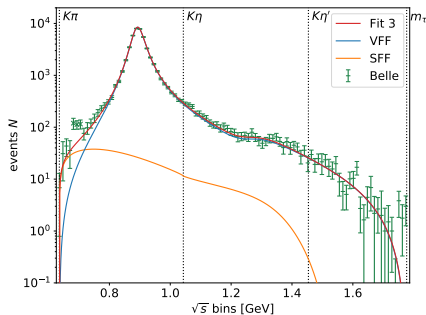
- S -wave parametrised as just explained
- P -wave parametrised by conventional RChPT model [Antonelli et al., 2013]
- further constraints:
 - **normalisation** of $F_S^{\pi K}(0)$, $F_S^{\eta' K}(0)$ fixates $c_1^{(0)}$, $c_2^{(0)}$
 - Callan-Treiman (CT) point: $F_S(\Delta_{\pi K}) = \frac{F_K}{F_\pi} + \Delta_{CT}$
- further considerations:
 - $K_0^*(1950)$ lies outside τ decay region, $\alpha^{(2)}$ poorly constrained
 - higher polynomial terms (i.e. $c_1^{(1)}$) change high-energy behaviour
- four fit scenarios:

$$\alpha^{(2)} = \begin{cases} \text{free} & , \text{fit 1 \& 2} \\ 0 \text{ (fixed)} & , \text{fit 3 \& 4} \end{cases}, \quad c_1^{(1)} = \begin{cases} \text{free} & , \text{fit 2 \& 4} \\ 0 \text{ (fixed)} & , \text{fit 1 \& 3} \end{cases}$$

Fit to $\tau \rightarrow \nu_\tau K \pi$

- data from Belle [Epifanov et al., 2007]
- compare to Breit-Wigner parametrisation from Belle

red. χ^2 : [0.98, 1.07]



- $\tau \rightarrow \nu_\tau K \pi$ dominated by $K^*(892)$ (P -wave)
- overlapping $K_0^*(1430)$ (S -wave) and $K^*(1410)$ (P -wave)

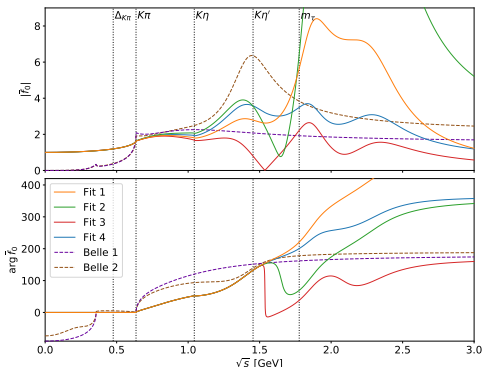
All four fits are **equally suitable**, only very slight variations

Scalar form factor

- scalar form factor F fixed by scattering up to source term M :

$$M_i = \sum_k c_i^{(k)} s^k - \sum_r g_i^{(r)} \frac{s - s_0}{(s - m_{(r)}^2)(s_0 - m_{(r)}^2)} \alpha^{(r)}$$

- different fit scenarios: $m_\tau < m_{K_0^*(1950)}$; $M_i \rightarrow s^{k_{\max}}$



$$\alpha^{(2)} = \begin{cases} \text{free} & , 1 \ \& \ 2 \\ 0 & , 3 \ \& \ 4 \end{cases}, \quad c_1^{(1)} = \begin{cases} \text{free} & , 2 \ \& \ 4 \\ 0 & , 1 \ \& \ 3 \end{cases}$$

- correct phase by construction
- fulfils unitarity & analyticity
- correct normalisation
- in contrast to Breit-Wigner
- further distinction requires additional data

Further Results

- **Branching Ratio:** $\text{BR}(\tau \rightarrow K_S \pi \nu_\tau) = 4.35(10) \times 10^{-3}$
agrees at 1.5σ with PDG [Zyla et al., 2020] and [Ryu et al., 2014]
- estimate on **BSM CP asymmetry** supports [Cirigliano et al., 2018]

$$A_{CP}^{\tau, \text{BSM}} = -0.034(14) \text{ Im } c_T \quad (c_T, \text{ Tensor Wilson coefficient})$$

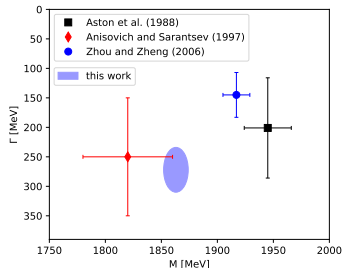
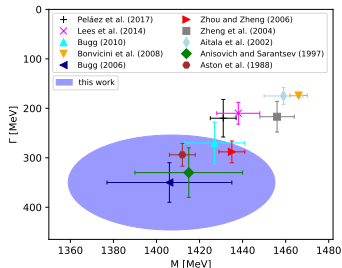
- **pole extraction** via Padé approximants:

$$m_{K_0^*(1430)} = 1408(4)(47) \text{ MeV},$$

$$m_{K_0^*(1950)} = 1863(11)(4) \text{ MeV},$$

$$\Gamma_{K_0^*(1430)} = 360(14)(96) \text{ MeV}$$

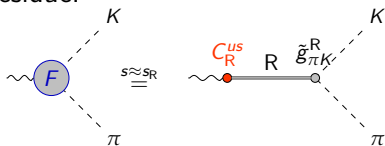
$$\Gamma_{K_0^*(1950)} = 272(38)(8) \text{ MeV}$$



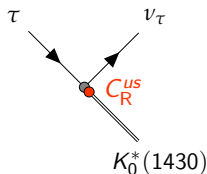
Further Results

- coupling of scalar resonance C_R^{us} via residue:

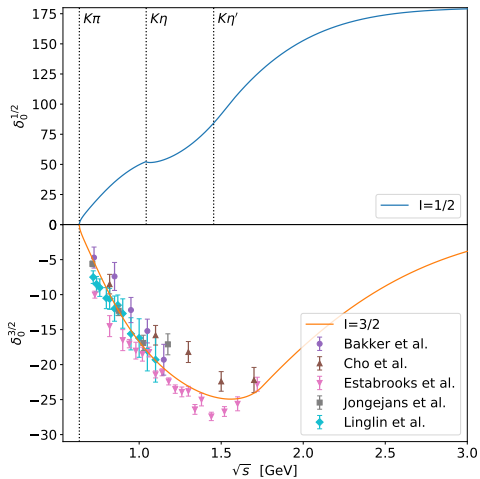
$$F = \sqrt{\frac{2}{3}} \frac{\tilde{g}_{\pi K}^R C_R^{us}}{s_R - s}$$



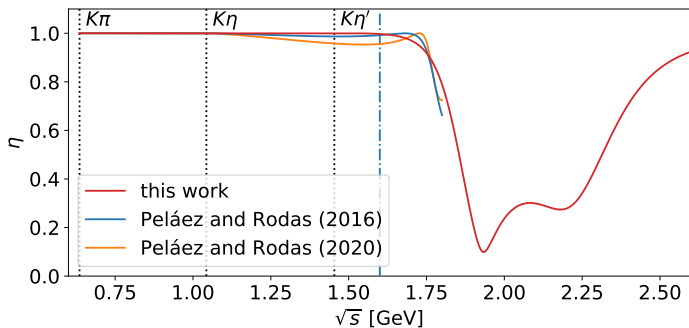
- $\text{BR}(\tau \rightarrow K_0^*(1430)\nu_\tau) \sim \left| C_{K_0^*(1430)}^{us} \right|^2$
- $\text{BR}(\tau \rightarrow K_0^*(1430)\nu_\tau) < 1.6 \times 10^{-4}$



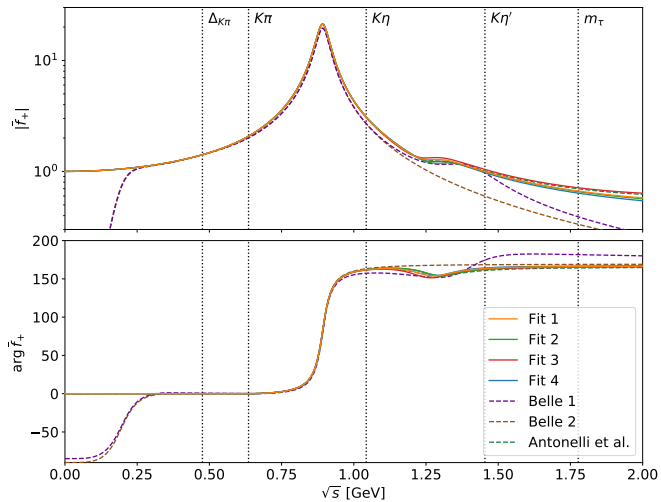
Plots I



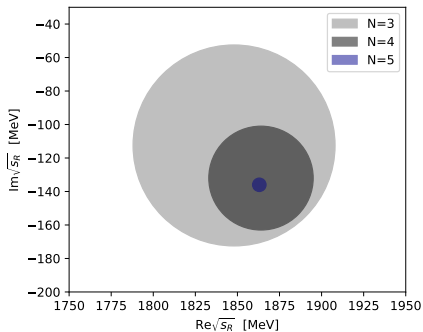
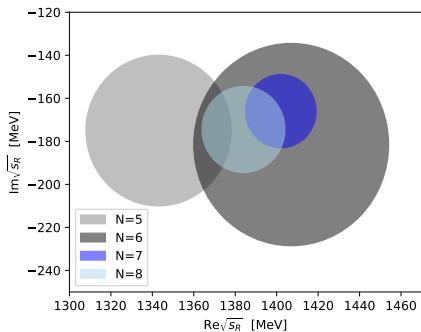
Plots II



Plots III



Plots IV



Tables I

Parameter	Value
$g_1^{(1)}$ [GeV]	2.898(29)
$g_2^{(1)}$ [GeV]	-0.25(35)
$g_1^{(2)}$ [GeV]	2.14(17)
$g_2^{(2)}$ [GeV]	7.70(64)
$\tilde{M}_{(1)}$ [GeV]	1.5708(33)
$\tilde{M}_{(2)}$ [GeV]	2.133(36)
#data points	112
#variables	6
χ^2	370.8
$\chi^2 / \#d.o.f.$	3.50

Tables II

Parameter	Fit 1	Fit 2	Fit 3	Fit 4
λ	0.753(11)	0.7440(94)	0.7617(87)	0.746(11)
$\alpha^{(1)}$ [GeV]	-0.35(26)	-0.28(21)	0.035(40)	-0.42(19)
$\alpha^{(2)}$ [GeV]	1.9(1.3)	-4.3(3.9)	0 (fixed)	0 (fixed)
$c_1^{(1)}$ [GeV ⁻²]	0 (fixed)	-0.65(34)	0 (fixed)	-0.25(11)
$\tilde{M}_{K^*(892)}$ [MeV]	943.71(57)	943.26(53)	944.04(52)	943.40(54)
$\tilde{\Gamma}_{K^*(892)}$ [MeV]	67.15(88)	66.46(82)	67.61(80)	66.69(82)
$\tilde{M}_{K^*(1410)}$ [MeV]	1355(34)	1381(39)	1354(15)	1357(24)
$\tilde{\Gamma}_{K^*(1410)}$ [MeV]	205(100)	205(100)	229(22)	176(35)
β	-0.032(16)	-0.029(12)	-0.0418(48)	-0.0251(75)
#data points	97+1	97+1	97+1	97+1
#variables	8	9	7	8
χ^2	93.1	87.4	97.7	89.4
$\chi^2/\#\text{d.o.f.}$	1.03	0.98	1.07	0.99

Tables III

$$\begin{aligned}
 \text{BR}(\tau \rightarrow K_S \pi \nu_\tau) \Big|_{\text{Fit 1}} &= 4.334(66)(25) \times 10^{-3}, \\
 \text{BR}(\tau \rightarrow K_S \pi \nu_\tau) \Big|_{\text{Fit 2}} &= 4.390(48)(26) \times 10^{-3}, \\
 \text{BR}(\tau \rightarrow K_S \pi \nu_\tau) \Big|_{\text{Fit 3}} &= 4.284(35)(25) \times 10^{-3}, \\
 \text{BR}(\tau \rightarrow K_S \pi \nu_\tau) \Big|_{\text{Fit 4}} &= 4.377(49)(26) \times 10^{-3}, \tag{1}
 \end{aligned}$$

$$\begin{aligned}
 \text{BR}(\tau \rightarrow K_S \pi \nu_\tau) &= 4.35(6)_{\text{st}}(3)_{\text{norm}}(7)_{\text{sys}} \times 10^{-3} \\
 &= 4.35(10) \times 10^{-3}. \tag{2}
 \end{aligned}$$

Tables IV

$\sqrt{s_R^{(5)}} \text{ [MeV]}$	$1863(11)(4) - i 136(19)(4)$
$\Delta_{\text{sys}}^{(5)} \text{ [MeV]}$	4
$\sqrt{s_0^{(5)}} \text{ [GeV]}$	1.86
$\text{mod} \left(\tilde{g}_{\pi K}^{K_0^*(1950)} \right) \text{ [GeV]}$	4.32(35)(8)
$\text{arg} \left(\tilde{g}_{\pi K}^{K_0^*(1950)} \right)$	-0.20(3)(1)
$\Gamma_{K_0^*(1950) \rightarrow \pi K} \text{ [MeV]}$	184(19)(4)
$\Gamma_{K_0^*(1950) \rightarrow \pi K} / \Gamma_{\text{tot}}$	0.70(7)(2)

Results of Padé extractions at $N = 5$ for the $K_0^*(1950)$ including statistical (first bracket) and systematic uncertainties (second bracket).

Tables V

$\sqrt{s_R^{(6)}} \text{ [MeV]}$	$1408(4)(47) - i 180(7)(47)$
$\Delta_{\text{sys}}^{(6)} \text{ [MeV]}$	47
$\sqrt{s_0^{(6)}} \text{ [GeV]}$	1.36
$\text{mod} \left(\tilde{g}_{\pi K}^{K_0^*(1430)} \right) \text{ [GeV]}$	4.96(14)(78)
$\text{arg} \left(\tilde{g}_{\pi K}^{K_0^*(1430)} \right)$	0.06(1)(4)
$\Gamma_{K_0^*(1430) \rightarrow \pi K} \text{ [MeV]}$	304(8)(42)
$\Gamma_{K_0^*(1430) \rightarrow \pi K} / \Gamma_{\text{tot}}$	0.87(2)(11)

Results of Padé extractions at $N = 6$ for the $K_0^*(1430)$ including statistical (first bracket) and systematic uncertainties (second bracket).

Tables VI

	$ C_{K_0^*(1430)}^{us} $ [GeV]	$\text{BR}(\tau \rightarrow K_0^*(1430)\nu_\tau)$
Fit 1	0.23(4)	$0.31(11) \times 10^{-4}$
Fit 2	0.37(8)	$0.78(38) \times 10^{-4}$
Fit 3	0.11(2)	$0.07(3) \times 10^{-4}$
Fit 4	0.31(5)	$0.55(20) \times 10^{-4}$
Other theoretical determinations:		
Ref. [Maltman, 1999]	0.37	0.79×10^{-4}
Ref. [El-Bennich et al., 2009]	0.28	0.45×10^{-4}
Experiment:		
Ref. [Barate et al., 1999]	< 0.93	$< 5 \times 10^{-4}$

Formulas I

Further formulas from [\[Von Detten et al., 2021\]](#):

$$T_0 = \begin{pmatrix} \frac{1}{\rho_1} \sin \delta_0 e^{i\delta_0} & 0 \\ 0 & 0 \end{pmatrix}. \quad (3)$$

$$\Omega = \mathbb{1} + T_0 G \quad (4)$$

$$\Omega = \begin{pmatrix} \Omega_{11} & 0 \\ 0 & 1 \end{pmatrix}, \quad \Omega_{11} = \exp \left(\frac{s}{\pi} \int_{s_{\text{th}}}^{\infty} dz \frac{\delta_0(z)}{z(z-s)} \right). \quad (5)$$

$$\text{disc} \Omega_{if} = 2i (T_0^*)_{im} \rho_m \Omega_{mf}, \quad (6)$$

Formulas II

$$T_R = \Omega t_R \Omega^T \quad (7)$$

$$t_R = V_R + V_R \Sigma t_R, \quad (8)$$

$$\Sigma = G \Omega \quad (9)$$

$$\Sigma_{ij}(s) = \frac{s}{2\pi i} \int_{s_{th}}^{\infty} dz \frac{\text{disc} \Sigma_{ij}(z)}{z(z-s)}, \quad (10)$$

$$\text{disc} \Sigma_{if} = \Omega_{im}^\dagger \text{disc} G_{mm} \Omega_{mf}. \quad (11)$$

Formulas III

$$\hat{T}_{if} = \rho_i \left(T^{\frac{1}{2}} + T^{\frac{3}{2}}/2 \right)_{if}. \quad (12)$$

$$\delta_0(s) = L - (L - \delta_0(s_m)) \exp \left(-\frac{(s-s_m)\delta'_0(s_m)}{L-\delta_0(s_m)} \right) \quad (13)$$

$$\begin{aligned} \frac{d\Gamma}{d\sqrt{s}} &= \frac{c_\Gamma}{s} \left(1 - \frac{s}{m_\tau^2} \right)^2 \left(1 + 2\frac{s}{m_\tau^2} \right) q_{\pi K} \\ &\times \left(q_{\pi K}^2 |\bar{f}_+|^2 + \frac{3\Delta_{\pi K}^2}{4s(1 + 2\frac{s}{m_\tau^2})} |\bar{f}_0|^2 \right), \end{aligned} \quad (14)$$

$$c_\Gamma = \frac{G_F^2 m_\tau^3}{96\pi^3} S_{EW}^\tau (|V_{us}| f_+(0))^2 (1 + \delta_{EM}^{K\tau})^2, \quad (15)$$

Formulas IV

$$\begin{aligned}
 \langle \bar{K}^0(p_K)\pi^-(p_\pi)|\bar{s}\gamma^\mu u|0\rangle &= (p_K - p_\pi)^\mu f_+(s) \\
 &\quad + (p_K + p_\pi)^\mu f_-(s), \\
 \langle \bar{K}^0(p_K)\pi^-(p_\pi)|\bar{s}u|0\rangle &= \frac{\Delta_{\pi K}}{m_s - m_u} f_0(s),
 \end{aligned} \tag{16}$$

$$f_-(s) = \frac{\Delta_{\pi K}}{s} (f_0(s) - f_+(s)). \tag{17}$$

$$\bar{f}_+(s) = \frac{f_+(s)}{f_+(0)}, \quad \bar{f}_0(s) = \frac{f_0(s)}{f_+(0)}. \tag{18}$$

$$\bar{f}_0(\Delta_{\pi K}) = \frac{F_K}{F_\pi} + \Delta_{CT}, \tag{19}$$

Formulas V

$$A_{CP}^{\tau, \text{BSM}} = \frac{\text{Im } c_T}{\Gamma_\tau \text{BR}(\tau \rightarrow K_S \pi \nu_\tau)} \quad (20)$$

$$\times \int_{s_{\pi K}}^{m_\tau^2} ds' \kappa(s') |f_+(s')| |B_T(s')| \sin(\delta_+(s') - \delta_T(s')),$$

$$\sqrt{s_R} = M_R - i \frac{\Gamma_R}{2}. \quad (21)$$

$$\bar{f}_0(s) = \sqrt{\frac{2}{3}} \frac{\tilde{g}_{\pi K}^R C_R^{us}}{s_R - s}, \quad \Gamma(\tau \rightarrow R \nu_\tau) = \frac{6\pi^2 c_T \Delta_{\pi K}^2}{M_R^4} \left(1 - \frac{M_R^2}{m_\tau^2}\right)^2 |C_R^{us}|^2 \quad (22)$$

Analyticity and unitarity

- S -matrix describing scattering processes:

$$|\text{out}\rangle = S |\text{in}\rangle$$

- T -matrix is non-trivial part of S -matrix:

$$S = 1 + iT$$

Analyticity and unitarity

- S -matrix describing scattering processes:

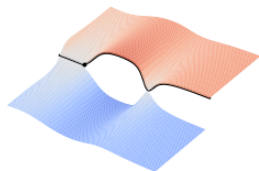
$$|\text{out}\rangle = S |\text{in}\rangle$$

- T -matrix is non-trivial part of S -matrix:

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- **analyticity up to cut(s)** on the real axis
- Schwartz reflection principle:

$$T(s^*) = T(s)^* \Rightarrow \text{disc } T = 2i \text{Im } T$$



imag. part of 1st sheet
(with right-hand cut (RHC))

Analyticity and unitarity

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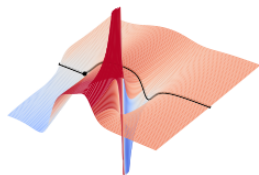
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- resonance structures from **poles on the second sheet**



imag. part of 1st & 2nd sheet
(with pole on 2nd sheet)

Analyticity and unitarity

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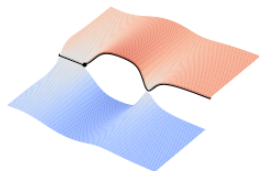
$$S = 1 + iT$$

- **analyticity up to cut**(s) on the real axis
- Schwartz reflection principle:

$$T(s^*) = T(s)^* \Rightarrow \text{disc } T = 2i \text{Im } T$$

- resonance structures from **poles on the second sheet**
- **unitarity** of the S -Matrix (above threshold):

$$S^\dagger S = 1 \Rightarrow \text{Im}(T^{-1}) = -\sigma = -\frac{q\pi K}{\sqrt{s}} \quad (\text{phase space})$$

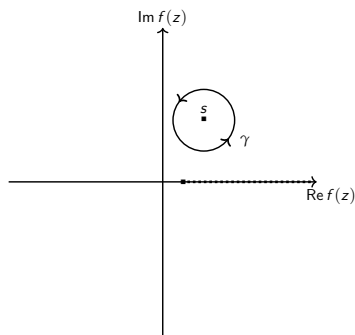


imag. part of 1st sheet
(with right-hand cut (RHC))

Dispersion theory

- Cauchy's integral formula:

$$f(s) = \frac{1}{2\pi i} \oint_{\gamma} \frac{f(z)}{z-s} dz$$



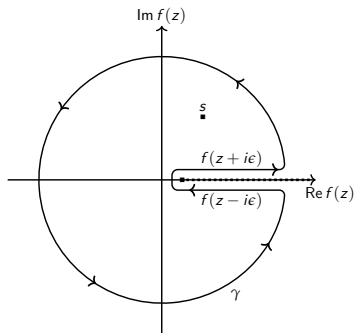
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Dispersion theory

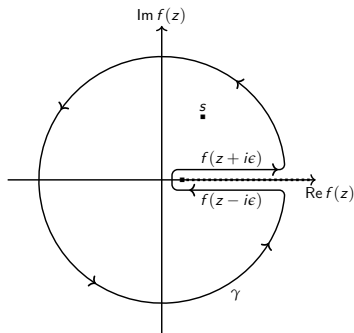
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- if $f(z) \xrightarrow{|z| \rightarrow \infty} 0$



Dispersion theory

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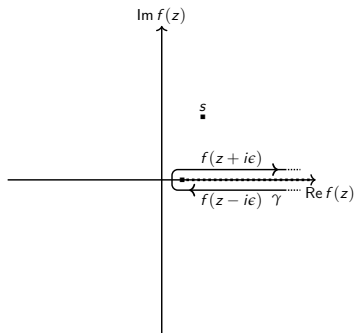
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- **dispersion relation:**

$$f(s) = \frac{1}{2\pi i} \int_{s_0}^{\infty} \frac{\text{disc}f(z)}{z-s} dz$$



Dispersion theory

- Cauchy's integral formula:

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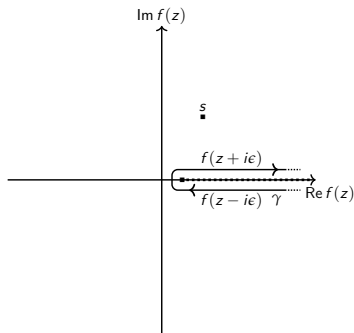
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$$f(s) = \frac{1}{2\pi i} \int_{s_0}^{\infty} \frac{\text{disc}f(z)}{z-s} dz$$

- if $\tilde{f}(z) = \frac{f(z)-f(\tilde{s})}{z-\tilde{s}} \xrightarrow{|z| \rightarrow \infty} 0$ with $\tilde{s} < s_0$



Dispersion theory

- Cauchy's integral formula:

$$f(s) = \frac{1}{2\pi i} \oint_{\gamma} \frac{f(z)}{z-s} dz$$

- discontinuity:

$$\text{disc}f(z) := \lim_{\epsilon \rightarrow 0^+} (f(z+i\epsilon) - f(z-i\epsilon))$$

- dispersion relation:**

$$f(s) = \frac{1}{2\pi i} \int_{s_0}^{\infty} \frac{\text{disc}f(z)}{z-s} dz$$

- once subtracted dispersion relation:

$$f(s) = f(\tilde{s}) + \frac{s-\tilde{s}}{2\pi i} \int_{s_0}^{\infty} \frac{\text{disc}f(z)}{(z-s)(z-\tilde{s})} dz$$

