Precision measurements on dipole moments of the τ and hadronic multi-body final states

Fabian Krinner, Stephan Paul

Max Planck Institut für Physik

TAU2021

τ dipole moments

- Coupling of $\tau^\pm$ to photons via: $\bar{v}_{\tau^+}\Gamma^\mu(q^2)u_{\tau^-}$ with

$$\Gamma^\mu(q^2) = -ieQ_\tau\left[\gamma^\mu F_1(q^2) + \frac{\sigma^{\mu\nu}q_\nu}{2m_\tau}\left(iF_2(q^2) + F_3(q^2)\gamma^5\right)\right]$$

- $F_1(q^2)$ Dirac form factor $F_1(0) = 1$
- $F_2(q^2)$ Pauli form factor *MDM* $F_2(0) = a_\tau$
- $F_3(q^2)$: EDM: $F_3(0) = d_\tau \frac{2m_\tau}{eQ_\tau}$
- Neither shown nor discussed $F_4(q^2)$ anapole moment

S. Eidelman, D. Epifanov, M. Fael, L. Mercolli, M. Passera, arXiv:1601.07987

- Pair production

$$e^- + e^+ \rightarrow \tau^- + \tau^+$$

- Disclaimer $F_{2/3}(q^2 = E^2_{\mathrm{CM}} = m^2_{\Upsilon(4S)})$, not actual dipole moments

- Production described by spin-density matrix

$$\chi_{\lambda_-\lambda_+,\lambda'_-\lambda'_+} = \chi^{\mathrm{SM}}_{\lambda_-\lambda_+,\lambda'_-\lambda'_+} + \sum_{x \in \{\Re/\Im(F_{2/3})\}} x \cdot \chi^{x}_{\lambda_-\lambda_+,\lambda'_-\lambda'_+}$$

- Dipole moments alter spin-correlation of $\tau^{\pm}$-pair

- Access via angular distributions of decay products

Optimal observables

- Spin-density matrix contracted with decay modes

$$\mathcal{M}^2 = \sum_{\lambda_\pm^{(\prime)}} \chi_{\lambda_-\lambda_+,\lambda'_-\lambda'_+} D^-_{\lambda_-\lambda'_-} D^+_{\lambda_+\lambda'_+}$$

- Construct optimal observables

$$\mathcal{M}^2 = \mathcal{M}^2_{\text{SM}} + \sum_{x\in\{\Re/\Im(F_{2/3})\}} x \cdot \mathcal{M}^2_x \quad \Rightarrow \quad \mathrm{OO}_x = \frac{\mathcal{M}^2_x}{\mathcal{M}^2_{\text{SM}}}$$

- Get $\Re/\Im(F_{2/3})$ from expectation value of data set

$$\langle \mathrm{OO}_x \rangle_{\text{data}} \propto x$$

- Optimal observables appear also naturally in a $\log \mathcal{L}$ fit

M. Diehl and O. Nachtmann, Z. Phys.C62(1994) 397412

Optimal observables

Simple estimation of expected resolution

- First: Only 1×1 topologies: $\tau \rightarrow \{\pi\nu\,;\rho\nu\,;\mu\nu\bar{\nu}\,;\mathrm{e}\nu\bar{\nu}\}$
- Similar resolution for all 16 combinations (10^6 events):

$$\delta\Re(F_2) = 7.0 \times 10^{-4};$$
$$\delta\Im(F_2) = 7.2 \times 10^{-4};$$
$$\delta\Re(F_3) = 9.3 \times 10^{-4};$$
$$\delta\Im(F_3) = 5.2 \times 10^{-4}$$

- However: Escaping neutrino carries away kinematic information
  - Always: Average over two-fold ambiguity
  - For every leptonic decay:

$$\int \mathcal{M}^2 \mathrm{d}\phi \mathrm{d}\cos\theta \mathrm{d}m^2_{\nu\nu}$$

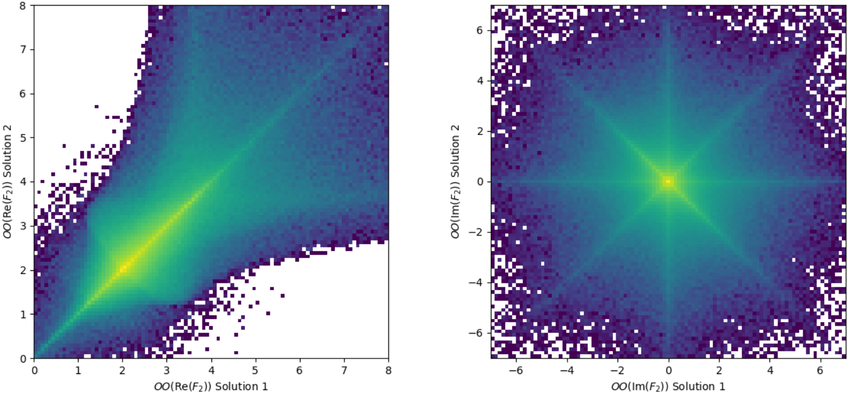
Optimal observables

Missing information

τ^- mode	τ^+ mode	$x_{\delta\Re(F_2)}$	$x_{\delta\Im(F_2)}$
$\pi^-\nu_\tau$	$\pi^+\bar{\nu}_\tau$	1.09	1.60
$\pi^-\nu_\tau$	$\rho^+\bar{\nu}_\tau$	1.11	1.19
$\pi^-\nu_\tau$	$e^+\bar{\nu}_\tau\nu_e$	2.07	1.75
$\pi^-\nu_\tau$	$\mu^+\bar{\nu}_\tau\nu_\mu$	2.06	1.72
$\rho^-\nu_\tau$	$\pi^+\bar{\nu}_\tau$	1.11	1.19
$\rho^-\nu_\tau$	$\rho^+\bar{\nu}_\tau$	1.11	1.26
$\rho^-\nu_\tau$	$e^+\bar{\nu}_\tau\nu_e$	2.03	1.79
$\rho^-\nu_\tau$	$\mu^+\bar{\nu}_\tau\nu_\mu$	2.04	1.79
$e^-\nu_\tau\bar{\nu}_e$	$\pi^+\bar{\nu}_\tau$	2.09	1.81
$e^-\nu_\tau\bar{\nu}_e$	$\rho^+\bar{\nu}_\tau$	2.03	1.75
$e^-\nu_\tau\bar{\nu}_e$	$e^+\bar{\nu}_\tau\nu_e$	3.45	2.28
$e^-\nu_\tau\bar{\nu}_e$	$\mu^+\bar{\nu}_\tau\nu_\mu$	3.73	2.28
$\mu^-\nu_\tau\bar{\nu}_\mu$	$\pi^+\bar{\nu}_\tau$	2.07	1.79
$\mu^-\nu_\tau\bar{\nu}_\mu$	$\rho^+\bar{\nu}_\tau$	2.03	1.72
$\mu^-\nu_\tau\bar{\nu}_\mu$	$e^+\bar{\nu}_\tau\nu_e$	5.83	2.28
$\mu^-\nu_\tau\bar{\nu}_\mu$	$\mu^+\bar{\nu}_\tau\nu_\mu$	3.11	2.32

Correlations of ambiguous solutions

$\pi^- \nu \times \pi^+ \bar{\nu}$



Additional hadronic channels

- Hadronic decays give best resolution
- Next highest branching fraction:

$$\tau^{\pm} \to 3\pi^{\pm} + \nu$$

- Branching: $9.02 \pm 0.05\%$

P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020) and 2021 update.

- Decay spin-density matrix

$$D_{\lambda\lambda'} = \mathcal{A}^*_{\lambda}\mathcal{A}_{\lambda'} \quad \text{with} \quad \mathcal{A}_{\lambda} \propto \bar{u}_{\nu}\gamma_{\mu}(1-\gamma^5)u^{\lambda}_{\tau}J^{\mu}_{\text{had}} = \ell^{\lambda}_{\mu}J^{\mu}_{\text{had}}$$

- For π and ρ

$$J^{\mu}_{\pi} \propto p^{\mu}_{\pi}; \quad J^{\mu}_{\rho} \propto \text{BW}_{\rho}\left(s_{\pi\pi^0}\right)\left(p^{\mu}_{\pi} - p^{\mu}_{\pi^0}\right)_{\perp}$$

- $\Rightarrow 3\pi^{\pm}$: Model necessary

Dipole moments with wrong hadronic current

- Use hadronic model inspired by COMPASS

C. Adolph *et al.* [COMPASS], Phys. Rev. D **95** (2017) no.3, 032004 doi:10.1103/PhysRevD.95.032004 [arXiv:1509.00992 [hep-ex]].

- Generate toy data sets for $3\pi^{\pm}\nu \times \pi^{+}\bar{\nu}$ for

$$\Re/\Im(F_{2/3}) = 0.01$$

- Analyze with pure $\mathrm{a}_1\,[\rho\pi]_S$ model
  - Model overlap 78%
- Resulting values

$$\Re(F_2) = 0.0528 \pm 0.0005; \quad \Im(F_2) = 0.0109 \pm 0.0005$$

$$\Re(F_3) = 0.0090 \pm 0.0007; \quad \Im(F_3) = 0.0079 \pm 0.0003$$

- $\Rightarrow 3\pi^{\pm}$: good model necessary

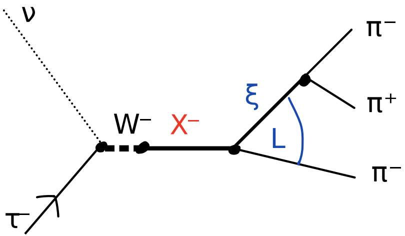
- Learn $\tau \to 3\pi + \nu$ amplitude from data
  - Perform amplitude analysis

$$J^{\mu}_{\text{had}} = \sum_{i \in \text{waves}} c_i j_i^{\mu}(p_1, p_2, p_3)$$

- Free complex-valued coefficients c_i
- Specific dependence on the phase-space variables $j_i^{\mu}(p_1, p_2, p_3)$

Isobar model

- Possible formulation: isobar model
- Waves given by:
  - Three pion state X^-
  - Known J^{PC} quantum numbers
  - Decays to isobar ξ^0 and π
  - Also known J^{PC}
  - Orbital angular momentum L
- Alternative non-isobaric waves (e.g. chiral models or combinations)



The decay $\tau \to 3\pi$

Partial wave currents

- Partial-wave current: f.s. $\pi_1^+, \pi_2^-, \pi_3^- \Rightarrow p_1^\mu, p_2^\mu$, and p_3^μ
  - $\mathrm{a}_1[\rho\pi]_S$

- Angular momenta encoded in:

$$\Xi_i^\mu = \left(\eta^{\mu\nu} - \frac{p_{123}^\mu p_{123}^\nu}{p_{123}^2}\right)(p_1 - p_i)_\nu$$

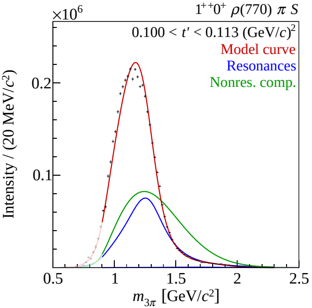
- Bose symmetrization

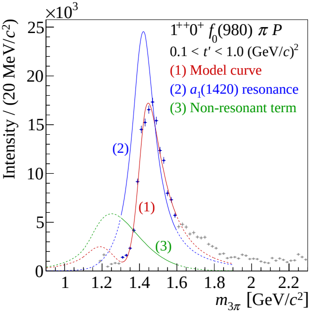
$$j_{[\rho\pi]_S}^\mu = \mathrm{BW}_{\mathrm{a}_1}(s_{123})\left(\mathrm{BW}_\rho(s_{12})\Xi_2^\mu + \mathrm{BW}_\rho(s_{13})\Xi_3^\mu\right)$$

- Dynamic amplitudes BW(s)

F. Krinner and S. Paul,[arXiv:2107.04295 [hep-ph]].

a_1 resonances





M. Aghasyan *et al.* [COMPASS], Phys. Rev. D **98** (2018) no.9, 092003 doi:10.1103/PhysRevD.98.092003 [arXiv:1802.05913 [hep-ex]].

C. Adolph *et al.* [COMPASS],Phys. Rev. Lett. **115** (2015) no.8, 082001 doi:10.1103/PhysRevLett.115.082001 [arXiv:1501.05732 [hep-ex]].

- $a_1(1260)$
- PDG:

$$m_{a_1} = 1,230 \pm 40\,\text{MeV}/c^2$$

$$\Gamma_{a_1} = 250 - 600\,\text{MeV}/c^2$$

- $a_1(1420)$
- Possible explanations:
  - Resonance
  - Triangle singularity
  - Interference with non-resonant processes

Form factors $F_{2/3}$ with improved hadronic model

- Improve overlap to 95%

$$\Re(F_2) = 0.0180 \pm 0.0005; \quad \Im(F_2) = 0.105 \pm 0.0005$$

$$\Re(F_3) = 0.0104 \pm 0.0006; \quad \Im(F_3) = 0.0099 \pm 0.0003$$

- Overlap to 99%

$$\Re(F_2) = 0.0116 \pm 0.0005; \quad \Im(F_2) = 0.0098 \pm 0.0005$$

$$\Re(F_3) = 0.0097 \pm 0.0006; \quad \Im(F_3) = 0.0102 \pm 0.0003$$

- Good hadronic model can recover F_3
- $\Im(F_2)$ not largely affected
- $\Re(F_2)$ critical

Summary & conclusion

- Measure the τ form factors F_2 and F_3 in τ-pair events
  - Kinematic distribution of the decay products
- Missing neutrino kinematics:
  - Hadronic events give better resolution
- 3rd largest hadronic mode: $\tau \to 3\pi^{\pm} + \nu_\tau$
- Dipole moments need proper hadronic model
- Amplitude analysis of $\tau \to 3\pi + \nu_\tau$
  - E.g. PWA in the isobar model
  - $\mathrm{a}_1(1260)$, $\mathrm{a}_1(1420)$
- F_3 and $\Im(F_2)$ can be improved
  - $\Re(F_2)$ too sensitive on hadronic model