# T violating effects in $\nu_{\tau}\left(\bar{\nu}_{\tau}\right)$-nucleon quasielastic scattering 

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## Introduction






 $\Delta S=0$ and strangeness changing $\Delta S=1$ weak currents. In the case of $\Delta S=1$ reactions, we have also studied $\operatorname{SU}(3)$ symmetry breaking effects.

## Quasielastic production of nucleons and hyperons

The $\nu_{\tau}\left(\bar{\nu}_{\tau}\right)$ induced quasielastic production on the free nucleon target are given by the reactions

$$
\nu_{\tau}\left(\bar{\nu}_{\tau}\right)(k)+N(p) \longrightarrow \tau^{\mp}\left(k^{\prime}\right)+N\left(p^{\prime}\right), \quad N=n, p
$$

$$
\bar{\nu}_{\tau}(k)+N(p) \longrightarrow \tau^{+}\left(k^{\prime}\right)+Y\left(p^{\prime}\right), \quad Y=\Lambda, \Sigma^{0}, \Sigma^{\prime}
$$

The transition matrix element is given by

$$
\mathcal{M}=\frac{G_{F}}{\sqrt{2}} a l^{\mu} J_{\mu}
$$

where $a=\cos _{c}\left(\sin _{c}\right)$ for $\Delta S=0$ (1) processes
The leptonic ( $l^{\mu}$ ) and the hadronic ( $J_{\mu}$ ) currents are defined as
$l^{\mu}=\bar{u}\left(k^{\prime}\right) \gamma^{\mu}\left(1 \mp \gamma_{5}\right) u(k)$,
$(+)$ is for $\nu_{\tau}\left(\bar{\nu}_{\tau}\right)$
$J_{\mu}=\bar{u}\left(p^{\prime}\right)\left[\gamma_{\mu} f_{1}\left(Q^{2}\right)+i \sigma_{\mu \nu} \frac{q^{\nu}}{M+M_{Y}} f_{2}\left(Q^{2}\right)+\frac{2 q_{\mu}}{M+M_{Y}} f_{3}\left(Q^{2}\right)\right.$

$$
\left.\gamma_{\mu} \gamma_{5} g_{1}\left(Q^{2}\right)-i \sigma_{\mu \nu} \gamma_{5} \frac{q^{\nu}}{M+M_{Y}} g_{2}\left(Q^{2}\right)-\frac{2 q_{\mu} \gamma_{5}}{M+M_{Y}} g_{3}\left(Q^{2}\right)\right] u(p
$$

$f_{1,2}\left(Q^{2}\right)$ are determined in terms of the electromagnetic form factors while $f_{3}\left(Q^{2}\right)=0$ due to CVC and G invariance.
The axial vector and the weak electric form factors $g_{1,2}\left(Q^{2}\right)$ are determined in terms of $g_{A, 2}\left(Q^{2}\right)$, which are parameterized in the dipole form as

$$
g_{i}\left(Q^{2}\right)=g_{i}(0)\left[1+\frac{Q^{2}}{M_{i}^{2}}\right]^{-2} ; i=A, 2
$$

with $g_{A}(0)=1.267, g_{2}(0)=g_{2}^{I}(0)\left(g_{2}^{R}(0)\right)$ shows the presence of second class currents with(without) T-invariance, and $M_{A}=M_{2}=1.026$ GeV . For $g_{3}\left(Q^{2}\right)$, the parameterization given by Nambu is used, i.e.

$$
g_{3}\left(Q^{2}\right)=\frac{\left(M+M_{Y}\right)^{2}}{2\left(m_{K}^{2}+Q^{2}\right)} g_{1}\left(Q^{2}\right)
$$

The $Q^{2}$ distribution is written as

$$
\frac{d \sigma}{d Q^{2}}=\frac{G_{F}^{2} a^{2}}{8 \pi M^{2} E_{\nu_{\tau}\left(\bar{\nu}_{\tau}\right)}^{2}} N\left(Q^{2}\right)
$$

where the expression of $N\left(Q^{2}\right)$ is given in Ref. [3].
$\sigma, \bar{P}_{L}\left(E_{\nu_{\tau}}\right)$ and $\bar{P}_{P}\left(E_{\nu_{\tau}}\right)$ versus $E_{\nu_{\tau}}$ for the process $\nu_{\tau}+n \longrightarrow \tau^{-}+p$ when $\tau^{-}$is
polarized. The results are shown for the different values of $g_{2}^{R}(0)=0, \pm 1$.




## SU(3) symmetry breaking effects

The SU(3) symmetry breaking effects are incorporated following the works of Faessler et al. [4] and Schlumpf [5]. The main features of these models are summarized as
F Faessler et al. have studied the $\operatorname{SU}(3)$ symmetry breaking effects on $f_{2}\left(Q^{2}\right)$ and $g_{1}\left(Q^{2}\right)$ form factors. $f_{1}\left(Q^{2}\right)$ recieves no contribution at the leading order because of the AdemolloGatto theorem and $g_{3}\left(Q^{2}\right)$ recieves the $\mathrm{SU}(3)$ breaking effects via $g_{1}\left(Q^{2}\right)$

$$
\begin{aligned}
\mathcal{F}^{p \Lambda}(0) & =-\sqrt{\frac{3}{2}}\left(F+\frac{D}{3}+\frac{1}{9}\left(H_{1}-2 H_{2}\right.\right. \\
& \left.\left.-3 H_{3}-6 H_{4}\right)\right) \\
\mathcal{F}^{n \Sigma^{-}}(0) & =D-F-\frac{1}{3}\left(H_{1}+H_{3}\right) .
\end{aligned}
$$

© Schlumpf [5] has studied SU(3) symmetry breaking in the hadronic current containing axial $f_{1}\left(Q^{2}\right)$ and axial vector $g_{1}\left(Q^{2}\right)$ form factors using relativistic quark model. The modified $f_{1}\left(Q^{2}\right)$ and $g_{1}\left(Q^{2}\right)$ form factors are given by
$f_{1}^{\prime}\left(Q^{2}\right)=\alpha f_{1}\left(Q^{2}\right), \quad g_{1}^{\prime}\left(Q^{2}\right)=\beta g_{1}\left(Q^{2}\right)$
Polarization components of the final lepton


## Polarization of the final lepton

If one assumes the final lepton to be polarized, then the polarization 4 -vector $\left(\zeta^{\top}\right)$ is written as

$$
\zeta^{\tau}=\frac{\operatorname{Tr}\left[\gamma^{\tau} \gamma_{5} \rho_{f}\left(k^{\prime}\right)\right]}{\operatorname{Tr}\left[\rho_{f}\left(k^{\prime}\right)\right]},
$$

and the spin density matrix for the final lepton $\rho_{f}\left(k^{\prime}\right)$ is given by $\rho_{f}\left(k^{\prime}\right)=\mathcal{J}^{\alpha \beta} \operatorname{Tr}\left[\Lambda\left(k^{\prime}\right) \gamma_{\alpha}\left(1 \pm \gamma_{5}\right) \Lambda(k) \tilde{\gamma}_{\beta}\left(1 \pm \tilde{\gamma}_{5}\right) \Lambda\left(k^{\prime}\right)\right]$,
with $\tilde{\gamma}_{\alpha}=\gamma^{0} \gamma_{\alpha}^{\dagger} \gamma^{0}$ and $\tilde{\gamma}_{5}=\gamma^{0} \gamma_{5}^{\dagger} \gamma^{0}$. The polarization vector $\vec{\zeta}$ can be rewritten as

$$
\vec{\zeta}=\zeta_{L} \hat{e}_{L}^{l}+\zeta_{P} \hat{e}_{P}^{l}+\zeta_{T} \hat{e}_{T}^{l}
$$

where $\hat{e}_{P}^{l}, \hat{e}_{L}^{l}$ and $\hat{e}_{T}^{l}$ are the unit vectors corresponding to the perpendicular, longitudinal and transverse directions along the momentum of the final lepton and are given as

$$
\hat{e}_{L}^{l}=\frac{\vec{k}^{\prime}}{\left|\overrightarrow{k^{\prime}}\right|}, \quad \hat{e}_{P}^{l}=\hat{e}_{L}^{l} \times \hat{e}_{T}^{l}, \quad \hat{e}_{T}^{l}=\frac{\vec{k} \times \vec{k}^{\prime}}{\left|\vec{k} \times \vec{k}^{\prime}\right|},
$$

with $\zeta_{L, P, T}\left(Q^{2}\right)=\vec{\zeta} \cdot \hat{e}_{L, P, T}^{l}$
The longitudinal $P_{L}^{L}\left(Q^{2}\right)$, perpendicular $P_{P}^{L}\left(Q^{2}\right)$ and transverse $P_{T}^{l}\left(Q^{2}\right)$ components of the polarization vector are obtained as:

The expressions of $A^{l}\left(Q^{2}\right), B^{l}\left(Q^{2}\right), C^{l}\left(Q^{2}\right)$ are given in Refs. [3].
The real value of $g_{2}(0)$ gives $G$ violation keeping $T$ conserved. The transverse component of polarization, which arises due to the interference terms of the first and the second class current, vanishes when $g_{2}(0)$ is taken to be purely real. The imaginary value of $g_{2}(0)$ gives $G$ violation as well as $T$ violation.
$\sigma, \bar{P}_{L}\left(E_{\bar{\nu}_{\tau}}\right)$ and $\bar{P}_{P}\left(E_{\bar{\nu}_{\tau}}\right)$ versus $E_{\bar{\nu}_{\tau}}$ for the process $\bar{\nu}_{\tau}+p \longrightarrow \tau^{+}+n$ when $\tau^{+}$is polarized. The results are shown for the different values of $g_{2}^{R}(0)=0, \pm 1$.
$\sigma, \bar{P}_{L}\left(E_{\bar{\nu}_{\tau}}\right)$ and $\bar{P}_{P}\left(E_{\bar{\nu}_{\tau}}\right)$ versus $E_{\bar{\nu}_{\tau}}$ for the process $\bar{\nu}_{\tau}+p \longrightarrow \tau^{+}+\Lambda$. The calculations have been performed with $\operatorname{SU}(3)$ symmetry, and its breaking effects parameterized by Faessler et al. [4] and Schlumpf [5].

$\sigma, \bar{P}_{L}\left(E_{\bar{\nu}_{\tau}}\right)$ and $\bar{P}_{P}\left(E_{\bar{\nu}_{\tau}}\right)$ versus $E_{\bar{\nu}_{\tau}}$ for the process $\bar{\nu}_{\tau}+n \longrightarrow \tau^{+}+\Sigma^{-}$. The calculations have been performed with $\operatorname{SU}(3)$ symmetry, and its breaking effects parameterized by

Faessler et al. [4] and Schlumpf [5].



## Conclusions

In the case of $\Delta S=0$ processes, there is a strong dependence of $M_{A}$ on $\sigma\left(E_{\nu_{-}\left(\bar{\nu}_{-}\right)}\right)$for both neutrino as well as antineutrino induced reactions. The polarization observables show both neutrino as well as and
mild dependence on $M_{A}$.

The effect of the second class current is appreciable for $g_{2}^{R}(0)=+1$ in the case of $\sigma\left(E_{\nu_{\tau}\left(\bar{\nu}_{\tau}\right)}\right)$ both for the neutrino as well as antineutrino induced processes.
In the case of antineutrino induced reactions, there is an appreciable dependence of the polarization observables on $g_{2}^{R}(0)$, while in the case of neutrino induced reactions, the polarization observables show mild dependence on $g_{2}^{R}(0)$.
In the case of $\Delta S=1$ processes, the total, differential cross section and the polarization observables are quite sensitive to: $(i)$ the different parameterizations of the vector form factors and the pseudoscalar form factor, ( $i i$ ) the choice of $M_{A}$ and (iii) non-zero value of $g_{2}^{R}(0)$.

There is a significant variation in $P_{L}$ and $P_{P}$, on the different parameterizations of the SU(3) symmetry breaking, while the effect of $\operatorname{SU}(3)$ symmetry breaking is not much in the case of total and differential cross sections

## References

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[2] A. Fatima, M. Sajjad Athar and S. K. Singh, [arXiv:2106.14590 [hepph]].
[3] A. Fatima, M. Sajjad Athar and S. K. Singh, Phys. Rev. D 98, 033005 (2018)
[4] A. Faessler et al., Phys. Rev. D 78, 094005 (2008)
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$$
\begin{aligned}
& P_{L}^{l}\left(Q^{2}\right)=\frac{m_{\tau} A^{l}\left(Q^{2}\right) \vec{k} \cdot \vec{k}^{\prime}+B^{l}\left(Q^{2}\right)\left|\vec{k}^{\prime}\right|^{2}}{E_{k^{\prime}}}, \\
& P_{P}^{l}\left(Q^{2}\right)=\frac{\left.A^{\prime}\left(Q^{2}\right)|\vec{k}|^{2}|\vec{k}|^{2}-\left(\vec{k} \cdot \vec{k}^{\prime}\right)^{2}\right]}{N\left(Q^{2}\right)|\vec{k} \prime|\left|\vec{k} \times \vec{k}^{\prime}\right|}, \\
& P_{T}^{l}\left(Q^{2}\right)=\frac{\left.C^{l}\left(Q^{2}\right) M\left[\mid \vec{k} \cdot \vec{k}^{\prime}\right)^{2}-|\vec{k}|^{2}\left|\vec{k}^{\prime}\right|^{2}\right]}{N\left(Q^{2}\right)\left|\vec{k} \times \vec{k}^{\prime}\right|} .
\end{aligned}
$$

