

# Monte Carlo Event Generator updates, for $\tau$ pair events at Belle II energies

Sw. Banerjee<sup>a</sup>, D. Biswas<sup>a</sup>, T. Przedzinski<sup>b</sup>, Z. Was\*

<sup>a</sup> University of Louisville, Kentucky, USA

<sup>b</sup> Institute of Physics, Jagellonian University, Cracow, Poland

\*Speaker; Institute of Nuclear Physics, Polish Academy of Sciences, Cracow, Poland

- **(1) Announcement:** installation in `basf2` system of Belle2  
`KKMC+tauola+photos.f+photospp` works.
- **(2)** Technical aspects: (i) presamplers in  $\tau$  decay phase space parametrization (ii) mixed FORTRAN c++ solutions.
- **(3)** bremsstrahlung in decays and additional pair emissions with *c* version of `photos` Monte Carlo; `photospp`. Pair emission through virtual photon. Also through exotic virtual scalar, pseudo-scalar or vector.
- **(4)** Updates on  $\tau$  decays.
- **(5)** Status of `KKMC`.
- **(6)** Summary/outlook/limitations.

**(general)**

1. `photospp` and `tauola`: phase-space for narrow resonances to lepton pairs ME.
  - (a) Tests with semianalytical calculations
  - (b) Tests with matrix element exact phase space, event samples...
  - (c) ...from `KORALW` for FSR QED pair emission in Z decay,
  - (d) ... from `MadGraph` for  $\tau\tau$  scalar( $\rightarrow$  lepton pair) sample (solution like for ML used for Higgs parity observables Phys.Rev.D 103 (2021), 036003 ).
2. Tests evolve into physics extensions, **but are important for technical side.**
3. Initial step for work on practical factorization and matching with photon emissions.
4. Issues with configuration/makefile systems.

**(code in C, interfaces in C++)**

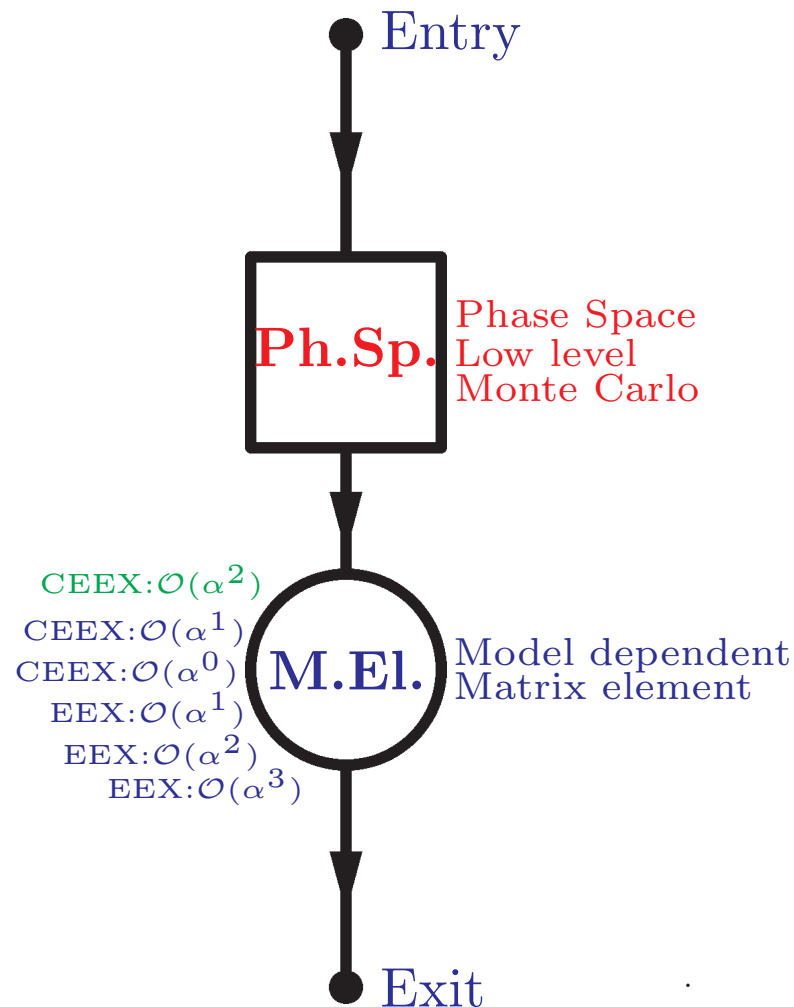
1. it features not only photon but also lepton pair emissions, all with rigorous exact phase space, but approximations in matrix element parts
2. they can be from QED, but also from dark photons/scalars [prompt/long-lived]
3. this work with `KKMC+tauola+photos.f` events
4. and can be used for simulation of decay channels with such extra pairs,
5. also for additional objects thus final states of  
 $Z/\gamma^* \rightarrow \tau\tau X_{scalar/pseudoscalar/vector}$  with no restrictions on  $\tau$  decay channels and with  $\tau^+\tau^-$  spin correlations, also in case of  $X \rightarrow ee(\mu\mu)$ .
6. References: S. Antropov, “High Precision Lepton Pair Bremsstrahlung with PHOTOS”, Acta Phys. Pol. B 51 (2020) 6, 1221

# (4) updates on tauola

**(physics)**

1. new channels new matrix elements
2. establishing defaults
3. step back with pointers to users currents and ME
4. but modularity of segments to migrate to C and those to C++ preserved, to follow the same path as `photospp`
5. no progress on i/o compatibility with fitting software.
6. References: S. Antropov, et. al., “TAUOLA Update for Decay Channels with  $e^+e^-$  Pairs in the Final State”, e-Print: 1912.11376 [hep-ph]

To assure precision “matrix element  $\times$  full phase space” is good to use



- Phase-space module produce “raw events” of raw matrix elements.
- Library of models (often perturbative results ) provide input for “model weight”: ratio of model to raw  $ME^2$ . Monte Carlo may work as formal integrator over required phase space region.
- **As a matter of principle we use exact phase space and explicit form of matrix elements.**
- Convenient for ML training samples.
- *For the programs to be efficient we need “raw event” distribution to approximate peaks of the final one.* Narrow width limit is indispensable for many analytical tests.

To illustrate, **4-body phase space** from Appendix A of Reference: S. Antropov, et. al., “Extra lepton pair emission corrections to Drell-Yan processes in PHOTOS and SANC”, Acta Phys.Polon.B 48 (2017) 1469, e-Print: 1706.05571 [hep-ph]

$$\begin{aligned}
\Omega &= \int \frac{d^3 q_1}{2(q_1)_0 (2\pi)^3} \cdot \frac{d^3 q_2}{2(q_2)_0 (2\pi)^3} \cdot \frac{d^3 p}{2p_0 (2\pi)^3} \cdot \frac{d^3 p'}{2p'_0 (2\pi)^3} (2\pi)^4 \delta^4(R - p - p' - q_1 - q_2) = \\
&= \int d^4 q d^4 Q \frac{d^3 q_1}{2(q_1)_0 (2\pi)^3} \cdot \frac{d^3 q_2}{2(q_2)_0 (2\pi)^3} \cdot \frac{d^3 p}{2p_0 (2\pi)^3} \cdot \frac{d^3 p'}{2p'_0 (2\pi)^3} (2\pi)^4 \times \\
&\times \delta^4(R - p - p' - q_1 - q_2) \delta^4(q - q_1 - q_2) \delta^4(Q - p - p')
\end{aligned} \tag{1}$$

$$\int \frac{d^3 q_1}{2(q_1)_0} \frac{d^3 q_2}{2(q_2)_0} \delta^4(q - q_1 - q_2) = \int \frac{|\vec{q}_1| d \cos \theta_{q_1} d\phi_{q_1}}{4\sqrt{q^2}},$$

$$\int \frac{d^3 p}{2(p)_0} \frac{d^3 p'}{2(p')_0} \delta^4(Q - p - p') = \int \frac{|\vec{p}| d \cos \theta_p d\phi_p}{4\sqrt{p^2}},$$

$$\int d^4 q d^4 Q \delta^4(R - Q - q) = \int (d \cos \theta_q d\phi_q) dM_Q^2 dM_q^2 \frac{\sqrt{\lambda}}{8s}$$

We get finally:

$$\Omega = \frac{1}{(2\pi)^8} \int dM_q^2 dM_Q^2 d \cos \theta_{q_1} d\phi_{q_1} d \cos \theta_p d\phi_p d \cos \theta_q d\phi_q \frac{1}{8} \sqrt{1 - \frac{4\mu^2}{q^2}} \frac{1}{8} \sqrt{1 - \frac{4m^2}{Q^2}} \frac{\sqrt{\lambda(s, M_Q^2, M_q^2)}}{8s}. \tag{2}$$

1. We have invariant masses  $M_q, M_Q$  available for variable changes with Jacobians of minimum where matrix element peak, e.g.:

$$\begin{aligned}\alpha_{min/max} &= \arctan \frac{M_{Q,min/max}^2 - M_R^2}{\Gamma_R M_R}, \\ \alpha &= \alpha_{min} + (\alpha_{max} - \alpha_{min}) \cdot x, \\ M_Q^2 &= M_R^2 + \Gamma_R M_R \tan \alpha,\end{aligned}\tag{3}$$

2. Jacobian of this is used to cancel matrix element enhancements due to resonance propagators (or near-real photon).
3. Angular variables can be used for collinear singularities or matrix elements.
4. **Parametrizations of that type are used in tauola and photos for phase-space with pair of light leptons added or not.** In practice, multi-channel generators are necessary. Jacobians of parametrisations are combined. That is used for lepton pairs from virtual photons or dark scalars/vectors.
5. Why not use automated solutions? In part, because one want to use narrow width limit for numerical tests. These are necessary for our tests with analytical semi-analytical results. **These results are useful to build intuitions.**

1. One can easily compare Monte Carlo samples generated by different programs. To minimize human mistakes we have used our `MC-tester` package, *Comput.Phys.Commun.* 182 (2011) 77, which define histograms automatically.
2. We have used KORALW (*Comput.Phys.Commun.* 94 (1996) 216) to generate samples of 4-fermion final states from Z boson decays. When KORALW matrix element installed into `photos` no `photos-KORALW` differences.
3. But this required special non universal choice of event sample configuration.
4. later to identify optimal choice of the kernel used in `photos` Monte Carlo. That is kernel which is close to complete matrix element, but has some factorization in it, that is why can be used also if additional photons are present.
5. Tests were useful for implementation of dark photon channels into `tauola`
6. Similar comparisons of `MadGraph` and `KKMC+photos` for  $e^+e^- \rightarrow \tau^+\tau^- X$ ,  $X \rightarrow l\bar{l}$ . Adaptation of `photospp` kernel, but approximation to `MadGraph`. Advantage: all  $\tau$  decay channels, spin correlations, bremsstrahlung included.



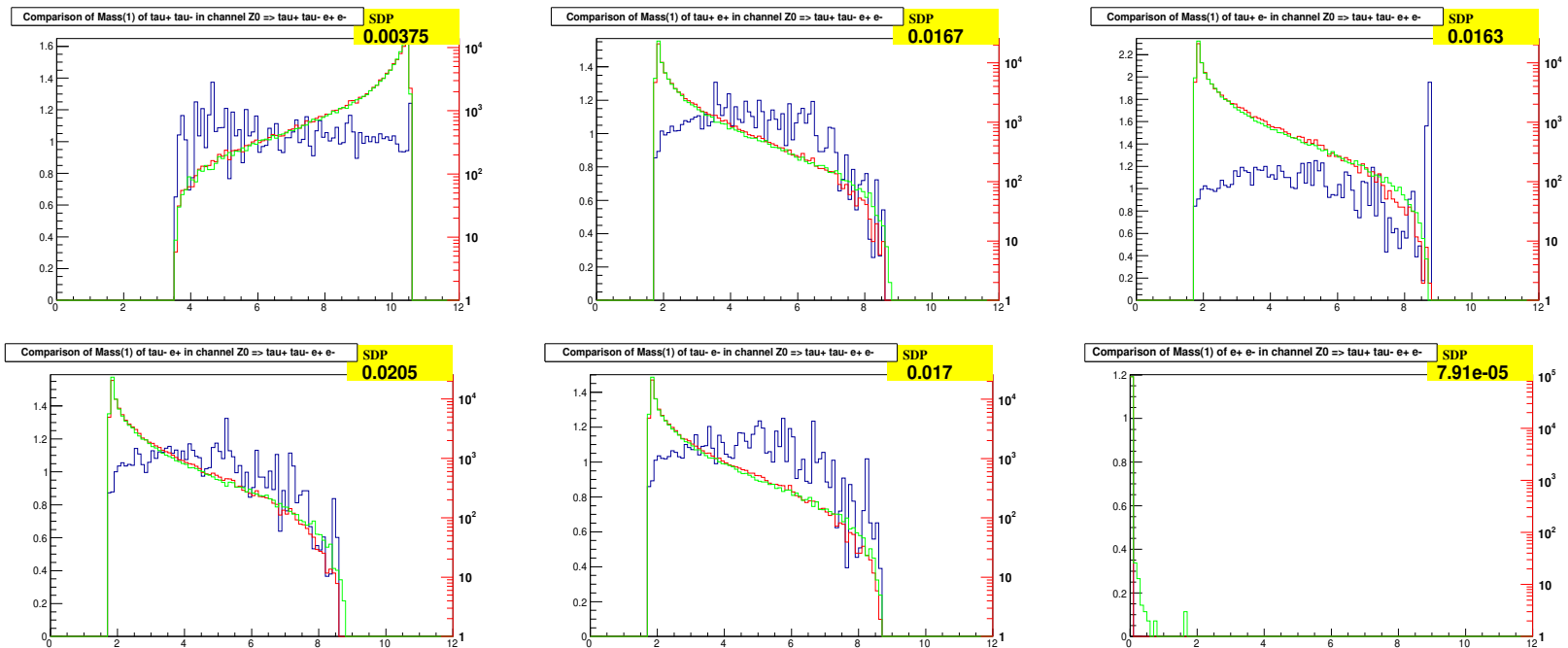
# (Tests with other Monte Carlo programs)

$$e^-e^+ \rightarrow \tau^- \tau^+ \phi_{\text{Dark Scalar}} (50 \text{ MeV}) (\rightarrow e^-e^+)$$

Red lines: 50K events from MadGraph (MG5)

Green lines: 100K events from KKMC+photos

First check: ISR bremsstrahlung effects turned off in both generators

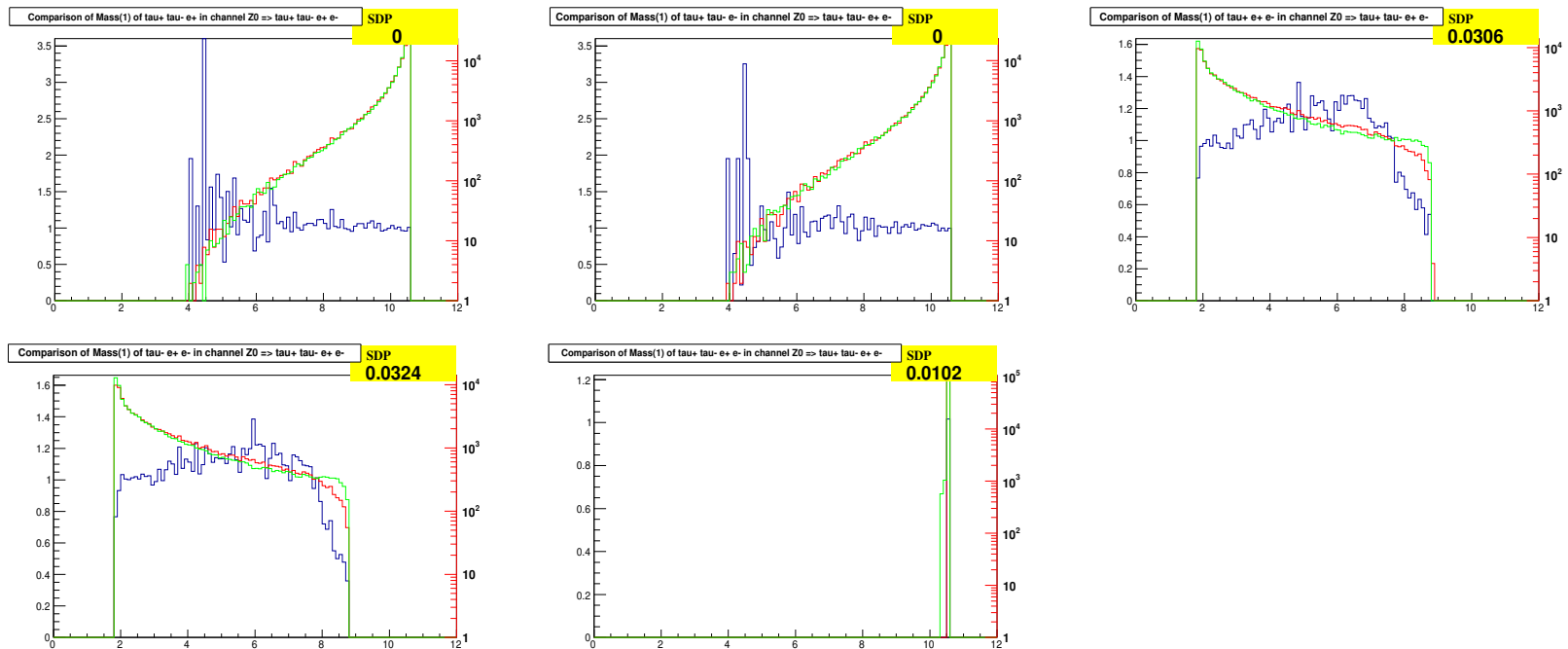


$$e^-e^+ \rightarrow \tau^- \tau^+ \phi_{\text{Dark Scalar}} (50 \text{ MeV}) (\rightarrow e^-e^+)$$

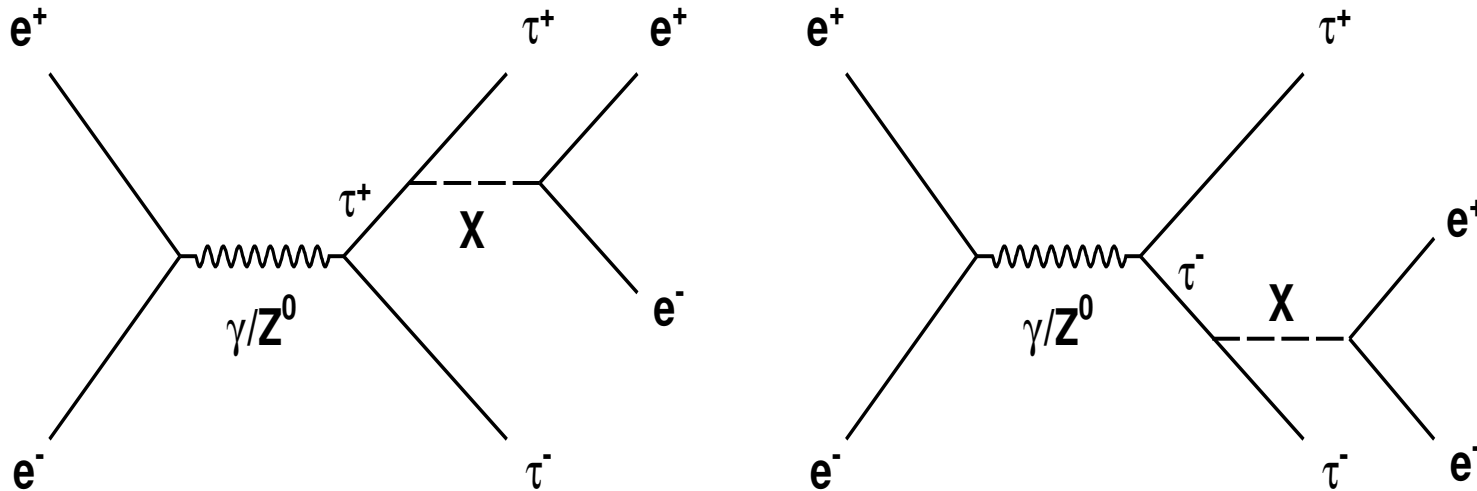
Red lines: 50K events from MadGraph (MG5)

Green lines: 100K events from KKMC+photos

First check: ISR bremsstrahlung effects turned off in both generators

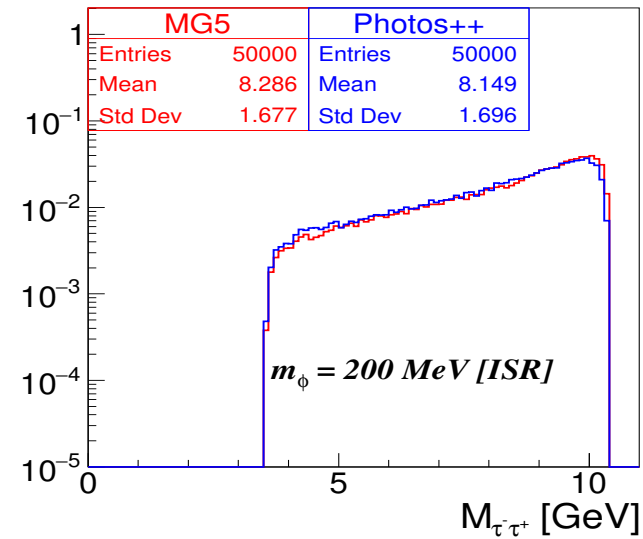
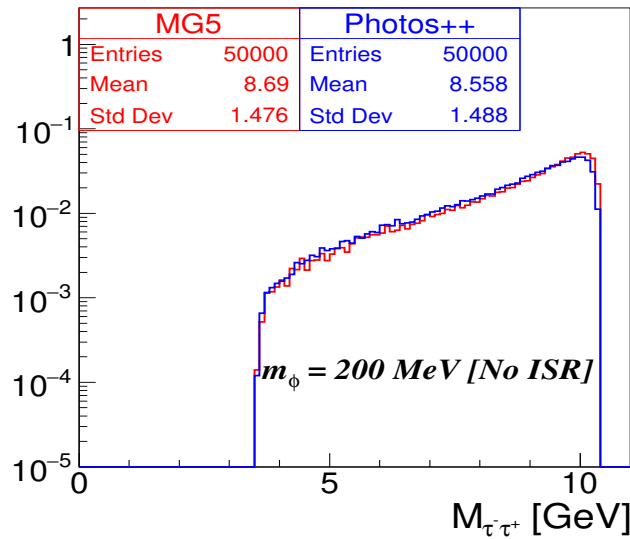
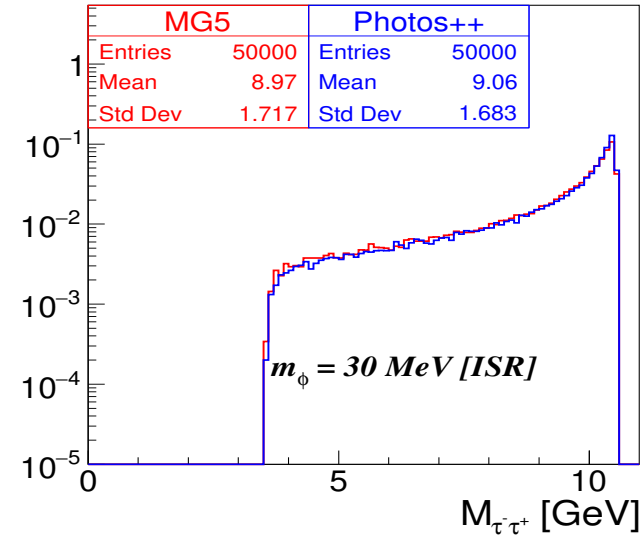
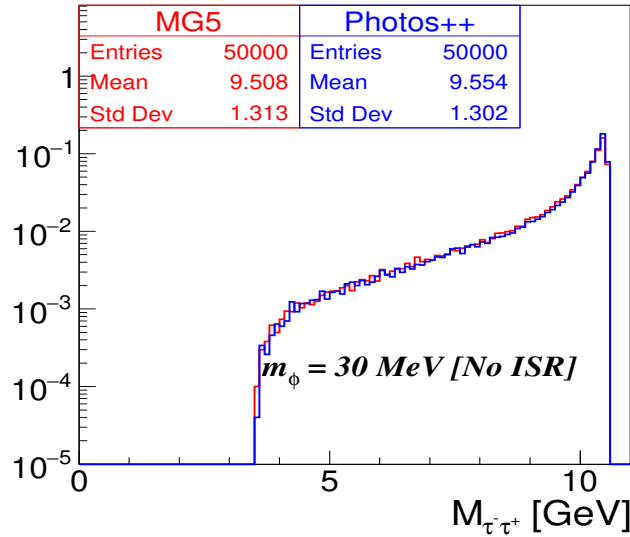


1. Since Ref. Eur.Phys.J.C 50 (2007) 53, `photos` Monte Carlo for radiative corrections in decays has its part calculating matrix elements separated.
2. First for Z and W decay, later for decays like  $B^- \rightarrow \pi^0 K^-$ ,  $B^0 \rightarrow \pi^+ K^-$  or  $\gamma^* \rightarrow \pi^- \pi^+$ , this provided technical steps necessary to introduce algorithm for lepton pair emission, relying on matrix element too. It is implemented in successor of FORTRAN `photos` the `photospp` in C++.
3. **Previous 3 slides of automated test:** comparisons between MadGraph and KKMC+`photospp` samples for  $e^+ e^- \rightarrow \tau^+ \tau^- X$ , are not technical only, but the result of work on matrix element.
4. In case of `photospp` adaptation/interpolation of matrix element to configurations where additional photons are present was needed.
5. Only dominant part of Matrix Element generated by MadGraph was used.
6. Its identification was not as formal as in case of lepton pair from virtual photon.
7. Then Gauge invariance and symmetries were used to identify parts. Now it was rather analogy and similarity for singular terms.
8. Spin of X is zero, and not 1. Back of the envelope calculations to identify differences.

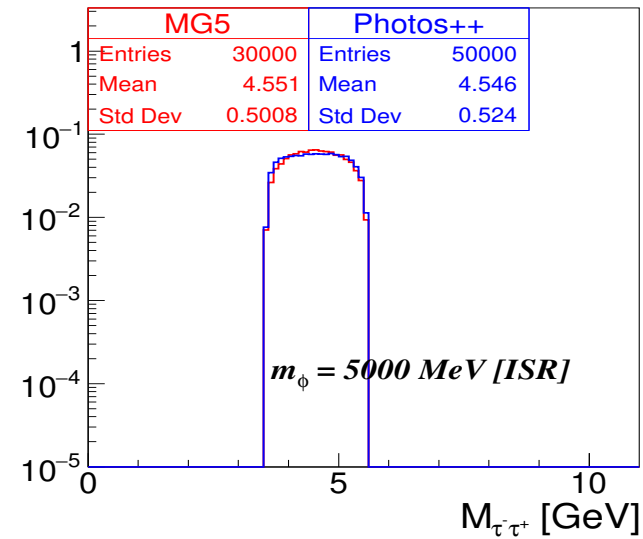
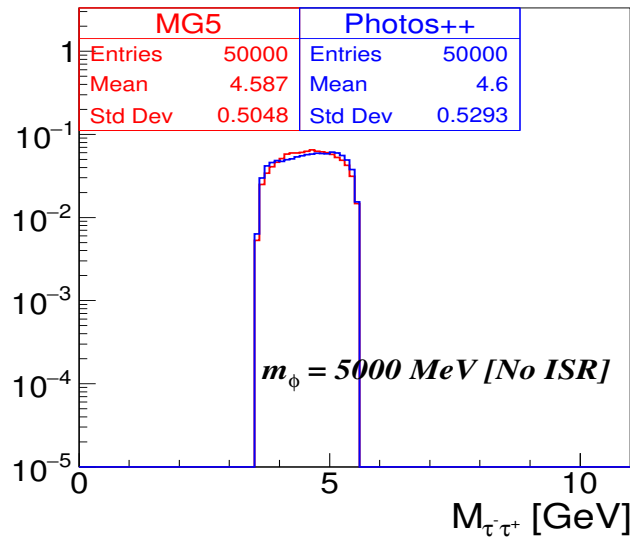
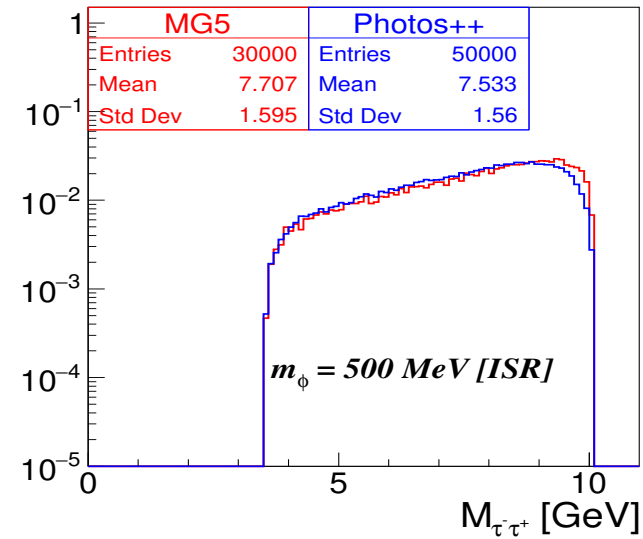
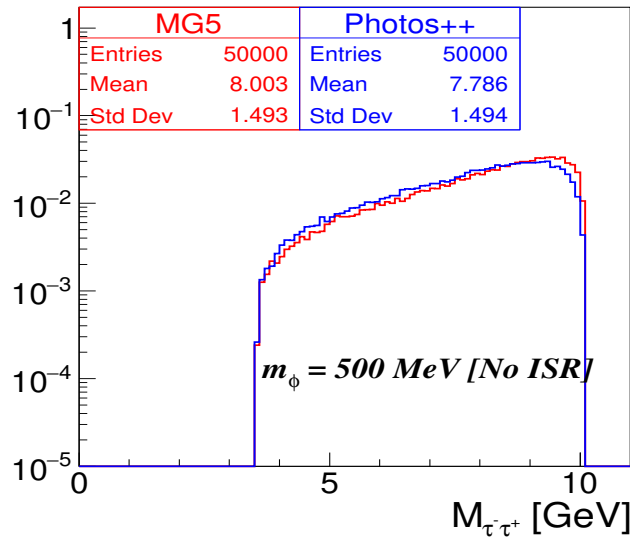


Dominant Feynman diagrams for  $e^-e^+ \rightarrow \tau^-\tau^+X(\rightarrow e^-e^+)$  as installed in MadGraph. For our applications dominant part, which can be used together with bremsstrahlung is used. It is an approximation, but validated. Opens gate for simultaneous inclusion of large QED effects. Note that spin state of  $\tau$  flips if  $X$  is scalar. That need to be taken into account for  $\tau - \tau$  spin correlations.

$$e^-e^+ \rightarrow \tau^-\tau^+\phi_{\text{Dark Scalar}} (\rightarrow e^-e^+)$$



$$e^-e^+ \rightarrow \tau^-\tau^+ \phi_{\text{Dark Scalar}} (\rightarrow \mu^-\mu^+)$$

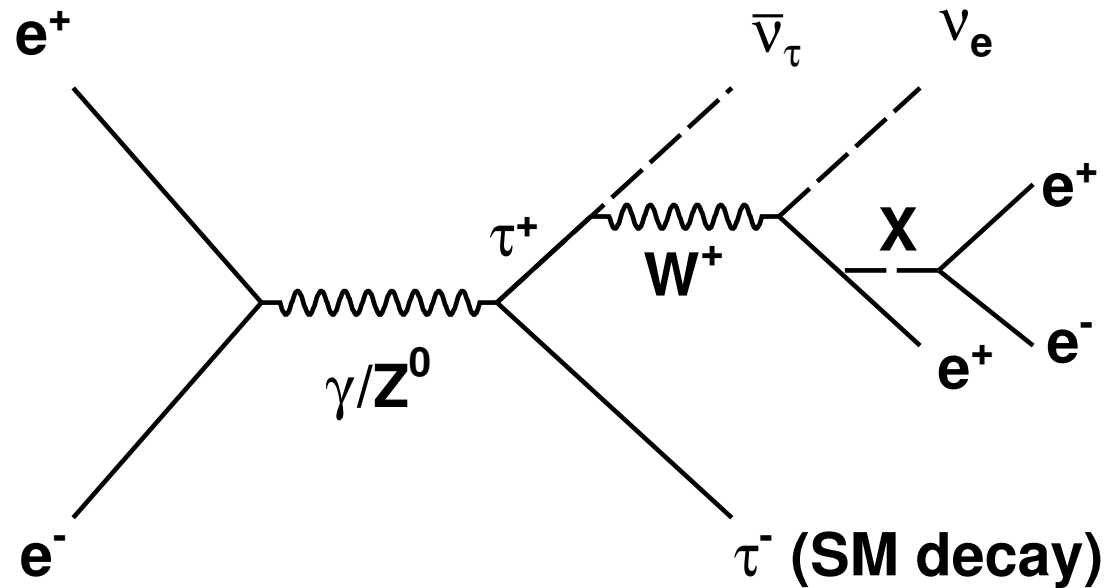


1. This work was quick, because of previous steps and installation into `photospp` possibility to generate lepton pairs:
  - (a) produced through virtual photon Comput.Phys.Commun. 199 (2016) 86  
1912.11376
  - (b) produced through narrow vector resonance (like dark photon) 1912.11376  
footnote 9
2. These changes were introduced first to `tauola`,
3. Both programs have similar phase space parametrisations which is exact.

For simulations in Belle II large number of channels prepared:

1. Total number of decay channels: 278
2. 2 body neutrinoless non SM decays: 58
3. 3 body neutrinoless non SM decays: 46
4. Number of generic SM decay channels : 92 [initialized with PDG2020 Branching Fractions]. For matrix elements choices from older versions of parametrisations were taken, but ...
  - (a) ...for high precision data, new parametrisations will be needed.
  - (b) Theoretical uncertainty of models is worse than quality of data.
5. New decay modes with lepton pair radiated from SM photons or from Dark photon with mass  $\in [50, 1500]$  MeV (ME cross-validated with MadGraph)
  - $\tau^- \rightarrow \nu_\tau \bar{\nu}_\ell \ell^- \ell^+ \ell^-$  [see next page]
  - $\tau^- \rightarrow \nu_\tau \pi^- \ell^+ \ell^-$





Once we could rely on tests for SM pairs, we could include safely  $X \rightarrow e^- e^+$ .

Feynman diagram for  $e^- e^+ \rightarrow \tau^-$  (SM decay)  $\tau^+ (\rightarrow \bar{\nu}_\tau \nu_e e^+ X (\rightarrow e^- e^+))$   
 process with X emitted in  $\tau^+$  decay.

**General formalism for semileptonic decays**

- Matrix element used in TAUOLA for semileptonic decay

$$\tau(P, s) \rightarrow \nu_\tau(N) X$$

$$\mathcal{M} = \frac{G}{\sqrt{2}} \bar{u}(N) \gamma^\mu (v + a\gamma_5) u(P) J_\mu$$

Matrix element squared contracted with spin density matrix:

$$|\mathcal{M}|^2 = G^2 \frac{v^2 + a^2}{2} (\omega + H_\mu s^\mu)$$

$$\omega = P^\mu (\Pi_\mu - \gamma_{va} \Pi_\mu^5)$$

$$H_\mu = \frac{1}{M} (M^2 \delta_\mu^\nu - P_\mu P^\nu) (\Pi_\nu^5 - \gamma_{va} \Pi_\nu)$$

$$\Pi_\mu = 2[(J^* \cdot N) J_\mu + (J \cdot N) J_\mu^* - (J^* \cdot J) N_\mu]$$

$$\Pi^{5\mu} = 2 \operatorname{Im} \epsilon^{\mu\nu\rho\sigma} J_\nu^* J_\rho N_\sigma$$

$$\gamma_{va} = -\frac{2va}{v^2 + a^2}$$

- $J_\mu$  the current; depends on the momenta of all hadrons and is expected to be build from Lorentz invariants, vectors or tensors.

- For two scalar decay channels all is rather simple: single distribution in  $Q^2$  (in cases with kaons also in  $\cos \theta_{\pi^\pm}$ ) to be measured and fitted with:

$$J^\mu = (p_{\pi^\pm} - p_{\pi^0})^\mu F_V(Q^2) + (p_{\pi^\pm} + p_{\pi^0})^\mu F_S(Q^2) \quad (F_S \simeq 0).$$

- For 3-scalar channels: 4 complex function of 3 variables to fit. Role of theoretical assumptions (oversimplifications?) is essential. Agreement on 1-dim distribution is just a consistency check.
- No go for model independent measurements? Use of all dimensions for data distributions: invariant masses  $Q^2$ ,  $s_1$ ,  $s_2$  as arguments of form-factors. Angular asymmetries help to separate currents: scalar  $J_4^\mu \sim Q^\mu = (p_1 + p_2 + p_3)^\mu$ , vector  $J_1^\mu \sim (p_1 - p_3)^\mu|_{\perp Q}$  and  $J_2^\mu \sim (p_2 - p_3)^\mu|_{\perp Q}$  and finally pseudovector  $J_5^\mu \sim \epsilon(\mu, p_1, p_2, p_3)$ .
- Possible if: (i) enough data, (ii) absolute precision, (iii) no background, (iv) full detector coverage can assured. We need that for orthogonality conditions.
- It is a challenge. Long time ago, for Cleo it was easier,  $\tau$  were produced nearly at rest.

1. Hadronic currents, or decay matrix elements in principle can be provided by the user, with c++ pointer, but this possibility is temporarily postponed in basf2 as it require software fine tunings of the collaboration libraries.
2. Note, that in contrary to old tauola paper Comp. Phys. Commun. 76 (1993) 361, separation into hadronic current and SU(2) Clebsh-Gordan coefficients and couplings is not necessarily useful in all possible models.
3. Also coefficients for  $\sqrt{1/n!}$  need to be watched as they are decay channel specific, thus also user choice input is needed.
4. Solutions similar to the one of ML for higgs CP in  $h \rightarrow \tau\tau$  are to be expected.
5. From our experience, Phys.Rev.D 103 (2021) 3, 036003, use of weighted events may be then convenient and helpful even for fits.
6. **Useful projects ahead in that domain ?**

## The last three years...

1. Final states with dark scalar/photon decaying to the lepton pair introduced in  $\tau$  pair production and decay.
2. Effort on channels parametrisations, extended list of channels in `basf2` of Belle2 collaboration.
3. Steps for language change in `tauola` prepared
4. parts for migration into C, parts for migration into C++ are already separated.
5. C++ version of `photos` installed.
6. C++ version of KKMC already exist, work focus – FCC.
7. Arrangements for fits and ML solutions, not much progress but are kept in mind.
8. I hope to provide help.